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ELECTORAL UNCERTAINTY, THE DEFICIT BIAS AND THE ELECTORAL CYCLE IN A NEW KEYNESIAN ECONOMY

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Electoral Uncertainty, the Deficit Bias and the Electoral Cycle in a New Keynesian Economy

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Abstract: Recent attempts to incorporate optimal fiscal policy into New Keynesian models subject to nominal inertia, have tended to assume that policy makers are benevolent and have access to a commitment technology. A separate literature, on the New Political Economy, has focused on real economies where there is strategic use of policy instruments in a world of political conflict. In this paper we combine these literatures and assume that policy is set in a New Keynesian economy by one of two policy makers facing electoral uncertainty (in terms of infrequent elections and an endogenous voting mechanism). The policy makers generally share the social welfare function, but differ in their preferences over fiscal expenditure (in its size and/or composition).

We use this model to examine three issues that arise from either literature. First, we consider the extent to which electoral competition gives rise to a debt or deficit bias, as one party seeks to win elections and tie the hands of a potential successor, when all debt is defined in nominal terms. Second, we examine the extent and nature of the electoral cycle introduced by having two parties reflecting different preferences over either the composition or amount of government spending. Third, we examine whether electoral competition has any impact on the conventional business cycle stabilisation policy, compared to the standard analysis that assumes a single benevolent government.

JEL Codes: E62, E63

Keywords: New Keynesian Model; Government Debt; Monetary Policy; Fiscal Policy, Electoral Uncertainty, Time Consistency.

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1 Overview

In recent years the New Keynesian literature has integrated fiscal policy and debt into an analysis of optimal policy where policymakers maximise a measure of social welfare derived from agents' utility. Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004) show that, under commitment, the steady-state level of debt should follows a random walk, while Leith and Wren-Lewis (2007) show that, under discretion, debt quickly returns to its initial level under optimal policy. However the common assumption in these and other papers in the literature is that there is a single, benevolent policy maker. This ignores the results of the New Political Economy literature, which has investigated the implications of electoral competition amongst partisan politicians for both the level of government debt and macroeconomic dynamics in the form of an electoral cycle.

In this paper we extend a benchmark New Keynesian model to incorporate partisan politicians engaged in electoral competition. We assume that households can be split into two types, according to their preferences for both the size and composition of government expenditure. Probabilistic voting as in Lindbeck and Weibull (1987) then implies that there can be random, but endogenous, shifts in the median voter resulting in election of parties representing one or other of the two groups. When in power a particular party will seek to implement policy which maximises the welfare of their electoral base, but in doing so will consider the influence their actions have on their future electoral success and on policies adopted by the other party should they happen to be elected in the future. They can influence these actions by affecting the stock of debt inherited by the other party in the future. Therefore the political economy elements we add encompass those of Alesina and Tabellini (1990), Persson and Svensson (1989) and Aghion and Bolton (1990).

We use this model to examine three issues that arise from either literature. First, we consider the extent to which electoral competition gives rise to a debt or deficit bias, as one party seeks to tie the hands of a potential successor. Second, we examine the extent and nature of the electoral cycle introduced by having two parties reflecting different preferences over either the composition or amount of government spending. Third, we examine whether electoral competition has any impact on the conventional business cycle stabilisation policy, compared to the standard analysis that assumes a single benevolent government.

Alesina and Tabellini (1990) and Persson and Svensson (1989) introduced the idea that electoral competition could lead to debt or deficit bias (i.e. a level of debt that was higher than the one that would be chosen by a single benevolent social planner). There are a number of papers building on this earlier literature and exploring time-consistent fiscal policy in a setting where there is political conflict. This often takes place in the context of real models featuring distributional conflict in an overlapping generations framework (see, for example, Song et al (2007) and Hassler et al. (2005)), or a heterogeneous initial wealth distribution in a neoclassical growth model (for example, Krusell et al. (1997)). Debertoli and Nunes (2007) explore the impact on government debt of exogenous political turnover in a real infinite horizon model with endogenous government spending. However these are typically real models, whereas in all advanced industrial economies the majority of government debt is defined in nominal terms. As Lucas and Stokey (1983) show, if debt is nominal and inflation is costless, then inflation surprises can be used to costlessly deflate the real value of debt, implying that debt cannot be used strategically as a state variable, so no debt bias will arise on this account. The question we address is whether sticky prices, which imply that inflation has welfare costs, are enough to reintroduce a significant amount of debt bias of the type explored in real models.

While the idea of an electoral cycle is a familiar one, the nature and degree of such cycles in microfounded New Keynesian models remains largely unexplored. Not only do we show how changes in power between parties that favour big or small government can lead to important changes in output and inflation, but we compare the welfare costs of these electoral cycles with the costs associated with standard business cycle shocks. We also investigate whether the strategic use of policy to both tie the hands of their political opponents and influence the endogenous outcome of future elections has a significant impact on the extent to which governments attempt to stabilise business cycle shocks.

The plan of the paper is as follows. In Section 2 we outline our model in which households supply labour to imperfectly competitive firms who are only able to change prices at random intervals of time. Households are split into two groups with different preferences for the composition and/or size of government spending. In Section 3 we derive a second-order approximation to welfare for these consumers and contrast that with social welfare. In Section 4, we describe the nature of the electoral game and optimal time-consistent policy in the face of this electoral uncertainty, where political parties represent the interests of one of the two types of household. This then informs the simulation results in section 5, which reveal that political parties operating in a New Keynesian economy, facing electoral uncertainty and constrained to be time consistent in their policy actions, will generate a costly electoral cycle with a suboptimal level of government debt which ties the hands of their opponents and influences the outcome of the electoral game.

2 The Model

This section outlines our economy. Other than the political economy aspects of our analysis[1], the model is a standard New Keynesian model, but augmented to include the government's budget constraint where government spending is financed by distortionary taxation and/or borrowing. This basic set-up is similar to that in Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004) but with some differences. Specifically, we split government spending into two types and allow policy makers to vary such spending in a way which they find optimal, rather than simply treating government spending as an exogenous flow

[1] Section 4 below, describes the political aspects of the policy problem in detail.

which must be financed. However, the main innovation of the paper lies in introducing additional political conflict to the policy problem. This is achieved by assuming that there are two political parties representing two representative households with differing preferences over the size and/or composition of government spending. These parties alternate in power through a probabilistic voting mechanism (Lindbeck and Weibull, 1987) which generates fluctuations in the median voter such that electoral outcomes are stochastic, but endogenous. This gives rise to the possibility of policy makers using debt strategically, both to win elections and influence their opponent's behaviour and leads to the generation of electorally driven economic fluctuations. We examine the households' problem initially, before turning to the firms' problem.

2.1 Households

There are a continuum of households of size one. However, the households split evenly into two types depending on their tastes for government spending. We shall assume full asset markets, such that, through risk sharing, they will face the same budget constraint. As a result the typical household will seek to maximise the following objective function,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t, G_t^A; G_t^B) \tag{1}$$

where C, G^A, G^B and N are a consumption aggregate, two types of public goods aggregate, and labour supply respectively.

The consumption aggregate is defined as[2]

$$C = (\int_0^1 C(j)^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

where j denotes the good's type or variety. The public goods aggregates take the same form[3]

$$\begin{aligned} G^A &= (\int_0^1 G^A(j)^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}}, \text{ and,} \\ G^B &= (\int_0^1 G^B(j)^{\frac{\epsilon-1}{\epsilon}} dj)^{\frac{\epsilon}{\epsilon-1}} \end{aligned}$$

The elasticity of substitution between varieties $\epsilon_t > 1$ is assumed to time varying as we wish to allow for iid cost-push/mark-up shocks.

[2] We drop the time subscript when all variables in an expression are dated in the same period and there is no possibility of confusion.

[3] The assumption that private and public consumption baskets take the same form is common in the literature, since it makes aggregation of demand for individual firms' products across the private and public sectors straightforward. We make the same assumption in further separating public consumption into two types for the same reason.

The budget constraint at time t is given by

$$\int_0^1 P_t(j)C_t(j)dj + E_t\{Q_{t,t+1}D_{t+1}\} = \Pi_t + D_t + W_tN_t(1-\tau_t) - T_t$$

where $P_t(j)$ is the price of variety j , D_{t+1} is the nominal payoff of the portfolio held at the end of period t, Π is the representative household's share of profits in the imperfectly competitive firms, W are wages, τ is an wage income tax rate, and T are lump sum taxes. $Q_{t,t+1}$ is the stochastic discount factor for one period ahead pay-offs.

Households must first decide how to allocate a given level of expenditure across the various goods that are available. They do so by adjusting the share of a particular good in their consumption bundle to exploit any relative price differences - this minimises the costs of consumption. Optimisation of expenditure for any individual good implies the demand function given below,

$$C(j) = (\frac{P(j)}{P})^{-\epsilon}C$$

where we have a price index given by

$$P = (\int_0^1 P(j)^{1-\epsilon}dj)^{\frac{1}{1-\epsilon}}$$

The budget constraint can therefore be rewritten as

$$P_tC_t + E_t\{Q_{t,t+1}D_{t+1}\} = D_t + W_tN_t(1-\tau_t) - T_t \tag{3}$$

where $\int_0^1 P(j)C(j)dj = PC$.

2.1.1 Households' Intertemporal Consumption Problem

The first of the households intertemporal problems involves allocating consumption expenditure across time. For tractability assume that (1) takes the specific form for household type i, $i = 1, 2$.

$$E_0 \sum_{t=0}^{\infty} \beta^t (\frac{C_t^{1-\sigma}}{1-\sigma} + \chi_i^A \frac{(G_t^A)^{1-\sigma}}{1-\sigma} + \chi_i^B \frac{(G_t^B)^{1-\sigma}}{1-\sigma} - \frac{N_t{}^{1+\varphi}}{1+\varphi}) \tag{4}$$

It is differences in the χ_i^A and χ_i^B parameters that will be the source of political conflict to the extent that these differ across the two household types and the two parties that represent them. We leave the exact nature of that heterogeneity unspecified such that we can consider differences in preferences over the composition of government spending as in Alesina and Tabellini (1990) or government size as in Persson and Svennson (1989).

We can then maximise utility subject to the budget constraint (3) to obtain the optimal allocation of consumption across time,

$$\beta(\frac{C_t}{C_{t+1}})^{\sigma}(\frac{P_t}{P_{t+1}}) = Q_{t,t+1}$$

Taking conditional expectations on both sides and rearranging gives

$$\beta R_t E_t\{(\frac{C_t}{C_{t+1}})^{\sigma}(\frac{P_t}{P_{t+1}})\} = 1 \tag{5}$$

where $R_t = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross return on a riskless one period bond paying off a unit of currency in $t+1$. This is the familiar consumption Euler equation which implies that consumers are attempting to smooth consumption over time such that the marginal utility of consumption is equal across periods (after allowing for tilting due to interest rates differing from the households' rate of time preference).

A log-linearised version of (5) can be written as

$$\widehat{C}_t = E_t\{\widehat{C}_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\}) \tag{6}$$

where hatted variables denote percentage deviations from steady-state, $r_t = R_t - \rho$ where $\rho = \frac{1}{\beta} - 1$, and $\pi_t = p_t - p_{t-1}$ is price inflation.

The second foc relates to their labour supply decision and is given by,

$$(1-\tau)\left(\frac{W}{P}\right) = N^{\varphi}C^{\sigma}$$

Log-linearising implies,

$$-\frac{\overline{\tau}}{1-\overline{\tau}}\widehat{\tau} + \widehat{w} = \varphi\widehat{N} + \sigma\widehat{C}$$

2.2 Allocation of Government Spending

The allocation of government spending across individual goods is determined by minimising total costs, $\int_0^1 P(j)(G^A(j) + G^B(j))dj$. Given the form of the baskets of public goods this implies,

$$\begin{aligned} G^A(j) &= (\frac{P(j)}{P})^{-\epsilon}G^A \\ G^B(j) &= (\frac{P(j)}{P})^{-\epsilon}G^B \\ G &= G^A + G^B \end{aligned}$$

2.3 Firms

The production function is linear, so for firm j

$$Y(j) = AN(j) \tag{7}$$

where $a = \ln(A)$ is time varying and stochastic. While the demand curve they face is given by,

$$Y(j) = (\frac{P(j)}{P})^{-\epsilon}Y$$

where $Y = \left[\int_0^1 Y(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$. The objective function of the firm is given by,

$$\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t+s}\left[P(j)_t Y(j)_{t+s} - W_{t+s}\frac{Y(j)_{t+s}(1-\varkappa)}{A_{t+s}}\right] \quad (8)$$

where θ_p is the probability that the firm is unable to change its price in a particular period and $\varkappa$ is a time-invariant employment subsidy which can be used to eliminate the steady-state distortion associated with monopolistic competition and distortionary income taxes. Profit maximisation then implies that firms that are able to change price in period t will select the following price,

$$P_t^* = \frac{\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t+s}\left[\epsilon_t W_{t+s} P_{t+s}^{\epsilon_t}\frac{Y_{t+s}}{A_{t+s}}\right]}{\sum_{s=0}^{\infty}(\theta_p)^s Q_{t,t+s}\left[(\epsilon_t-1)P_{t+s}^{\epsilon_t} Y_{t+s}(1-\varkappa)\right]}$$

Leith and Wren-Lewis (2007) demonstrate that log-linearisation of this pricing behaviour implies a New Keynesian Phillips curve for price inflation which is given by,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda(\widehat{mc}_t + \widehat{\mu}_t)$$

where $\lambda = \frac{(1-\theta_p\beta)(1-\theta_p)}{\theta_p}$, $\widehat{mc} = -a + \widehat{w}$ are the real log-linearised marginal costs of production, and $\widehat{\mu}_t = \ln(\frac{\epsilon_t}{\epsilon_t - 1}) - \ln(\frac{\overline{\epsilon}}{\overline{\epsilon}-1})$ is a mark-up shock representing the temporary deviation of the desired mark-up from its steady-state value.

2.4 Equilibrium

Goods market clearing requires, for each good j,

$$Y(j) = C(j) + G^A(j) + G^B(j) \quad (9)$$

which allows us to write,

$$Y = C + G$$

where aggregate output is defined as, $Y = [\int_0^1 Y(j)^{\frac{\epsilon_t-1}{\epsilon_t}} dj]^{\frac{\epsilon_t}{\epsilon_t-1}}$. Log-linearising implies

$$\begin{aligned}\widehat{Y} &= \theta\widehat{C} + (1-\theta)\widehat{G} \\ \widehat{G} &= \gamma\widehat{G}^A + (1-\gamma)\widehat{G}^B\end{aligned}$$

where we define $\theta = \frac{\overline{C}}{\overline{Y}}$ and $\gamma = \frac{\overline{G^A}}{\overline{G}}$. These steady-state shares will be related to deeper preference parameters below.

2.5 Government Budget Constraint

Combining the series of the representative consumer's flow budget constraints, (3), with borrowing constraints that rule out Ponzi-schemes, gives the intertemporal budget constraint (see Woodford, 2003, chapter 2, page 69),

$$\sum_{T=t}^{\infty} E_t[P_T C_T] \leq D_t + \sum_{T=t}^{\infty} E_t[Q_{t,T}(\Pi_T + W_T N_T(1-\tau_T) - T_T)]$$

Noting the equivalence between factor incomes and national output,

$$PY = WN + \Pi - \varkappa WN$$

and the definition of aggregate demand, we can rewrite the private sector's budget constraint as,

$$D_t = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(P_T G_T - W_T N_T(\tau_T - \varkappa) - T_T)]$$

In order to focus on any deficit bias and time-inconsistency problems associated with the introduction of electoral uncertainty in a New Keynesian model with debt and distortionary taxation we follow Rotemberg and Woodford (1997) and later authors and introduce a steady-state subsidy. This subsidy offsets, in a benchmark steady-state, the distortions caused by distortionary taxation and imperfect competition in price setting, and removes the usual desire on the part of policy makers to raise output above its natural level to compensate for these distortions. In other words, this subsidy ensures that our benchmark steady state is efficient, such that any desire to deviate from that steady-state is solely being driven by the biases and distortions caused by heterogeneity in preferences over government spending and the electoral cycle. The steady state subsidy is financed by lump-sum taxation.[4] We shall assume that both the level of the subsidy and the associated level of lump-sum taxation cannot be altered from this steady state level, so that any changes in the government's intertemporal budget constraint have to be financed by changes in distortionary taxation or government spending or accomodated through monetary policy. This implies that $W_T N_T \varkappa = T_T$ in our economy at all points in time, allowing us to simplify the budget constraint to,

$$D_t = -\sum_{T=t}^{\infty} E_t[Q_{t,T}(P_T G_T - W_T N_T \tau_T)]$$

i.e. distortionary taxation and spending adjustments are required to service government debt as well as stabilise the economy. Rewriting in real terms and noting that government debt is dated at the beginning of the period,

$$\frac{B_t}{P_{t-1}}\frac{P_{t-1}}{P_t} = \sum_{T=t}^{\infty} E_t[\beta^{T-t}(\frac{C_t}{C_T})^{\sigma}(w_T N_T \tau_T - G_T)]$$

where real debt is defined as, $b_t \equiv \frac{B_t}{P_{t-1}}$ and its initial steady-state is given by,

$$\overline{b} = \frac{\overline{w}\overline{N}\overline{\tau} - \overline{G}}{1-\beta}$$

[4] An alternative means of ensuring the steady-state was efficient with a positive stock of government debt, but without recourse to a lump sum tax would be to allow the policy maker to apply a time invariant distortionary tax to leisure - see, for example, Bilbiie et al (2008).

Log-linearising around this steady-state,

$$\begin{aligned}\widehat{b}_t - \pi_t - \sigma(\widehat{C}_t) &= \beta\widehat{b}_{t+1} - E_t\{\pi_{t+1} + \sigma\widehat{C}_{t+1}\} \\ &\quad +[-\sigma(1-\beta)(\widehat{C}_t) + \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}(\widehat{w}_t + \widehat{N}_t + \widehat{\tau}_t) - \frac{\overline{G}}{\overline{b}}\widehat{G}_t]\end{aligned} \tag{10}$$

Appendix 1 defines the steady-state ratios contained in this log-linearisation as a function of model parameters and the initial steady-state debt-GDP ratio.

3 Policy Makers' Objectives

In order to derive objective functions for policy analysis we proceed in the following manner. Firstly, we consider the social planner's problem. We then contrast this with the outcome under flexible prices in order to determine the level of the steady-state subsidy required to ensure the model's initial steady-state is socially optimal when the government implements a plan for government expenditure consistent with that chosen by the social planner. We then construct a quadratic approximation to utility in our sticky-price/distortionary tax economy which is driven by the extent to which endogenous variables differ from the efficient equilibrium due to the nominal inertia, tax distortions and electoral uncertainty present in the model. We contrast this social objective function with the objective function adopted by either policy maker when they represent their constituencies. This will imply that a given policy maker has a different target for each expenditure gap and that the weight attached to that target differs across policy makers. Finally, we recast our model in terms of the 'gap' variables contained within our social welfare metric.

3.1 The Social Planner's Problem

The social planner is not constrained by the price mechanism and simply maximises the average of the two households' utilities, (4), subject to the technology, (7), and resource constraints, (9). Therefore the social planner's objective function is given by,

$$\left(\frac{C_t^{1-\sigma}}{1-\sigma} + \chi^A\frac{(G_t^A)^{1-\sigma}}{1-\sigma} + \chi^B\frac{(G_t^B)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}\right) \tag{11}$$

where $\chi^A = \frac{1}{2}(\chi_i^A + \chi_j^A)$ and $\chi^B = \frac{1}{2}(\chi_i^B + \chi_j^B)$ are the average household preference weights attached to the two types of government spending basket.

This yields the following first order conditions,

$$\begin{aligned}(C_t^*)^{-\sigma} &= \frac{1}{2}(\chi_i^A + \chi_j^A)(G_t^{A*})^{-\sigma} \\ (C_t^*)^{-\sigma} &= \frac{1}{2}(\chi_i^B + \chi_j^B)(G_t^{B*})^{-\sigma}\end{aligned}$$

$$(C_t^*)^{-\sigma} - Y_t^{*\varphi}A_t^{-(1+\varphi)} = 0$$

where we introduce the '*' superscript to denote the efficient level of that variable. These can be log-linearised around the deterministic efficient steady-state, and given the national accounting identity we obtain,

$$\widehat{Y}_t^* = (\frac{1+\varphi}{\sigma+\varphi})a_t$$

and,

$$\widehat{Y}_t^* = \widehat{C}_t^* = \widehat{G}_t^* = \widehat{G}_t^{A*} = \widehat{G}_t^{B*}$$

3.2 Flexible Price Equilibrium

Appendix 1 derives the subsidy, $\varkappa$, required for the flexible price equilibrium to reproduce the efficient steady state. If any government implements its spending plans in line with the social planner's problem in steady-state then the flex price steady-state conditional on the initial fiscal position is the same as the efficient output level. Appendix 1 also defines the steady-state ratios contained in the log-linearised budget constraint, (10), as a function of model parameters and the initial steady-state debt-GDP ratio.

