Vertical Integration, Missing Middle and Investor Protection in Developing Countries

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Abstract

The industrial organization of developing countries is characterized by: i) pervasive use of subcontracting arrangements among small firms, ii) "missing middle" in the firm size distribution, and iii) financially constrained firms. This paper studies an incomplete contract model in which the integration decision is chosen to maximize the returns of two vertically related projects to an external investor. The model jointly determines the financing, size and organization of firms. Vertical integration trades-off the benefits of joint liability against the cost of rendering the supply chain more opaque from the point of view of investors. The model shows that vertical integration is more likely to arise for intermediate levels of investor protection and that better contract enforcement reduces vertical integration only if financial markets are sufficiently developed. Moreover, the firm size distribution is more likely to display a missing middle in industries which favor vertical integration. The model sheds light on the industrial organization of developing countries showing that the motives for vertical integration are not necessarily higher in those countries.

Keywords: Vertical Integration, Missing Middle, Industrial Development, Financial Constraints, Joint Liability, Trade Credit, Community Based Industries.

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1 Introduction

This paper is motivated by three sets of facts about differences in industrial structure across countries. First, evidence of widespread subcontracting arrangements among small firms in the developing world (see e.g. the footwear industry in Taiwan (Levy (1990)), Mexico (Woodruff (2002)) and Brazil (Schmitz (1995)) contrasts with the intuition that, in response to input markets failures and poor contract enforcement, firms should tend to be larger and more vertically integrated in those countries (see e.g. Khanna and Palepu (1997, 2000)). Second, the firms size distribution in the typical developing country displays a "missing middle": the vast majority of workers is employed in very small firms (typically with less than five employees) with the rest working for very large firms. This is in stark contrast with the industrial structure in rich countries where a significant share of workers is employed by large and medium-size enterprises (see e.g. Snodgrass and Biggs (1996) and Cull et al. (2005)). Third, there is a large literature documenting significant cross-country differences in degrees of financial market development as well as substantial evidence that underdeveloped financial markets seriously constrain the investment decisions of firms in developing countries.\(^1\)

This paper explores the connections between borrowing constraints, vertical integration and the "missing middle" in a model in which the financing decision is jointly determined along with the organization of two vertically related projects.\(^3\) It studies an incomplete contract model in which a seller produces a good that can be used by a buyer, or sold on a spot market. The seller and the buyer have no cash and need to borrow from an external investor. Since entrepreneurs can steal part of the profits of their firms, however, they face borrowing constraints. The choice between vertical integration and non-integration is taken to maximize the pledgeable income (i.e. the expected returns that can be credibly promised to the investor) of the two projects. The model

\(^1\)Acemoglu et al. (2006) show that this anecdotal observation cannot be explained by differences in industrial composition: developing countries have relatively larger shares of firms in industries that are relatively more vertically integrated in richer countries. Rajan and Zingales (1995), Macchiavello (2006) and Kim and Shin (2007) provide further evidence that systematic differences in the degree of vertical integration across countries correlate with the degree of financial development.

\(^2\)See e.g. Banerjee and and Duflo (2004) for an excellent survey of evidence on borrowing constraints.

\(^3\)The model is meant to provide a framework to think about small and medium sized enterprises, for which borrowing constraints are relatively more likely to be important. In doing so, we do not intend to deny that other characteristics of the business environment in developing countries (such as low skills in the labor force, labor regulation and other reasons pushing firms in the informal sector) have potentially large effects on the size of firms and on the incentives to vertically integrate.
First, the paper compares pledgeable income under vertical integration and under non-integration. Vertical integration trades-off the benefits of joint liability against the cost of rendering the supply chain more opaque from the point of view of the external investor. Second, the model underlines the role of financial constraints as a barrier to the creation and expansion of firms. Low investor protection has an ambiguous effect on vertical integration: on the one hand it favors vertical integration because of the need to reduce ex-post incentives problems between buyers and sellers. On the other hand low investor protection biases the financing decision towards smaller, and therefore less vertically integrated, firms. Third, as a direct implication of these two observations, the distribution in firms size is more likely to display a missing middle in industries which favor vertical integration.

The model also considers the role of contractual imperfections between the buyer and the seller of the intermediate input. It shows that better contract enforcement (either formally provided through courts, or informally by community ties) has a negative impact on vertical integration only when financial markets are relatively developed. These predictions shed light on patterns in the industrial organization of developing countries and suggest that the relationship between (capital and input) market development and vertical integration is a complex one. The motives for vertical integration are not necessarily higher in developing countries.

This paper contributes to the literature on the relationship between financial development and the industrial organization of developing countries. Banerjee (2004) and Banerjee and Munshi (2004) present insightful anecdotal evidence on the relationship between financial constraints and vertical integration in industries based within communities in India. The model in this paper provides an analytical treatment of some of the issues treated in those papers. Mookerjee (1999) also provides a discussion of the costs and benefits of vertical integration in less developed countries, but he focusses on the role of uncertainty in input supply. More recently, Kranton and Swamy (2006) study the microeconomics of exporting in a model that also features multiple hold-up problems between various actors along the supply chain (exporters, agents and producers). They discuss reasons why vertical integration might not be feasible in institutionally poor environments, which

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4 This trade-off generates predictions that are consistent with received wisdom and existing empirical evidence on vertical integration. For example, it predicts that vertical integration is preferred in the presence of specificity and investments associated with financial outlays.
complements the insights of this paper. Moreover, they provide an analysis of putting-out systems, a hybrid organizational form closely related to some of the discussion in this paper. Finally, this paper is related to the large theoretical literature on microfinance and joint liability contracts across firms in developing countries (see e.g. Ghatak and Guinnane (1999) and Ghatak and Kali (2001)). An important difference, however, is that in our context the joint liability of the two productive units is linked to an input transaction.

This work is also related to the large literature on the theory of the firm. While we do not intend to downplay the importance of non-contractible investments in determining the vertical integration decision (a view formalized in the property right approach (see e.g. Hart (1995)), this paper emphasizes how vertical integration affects the ex-post (governance) relationship between the external investor and the entrepreneurs, and is therefore closer in spirit to transaction costs approaches to the boundaries of the firm (see e.g. Williamson (1971)). In defining a firm as a nexus of contracts characterized by centralized allocation of control rights and joint liability, the paper borrows from the legal literature (see e.g. Cheung (1983), Hausmann and Kraakman (2001)) as well as from the work of business historians (see e.g. Lamoreaux (1998)).5

The paper is organized as follows. Section 2 presents the model and the main assumptions. Section 3 derives the pledgeable income under vertical integration and under non-integration, and compares the two organizational forms. Section 4 characterizes the full financing decision and establishes the connection between the choice of vertical integration under financial constraints and the "missing middle". Section 5 provides an extension that considers the role of contractual frictions associated with the intermediate input transaction. Section 6 discusses joint liability contracts and areas for future research. Section 7 offers some concluding remarks.

2 The Model

Set up

Consider two managers, a buyer and a seller, $j \in \{d, u\}$ respectively in charge of two different projects: a downstream ($d$) unit and an upstream ($u$) unit. The upstream unit produces a good

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5The trade-off associated with vertical integration is reminiscent of the informal discussions in Williamson (1971) and Holmstrom (1999). This paper is also closely related to the literatures on trade credit (see e.g. Burkart and Ellingsen (2004)) and internal capital markets (see Stein (2003) for an excellent survey). The focus on the link between vertical integration and the institutional environment distinguishes this paper from those literatures.
that can be used by the downstream unit, or sold to an external market. The two managers are aware of the possibility that certain features of the input may make it best suited to be traded on the spot market, but they cannot foresee in advance the nature of these features, and hence cannot write an ex-ante contract which is contingent on the nature of ex-post trade.

