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DELEGATION AND COMMITMENT IN DURABLE GOODS MONOPOLIES

Tarek Coury and Vladimir P. Petkov

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Tarek Coury† Vladimir P. Petkov‡

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Abstract

This paper studies a simultaneous-move finite-horizon delegation game in which the principal of a durable goods monopoly entrusts pricing decisions to a manager who enjoys consuming her monetary rewards but dislikes production effort. The delegation contract allows for continual interference with managerial incentives: in each period the principal rewards the manager according to her performance. We show that when the cost of delegation is low relative to profits, the principal can attain the precommitment price plan in a perfect rational expectations equilibrium. The paper analyzes the robustness of this result under alternative specifications of timing and objectives. We also provide a numerical characterization of the equilibrium strategies for the case of linear-quadratic payoffs.

Keywords: durable goods monopoly; delegation; perfect rational expectations equilibrium

JEL: L12, D42, C73

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†University of Oxford, Department of Economics, Manor Road Building, Manor Road, Oxford, OX1 3UQ, United Kingdom. Email: tarek.coury@economics.oxford.ac.uk

‡School of Economics and Finance, Victoria University of Wellington, Wellington, New Zealand. Email: vladimir.petkov@vuw.ac.nz
1 Introduction

In many dynamic decision problems, credible commitment to future policies can affect intertemporal trade-offs by influencing the expectations of forward-looking agents. The issue of dynamic consistency has gained particular prominence in the context of durable goods. In his pioneering work, Coase (1972) studies the consequences of rational expectations for market power. He argues that buyers of durable goods correctly anticipate subsequent price reductions. This motivates them to substitute current for future consumption and decreases current demand. The implication for firms is that the ability to precommit to future conduct would be profitable, since it enables them to maintain higher market prices.

Our paper explores the separation of ownership and management in durable goods monopolies and demonstrates its effectiveness as an instrument of intertemporal commitment. We analyze an infinite-horizon game between the principal (the owner of the firm) and the manager (the principal’s agent). The manager dislikes production effort but enjoys consuming her monetary rewards. Delegation is modelled not as a one-shot event, but as a dynamic process where the players interact in all periods. Since both parties can manipulate their opponent’s future incentives, commitment is non-trivial. Nevertheless, delegation can still resolve the time-inconsistency problem by decoupling the principal’s instantaneous payoff from her future strategies.

Commitment through delegated management offers several advantages over alternative methods proposed previously: i) it does not require legal enforceability; and ii) it circumvents the moral hazard issues associated with renting. Thus, delegation can mitigate dynamic inconsistencies even when other instruments are infeasible or costly. Moreover, the majority of medium and large firms are already structured in a way that establishes clear-cut separation of management and ownership. Therefore, the existing corporate structure implies wide
availability of this commitment technology.

Our main result states that when the cost of delegation is low, (e.g. managerial wages are negligible relative to gross profits), the principal can motivate the manager to choose the precommitment price path in a perfect rational expectations equilibrium (PREE). The adoption of feedback Nash strategies ensures that policies will be dynamically consistent: the players will find it optimal to adhere to the precommitment price sequence in all periods and for all histories. Thus, firms can credibly implement precommitment pricing without requiring legally enforceable contracts to constrict their future choices. Since implementation does not rely on trigger strategies, decision makers only need to know the current state, and they can be arbitrarily impatient. Furthermore, our results do not depend on the functional form of profits and managerial utility.

We also consider the implications of delegation costs and alternative timing.

- While the principal’s concern with management costs will distort the price plan away from the precommitment optimum, delegation maintains its precommitment function. We provide a condition under which equilibrium remuneration is low, enabling the principal to improve over the no-delegation PREE.

- Sequential-move costless delegation fails to decouple current profits from the principal’s future choices, and is therefore unable to resolve the dynamic inconsistency. The equilibrium price path is identical to the time-consistent plan without delegation.