3.3 Social Welfare

Appendix 1(2) derives the quadratic approximation to utility

$$\begin{aligned}\Gamma &= -\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{\sigma\theta(\widehat{C}_t-\widehat{C}_t^*)^2+\sigma(1-\theta)\gamma(\widehat{G}_t^A-\widehat{G}_t^{A*})^2\\ &\quad +\sigma(1-\theta)(1-\gamma)(\widehat{G}_t^A-\widehat{G}_t^{A*})^2+\varphi(\widehat{Y}_t-\widehat{Y}_t^*)^2+\frac{\epsilon}{\lambda}\pi_t^2\}+tip+O[2]\end{aligned}$$

where *tip* refers to 'terms independent of policy' and $O[2]$ captures terms of order greater than 2. It contains quadratic terms in price inflation reflecting the costs of price dispersion induced by inflation in the presence of nominal inertia, as well as terms in the consumption, government spending and output gaps i.e. the difference between the actual value of the variable and its optimal value. The weights attached to each element are a function of deep model parameters.

However, we wish to consider how policy is affected by political parties having welfare functions which differ from the social optimum due to heterogeneity in preferences over government spending. Therefore, we need to define the equivalent welfare measures adopted by our political parties when they solely represent the interests of one of the two types of household present in our economy. The difference between social welfare and the objective function adopted by party i is given by,

$$\begin{aligned}\Gamma_i &= \Gamma+(\chi_i^A-\chi^A)(\overline{G}^A)^{1-\sigma}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{2\widehat{G}_t^A+(1-\sigma)(\widehat{G}_t^A)^2\}\\ &\quad +(\chi_i^B-\chi^B)(\overline{G}^B)^{1-\sigma}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{2\widehat{G}_t^B+(1-\sigma)(\widehat{G}_t^B)^2\}\end{aligned}$$

where $\chi^A = \frac{\chi_1^A + \chi_1^A}{2}$ and, $\chi^B = \frac{\chi_1^B + \chi_2^B}{2}$ are the average weights across the two household types. Using the steady-state relationship,

$$\begin{aligned}
\chi^A(\overline{G}^A)^{1-\sigma} &= \overline{N}^{1+\varphi}(1-\theta)\gamma \\
\chi^B(\overline{G}^B)^{1-\sigma} &= \overline{N}^{1+\varphi}(1-\theta)(1-\gamma)
\end{aligned}$$

this can be rewritten as,

$$\begin{aligned}
\Gamma_i &= -\frac{1}{2}\overline{N}^{1+\varphi}E_0\sum_{t=0}^{\infty}\beta^t\{\sigma\theta(\widehat{C}_t - \widehat{C}_t^*)^2 + \Omega_i^A(\widehat{G}_t^A - \widehat{G}_t^{A*} - \widehat{G}_t^{ATi})^2 \\
&\quad + \Omega_i^B(\widehat{G}_t^B - \widehat{G}_t^{B*} - \widehat{G}_t^{BTi})^2 + \varphi(\widehat{Y}_t - \widehat{Y}_t^*)^2 + \frac{\epsilon}{\gamma}\pi_t^2\} \qquad (12)
\end{aligned}$$

where $\Omega_i^A = (1-\theta)\gamma(\sigma - \frac{\chi_i^A - \chi^A}{\chi^A}(1-\sigma))$ and $\Omega_i^B = (1-\theta)(1-\gamma)(\sigma - \frac{\chi_i^B - \chi^B}{\chi^B}(1-\sigma))$ are the transformed weights on the government spending gaps for policy maker i and,

$$\begin{aligned}
\widehat{G}_t^{ATi} &= \frac{(\chi_i^A - \chi^A)}{\chi^A - \chi_i^A(1-\sigma)} + \frac{(\chi_i^A - \chi^A)(1-\sigma)}{\chi^A - \chi_i^A(1-\sigma)}\widehat{G}_t^{A*} \\
\widehat{G}_t^{BTi} &= \frac{(\chi_i^B - \chi^B)}{\chi^B - \chi_i^B(1-\sigma)} + \frac{(\chi_i^B - \chi^B)(1-\sigma)}{\chi^B - \chi_i^B(1-\sigma)}\widehat{G}_t^{B*}
\end{aligned}$$

are the policy maker specific targets. They reflect a preference driven constant desire to move away from the socially optimal level of government spending as well as an element reflecting the desire to alter that target in the face of shocks.

Since these party specific targets will drive the incentives facing governments to utilise debt strategically it is helpful to explore how different types of heterogeneity affect these targets. Assume that $\chi^A = \chi^B$ so that the social planner would choose the same level of spending of each basket of goods, implying $\gamma = 1/2$, and that these average taste parameters are held constant. Consider the constant elements first, which capture the desire to deviate from the socially optimal government spending plan in the absence of shocks. If we introduce a symmetrical composition heterogeneity in party tastes then $\chi_1^A - \chi^A = \chi^A - \chi_2^A > 0$ and $\chi_2^B - \chi^B = \chi^B - \chi_1^B > 0$. That is party 1 prefers basket A to basket B and party 2 prefers the opposite. This implies the following inequalities,

$$\begin{aligned}
\frac{(\chi_i^A - \chi^A)}{\chi^A - \chi_i^A(1-\sigma)} &< (>) - \frac{(\chi_i^B - \chi^B)}{\chi^B - \chi_i^B(1-\sigma)} \\
\text{when } \sigma &> (<)1,\ \chi_i^A - \chi^A > 0 \text{ and } \chi_i^B - \chi^B < 0
\end{aligned}$$

In other words, when $\sigma > 1$ each party wishes to cut spending on its least preferred basket by more than it wishes to increase spending on its preferred basket, while when $\sigma < 1$ the opposite is true. The σ parameter is identical to the concavity index, $-H_{GG}(G)/(H_G(G))^2$ for each government spending utility felicity, $H(G) = \frac{(G_t)^{1-\sigma}}{1-\sigma}$. A value of $\sigma > 1$ implies that the concavity of the felicity is increasing in G, such that the different government consumption baskets become greater substitutes as the overall level of spending is increased. Accordingly, parties implement more similar policies when resources are scarce, and indulge in political conflict when resources are abundant. Therefore, when $\sigma > 1$ we find a general desire to reduce government spending and achieve a minimal provision of both types of government spending. This drives the deficit bias we shall observe below. Persson and Tabellini (2000) find that a similar condition emerges in the analysis of a variety of political conflicts.

Similarly when we consider a size heterogeneity such that $\chi_1^A - \chi^A = \chi^A - \chi_2^A > 0$ and $\chi_1^B - \chi^B = \chi^B - \chi_2^B > 0$, then the following results hold,

$$\frac{(\chi_1^C - \chi^C)}{\chi^C - \chi_1^C(1-\sigma)} < (>) - \frac{(\chi_2^C - \chi^C)}{\chi^C - \chi_2^C(1-\sigma)} \text{ when } \sigma > (<)1 \text{ where } C = A, B$$

This implies that the party preferring 'small' government wishes to cut spending by more than the 'large' government party wishes to increase it whenever $\sigma > 1$.

Finally, the desired response to shocks is also affected by any heterogeneity, such that,

$$\frac{(\chi_i^C - \chi^C)(1-\sigma)}{\chi^C - \chi_i^C(1-\sigma)} < (>)0 \text{ when } \sigma > 1,\ \chi_i^C > (<)\chi^C \text{ and } C = A, B$$

and, when $\sigma > 1$, parties utilise their least preferred instruments to respond to shocks.

It is helpful to rewrite the per-period loss function for party i in the following form before computing the time consistent solution to the policy problem,

$$\begin{aligned} & \sigma\theta(\widehat{C}_t^i - \widehat{C}_t^*)^2 + \Omega_i^A(\widehat{G}_t^{Ai} - \widehat{G}_t^{A*} - \widehat{G}_t^{ATi})^2 + \Omega_i^B(\widehat{G}_t^{Bi} - \widehat{G}_t^{B*} - \widehat{G}_t^{BTi})^2 \\ & +\varphi(\widehat{Y}_t^i - \widehat{Y}_t^*)^2 + \frac{\epsilon}{\gamma}(\pi_t^i)^2 + tip \\ = \; & \boldsymbol{\pi}_t^i \mathbf{R} \boldsymbol{\pi}_t^i + (\mathbf{u}_t^i - \mathbf{u}_t^{i*})' \mathbf{Q}^i (\mathbf{u}_t^i - \mathbf{u}_t^{i*}) + tip + O[2] \end{aligned}$$

$$\begin{aligned} \text{where } \mathbf{R} &= \left[\frac{\varepsilon}{\lambda}\right] \text{ and } \mathbf{Q}^i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & Z_i^C & Z_i^{CA} & Z_i^{CB} \\ 0 & 0 & Z_i^A & Z_i^{AB} \\ 0 & 0 & 0 & Z_i^B \end{bmatrix} \\ Z_i^C &= \sigma\theta + \varphi\theta^2 \\ Z_i^A &= \Omega_i^A + \varphi(1-\theta)^2\gamma^2 \\ Z_i^B &= \Omega_i^B + \varphi(1-\theta)^2(1-\gamma)^2 \\ Z_i^{CA} &= 2\varphi\theta(1-\theta)\gamma \\ Z_i^{CB} &= 2\varphi\theta(1-\theta)(1-\gamma) \\ Z_i^{AB} &= 2\varphi(1-\theta)^2\gamma(1-\gamma) \end{aligned}$$

and the vector of controls and targets are given by,

$$\mathbf{u}_t^i = \begin{bmatrix} \widehat{\tau}_t^i - \widehat{\tau}_t^* \\ \widehat{C}_t^i - \widehat{C}_t^* \\ \widehat{G}_t^{Ai} - \widehat{G}_t^{A*} \\ \widehat{G}_t^{Bi} - \widehat{G}_t^{B*} \end{bmatrix} \text{ and } \mathbf{u}_t^{i*} = \overline{\mathbf{u}}^{i*} + \mathbf{K1}^i \mathbf{S}_{t-1} + \mathbf{K3}^i \xi_t$$

where $\overline{\mathbf{u}}^{i*}$, $\mathbf{K1}^i$ and $\mathbf{K3}^i$ are defined in Appendix 2(1).

3.4 Gap variables

We have derived welfare based on various gaps, so we now proceed to rewrite our model in terms of the same gap variables to facilitate derivation of optimal policy. The consumption Euler equation can be written in gap form as,

$$(\widehat{C}_t - \widehat{C}_t^*) = E_t\{(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*)\} - \frac{1}{\sigma}((r_t - r_t^*) - E_t\{\pi_{t+1}\})$$

where $r_t^* = \sigma \frac{1+\varphi}{\sigma+\varphi}(E_t\{a_{t+1}\} - a_t)$ is the natural/efficient rate of interest. (This comes from the fact that $\widehat{C}_t^* = \widehat{Y}_t^*$ and the definition of the efficient level of output).

While the NKPC can be written in gap form as,

$$\pi_t = \beta E_t \pi_{t+1} + \lambda(\varphi(\widehat{Y}_t - \widehat{Y}_t^*) + \sigma(\widehat{C}_t - \widehat{C}_t^*) + \frac{\overline{\tau}}{1-\overline{\tau}}(\widehat{\tau}_t - \widehat{\tau}_t^*))$$

where, following Benigno and Woodford (2003) we define, $\frac{\overline{\tau}}{1-\overline{\tau}}\ \widehat{\tau}_t^* = \widehat{\mu}_t$. In other words we are defining our 'efficient' tax rate as the tax rate required to perfectly offset the impact of a cost-push shock.[5] If we had access to a lump-sum tax to finance the budget deficit then this would be the optimal tax rate. However, given the need to finance the government liabilities through distortionary taxation, actual tax rates are likely to deviate from the level required to perfectly offset shocks. Appendix 1 rewrites the budget constraint in gap form as,

$$\widehat{b}_t - \pi_t - \sigma(\widehat{C}_t - \widehat{C}_t^*) = \beta \widehat{b}_{t+1} - \beta E_t\{\pi_{t+1} + \sigma(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*)\} + ps_t - f_t - \sigma(1-\beta)(\widehat{C}_t - \widehat{C}_t^*)$$

with the primary surplus defined as,

$$ps_t = \frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}[(1+\varphi)(\widehat{Y}_t - \widehat{Y}_t^*) + \frac{1}{1-\overline{\tau}}(\widehat{\tau}_t - \widehat{\tau}_t^*) + \sigma(\widehat{C}_t - \widehat{C}_t^*)] - \frac{\overline{G}}{\overline{b}}(\widehat{G}_t - \widehat{G}_t^*) \quad (13)$$

and

$$f_t = -(\sigma(1-\beta\rho_a) + (1-\sigma)(1-\beta))\frac{(1+\varphi)}{\sigma+\varphi}a_t - \frac{\overline{w}\overline{N}}{\overline{b}}\widehat{\mu}_t$$

capturing the extent to which the various shocks hitting our model have fiscal consequences. Also recall the composition of government spending,

$$\widehat{G}_t - \widehat{G}_t^* = \gamma(\widehat{G}_t^A - \widehat{G}_t^{A*}) + (1-\gamma)(\widehat{G}_t^B - \widehat{G}_t^{B*})$$

[5] It should be noted that we could define the tax 'gap' as being the actual tax rate relative to any benchmark tax rate we choose, such as, for example, the initial steady-state tax rate. However, it is convenient to define the gap relative to the tax rate which offsets the impact of a cost-push shock on inflation.

4 The Electoral Game and Time Consistent Policy

We now examine the time-consistent solution to the policy problem under electoral uncertainty. We focus on the time-consistent solution as the existence of endogenous, but stochastic, switches in the identity of the governing party are assumed to make it difficult for individual policy makers to credibly commit to future policy actions. We further assume that elections occur after a random interval of time, such that the probability of observing an election in a given time period is a constant, e. This is different from most of the literature which assumes that there is an election in every period. There are several reasons why we allow for a random election probability. The first is that we have developed a sticky-price business cycle model where the natural interpretation of a time period is one quarter year, and it is clearly unrealistic to assume that elections occur at this frequency. Secondly, adopting an election probability rather than assuming fixed term elections is more tractable. Under the assumption that there is a constant probability of facing an election, economic agents forecasts of future economic policies will be conditional on who happens to be the incumbent. If we were to adopt a fixed term election structure, economic agents' forecasts of the future would not only be conditional on who was incumbent, but also on how many periods we were from the next election.

Accordingly, we can define the probability of party i being in power in the next period as follows,

$$q(i \mid j)_t = eq(i)_t \text{ for } i \neq j,\ i,j = 1,2$$
$$q(i \mid i)_t = (1-e) + eq(i)_t \text{ for } i = 1,2$$

where $q(i \mid j)_t$ reflects the probability of party i obtaining power, given that party j is the current incumbent, and $q(i \mid i)_t$ gives the probability of party i obtaining power given that i is incumbent. Since there is not an election every period, there is a clear advantage from being in power. $q(i)_t$ then captures the probability that, given an election has been called in period t, party i wins that election. $q(j)_t = 1-q(i)_t$ is the complementary probability that party j wins the election. These election victory probabilities shall be endogenously determined through a process of probabilistic voting which shall be outlined below. Before detailing the probabilistic voting mechanism it is helpful to outline the policy problem facing policy maker i.

The policy problem is fully described in Appendix 2, however in order to outline the endogenous voting mechanism we can characterise policymaker i's policy problem as follows,

$$V^i(\mathbf{S}_{t-1};\xi_t) = \underset{\mathbf{u}_t^i}{Min}(\boldsymbol{\pi}_t^i \mathbf{R} \boldsymbol{\pi}_t^i + (\mathbf{u}_t^i - \mathbf{u}_t^{i*})' \mathbf{Q}^i (\mathbf{u}_t^i - \mathbf{u}_t^{i*}) + \beta E_t C^i(\mathbf{S}_t^i; \boldsymbol{\xi}_{t+1})) \quad (14)$$

subject to,

$$\boldsymbol{\pi}_t^i = \mathbf{C0}^i + \mathbf{C1}^i \mathbf{S}_{t-1} + \mathbf{C2}^i \mathbf{u}_t^i + \mathbf{C3}^i \boldsymbol{\xi}_t$$

$$\mathbf{S}_t^i = \mathbf{D0}^i + \mathbf{D1}^i\mathbf{S}_{t-1} + \mathbf{D2}^i\mathbf{u}_t^i + \mathbf{D3}^i\boldsymbol{\xi}_t$$

where $V^i(\mathbf{S}_{t-1};\xi_t)$ is the value function for policy maker i, $E_tC^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1}))$ is the expected discounted value of future payoffs from the continuation game facing player i, $\mathbf{CJ}^i$ and $\mathbf{DJ}^i$,with $J = 1, 2, 3$, are the coefficient matrices defined in Appendix 2 after exploiting the linear-quadratic form of the problem to eliminate expectations. The i superscript denotes who is the incumbent. Aside from affecting the choice of control variables, $\mathbf{u}_t^i$, the coefficient matrices are also indexed by i since who is incumbent affects economic agents' forecasts of the future and these forecasts are embedded in the coefficient matrices as described in Appendix 2. $\mathbf{S}_{t-1} = \begin{bmatrix} \widehat{b}_t \\ a_{t-1} \\ \mu_{t-1} \end{bmatrix}$, $\mathbf{u}_t^i = \begin{bmatrix} \widehat{\tau}_t^i - \widehat{\tau}_t^* \\ \widehat{C}_t^i - \widehat{C}_t^* \\ \widehat{G}_t^{Ai} - \widehat{G}_t^{A*} \\ \widehat{G}_t^{Bi} - \widehat{G}_t^{B*} \end{bmatrix}$ and $\mathbf{u}_t^{i*} = \overline{\mathbf{u}}^{i*} + \mathbf{K}^i\mathbf{S}_t$ $= \overline{\mathbf{u}}^{i*} + \mathbf{K1}^i\mathbf{S}_{t-1} + \mathbf{K3}^i\boldsymbol{\xi}_t$ are the vectors of state, control and policy maker specific target variables respectively, while $\boldsymbol{\xi}_t$ is a vector of iid innovations to the model's shock processes.

The value of the continuation game depends on whether or not party i is re-elected in the next period,

$$E_tC^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1}) = q(i \mid i)_{t+1}E_tV^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1}) + q(j \mid i)_{t+1}E_tW^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})$$

where the expected pay-offs when party i is out of power are given by,

$$W^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) = \boldsymbol{\pi}_t^j\mathbf{R}\boldsymbol{\pi}_t^j + (\mathbf{u}_t^j - \mathbf{u}_t^{i*})'\mathbf{Q}^i(\mathbf{u}_t^j - \mathbf{u}_t^{i*}) + \beta E_tC^i(\mathbf{S}_t^j;\boldsymbol{\xi}_{t+1})$$

Note that the latter expression computes the flow benefits to party i given the policies implemented by party j, and values the continuation game conditional on the level of debt left by party j highlighting the scope for using debt strategically. Further strategic considerations are introduced by endogenising the election victory probability, $q(i)_t$.

4.1 Voting Behaviour

Parties are assumed to pursue the economic interests of the households they represent. That is party i possesses an economic objective function which reflects the economic preferences of household i. However, in order to introduce probabilistic voting behaviour, we also assume that individual members of an particular household also care about other non-economic factors, such that for individual k of household i they receive a reduction in losses of $(\sigma^{ik} + \delta)$ when party 1 is elected, and zero otherwise. σ^{ik} is a zero mean random variable uniformly distributed with density Ψ^i, $i = 1, 2$. δ is also uniformly distributed with mean 0 and density Λ.

The timing of events is repetitive and can be summarised as in Figure 1, beginning from the point at which economic agents form their expectations of the next period, t, conditional on knowing who is incumbent and the policies they have implemented in period t-1. We then enter period t and there is a draw from the distribution which determines whether or not there will be an election. Upon that signal being positive, with probability e, there is a draw of the voter preference shocks which affects the outcome of the probabilistic voting, namely the general preference towards party i across all voters, δ, and the individual specific preferences towards party 1 of voters of households 1 and 2, respectively, σ^{ik} . These preference parameters remain in place until there is another signal to hold an election. Accordingly, $\frac{(\sigma^{ik}+\delta)}{1-\beta(1-e)}$captures the expected discounted non-economic benefit to voter k of household i of party 1 winning the election. Since these variables are redrawn at each election, their expected value prior to an election is zero. Therefore, even if a particular draw of voter preference shocks has meant that voters were very content to see party i elected, this will not affect future election outcomes and will not affect incumbent behaviour.

Voters then decide how to vote, comparing the expected economic and non-economic benefits of electing either party, such that voter k of household 1 will vote for party 1 whenever,

$$E_{t-1}V^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) - \frac{(\sigma^{1k}+\delta)}{1-\beta(1-e)} < E_{t-1}W^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$$

In other words the voter compares the expected discounted losses associated with party 1 being in power, $E_{t-1}V^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$, after adjusting for the discounted idiosyncratic, σ^{1k}, and general, δ, non-economic benefits of party 1 being in power, with the losses he would experience if party 1 was out of power, $E_{t-1}W^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$. Since, the voting takes place prior to the realisation of the innovations to the economic shock processes in period t, the expected value of these losses is based on information available at the end of period t-1.[6]

Given these trade-offs facing voters, the swing voter of household 1 is defined as,

$$\sigma^1 = (1-\beta(1-e))\left(E_{t-1}V^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) - E_{t-1}W^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right) - \delta$$

While the swing voter of household 2 is given by,

$$\sigma^2 = (1-\beta(1-e))\left(E_{t-1}W^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) - E_{t-1}V^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right) - \delta$$

As a result party 1's vote share is given by,

$$v^1 = \frac{1}{2}\Psi^1(\frac{1}{2\Psi^1} - \sigma^1) + \frac{1}{2}\Psi^2(\frac{1}{2\Psi^2} - \sigma^2)$$

The first element captures the votes from household 1 and the second from household 2. Note that since δ is a random variable σ^1 and σ^2 are also random.