The production process generates final cash flows $V$. The input can be purchased (resp. sold) on the spot market at price $p'$ (resp. $p''$). There is ex-ante uncertainty over the prevailing input market conditions. To simplify, assume that with probability $\pi$ the input can be purchased at price $p' = \overline{p}$, otherwise $p' = \underline{p} < \overline{p}$. For simplicity, let us assume that $p'' = \gamma p'$ with $\gamma < 1$. The upstream manager can always produce an appropriate input at cost $c$. A fraction $\mu$ of the costs is monetary and corresponds to a financial outlay borne by the owner (for example, it could correspond to the purchase of tools). The remaining fraction $1 - \mu$, however, is an effort cost that cannot be transferred and is borne by the manager, for example the opportunity cost of working to produce the intermediate input. Let $c$ be such that $\underline{p} < c < \overline{p} < V$. When $p = \underline{p} < c$, the upstream manager is not cost effective, and the input should be procured on the market. When $p = \overline{p} > c$, however, the upstream manager is cost effective and, if $\gamma < 1$, it is strictly more profitable for the input produced by the upstream manager to be sold to the downstream manager. The parameters $\pi$ and $\gamma$ therefore capture the specificity of the relationship.

As commonly assumed in the incomplete contracts literature, the realization of the state of nature is observable but not verifiable: it is observed by the two managers but not by third parties such as investors and courts.\footnote{Prices cannot be verified by courts because the exact features of the required input cannot be described. The court observes many input prices on the market, but does not know which input is appropriate.}

Ownership

Ownership determines residual control rights over the use of the input. We focus on two different configurations. Under non-integration the two units are separately owned and managed firms. In the absence of a previous enforceable contract, two independent firms trade with each other if and only if the two owners agree on a suitable price $P$ for the input. Under vertical integration the owner of the firm has to pay a wage $w$ to the upstream manager, but can impose by fiat whether the two divisions of the integrated firms should trade together or trade on the spot market.\footnote{To fix ideas, I focus on backward vertical integration, in which the manager of the upstream unit is an employee of the manager of the downstream unit (the owner).} Both $P$ and
\(w\) are negotiated through an efficient ex-post (Nash) bargaining process, in which the downstream manager receives a fraction \(\beta\) of the surplus, while the remaining fraction \(1 - \beta\) goes to the upstream manager.

Control over financial streams is transferred with ownership. We assume that the owner of a firm can hide and keep a fraction \((1 - \varphi) > 0\) of the monetary profits (cash) by making them unverifiable. If she does so, investors are not repaid.\(^8\) The parameter \(\varphi\) is a proxy for the degree of external investor protection in the economy: the higher \(\varphi\) the harder it is to steal cash and therefore the higher is the protection enjoyed by external investors.\(^9\)

The model assumes that the owner of the firm has to repay her employee and other input before she can steal profits. In other words, in stark contrast with repayment to an investor lending capital, there is no ex-post moral hazard to repay input providers and workers. This assumption is made to focus initial attention to the financial motives (and consequences) of vertical integration in Section 3 and 4. Section 5 relaxes the assumption and considers the more general case in which buyers can avoid payment to input suppliers as well.

In the remainder of the text, the downstream manager will also be called buyer under non-integration and owner under vertical integration. Similarly, the upstream manager will also be called seller under non-integration and worker (or employee) under vertical integration.

\textit{Initial Contract and Timing of Events}

The two managers have no cash and need to borrow in order to finance the start up costs of the two projects. These costs are denoted by \(k_d\) and \(k_u\) for the downstream and upstream unit respectively. If either of the two managers fail to get capital to finance her project, she can start a smaller project that requires no initial capital disbursement (she could, for instance, become self employed in the informal sector). To simplify, we normalize the payoff of starting such a business with no capital to be equal to zero. Since we are interested in determining i) which organizational form raises more external funds and ii) which projects can be financed by external investors, the

\(^8\)This assumption is stronger than necessary and it is done for the sake of simplicity. What is important for the analysis is that repayments to investors cannot be made fully contingent on the realization of profits. Moreover, as it will become clear from inspection of condition 3, the assumption of a linear stealing technology is made to avoid a bias in favor (resp. against) vertical integration purely originating from increasing (resp. decreasing) returns in such technology.

\(^9\)We assume that control over financial streams cannot be (fully) separated from ownership because (part of) the returns cannot be verified. For example, ownership entails the right to sign contracts with third parties that could be used to generate private benefits for the owner.
analysis assumes that there is a unique risk neutral investor who has all the ex-ante bargaining power. Contracts are thus signed to maximize the pledgeable income of the two projects, subject to the participation constraints of the two managers.\textsuperscript{10}

Only simple debt-like contracts are feasible. The investor holds a debt-like claim $B$ over the profits of a firm. When the firm is integrated there is a unique $B$. When the two firms are not integrated, the investor holds claims $B_d$ and $B_u$ on the profits of the downstream and upstream firms respectively. The monitoring costs associated with equity-like contracts are assumed to be prohibitively high.

To summarize, the timing of events is as follows (see Figure 1). At date 0 contracts are signed. These are the financial contracts signed with the external investor, and the creation of an integrated firm or of two separate firms (the allocation of control rights). At date 1, the state of nature is realized and observed by the managers. Given the ownership configuration, the two managers bargain over the input transaction. At date 2 profits are realized. Owners decide whether to hide profits or not. Finally, if profits have not been hidden, existing financial contracts are executed.

We focus for simplicity on the case $\beta = 1$ and $\gamma \simeq 1$ and relegate to the end of Section 6 a short discussion of the more general case. In order to avoid a taxonomy of cases that would not add to the intuition of the model we focus on the case $(1 - \varphi) (\overline{p} - \mu c) > (1 - \mu)c$. This corresponds to the case in which investor protection is relatively low, and the financial contract leaves some rents to

\textsuperscript{10}The two managers might also try to maximize the amount they borrow in order to transfer rents according to the initial distribution of bargaining power.
the upstream manager whenever non-integration is chosen as organizational form. This condition is necessarily verified when \( \mu = 1 \). The results for the case \( (1 - \varphi)(\overline{p} - \mu c) < (1 - \mu)c \) correspond to the case \( \mu = 1 \) in the main text.

3 Pledgeable Incomes

Pledgeable Income under Vertical Integration

Under vertical integration the investor chooses the debt level \( B \) in order to maximize the pledgeable income of the integrated firm. With probability \( \pi \) the market price for an appropriate input is \( p = \overline{p} > c \). The employee should produce the input at cost \( c \), and the two divisions of the integrated firm should trade together. The owner of the firm bargains with the employee over the wage \( w \). Since the downstream manager is the owner of the firm, the employee cannot produce the input and sell it on the market at price \( \gamma \overline{p} \), and therefore her outside option in the bargaining game is equal to zero. Focussing on the case in which the downstream manager has all the bargaining power (i.e. \( \beta = 1 \)), she offers a wage \( w = (1 - \mu)c \) that exactly compensates the employee for the costs associated with effort, and the offer is accepted. The owner of the firm also purchases the necessary tools and input required by the employee to produce the input at cost \( \mu c \).

The monetary profits (gross of debt repayments) of the firm are given by \( \Pi(\overline{p}) = V - c \). As owner of the firm the downstream manager repays debt \( B \) if this is more profitable than hiding the monetary profits, and keeping a fraction \( 1 - \varphi \) of them. This happens if \( \Pi(\overline{p}) - B \geq (1 - \varphi) \Pi(\overline{p}) \), i.e. if \( B \leq \varphi(V - c) \).

Similarly, with probability \( 1 - \pi \) the market price for an appropriate input is \( p = \underline{p} < c \). The input is procured on the spot market at price \( \underline{p} \). The monetary profits (gross of debt repayments) of the firm are given by \( \Pi(\underline{p}) = V - \underline{p} \), so that the owner of the firm repays debt \( B \) if and only if \( B \leq \varphi(V - \underline{p}) \).