There is substantial literature on durable goods monopolies originating from the work of Coase (1972), in which he hypothesizes that rational expectations will force the seller to saturate the market at all dates. Driskill and McCaffery (2001) show that habit formation has similar implications for market power. Some subsequent research, which includes Stokey
investigates the validity of the Coase conjecture. Another strand of literature explores commitment technologies and their effect on firm conduct. Bulow (1982) shows that renting can eliminate the time consistency problem by severing the intertemporal link between periods. Furthermore, Bulow (1986) argues that planned product obsolescence can be used to weaken future incentives to reduce prices. Other commitment tools include guaranteed buy-back, destroying production capacity, and building reputation for maintaining high prices.

While delegation has been overlooked in the context of durable goods monopolies, its commitment value has been recognized in the dynamic oligopoly literature. Sklivas (1987) and Fershtman and Judd (1987) analyze an oligopoly game in which principals entrust output or price decisions to managers whose compensation is tied to both sales and profits. They show that: i) the separation of ownership from management is individually rational; and ii) equilibrium contracts will strategically distort managerial incentives away from profit maximization. The link between corporate and industry structure is further studied by, among others, Miller and Pazgal (2001), Basu (1994), Baye, Crocker and Ju (1996). These papers suggest that the rationale for delegation is based on the strategic nature of market competition: contracts are used to obtain an “instantaneous” first-mover advantage. We assign a somewhat different role to this instrument. In our model, delegation is used as an intertemporal commitment device: it allows current decision makers to attain future goals.

Furthermore, the present paper may also shed light on the time consistency of policies in the macroeconomics literature. Kydland and Prescott (1977) first recognize that central banks conducting monetary policy have a commitment problem which gives rise to an inflationary bias. They show that welfare can be improved if the social planner foregoes discretion and adopts rules that limit her freedom of choice. Rogers (1987) analyzes the time
consistency of fiscal policy. Rogoff (1985) focuses on delegation as an institutional remedy to the problem outlined by Kydland and Prescott. He demonstrates that the appointment of an independent central banker whose preferences differ from the government’s (e.g. she puts an emphasis on inflation-rate stabilization) can be used for credible commitment. Our delegation model differs from Rogoff (1985) in several key aspects:

- **Dynamic delegation**: unless completely isolated from decision making, a time-inconsistent principal will have an incentive to continually interfere with subsequent management. We account for this by examining a dynamic relationship involving repeated interactions. Unlike Rogoff’s central banker, the manager in our model is not independent: When determining remuneration, the principal takes into account past pricing decisions.

- **Irrelevance of managerial preferences**: Rogoff’s one-shot delegation model requires identifying an agent with specific “socially optimal” preferences. In contrast, our model allows the principal to attain the optimal precommitment policy path in a PREE irrespective of managerial utility, provided that delegation costs are not a big concern.

The remainder of the paper is organized as follows: Section 2 defines the industry structure, technology and preferences; it also describes the interactions between the principal and the manager. In Section 3, we characterize two benchmarks: the precommitment price plan and the time consistent price plan without delegation. The PREE of the costless delegation game is derived in Section 4. Its properties are illustrated with a linear-quadratic example. In Section 5, we analyze the robustness of the results to changes in payoffs, timing and decision variables. Section 6 concludes the paper.
2 Setup

2.1 Demand and Industry Structure

Consider suppose that overlapping generations of two-period lived consumers have a lifetime endowment of a numeraire good equal to $\varepsilon$. They derive utility from the consumption of a numeraire $m$ and a durable good $x$. The quantities of $x$ consumed during infancy and maturity are denoted by $x_y$ and $x_o$, respectively. The lifetime utility of a representative agent born in period $t$ is:

$$
\phi(m^t, x^t_y, x^t_{o+1}) = m^t + v(x^t_y) + \beta \tilde{v}(x^t_{o+1} + \lambda x^t_y),
$$

where $\tilde{v}' > 0, v'' < 0, \tilde{v}'' < 0, v''' \geq 0, \tilde{v}''' \geq 0$. The parameter $\lambda \in (0, 1)$ captures the depreciation of the durable good. Note that $\partial^2 \phi / \partial x^t_y \partial x^t_{o+1} = \beta \lambda \tilde{v}''(x^t_{o+1} + \lambda x^t_y) < 0$: a positive value of $\lambda$ implies that current and future consumption levels exhibit intertemporal substitutability.