[6] An alternative timing assumption would be to allow voters to observe period t's economic shocks prior to voting. This would not affect the strategic behaviour of the policy maker in the previous period, t-1, since he expects these shocks to be, on average, zero. However, it would add additional volatility to the electoral outcome as economic shocks affect voter choices.

Party 1's probability of winning becomes,

$$
\begin{aligned}
q(1)_t &= \Pr[\frac{1}{2}\Psi^1(\frac{1}{2\Psi^1}-\sigma^1)+\frac{1}{2}\Psi^2(\frac{1}{2\Psi^2}-\sigma^2)\geqslant\frac{1}{2}] \\
&= \Pr[\frac{1-\beta(1-e)}{\Psi^1+\Psi^2}\left(\begin{array}{c} -\frac{1}{2}\Psi^1\left(E_{t-1}V^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)-E_{t-1}W^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right) \\ -\frac{1}{2}\Psi^2\left(E_{t-1}W^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)-E_{t-1}V^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right)\end{array}\right)\geqslant\delta] \\
&= \frac{1}{2}-z^1\left(E_{t-1}V^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)-E_{t-1}W^1(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right) \\
&\quad +z^2\left(E_{t-1}V^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)-E_{t-1}W^2(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)\right)
\end{aligned}
$$

where $z^i = \frac{\Lambda(1-\beta(1-e))}{\Psi^i+\Psi^j}\Psi^i$, $i = 1, 2$. Therefore the probability of winning the election depends upon the expected relative costs to voters of their natural party being out of power. As the expected losses of household i when party i is out of power, $E_{t-1}W^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$, rise relative to the losses they expect to experience when party i is in power, $E_{t-1}V^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t)$, fewer voters within household i will be tempted to switch allegiance to party j for a given draw of voter preference shocks. Furthermore, the weights attached to the different expected losses when a particular party is in or out of power, z_i , are dependent upon the relative densities of the voter preference shocks within each household, Ψ^i, $i = 1, 2$. As the density is increased voters are more homogeneous within each household. Therefore, when $z_i > z_j$, the voters within household i are more homogeneous than those within household j. This implies that there are fewer swing voters in household i and policy makers will find it relatively easy to tempt voters from household j to switch allegiance to party i creating an electoral bias towards party i, cet. par.

There are several differences between this set-up and that which would emerge in the absence of electoral uncertainty and heterogeneity in preferences across households. Firstly, the heterogeneity gives rise to targets for control variables which capture the desire to deviate from the socially optimal level of such variables. Secondly, the value of the continuation game must take account of the likely behaviour of one's political rival to the extent that he/she is likely to be elected. The possibility that one's rival is elected and their behaviour if elected are affected by the actions of the incumbent policy maker since the debt passed from one period to the next affects the trade-offs facing voters and constrains the actions of any subsequent policy-maker. Debt can therefore be used as a strategic tool to both affect the likely outcome of elections and tie the hands of future policy makers.

The solution proceeds by 'guessing' the form of the pay-offs when each party is in or out of power and using this to solve the policy problem conditional on these undetermined coefficients, as well as those involved in relating expectations to state-variables. Substituting this solution into the model and the Bellman equation allows us to solve for the undetermined coefficients and complete the description of policy for both parties. Full details of this procedure are given in Appendix 3.[7]

[7] For the special case of $e = 1$ and $z_1 = z_2 = 0$, it is possible to solve the policy problem

5 Numerical Results

In this section we explore the extent of biases introduced by heterogeneity in party preferences over the composition and size of government spending in the face of electoral uncertainty. We compare the welfare costs of these biases and election-induced fluctuations with those arising from more conventional shock processes (namely, technology and mark-up shocks).

5.1 Parameterisation

Following the econometric estimates in Leith and Malley (2005) we adopt the following parameter set, $\varphi = 1$, $\sigma = 2, \mu = 1.2$, $\overline{\epsilon} = 6$, $\beta = 0.99$. We assume $\chi^A = \chi^B = 4/9$ which implies, in line with Gali (1994), that the share of government consumption in GDP, $1 - \theta = 0.25$ and the optimal size of the two government spending baskets is the same, $\gamma = 1/2$. In our benchmark simulations we assume a degree of price stickiness of $\theta_p = 0.75$, which implies that an average contract length of one year, and an initial debt-GDP ratio of 60%. However, we also explore the implications of alternative assumptions regarding the degree of price stickiness below.

As noted above, our assumption that $\sigma > 1$ implies that the utility felicities for private and public consumption are increasingly concave in their arguments, and this is crucial in creating the incentives to move the economy towards a deficit bias. While this parameter can be difficult to estimate using single equation instrumental variable approaches (see, for example, Yogo(2004)), in the context of systems estimation of general equilibrium policy analysis models it tends to have an estimated value in excess of one (an intertemporal elasticity of substitution less than 1) since the impact of interest rate changes on output is too great otherwise. For example, our assumed value of 2 is in line with GMM-systems estimation of a New Keynesian description of the US and Euro-area economies in Leith and Malley (2005) and the Bayesian posterior distributions of Smets and Wouters (2005).

To compare the welfare costs of any electoral cycle and deficit bias to business cycle costs, we need to parameterise the productivity and mark-up shocks. The productivity shock follows the following pattern,

$$a_t = \rho_a a_{t-1} + \xi_t$$

where we adopt a degree of persistence in the productivity shock of $\rho_a = 0.99$ and a standard deviation of the productivity shock of $\sigma_\xi = 0.01$. This is similar to the productivity process in Ireland (2004), although he adopts a unit-root

analytically using the symbolic maths package Maple 10. The uniqueness of this solution implies that the policy problem does not suffer from the multiplicity of equilibrium found in Blake and Kirsanova (2008). Outside of this special case, the policy problem is solved using the non-linear numerical solution algorithms of Matlab. When applicable, both approaches yield the same results. Furthermore, solving the model without conflict using these approaches yields identical results to the discretionary solution of the same model using the Matlab code of Soderlind (1999).

process in technology which we render stationary as in Smets and Wouters (2005). We adopt an iid mark-up shock with a standard deviation of 0.0175 , where we have rescaled the persistent shock in Ireland (2004) to match the unconditional variance of the mark-up process.

We also need to consider plausible degrees of composition and size heterogeneity. The cross-country evidence on such effects is mixed - see, for example, the meta-analysis of Imbeau *et al.* (2001) of studies which explore the link between the party composition of government and policy outcomes. Therefore, in order to measure the possible magnitude of composition and size heterogeneities in a manner which would allow us to calibrate our model, we examined data from US federal government spending[8] broken down into spending components between 1970 and 2007, and scaled by GDP. We then quadratically detrended this data and regressed the residuals on a dummy based on the outcome of the presidential elections. The results are given in Table 1. Here there are negligible party effects on all categories except for Defense and Total Health, where Republicans typically raise defence spending by 0.785% of GDP relative to the level chosen by the Democrats, while, in the case of health spending, Democrats tend to spend 0.24% of GDP more than the Republicans.

Table 1 - Party Differences in Categories of Government Spending.

Category	Coefficient on Democrat Dummy	t-ratio
Defense	-0.7854	-3.3348*
Education	0.0522	1.0412
Total Health	0.2370	5.2125*
Transport	-0.0032544	-0.163
Other	-0.05841	-0.98071

While these results suggest that there is both a composition and size component to heterogeneity in party preferences, given the wide range of conflicting empirical evidence on this issue it is difficult to reach a clear conclusion on the exact size of these effects. In light of this, we consider composition and size heterogeneity separately, with relatively modest measures of heterogeneity in both cases. We adopt the following parameters when consider the composition heterogeneity, $\chi_1^A = \chi_2^B = \frac{416}{900}$ and $\chi_2^A = \chi_1^B = \frac{384}{900}$. This implies that party 1 will drive spending on basket A 0.5% of GDP higher than party 2. Similarly, party 2 will attempt to drive spending on basket B 0.5% of GDP higher than party 1. While in the case of size heterogeneity we assume $\chi_1^A = \chi_1^B = \frac{422}{900}$ and $\chi_2^A = \chi_2^B = \frac{378}{900}$. This then implies that party 1 will drive spending on baskets A and B 0.5% of GDP higher than party 2.

We also set the weights in the election success probability equation equal to $z_1 = z_2 = 10$. This implies an election success probability of roughly 0.5 under both forms of political conflict, with a slight bias towards the 'big' government

[8] The data was taken from Table 3.1 of the Historical Tables of the US Budget for the Fiscal Year 2009 and refers to 'Outlays by Superfunction and Function 1940-2013', where the data beyond 2007 are estimates.

party when considering size heterogeneity. This is in line with the average relative popular vote shares of the Republican and Democrat candidates in US presidential elections where, between 1948 and 2008, the relative vote shares are 51% and 49% for the Republican and Democratic candidates, respectively. Finally, in our benchmark calibration, we assume that the probability of an election being called, e, of $e = 1/16$ which implies an expected electoral cycle of 4 years.

We also consider the sensitivity of our results to alternative degrees of price stickiness, voter homogeneity, frequency of elections and political polarisation.

5.2 The Deficit Bias and the Electoral Cycle

We begin this subsection by considering the nature of the electoral cycle when parties differ in their preferences over the size of government, but not the composition of government spending. We then look at the extent of deficit bias in a stochastic steady state. Both discussions are then repeated for the case of conflict over composition rather than size. The results are then summarised by looking at the welfare cost of political conflict. Finally we consider how political conflict alters the response of government to standard business cycle shocks.

In looking at conflict over the size of government, we consider the following pattern of preferences, $\chi_1^A = \chi_1^B = \frac{422}{900}$ and $\chi_2^A = \chi_2^B = \frac{378}{900}$, which implies that party 1 will drive spending on baskets A and B 0.5% of GDP higher than party 2. Figure 2 details the impulse responses of a hypothetical draw from the election and voter taste distributions, where the parties happen to alternate in power over a four year cycle. It should be noted that within these impulse responses economic agents and political parties are aware of the probabilities of an election being called and form expectations over the endogenous probability that a specific party will be elected within that election, as well as the nature of the political game described above. All economic variables are measured as a percentage deviation from the steady-state chosen by a benevolent social planner, such that any non-zero economic variables implies a cost due to the electoral cycle.

A key to understanding the nature of the electoral cycle revealed in Figure 2 is the potential time inconsistency concerning inflation and debt. The big government party wants permanently higher government spending, and this can be achieved either through permanently higher taxes, or through a reduction in the level of debt which lowers debt interest payments. Debt can be reduced through higher output (which implies an expansion in the tax base), higher inflation, lower interest rates or a temporary tax hike, but higher inflation is costly so there is a limit to how much debt can be reduced through this means. In addition, the need to follow a time-consistent policy means that policymakers implement policies which rapidly adjust the debt since slower adjustment would imply that the policy maker was ignoring incentives to introduce further policy surprises. This rapid adjustment of debt under an optimal discretionary policy, and indeed the small overshooting observed in Figure 2, is analysed in detail in Leith and Wren-Lewis (2007) for a benevolent policy maker.[9]

We therefore see a sharp increase in output and inflation immediately the big government party is elected, but these increases last only for a single period. Inflation rises partly through an easing of monetary policy (lower interest rates also help to reduce debt directly), and partly because of higher tax rates which raise marginal costs. Tax rates fall back after the first period the party gains power, but to a higher level than under the small government administration. We consider the welfare costs of these fluctuations in macroeconomic aggregates below.

To illustrate the role of price rigidity in generating this cycle, Figure 3 repeats exactly the same experiment, but where the degree of price stickiness is significantly lower, with the probability of no-price change within a quarter given by $\theta_p = 0.05$, relative to the benchmark, $\theta_p = 0.75$. With more flexible prices political parties use surprise inflation to a greater degree to achieve their desired debt levels. The behaviour of taxes illustrate that temporary changes in taxes and inflation are close substitutes as means of changing debt levels, so that when costs of generating inflation are much lower, tax rates actually move in the opposite direction. In this relatively more flexible economy, the fluctuations in private consumption, output and the endogenous election victory probability are also lower over the electoral cycle.

We now turn from the electoral cycle to the issue of deficit bias, by considering the stochastic steady state of the economy (i.e. the level around which our economy fluctuates in response to economic and electoral shocks). In the real economy models in, for example, Alesina and Tabellini (1990) and Persson and Svensson (1989), deficit biases can arise because there is an incentive for the incumbent party to increase debt to tie the hands of the other party should they later win power. With nominal debt and costless inflation debt cannot be used in this strategic manner, but with Calvo contracts inflation becomes costly and the potential strategic use of debt re-emerges. In Figure 4 we compare the level of debt in the stochastic steady state with the level chosen by a benevolent social planner at different degrees of price stickiness. Figure 4 also plots the steady-states around which each party would fluctuate if they were lucky enough to remain in power indefinitely, but expected to win and lose elections in line with the endogenous probabilities generated by the model.

For most economic variables and levels of price stickiness, the stochastic steady state is close to the steady state chosen by a benevolent social planner. This implies that the magnitude of the overall debt bias is small in the case of conflict over the size of government spending. One interesting result is that when prices are relatively flexible, the steady state rate of inflation is mildly negative. This non-zero rate of inflation in steady-state reflects an inflationary bias driven by the desire of a party to manipulate public debt through surprise infla-

[9] Leith and Wren-Lewis (2007) find that the rapid stabilisation of debt under discretion occurs for plausible debt/gdp ratios and degrees of price stickiness. It is only when debt/gdp ratios are very low (<10%) that time-consistent policy implies gradual debt adjustment - these results are available upon request.

tion. In equilibrium each party's policies must be time-consistent and economic agents' expectations must be fulfilled ex ante (since there are random changes in governing party, ex post expectations will not be fulfilled unless one party is expected to be re-elected with a probability of 1 and there are no economic shocks). At low levels of price stickiness the big government party 1 would face a positive time-consistent rate of inflation as at lower rates of inflation they would wish to further inflate away any debt they inherit, while for the small government party 2 the opposite is true.[10] Since the small government party's desire to cut spending is greater than the big government's desire to increase it the aggregate inflationary bias is negative. As prices become more sticky, parties become increasingly reluctant to use policy surprises to influence debt and variations in tax rates become increasingly important in influencing debt.

We now turn to consider the case where there is heterogeneity in party preferences over the composition of government spending, and not the size of government. For our benchmark calibration, where each parties' voters are equally homogeneous, the two parties are otherwise symmetrical, such that party 1's voters typically dislike party 2's policies to the same degree as party 2's voters dislike party 1's policies. The solid line in Figure 5 details the response of key variables to the same hypothetical electoral cycle considered in Figure 2. While there is a clear fluctuation in spending over the two government consumption baskets as parties alternate in power, this does not induce any fluctuations in aggregate demand, as each basket is comprised of the same goods. Nevertheless, there is deficit bias with suboptimally high levels of government debt, and a negative rate of inflation for reasons discussed above. The higher level of debt serves to achieve each party's objective of reducing spending on their least preferred basket by more than it increases spending on their most preferred basket, which allows private consumption to be crowded in, although taxes rise and output falls overall. However the size of the deficit bias generated is still quite small.

The green dashed line in Figure 5 considers the same electoral cycle but where the voters of party 1 are more homogeneous than those of party 2 ($z_1 = 15$ and $z_2 = 5$) so that party 1 enjoys a significantly higher probability of winning any election, ceteris paribus, since party 2's natural voters can be tempted to switch to party 1. Introducing this heterogeneity implies that the economy begins to be subject to a clearer electoral cycle with fluctuations in response to changes in the governing party. In the asymmetric case the election success probability is biased towards party 1, which means that party 2 is more interested in disciplining party 1 than vice versa. As a result, party 2 issues more debt than party 1 and we now observe fluctuations in aggregate variables. Despite these

[10] Since expectations are a weighted average of what potential political parties would do in power, where the weights reflect the endogenous probability of each party being in power in the next period, there will perpetually be ex post policy surprises even if an incumbent is re-elected. Since the incumbent has an in-built electoral advantage (the details of which are discussed below), expectations of future policy will tend to be dominated by their expected actions. Accordingly, when the governing party changes they can achieve large inflation/deflation surprises relative to expectations.

fluctuations, the average debt level is lower than under the symmetric case.

Figure 6 repeats Figure 4 for the case of heterogeneity in party preferences over the composition rather than the size of government spending, where the voters of each party are equally homogeneous. For aggregate variables the stochastic steady state coincides with the steady state chosen by each party if they remained in power, as party preferences only differ over the composition of spending. Figure 6 clearly shows how the steady-state debt stock rises as the degree of price stickiness increases, capturing the fact that issuing debt becomes a more effective means of tying the hands of one's opponent when the use of surprise inflation to manipulate debt is less desirable. However, as with the case involving conflict over government size, the magnitude of the deficit bias remains small, even when prices are very inflexible.

Our results so far indicate that conflict over the size of government produces a noticeable electoral cycle. Conflict over either size or composition leads to deficit bias, but the scale of this bias is not large. Table 2 examines the welfare cost of either form of electoral conflict. Under our benchmark calibration the costs of the size and composition heterogeneities amount to 2% and 1% of steady-state consumption respectively. The large size of these welfare costs have an immediate implication, which is that voters will tend to prefer incumbent parties because changes in government are costly. This impact on electoral probabilities (the q(i) variable) can be clearly observed in Figures 2 and 5. The importance of this effect is examined in the next subsection.

Table 2 - Welfare Costs of Deficit Biases, Electoral Cycles and Economic Shocks[11]

		Economic Shocks	No Economic Shocks
No Conflict, $\theta_p = 0.75$		0.092%	0%
$e = 1$, Exogenous $q(i) = 1/2$	Composition Heterogeneity	1.103%	1.001%
$\theta_p = 0.75$	Size Heterogeneity	1.839%	1.737%
$e = 1/16$, Exogenous $q(i) = 1/2$	Composition Heterogeneity	1.100%	1.001%
$\theta_p = 0.75$	Size Heterogeneity	2.151%	2.055%
$e = 1/16$, Endogenous $q(1)$:	Composition Heterogeneity	1.100%	1.001%
$z_1 = z_2 = 10$, $\theta_p = 0.75$	Size Heterogeneity	2.154%	2.056%
$e = 1/16$, Endogenous $q(1)$:	Composition Heterogeneity		1.001%
$z_1 = z_2 = 10$, $\theta_p = 0.05$	Size Heterogeneity		1.866%

In the case of composition heterogeneity, virtually all the welfare cost of the electoral cycle derives from the direct costs of the fluctuations in government

[11] Welfare is expressed as a percentage of steady-state consumption.

spending rather than the costs of the deficit bias and its attendant deflation bias. In contrast, when the conflict is over the size of government, the direct costs of the fluctuations in government spending amount to around 1.5% of steady-state consumption. The remaining 0.5% arises mainly from the electoral cycle, with a small contribution from the deficit bias itself. If we reduce the probability of no-price change within a quarter from $\theta_p = 0.75$ to $\theta_p = 0.05$ then the total costs of conflict over government size falls from 2.06% to 1.87% of steady-state consumption, even though the direct costs of fluctuations in government spending over the electoral cycle are unaffected. We can therefore conclude that sticky prices are responsible for a significant proportion of the indirect costs of the electoral cycle in the presence of conflict over the size of government.

How do these costs relate to the business cycle costs more frequently analysed in the context of New Keynesian models? The welfare costs of our technology and mark-up shocks are equivalent to 0.09% of steady-state consumption in the absence of political conflict, rising to 0.1% under composition and size heterogeneity. (The reason for this small difference is discussed below.) We can therefore conclude that the welfare costs of movements in output and inflation induced by conflict over the size of government are at least as large as the cost of typical business cycle shocks in New Keynesian models.[12]

Does political conflict also alter how governments respond to business cycle shocks? This is examined in Figures 7 and 8. Figure 7 gives the impulse response to a 1% technology shock with party 1 and party 2 in power, respectively, where they differ in their preferences over the composition of government spending. The figure reveals that party 1 does move spending on basket A by more in its initial response to the shock. However, the size of the effect is small. The response in the absence of political conflict is not very different. Figure 8 does the same, but with a size heterogeneity. In this case, there are some differences across parties in terms of the impulse response of all variables, and this accounts for the difference in the welfare cost of these shocks with and without political conflict noted above. However, this difference itself, and the differences in responses shown in Figures 7 and 8, are not large. This suggests one important final conclusion, which is that analysis of the optimal policy response to business cycles can be effectively separated from the analysis of electoral cycles.

5.3 Electoral Uncertainty

In this section we explore the impact of changing the parameterisation of the electoral elements of our model. We first consider the importance of allow-

[12] Canzoneri et al (2008) evaluate the costs of business cycles in sticky wage/price New Keynesian models under different interest rate rules. Our welfare costs of economic shocks are comparable to theirs once it is noted that in our economy wages are flexible, policy makers have to ensure fiscal solvency and cannot commit future policy, but subject to those constraints, are effectively optimally responding to such shocks and have access to fiscal instruments as well as monetary policy instruments.

ing election probabilities to be endogenous. We then consider the impact of changing the homogeneity of voters naturally voting for a particular party. A subsequent discussion looks at the impact of varying the probability of an election being called. Finally, we consider the impact of increasing the degree of political polarisation. None of this analysis overturns the key conclusions from the previous section, but does highlight key aspects of the electoral game.

