INFORMATION-SHARING BETWEEN COMPETITION AUTHORITIES: THE CASE OF A MULTINATIONAL MERGER

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Abstract

The increasing number of antitrust cases that affect more than one country calls for more active cooperation between competition authorities. I analyse the impact of exchange of confidential information between two authorities deciding on a multinational merger. The authorities want to clear the merger if the information sent by the firm suggests that the expected welfare in their country will be enhanced and the firm can secretly manipulate the precision with which it transmits this information. The authorities differ in their leniency towards the merger and we focus on the cases where the authorities disagree about the decision.

Under no information-sharing, the firm chooses an extreme level of precision: very high (low) for the most (least) lenient authority. Under information-sharing, the firm is restricted to choose the same precision for both authorities. The firm’s choice depends on the level of cooperation in the decision-making between the countries. If the authorities exert their veto power, the firm always uses the lowest level of precision. If the authorities also cooperate in the decision-making, the firm’s choice of precision may be non-monotonic in the average welfare implications and intermediate levels of precision are chosen.

Other situations where the model can be applied abound in industrial organisation and political economy.

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1 Introduction

“Most officials believe that the issue of confidentiality is the chief limitation of enforcement cooperation agreements and hence it is submitted that the majority of effort should be concentrated on overcoming this particular obstruction to effective cooperation between antitrust agencies.”


With globalisation, competition has had an increasingly international dimension. A clear example is the existence of international cartels, such as the vitamins cartel which took place between January 1990 and February 1999, as well as the increasing number of mergers that involve more than one jurisdiction.¹ Both examples suggest the need to enhance cooperation between competition agencies.²

There has been a proliferation of multilateral platforms where various policy issues are discussed such as the ICN (International Competition Network) and the OECD (Organisation for Economic Co-operation and Development). These policy forums produce useful instruments³; nonetheless, their usefulness is constrained by their non-binding aspect. Similarly, many bilateral agreements have emerged. As pointed out by the quotation above, one of the main limitations of these agreements is the impossibility of exchanging confidential information between competition authorities. One of the most prominent examples of this type of agreement is the one between the E.U. and the U.S.⁴

The issue of exchange of confidential information has been raised in many occasions. In 2002, Advanced Micro Devices (AMD) requested court documents, gathered during a U.S. antitrust case against Intel several years before, to be transferred to the European Commission as support for a complaint against Intel. AMD believed that

¹UNCTAD (2000) shows that the share of cross-border mergers increased to 78% of the world FDI in the late 1990’s.
²For instance, the U.S. agencies have about 120 mergers’ notifications to foreign governments in a two-year period. In some 64 of them there is additional contact with the foreign agency where publicly-available information is exchanged. In about 40 cases the agencies engage in some level of cooperation, but confidential business information is only exchanged if a waiver is granted, which happens in some 16 cases. See OECD (2003).
³Such as those introduced by the OECD to deal with exchange of information in hard-core cartels: http://www.oecd.org/dataoecd/1/33/35590548.pdf.
many of the issues in the U.S. case were similar to the questions under investigation by the E.C. This request was made under 28 U.S.C. § 1782, which allows a district court to order the production of documents "for use in a proceeding in a foreign or international tribunal" upon request by "any interested person". This request was first denied by the district court and then reversed by the Ninth Circuit Court of Appeals.\textsuperscript{5}

A minority of competition policy agreements expressly provide for the exchange of confidential information. Within this group we find the bilateral agreement between the U.S. and Australia,\textsuperscript{6} the trilateral agreement between Iceland, Norway and Denmark\textsuperscript{7} and, more recently, the agreement between competition authorities of the European Member States.\textsuperscript{8}

The goal of this paper is to explore the information-sharing agreement’s impact on the incentives of the firm to provide precise information when undertaking a multinational merger.

We start by analysing the case where the firm ("he") deals with a single authority ("she") or, equivalently, when the authorities do not share information. The welfare implications of the merger correspond to an underlying real-valued state variable. In order to have the merger assessed, the multinational firm has to provide information about this variable to the competition authority. Neither the firm nor the authority have private information\textsuperscript{9} concerning this variable and the sole mechanism available to transmit it is through a noisy signal\textsuperscript{10}. The authority only observes a random realisation of the signal. The signal is unbiased, that is, the firm cannot misrepresent the merger’s welfare implications on average. However, the firm can choose to secretly manipulate the precision with which he transmits this information to the authority at no cost. We model this by allowing the firm to choose the variance\textsuperscript{11} of the signal, for instance, by adding or substracting relevant documents. The choice of the noise is unobservable because the authority does not actually know how much

\begin{itemize}
\item \textsuperscript{5}In the Supreme court opinion two justices concurred with the judgment, one dissented, and one took no part in it. Intel Corp. v. Advanced Micro Devices, Inc., No. 02-572, available at: www.supremecourtus.gov/opinions/03pdf/02-572.pdf
\item \textsuperscript{6}Agreement on Mutual Antitrust Enforcement Assistance, available at www.apeccp.org.tw/doc/USA/Cooperation/usaus7.htm
\item \textsuperscript{7}www.globalcompetitionforum.org/regions/europe/Denmark/Agreements1.pdf
\item \textsuperscript{9}At the end of the paper, we discuss how the results change if we allow the firm to have private information about this variable.
\item \textsuperscript{10}The signal has a normal additive structure.
\item \textsuperscript{11}Noise, variance and lack of precision are interchangeable.
\end{itemize}
information the firm has. A larger quantity of information does not necessarily imply more precision as some of it may be irrelevant.

The policy decision (to clear the merger or not) depends only on the realisation of the signal. The authority adopts a cut-off rule whereby, if the realisation of the signal is above some threshold, she clears the merger and she blocks it otherwise.

We find that the optimal variance chosen by the firm does not depend on how good or bad the average merger is (i.e. how far the average merger is above or below the policy threshold) but rather on whether it is good or bad (i.e. above or below the threshold). In particular, if the average merger is bad, the firm chooses the risky strategy of the largest variance to have more chances to be thought good. By contrast, if the average merger is good, the firm chooses the lowest variance to increase the likelihood of obtaining a high realisation of the signal. Furthermore, this strategy does not change as a function of the threshold depending on whether the authority can commit to it ex-ante. We also establish the optimal response (i.e. policy threshold) of the authority to the firm’s behaviour. If the authority cannot commit to a policy ex-ante, the optimal threshold when the average merger is welfare detrimental (enhancing) is stricter (more lenient) than the full information threshold. When there is uncertainty about the undesirability of the merger, the authority’s ability to commit (for instance, by issuing detailed guidelines) makes her set a more lenient threshold in order to induce the firm to provide more precise information. Otherwise, the authority sets the ex-post optimal threshold of the no-commitment case.

Then, we analyse an international merger, where a multinational wants to undertake a merger that needs the approval of two countries at the same time. We consider the case where one authority is more lenient than the other (i.e. their policy thresholds differ), either because their tolerance to mergers differ or because the same underlying state variable has different welfare implications in the two countries. Information-sharing means that the authorities receive the same realisation of the signal sent by the multinational, so the multinational is left to make a unique choice of variance for both countries. We explore the impact of the information-sharing regime on the incentives of the firm to provide precise information\textsuperscript{12}.

When the authorities share information, we find that the agreement only has an impact on the firm’s behaviour when the average merger is "conflicting", that is, it is good for one country but bad for the other. This impact will depend on

\textsuperscript{12}We do not consider how the policy thresholds change following the agreement.
the particular rule that the authorities use to deal with the case of disagreement (i.e. when the realisation of the signal lies between their thresholds). We consider different possible rules to deal with disagreement.

If the authorities have veto power (i.e. each country can unilaterally block the merger), then information-sharing induces the firm to send a very imprecise signal, making the more lenient country strictly worse off.

Another possibility is for cooperation to go beyond the information-sharing stage, by having some informal bargaining/persuasion process (not explicitly modelled in this paper) taking place between the competition authorities. We model the outcome of the bargaining process as the merger being approved with some probability in case of disagreement.

First, we take this probability as being exogenous, which may be interpreted as the authorities’ relative bargaining power. The choice of the variance in this case may be non-monotonic in the expected merger, and intermediate values of variance may be chosen in equilibrium. When the average merger is relatively bad for the less lenient country, the firm chooses either low variance or high variance. In particular, if the countries have equal bargaining power and the distance between authorities’ policies is very large (or the lowest variance is low enough), the firm first chooses high variance in order to maximise the chances to be above the more lenient threshold (where the merger is cleared for sure) and, as the average merger improves, he switches to low variance in order to maximise the chances of being cleared at least with some probability. As the average merger gets less welfare detrimental for the stricter country, it becomes safer to play a riskier strategy by increasing the variance to an intermediate level. Furthermore, this variance decreases as the average merger approaches to the policy threshold of the stricter country. This is because the chances of having the merger cleared are already high and, by reducing the variance, the probability of having it blocked by both authorities is decreased.

We also consider the case where the probability of clearing the merger depends on the particular realisation of the signal. This captures the idea that the less lenient country is more willing to clear the merger if the realisation falls near its threshold as compared to when it falls very far from it. In particular, we consider the case where this probability is increasing and linear in the signal realisation. From the point of view of the firm, it is as if there were a unique but uncertain threshold. Because of the linearity, the firm considers the expected threshold and behaves as in the case of one authority with respect to this threshold.
To sum up, if authorities exert their veto power, then sharing information is a bad idea because the firm will send very imprecise information. Further cooperation modifies the firm’s payoff structure in such a way that the firm makes a greater use of intermediate and lower levels of noise (as compared to the veto power case) and, therefore, it can be potentially good if the lenient country is not always the same one.