If $p^t$ and $p^t_{e+1}$ are the current and expected future price, the budget constraint is given by:

$$
m^t + p^t x^t_y + \beta p^t_{e+1} x^t_{o+1} = \varepsilon.
$$

In the remainder of the paper, we impose rational expectations: $p^t_{e+1} = p^{t+1}$.

If prices are positive and bounded and $\lambda$ is sufficiently small, we have that $x^t_y > 0, x^t_{o+1} > 0, \forall t$. A standard utility maximization exercise yields

$$
x^t_y = \tilde{v}^{-1}(p^t - \lambda \beta p^t_{e+1}), \quad x^t_{o+1} = \tilde{v}^{-1}(p^{t+1} - \lambda \beta p^{t+1}).
$$
Thus, the period-\(t\) demand \(x_t\) for the durable good is:

\[
x_t(p_t^{-1}, p_t, p_{t+1}) = x_y + x_o = v^{-1}(p_t - \lambda \beta p_{t+1}) - \lambda v^{-1}(p_t - \lambda \beta p_t) + \tilde{v}^{-1}(p_t). \tag{1}
\]

The properties of \(v\) and \(\tilde{v}\) guarantee that current demand is decreasing in \(p_t\) and concave in \(p_t\) and \(p_{t+1}\). Also, note that \(\partial x_t/\partial p_{t+1} = -\lambda \beta/\tilde{v}''(x_o + \lambda x_y) > 0\). Intertemporal substitutability implies that expectations of a subsequent price drop will boost planned consumption in the future, leading buyers to decrease current purchases.

On the production side, in each period the market is served by a single producer with a cost function \(C(x_t)\) and discount factor \(\delta\). As in Kahn (1986), we assume that \(C''(x_t) > 0\). The monopolist’s period-\(t\) profit is therefore given by:

\[
\pi_t = p_t x_t(p_t^{-1}, p_t, p_{t+1}) - C(x_t(p_t^{-1}, p_t, p_{t+1})) = \pi(p_t^{-1}, p_t, p_{t+1}). \tag{2}
\]

### 2.2 Delegation of Management

Now suppose that, for a compensation of \(w_t\), pricing decisions can be entrusted to a manager who experiences disutility from production effort \(x_t\), but enjoys consumption \(c_t\). The manager may experience habit persistence: her marginal utility can be affected by previous consumption. Thus, her period-\(t\) payoff is defined as \(u_t = u(x_t, c_t - \gamma c_t^{-1})\), where \(u\) satisfies the Inada conditions and \(\partial u/\partial c_t > 0, \partial^2 u/\partial (c_t)^2 < 0, \partial u/\partial x_t < 0, \partial^2 u/\partial (x_t)^2 < 0\). The parameter \(\gamma \geq 0\) captures the degree of habit persistence.\(^1\) For simplicity assume that the manager cannot borrow or save: \(c_t \equiv w_t, \forall t\). Her consumption decisions are endogenized in Section 5.

\(^1\)Our results also hold when \(\gamma = 0\). In that case, \(w_t^{-1}\) is not directly payoff relevant. However, if the players believe that current remuneration will affect future prices, they will still treat \(w_t^{-1}\) as a state variable. In an infinite-horizon PREE, those beliefs will be self-fulfilling. Furthermore, if the manager has access to credit markets, then \(w_t^{-1}\) becomes payoff-relevant even without habit persistence (see Section 5.3).
With some abuse of notation her utility can be written as 
\[ u^t = u(p^{t-1}, p^t, p^{t+1}, w^t - \gamma w^{t-1}). \]

In the beginning of the game, the principal sets both \( w^1 \) and \( p^1 \). In each of the subsequent periods, the principal and the manager simultaneously and non-cooperatively choose respectively the current remuneration \( w^t \) and price \( p^t \). Then buyers purchase the durable good. The delegation contract is of infinite duration and specifies severance payments that are high enough to deter the principal in the future from firing the manager or shutting down.