In Figure 2, which examined heterogeneity in party preferences over the size of government spending, election probabilities were endogenous. Figure 9 repeats the experiment from Figure 2, and compares the outcome of an endogenous election victory probability (solid line) with an exogenous probability equal to $q(1) = 0.5$ (dashed line). Changes in the endogenous election victory probability have relatively little impact on the macroeconomic outcomes of our hypothetical electoral cycle. In other words, political parties make little effort to adjust their policies to achieve additional electoral success. One possible reason for this is that there is only a relatively small chance that there will be an election in a given period. Extending the model to allow for fixed term elections might encourage parties to maximise their probabilities of electoral success when an election is imminent. Nevertheless, the effect of plausible variations in the electoral success probability on debt are relatively small, possibly explaining the empirical results of Lambertini (2009) who fails to find that opinion poll data Granger causes fiscal deficits.

In Figure 10 we explore how varying the homogeneity of voters naturally voting for a particular party affects the nature of the stochastic steady-state of our model when parties disagree about the appropriate size of government spending. Rather than plot the voter density parameters against steady-state outcomes, we plot outcomes against the endogenous election victory probabilities that emerge from varying these parameters. If we increase the probability of party 1's victory steady-state debt is reduced. Party 1 desires large government which requires additional resources, and as we showed in the previous section lower debt service costs allow the party to increase government spending. It is interesting to note that this surplus doesn't emerge at exactly $q(1) = 1/2$ as party 2's desire to reduce spending is greater than party 1's desire to increase it.

Figure 10 also plots the steady-state around which each party would fluctuate, if they were lucky enough to remain in power. The big government party would support increased debt for electoral probabilities $q(1) < 0.45$. This party never wants to issue as much debt as the other, but does not attempt to completely reverse their policies. There are two constraints on the big government party preventing them achieving the lower level of debt they would choose if permanently in power - firstly, their current policy choices will depend upon inflation and consumption expectations which partly depend upon the expected actions of their opponents and, secondly, in a sticky price environment, any attempt by their opponent to undo the strategic use of debt will give rise to aggregate fluctuations which are costly to the incumbent. This explains why the fluctuations in debt across the parties are small relative to the overall deficit/surplus bias. The steady-state rate of inflation is slightly positive when the small government/large debt party is more likely to be in power. This reflects the fact that the weaker party has the greatest desire to undo the discipline imposed on them so that their desire to inflate away the debt they would inherit if they were lucky enough to win an election dominates expectations.

We now consider the impact of varying the probability of an election being called, e. Intuitively, as we increase this probability the political system could be viewed as being less stable as governments are more prone to dissolution. Figure 11 contrasts our hypothetical 4 year average electoral cycle ($e = 1/16$) to one in which elections are held every period ($e = 1$). With a more frequent electoral cycle, the small government party still seeks to increase debt by tightening monetary policy and cutting taxes, but the magnitude of fluctuations are significantly smaller. Additionally, fluctuations in the endogenous election victory probability are effectively eliminated and there ceases to be an advantage to being an incumbent. This is because, with elections being held each period, expectations are not weighted towards the expected policies of the incumbent rather than the opposition, meaning that voters do not suffer relatively costly policy surprises if they move their votes away from the incumbent. Accordingly, less stable electoral systems which induce frequent elections also imply smaller (but more frequent) electoral cycle fluctuations.

We also consider the impact of variations in the probability that an election is called on the stochastic steady state of the model in Figure 12. Here, the more frequent the election the more similar the debt levels around which each party would fluctuate when in power. Interestingly, it is only at relatively low probabilities of an election being called that the small government/large debt party faces a negative steady-state rate of inflation (capturing their desire to further increase debt through surprise deflations). As elections become more frequent, agents expectations attach a greater weight to the policies likely to be pursued by the party currently in opposition, such that the steady-state positive inflation which the small government/large debt party would fluctuate around still implies an ex post surprise deflation since the opposition was expected to inflate by more prior to the outcome of any electoral competition being known. As noted above the election victory advantage of the incumbent is eliminated quite quickly as the frequency of elections is increased and voters are less concerned about electorally induced fluctuations.

We can examine a similar sensitivity analysis when the source of the political conflict is heterogeneity in party preferences over the composition of govt spending. As we vary the relative homogeneity of each party's natural constituency the endogenous probability of election victory varies. We plot the stochastic steady-state of the model against that endogenous probability in Figure 13. Here the deficit bias (and the attendant deflationary bias) is at a maximum when $q(1) = 1/2$ since this maximises electoral turnover. As either party's probability of electoral success increases, the need to tie the hands of the other party falls. Figure 13 also plots the steady-state levels of variables around which each party would fluctuate if they were lucky enough to remain in power. These are calculated by imagining a party i being permanently in power, but expecting to lose/regain power with probability $q(i)$ should an election be called. The figure reveals that debt is slightly higher for the low electoral success party, which also tends to pursue lower consumption and inflation as policy is tightened to raise debt. However, the effect of very large variations in the electoral success probability on debt is relatively small, consistent with the empirical results of Lambertini (2009) noted above.

In Figure 14 we consider the same analysis for variations in the probability of an election being called, e. Unlike the case of heterogeneity in party preferences over the size of government spending, heterogeneity in preferences over the composition of government spending does not imply differences in the extent of policy surprises whenever an incumbent loses an election, since each party is expected to deliver the same macroeconomic outcomes under our benchmark calibration. As a result, increasingly frequent elections mean that any policymaker expects to be replaced by its opponent more quickly, and is therefore more interested in disciplining that opponent by leaving a higher level of government debt. As a result we see the deficit and deflationary biases increasing in the probability of an election being called.

How do these aspects of the political process affect the welfare costs of the electoral cycle? Table 2, above, also looks at the impact of endogenising the election success probability and changing the frequency of the electoral cycle on the welfare costs of the electoral cycle. In the case of conflict over the composition of government spending changing the frequency of elections has a negligible impact on welfare, while in the case of conflict over the size of government spending, the additional fluctuations one suffers from more frequent governing party turnover is costly. However, endogenising the election victory probability has no impact, under our benchmark calibration, on the welfare consequences of conflict over the composition of government spending and negligible impact when the political conflict is over the size of government spending.

Finally, we consider the impact of increasing the degree of political polarisation on the size of the distortions identified above. In the benchmark calibration the extent of heterogeneity across the political parties was quite muted. Despite that the welfare costs of the electoral cycle were large. In Figure 15 we assess how the steady-state of our economy is affected by increasing the extent of heterogeneity across the political parties over the size of government spending. As before, the figure also plots the steady-state around which each party would fluctuate if they were lucky enough to remain in power (although they do not expect to remain so). Here we can see that increasing the extent of size heterogeneity across parties increasingly induces a deficit bias, with an associated deflationary bias at which policy makers are no longer tempted to deflate to increase the debt inherited by their opponent. The figure also implies that there would be sizeable fluctuations in debt, taxation, consumption and government spending over the course of the electoral cycle.

Figure 16 considers increasingly levels of political polarisation over the composition of government spending. In the absence of an electoral advantage of one party over another, there are no economic fluctuations over the course of the electoral cycle, other than in the composition of government spending. However, as conflict over the composition is increased there is a sharp increase in the size of the deficit and deflation biases associated with political conflict.

6 Conclusions

In this paper we have combined two separate literatures: the New Keynesian analysis of monetary and fiscal policy which has typically assumed the existence of a single benevolent policy maker, and the New Political Economy analysis of political conflict over fiscal policy which has usually taken place in the context of real economies. In our model governments can use policy strategically to both tie the hands of their political opponents and influence the endogenous outcome of future elections. At the same time economic agents must form expectations as to how the respective policy makers are likely to behave in the future and attach probabilities to the likelihood of the respective parties gaining power. This takes place in an environment where prices are sticky and so manipulating inflation (and hence the real value of nominal debt) is costly.

Our first key result is that realistic political conflicts between parties over the size of government can generate significant electoral cycles in output and inflation. Nominal debt and sticky prices combined with the need for policy to be time consistent gives this electoral cycle a distinct character. The party of big government, when elected, induces through monetary policy and higher taxes an immediate but temporary increase in output and inflation. This increase in inflation, together with lower interest rates and higher taxes, produces a rapid reduction in the real stock of debt. Higher government spending is then financed by lower debt interest payments as well as higher taxes.

This electoral cycle generates large reductions in social welfare. Although most of this social cost is due to fluctuations in the size of government, a significant part is accounted for by the movements in output and inflation described above. We have shown that these macroeconomic costs are at least as large as the costs normally associated with business cycle shocks in this New Keynesian model. Because electors try and avoid such costs, this gives incumbent parties a significant electoral advantage.

Although government debt is (realistically) defined in nominal terms in our model, sticky prices mean that changes in inflation are costly, and this reopens the possibility that debt could be used strategically to tie the hands of successive governments. Our second key result is that although this effect occurs, its impact on both the stock of debt and social welfare is not large over a wide range of degrees of price rigidity. This is true if political conflict occurs over the size of government, or over its composition. This positive deficit bias is accompanied by a negative inflation bias, but again its size is small. This suggests that explanations for the substantial deficit bias seen in most OECD countries over the last few decades must look to other explanations besides the conflict over the size or composition of government spending examined here.

Our final important result is reassuring if unexciting. We find that an environment of political conflict has very little effect on the way governments respond to typical business cycle shocks. This suggests that conventional New

Keynesian analysis of business cycle stabilisation policy that assumes a single benevolent government also applies to more realistic settings involving heterogeneity in preferences over the size or composition of government spending.

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Appendix 1 - Deriving Policy Objectives

(1) Flexible Price Equilibrium

Profit-maximising behaviour implies that firms will operate at the point at which marginal costs equal marginal revenues,

$$\begin{aligned} -\ln(\mu_t) &= mc_t \\ \left(1-\frac{1}{\epsilon_t}\right) &= \frac{(1-\varkappa)}{(1-\tau_t)}(N_t^n)^{(\varphi)}A_t^{-1}(C_t^n)^{\sigma}\xi_t^N \end{aligned}$$

In the initial steady-state this reduces to,

$$\left(1-\frac{1}{\overline{\epsilon}}\right) = \frac{(1-\varkappa)}{(1-\overline{\tau})}(\overline{N}^n)^{\varphi}(\overline{C}^n)^{\sigma}$$

If the subsidy $\varkappa$ is given by

$$(1-\varkappa) = (1-\frac{1}{\overline{\epsilon}})(1-\overline{\tau})$$

then

$$(\overline{C}^n)^{-\sigma} = (\overline{N}^n)^{\varphi}$$

which is identical to the optimal level of employment in the efficient steady-state. Given the steady-state government spending rule,

$$\frac{\overline{G}}{\overline{Y}} = (1+(\chi^A)^{-\frac{1}{\sigma}}+(\chi^B)^{-\frac{1}{\sigma}})^{-1}$$

and,

$$\frac{\overline{G}^A}{\overline{G}} = \gamma = [\left(\frac{\chi^A}{\chi^B}\right)^{-\frac{1}{\sigma}}+1]^{-1}$$

the steady-state level of output is given by,

$$\overline{Y} = \overline{N} = (1+(\chi^A)^{\frac{1}{\sigma}}+(\chi^B)^{\frac{1}{\sigma}})^{\frac{\sigma}{\sigma+\varphi}}$$

and, if the subsidy is in place, then the steady-state real wage is given by,

$$\overline{w} = \frac{1}{1-\overline{\tau}}$$

The steady-state tax rate required to support a given debt to GDP ratio is given by,

$$\overline{\tau} = \frac{(1-\beta)\frac{\overline{B}}{\overline{Y}}+\frac{\overline{G}}{\overline{Y}}}{1+(1-\beta)\frac{\overline{B}}{\overline{Y}}+\frac{\overline{G}}{\overline{Y}}}$$

and the tax revenues relative to debt this implies are given by,

$$\frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}} = \frac{\frac{\overline{\tau}}{1-\overline{\tau}}}{\frac{\overline{B}}{\overline{Y}}}$$

This is enough to define all log-linearised relationships dependent on model parameters and the initial debt to GDP ratio.

(2) Derivation of Welfare

Average household utility in period t is

$$\left(\frac{C_t^{1-\sigma}}{1-\sigma} + \frac{1}{2}(\chi_i^A + \chi_j^A)\frac{(G_t^A)^{1-\sigma}}{1-\sigma} + \frac{1}{2}(\chi_i^B + \chi_j^B)\frac{(G_t^B)^{1-\sigma}}{1-\sigma} - \frac{N_t{}^{1+\varphi}\xi_t^N}{1+\varphi}\right)$$

Before considering the elements of the utility function we need to note the following general result relating to second order approximations,

$$\frac{Y_t - Y}{Y_t} = \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + O[2]$$

where $\widehat{Y}_t = \ln(\frac{Y_t}{Y})$, $O[2]$ represents terms that are of order higher than 2 in the bound on the amplitude of the relevant shocks. This will be used in various places in the derivation of welfare. Now consider the second order approximation to the first term,

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \overline{C}^{1-\sigma}\left(\frac{C_t - \overline{C}}{\overline{C}}\right) - \frac{\sigma}{2}\overline{C}^{1-\sigma}\left(\frac{C_t - \overline{C}}{\overline{C}}\right)^2 + tip + O[2]$$

where tip represents 'terms independent of policy'. Using the results above this can be rewritten in terms of hatted variables,

$$\frac{C_t^{1-\sigma}}{1-\sigma} = \overline{C}^{1-\sigma}\{\widehat{C}_t + \frac{1}{2}(1-\sigma)\widehat{C}_t^2\} + tip + O[2]$$

Similarly for the terms in government spending,

$$\chi^A\frac{(G_t^A)^{1-\sigma}}{1-\sigma} = \chi^A(\overline{G}^A)^{1-\sigma}\{\widehat{G}_t^A + \frac{1}{2}(1-\sigma)(\widehat{G}_t^A)^2\} + tip + O[2]$$

$$\chi^B\frac{(G_t^B)^{1-\sigma}}{1-\sigma} = \chi^B(\overline{G}^B)^{1-\sigma}\{\widehat{G}_t^B + \frac{1}{2}(1-\sigma)(\widehat{G}_t^B)^2\} + tip + O[2]$$

The final term in labour supply can be written as,

$$\frac{N_t^{1+\varphi}\xi_t^N}{1+\varphi} = \overline{N}^{1+\varphi}\{\widehat{N}_t + \frac{1}{2}(1+\varphi)\widehat{N}_t^2 + \widehat{N}_t\widehat{\xi}_t^N\} + tip + O[2]$$

Now we need to relate the labour input to output and a measure of price dispersion. Aggregating the individual firms' demand for labour yields,

$$N = (\frac{Y}{A})\int_0^1 (\frac{P_H(i)}{P_H})^{-\epsilon_t} di$$

It can be shown (see Woodford, 2003, Chapter 6) that

$$\begin{aligned} \widehat{N} &= \widehat{Y} - a + \ln[\int_0^1 (\frac{P(i)}{P})^{-\epsilon_t} di] \\ &= \widehat{Y} - a + \frac{\epsilon}{2} var_i\{p(i)\} + O[2] \end{aligned}$$

so we can write

$$\begin{aligned} \frac{N_t^{1+\varphi}}{1+\varphi} &= \overline{N}^{1+\varphi}\{\widehat{Y}_t + \frac{1}{2}(1+\varphi)\widehat{Y}_t^2 - (1+\varphi)\widehat{Y}_t a_t + \frac{\epsilon}{2} var_i\{p_t(i)\}\} \\ &\quad + tip + O[2] \end{aligned}$$

Using these expansions, individual utility can be written as

$$\begin{aligned} \Gamma_t &= \overline{C}^{1-\sigma}\{\widehat{C}_t + \frac{1}{2}(1-\sigma)\widehat{C}_t^2\} \\ &\quad + \chi^A(\overline{G}^A)^{1-\sigma}\{\widehat{G}_t^A + \frac{1}{2}(1-\sigma)(\widehat{G}_t^A)^2\} \\ &\quad + \chi^B(\overline{G}^B)^{1-\sigma}\{\widehat{G}_t^B + \frac{1}{2}(1-\sigma)(\widehat{G}_t^B)^2\} \\ &\quad - \overline{N}^{1+\varphi}\{\widehat{Y}_t + \frac{1}{2}(1+\varphi)\widehat{Y}_t^2 - (1+\varphi)\widehat{Y}_t a_t \\ &\quad + \frac{\epsilon}{2} var_i\{p_t(i)\}\} \\ &\quad + tip + O[2] \end{aligned}$$

Using second order approximation to the national accounting identity,

$$\theta\widehat{C}_t = \widehat{Y}_t - (1-\theta)\widehat{G}_t - \frac{1}{2}\theta\widehat{C}_t^2 - \frac{1}{2}(1-\theta)\widehat{G}_t^2 + \frac{1}{2}\widehat{Y}_t^2 + O[2]$$

and,

$$\gamma\widehat{G}_t^A = \widehat{G}_t - (1-\gamma)\widehat{G}_t^B - \frac{1}{2}\gamma(\widehat{G}_t^A)^2 - \frac{1}{2}(1-\gamma)(\widehat{G}_t^B)^2 + \frac{1}{2}\widehat{G}_t^2 + O[2]$$

With the steady-state subsidy in place and government spending chosen optimally, the following conditions hold in the initial steady-state,

$$\overline{C}^{1-\sigma} = \overline{N}^{1+\varphi}\theta$$

and,

$$\begin{aligned} \chi^A(\overline{G}^A)^{1-\sigma} &= \overline{N}^{1+\varphi}(1-\theta)\gamma \\ \chi^B(\overline{G}^B)^{1-\sigma} &= \overline{N}^{1+\varphi}(1-\theta)(1-\gamma) \end{aligned}$$

Which allows us to eliminate the levels terms and rewrite welfare as,

$$\begin{aligned}\Gamma_t &= \overline{C}^{1-\sigma}\{-\frac{1}{2}\sigma\widehat{C}_{tt}^2\} + \chi^A(\overline{G}^A)^{1-\sigma}\{-\frac{1}{2}\sigma(\widehat{G}_t^A)^2\} + \chi^B(\overline{G}^B)^{1-\sigma}\{-\frac{1}{2}\sigma(\widehat{G}_t^B)^2\} \\ &-\overline{N}^{1+\varphi}\{\frac{1}{2}\varphi\widehat{Y}_t^2 - (1+\varphi)\widehat{Y}_t a_t + \frac{\epsilon}{2}var_i\{p_t(i)\}\} \\ &+tip + O[2]\end{aligned}$$

We now need to rewrite this in gap form using the focs for the social planner to eliminate the term in the technology shock,

$$\begin{aligned}\Gamma_t &= -\overline{N}^{1+\varphi}\frac{1}{2}\{\sigma\theta(\widehat{C}_t - \widehat{C}_t^*)^2 + \sigma(1-\theta)\gamma(\widehat{G}_t^A - \widehat{G}_t^{A*})^2 + \sigma(1-\theta)(1-\gamma)(\widehat{G}_t^B - \widehat{G}_t^{B*})^2 \\ &+\varphi(\widehat{Y}_t - \widehat{Y}_t^*)^2 + \epsilon var_i\{p_t(i)\}\} + tip + O[2]\end{aligned}$$

Using the result from Woodford (2003) that

$$\sum_{t=0}^{\infty}\beta^t var_i\{p_t(i)\} = \frac{\theta}{(1-\theta)(1-\theta\beta)}\sum_{t=0}^{\infty}\beta^t\pi_t^2 + t.i.p + O[2]$$

we can write the discounted sum of utility as,

$$\begin{aligned}\Gamma = -\overline{N}^{1+\varphi}\frac{1}{2}E_0\sum_{t=0}^{\infty}\beta^t\{\sigma\theta(\widehat{C}_t - \widehat{C}_t^*)^2 \quad &+\sigma(1-\theta)\gamma(\widehat{G}_t^A - \widehat{G}_t^{A*})^2 + \sigma(1-\theta)(1-\gamma)(\widehat{G}_t^B - \widehat{G}_t^{B*})^2 \\ +\varphi(\widehat{Y}_t - \widehat{Y}_t^*)^2 + \frac{\epsilon}{\lambda}\pi_t^2\} \quad &+tip + O[2]\end{aligned}$$

(3) The budget constraint using gap variables

The log-linearised budget constraint is given by,

$$\begin{aligned}\widehat{b}_t - \pi_t - \sigma\widehat{C}_t &= \beta\widehat{b}_{t+1} - \beta E_t\{\pi_{t+1} + \sigma\widehat{C}_{t+1}\} \\ &+\frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}(\widehat{w}_t + \widehat{N}_t + \widehat{\tau}_t) - \frac{\overline{G}}{\overline{b}}\widehat{G}_t - \sigma(1-\beta)\widehat{C}_t\end{aligned}$$

Using the labour supply function to eliminate real wages and the definition of efficient output to eliminate the technology shock,

$$\begin{aligned}\widehat{b}_t - \pi_t - \sigma\widehat{C}_t &= \beta\widehat{b}_{t+1} - \beta E_t\{\pi_{t+1} + \sigma\widehat{C}_{t+1}\} \\ &-\sigma(1-\beta)\widehat{C}_t - \frac{\overline{G}}{\overline{b}}\widehat{G}_t \\ &+\frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}((1+\varphi)(\widehat{Y}_t - \widehat{Y}_t^*) + \frac{1}{1-\overline{\tau}}\widehat{\tau}_t + \sigma(\widehat{C}_t - \widehat{C}_t^*) + \widehat{Y}_t^*)\end{aligned}$$

Gapping the remaining variables and combining shock terms,

$$\begin{aligned}\widehat{b}_t - \pi_t - \sigma(\widehat{C}_t - \widehat{C}_t^*) &= \beta E_t\{\widehat{b}_{t+1} - \pi_{t+1} - \sigma(\widehat{C}_{t+1} - \widehat{C}_{t+1}^*)\} - f_t \\ &-\sigma(1-\beta)(\widehat{C}_t - \widehat{C}_t^*) - \frac{\overline{G}}{\overline{b}}(\widehat{G}_t - \widehat{G}_t^*) \\ &+\frac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}}((1+\varphi)(\widehat{Y}_t - \widehat{Y}_t^*) + \frac{1}{1-\overline{\tau}}(\widehat{\tau}_t - \widehat{\tau}_t^*) + \sigma(\widehat{C}_t - \widehat{C}_t^*))\end{aligned}$$

where

$$f_t = -(\sigma(1-\beta\rho_a) + (1-\sigma)(1-\beta))\frac{(1+\varphi)}{\sigma+\varphi}a_t - \frac{\overline{w}\overline{N}}{\overline{b}}\mu_t$$

captures the fiscal consequences of the various shocks hitting the economy.