1.1 Related literature

This is a signal-jamming model, like the ones used in the career concerns literature\(^{13}\), where the firm jams the signal, not by manipulating its mean but by changing its variance, and hence its information content.

The choice of variability as a strategic variable has been considered in a large variety of setups. For instance, Anderson and Cabral (2007) analyse a model of R&D races where the two contestants, taking as given the level of R&D expenditure, need to choose a level of risk. Tsetlin et al. (2004) consider the choice of variability of the performance distribution in a multi-round contest. In the compensation literature, Gaba and Kalra (1999) introduce the level of dispersion of the probability distribution of sales as a choice variable besides the level of effort. The general conclusion of all these papers is that the players that are at a disadvantage tend to optimally choose more risky strategies than those who are in a favourable position. We also find this "gambling for resurrection" behaviour in our national merger framework. However, this behaviour may disappear in the multinational merger case if authorities decide to cooperate both in the decision making as well as at the information-sharing stage.

Johnson and Myatt (2006) also consider the incentives of a firm to provide its potential customers with more or less precise information. They show how the seller’s supply of information affects the shape of the distribution of buyers’ expected valuations and hence generates rotations of the demand curve. These rotations generate a convexity in the monopolist’s profits, which explains the optimality of the monopolist’s extreme choices of variance in Lewis and Sappington (1994). Contrary to Johnson and Myatt (2006), in our model there is another strategic player: the competition authority. In particular, our firm will choose a level of precision in order to maximise the chances that the realisation of the signal falls above the policy

\(^{13}\)See Holmstrom (1982, 1999) and Dewatripont et al. (1999).
threshold level chosen by the authority. On the other hand, the authority will choose the threshold in order to maximise the (ex-post or ex-ante) expected welfare. In the Johnson and Myatt paper, the monopolist is choosing both the precision and the threshold (i.e. the price) in order to maximise the ex-post expected profits.

As far as we are aware, another distinctive feature of our model with respect to the previous work is that the noise (and hence, the distribution of the signal) is not observed by the receiver. In our framework, observability of the noise renders the problem trivial because the competition authority could simply condition the policy threshold on the noise and block any merger with an imprecise report.

Finally, this paper considers the consequences of information-sharing between receivers/principals and hence it is related to the literature that compares private with public communication. However, comparisons with this literature are difficult because, so far, its focus has been on frameworks of mechanism design (see, for instance, Calzolari and Pavan (2006), Maier and Ottaviani (2009)), cheap talk (see Farrell and Gibbons(1989)), and signalling (see Gertner, Gibbons and Scharfstein (1988) and Spiegel and Spulber (1997)).

The paper proceeds as follows. Section 2 introduces the model. Section 3 studies the national merger. This section first presents the results for the no-commitment case and then highlights the differences resulting from the authority’s ability to commit. Section 4 introduces and analyses the multinational merger setup. Section 5 discusses the consequences of the firm having private information. Finally, Section 6 concludes. All the proofs can be found in the Appendix.

2 The model

We consider a multinational firm (M) proposing a merger that must be cleared under the competition law of the country. The welfare consequences of the merger can be summarised in the real-valued state variable $\theta$, with support on $[-\infty, +\infty]$. The mergers with $\theta > 0$ are welfare enhancing while the ones with $\theta < 0$ are welfare detrimental. Furthermore, the higher the $\theta$, the more desirable it is for the country to have the merger cleared by the competition regulator (R). If R were to observe the true $\theta$ (full information framework), she would like to clear the merger with probability one whenever $\theta \geq 0$ and block it otherwise. The multinational always has a positive net benefit normalised to 1 from the merger\footnote{\footnotetext{Otherwise, he would not have proposed it in the first place.}} and, therefore, he
would like to have the merger cleared for any \( \theta \). The firm does not have private information about \( \theta \). For instance, the firm may be giving information about the merger’s market effect or the merger’s effect on third parties, for which the firm may not have more information than the competition authority. The parameter \( \theta \) is distributed according to a Normal distribution, \( f(\theta) \), with mean \( \mu \) and variance \( \eta^2 \) and this distribution is common knowledge.

The actions available to the players are the following. M sends a message containing the information about \( \theta \) to R but he does so through a noisy signal. In particular, the signal has the following form:

\[
S = \theta + \epsilon
\]

where \( \epsilon \) is a random variable distributed according to a Normal distribution with mean zero and variance \( V \). M cannot choose to send a message that differs from the true \( \theta \) (i.e. lying is not possible and therefore, the signal should be on average equal to the true \( \theta \)), but he can secretly choose \( V \), which reflects the precision with which the message is sent (the higher the \( V \), the less informative is the realisation of the signal about \( \theta \)). For instance, M can add or subtract relevant documents and this choice is secret because R does not know which documents M has. For simplicity, we assume that M can only choose \( V \) within this interval \([V_L, V_H]\), where \( 0 < V_L < V_H \). Therefore, the distribution of the signal \( S \), \( g(s, V) \), is Normal with mean \( \mu \) and variance \( \eta^2 + V \) and \( G(s, V) \) is the cumulative distribution.

R observes the realisation of the signal \( s \), updates her beliefs about \( \theta \), and chooses the appropriate probability of clearance \( p(s) \). There is no loss of generality in the monotone likelihood ratio property (MLRP) holds in the setup. This is a sufficient con-
considering the case where \( p(s) \) is a cut-off rule, where \( \hat{s} \) is optimally determined by R:

\[
p(s) = \begin{cases} 
0 & \text{if } s < \hat{s} \\
1 & \text{if } s \geq \hat{s}
\end{cases}
\]

The timing of the game depends on whether R has the ability to commit to a particular threshold before the information is transmitted by M. Under no commitment, the timing is as follows. First, M chooses the variance with which he sends the signal. A realisation of the signal, \( s \), is observed by R. Given this realisation, R updates her beliefs about \( \theta \) and decides which policy threshold \( \hat{s} \) to use. Finally, the payoffs are realised. The solution of this model is a Nash equilibrium where the conjectures of each of the players are correct in equilibrium. Under commitment, first R chooses a policy threshold \( \hat{s} \) and M then chooses \( V \). A realisation of the signal, \( s \), is observed by R. Given this realisation, R clears the merger whenever \( s \) is above \( \hat{s} \). Finally, the payoffs are realised. We solve this model backwards.

3 Benchmark: national merger

3.1 No-commitment regime

3.1.1 The problem of the firm

The firm always benefits from the merger and, therefore, chooses the level of precision that maximises the probability of having the merger cleared. M makes a conjecture about the policy threshold used by R, \( \hat{s}^c \), where the superscript stands for conjecture. Using Bayes rule and the assumption on the form of \( p(s) \), M maximises the probability of having the merger cleared:

\[
P(V) = \int_{\hat{s}^c}^{+\infty} g(s, V) \, ds
\]

The objective function is decreasing in \( V \) whenever the average merger is a good condition for a cut-off rule to be optimal if the authority cannot commit to a threshold ex-ante (see Milgrom (1981)). It is not clear whether there is any loss of generality in the case where the authority can commit to a policy, but for comparison purposes, we restrict ourselves to a cut-off rule also in this case.

\[R \] does not infer the noise used based on the unique realisation of the signal. This assumption is not restrictive because, in the no-commitment case, R has point beliefs about \( V \) which will not be affected by letting R make inferences from \( s \). In the commitment case, it would be difficult for the authority to justify a punishment for an extreme positive realisation, given that all realisations can happen with positive probability and MLRP holds.
merger (i.e., $\hat{s}^c < \mu$) and increasing in $V$ when the average merger is a bad merger (i.e., $\hat{s}^c > \mu$). Therefore, if $\hat{s}^c < \mu$, M will choose the minimum available variance $V_L$. Conversely, if $\hat{s}^c > \mu$, M will choose the maximum available variance $V_H$.

**Lemma 1** Under no-commitment, the optimal variance is:

$$V^*(\mu, \hat{s}^c) = \begin{cases} V_H & \text{if } \mu < \hat{s}^c \\ V_L & \text{if } \mu > \hat{s}^c \end{cases}$$

If $\hat{s}^c = \mu$, M is indifferent between any variance in $[V_L, V_H]$.

**Proof.** See Appendix. ■

For simplicity, we will assume that if $\hat{s}^c$ is exactly $\mu$ the firm will choose $V_L$.

The intuition supporting this optimal strategy is that, by increasing the variance, an expected bad merger obtains more chances to be thought good. By contrast, decreasing the variance cuts down an expected good merger’s chance of being considered bad. Therefore, the optimal $V$ does not depend on how far the particular $\mu$ is from $\hat{s}^c$, only on whether $\mu$ lies below or above $\hat{s}^c$. This is due to the "bang-bang" payoff structure created by the cut-off rule. This "gambling for resurrection" result is in line with the findings of the literature that considers the choice of variability as a strategic variable\(^{21}\).