The investor trades-off a higher debt level \( B \) with a higher probability that the debt is repaid. The investor can set \( B = \varphi(V - c) \) and be repaid regardless of the state of the world, or set \( B = \varphi(V - \underline{p}) \) and be repaid only with probability \( 1 - \pi \). The following proposition summarizes the previous discussion.
Proposition 1  The pledgeable income of an integrated firm is given by

\[ P_{\text{int}} = \varphi \max\{ (V - c), (1 - \pi)(V - p) \} \]  

The pledgeable income of an integrated firm is i) increasing in the degree of investors’ protection \( \varphi \), ii) increasing in downstream cash flows \( V \), iii) decreasing in the (expected) cost of the input (\( c \), \( \pi \) and \( p \)) and iv) independent of the composition of input costs \( \mu \).

Pledgeable Income under Non-Integration

Under non-integration the two units are two independent firms managed by two separate owners. The investor chooses debt levels \( B_d \) and \( B_u \) in order to maximize the joint pledgeable income of the two non-integrated firms.

With probability \( 1 - \pi \) the market price for an appropriate input is \( p = p < c \). The input should be procured on the spot market at price \( p \). The monetary profits (gross of debt repayments) of the downstream and upstream firm are respectively given by \( \Pi_d(p) = V - p \) and \( \Pi_u(p) = 0 \). The owner of the downstream firm repays debt \( B_d \) if and only if \( B_d \leq \varphi(V - p) \), while the owner of the upstream firm never repays debt.

With probability \( \pi \) the market price for an appropriate input is \( p = p > c \). The two firms should trade together. The owners bargain over the price \( P \) for the input. Since we focus on the case in which the downstream manager gets all the surplus (i.e. \( \beta = 1 \)) and the value of the input inside the relationship is approximately equal to its sale value on the market (i.e. \( \gamma \simeq 1 \)), the buyer proposes a price \( P = \gamma p \simeq p > c \) for the input, and the offer is accepted by the seller. The monetary profits (gross of debt repayments) of the downstream and upstream firm are respectively given by \( \Pi_d(p) = V - p \) and \( \Pi_u(p) = p - \mu c \). The buyer repays debt if and only if \( B_d \leq \varphi(V - p) \).

Similarly the seller repays debt if and only if \( B_u \leq \varphi(p - \mu c) \). Note that the seller monetary payoff is given by \( (p - \mu c) - B_u \) which is strictly larger than her effort cost \( (1 - \mu)c \), under the assumption \( (1 - \varphi)(p - \mu c) > (1 - \mu)c \). \(^{11}\)

As for the case of integration, the investor trades off a higher debt level \( B_i \) in each firm \( i \in \{d, u\} \) with a higher probability that debt is repaid. For the upstream firm, the optimal debt level is

\(^{11}\)If \( (1 - \varphi)(p - \mu c) < (1 - \mu)c \) the participation constraint of the upstream manager is binding and \( B_u = p - c \). The pledgeable income of the upstream firm ceases to depend on \( \mu \), and the analysis is as in the main text for the special case \( \mu = 1 \).
obviously given by $B_u = \varphi (\overline{p} - \mu c)$. This debt is repaid with probability $\pi$, and the pledgeable income of the upstream firm is given by $P_u = \pi \varphi (\overline{p} - \mu c)$. The pledgeable income of the downstream firm is given by $P_d = \varphi \max\{(V - \overline{p}), (1 - \pi)(V - \overline{p})\}$.

The following proposition summarizes the previous discussion.

**Proposition 2** The pledgeable income of two non-integrated firms is given by

$$P_{ni} = \varphi \left[ \max\{(V - \overline{p}), (1 - \pi)(V - \overline{p})\} + \pi (\overline{p} - \mu c) \right] \quad (2)$$

The pledgeable income of two independent firms is i) increasing in the degree of investors’ protection $\varphi$, ii) increasing in downstream cash flows $V$, iii) decreasing in the expected cost of the input ($c$, $\pi$, $\overline{p}$ and $\overline{p}$), and iv) decreasing in the share of monetary costs borne by the owner $\mu$. In contrast to the integration case, the pledgeable income depends on $\overline{p}$ and $\mu$. Under non-integration the pledgeable income depends on $\overline{p}$ because when the two firms trade together the price prevailing in the input market $p = \overline{p}$ pins down, through bargaining, the division of surplus between the two firms. It also depends on $\mu$ because of a fundamental accounting difference between vertical integration and non-integration. While value added and profits are equal along the chain under the two organizational forms, the monetary profits are not. This is because under integration the non-monetary costs $(1 - \mu)c$ are transformed into a monetary disbursement corresponding to a wage, while under non-integration they are not.

Under non-integration, the outcome of the bargaining process $P$, and therefore the division of surplus between the two firms, does not depend on the debt levels $B_d$ and $B_u$. In fact, the pledgeable income of the non-integrated structure is simply given by the sum of the pledgeable incomes of the two firms, i.e. $P_{ni} = P_d + P_u$. Moreover, since the buyer has all the ex-post bargaining power ($\beta = 1$) and the input can be sold and procured on the market at approximately the same price ($\gamma \approx 1$), the pledgeable income of firm $d$ is independent of whether firm $u$ has been financed or not (and vice versa). As a consequence, all financial externalities between the two separate firms have been removed, and the optimal financial contract can be obtained independently for each firm: different investors financing $u$ and $d$ separately would therefore sign the same financial contracts. The assumption of a unique investor is therefore immaterial for the results in this setting.
Comparison of the two structures

The next proposition provides necessary and sufficient conditions under which vertical integration delivers higher pledgeable income than non-integration.

**Proposition 3** \( P_{\text{int}} \geq P_{\text{ni}} \) if and only if

\[
\min\{\pi (V - \overline{p}) + (1 - \pi)p, (1 - \pi)\overline{p}\} \geq (1 - \mu \pi)c \tag{3}
\]

**Proof:**

Note first that \( P_{\text{ni}} \) and \( P_{\text{int}} \) are strictly increasing, convex and piecewise linear in \( V \) and that their slopes with respect to \( V \) are either equal to \( \varphi \) or to \( \varphi(1 - \pi) \). This implies that the two curves will cross at most once. When \( V \) is sufficiently large, we have \( P_{\text{int}} > P_{\text{ni}} \) if and only if \( (\overline{p} - c) \geq \pi (\overline{p} - \mu c) \) (denote this as condition \( N \)). When, however, \( V \) is sufficiently low, we have \( P_{\text{int}} < P_{\text{ni}} \). The two curves cross exactly once if \( (\overline{p} - c) \geq \pi (\overline{p} - \mu c) \), then \( (V - c) > (V - \overline{p}) + \pi \overline{p} \) and since \((1 - \pi)(V - \overline{p}) + \pi(\overline{p} - c) > (1 - \pi)(V - \overline{p}) \), the two curves must cross when \( P_{\text{int}} = \varphi(V - c) \) and \( P_{\text{ni}} = \varphi((1 - \pi)(V - \overline{p}) + \pi(\overline{p} - \mu c)) \). Substituting these expressions in the condition \( P_{\text{int}} > P_{\text{ni}} \), rearranging terms, and considering condition \( N \) above, gives the expression in the proposition. ■

Depending on parameters, the pledgeable income might be higher either under vertical integration or under non-integration. The costs and benefits of vertical integration depend on the relative magnitudes of various effects: a positive joint liability effect, against two negative accounting and demonitoring effects.

To see why vertical integration has a positive joint liability effect, note that since \( V \) is fixed, the two units have negatively correlated returns. When \( p = p_\overline{} \), for instance, the returns are highest for the downstream unit, and lowest for the upstream one. The opposite is true when \( p = \overline{p} \). By setting \( B = \varphi(V - c) \) for an integrated firm, the investor can make sure that this amount is repaid in both states of the world. In contrast, under non-integration, investors can never be repaid by both firms in both states of the world.

The benefits of joint liability come with costs. First, vertical integration comes with a negative accounting effect. As noted above, under vertical integration a fraction of the non-monetary effort costs \((1 - \mu)c\) are transformed into a monetary flow associated with the payment of a wage, while
under non-integration this is not the case. Therefore the monetary profits under non-integration are higher than under vertical integration, and the pledgeable income tends to be correspondingly higher.