Appendix 2 - Time Consistent Policy with Targets and Electoral Uncertainty

(1) Deriving the Bellman equation

The first problem we face is in formulating a recursive problem when our model contains expectations of the future value of variables, in particular consumption, $E_t c^g_{t+1}$ and inflation, $E_t \pi_{t+1}$. However, since we have a linear-quadratic form for our problem we can hypothesize a solution for these endogenous variables of the form, conditional on party i being the incumbent,

$$E_{t-1}(c^g_t \mid i) = \mathbf{G0}^i + \mathbf{G1}^i \mathbf{S}_{t-1}$$

$$E_{t-1}(\pi_t \mid i) = \mathbf{F0}^i + \mathbf{F1}^i \mathbf{S}_{t-1} \qquad (15)$$

where $\mathbf{G0}^i = [g0^i]$, $\mathbf{G1}^i = \begin{bmatrix} g1^i & g2^i & g3^i \end{bmatrix}$ and $\mathbf{F0}^i = [f0^i]$ and $\mathbf{F1}^i = \begin{bmatrix} f1^i & f2^i & f3^i \end{bmatrix}$ are two $1x3$ vectors of undefined constants and $\mathbf{S}_{t-1} = \begin{bmatrix} \widehat{b}_t \\ a_{t-1} \\ \mu_{t-1} \end{bmatrix}$ is the vector of state variables. Note that in forming these expectations economic agents do not know who is going to be in power, however they do know who the incumbent is, the exogenous probability that there will be an election, e, and the probability that party i will win that election, $q(i)$ and the corresponding probability that party j, $i \neq j$, will win, $q(j) = 1 - q(i)$. Unless elections occur in every period, there is an electoral advantage to being an incumbent such that we must condition the expectations on who is incumbent at the time the expectations are formed. The constants reflect the influence of the parties' targets on expectations of future policy independent of the current state of the economy, but conditional on who is the incumbent at the time the expectations are formed.

Using the former of these we can write the equations describing the evolution of the state variables[13] as conditional on the assumption that party i is currently in power,

$$\mathbf{B0S}^i_t = \mathbf{B1S}_{t-1} + \mathbf{B2u}^i_t + \mathbf{B3}\boldsymbol{\xi}_t + \mathbf{B4}^i$$

$$\mathbf{B0}^i = \begin{bmatrix} B0^i_{1,1} & B0^i_{1,2} & B0^i_{1,3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

where $B0^i_{1,1} = \beta - \sigma\beta g1^i$, $B0^i_{1,2} = (\sigma(1-\rho_a\beta) - (\sigma-1)(1-\beta))\dfrac{(1+\varphi)}{\sigma+\varphi} - \sigma\beta g2^i$

and $B0^i_{1,3} = \dfrac{\overline{w}\overline{N}\overline{\tau}}{\overline{b}} - \sigma\beta g3^i$.

[13] In this section we make the empirically plausible assumption that debt is denominated in nominal terms.

$$\mathbf{B1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \rho_a & 0 \\ 0 & 0 & \rho_\mu \end{bmatrix}, \mathbf{B2} = \begin{bmatrix} B2_{1,1} & B2_{1,1} & B2_{1,3} & B2_{1,4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{B4}^i = \begin{bmatrix} \sigma\beta g0^i \\ 0 \\ 0 \end{bmatrix}$$

$$\text{where } B2_{1,1} = \frac{(\varphi\theta+\sigma)(-(1+\lambda)\frac{\overline{B}}{\overline{Y}} - (1-\theta)) - \theta(1-\theta) - (1-\beta)\theta\frac{\overline{B}}{\overline{Y}} + \beta\varphi\theta\frac{\overline{B}}{\overline{Y}}}{\frac{\overline{B}}{\overline{Y}}},$$

$$B2_{1,2} = \frac{(-(2-\theta) - (1-\beta+\lambda)\frac{\overline{B}}{\overline{Y}})((1-\beta)\frac{\overline{B}}{\overline{Y}} + (1-\theta))}{\frac{\overline{B}}{\overline{Y}}} \text{ and,}$$

$$B2_{1,3} = \frac{((1+\varphi)(\theta-(1-\beta)\frac{\overline{B}}{\overline{Y}}) - \varphi(1+\lambda\frac{\overline{B}}{\overline{Y}}))(1-\theta)\gamma}{\frac{\overline{B}}{\overline{Y}}}$$

$$B2_{1,4} = \frac{((1+\varphi)(\theta-(1-\beta)\frac{\overline{B}}{\overline{Y}}) - \varphi(1+\lambda\frac{\overline{B}}{\overline{Y}}))(1-\theta)(1-\gamma)}{\frac{\overline{B}}{\overline{Y}}}$$

$\mathbf{u}_t^i = \begin{bmatrix} \widehat{\tau}_t^i - \widehat{\tau}_t^* \\ \widehat{C}_t^i - \widehat{C}_t^* \\ \widehat{G}_t^{Ai} - \widehat{G}_t^{A*} \\ \widehat{G}_t^{Bi} - \widehat{G}_t^{B*} \end{bmatrix}$ is the vector of control variables in gap form and $\boldsymbol{\xi}_t$ is a vector of iid shocks to our shock processes. This allows us to rewrite the equation of motion for the state variables as,

$$\mathbf{S}_t^i = \mathbf{D0}^i + \mathbf{D1}^i\mathbf{S}_{t-1} + \mathbf{D2}^i\mathbf{u}_t^i + \mathbf{D3}^i\boldsymbol{\xi}_t \tag{16}$$

where

$$\begin{aligned} \mathbf{D0}^i &= \left[\mathbf{B0}^i\right]^{-1}\mathbf{B4}^i \\ \mathbf{D1}^i &= \left[\mathbf{B0}^i\right]^{-1}\mathbf{B1} \\ \mathbf{D2}^i &= \left[\mathbf{B0}^i\right]^{-1}\mathbf{B2} \\ \mathbf{D3}^i &= \left[\mathbf{B0}^i\right]^{-1}\mathbf{B3} \end{aligned}$$

Similarly we can write the evolution of inflation as follows,

$$E_t\boldsymbol{\pi}_{t+1} = \mathbf{A1}\boldsymbol{\pi}_t^i + \mathbf{A2}\mathbf{u}_t^i \tag{17}$$

where

$$\mathbf{A1} = \begin{bmatrix} \frac{1}{\beta} \end{bmatrix} \text{ and,}$$

$$\mathbf{A2} = \begin{bmatrix} -\frac{\lambda(\varphi\theta+\sigma)}{\beta} & -\frac{\lambda((1-\beta)\frac{\overline{B}}{\overline{Y}}+(1-\theta))}{\beta} & -\frac{\lambda\varphi(1-\theta)\gamma}{\beta} & -\frac{\lambda\varphi(1-\theta)(1-\gamma)}{\beta} \end{bmatrix}$$

Leading equation (15) forward one period and utilising the equation describing the evolution of the state variables, we can write,

$$\mathbf{F0}^i + \mathbf{F1}^i\mathbf{D0}^i + \mathbf{F1}^i\mathbf{D1}^i\mathbf{S}_{t-1} + \mathbf{F1}^i\mathbf{D2}^i\mathbf{u}_t^i + \mathbf{F1}^i\mathbf{D3}^i\boldsymbol{\xi}_t = \mathbf{A1}\boldsymbol{\pi}_t^i + \mathbf{A2}\mathbf{u}_t^i$$

Solving for inflation,

$$\boldsymbol{\pi}_t^i = \mathbf{C0}^i + \mathbf{C1}^i\mathbf{S}_{t-1} + \mathbf{C2}^i\mathbf{u}_t^i + \mathbf{C3}^i\boldsymbol{\xi}_t$$

where

$$\begin{aligned}
\mathbf{C0}^i &= [\mathbf{A1}]^{-1}[\mathbf{F1}^i\mathbf{D0}^i + \mathbf{F0}^i] \\
\mathbf{C1}^i &= [\mathbf{A1}]^{-1}[\mathbf{F1}^i\mathbf{D1}^i] \\
\mathbf{C2}^i &= -[\mathbf{A1}]^{-1}[\mathbf{A2} - \mathbf{F1}^i\mathbf{D2}^i] \\
\mathbf{C3}^i &= [\mathbf{A1}]^{-1}[\mathbf{F1}^i\mathbf{D3}^i]
\end{aligned}$$

These allow us to derive the optimisation of policymaker i as follows,

$$V^i(\mathbf{S}_{t-1};\xi_t) = \underset{\mathbf{u}_t^i}{Min}(\boldsymbol{\pi}_t^i\mathbf{R}\boldsymbol{\pi}_t^i + (\mathbf{u}_t^i - \mathbf{u}_t^{i*})'\mathbf{Q}^i(\mathbf{u}_t^i - \mathbf{u}_t^{i*}) + \beta E_t C^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})) \quad (18)$$

subject to,

$$\boldsymbol{\pi}_t^i = \mathbf{C0}^i + \mathbf{C1}^i\mathbf{S}_{t-1} + \mathbf{C2}^i\mathbf{u}_t^i + \mathbf{C3}^i\boldsymbol{\xi}_t$$
$$\mathbf{S}_t^i = \mathbf{D0}^i + \mathbf{D1}^i\mathbf{S}_{t-1} + \mathbf{D2}^i\mathbf{u}_t^i + \mathbf{D3}^i\boldsymbol{\xi}_t$$

and if party j is out of power,

$$W^j(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) = (\boldsymbol{\pi}_t^i\mathbf{R}^j\boldsymbol{\pi}_t^i + (\mathbf{u}_t^i - \mathbf{u}_t^{j*})'\mathbf{Q}^j(\mathbf{u}_t^i - \mathbf{u}_t^{j*})) + \beta E_t C^j(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})$$

where $\mathbf{S}_{t-1} = \begin{bmatrix} \widehat{b}_t \\ a_{t-1} \\ \mu_{t-1} \end{bmatrix}$, $\mathbf{u}_t^i = \begin{bmatrix} \widehat{\tau}_t^i - \widehat{\tau}_t^* \\ \widehat{C}_t^i - \widehat{C}_t^* \\ \widehat{G}_t^{Ai} - \widehat{G}_t^{A*} \\ \widehat{G}_t^{Bi} - \widehat{G}_t^{B*} \end{bmatrix}$ and $\mathbf{u}_t^{i*} = \overline{\mathbf{u}}^{i*} + \mathbf{K}^i\mathbf{S}_t$ are the vectors of state and control variables respectively. While $\boldsymbol{\xi}_t$ is a vector of iid innovations to the model's shock processes and $\mathbf{u}_t^{i*}$ is the vector of party-specific target variables defined as,

$$\overline{\mathbf{u}}^{i*} = \begin{bmatrix} 0 \\ \overline{C}^{Ti} \\ \overline{G}^{ATi} \\ \overline{G}^{BTi} \end{bmatrix} \text{ and } \begin{bmatrix} \overline{C}^{Ti} \\ \overline{G}^{ATi} \\ \overline{G}^{BTi} \end{bmatrix} = \begin{bmatrix} 2Z_i^C & Z_i^{CA} & Z_i^{CB} \\ Z_i^{CA} & 2Z_i^A & Z_i^{AB} \\ Z_i^{CB} & Z_i^{AB} & 2Z_i^B \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 2\Omega_i^A \frac{(\chi_i^A - \chi^A)}{\chi^A - \chi_i^A(1-\sigma)} \\ 2\Omega_i^B \frac{(\chi_i^B - \chi^B)}{\chi^B - \chi_i^B(1-\sigma)} \end{bmatrix}$$

$$\begin{aligned}
\mathbf{K}^i &= \begin{bmatrix} 0 \quad 0 \quad 0 \\ \widetilde{\mathbf{K}}^i \end{bmatrix} \\
\widetilde{\mathbf{K}}^i &= \begin{bmatrix} 2Z_i^C & Z_i^{CA} & Z_i^{CB} \\ Z_i^{CA} & 2Z_i^A & Z_i^{AB} \\ Z_i^{CB} & Z_i^{AB} & 2Z_i^B \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2\Omega_i^A \frac{(\chi_i^A - \chi^A)(1-\sigma)}{\chi^A - \chi_i^A(1-\sigma)} \frac{1+\varphi}{\sigma+\varphi} & 0 \\ 0 & 2\Omega_i^B \frac{(\chi_i^B - \chi^B)(1-\sigma)}{\chi^B - \chi_i^B(1-\sigma)} \frac{1+\varphi}{\sigma+\varphi} & 0 \end{bmatrix}
\end{aligned}$$

Notice that the targets are given by,

$$\mathbf{u}_t^{i*} = \overline{\mathbf{u}}^{i*} + \mathbf{K}^i \mathbf{S}_t$$

however, the $\mathbf{K}^i$ matrix only implies a dependence of the target variables on the exogenous shock processes. Accordingly we can rewrite the targets as,

$$\mathbf{u}_t^{i*} = \overline{\mathbf{u}}^{i*} + \mathbf{K1}^i \mathbf{S}_{t-1} + \mathbf{K3}^i \xi_t$$

where $\mathbf{K1}^i = \mathbf{K}^i \begin{bmatrix} 0 & 0 & 0 \\ 0 & \rho_a & 0 \\ 0 & 0 & \rho_\mu \end{bmatrix}$ and $\mathbf{K3}^i = \mathbf{K}^i$.

(2)The Form of the Continuation Game

Given that the problem is linear-quadratic the value function for player i, $V^i(\mathbf{S}_{t-1}; \boldsymbol{\xi}_t)$ and the discounted pay-offs if he is out of power, $W^i(\mathbf{S}_{t-1}; \boldsymbol{\xi}_t)$ are given by,

$$\begin{aligned} V^i(\mathbf{S}_{t-1}; \boldsymbol{\xi}_t) &= \mathbf{\Phi 0}^i + \mathbf{\Phi 1}^i \mathbf{S}_{t-1} + \mathbf{S}_{t-1}' \mathbf{\Phi 2}^i \mathbf{S}_{t-1} + \mathbf{\Phi 3}^i \boldsymbol{\xi}_t + \mathbf{S}_{t-1}' \mathbf{\Phi 4}^i \boldsymbol{\xi}_t + \boldsymbol{\xi}_t' \mathbf{\Phi 5}^i \boldsymbol{\xi}_t \\ W^i(\mathbf{S}_{t-1}; \boldsymbol{\xi}_t) &= \boldsymbol{\mu 0}^i + \boldsymbol{\mu 1}^i \mathbf{S}_{t-1} + \mathbf{S}_{t-1}' \boldsymbol{\mu 2}^i \mathbf{S}_{t-1} + \boldsymbol{\mu 3}^i \boldsymbol{\xi}_t + \mathbf{S}_{t-1}' \boldsymbol{\mu 4}^i \boldsymbol{\xi}_t + \boldsymbol{\xi}_t' \boldsymbol{\mu 5}^i \boldsymbol{\xi}_t \end{aligned}$$

where $\mathbf{\Phi J}^i$ and $\boldsymbol{\mu}\mathbf{J}^i$ with $J = 0, 1, 2$ are matrices of unknown coefficients for player i's value and payoff functions. However, the value of the continuation game, not only directly depends on these payoffs, but also indirectly through their impact on the probability of election victory. Since we are focusing on linear strategies we need only consider a 2nd order approximation to the continuation game,

$$E_t C^i(\mathbf{S}_t^j; \boldsymbol{\xi}_{t+1}) = \boldsymbol{\beta 0}^{i|j} + \boldsymbol{\beta 1}^{i|j} \mathbf{S}_t + \mathbf{S}_t' \boldsymbol{\beta 2}^{i|j} \mathbf{S}_t + O[2]$$

where i indicates from which party's perspective we are evaluating the continuation game, j denotes who is setting policy, $O[2]$ refers to terms which are greater than second order and,

$$\begin{aligned} \boldsymbol{\beta 0}^{i|j} &= \frac{1}{2} e \widetilde{\mathbf{\Phi 0}}^i + (1 - \frac{1}{2} e) \widetilde{\boldsymbol{\mu 0}}^i \\ &+ e(-z^i \left(\widetilde{\mathbf{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i \right) + z^j \left(\widetilde{\mathbf{\Phi 0}}^j - \widetilde{\boldsymbol{\mu 0}}^j \right))(\widetilde{\mathbf{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i) \end{aligned}$$

$$\begin{aligned} \boldsymbol{\beta 0}^{i|i} &= (1 - \frac{1}{2} e) \widetilde{\mathbf{\Phi 0}}^i + \frac{1}{2} e \widetilde{\boldsymbol{\mu 0}}^i \\ &+ e(-z^i \left(\widetilde{\mathbf{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i \right) + z^j \left(\widetilde{\mathbf{\Phi 0}}^j - \widetilde{\boldsymbol{\mu 0}}^j \right))(\widetilde{\mathbf{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i) \end{aligned}$$

$$\begin{aligned}\boldsymbol{\beta 1}^{i|j} &= \frac{1}{2}e\boldsymbol{\Phi 1}^i + (1-\frac{1}{2}e)\boldsymbol{\mu 1}^i \\ &+e(-z^i\left(\widetilde{\boldsymbol{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i\right) + z^j\left(\widetilde{\boldsymbol{\Phi 0}}^j - \widetilde{\boldsymbol{\mu 0}}^j\right))(\widetilde{\boldsymbol{\Phi 1}}^i - \widetilde{\boldsymbol{\mu 1}}^i) \\ &+e(-z^i\left(\boldsymbol{\Phi 1}^i - \boldsymbol{\mu 1}^i\right) + z^j\left(\boldsymbol{\Phi 1}^j - \boldsymbol{\mu 1}^j\right))(\widetilde{\boldsymbol{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i)\end{aligned}$$

$$\begin{aligned}\boldsymbol{\beta 1}^{i|i} &= (1-\frac{1}{2}e)\boldsymbol{\Phi 1}^i + \frac{1}{2}e\boldsymbol{\mu 1}^i \\ &+e(-z^i\left(\widetilde{\boldsymbol{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i\right) + z^j\left(\widetilde{\boldsymbol{\Phi 0}}^j - \widetilde{\boldsymbol{\mu 0}}^j\right))(\widetilde{\boldsymbol{\Phi 1}}^i - \widetilde{\boldsymbol{\mu 1}}^i) \\ &+e(-z^i\left(\boldsymbol{\Phi 1}^i - \boldsymbol{\mu 1}^i\right) + z^j\left(\boldsymbol{\Phi 1}^j - \boldsymbol{\mu 1}^j\right))(\widetilde{\boldsymbol{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i)\end{aligned}$$

$$\begin{aligned}\boldsymbol{\beta 2}^{i|j} &= \frac{1}{2}e\boldsymbol{\Phi 2}^i + (1-\frac{1}{2}e)\boldsymbol{\mu 2}^i \\ &+e(-z^i\left(\widetilde{\boldsymbol{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i\right) + z^j\left(\widetilde{\boldsymbol{\Phi 0}}^j - \widetilde{\boldsymbol{\mu 0}}^j\right))(\widetilde{\boldsymbol{\Phi 2}}^i - \widetilde{\boldsymbol{\mu 2}}^i) \\ &+e(-z^i\left(\boldsymbol{\Phi 2}^i - \boldsymbol{\mu 2}^i\right) + z^j\left(\boldsymbol{\Phi 2}^j - \boldsymbol{\mu 2}^j\right))(\widetilde{\boldsymbol{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i) \\ &+e(-z^i\left(\boldsymbol{\Phi 1}^i - \boldsymbol{\mu 1}^i\right)' + z^j\left(\boldsymbol{\Phi 1}^j - \boldsymbol{\mu 1}^j\right)')(\boldsymbol{\Phi 1}^i - \boldsymbol{\mu 1}^i)\end{aligned}$$

$$\begin{aligned}\boldsymbol{\beta 2}^{i|i} &= (1-\frac{1}{2}e)\boldsymbol{\Phi 2}^i + \frac{1}{2}e\boldsymbol{\mu 2}^i \\ &+e(-z^i\left(\widetilde{\boldsymbol{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i\right) + z^j\left(\widetilde{\boldsymbol{\Phi 0}}^j - \widetilde{\boldsymbol{\mu 0}}^j\right))(\widetilde{\boldsymbol{\Phi 2}}^i - \widetilde{\boldsymbol{\mu 2}}^i) \\ &+e(-z^i\left(\boldsymbol{\Phi 2}^i - \boldsymbol{\mu 2}^i\right) + z^j\left(\boldsymbol{\Phi 2}^j - \boldsymbol{\mu 2}^j\right))(\widetilde{\boldsymbol{\Phi 0}}^i - \widetilde{\boldsymbol{\mu 0}}^i) \\ &+e(-z^i\left(\boldsymbol{\Phi 1}^i - \boldsymbol{\mu 1}^i\right)' + z^j\left(\boldsymbol{\Phi 1}^j - \boldsymbol{\mu 1}^j\right)')(\boldsymbol{\Phi 1}^i - \boldsymbol{\mu 1}^i)\end{aligned}$$

where $\widetilde{\boldsymbol{\Phi 0}}^i = \boldsymbol{\Phi 0}^i + tr[\boldsymbol{\Sigma\Phi 5}^i]$ and $\widetilde{\boldsymbol{\mu 0}}^i = \boldsymbol{\mu 0}^i + tr[\boldsymbol{\Sigma\mu 5}^i]$. In other words, the second order approximation to the continuation game takes account of the impact of debt (and other state variables) on both the expected pay-offs to each party when in and out of power, but also factors in the repercussions of this for the probability of electoral success.