### 3.1.2 The problem of the competition authority

Given the realisation $s$ and the conjecture that $R$ forms about M’s choice $V^c$, she needs to decide whether to clear the merger or not. The ex-post expected welfare of the merger is:

$$W(s, \hat{s}^c) = \int_{-\infty}^{+\infty} \theta f(\theta|s, V^c(\mu, \hat{s}^c)) d\theta$$

If $W(s, \hat{s}^c)$ is positive, $R$ will clear the merger. Conversely, if $W(s, \hat{s}^c)$ is negative, $R$ will block the merger. The authority determines the threshold $\hat{s}$ so that she is indifferent between blocking or clearing the merger, that is, so that $W(s, \hat{s}^c)$ is zero:

$$W(s, \hat{s}^c) \big|_{s=\hat{s}(\hat{s}^c)} = \frac{\hat{s}(\hat{s}^c) \eta^2 + \mu V^c(\mu, \hat{s}(\hat{s}^c))}{\eta^2 + V^c(\mu, \hat{s}(\hat{s}^c))} = 0$$

\(^{21}\) See Section 1.1.
Denote the solution of equation (1) as $\hat{s}(\hat{s}^*)$. The equilibrium threshold $\hat{s}^*$ is, then, the fixed point of this solution:

$$\hat{s}(\hat{s}^*) = \hat{s}^*$$

**Proposition 1** Under no commitment, the equilibrium threshold $\hat{s}^*$ is:

$$\hat{s}^* = \begin{cases} \frac{-\mu V_H}{\eta^2} & \text{if } \mu < 0 \\ \frac{-\mu V_L}{\eta^2} & \text{if } \mu \geq 0 \end{cases}$$

**Proof.** It is straightforward to solve equation (1). □

Figure 1 depicts the optimal threshold $\hat{s}^*$ as a function of the average merger $\mu$.\textsuperscript{22} If the merger is on average welfare detrimental ($\mu < 0$), the authority sets a policy threshold above the merger type that leaves welfare unaffected. The intuition behind this is that, if the bulk of mergers are bad mergers, then the authority increases the standard of proof of a good merger by setting the policy threshold above the neutral merger $0$. Conversely, when the average merger is welfare enhancing ($\mu > 0$), the authority sets a very lenient standard of proof, tolerating even negative signals.

Finally, note that as $V_L$ tends to 0, the report becomes extremely informative and $\hat{s}^*$ increases to zero (i.e. R becomes stricter with the on average good mergers as the probability of "unlucky realisations", that is, $s$ different from $\theta$, decreases). Similarly, as $V_H$ tends to $+\infty$, the report submitted by M becomes uninformative and $\hat{s}^*$ tends to $+\infty$, so all the mergers that may raise competition issues will on average be blocked.

### 3.2 Commitment regime

#### 3.2.1 The problem of the firm

Suppose that R can commit in advance to a policy threshold $\hat{s}$, for instance, by issuing very detailed merger guidelines. Then, M chooses $V$ taking into account the actual $\hat{s}$ instead of the conjecture $\hat{s}^c$. The problem is identical to the one solved in Section 3.1.1 and therefore it is omitted here.

\textsuperscript{22}By Lemma 1, above the 45° line, $\hat{s}^* > \mu$, so the firm chooses $V_H$ and below the 45° line, $\hat{s}^* \leq \mu$, so M chooses $V_L$. By Proposition 1, the optimal threshold, $\hat{s}^*$, has the opposite sign to the average merger and increases in absolute value with the average merger and the variance chosen M.
Lemma 2 Under commitment, the optimal variance is:

\[ V^*(\mu, \hat{s}) = \begin{cases} 
V_H & \text{if } \mu < \hat{s} \\
V_L & \text{if } \mu > \hat{s} 
\end{cases} \]

If \( \hat{s} = \mu \), M is indifferent between any variance in \([V_L, V_H]\).

Proof. It is omitted as it is the same as Lemma 1. ■

Again, we break indifference by assuming that M will choose \( V_L \).

3.2.2 The problem of the competition authority

The competition authority chooses \( \hat{s} \) to maximise the ex-ante expected welfare, taking into account the behaviour of the firm:

\[
EW(\hat{s}, V^*(\mu, \hat{s})) = \int_{\hat{s}}^{+\infty} \left( \frac{s\eta^2 + \mu V^*(\mu, \hat{s})}{\eta^2 + V^*(\mu, \hat{s})} \right) g(s, V^*(\mu, \hat{s})) ds
\]

subject to : \[ V^*(\mu, \hat{s}) = \begin{cases} 
V_H & \text{if } \mu < \hat{s} \\
V_L & \text{if } \mu \geq \hat{s} 
\end{cases} \]
where the term in brackets is the expected welfare given a realisation $s$, $W(s, \bar{s})$.\footnote{The objective function is equivalent to $\int_{-\infty}^{+\infty} \theta f(\theta|s, V^*(\mu, \bar{s}))d\theta$, where $f(\theta|s, V^*(\mu, \bar{s}))$ is the posterior distribution of $\theta$.}

**Proposition 2** There exist $\bar{\mu}$ such that, under commitment, the optimal threshold $\hat{s}^{**}$ is:

$$
\hat{s}^{**} = \begin{cases} 
\frac{-\mu V_H}{\sigma^2} & \text{if } \mu < \bar{\mu} \\
\mu & \text{if } \mu \in [\bar{\mu}, 0] \\
\frac{-\mu V_L}{\sigma^2} & \text{if } \mu > 0 
\end{cases}
$$

$\hat{s}^{**}$ is non-monotonic in the expected welfare and $\bar{\mu}$ strictly increases (decreases) with $V_L$ ($V_H$).

**Proof.** See Appendix.

As in Figure 1, we depict $\hat{s}^{**}$ as a function of $\mu$ in Figure 2.

Note that the authority values the level of precision of the signal as this allows her to make fewer mistakes when deciding whether to clear the merger or not.

When the average merger is welfare enhancing ($\mu > 0$), by Lemma 2, M chooses $V_L$. Since, this is as informative as the report can be, R sets the ex-post optimal threshold found in Section 3.1.2.
By the same Lemma, when the average merger is welfare detrimental, M chooses $V_H$. This introduces a new trade-off for the competition authority because she may decide to distort the ex-post optimal policy in order to extract more precise information from the firm. Indeed, we find that, when there is substantial uncertainty about the undesirability of the average merger ($\mu$ is negative and close to zero, i.e., $\mu \in [\bar{\mu}, 0]$), then R gains from committing to a more lenient threshold (not only more lenient than the ex-post optimal threshold but also than the full information threshold) in order to induce M to reduce his equilibrium variance from $V_H$ to $V_L$. On the other hand, if the merger is on average clearly welfare detrimental ($\mu < \bar{\mu}$), the authority will again set the ex-post optimal threshold found in Section 3.1.2.

Therefore in some cases, the authority, by gaining commitment power, decides to distort her policy by decreasing her standards (in an ex-post non-optimal way) in order to fix the information problem (i.e. to improve the informativeness of the signal).

**Corollary 1** *Policy comparison:*

$\hat{s}^* = \hat{s}^{**}$ $\forall \mu \in (-\infty, \bar{\mu}) \cup (0, +\infty)$ and $\hat{s}^* > \hat{s}^{**}$ $\forall \mu \in [\bar{\mu}, 0]$.

Finally, note that if $V_L$ increases, the range of mergers for which the authority is more lenient than ex-post optimal, $[\bar{\mu}, 0]$, shrinks as there is less gain from obtaining a precise signal. Similarly, this range will expand if $V_H$ increases.

## 4 Multinational merger

In this section we consider the situation where a multinational wants to undertake the same merger in two different countries (or jurisdictions) and these countries differ in terms of their policies. Without loss of generality, we assume that Country 1 will block the merger if the signal is below $\hat{s}_1$ and clear it otherwise, while Country 2 will do the same using the threshold $\hat{s}_2$, where $\hat{s}_1 < \hat{s}_2$.

The difference in thresholds can be interpreted as Country 1 being in general more lenient in its merger policy than Country 2.\(^{24}\) For instance, if the welfare function of Country $i$ is $a_i + b_i\theta$, where $i = 1, 2$, then $a_1 > a_2 > 0$ and $b_2 > b_1 > 0$.

\(^{24}\)Another possible justification is that the merger is expected to have different competitive effects, for instance because the market concentration levels or the likelihood of coordinated interaction are different.
where $a_i$ could be interpreted as how lenient is the authority and $b_i$ how sensitive is the authority to the welfare’s changes.\textsuperscript{25}

In practice, some mergers may raise competition issues at a national (or sub-national) level that can be solved by a local action (for instance, to impose a remedy that forces the firm to undertake a national divestiture if the merger is cleared somewhere else) without the need to reach an agreement with other competition authorities. In this paper we focus on the type of mergers that do need the agreement of all the jurisdictions in order for the merger to happen. An example of such a merger was the one proposed between two U.S.-based companies, General Electric and Honeywell, with the E.U. prohibiting the merger,\textsuperscript{26} and the U.S. Department of Justice approving it\textsuperscript{27}.\textsuperscript{28}

Since what drives the variance decision is how $\mu$ compares with the threshold, we will focus on the case where the information-sharing agreement would make a difference in the firm’s choice. This occurs when there is a conflicting merger in the sense that an average merger is good for Country 1 but bad for Country 2, that is, $\tilde{s}_1 < \mu < \tilde{s}_2$.\textsuperscript{29}

When the competition authorities do not share information about the firm, the problem of the multinational is separable and we are back to the national merger case. The multinational needs to convince each authority individually, regardless of the way in which authorities reach an agreement ex-post. In other words, he chooses a level of precision for each country so as to maximise the probability of having the merger cleared in each country. By Lemma 1, the firm will choose $V_L$ in Country 1 and $V_H$ in Country 2.

When the authorities share information, they receive the same realisation of the signal, thus the problem of the multinational is no longer separable. The multina-

\textsuperscript{25}It is easy to check that the resulting equilibrium threshold under no-commitment is:

$$\tilde{s}_1 = \begin{cases} 
-\mu \frac{V_H}{\eta} - \frac{a_i}{\eta} \frac{\eta^2 + V_H}{\eta^2} & \text{if } \mu < -\frac{a_i}{\eta} \\
-\mu \frac{V_L}{\eta} - \frac{a_i}{\eta} \frac{\eta^2 + V_L}{\eta^2} & \text{if } \mu \geq -\frac{a_i}{\eta}
\end{cases}$$

Under commitment there would be a region of $\mu$ where both authorities will coincide in setting $\tilde{s}_i = \mu$.