Second, vertical integration has a negative demonitoring effect with respect to non-integration. The investor faces a sort of moral hazard in team problem associated with the possibility of stealing monetary profits: non-integration improves over integration by making available a second performance measure. In particular, should the downstream firm default (which might occur in state $p = \bar{p}$) under non-integration the investors will at least be repaid out of the profits of the upstream firm.

The accounting effect comes from the presence of the non-monetary effort costs $(1 - \mu)c$. This can be seen by noting that the condition in 3 is more likely to be verified if $\mu$ is large. The accounting effect is consistent with the commonly held view that independent ownership is better when human capital investments are important, while coordination of capital investment is better achieved through centralized ownership (see e.g. Holmstrom and Tirole (1991)).

Are the joint liability and demonitoring effects plausible as well? In order to answer this question we briefly discuss the setting in which the accounting effect is shut down (i.e. the case $\mu = 1$).

**Corollary** Suppose $\mu = 1$. Then $P_{int} \geq P_{ni}$ if and only if

$$\pi (V - \bar{p}) \geq (1 - \pi)(c - \bar{p})$$

(4)

As already noted, the condition in 4 enlarges the set of parameters for which integration is more likely to deliver higher pledgeable income. The condition in 4 is more likely to be satisfied for higher degrees of specificity linking the two projects (i.e. if $\pi$ is high). When $\pi$ is high, the two units are likely to trade with each other and therefore the indirect monitoring generated by market transactions is of little value. The model suggests that firms that use specific inputs that are geographically localized and costly to transport should be jointly owned and financed, i.e. vertically integrated. This prediction is consistent with anecdotal as well as formal evidence (see e.g. Whinston (2003)) and echoes the property rights result that complementary assets should be jointly owned, although assets complementarity is defined differently here. As a consequence of this result, in a cross section of firms, two divisions within an integrated firm are more likely to
trade with each other than two separated firms. Since assets with high \( \pi \) will be owned, financed and managed together, the model suggests that vertically integrated firms are inherently different from non-integrated firms. This selection effect is consistent with the evidence in Mullainhathan and Scharfstein (2001). These considerations suggest that the combination of the monitoring and joint liability effects delivers plausible predictions.\(^{12}\)

The monitoring and accounting effects capture the intuition that the bargaining process associated with market transactions might be an indirect source of information from the point of view of external investors. These effects echo the informal discussion in Williamson (1971), in which it is argued that "(...) unable to monitor the performance of large, complex organizations in any but the crudest way (...) investors demand larger returns as finance requirements become progressively greater, ceteris paribus". The model suggests that vertically integrated firms are more complex and inherently more difficult to monitor because they bring the bargaining process within firm's boundaries.\(^{13}\)

Following the seminal work of Adelman (1955), a commonly used index to measure vertical integration is given by the ratio of value added \( VA \) over sales revenues \( V \), i.e. \( VI = \frac{VA}{V} \), where value added is defined as the difference between revenues and material input costs (i.e. the value added is defined as the sum of profits plus payments to labor and capital). The main idea beyond the index is that, ceteris paribus, a vertically integrated firm has higher value added, since more stages of the production process are performed in-house. The model nicely captures the foundation for the index.

**Remark** Under constant value added along the chain across organizational forms (i.e. \( VA_{int} = VA_{ni} \)), \( VI_{int} > VI_{ni} \)

**Proof:**

Consider first the case \( p = \overline{p} \). To see that the value added is constant under the two organizational forms note that under integration \( VA_{int}(\overline{p}) = V - \mu c \) while under non-integration

\(^{12}\)The binary structure of the model and the linear "stealing" technology imply that the conditions 3 and 4 do not depend on the degree of investors' protection \( \phi \). Obviously, decreasing (resp. increasing) returns in the stealing technology would make the pledgeable income higher under vertical integration (resp. non-integration).

\(^{13}\)Related to the monitoring effect, Garcia-Appendini (2006) provides some evidence that banks base their credit-granting decisions on information about the trade credit relationships between the firm and its input suppliers. Information on trade credit can be obtained from information brokers or directly from a firm's suppliers. In both cases, this information becomes unavailable for at least one of the two divisions if the firm is vertically integrated.
\[ VA_{ni}(\overline{p}) = VA_d(\overline{p}) + VA_u(\overline{p}) \] and since \[ VA_d(\overline{p}) = (V - \overline{p}) \] and \[ VA_u(\overline{p}) = (\overline{p} - \mu c) \] we have \[ VA_{ni}(\overline{p}) = V - \mu c. \] The indexes are therefore \[ VI_{int}(\overline{p}) = (1 - \frac{\mu c}{\overline{p}}) > VI_{ni} = \frac{(1 - \frac{\overline{p}}{V}) + (1 - \frac{\overline{p}}{V})}{2}. \]

Consider now the case \( p = \overline{p}. \) We have \[ VA_{int}(p) = VA_{ni}(p) = V - p. \] As \( VA_u(p) = 0. \) It follows that \( VI_{int}(\overline{p}) = VI_{ni}(\overline{p}). \)

Finally, the fact that any convex combination of the two cases implies \( VA_{int} = VA_{ni} \) and \( VI_{int} > VI_{ni} \) concludes the proof.

4 Financing Decision and Missing Middle

So far the analysis has compared pledgeable incomes under vertical integration and under non-integration. This section characterizes the optimal financing decision from the point of view of the investor. It focuses on the special case in which \( \mu = 1 \) and the assumption \( (1 - \varphi)(\overline{p} - \mu c) > (1 - \mu)c \) holds for all \( \varphi. \) This is done for the sole sake of clarity: under the more general case the analysis goes through unchanged provided \( \varphi \) is low enough for the condition to hold.

Before we characterize the optimal investment decision from the point of view of the investor, note that there exist profitable investment opportunities that cannot be financed because of low investor protection. This easily follows from the fact that \( \max\{P_{int}, P_{ni}\} < V - C, \) where \( C = \pi c + (1 - \pi)p \) is the level of expected costs.

The investor can choose among projects with different sizes and returns. She could choose one "large" project, in which case she finances the entire value chain (i.e. the two units \( d \) and \( u \)) under the umbrella of a single vertically integrated firm. She could also decide to finance both units organizing the value chain with two independent firms. Let us denote those two options by the subscript \( int \) and \( ni \) respectively. Alternatively, she can choose one of two "small" projects, i.e. financing only one of the two units (either \( u \) or \( d \)), if this delivers higher expected returns. The subscripts \( d \) and \( u \) will denote those two projects. Since, as noted in the previous section, the pledgeable income of two separated firms \( P_{ni} \) is given by the sum of the pledgeable incomes of the two units, i.e. \( P_{ni} = P_u + P_d, \) the option \( ni \) available to the investor is simply the sum of the two projects \( d \) and \( u. \) The set of available options is given by \( \mathcal{P} = \{d, u, int, ni\}. \)

To characterize the optimal investment decision, let us denote the fixed costs associated with the downstream and upstream units with \( k_d \) and \( k_u \) respectively, and let \( k = k_u + k_d \) be the fixed
costs to finance the entire value chain, regardless of organizational form, i.e. \( k_{\text{int}} = k_{\text{ni}} = k \). The (net) present value \( PV_p \) of a project \( p \in \mathcal{P} \) is given by the difference between the expected returns from the project and the corresponding fixed costs. The expected returns are simply given by the pledgeable incomes derived in the previous section, for the various cases. Therefore, \( PV_p = P_p - k_p \).

The investor chooses the option \( p \in \mathcal{P} \) that delivers the highest net present value.

A necessary and sufficient condition for vertical integration to be chosen by the investor is that

\[
P_{\text{int}} - k \geq \max\{P_u - k_u, 0\} + \max\{P_d - k_d, 0\} \geq P_{\text{ni}} - k
\]  

From this condition, the following proposition follows.

**Proposition 4** If \( P_{\text{int}} \geq P_{\text{ni}} \), there exist unique \( \varphi^* \) and \( \varphi^{**} \) such that vertical integration is chosen if and only if \( \varphi \geq \varphi^* \) and no firm is financed if \( \varphi \leq \min\{\varphi^*, \varphi^{**}\} \);

If \( P_{\text{int}} < P_{\text{ni}} \) vertical integration is never chosen. Moreover, there exist unique \( \varphi' \geq \varphi'' \), such that non-integration is chosen if and only if \( \varphi \geq \varphi' \) and no firm is financed if \( \varphi \leq \varphi'' \).