The incumbent policymaker therefore takes account of the impact a marginal increase in debt has on the value of the continuation game,

$$\frac{\partial E_t C^i(\mathbf{S}_t^i; \boldsymbol{\xi}_{t+1})}{\partial \mathbf{S}_t^i} = \boldsymbol{\beta 1}^{i|i\prime} + (\boldsymbol{\beta 2}^{i|i\prime} + \boldsymbol{\beta 2}^{i|i})\mathbf{S}_t^i$$

where the debt affects not only the payoffs when each party is in or out of power, but, as a consequence, also the probability of each party winning any election that takes place.

Appendix 3 - Solving the Bellman equation

(1)The first-order conditions conditional on 'guesses'

The first-order conditions with respect to the control variables chosen by policy maker i from solving (18) are then given by,

$$2\mathbf{C2}^{i\prime}\mathbf{R}\boldsymbol{\pi}_t^i + (\mathbf{Q}^i + \mathbf{Q}^{i\prime})(\mathbf{u}_t^i - \mathbf{u}_t^{i*}) + \beta\mathbf{D2}^{i\prime}\frac{\partial E_t C^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})}{\partial\mathbf{S}_t^i} = 0$$

Using the definition of the targets for player i, $\mathbf{u}_t^{i*} = \overline{u}^{i*} + \mathbf{K1}^i\mathbf{S}_{t-1} + \mathbf{K3}^i\boldsymbol{\xi}_t$ we can write this as,

$$2\mathbf{C2}^{i\prime}\mathbf{R}\boldsymbol{\pi}_t^i + (\mathbf{Q}^i + \mathbf{Q}^{i\prime})(\mathbf{u}_t^i - \overline{\mathbf{u}}^{i*} - \mathbf{K1}^i\mathbf{S}_{t-1} - \mathbf{K3}^i\boldsymbol{\xi}_t) + \beta\mathbf{D2}^{i\prime}\frac{\partial E_t C^i(\mathbf{S}_t^i;\boldsymbol{\xi}_{t+1})}{\partial\mathbf{S}_t^i} = 0$$

Using the equation of motion the vector of state variables, the relationship between inflation and state variables and the 'guess' for the value of the continuation game, this can be written as,

$$\begin{aligned}
& 2\mathbf{C2}^{i\prime}\mathbf{R}[\mathbf{C0}^i + \mathbf{C1}^i\mathbf{S}_{t-1} + \mathbf{C2}^i\mathbf{u}_t^i + \mathbf{C3}^i\boldsymbol{\xi}_t] \\
& +(\mathbf{Q}^i + \mathbf{Q}^{i\prime})(\mathbf{u}_t^i - \overline{\mathbf{u}}^{i*} - \mathbf{K1}^i\mathbf{S}_{t-1} - \mathbf{K3}^i\boldsymbol{\xi}_t]) \\
& +\beta\mathbf{D2}^{i\prime}[\boldsymbol{\beta}\mathbf{1}^{i|i\prime} + (\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})[\mathbf{D0}^i + \mathbf{D1}^i\mathbf{S}_{t-1} + \mathbf{D2}^i\mathbf{u}_t^i + \mathbf{D3}^i\boldsymbol{\xi}_t]] \\
= \; & 0
\end{aligned}$$

and solved for control variables,

$$\mathbf{u_t^i} = -[\mathbf{U2}^i]^{-1}\mathbf{U0}^i - [\mathbf{U2}^i]^{-1}\mathbf{U1}^i\mathbf{S}_{t-1} - [\mathbf{U2}^i]^{-1}\mathbf{U3}^i\boldsymbol{\xi}_\mathbf{t}$$

where

$$\begin{aligned}
\mathbf{U0}^i &= 2\mathbf{C2}^{i\prime}\mathbf{RC0}^i - (\mathbf{Q}^i + \mathbf{Q}^{i\prime})\overline{u}^{i*} + \beta\mathbf{D2}^{i\prime}[\boldsymbol{\beta}\mathbf{1}^{i|i\prime} + (\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})\mathbf{D0}^i] \\
\mathbf{U1}^i &= [2\mathbf{C2}^{i\prime}\mathbf{RC1}^i - (\mathbf{Q}^i + \mathbf{Q}^{i\prime})\mathbf{K1}^i + \beta\mathbf{D2}^{i\prime}(\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})\mathbf{D1}^i] \\
\mathbf{U2}^i &= [2\mathbf{C2}^{i\prime}\mathbf{RC2}^i + (\mathbf{Q}^i + \mathbf{Q}^{i\prime}) + \beta\mathbf{D2}^{i\prime}(\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})\mathbf{D2}^i] \\
\mathbf{U3}^i &= [2\mathbf{C2}^{i\prime}\mathbf{RC3}^i - (\mathbf{Q}^i + \mathbf{Q}^{i\prime})\mathbf{K3}^i + \beta\mathbf{D2}^{i\prime}(\boldsymbol{\beta}\mathbf{2}^{i|i\prime} + \boldsymbol{\beta}\mathbf{2}^{i|i})\mathbf{D3}^i]
\end{aligned}$$

The solution for inflation is now given as,

$$\boldsymbol{\pi}_t^i = \mathbf{P0}^i + \mathbf{P1}^i\mathbf{S}_{t-1} + \mathbf{P3}^i\boldsymbol{\xi}_\mathbf{t}$$

where,

$$\begin{aligned}
\mathbf{P0}^i &= \mathbf{C0}^i - \mathbf{C2}^i[\mathbf{U2}^i]^{-1}\mathbf{U0}^i \\
\mathbf{P1}^i &= \mathbf{C1}^i - \mathbf{C2}^i[\mathbf{U2}^i]^{-\mathbf{1}}\mathbf{U1}^i \\
\mathbf{P3}^i &= \mathbf{C3}^i - \mathbf{C2}^i[\mathbf{U2}^i]^{-\mathbf{1}}\mathbf{U3}^i
\end{aligned}$$

(2)Equating the Undetermined Coefficients in the Forecasting Equations

We are now in a position to obtain our first set of equations to solve for the forecast guess parameters. Taking expectations of the optimal relationship between the controls and state variables, conditional on party i being in power,

$$\begin{aligned} E_{t-1}(\mathbf{u}_t \quad | \quad i) &= q(i \mid i)_t E_{t-1}\mathbf{u}_t^i + q(j \mid i)_t E_{t-1}\mathbf{u}_t^j \\ &= \widetilde{\mathbf{G0}}^i + \widetilde{\mathbf{G1}}^i \mathbf{S}_{t-1} \end{aligned}$$

where,

$$\begin{aligned} \widetilde{\mathbf{G0}}^i &= -[1-e+e(\frac{1}{2}-z^i\left(\widetilde{\mathbf{\Phi 0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)+z^j\left(\widetilde{\mathbf{\Phi 0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))][\mathbf{U2}^i]^{-1}\mathbf{U0}^i \\ &\quad -[e(\frac{1}{2}+z^i\left(\widetilde{\mathbf{\Phi 0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)-z^j\left(\widetilde{\mathbf{\Phi 0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))][\mathbf{U2}^j]^{-1}\mathbf{U0}^j \end{aligned}$$

and

$$\begin{aligned} \widetilde{\mathbf{G1}}^i &= -[1-e+e(\frac{1}{2}-z^i\left(\widetilde{\mathbf{\Phi 0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)+z^j\left(\widetilde{\mathbf{\Phi 0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))][\mathbf{U2}^i]^{-1}\mathbf{U1}^i \\ &\quad -[e(\frac{1}{2}+z^i\left(\widetilde{\mathbf{\Phi 0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)-z^j\left(\widetilde{\mathbf{\Phi 0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))][\mathbf{U2}^j]^{-1}\mathbf{U1}^j \\ &\quad -e([\mathbf{U2}^i]^{-1}\mathbf{U0}^i-[\mathbf{U2}^j]^{-1}\mathbf{U0}^j)[-z^i\left(\mathbf{\Phi 1}^i-\boldsymbol{\mu}\mathbf{1}^i\right)+z^j\left(\mathbf{\Phi 1}^j-\boldsymbol{\mu}\mathbf{1}^j\right)] \end{aligned}$$

The second row of this relationship can then be equated to the guess parameters in

$$E_{t-1}(c_t^g \mid i) = \mathbf{G0}^i + \mathbf{G1}^i\mathbf{S}_{t-1}$$

Similarly, we can take expectations of the expression for inflation to obtain,

$$\begin{aligned} E_{t-1}[\boldsymbol{\pi}_t \quad | \quad i] &= q(i \mid i)_t E_{t-1}\boldsymbol{\pi}_t^i + q(j \mid i)_t E_{t-1}\boldsymbol{\pi}_t^j \\ &= \widetilde{\mathbf{P0}}^i + \widetilde{\mathbf{P1}}^i \mathbf{S}_{t-1} \end{aligned}$$

where

$$\begin{aligned} \widetilde{\mathbf{P0}}^i &= [1-e+e(\frac{1}{2}-z^i\left(\widetilde{\mathbf{\Phi 0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)+z^j\left(\widetilde{\mathbf{\Phi 0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))]P\mathbf{0}^i \\ &\quad +[e(\frac{1}{2}+z^i\left(\widetilde{\mathbf{\Phi 0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)-z^j\left(\widetilde{\mathbf{\Phi 0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))]\mathbf{P0}^j \end{aligned}$$

and

$$\begin{aligned} \widetilde{\mathbf{P1}}^i &= [1-e+e(\frac{1}{2}-z^i\left(\widetilde{\mathbf{\Phi 0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)+z^j\left(\widetilde{\mathbf{\Phi 0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))]P\mathbf{1}^i \\ &\quad +[e(\frac{1}{2}+z^i\left(\widetilde{\mathbf{\Phi 0}}^i-\widetilde{\boldsymbol{\mu}\mathbf{0}}^i\right)-z^j\left(\widetilde{\mathbf{\Phi 0}}^j-\widetilde{\boldsymbol{\mu}\mathbf{0}}^j\right))]\mathbf{P1}^j \\ &\quad +e(\mathbf{P0}^i-\mathbf{P0}^j)[-z^i\left(\mathbf{\Phi 1}^i-\boldsymbol{\mu}\mathbf{1}^i\right)+z^j\left(\mathbf{\Phi 1}^j-\boldsymbol{\mu}\mathbf{1}^j\right))] \end{aligned}$$

which can then be equated with the elements in,

$$E_{t-1}(\pi_t \mid i) = \mathbf{F0}^i + \mathbf{F1}^i \mathbf{S}_{t-1}$$

(3)Equating the Undetermined Coefficients in the Continuation Game

It is helpful to write the evolution of the vector of state variables.

$$\begin{aligned} \mathbf{S}_t^i &= \mathbf{D0}^i + \mathbf{D1}^i \mathbf{S}_{t-1} + \mathbf{D2}^i \mathbf{u}_t^i + \mathbf{D3}^i \boldsymbol{\xi}_t \\ &= \mathbf{J0}^i + \mathbf{J1}^i \mathbf{S}_{t-1} + \mathbf{J3}^i \boldsymbol{\xi}_t \end{aligned}$$

where

$$\begin{aligned} \mathbf{J0}^i &= \mathbf{D0}^i - \mathbf{D2}^i [\mathbf{U2}^i]^{-1} \mathbf{U0}^i \\ \mathbf{J1}^i &= \mathbf{D1}^i - \mathbf{D2}^i [\mathbf{U2}^i]^{-1} \mathbf{U1}^i \\ \mathbf{J3}^i &= \mathbf{D3}^i - \mathbf{D2}^i [\mathbf{U2}^i]^{-1} \mathbf{U3}^i \end{aligned}$$

Using the definition,

$$\mathbf{u}_t^{i*} = \overline{\mathbf{u}}^{i*} + \mathbf{K1}^i \mathbf{S}_{t-1} + \mathbf{K3}^i \xi_t$$

It is convenient to define,

$$\mathbf{u_t^i} - \mathbf{u_t^{j*}} = \widetilde{\mathbf{U0}}^{ij} + \widetilde{\mathbf{U1}}^{ij} \mathbf{S}_{t-1} + \widetilde{\mathbf{U3}}^{ij} \boldsymbol{\xi}_t$$

where

$$\begin{aligned} \widetilde{\mathbf{U0}}^{ij} &= -[[\mathbf{U2}^i]^{-1} \mathbf{U0}^i + \overline{\mathbf{u}}^{\mathbf{j}*}] \\ \widetilde{\mathbf{U1}}^{ij} &= -[[\mathbf{U2}^i]^{-1} \mathbf{U1}^i + \mathbf{K1}^{\mathbf{j}}] \\ \widetilde{\mathbf{U3}}^{ij} &= -[[\mathbf{U2}^i]^{-1} \mathbf{U3}^i + \mathbf{K3}^{\mathbf{j}}] \end{aligned}$$

We now need to find expressions to solve for the 'guessed' parameterisation of the continuation game.

$$\begin{aligned} E_t C^i(\mathbf{S}_t^i; \boldsymbol{\xi}_{t+1}) &= \boldsymbol{\beta}\mathbf{0}^{i|i} + \boldsymbol{\beta}\mathbf{1}^{i|i} \mathbf{S}_t^j + \mathbf{S}_t^{j\prime} \boldsymbol{\beta}\mathbf{2}^{i|i} \mathbf{S}_t^j \\ &= \boldsymbol{\beta}\mathbf{0}^{i|i} + \boldsymbol{\beta}\mathbf{1}^{i|i} \mathbf{J0}^i + \mathbf{J0}^{i\prime} \boldsymbol{\beta}\mathbf{2}^{i|i} \mathbf{J0}^i \\ &\quad + [\boldsymbol{\beta}\mathbf{1}^{i|i} \mathbf{J1}^i + \mathbf{J0}^{i\prime} (\boldsymbol{\beta}\mathbf{2}^{i|i} + \boldsymbol{\beta}\mathbf{2}^{i|i'}) \mathbf{J1}^i] \mathbf{S}_{t-1} \\ &\quad + \mathbf{S}_{t-1}' \mathbf{J1}^{i\prime} \boldsymbol{\beta}\mathbf{2}^{i|i} \mathbf{J1}^i \mathbf{S}_{t-1} + tis + O[2] \end{aligned}$$

where *tis* denotes non-constant terms independent of the vector of state variables. Similarly we can define the value of the continuation game for party i given that party j has been in power,

$$\begin{aligned} E_t C^i(\mathbf{S}_t^j; \boldsymbol{\xi}_{t+1}) &= \boldsymbol{\beta}\mathbf{0}^{i|j} + \boldsymbol{\beta}\mathbf{1}^{i|j} \mathbf{S}_t^j + \mathbf{S}_t^{j\prime} \boldsymbol{\beta}\mathbf{2}^{i|j} \mathbf{S}_t^j \\ &= \boldsymbol{\beta}\mathbf{0}^{i|j} + \boldsymbol{\beta}\mathbf{1}^{i|j} \mathbf{J0}^j + \mathbf{J0}^{j\prime} \boldsymbol{\beta}\mathbf{2}^{i|j} \mathbf{J0}^j \\ &\quad + [\boldsymbol{\beta}\mathbf{1}^{i|j} \mathbf{J1}^j + \mathbf{J0}^{j\prime} (\boldsymbol{\beta}\mathbf{2}^{i|j} + \boldsymbol{\beta}\mathbf{2}^{i|j\prime}) \mathbf{J1}^j] \mathbf{S}_{t-1} \\ &\quad + \mathbf{S}_{t-1}' \mathbf{J1}^{j\prime} \boldsymbol{\beta}\mathbf{2}^{i|j} \mathbf{J1}^j \mathbf{S}_{t-1} + tis + O[2] \end{aligned}$$

Using these results the Bellman equation can be rewritten as,

$$\begin{aligned}
V^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) &= \mathbf{P0}^{i'}\mathbf{RP0}^i + \widetilde{\mathbf{U0}}^{ii\prime}\mathbf{Q}^i\widetilde{\mathbf{U0}}^{ii} + \beta(\boldsymbol{\beta}\mathbf{0}^{i|i} + \boldsymbol{\beta}\mathbf{1}^{i|i}\mathbf{J0}^i + \mathbf{J0}^{i\prime}\boldsymbol{\beta}\mathbf{2}^{i|i}\mathbf{J0}^i) \\
&+[\mathbf{P0}^{i\prime}(\mathbf{R}+\mathbf{R}')\mathbf{P1}^i]\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\mathbf{P1}^{i\prime}\mathbf{RP1}^i\mathbf{S}_{t-1} \\
&+[\widetilde{\mathbf{U0}}^{ii\prime}(\mathbf{Q}^i+\mathbf{Q}^{i\prime})\widetilde{\mathbf{U0}}^{ii}]\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\widetilde{\mathbf{U1}}^{ii\prime}\mathbf{Q}^i\widetilde{\mathbf{U1}}^{ii}\mathbf{S}_{t-1} \\
&+\beta[\boldsymbol{\beta}\mathbf{1}^{i|i}\mathbf{J1}^i + \mathbf{J0}^{i\prime}(\boldsymbol{\beta}\mathbf{2}^{i|i} + \boldsymbol{\beta}\mathbf{2}^{i|i\prime})\mathbf{J1}^i]\mathbf{S}_{t-1} + \beta\mathbf{S}'_{t-1}\mathbf{J1}^{i\prime}\boldsymbol{\beta}\mathbf{2}^{i|i}\mathbf{J1}^i\mathbf{S}_{t-1} \\
&+tis + +O[2]
\end{aligned}$$

The pay-offs for a party out of power can be written as,

$$\begin{aligned}
W^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) &= \mathbf{P0}^{j'}\mathbf{RP0}^j + \widetilde{\mathbf{U0}}^{ji\prime}\mathbf{Q}^i\widetilde{\mathbf{U0}}^{ji} + \beta(\boldsymbol{\beta}\mathbf{0}^{i|j} + \beta 1^{i|j}\mathbf{J0}^j + \mathbf{J0}^{j\prime}\beta 2^{i|j}\mathbf{J0}^j) \\
&+[\mathbf{P0}^{j\prime}(\mathbf{R}+\mathbf{R}')\mathbf{P1}^j]\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\mathbf{P1}^{j\prime}\mathbf{RP1}^j\mathbf{S}_{t-1} \\
&+[\widetilde{\mathbf{U0}}^{ji\prime}(\mathbf{Q}^i+\mathbf{Q}^{i\prime})\widetilde{\mathbf{U1}}^{ji}]\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\widetilde{\mathbf{U1}}^{ji\prime}\mathbf{Q}^i\widetilde{\mathbf{U1}}^{ji}\mathbf{S}_{t-1} \\
&+\beta[\boldsymbol{\beta}\mathbf{1}^{i|j}\mathbf{J1}^j + \mathbf{J0}^{j\prime}(\boldsymbol{\beta}\mathbf{2}^{i|j} + \boldsymbol{\beta}\mathbf{2}^{i|j\prime})\mathbf{J1}^j]\mathbf{S}_{t-1} + \beta\mathbf{S}'_{t-1}\mathbf{J1}^{j\prime}\boldsymbol{\beta}\mathbf{2}^{i|j}\mathbf{J1}^j\mathbf{S}_{t-1} \\
&+tis + O[2]
\end{aligned}$$

These can then be equated with the corresponding terms from the value function guesses,

$$\begin{aligned}
V^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) &= \widetilde{\boldsymbol{\Phi}\mathbf{0}}^i + \boldsymbol{\Phi}\mathbf{1}^i\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\boldsymbol{\Phi}\mathbf{2}^i\mathbf{S}_{t-1} + tis \\
W^i(\mathbf{S}_{t-1};\boldsymbol{\xi}_t) &= \widetilde{\boldsymbol{\mu}\mathbf{0}}^i + \boldsymbol{\mu}\mathbf{1}^i\mathbf{S}_{t-1} + \mathbf{S}'_{t-1}\boldsymbol{\mu}\mathbf{2}^i\mathbf{S}_{t-1} + tis \text{ where } i = 1,2
\end{aligned}$$

which completes the description of optimal policy for both parties.