\textsuperscript{26}GE/Honeywell, Case No COMP/M.2220, European Commission Decision available at http://www.ec.europa.eu/competition/mergers/cases/decisions/m2220_en.pdf

\textsuperscript{27}Subject to GE divesting Honeywell’s helicopter engine business and licensing a new competitor to maintain and repair certain Honeywell engines.

\textsuperscript{28}See Muris (2001) for more detail.

\textsuperscript{29}Note that if $\mu < \tilde{s}_1$ (or $\tilde{s}_2 < \mu$), the firm chooses $V_H$ (or $V_L$) in both countries and the information-sharing agreement makes no difference.
tional needs to choose a unique level of precision so as to maximise the probability of having the merger cleared. This probability will depend on the process by which a final decision on the international merger is reached and we consider several possibilities below. In what follows, we analyse the impact that information-sharing has on the behaviour of the firm, keeping the competition authorities policies fixed, that is, we consider the firm’s reaction to \((\hat{s}_1, \hat{s}_2)\).

4.1 Information-sharing with veto power

Consider first the case where the authorities only cooperate in exchanging information but not in their decision process, i.e. the authorities use their veto power. The only disagreement that can arise is that Country 1 wants to clear the merger, while Country 2 does not want this. Country 2 using its veto power translates into the merger being cleared only if both competition authorities agree that the merger should be cleared (i.e. if the signal is above \(\hat{s}_2\)) and as a result the firm will choose \(V_H\).

Therefore, Country 2 will be indifferent between signing or not signing the agreement, while Country 1 will be strictly worse off because it will receive less precise information.

4.2 Information-sharing with cooperation in the decision-making

We turn now to the case where there is a bargaining or persuasion process taking place between the authorities, for example, due to the repeated interaction between them. We model this decision as taking place in two stages: first, the competition authorities decide unilaterally whether they should clear the merger and, then, if the decisions differ, they discuss their arguments until they reach an agreement.\(^30\) From the point of view of the firm, the outcome of this bargaining is a conflicting merger being cleared with some probability. We consider two ways of how this probability is determined.

\(^30\)For instance, in the WorldCom / Sprint merger review, the co-operation between the Europolitan Commission and the US Department of Justice involved such an extensive sharing of information (thanks to a confidentiality waiver granted by the parties) that allowed both case teams to discuss in-depth the merits of the case and to reach consistent assessments of the competitive impact of the transaction on the area of joint concern. For this, and many more examples where information-sharing has lead to a more active cooperation that have resulted in decisions to clear mergers, see OECD (2001).
4.2.1 Constant probability

We first analyse the case where, in case of disagreement, the merger is cleared with probability $\alpha$ (even though Country 2 does not want this) and blocked with probability $1 - \alpha$. The parameter $\alpha$ can be interpreted as a measure of the power of the authority in Country 1 versus the one in Country 2. $M$ maximises the probability of having the merger cleared:

$$P^\alpha(V) = \alpha \int_{\bar{s}_1}^{\bar{s}_2} g(s, V) \, ds + \int_{\bar{s}_2}^{+\infty} g(s, V) \, ds$$

for $\alpha \in (0, 1)$.

Note that the first part of his objective function is decreasing in $V$, while the second part is increasing in $V$. In other words, when the firm chooses a high variance, this decreases the mass of the probability distribution of the signal in the interval $[\bar{s}_1, \bar{s}_2]$, but at the same time increases the mass in the tails, so the chances that the realisation lies in the interval $[\bar{s}_2, +\infty)$ increase. Therefore the firm faces a trade-off between choosing $V$ so as to maximise the probability in the interval $[\bar{s}_1, \bar{s}_2]$ or in the interval $[\bar{s}_2, +\infty)$.

Denote the middle point of the interval $[\bar{s}_1, \bar{s}_2]$ by $s_M = \frac{\bar{s}_1 + \bar{s}_2}{2}$. The behaviour of the multinational for $\mu \in [\bar{s}_1, \bar{s}_2]$ is summarised in the next Proposition.

**Proposition 3** For $\mu \in [\bar{s}_1, \bar{s}_2]$, the choice of variance may be non-monotonic in $\mu$. In particular, for $\mu \in [\bar{s}_1, s_M)$, $M$ chooses either the minimum or maximum variance. For $\mu \in (s_M, \bar{s}_2]$, $M$ chooses some intermediate variance $V(\mu, \alpha) = \frac{(\bar{s}_2 - \mu)^2 - (\bar{s}_2 - s_M)^2}{2 \ln \left( \frac{1 - \alpha}{\alpha(\bar{s}_2 - s_M)} \right)} - \eta^2$ (provided $V(\mu, \alpha) \in (V_L, V_H)$).

**Proof.** See Appendix.

This result relies on the fact that the curvature of the firm’s objective function depends on $\mu$. In particular, the objective function is quasi-convex in $V$ for $\mu \in (\bar{s}_1, s_M)$ and quasi-concave in $V$ for $\mu \in (s_M, \bar{s}_2)$.

Thus, for $\mu \in (\bar{s}_1, s_M)$, $M$ will choose either $V_H$ for all $\mu$ or it will start with $V_H$ and then switch to $V_L$ if the following condition holds:

$$\frac{1 - G(\bar{s}_2; V_H) - (1 - G(\bar{s}_2; V_L))}{\alpha [G(\bar{s}_2; V_L) - G(s_M; V_L) - (G(\bar{s}_2; V_H) - G(s_M; V_H))]} < 0$$

where $G(s; V)$ denotes the probability of having the merger cleared.
This condition states that, by choosing $V_L$, as opposed to $V_H$, M loses chances of being cleared with probability 1 but may increase the chances of being cleared with probability $\alpha$ if $\mu$ is close enough to $s_M$ and depending on the function $c(V_L, V_H)$, which determines how far away from $\mu$ is the crossing point between $g(s; V_L)$ and $g(s; V_H)$.

The following two corollaries state the precise variance that the firm chooses, when the authorities have equal bargaining power, that is, for $\alpha = \frac{1}{2}$.

**Corollary 2** For $\mu \in (\hat{s}_1, s_M)$ and $\alpha = \frac{1}{2}$, the optimal variance is either $V_L$ or $V_H$. A sufficient condition for M to choose $V_H$ for any $\mu \in (\hat{s}_1, s_M)$ is:

$$\hat{s}_2 - \hat{s}_1 < c(V_L, V_H)$$

where $c(V_L, V_H)$ increases with $V_H$ and $V_L$.

There exists a threshold $\mu$ such that a sufficient condition for M to first choose $V_H$ until $\mu = \mu$ and then switch to $V_L$ for $\mu \in (\mu, s_M)$ is:

$$c(V_L, V_H) < \mu - \hat{s}_1$$

**Proof.** See Appendix.

**Corollary 3** For $\mu \in (s_M, \hat{s}_2)$ and $\alpha = \frac{1}{2}$, the optimal variance is:

$$V(\mu, \frac{1}{2}) = \max \left\{ \min \left\{ \frac{(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2}{2 \ln \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right)} - \eta^2, V_H \right\}, V_L \right\}$$

and $\frac{dV(\mu, \frac{1}{2})}{d\mu} < 0$.

**Proof.** See Appendix.

Figure 3 depicts this result.

Note from Corollary 2 that for a given pair of policy thresholds $\{\hat{s}_1, \hat{s}_2\}$, the larger $V_H$, the larger $c(V_L, V_H)$ and hence, the more likely it is that we are in the case where the firm chooses $V_H$ for all $\mu \in (\hat{s}_1, s_M)$. In the same way, the smaller $V_L$ or the larger the conflict between authorities $(\hat{s}_2 - \hat{s}_1)$, the more likely it is that we are in the regime where the firm chooses first $V_H$ and then $V_L$. Therefore, if the authorities have equal bargaining power and their conflict is large (or it is possible to send very precise reports), the choice of the variance for the conflicting mergers
is non-monotonic in the average merger. This non-monotonicity is the result of the trade-off between maximising the probability at the interval $[\hat{s}_1, \hat{s}_2]$ or at the interval $[\hat{s}_2, +\infty)$.

The intuition behind the choice of variance is the following. For the average merger $\mu$ between $\hat{s}_1$ and $s_M$, the objective function is quasi-convex in $V$. Therefore, the firm chooses an extreme variance. First consider the average merger at the left endpoint, $\mu = \hat{s}_1$. For this $\mu$, the probability that the realisation of the signal is above $\hat{s}_1$ is $\frac{1}{2}$ for all the values of $V$. However, the larger the variance, the higher the probability of obtaining a realisation above $\hat{s}_2$ where the merger is cleared with probability 1 rather than $\frac{1}{2}$. Therefore, he will choose $V_H$. By continuity, the same intuition holds for the average mergers that are in between $\hat{s}_1$ and $\overline{\mu}$, where $\overline{\mu}$ is defined in equation (15). If the minimum variance is low enough (or the conflict between authorities is high enough), then it is optimal for the firm to switch to $V_L$ after $\overline{\mu}$ in order to maximise the chances of being cleared at least by Country 1. It is at this point that the new trade-off becomes effective.