The manager’s lifetime payoff is \( U^\tau = \sum_{t=\tau}^{\infty} \delta^{t-1} u^t \). To focus on the commitment properties of delegation, Section 4 ignores management costs: We assume that the principal maximizes the discounted stream of gross profits \( \Pi^\tau = \sum_{t=\tau}^{\infty} \delta^{t-1} \pi^t \). The assumptions of costless delegation and simultaneous choice are relaxed in Section 5.

3 Direct Pricing

First, consider the problem of a monopoly which cannot resort to delegation, so all pricing decisions are made directly by the principal. The necessary conditions that characterize the profit maximizing price sequences are derived in Appendix A.

Suppose that in period 1, the monopolist can precommit to a lifetime sequence of future prices. The optimal plan \( \{p^t\}_{t=1}^{\infty} \) then satisfies

\[ \pi_2^t + \delta \pi_1^{t+1} = 0, \quad t = 1 \]
\[ \pi_3^t + \delta \pi_2^{t+1} + \delta^2 \pi_1^{t+2} = 0, \quad t \geq 2, \]

where \( i \) denotes the partial derivative of the instantaneous payoff function with respect to the \( i \)-th argument (e.g. \( \pi_1^t = \partial \pi(p^{t-1}, p^t, p^{t+1})/\partial p^{t-1} \)). Since (3) and (4) are obtained through
unconstrained maximization, these prices yield the highest possible lifetime profit. However, a policy which follows (4) is dynamically inconsistent. If the monopolist reoptimizes in a later period $\tau \geq 2$, the revised price $p^\tau$ will solve (3) instead of (4) and the new plan will diverge from the earlier one. Since future decision makers do not internalize the effect of their choices on past profits, they will make a downward adjustment of the prices prescribed by (4).

When precommitment is not feasible, sophisticated agents will take into account future temptations to deviate. The discrepancy between the firm’s current and future objectives suggests that decision making should be modelled as a game between a sequence of players representing the monopolist’s “selves” associated with each period. We focus on the perfect rational expectations equilibrium of this intra-personal game, where strategies are feedback Nash: $p^t = f(p^{t-1})$. Furthermore, the analysis is restricted to PREE in differentiable strategies. The differentiability requirement is intuitively appealing and helps eliminate potential indeterminacies.$^2$

Suppose that the decision maker expects her future choices to adhere to a strategy function (or an “expectations function”) $f_e(p)$. Let $f(p)$ be the optimal pricing strategy. Expectations are fulfilled on the equilibrium path: $f_e(p^t) \equiv f(p^t)$, $\forall t \geq 1$. To ensure the time consistency of the price plan, we formulate the monopolist’s problem recursively. Payoff maximization requires that her strategy satisfies the Bellman equation:

$$V(p^{t-1}) = \max_{p^t} \{ \pi(p^{t-1}, p^t, f(p^t)) + \delta V(p^t) \} \text{ for all } t \geq 1.$$  \hspace{1cm} (5)

\hspace{1cm}$^2$Stokey (1981) demonstrates that if strategies are extended to discontinuous functions, there exists an infinite number of PREE. However, she argues that they are difficult to justify from an economic point of view, because discontinuous expectations seem unrealistic. Furthermore, our equilibrium is the limit of the finite horizon PREE as time extends to infinity. Klein, Krussel and Rios-Rull (2002) also note that differentiability enables us to obtain a set of necessary conditions with a simple economic interpretation.
Appendix A uses dynamic programming to show that $f(p)$ will solve the generalized Euler equation:

$$\pi_t^2 + f'(p_t)\pi_t^3 + \delta\pi_{t+1} = 0 \text{ for all } t \geq 1. \quad (6)$$

If the period-$t$ decision maker is unable to precommit, she expects $p_{t+1}$ to be set suboptimally low. Thus, she will compensate by reducing current prices in order to boost period-$t$ demand. As a result, time consistent prices will be below the precommitment optimum. This in turn implies that the decision maker will value commitment devices which can increase profits by enabling her to attain the plan defined by (3) and (4).

4 Delegated Pricing

This section analyzes the commitment properties of delegation in durable goods monopolies. In particular, we focus on the simultaneous-move costless delegation game described above, in which the principal entrusts pricing decisions to a manager who receives remuneration in exchange for her effort. The simultaneous choice of prices and wages captures the idea that when setting the current wage, the principal cannot directly observe managerial effort exerted in that period.