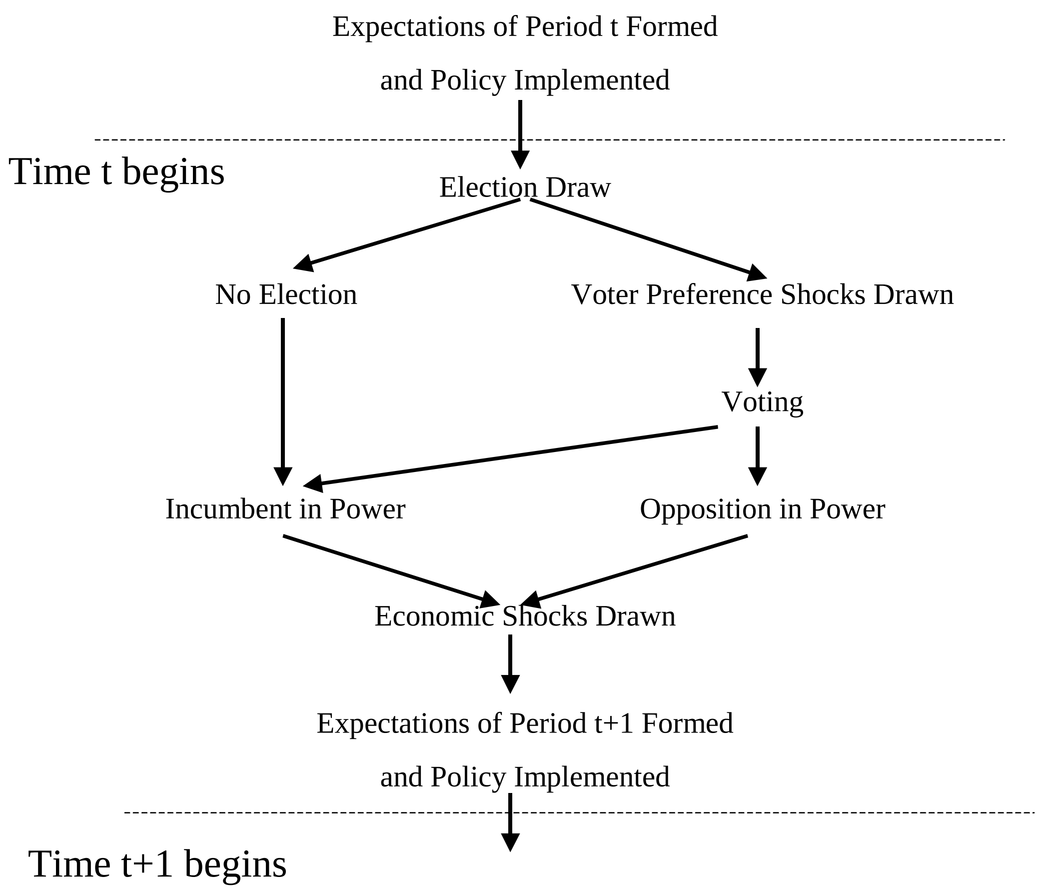

Figure 1: Timing of Events

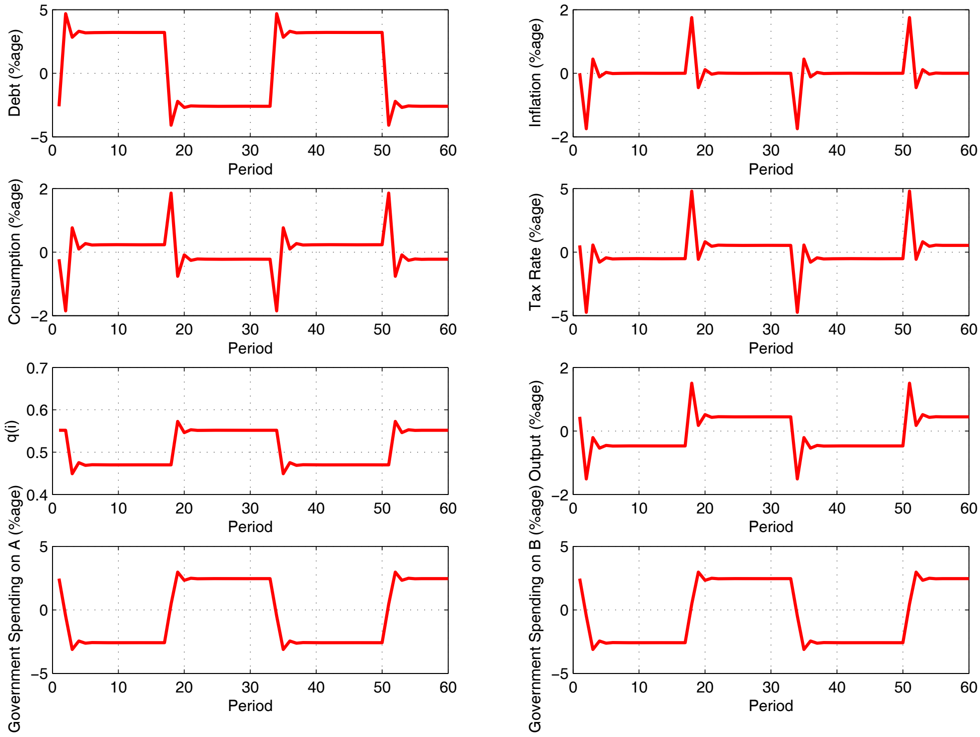

Figure 2: Impulse response with size heterogeneity and an electoral cycle.

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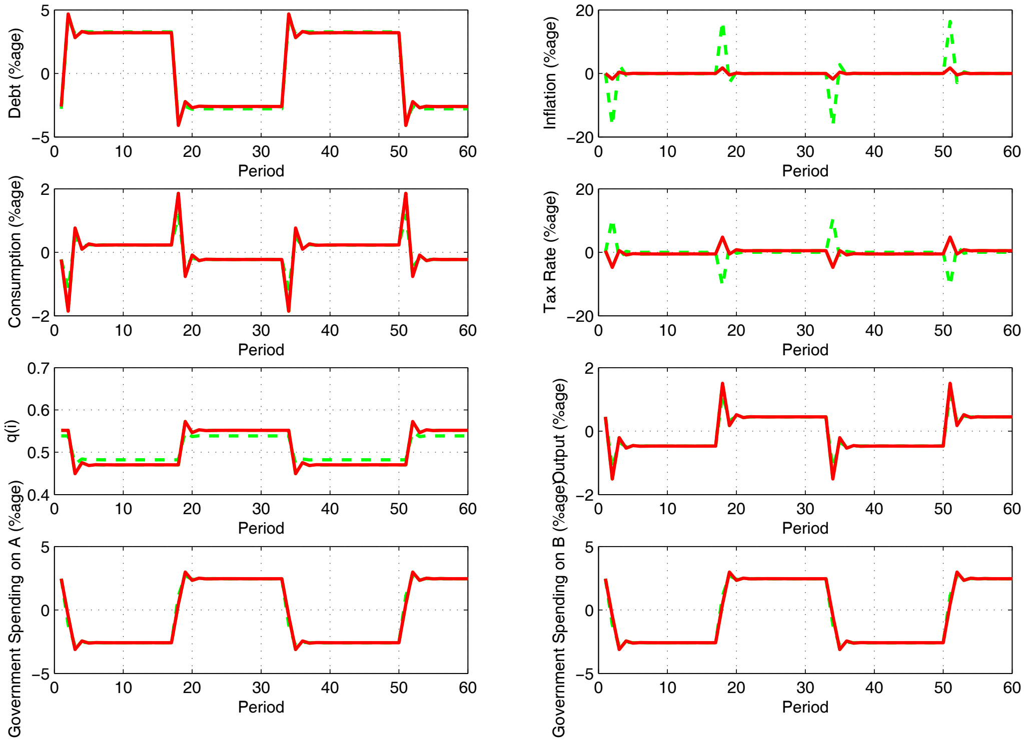

Figure 3: Impulse Response with size heterogeneity and different degrees of price stickiness.

Notes to Figure - solid red line, benchmark price stickiness, $\theta_p = 0.75$; more flexible prices, $\theta_p = 0.05$.

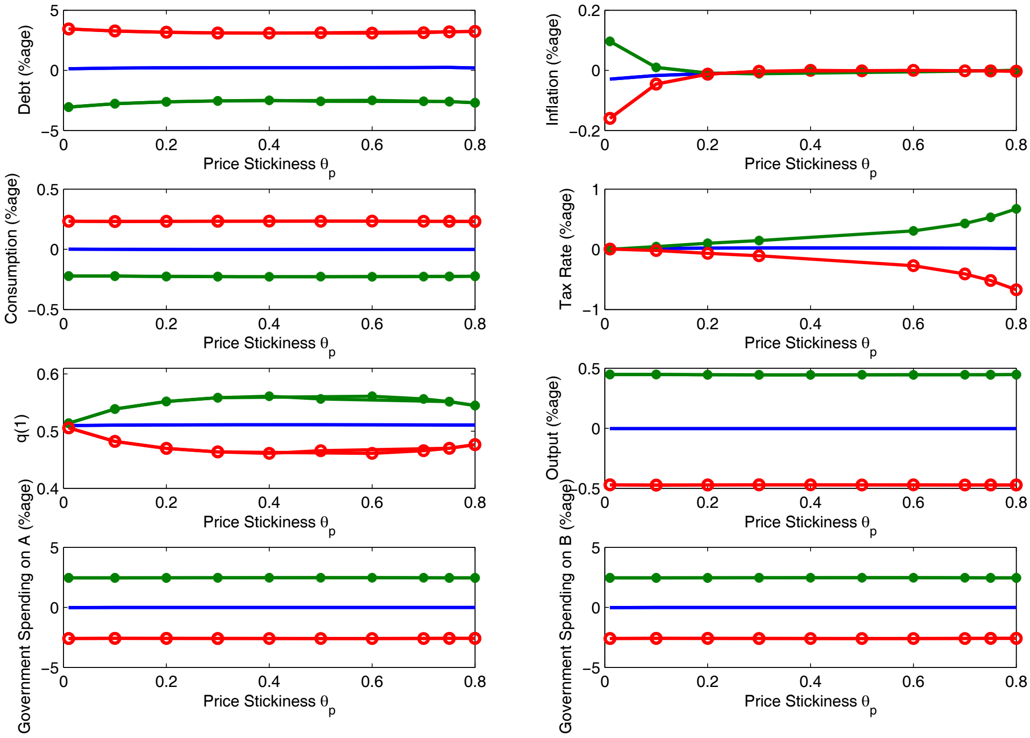

Figure 4: Steady-state with size heterogeneity as a function of price stickiness.

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Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

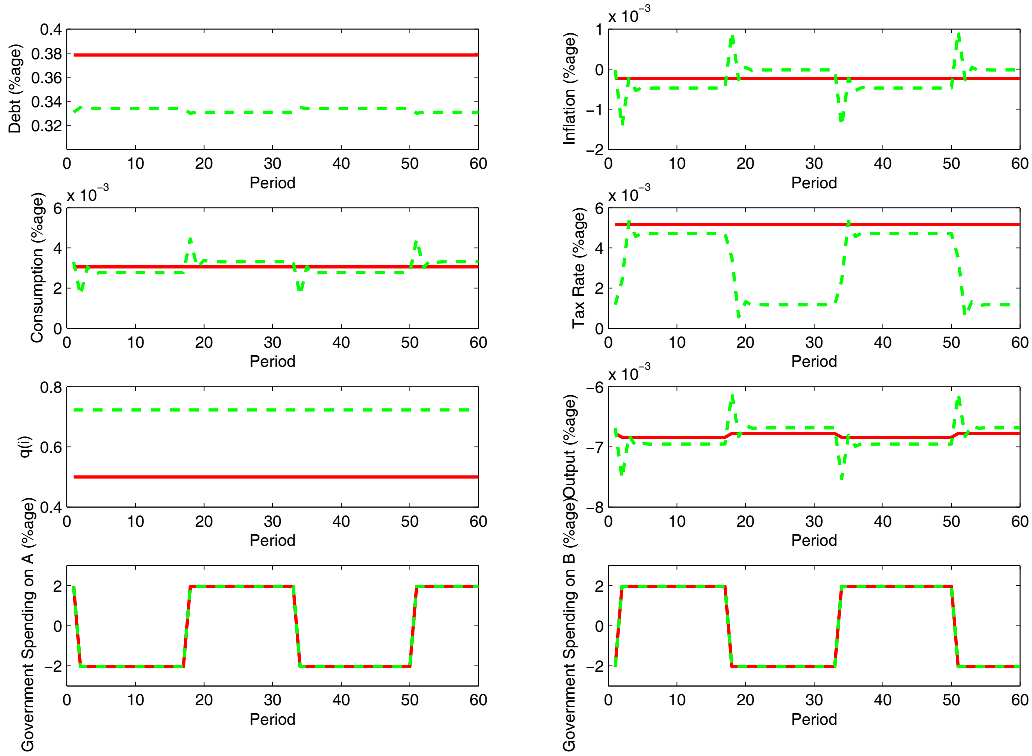

Figure 5: Impulse response with composition heterogeneity.

Notes to Figure - solid red line, symmetrical voters, $z_1 = z_2 = 10$; dashed green line, asymmetrical voting, $z_1 = 15$ and $z_2 = 5$.

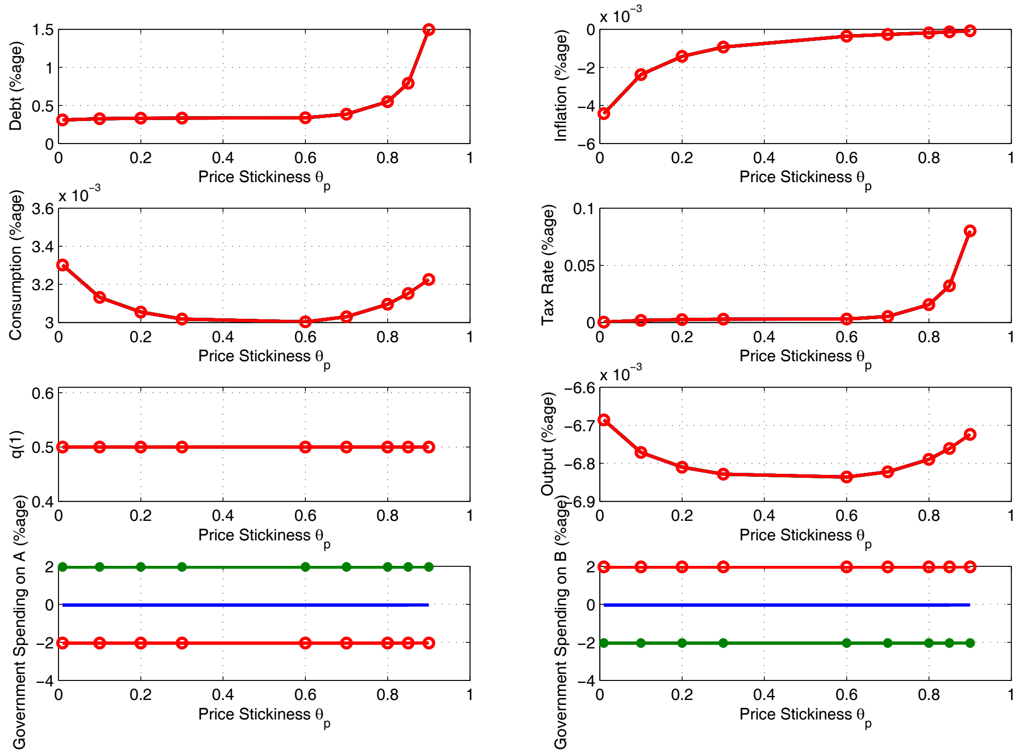

Figure 6: Steady-state with composition heterogeneity as a function of the degree of price stickiness.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

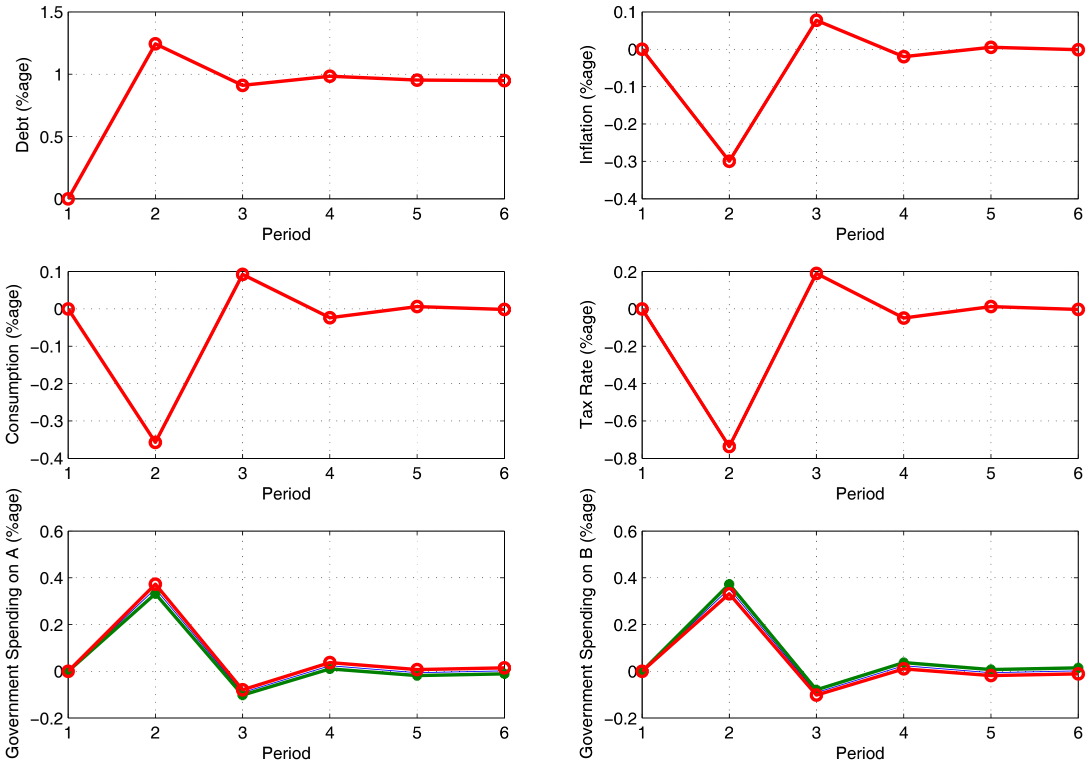

Figure 7: Impulse response to 1% technology shock with compositional heterogeneity.

Notes to Figure - green line with stars party 1's response, red line with hollow circles i party 2's response.

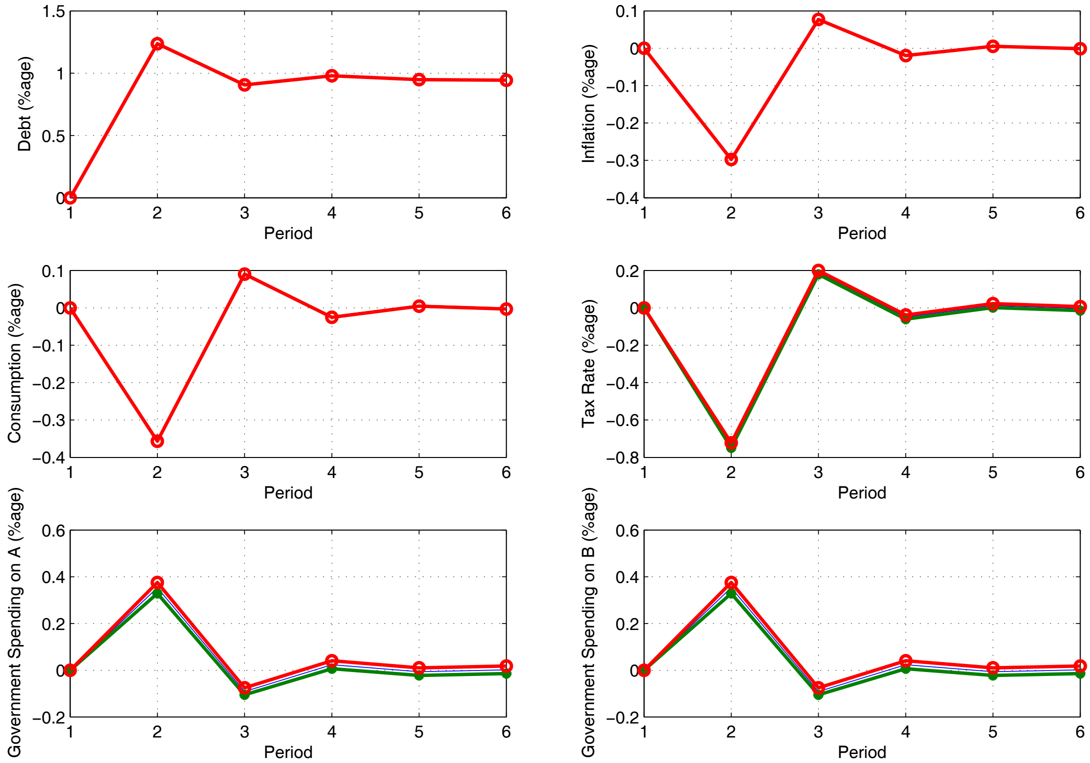

Figure 8: Impulse response to 1% technology shock with size heterogeneity.