The firm with an average merger just in the middle, $\mu = s_M$, is indifferent between any variance. The reason for this is that the probability of having the merger cleared is $\frac{1}{2}$ for all possible values of $V$ due to the symmetry of the probability function at this point.

As $\mu$ exceeds $s_M$, the objective function of the firm becomes quasi-concave in $V$. 

Figure 3: Optimal choice of variance with constant probability
Because of the proximity of $\mu$ with the interval $[\hat{s}_2, +\infty)$, it becomes safer to play a riskier strategy by increasing $V$ (without reaching the maximum level). Finally, as $\mu$ keeps increasing towards $\hat{s}_2$, the value of this optimal intermediate variance decreases. This is because the chances of having the realisation in the interval $[\hat{s}_1, +\infty)$ are already high and, by reducing the variance, the probability of having the realisation in the tail $(-\infty, \hat{s}_1)$ is decreased. The variance decreases until it hits the minimum level at $\bar{\mu}$, defined in equation (16).

Finally, when the average merger is at the right endpoint, $\mu = \hat{s}_2$, the probability of having a realisation of the signal below $\hat{s}_2$ is $\frac{1}{2}$; however the higher the variance, the higher the probability that the realisation of the signal is below $\hat{s}_1$ where the merger is blocked, rather than cleared with probability $\frac{1}{2}$. As a result, the firm chooses $V_L$.

The information-sharing agreement has modified the payoff structure, which is no longer "bang-bang" as in Section 3. As a result, the firm makes a greater use of intermediate levels of variability in a non-monotonic way.

4.2.2 Increasing probability

Now we consider the case where the bargaining power of Country 1 increases monotonically with the particular realisation $s$. This reflects the fact that the closer $s$ is to $\hat{s}_2$, the less reluctant will Country 2 be about clearing the merger as compared to a realisation $s$ very close to $\hat{s}_1$. For simplicity, we consider the case of a linear probability function $\rho(s) = \frac{s - \hat{s}_1}{\hat{s}_2 - \hat{s}_1}$.

Note that $\rho(\hat{s}_1) = 0$ and that $\rho(\hat{s}_2) = 1$. The expected probability of clearance then becomes:

$$P^e(V) = \int_{\hat{s}_1}^{\hat{s}_2} \rho(s)g(s, V)ds + \int_{\hat{s}_2}^{+\infty} g(s, V)ds$$

**Proposition 4** The optimal variance chosen by $M$ is:

$$V^*(\mu, s_M) = \begin{cases} V_H & \text{if } \mu < s_M \\ V_L & \text{if } \mu > s_M \end{cases}$$

If $\mu = s_M$, then $M$ is indifferent between any variance in $[V_L, V_H]$.

**Proof.** See Appendix. \[ \]

See Figure 4. From the point of view of the firm, it is as if there were a unique threshold but there is uncertainty about where exactly this lies. Because of the

\[31\]The results of this Section will change if $\rho(s)$ is not linear.
Figure 4: Optimal choice of variance with increasing probability

linearity of $\rho(s)$, the firm takes the expectation and behaves as in the national merger case with this unique threshold. Therefore, the firm will use high variance whenever he thinks the merger is not going to pass through ($\mu < s_M$) and low variance otherwise.

If the average merger is "relatively" bad (below $s_M$), then Country 2 is strictly worse off by signing the agreement as it will receive less precise information. However, if the average merger is "relatively" good (above $s_M$), then signing the agreement and cooperating in the decision-making makes the firm use very precise information, making Country 1 strictly better off.

4.3 Information-sharing with cooperation in the policy-making

So far, we have assumed that each authority is fixing its policy threshold unilaterally. In this section, we consider the situation where the authorities fix a unique threshold, $\widehat{s}_J$, for both countries, for instance, in the European case, this would correspond to the European Commission setting a common policy. This policy will be chosen so that their joint ex-post welfare is maximised:

$$W(s, \widehat{s}_J) \big|_{s=\widehat{s}} = \gamma \left[ a_1 + b_1 \left( \frac{s \eta^2 + \mu V^*(\mu, s)}{\eta^2 + V^*(\mu, s)} \right) \right] + (1 - \gamma) \left[ a_2 + b_2 \left( \frac{s \eta^2 + \mu V^*(\mu, s)}{\eta^2 + V^*(\mu, s)} \right) \right]$$

where $\gamma$ and $1-\gamma$ are the weights given to the welfare of Country 1 and 2, respectively. It is easy to check that, given the firm's behaviour in Lemma 2, the optimal threshold
is:

\[
\tilde{s}^J = \begin{cases} 
-\mu \frac{V_a}{\eta^2} - \frac{\gamma a_1 + (1-\gamma)a_2}{\gamma b_1 + (1-\gamma)b_2} \frac{\eta^2 + V_M}{\eta^2} & \text{if } \mu < -\frac{\gamma a_1 + (1-\gamma)a_2}{\gamma b_1 + (1-\gamma)b_2} \\
-\mu \frac{V_L}{\eta^2} - \frac{\gamma a_1 + (1-\gamma)b_2}{\gamma b_1 + (1-\gamma)b_2} \frac{\eta^2 + V_L}{\eta^2} & \text{if } \mu \geq -\frac{\gamma a_1 + (1-\gamma)b_2}{\gamma b_1 + (1-\gamma)b_2}
\end{cases}
\]

Note that \(\tilde{s}^J\) will coincide with \(s_M\) in the increasing probability case if \(\gamma = \frac{b_2}{b_2 + b_1}\).

5 Extension: the case of private information\(^{32}\)

In this Section, we explore the robustness of our results in the assumption that the firm is not more informed than the competition authority about the merger’s welfare effects.

If we assume that \(\theta\) is the private information of the firm then, qualitatively, the firm’s behaviour does not change. The optimal variance chosen by the firm will depend on the merger’s type \(\theta\) in the same way that it was depending before on the average merger \(\mu\). In particular, in the case of the national merger, a firm with a bad merger will choose the risky strategy of high variance, while a firm with a good merger will choose low variance to increase the likelihood of obtaining a high realisation of the signal. In the same way, in the case of the multinational merger, the information-sharing agreement will have no impact on the types of merger that are either good or bad for both countries at the same time. However, for those firms whose merger is good for one country but bad for the other, the choice of the variance may be non-monotonic in their type in the same way as was found in Proposition 3.

However, with this new assumption, there is a significant change in the authority’s behaviour because the monotone likelihood ratio property (MLRP) does not generally hold, due to the way in which the firm optimally responds to the threshold rule. In particular, a very large \(s\) is more likely to come from a (bad) merger below the threshold because the distribution of the signal sent by such a merger has fatter tails. If the MLRP does not hold, then the cut-off rule may not be optimal. However, if we nonetheless restrict the authority to using the cut-off rule, then we can show that the equilibrium threshold, when the average merger is welfare detrimental, is stricter than the full information threshold and, contrary to what we found in Section 3, it does not change with the authority’s ability to commit.

The intuition for this result is as follows. There are two types of effects following an increase in the commitment threshold, \(\tilde{s}^{**}\): the direct and the strategic effect.

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\(^{32}\)The proofs for this section are available upon request.
The direct effect is the result from the trade-off between the benefit of decreasing the clearance probability of a bad merger against the cost of decreasing the clearance probability of a good merger. The direct benefit can be interpreted as a type II error of clearing a merger that should be blocked. By increasing $\hat{s}^{**}$ we make this error less likely. Similarly, the direct cost can be interpreted as a type I error of blocking a merger when in fact it should be allowed and by increasing $\hat{s}^{**}$ we make this error more likely.

The strategic (or indirect) effect is the result of the change in the firm’s strategy resulting from moving merger types from above to below the threshold (i.e. when the firm switches from $V_L$ to $V_H$, a good type $\theta$ has less chances of being cleared). Given that the authority values precision, as this allows her to take more informed decisions, the fact that in this new framework she chooses the same threshold regardless of whether or not she can commit to a policy may be puzzling. It seems natural to think that the mechanism highlighted in the Corollary 1 would still apply (that is, if under no commitment, the authority sets a threshold $\hat{s}^*$, she would gain from committing to a lower threshold $\hat{s}^{**}$ as this would induce the types in the interval $[\hat{s}^{**}, \hat{s}^*]$ to reduce their equilibrium variance from $V_H$ to $V_L$). However, when the authority is considering a marginal decrease in the threshold, she only takes into account the strategic effects of the marginal type $\hat{s}^*$ which consists in decreasing the variance from $V_H$ to $V_L$. However, for $\hat{s}^*$ (the type at the margin) the probability that the realisation of the signal is above $\hat{s}^*$ is $\frac{1}{2}$ in both cases. Therefore, since the marginal change in the firm’s behaviour generated by the ability to commit cancels out, the criterion used to set such a threshold is the same under both regimes. In particular, by lowering the threshold, the authority only trades off the increase in the type II error and the decrease in the type I error (i.e. the direct effect).

6 Conclusions

The goal of this paper has been to study the impact of an information-sharing agreement on the incentives of the firm to provide precise information to the authorities. We find that the agreement has no impact on the firm with an average merger that is welfare enhancing for both countries at the same time. This would explain why in some cases, where the firm is sure that a merger does not raise competition issues, he voluntarily grants a confidentiality waiver to the authorities. However, the agreement affects the behaviour of the firm whose average merger is welfare enhance-
ing for one country but welfare detrimental for the other. The impact in terms of information provision will depend on how authorities reach an agreement in the case of disagreement.

If there is no further cooperation and authorities exert their veto power, then the firm sends very imprecise information to both authorities. If the authorities cooperate further, and if they have equal bargaining power, then the choice of the variance may be non-monotonic in the average merger. Furthermore, despite the choice of variance being costless, intermediate levels of precision are chosen in equilibrium. Finally, if the probability depends linearly on the particular report then the firm behaves as in the national merger case where the new cut-off rule is the expectation of the countries’ rules.