**Proof:**

Consider first the case \( P_{\text{int}} \geq P_{\text{ni}} \). Since \( P_{\text{int}} \) and \( P_{\text{ni}} \) are both strictly increasing functions of \( \varphi \), there exists a unique \( \varphi^* \) such that \( P_{\text{int}}(\varphi^*) - k = 0 \) and integration is not chosen if \( \varphi < \varphi^* \).

Obviously \( P_{\text{ni}}(\varphi^*) - k \leq 0 \), and therefore the two firms are never financed in a non-integrated way. Define \( \varphi_d^* \) and \( \varphi_u^* \) as the thresholds of \( \varphi \) at which \( P_d(\varphi_d^*) - k_d = 0 \) and \( P_u(\varphi_u^*) - k_u = 0 \) respectively. Since \( P_{\text{ni}} = P_d + P_u \leq P_{\text{int}} \) it follows that \( \max\{\varphi_d^*, \varphi_u^*\} \geq \varphi^* \), with equality holding if and only if \( P_{\text{ni}} = P_{\text{int}} \) and \( \varphi_d^* = \varphi_u^* \). Defining \( \varphi^{**} = \min\{\varphi_d^*, \varphi_u^*\} \), and noting that \( \varphi^{**} \leq \varphi^* \) depending on parameters, concludes the proof of the first part of the proposition.

The proof of the second part of the proposition is very similar. Obviously, there exists a unique \( \varphi' = \max\{\varphi_d^*, \varphi_u^*\} \) such that if \( \varphi \geq \varphi' \) two firms are financed, and non-integration is chosen. Similarly, defining \( \varphi'' = \min\{\varphi_d^*, \varphi_u^*, \varphi^*\} \), if \( \varphi \leq \varphi'' \) no firm is financed. For \( \varphi \in (\varphi', \varphi'') \) one firm is financed (either the downstream, the upstream firm or a vertically integrated firm). However, since \( P_{\text{ni}} = P_d + P_u > P_{\text{int}} \) simple algebra shows that \( \min\{\varphi_d^*, \varphi_u^*, \varphi^*\} \neq \varphi^* \), and therefore vertical integration never arises in equilibrium. This concludes the proof. \( \blacksquare \)

If \( P_{\text{ni}} \geq P_{\text{int}} \), vertical integration is never chosen. If both units \( u \) and \( d \) deliver positive net
present values, i.e. \( P_d > k_d \) and \( P_u > k_u \), the investor finances a non-integrated value chain. If either of the two projects has a negative net present value, the investor does not finance it. If, however, \( P_{int} > P_{ni} \), vertical integration is chosen if it delivers higher returns than any combination of the two smaller projects \( d \) and \( u \): \( P_{int} > P_{ni} \) is necessary but not sufficient for vertical integration to emerge.

The financial constraints implied by low investor protection can take different forms depending on the type of industry. In industries in which non-integration delivers the highest pledgeable income for the entire value chain, financial constraints imply that for relatively high \( \varphi \) two vertically disintegrated firms are financed, for intermediate \( \varphi \) only one firm is financed and, finally, for low \( \varphi \) no firm is financed at all. In this type of industry financial constraints mainly affect the creation of firms. In industries in which vertical integration delivers the highest pledgeable income, financial constraints imply that for sufficiently high \( \varphi \) a large vertically integrated firm is created while for relatively low \( \varphi \) no firm is created. There might be intermediate values of \( \varphi \) such that a single smaller firm is financed, but this does not need to be the case. In this type of industry financial constraints mainly affect the size of firms, since low investor protection biases the financing decision towards relatively smaller projects and financing a relatively small firm might maximize an investor's returns. This result immediately follows from the fact that only a fraction \( \varphi \) of the returns of a project is pledgeable. As a corollary to this observation, vertical integration becomes less likely when external investor protection is low.\(^\text{14}\)

A striking difference between the economic organization in rich and poor countries is the so-called "missing middle" in the distribution of firm size. In the typical developing country a very high share of workers are employed in enterprises with less than 5 employees and (almost) no capital investment, while most of the rest work for very large firms, leaving little room for employment in medium sized firms. This is in stark contrast with the richest countries where a significant share of workers is employed by large and medium-size enterprises.

The proposition sheds some light on the relationship between vertical integration, financial constraints and the missing middle. Since the outside option of the managers is given by starting a very small firm with no capital investment, a "missing middle" arises when investors finance large

\(^{14}\)Acemoglu et al. (2006) and Macchiavello (2006) present strong evidence that the relationship between financial development and vertical integration depends on industry characteristics, although both papers focus on different mechanisms.
and vertically integrated firms or no firm at all. Formally, a "missing middle" is defined as:

**Definition**

Consider heterogeneous projects with respect to \( \phi \). A "missing middle" arises when either large, vertically integrated firms are financed or no project at all.

The definition introduces some heterogeneity across projects in the degree of investor protection \( \phi \). This heterogeneity could originate from small technological differences of projects within industries (e.g. location) or from idiosyncratic differences in the relationship between the managers and the investor (e.g. due to kinship membership). The condition under which a missing middle happens is given in the following proposition.

**Proposition 5** A missing middle arises if

\[
\max\left\{\frac{(V - \pi)}{k_d}, \frac{(1 - \pi)(V - p)}{k_d}, \frac{\pi (\varphi - c)}{k_u}\right\} \leq \frac{(V - c)}{k}
\]

**Proof:**

A necessary condition for a missing middle to arise is \( P_{int}(\phi) > P_{ni}(\phi) \). Since this condition implies that \( \frac{\partial P_{int}(\phi)}{\partial \phi} > \frac{\partial P_{ni}(\phi)}{\partial \phi} \) > 0, a missing middle exists if both \( P_{int} - k = P_d - k_d \) and \( P_{int} - k = P_u - k_u \) imply \( P_{int} - k < 0 \). Denote by \( R_p = \frac{P_p}{\phi} \), for \( p \in \{d, u, int\} \). The condition
\[ P_{\text{int}} - k = P_d - k_d \text{ implies } P_{\text{int}} - k < 0 \text{ if } \frac{R_{\text{int}}}{k} > \frac{R_d}{k_d}. \] Similarly, it must be \( \frac{R_{\text{int}}}{k} > \frac{R_u}{k_u} \) for \( P_{\text{int}} - k = P_u - k_u \text{ to imply } P_{\text{int}} - k < 0 \). Therefore a missing middle exists if \( \frac{R_{\text{int}}}{k} > \max\{\frac{R_u}{k_u}, \frac{R_d}{k_d}\} \), which is the expression in the proposition. This concludes the proof.\]

The condition in the proposition implies \( P_{\text{int}} > P_{\text{ni}} \). Figure 2 illustrates how a "missing middle" is, ceteris paribus, more likely to arise in industries in which vertical integration delivers higher returns to the external investor: it shows on the x-axis the degree of investor protection \( \varphi \) (which varies across countries) and on the y-axis the level of downstream cash flows \( V \) (or any other industry characteristics favouring vertical integration). Since, as documented by Acemoglu et al. (2006), developing countries have relatively more firms in industries which tend to have higher degrees of vertical integration in the U.S., the model suggests that a "missing middle" in the distribution of firms size should be relatively more likely in developing countries.\(^{15}\) A "missing middle" is also, ceteris paribus, more likely to arise in industries in which value is added along the chain in relatively equal proportions across the two units (i.e. the returns of the two small projects \( d \) and \( u \) are relatively equal).

\section*{5 Institutional Environment and Vertical Integration}

In practice, the buyer and the seller rarely exchange cash for the input at the same time. Trade credit is extended by suppliers to buyers when inputs are paid at a later date (see e.g. Burkart and Ellingsen (2004) for an insightful model of trade credit). Conversely, putting-out systems emerge when buyers collect finished inputs after having supplied upstream producers with cash or material advances (see e.g. Kranton and Swamy (2006) for a model of putting-out systems inspired by textile manufacturing during the colonial period in India).