Notes to Figure - green line with stars party 1's response, red line with hollow circles party 2's response.

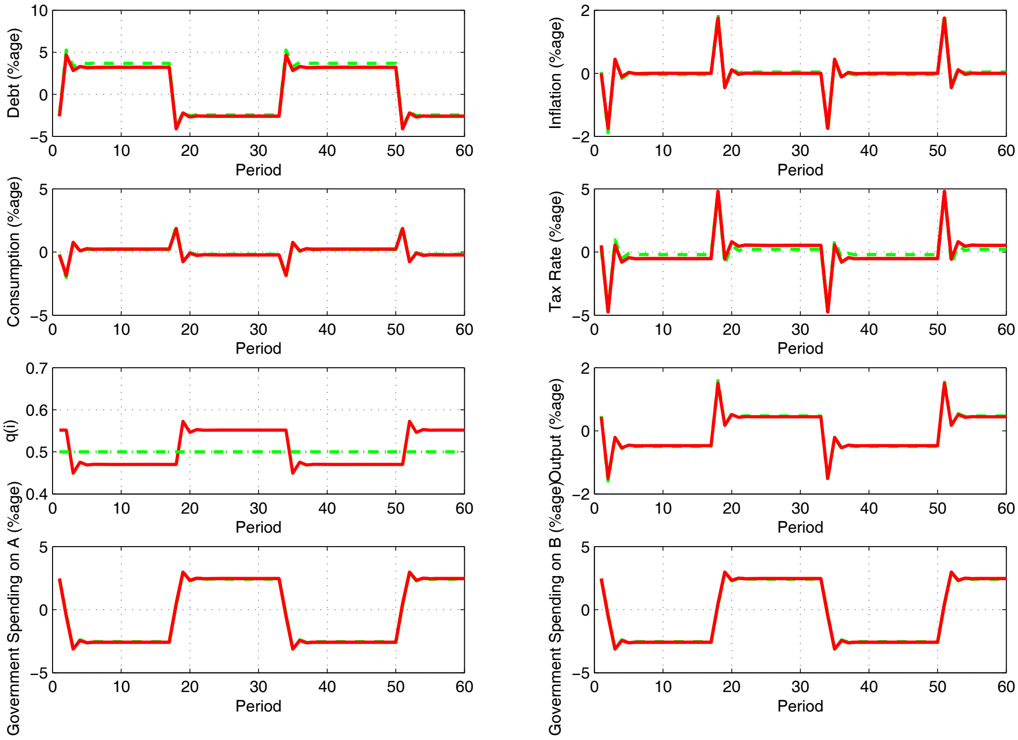

Figure 9: Impulse response with size heterogeneity and an electoral cycle with and without endogenous election victory probabilities.

Notes to Figure - solid red line, endogenous election victory probability; green dashhed line, exogenous election victory probability, q(i)=1/2.

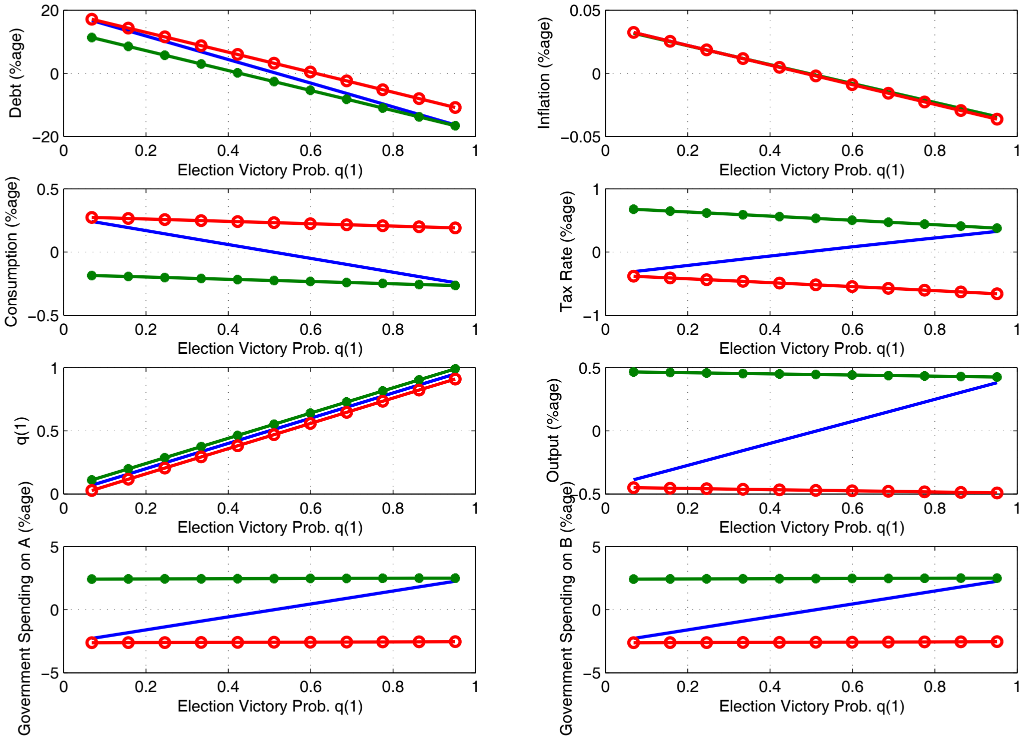

Figure 10: Steady-state with size heterogeneity as function of the (endogenous) election victory probability.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

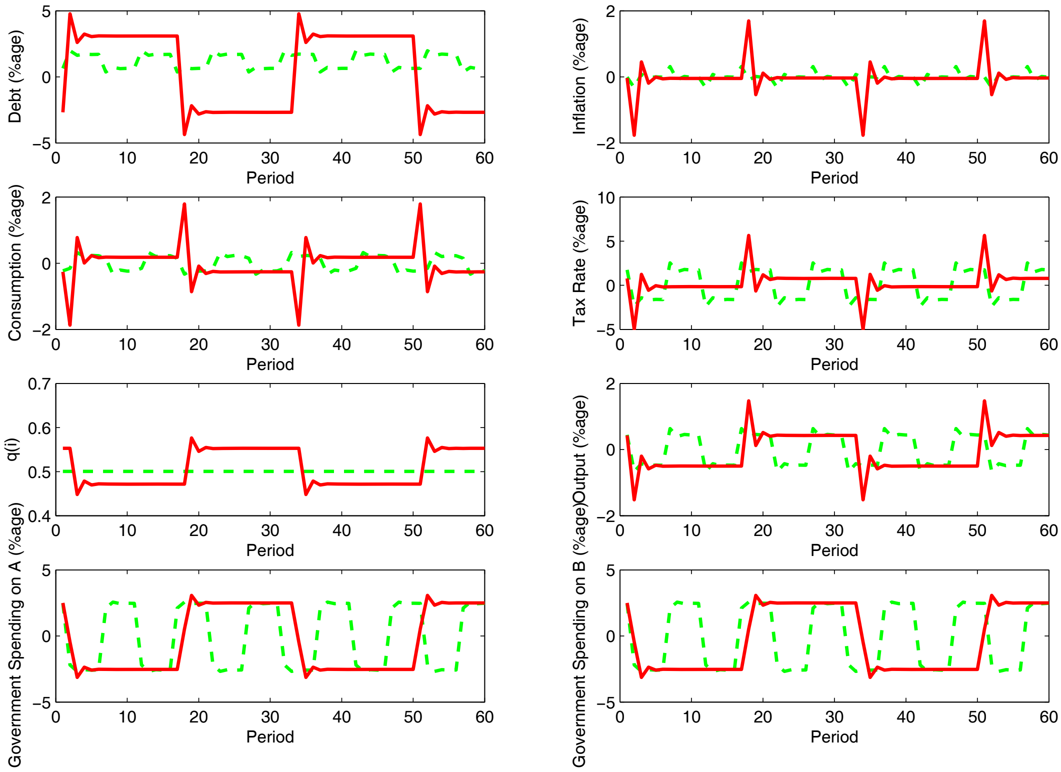

Figure 11: Impulse response under size heterogeneity with different election frequencies.

Notes to Figure - Expected electoral cycle of 4 years, e=1/16 - solid red line; election every quarter, e=1 - dashed green line.

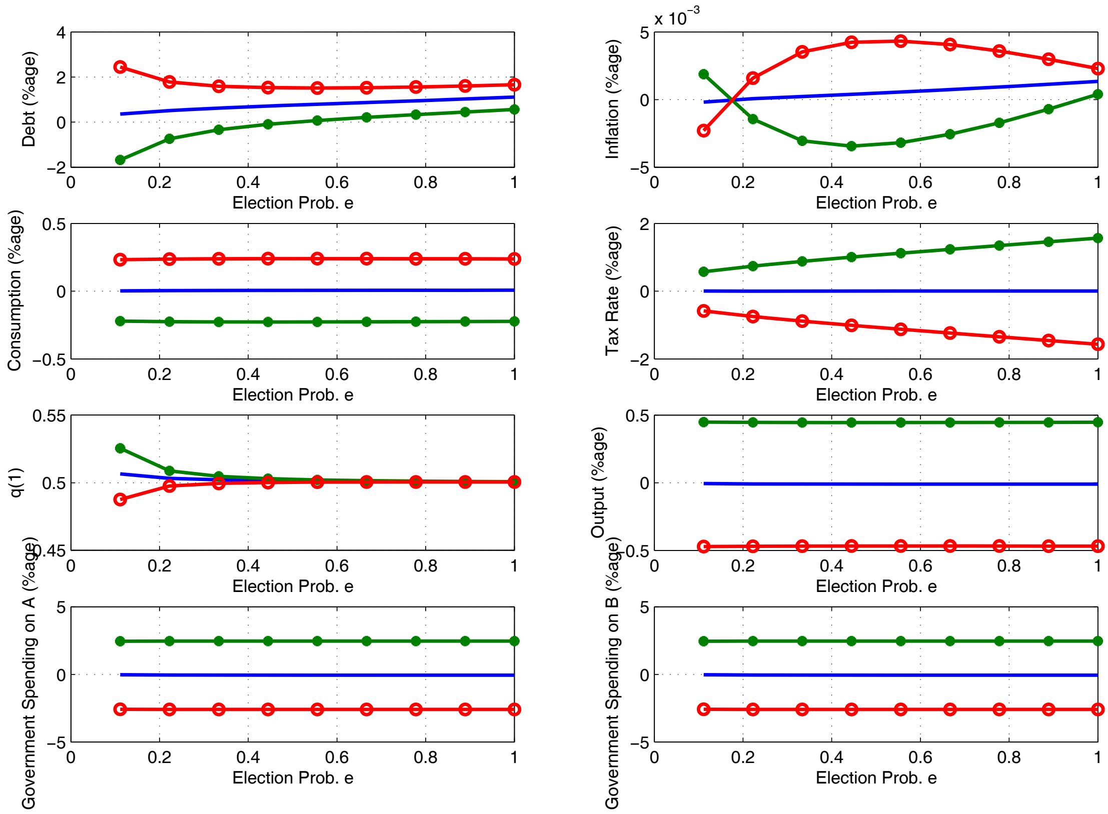

Figure 12: Steady-state with size heterogeneity as function of the election probability, e.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

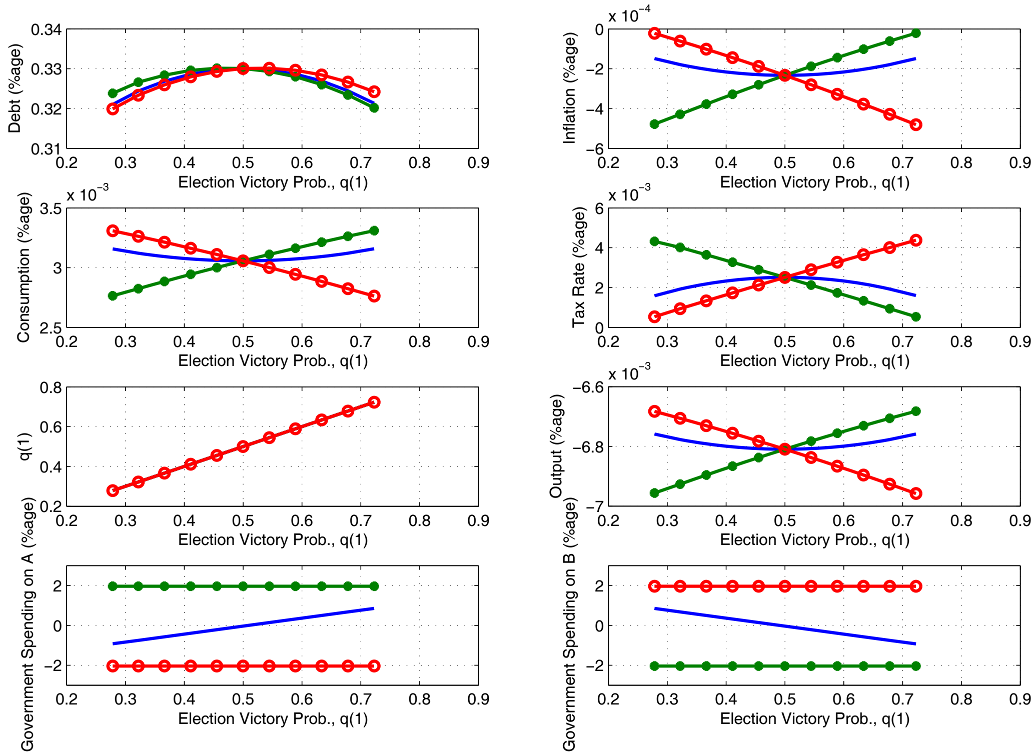

Figure 13: Steady-state with a composition heterogeneity as a function of the (endogenous) election victory probability.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

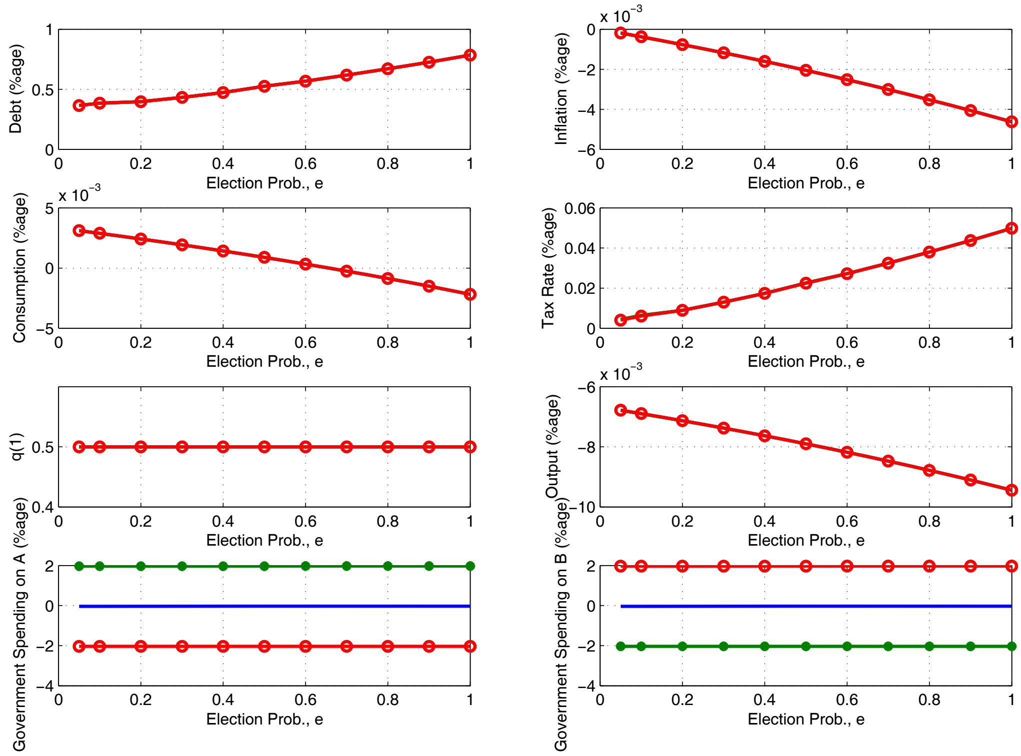

Figure 14: Steady-state with composition heterogeneity as a function of the election probability, e.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

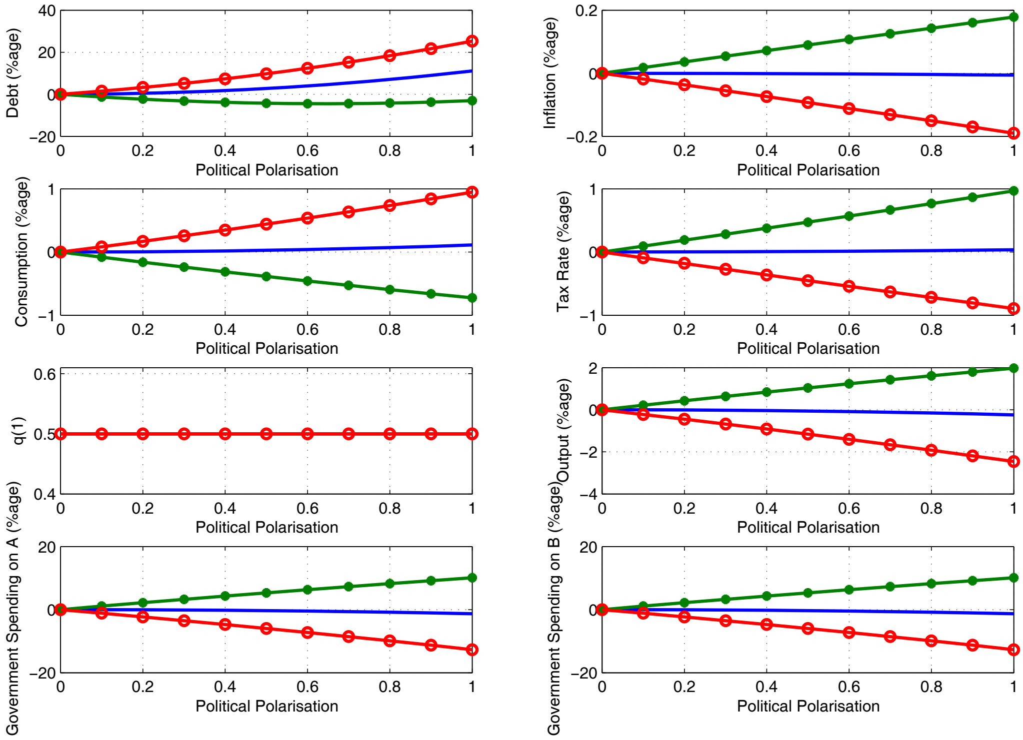

Figure 15: Steady-state with increasing conflict over government size.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.

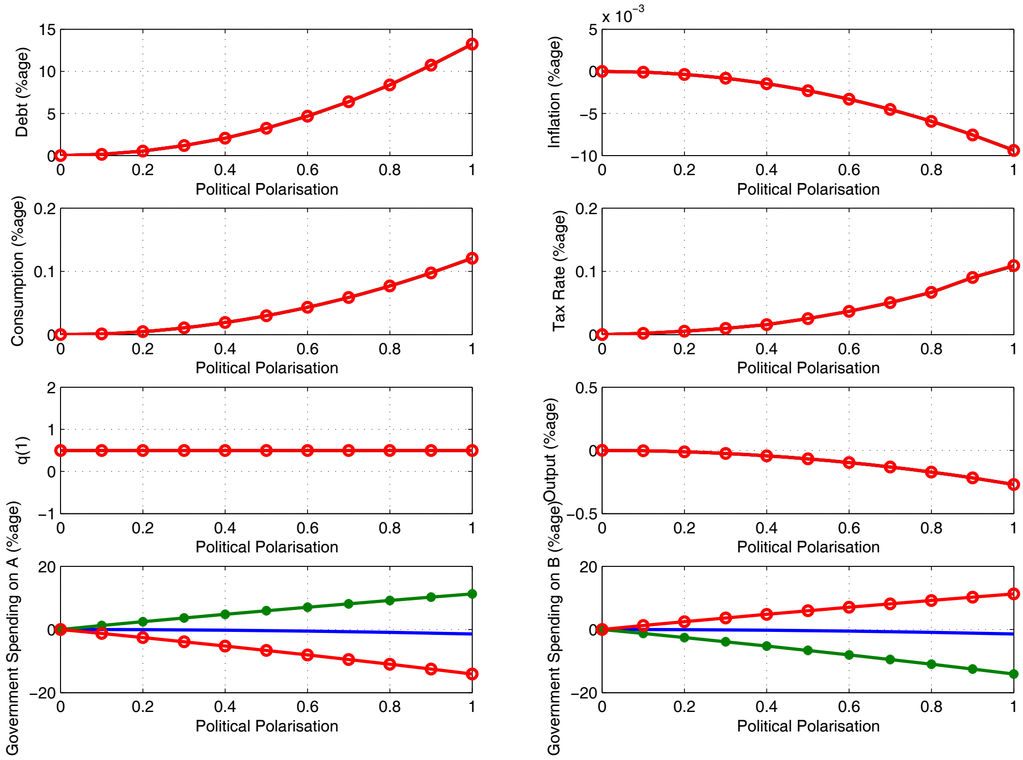

Figure 16: Steady-state with increasing conflict over the composition of government spending.

Notes to Figure - Stochastic steady-state - solid blue line; party 1 steady-state - green line with stars; and, party 2 steady-state - red line with hollow circles.