Given their policies, do the agencies benefit from the agreement? The authorities value the level of precision because this allows them to make more accurate decisions; therefore, sharing information but retaining their veto power makes the authorities strictly worse off. If there is further cooperation, the firm makes more use of intermediate and lower variances. Therefore, only the less lenient country benefits from the agreement because, before, she was receiving information with the minimum level of precision. The more lenient country will not benefit from the agreement because, without the agreement, she was receiving the information with the maximum level of precision. However, if the lenient country is not always the same one, information-sharing and cooperation can potentially be beneficial.

In our analysis, we ignore a very important cost attached to the exchange of confidential information: the danger of leakage of commercially sensitive information to third parties. This can be clearly an issue when cooperation involves competition agencies in countries where the law for protecting confidential information is weak or where the credibility of the agency is low. We also ignore many benefits. For instance, the exchange of information facilitates the discussion about the case and minimises the possibility of missing an issue that needs an enforcement action. In the merger cases, it also eliminates conflicting decisions and remedies.\footnote{A successful example of a merger involving cooperation between agencies that illustrates these points was the Holnam/Lafarge case. A waiver granted by the parties allowed the U.S. and Canadian agencies to effectively improve the coordination, which ended up in a more informed decision-making (see Valentine (2000)).}

A possible way to enrich the setup is to let each authority receive a different realisation from the same random signal\footnote{This could be the case if information-sharing happens at a late state of the investigation process,}. In this context, sharing information
increases the quantity of information on which to base their decisions and allows them to better infer the level of precision used by the firm. Thus, noise becomes more costly and firms with bad average mergers will have their behaviour affected. There is room for the more lenient authority to benefit from the agreement.

We compared the no-information-sharing regime with the information-sharing regime, keeping the policies of the competition authorities fixed, as we were interested in exploring the changes in the firm’s behaviour following the agreement. The other motivation for proceeding this way was that these agreements do not usually contemplate changes in policies. Therefore, one possible direction of future work would be to enrich the model by allowing the competition agencies to strategically adapt their policies to the new regime. This would allow us to assess the total impact of the information-sharing regime, which not only includes the change in the firm’s behaviour but also the change in the authority’s policy.

Finally, the lessons from this model can be applied to a variety of situations in industrial organisation and political economy. For instance, the model can apply to the information submitted to a sectorial regulator or central bank and a competition authority (as is the case with the mergers involving banks), to interviews in a recruitment process with two interviewers, to reforms submitted to bicameral parliaments or, more generally, to a politician trying to gain support for a policy in front of two audiences with different opinions about the policy. When thinking about the model more broadly, new questions arise. For instance, consider the case of a person providing information to several different receivers as is the case of a speaker to a congress. Which voting rule will make the speaker give a more precise information?

7 Appendix

Proof of Lemma 1. The first derivative of $P(V)$ with respect to $V$ is:

$$\frac{\partial P(V)}{\partial V} = \int_{s_e}^{+\infty} g(s,V) \frac{(s - \mu)^2 - (\eta^2 + V)}{2(\eta^2 + V)^2} ds$$

(4)

where the different signal’s realisations can be interpreted as the information obtained from the particular questions (interviews, questionnaires, etc.) carried out by each authority.
Using the fact that \( \frac{\partial g(s,V)}{\partial V} = \frac{1}{2} \frac{\partial^2 g(s,V)}{\partial s^2} \), we can integrate (4) to obtain:

\[
\frac{\partial P(V)}{\partial V} = \frac{1}{2} \left[ g(s,V) \frac{-(s - \mu)}{\eta^2 + V} \right]_{\hat{s}^c}^{+\infty} = \frac{g(\hat{s}^c, V)}{2(\eta^2 + V)} \left[ \hat{s}^c - \mu \right]
\]

When \( \hat{s}^c = \mu \) this derivative is zero, which implies that M is indifferent between any variance in the interval \([V_L, V_H]\). Similarly, when \( \hat{s}^c > \mu \) the derivative is positive, and therefore the objective function is increasing in the variance. As a result, M will choose the maximum available variance \( V_H \). Finally, when \( \hat{s}^c < \mu \) the derivative is negative and so M will choose \( V_L \).

**Proof of Proposition 2.** The logic of the proof is as follows. We first solve the authority’s problem ignoring the constraint about the firm’s behaviour. Then, we find under which conditions the constraint will be binding.

Let us then, start by ignoring the constraint about the firm’s behaviour; in which case, the first order condition for a given variance \( V^*(\mu, \hat{s}) \) is:

\[
\frac{\partial EW(\hat{s})}{\partial \hat{s}} = -\frac{\hat{s}^2 + \mu V^*(\mu, \hat{s})}{\eta^2 + V^*(\mu, \hat{s})} g(\hat{s}, V^*(\mu, \hat{s})) = 0 \quad (5)
\]

And the second order condition is:

\[
\frac{\partial^2 EW(\hat{s})}{\partial \hat{s}^2} = -\frac{\hat{s}^2 + \mu V^*(\mu, \hat{s})}{\eta^2 + V^*(\mu, \hat{s})} g(\hat{s}, V^*(\mu, \hat{s})) + \frac{\mu}{\eta^2 + V^*(\mu, \hat{s})} < 0
\]

The second order condition is locally satisfied because the first term is zero when the first order condition is satisfied and the second term is always negative.

To solve condition (5), note that this expression is zero only if \( \hat{s} \eta^2 + \mu V^*(\mu, \hat{s}) = 0 \), hence:

\[
\hat{s}^* = \frac{-\mu V^*(\mu, \hat{s})}{\eta^2} \quad (6)
\]

R values the level of precision of the signal. When \( \mu > \hat{s} \), by Lemma 2, the firm chooses \( V_L \). The signal has the maximum level of informativeness, therefore the constraint in (2) does not bind and the solution is (6). Conversely, when \( \mu < \hat{s} \), the firm chooses \( V_H \). This introduces the possibility for the unconstrained solution in (6) to be dominated by the constrained solution \( \hat{s}^* = \mu \) (by Lemma 2, this is the minimum threshold that induces the firm to choose \( V_L \) instead of \( V_H \)) due to the increase in the informativeness of the signal. In particular, the constrained solution
will be chosen if the authority’s objective function evaluated at this point is larger than evaluated at (6):

$$\int_{-\infty}^{+\infty} \frac{s_{\eta^2} + \mu V_H}{\eta^2 + V_H} g(s, V_H) \, ds < \int_{-\infty}^{+\infty} \frac{s_{\eta^2} + \mu V_L}{\eta^2 + V_L} g(s, V_L) \, ds \quad (7)$$

Let us show that (7) is satisfied for \( \mu > \tilde{\mu} \) and not satisfied for \( \mu < \tilde{\mu} \), where \( \tilde{\mu} \) is to be defined. Rewrite (7) as:

$$- \int_{-\infty}^{+\infty} \frac{s_{\eta^2} \partial g(s, V_H)}{\partial s} \, ds + \mu \int_{-\infty}^{+\infty} g(s, V_H) \, ds < - \int_{\mu}^{+\infty} \frac{s_{\eta^2} \partial g(s, V_L)}{\partial s} \, ds + \mu \int_{\mu}^{+\infty} g(s, V_L) \, ds$$

$$\mu \left[ \frac{1}{2} - G \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \right] < \eta^2 \left[ \frac{1}{\sqrt{2\pi(\eta^2 + V_L)}} - g \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \right]$$

We want to determine whether there exists some value of \( \mu \) in the interval \((-\infty, 0]\) for which (7) is satisfied. Note that (7) is trivially satisfied when \( \mu = 0 \) as the left-hand side is zero and \( g(0, V_L) - g(0, V_H) > 0 \). Conversely, when \( \mu \to -\infty \), the left-hand side tends to \(+\infty\) while the right-hand side tends to \( \frac{\eta^2}{\sqrt{2\pi(\eta^2 + V_L)}} \). Therefore the inequality is violated. This means that there exists at least one value of \( \mu, \tilde{\mu} \), such that the expected welfare is equal:

$$\int_{-\infty}^{+\infty} \frac{s_{\eta^2} + \tilde{\mu} V_H}{\eta^2 + V_H} g(s, V_H) \, ds = \int_{\tilde{\mu}}^{+\infty} \frac{s_{\eta^2} + \tilde{\mu} V_L}{\eta^2 + V_L} g(s, V_L) \, ds \quad (8)$$

Note that \( \tilde{\mu} \neq 0 \). In order to show that this value is unique, we need to show that the slope of the difference of expected welfare is strictly decreasing at \( \mu = \tilde{\mu} \):

$$\frac{\partial}{\partial \mu} \left( \mu \left[ \frac{1}{2} - G \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \right] - \eta^2 \left[ \frac{1}{\sqrt{2\pi(\eta^2 + V_L)}} - g \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \right] \right) \bigg|_{\mu = \tilde{\mu}}$$

$$= \frac{1}{2} - G \left( \frac{-\mu V_H}{\eta^2}, V_H \right) - \mu g \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \left[ \frac{-V_H}{\eta^2} - 1 \right] + \eta^2 g \left( \frac{-\mu V_H}{\eta^2}, V_H \right) \left[ \frac{-V_H}{\eta^2} \left( -\frac{-\mu V_H}{\eta^2} - \mu \right) \right] + \left( \frac{-\mu V_H}{\eta^2} - \mu \right)$$

$$= \frac{1}{2} - G \left( \frac{-\mu V_H}{\eta^2}, V_H \right) < 0$$

This expression is negative for \( \tilde{\mu} < 0 \), therefore there is a unique \( \tilde{\mu} \) for which
condition (8) holds.