\(^{15}\) This prediction of the model could be tested, subject to data availability, with the following empirical specification:

\[ \text{MM}_{ic} = \alpha + \mu_c + \eta_i + \beta \text{INT}_i * FD_c + \varepsilon_{ic} \]

where \( \text{MM}_{ic} \) is a measure capturing a missing middle in the distribution of firms size in industry \( i \) in country \( c \), \( \mu_c \) and \( \eta_i \) are country and industry fixed effects respectively, \( FD_c \) is a measure for the degree of financial development in country \( c \), and \( \text{INT}_i \) is a proxy for the tendency towards vertical integration in industry \( i \) (as proxied, for instance, by the degree of vertical integration in the US). Since the influential work by Rajan and Zingales (1998), this type of specification has been extensively used to disentangle the effects of various institutions on industry growth, structure, financing mode, trade patterns and vertical integration.
Under both systems (trade credit and putting-out), the contractual relationship between the buyer and the supplier could be plagued by similar agency problems as the one described in the model between the investor and the entrepreneurs. For instance, in the case of putting-out systems, the seller could use the cash or material advanced by the buyer to produce an input that would then be sold on the market. Under trade credit, the buyer could simply hide cash flows, avoiding repayment to investors as well as to suppliers (and/or employees).

This section discusses a simple extension of the model that allows the buyer to steal revenues avoiding repayment to the investor as well as to the supplier (or employee). The extension provides further insights on the relationship between vertical integration, trade credit and the institutional environment.16

The model is as in the previous section, the only difference being that entrepreneurs can, at some cost, avoid repaying input suppliers as well as external investors. After receiving the input at an agreed price \( \tilde{p} \) and producing final cash flows \( \tilde{V} \), assume that the entrepreneur can hide revenues and keep \((1 - \varphi)(\tilde{V} - \eta \tilde{p})\) for herself, with \( \eta \leq 1 \) parametrizing the degree through which it is possible to avoid repaying input suppliers. If \( \eta \to 1 \), an entrepreneur can only steal profits, as in the previous section. Conversely, if \( \eta \to 0 \), the entrepreneur can steal revenues, and completely avoid repaying suppliers.

The parameter \( \eta \) is related to the possibility of assuring repayment for the input and does not depend on the nature of the input transacted (e.g. labor vs. tools). The parameter \( \eta \) captures in a simple way the degree through which it is easy to avoid repaying suppliers of intermediate inputs, regardless of whether they are employees of the firm or arm’s length suppliers on the market. While \( \eta \) can be linked to the technology of production (for instance, if the input can be easily split in small components that are used at different dates, then \( \eta \to 1 \)), our preferred interpretation links \( \eta \) to features of the institutional environment in which the firms operate.

A first interpretation links \( \eta \) to the degree of contractual enforcement. In countries with relatively more efficient courts, it might be harder to avoid payments and circumvent the contractual obligation of paying suppliers. A second interpretation is that \( \eta \) proxies for the degree of informal monitoring mechanism available to suppliers. For example, within communities, it might be expected that \( \eta \) will be higher. Avoid repayment to suppliers and hiding profits might be at the same

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16 In contrast to the framework analyzed here, an analysis of putting-out systems requires the buyer to have cash to pay the seller in advance. Kranton and Swamy (2006) provides an insightful analysis of this organizational form.
time more difficult and more costly (in terms of loss of reputation) within a closed community.

The model can be solved following the same steps as in Section 3. Under vertical integration, with probability $\pi$ the two divisions of the integrated firm trade together and the owner of the firm bargains over the wage $w$ to be paid to her employee, the upstream manager. The assumed distribution of bargaining power implies that the owner of the firm offers a wage $w = (1 - \mu)c$ that exactly compensates the employee for the costs associated with effort, and the offer is accepted. The owner of the firm also purchases the necessary tools and input required by the employee to produce the input at cost $\mu c$. Given a debt level $B$, the owner repays the investor and her employee if $V - c - B \geq (1 - \varphi)(V - \eta c)$, i.e. if $B \leq \varphi V - \delta c$, where $\delta = (1 - \eta(1 - \varphi)) \geq \varphi$. Similarly, with probability $1 - \pi$ the market price for an appropriate input is $p = \underline{p} < c$, and the owner repays the investor and the external supplier if $B \leq \varphi V - \delta \underline{p}$. The investor trades-off a higher debt level $B$ with a higher probability that the debt is repaid. The only difference to the model in Section 3 is that the incentive compatibility constraint to induce repayment has to take into account the agency problem between the entrepreneur and her employee/supplier. The pledgeable income of an integrated firm is given by

$$P_{int}(\eta) = \max\{\varphi V - \delta c, (1 - \pi)(\varphi V - \delta \underline{p})\}$$

The analysis is very similar under non-integration. The pledgeable incomes of the downstream and upstream firms are respectively given by $P_d = \max\{\varphi V - \delta \underline{p}, (1 - \pi)(\varphi V - \delta \underline{p})\}$ and $P_u = \pi(\varphi \underline{p} - \delta \mu c)$. The pledgeable income of a non-integrated chain is given by the sum of the pledgeable incomes of the two independent firms,

$$P_{ni}(\eta) = \max\{\varphi V - \delta \underline{p}, (1 - \pi)(\varphi V - \delta \underline{p})\} + \pi(\varphi \underline{p} - \delta \mu c)$$

The pledgeable income under both structures is increasing in $\eta$ since, through the incentive compatibility constraint, the debt level has to take into account the incentive costs associated with repaying suppliers (or employees) as well.
Proposition 6  

\[ P_{int} \geq P_{ni} \text{ if and only if} \]

\[ \min\left(\frac{\pi \varphi}{1 - \eta(1 - \varphi)}(V - \bar{p}) + (1 - \pi)p, \left(1 - \frac{\pi \varphi}{1 - \eta(1 - \varphi)}\right)\bar{p}\right) \geq c(1 - \mu \pi) \quad (8) \]

Proof:

The proof of the proposition mimics the proof of Proposition 3. Note first that \( P_{ni} \) and \( P_{int} \) are strictly increasing, convex and piecewise linear in \( V \) and that their slopes with respect to \( V \) are either equal to \( \varphi \) or to \( \varphi(1 - \pi) \). This implies that the two curves will cross at most once. When \( V \) is sufficiently large, we have \( P_{int} > P_{ni} \) if and only if \( (\bar{p} - c) > \pi(\frac{\varphi}{c} - \mu c) \). When, however, \( V \) is sufficiently low, we have \( P_{int} < P_{ni} \). The two curves cross exactly once if \( (\bar{p} - c) > \pi(\frac{\varphi}{c} - \mu c) \) and never cross otherwise. If the two curves cross, they do so when \( P_{int} = \varphi V - \delta c \) and \( P_{ni} = (1 - \pi)(\varphi V - \delta \bar{p}) + \pi(\varphi \bar{p} - \delta \mu c) \). Rearranging terms gives the expression in the proposition.

The condition in 8 generalizes the condition in 3 for the case in which \( \eta \leq 1 \) (and therefore \( \delta \geq \varphi \)).

The conditions in 8 can be rewritten as

\[ \frac{1}{(1 - \varphi)} \left(1 - \frac{\varphi \pi \bar{p}}{\bar{p} - c + \pi \mu c}\right) = \eta(\varphi) \geq \eta(\varphi) = \frac{1}{1 - \varphi} \left(1 - \frac{\varphi \pi (V - \bar{p})}{c(1 - \mu \pi) - (1 - \pi)\bar{p}}\right) \quad (9) \]

Imperfections in the enforcement of contracts between the seller and the buyer \( \eta \) have an ambiguous effect on vertical integration. On the one hand, higher \( \eta \) reduces the costs associated with the accounting effect (which are given by \( \delta(1 - \mu)c \)). Moreover, since input transactions among independent firms tend to occur at higher prices, the rents necessary to give repayment incentives are higher under non-integration than under vertical integration. On the other hand, since the debt contract has to take into account the rents necessary to ensure the repayment of the supplier (and these rents are decreasing in \( \eta \)), higher \( \eta \) also reduces the benefits associated with joint liability. The first effect comes into play when \( \eta \) (and therefore pledgeable income) is relatively low. The second effect, in contrast, dominates when \( \eta \) (and therefore pledgeable income) is higher.