The comparative statics with respect to \( V_L \):

\[
\frac{d\mu}{dV_L} = \frac{\eta^2}{G \left( \frac{-\mu V}{\eta^2}, V_H \right) - \frac{1}{2}} 2(\eta^2 + V_L)\sqrt{2\pi(\eta^2 + V_L)} > 0
\]

and with respect to \( V_H \):

\[
\frac{d\mu}{dV_H} = \frac{g \left( \frac{-\mu V}{\eta^2}, V_H \right) \left[ \frac{\mu^2 + V_H + \eta^2}{2\pi(\eta^2 + V_H)} \right]}{\frac{1}{2} - G \left( \frac{-\mu V}{\eta^2}, V_H \right)} < 0
\]

noting that \( \bar{\mu} < 0 \) and that therefore \( G \left( \frac{-\mu V}{\eta^2}, V_H \right) - \frac{1}{2} > 0 \) give us the signs. \( \blacksquare \)

**Proof of Proposition 3.** We compute the first and the second derivative of the objective function in (3) in order to show that \( P^\alpha(V) \) is quasi-convex until \( \mu = s_M \) and then quasi-concave. The first derivative of \( P^\alpha(V) \) with respect to \( V \) is:

\[
\frac{\partial P^\alpha(V)}{\partial V} = \frac{\alpha}{2} \int_{\hat{s}_1}^{\hat{s}_2} \frac{\partial^2 g(s, V)}{\partial s^2} ds + \frac{1}{2} \int_{\hat{s}_2}^{+\infty} \frac{\partial^2 g(s, V)}{\partial s^2} ds
\]

\[
= \frac{\alpha}{2} \left[ \frac{\partial g(s, V)}{\partial s} \right]_{\hat{s}_1}^{\hat{s}_2} + \frac{1}{2} \left[ \frac{\partial g(s, V)}{\partial s} \right]_{\hat{s}_2}^{+\infty}
\]

\[
= \frac{1}{2(\eta^2 + V)} \left[ \alpha g(\hat{s}_1, V) (\hat{s}_1 - \mu) + (1 - \alpha) g(\hat{s}_2, V) (\hat{s}_2 - \mu) \right]
\]

where \((\hat{s}_1 - \mu)\) is negative and \((\hat{s}_2 - \mu)\) is positive. If \( \mu = \hat{s}_1 \), the derivative is positive so M chooses \( V_H \). Conversely, if \( \mu = \hat{s}_2 \), the derivative is negative so M chooses \( V_L \).

When \( \mu = s_M \), then:\(^{35}\)

\[
\frac{\partial P^\alpha(V)}{\partial V} = \frac{\hat{s}_2 - \hat{s}_1}{4(\eta^2 + V)} g(\hat{s}_2, V) [1 - 2\alpha]
\]

so the sign will depend on \( \alpha \).

From \( \frac{\partial P^\alpha(V)}{\partial V} = 0 \), we can find the "critical variance":

\[
V(\mu, \alpha) = \frac{(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2}{2\ln \left( \frac{(1-\alpha)(\hat{s}_2 - \mu)}{\alpha(\mu - \hat{s}_1)} \right)} - \eta^2
\]

\(^{35}\)Note that: \( g(\hat{s}_2, V) = g(\hat{s}_1, V) \) when \( \mu = s_M \).
The second derivative (evaluated at the first order condition) is:

\[
\frac{\partial^2 P^\alpha(V)}{\partial V^2} \bigg|_{\partial P^\alpha(V) = 0} \propto \alpha \frac{\partial g(\hat{s}_1, V)}{\partial V} (\hat{s}_1 - \mu) + (1 - \alpha) \frac{\partial g(\hat{s}_2, V)}{\partial V} (\hat{s}_2 - \mu)
\]

\[
= \alpha g(\hat{s}_1, V) \left( \frac{(\hat{s}_1 - \mu)^2 - (\eta^2 + V)}{2(\eta^2 + V)^2} \right) (\hat{s}_1 - \mu)
\]

\[
+(1 - \alpha) g(\hat{s}_2, V) \left( \frac{(\hat{s}_2 - \mu)^2 - (\eta^2 + V)}{2(\eta^2 + V)^2} \right) (\hat{s}_2 - \mu)
\]

\[
= \frac{1}{2(\eta^2 + V)^2} \left[ \alpha g(\hat{s}_1, V) (\hat{s}_1 - \mu)^3 + (1 - \alpha) g(\hat{s}_2, V) (\hat{s}_2 - \mu)^3 \right]
\]

\[
- \frac{1}{2(\eta^2 + V)^2} \left[ \alpha g(\hat{s}_1, V) (\hat{s}_1 - \mu) + (1 - \alpha) g(\hat{s}_2, V) (\hat{s}_2 - \mu) \right]
\]

\[
= \frac{1}{(\eta^2 + V)^2} \left[ \alpha g(\hat{s}_1, V) (\hat{s}_1 - \mu)^3 + (1 - \alpha) g(\hat{s}_2, V) (\hat{s}_2 - \mu)^3 \right]
\]

where the last equality follows from using the first order condition. In order to determine whether \( P^\alpha(V) \) is locally concave or convex, we need to determine the sign of the expression in brackets, which is proportional to:

\[
\frac{\partial^2 P^\alpha(V)}{\partial V^2} \propto \alpha \exp \left( \frac{-(\hat{s}_1 - \mu)^2}{2(\eta^2 + V)} \right) (\hat{s}_1 - \mu)^3
\]

\[
+ (1 - \alpha) \exp \left( \frac{-(\hat{s}_2 - \mu)^2}{2(\eta^2 + V)} \right) (\hat{s}_2 - \mu)^3
\]

\[
= -\alpha (\mu - \hat{s}_1)^3 \left( \frac{1 - \alpha}{\alpha (\mu - \hat{s}_1)} \right) \frac{-(\mu - \hat{s}_1)^2}{(\mu - \hat{s}_1)^2 - (\eta^2 + V)^2}
\]

\[
+(1 - \alpha) (\hat{s}_2 - \mu)^3 \left( \frac{1 - \alpha}{\alpha (\mu - \hat{s}_1)} \right) \frac{-(\hat{s}_2 - \mu)^2}{(\hat{s}_2 - \mu)^2 - (\eta^2 + V)^2}
\]

(10)

where the last equality follows from plugging in \( V(\mu, \alpha) \) from (9). The sign of (10) depends on how \( \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right)^3 \) compares to \( \left( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \right) \). When \( \mu \) equals \( s_M \), \( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \) is one and thus (10) is zero. When \( \mu \in (\hat{s}_1, s_M) \), \( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \) is larger than one and, therefore, (10) is positive. Conversely, when \( \mu \in (s_M, \hat{s}_2) \), \( \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1} \) is smaller than one and, thus, (10) is negative.

**Proof of Corollary 2.** First note that for \( \alpha = \frac{1}{2} \), (3) can be rewritten as:

\[
P^\frac{1}{2}(V) = 1 - \frac{1}{2} \left[ G(\hat{s}_1, V) + G(\hat{s}_2, V) \right]
\]

For \( \mu \in (\hat{s}_1, s_M) \), the optimal variance is either maximum or minimum variance. Whether this is one or the other depends on how the probability of clearance with
V_H compares to the one with V_L. In particular, M will choose V_L whenever:

\[ G(\hat{s}_1, V_H) + G(\hat{s}_2, V_H) > G(\hat{s}_1, V_L) + G(\hat{s}_2, V_L) \]  \hspace{1cm} (11)

Before we proceed, we state the following useful fact: if a function \( f(x) \) is symmetric around the point \( x = a \), then for any \( b \geq 0 \)

\[ \int_{a-b}^{+\infty} f(x)dx = \int_{-\infty}^{a+b} f(x)dx. \] \hspace{1cm} (12)

Let us use fact (12) to simplify condition (11). Note that \( \hat{s} \) is the middle point between \( b_s^2 \) and \( 2 \). Using the fact that the integrand is symmetric around \( \hat{s} \), by fact (12), the integral from \( -\infty \) to \( \hat{s} \) is equal to the integral from \( 2 - \hat{s} \) to \( +\infty \).

We can rewrite (11) as:

\[ G(\hat{s}_1, V_H) + 1 - G(2\mu - \hat{s}_2, V_H) > G(\hat{s}_1, V_L) + 1 - G(2\mu - \hat{s}_2, V_L) \]

\[ G(\hat{s}_1, V_H) - G(2\mu - \hat{s}_2, V_H) > G(\hat{s}_1, V_L) - G(2\mu - \hat{s}_2, V_L) \] \hspace{1cm} (13)

Note that \( 2\mu - \hat{s}_2 \leq \hat{s}_1 \) for all \( \mu \in (\hat{s}_1, s_M) \).

Two probability density functions, \( g(s, V_H) \) and \( g(s, V_L) \), intersect at two values of \( s \); to the left and to the right of \( \mu \). Define \( \tilde{s} \) as the smallest of these values:

\[ \tilde{s} = \mu - \sqrt{(V_L + \eta^2)(V_H + \eta^2)} \ln \left( \frac{V_H + \eta^2}{V_L + \eta^2} \right) \] \hspace{1cm} (14)

Rewrite \( \tilde{s} \) as \( \tilde{s} = \mu - c(V_L, V_H) \). It can be shown that \( c(V_L, V_H) \) increases with \( V_H \) and decreases when \( V_L \) decrease.

We are only able to determine M’s choices when \( \tilde{s} < 2\mu - \hat{s}_2 \) and when \( \hat{s}_1 < \tilde{s} \); therefore, the conditions that follow are sufficient. First we derive the condition for the firm to choose \( V_H \) for all the relevant \( \mu \), and then the one where the firm first chooses \( V_H \) and then switches to \( V_L \) as a function of \( \mu \).

\( V_H \) will be chosen for all \( \mu \in (\hat{s}_1, s_M) \) if \( \tilde{s} < 2\mu - \hat{s}_2 \) for the smallest possible \( \mu \), that is, \( \hat{s}_1 \):

\[ \hat{s}_2 - \hat{s}_1 < c(V_L, V_H) \]

\(^{36}\)If \( \tilde{s} \in (2\mu - \hat{s}_2, \hat{s}_1) \), we cannot say how the area under \( g(s, V_H) \) compares to the area under \( g(s, V_L) \) for the relevant range of integration.
To see why, consider Figure 5 which depicts a value of $\mu$ at the beginning of the relevant interval: $\mu = \hat{s}_1 + \Delta$, where $\Delta$ is small and positive. The area under $g(s, V_H)$ is smaller than under $g(s, V_L)$ for the relevant range of integration (from $2\mu - \hat{s}_2$ to $\hat{s}_1$) and by condition (13) the firm will choose $V_H$.

$V_L$ will be chosen if $\hat{s}_1 < \bar{s}$, or equivalently, if:

$$\hat{s}_1 + c(V_L, V_H) < \mu.$$  

Since $c(V_L, V_H) > 0$, this condition cannot hold for the smallest possible $\mu$ (i.e. $\hat{s}_1$ in the limit); thus, when $\mu$ is close to $\hat{s}_1$ M always chooses $V_H$. Consider now a value of $\mu$ at the end of the relevant interval: $\mu = s_M - \Delta$ such as the one depicted in Figure 6. The area under $g(s, V_H)$ is larger than under $g(s, V_L)$ for the relevant range of integration and by condition (13) the firm will choose $V_L$.

By continuity, there is a value $\bar{\mu}$ such that the firm is indifferent between $V_H$ and $V_L$:

$$G(\hat{s}_1, V_H) - G(2\bar{\mu} - \hat{s}_2, V_H) = G(\hat{s}_1, V_L) - G(2\bar{\mu} - \hat{s}_2, V_L) \quad (15)$$

To sum up, if $\hat{s}_1 < \bar{s}$, M chooses:

$$V^* \left( \mu, \frac{1}{2} \right) = \begin{cases} 
V_H & \text{if } \mu \in (\hat{s}_1, \bar{\mu}) \\
V_L & \text{if } \mu \in (\bar{\mu}, s_M)
\end{cases}$$

Figure 5: $\mu = \hat{s}_1 + \Delta$ and $\bar{s} < 2\mu - \hat{s}_2$
For this to be an equilibrium we need the following condition to hold:

\[ c(V_L, V_H) < \mu - \hat{s}_1 \]

**Proof of Corollary 3.** Take the optimal variance \( V(\mu, \alpha) \) in (9). Both the numerator and the denominator tend to zero as \( \mu \to s_M \), so using l’Hôpital rule:

\[
\lim_{\mu \to s_M} V\left(\mu, \frac{1}{2}\right) = \frac{- (\hat{s}_2 - \hat{s}_1) \mu - (\hat{s}_2 - \mu)}{\hat{s}_2 - \mu} \frac{\hat{s}_2 - \mu}{(\hat{s}_2 - \mu)(\hat{s}_2 - \mu)} = \frac{\hat{s}_1 - \hat{s}_2}{(\hat{s}_2 - \mu)(\hat{s}_2 - \mu)} = \frac{\hat{s}_1 - \hat{s}_2}{(\hat{s}_2 - \mu)(\hat{s}_2 - \mu)}
\]

\[
= (\hat{s}_2 - \mu)(\hat{s}_2 - \hat{s}_1) = \left(\frac{\hat{s}_2 - \hat{s}_1}{2}\right)^2
\]

The value of the variance at the opposite extreme, \( \hat{s}_2 \), is:

\[
\max \left\{ V\left(\hat{s}_2, \frac{1}{2}\right), V_L \right\} = \left\{ \frac{- (\hat{s}_2 - \hat{s}_1)^2}{2 \ln (0)} - \eta^2, V_L \right\} = V_L
\]

Therefore, there exists \( \bar{\mu} \) such that:

\[
V\left(\bar{\mu}, \frac{1}{2}\right) = \frac{(\hat{s}_2 - \bar{\mu})^2 - (\bar{\mu} - \hat{s}_1)^2}{2 \ln \left(\frac{\hat{s}_2 - \bar{\mu}}{\bar{\mu} - \hat{s}_1}\right)} - \eta^2 = V_L
\]  

(16)
We now show that $V(\mu, \frac{1}{2})$ is decreasing in $\mu$.

$$\frac{dV(\mu, \frac{1}{2})}{d\mu} = \frac{[-2(\hat{s}_2 - \mu) - 2(\mu - \hat{s}_1)] \ln \left(\frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1}\right)}{(2 \ln \left(\frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1}\right))^2} - 2 \left[(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2\right] \frac{\mu - \hat{s}_1}{\hat{s}_2 - \mu} \frac{-(\mu - \hat{s}_1)(\hat{s}_2 - \mu)}{(\mu - \hat{s}_1)^2}$$

$$= \frac{4(\hat{s}_1 - \hat{s}_2) \ln \left(\frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1}\right) - 2 \left[(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2\right] \frac{\mu - \hat{s}_1}{\hat{s}_2 - \mu} \frac{-(\mu - \hat{s}_1)(\hat{s}_2 - \mu)}{(\mu - \hat{s}_1)^2}}{(2 \ln \left(\frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1}\right))^2}$$

$$= (\hat{s}_1 - \hat{s}_2) \left[\frac{2 \ln \left(\frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1}\right)}{(2 \ln \left(\frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1}\right))^2} - \left[\frac{(\hat{s}_2 - \mu)^2 - (\mu - \hat{s}_1)^2}{(\hat{s}_2 - \mu)(\mu - \hat{s}_1)}\right]\right]$$

The first term is always negative and the denominator is always positive, therefore, the sign will depend on the term in square brackets. This term can be rewritten as:

$$2 \ln x - x + \frac{1}{x}$$

where $x = \frac{\hat{s}_2 - \mu}{\mu - \hat{s}_1}$. It is easy to see that this expression is positive when $0 < x < 1$ and negative when $x > 1$. Since we focus on $\mu \in (s_M, \hat{s}_2]$, the expression will always be positive and thus, $\frac{dV(\mu)}{d\mu} < 0$.

**Proof of Proposition 4.** The first derivative of $P^c(V)$ with respect to $V$ is:

$$\frac{\partial P^c(V)}{\partial V} = \frac{1}{2} \int_{\hat{s}_1}^{\hat{s}_2} \left(\frac{s - \hat{s}_1}{\hat{s}_2 - \hat{s}_1}\right) \frac{d^2g(s, V)}{ds^2} ds + \frac{1}{2} \int_{\hat{s}_2}^{+\infty} \frac{d^2g(s, V)}{ds^2} ds$$

$$= \left[\frac{1}{2} \left(\frac{s - \hat{s}_1}{\hat{s}_2 - \hat{s}_1}\right) \frac{dg(s, V)}{ds}\right]^{\hat{s}_2}_{\hat{s}_1} - \int_{\hat{s}_1}^{\hat{s}_2} \frac{1}{2} \left(\frac{s - \hat{s}_1}{\hat{s}_2 - \hat{s}_1}\right) \frac{dg(s, V)}{ds} ds + \frac{1}{2} \left[\frac{dg(s, V)}{ds}\right]_{\hat{s}_2}^{+\infty}$$

$$= -\frac{1}{2} g(\hat{s}_2, V) \frac{(\hat{s}_2 - \mu)}{V + \eta^2} - \frac{1}{2} \frac{g(\hat{s}_2, V)}{\hat{s}_2 - \hat{s}_1} \left[g(\hat{s}_2, V)\right]^{\hat{s}_2}_{\hat{s}_1} + \frac{1}{2} g(\hat{s}_2, V) \frac{(\hat{s}_2 - \mu)}{V + \eta^2}$$

$$= g(\hat{s}_1, V) - g(\hat{s}_2, V)$$

The firm will choose $V$ so that $\frac{\partial P^c(V)}{\partial V} = 0$, which happens when:

$$(\hat{s}_1 - \mu)^2 = (\hat{s}_2 - \mu)^2$$

(17)
If $\mu = s_M$, (17) holds for any $V$. Otherwise, (17) never holds. The firm sets $V_L$ or $V_H$ depending on the value of $\mu$. Note that $\frac{\partial P^c(V)}{\partial \mu}$ decreases with $\mu$:
\[
\frac{\partial^2 P^c(V)}{\partial V \partial \mu} \propto -\exp\left(-\frac{(\hat{s}_1 - \mu)^2}{2(\eta^2 + V)}\right)(\mu - \hat{s}_1) - \exp\left(-\frac{(\hat{s}_2 - \mu)^2}{2(\eta^2 + V)}\right)(\hat{s}_2 - \mu) < 0
\]

Therefore, $\frac{\partial P^c(V)}{\partial V}$ is positive for $\mu < s_M$ and the firm choose $V_H$ and it is negative for $\mu > s_M$ and the firm chooses $V_L$. ■

References