Since the two curves \( \eta(\varphi) \) and \( \eta(\varphi) \) are both decreasing in \( \varphi \) and never cross, contractual imperfections with external investors (\( \varphi < 1 \)) and with input suppliers (\( \eta < 1 \)) interact in a complex way. This complexity is illustrated in Figure 3, which shows on the vertical axis the
degree of contract enforcement between buyers and sellers $\eta$, and on the horizontal axis the degree of investor protection $\varphi$.

The two curves $\eta(\varphi)$ and $\eta(\varphi)$ illustrate the proposition and show that the effect of higher $\eta$ on the likelihood of vertical integration will tend to be positive at low levels of investor protection and negative at higher levels of investor protection. The model casts doubts on the view that vertical integration should be less prevalent in countries that have well functioning courts which enforce contracts and in industries based on closed communities. On this latter observation, there is a large literature documenting subcontracting within networks of small firms in community-based industrial districts (see e.g. Humphry (1995) for many examples from Latin America, and Brusco (1982) for Italy). Banerjee (2004) discusses examples of community based industries in India with various degrees of vertical integration. So, while the Stitched Garment industry in Calcutta is organized in relatively small but fully vertically integrated firms, the Knitted Garment industry in Tiruppur has firms belonging to the local dominant community fully vertically integrated, and firms owned by outsiders relatively disintegrated. Banerjee and Munshi (2004) suggest that the difference can be explained, at least in part, by differences in access to financing capital across the
two communities.\footnote{Acemoglu et al. (2006) and Macchiavello (2006) do not find that differences in formal contract enforcement institutions across countries correlate with the degree of vertical integration.}

Figure 3 also shows that, for a given level of \( \eta \), \( \varphi \) has an inverted-U shaped effect on the likelihood of vertical integration. This result is consistent with the empirical observation that corporate balance sheets in countries with developed financial systems have relatively larger items reflecting interactions among firms (see e.g. Rajan and Zingales (1995)) and with the lack of a linear relationship between vertical integration and financial development (see e.g. Acemoglu et al. (2006) and Macchiavello (2006)).

Figure 3 also combines the insights of Proposition 6 with the characterization of the financing decision in Section 4. As noted in Section 4, the condition \( P_{ni} > P_{mi} \) is necessary but not sufficient for vertical integration to arise in equilibrium. The Figure shows how vertical integration is more likely to emerge for intermediate levels of investor protection. This result comes from two contrasting effects: on the one hand, low levels of investor protection bias the financing decision towards smaller firms; while, on the other hand, high levels of investor protection make joint liability relatively less valuable.

Since, across countries, \( \varphi \) and \( \eta \) are likely to move together (countries with better courts enforcing contracts also protect the interests of external investors more) moving from the bottom-left corner towards the top-right corner of Figure 3 gives a sense of the changing industrial patterns as the institutional environment improves. In very underdeveloped institutional environments, there is no borrowing and firms operate with no capital. As the institutional environment improves, relatively small firms are created (in Figure 3 these are upstream firms). When the institutional environment reaches intermediate levels of development, the entire value chain is financed, under the umbrella of a single vertically integrated firm. As the institutional environment further improves, the value chain disintegrates leaving a "network" of firms interacting in relatively efficient input markets.\footnote{In the specific parametric configuration of Figure 2 neither a "missing middle" nor non-integration arises for intermediate values of \( \eta \) and \( \varphi \). This does not need to be the case. The likelihood of these various scenarios depends on the parameters \( k_u \) and \( k_d \).}
6 Joint Liability Contracts and Further Discussion

Joint Liability Contracts

The main advantage of vertical integration comes from joint liability. Under certain circum-
stances, the external investor could try to get the benefits of vertical integration without its costs
by having two separate companies signing joint liability contracts. Joint liability contracts between
independent firms can take the form of loan or mutual debt guarantees (as opposed to collateral)
and has been documented in several contexts (see e.g. Cull et al. (2005) for historical examples,
or Park and Shen (2003) for contemporary China).

This subsection briefly discusses joint liability contracts between two independent firms. The
model is as in the previous section, the only difference being that the initial contract signed at date
zero between the investor and the two managers also specifies "joint liability" payments, $L_d$ and
$L_u$, which are due by each firm in case of default of the other firm. Denote by $L_d$ (resp. $L_u$) the
amount of money that the downstream (resp. upstream) firm has to repay (on top of its original
debt $B_d$ (resp. $B_u$)) in case the upstream (resp. downstream) firm defaults.

For the sake of simplicity (and without losing further intuition), let us focus on the special case
$\mu = 1$ as in Section 4. The following proposition characterizes the optimal joint liability contract.

**Proposition 7** The optimal joint liability contract between non-integrated firms is given by $B_u = \varphi\pi - \delta c$, $B_d = \varphi V - \delta \varphi$, $L_d = \delta(\pi - \varphi)$ and $L_u = 0$.

The corresponding pledgeable income is given by

$$P^{JL}_{ni} = (\varphi V - \pi \delta c - (1 - \pi)\delta \varphi) - \pi(\delta - \varphi)\pi$$

(10)

**Proof:**

First note that the proposed contract is incentive compatible in both states of the world. When
$p = \varphi$ both firms repay since $B_u \leq \varphi\pi - \delta c$ and $B_d \leq \varphi V - \delta \varphi$. When, however, $p = \overline{p}$, the upstream
defaults while the downstream firm repays since $B_d + L_d = \varphi V - \delta \varphi$. With the proposed contract, the
pledgeable income of a non-integrated structure is given by $P^{JL}_{ni} = \pi(B_d + B_u) + (1 - \pi)(B_d + L_d) =
\varphi V - \pi \delta c - (1 - \pi)\delta \varphi - \pi(\delta - \varphi)\pi$, as reported in the proposition. To see why this is the optimal
contract, note that the pledgeable income is increasing in $B_d$, $B_u$ and $L_d$ and, in both states of
nature, all incentive compatibility constraints are binding. The investors solve a linear programming problem, whose solution will be at a corner. Obviously, $B_u$ cannot be increased, and there is no gain in increasing $L_u$, since the downstream firm never defaults. It remains to be checked that deviations in $B_d$ and $L_d$ are not profitable. Consider a reduction of $B_d$ by an amount $\varepsilon_b$ and an increase of $L_d$ by an amount $\varepsilon_l$. If $\varepsilon_l > \varepsilon_b$ the incentive compatibility constraint of the upstream firm is, in case of default, violated, and pledgeable income is reduced. If $\varepsilon_l < \varepsilon_b$ pledgeable income is also reduced, as $\varepsilon_l$ is only paid with probability $1 - \pi$. The opposite deviation (increasing $B_d$ by $\varepsilon_b$ and reducing $L_d$ by $\varepsilon_l$) is obviously unprofitable since it violates the incentive compatibility constraint when the upstream firm does not default. This concludes the proof.

Obviously, allowing for joint liability contracts increases the pledgeable income of two non-integrated firms. The first term in 10 is the highest pledgeable income for the value chain after taking into account the possibility of hiding monetary profits, and is given by the pledgeable revenues $\varphi V$ minus expected pledgeable costs $\delta \left( \pi c + (1 - \pi)p \right)$. The second term emerges from the accounting asymmetry between the input price $\varphi$ as a cost for the downstream firm and as a revenue for the upstream firm, and vanishes when $\eta = 1$.

A comparison of 6 and 10 reveals that vertical integration achieves higher pledgeable income than a joint liability contract between independent firms if

$$\pi(\delta - \varphi)\varphi > (1 - \pi)\delta(c - \varphi) \tag{11}$$

For sufficiently high $\eta$ (and therefore $\delta$ is approximately equal to $\varphi$) the condition can never be satisfied. This implies that joint liability contracts among independent firms are more likely to emerge in countries with relatively well developed court systems or in industries based on close community ties. This result echoes the discussion in the literature on microfinance and informal lending (see e.g. Ghatak and Guinnane (1999)) which emphasizes how a certain degree of social capital is necessary for joint liability contracts to be effective, as jointly liable entrepreneurs must be given not only the incentives, but also the means, to monitor each other to avoid default.

The optimal joint liability contract however has the property that the extra payment of the downstream firm in case of default of the upstream firm is higher than the debt level of the upstream firm: $L_d > B_u$ since $(\varphi - \delta)\varphi < 0 < \delta(c - \varphi)$. When this is the case, the investor is de facto holding
claims on the profits of the firm which are contingent on the identity of the contractual parties involved in the input transaction. This is obviously problematic, as the two firms have an incentive to "collude" and avoid the default of the upstream firm. For instance, when \( p = \overline{p} \), the two firms would have an incentive to pretend that the input was traded between the two firms at a price \( P = \overline{p} \) and share the rents given by \( (\delta - \varphi) \overline{p} + \delta (c - \overline{p}) \).

It is therefore convenient to compare the pledgeable income of the collusion-proof joint liability contract with the pledgeable income obtained under vertical integration. It can be shown that when the further constraint \( L_d \leq B_u \) is imposed the pledgeable income of two non-integrated firms under a joint liability contract is given by

\[
P_{\text{ni}}^{\text{JLCP}}(\delta) = \max\{\varphi V - \delta c - (\delta - \varphi)\overline{p}, (1 - \pi)(\varphi V - \delta \overline{p}) + \pi(\varphi \overline{p} - \delta c)\}
\]

This expression has to be compared with the pledgeable income of the vertically integrated firm in 6. Collusion-proof joint liability contracts increase the pledgeable income of two non-integrated firms by an amount equal to \( (1 - \pi)(\varphi \overline{p} - \delta c) \) when it is optimal to set \( B_d = \varphi V - \delta \overline{p} \). However, since \( (\delta - \varphi)\overline{p} \geq 0 \), we have that \( P_{\text{ni}}^{\text{JLCP}} \leq P_{\text{int}} \) in such a case. It follows that allowing for collusion-proof joint liability contracts does not change the condition for vertical integration given in 9. This suggests that the benefits of joint liability are better achieved under vertical integration: joint liability and centralized allocation of control rights are complements.

In practice, joint liability contracts are likely to have certain limitations that are not captured by this model. In particular, if certain activities are better organized under separate ownership (for example because of incentives considerations), joint liability contracts are likely to generate perverse effects precisely when these incentives considerations are more relevant: in the event of default of one firm, incentives to perform are reduced for the second firm (Besley and Coate (1995) make the point in the context of microfinance schemes).

The model defines a firm as an organization with joint liability and separate control. Loan and mutual debt guarantees are examples of hybrid organizational forms in which control is kept separate, while financial liability is (at least partially) joined. Business groups and pyramids are symmetric hybrid organizational form in which firms are jointly managed but are not jointly liable for their debt obligations. These alternative organizational forms are pervasive among large firms.
and conglomerates in developing countries (see e.g. Almeida and Wolfenzon (2006)). The current model, which aims at exploring issues in the financing and creation of relatively smaller firms, cannot explain these alternative organizational forms.\textsuperscript{19}

\textit{Further Discussion}

The model abstracts from a number of other factors that are likely to affect the integration decision of small and medium sized firms in developing countries. This section briefly discusses some of them, pointing towards areas left for future work.

First, we have abstracted from the possibility that, after default of one firm, investors can liquidate the initial investments recovering some funds. The model could be extended to consider this possibility. While it is possible to conjecture that the presence of fixed costs associated with monitoring and liquidation would favor vertical integration over non integration (only one firm has to be monitored), a countervailing effect might also be at play: vertical integration is best when the assets financed with the initial investment are more specific, a feature that can potentially reduce their value in the liquidation procedure.

Second, an often quoted constraint on firms in developing countries is uncertainty in input supply (see e.g. Mookerjee (1999)). Since less developed countries are likely to be characterized by more uncertain and volatile input markets, vertical integration could be relatively more prevalent in those countries. If better financial markets increase entry (and reduce uncertainty) in input markets, a further negative effect of financial development on vertical integration should be observed in the data. The model here focuses on one supply chain in isolation, and does not analyze industry equilibrium effects.\textsuperscript{20}

Third, the model has also abstracted from wealth effects (see e.g. Legros and Newman (1996, 2004)) in order to focus on the effects of the integration decision on the capacity to borrow. In the presence of financially constrained entrepreneurs, wealth is exchanged for control rights, and the vertical integration decision might be twisted in favor of the wealthier entrepreneurs. The exploration of the interaction between wealth effects and vertical integration in the context of this model is an important topic left for future research.

\textsuperscript{19} A natural extension of the model would allow the external investor to acquire some control over the value chain through equity participation. Certain forms of foreign direct investment and joint ventures between small local firms and multinationals in developing countries could then also be analyzed.

\textsuperscript{20} Macchiavello (2006) reports evidence consistent with this and other industry equilibrium effects.
Fourth, the model has taken technology as given. Entrepreneurs in developing countries might choose to operate relatively more labor intensive (lower $\mu$) or less specific (lower $\pi$) technologies in order to save on the fixed costs that need to be financed. In both cases, non-integration might be the most preferred organizational form.

Finally, the model has been solved for the case in which $\gamma = 1$ and $\beta = 1$. While the main results are not affected by focussing on this limiting case, further insights can be gained by considering the more general parametrization. When $\gamma < 1$, the two projects become linked by a common destiny and potential externalities arise. To see why this is the case, note that, when the upstream project has not been financed, the net present value of the downstream project is given by

$$PV_d^0 = V - \pi p - (1 - \pi)p - k_d$$

Similarly, when the downstream firm is not financed, the net present value of the upstream firm is given by

$$PV_u^0 = \pi (\gamma p - c) - k_u$$

The net present value of the entire chain is given by

$$PV_c = V - \pi c - (1 - \pi)p - k > PV_d^0 + PV_u^0$$

In other words, when $\gamma < 1$, the whole chain is worth more than the sum of its parts. Decentralized investors typically will not replicate the contracts signed by a sole investor, as the investor financing the upstream unit does not take into account the positive effect on the net present value of the downstream unit, and vice versa. These externalities might lead to further underinvestment and more vertical integration in the presence of inequality in borrowing capacity among entrepreneurs.

7 Conclusions

This paper explores the connections between borrowing constraints, vertical integration and the "missing middle" in a model in which financing decisions are jointly determined along with the organization of two vertically related projects. It shows that, from the financial point of view,
vertical integration trades-off the benefits of joint liability against the cost of rendering the supply chain more opaque to external investors. The model underlines the role of financial constraints as a barrier to the creation and expansion of firms. Low investor protection has an ambiguous effect on vertical integration: on the one hand it favors vertical integration because of the need to reduce ex-post incentives problems between buyers and sellers; on the other hand it biases the financing decision towards smaller, and therefore less vertically integrated, firms. As a corollary to this observation the firm size distribution is more likely to display a missing middle in industries which would otherwise favor vertical integration.

The model also considers the role of contractual imperfections between buyers and sellers of intermediate inputs. Better (formal or informal) contract enforcement has a negative impact on vertical integration only when financial markets are relatively developed. These predictions, while shedding light on the financing of small and medium sized firms in developing countries, suggest that the relationship between (capital and input) market development and vertical integration is a complex one: the motives for vertical integration are not necessarily higher in developing countries.

Future theoretical work should explore the effects of the interaction between financial and other constraints that firms face in less developed countries on industrial structure and supply chain performance. Eventually, this line of research will not only improve our understanding of firms and markets but will also lead to better design of policies aimed at fostering industrial upgrading and development.
References