Since both parties experience future temptations to deviate, delegation is modelled as a game between sequences of their contemporaneous “agents”. Again, we restrict the analysis to strategies that are differentiable functions of the current state. This ensures dynamic consistency: no player will want to unilaterally deviate at any point in the game for all states. Also, differentiability allows a direct comparison with the no-delegation PREE.

In any period $t \geq 2$, the state is given by $s^t = (p_{t-1}, w_{t-1})$. Let $w^t = g(s^t)$, $p^t = f(s^t)$ be the period-$t$ remuneration and pricing strategies. Rational expectations and payoff
maximization require that in equilibrium these strategies solve:

\[
\Pi(s') = \max_{w'} \{ \pi (p'^{-1}, f(s'), f(f(s'), w')) + \delta \Pi(f(s'), w') \}
\]

(7)

\[
V(s') = \max_{p'} \{ u (p'^{-1}, p', f(p'^{-1}, g(s'))), g(s') - \gamma w'^{-1}) + \delta V(p', g(s')) \}
\]

(8)

where:

\[
g(s') = \arg \max_{w'} \{ \pi (p'^{-1}, f(s'), f(f(s'), w')) + \delta \Pi(f(s'), w') \}
\]

(9)

\[
f(s') = \arg \max_{p'} \{ u (p'^{-1}, p', f(p'^{-1}, g(s'))), g(s') - \gamma w'^{-1}) + \delta V(p', g(s')) \}
\]

(10)

Note that the principal’s problem is well-defined: if the manager’s Nash feedback pricing strategy depends on past wealth, current remuneration can be used to affect current profits through the subsequent pricing decision.

**Definition 1** The perfect rational expectations equilibrium of the costless delegation game consists of value functions \( \Pi(s), V(s) \) which solve (7), (8) and strategy functions \( g(s), f(s) \) which are a fixed point of the mapping defined by (9), (10).

Consider the principal’s Bellman equation (7). The simultaneous choice of prices and wages implies that the principal’s period-\( t \) payoff \( \pi^t \) now depends only on her contemporaneous remuneration strategy \( w^t \). Subsequent wages do not have any repercussions for current profits: when the manager chooses the period-\( t+1 \) price \( p^{t+1} = f(p^t, w^t) \), the period-\( t+1 \) wage \( w^{t+1} \) is yet to be determined. Thus, delegation resolves the dynamic inconsistency problem by decoupling current profits from the principal’s future decisions.

Next we show that, if the cost of delegation is negligible, the principal can fine-tune
managerial monetary incentives to attain the first-best (i.e. precommitment) price path. This result is quite general and robust to changes in the assumptions regarding demand.

**Proposition 1** *The PREE strategies of the simultaneous-move costless delegation game $\Gamma$ beginning in period 2 satisfy the necessary conditions:*

\[
\pi^t_3 + \delta \pi^t_{2+1} + \delta^2 \pi^t_{1+2} = 0
\]  

(11)

\[
\begin{align*}
u^t_2 + & f_1(s^{t+1})u^t_3 + \delta u^t_{1+1} + \delta g_1(s^{t+1})(f_2(s^{t+2})u^t_{3+1} + u^t_{4+1} - \gamma \delta u^t_{1+2}) \\
& - \delta g_1(s^{t+1})g_2(s^{t+2}) g_1(s^{t+2}) (u^t_2 + f_1(s^{t+2})u^t_{3+1} + \delta u^t_{1+2}) = 0.
\end{align*}
\]

(12)

**Proof.** See Appendix B  ■

Condition (11) is the principal’s Euler equation. Given an initial condition, (11) is sufficient to pin down the PREE price plan of the delegation game. Note that (11) is the same as the benchmark precommitment condition (4). Therefore, from period 2 onward, the delegation game will generate the precommitment price sequence.⁵ The important distinction is that now this price plan emerges from the interactions of sophisticated players who use time consistent strategies. This result does not depend on the functional form of managerial preferences. Furthermore, the contract between the principal and the manager is self-enforcing: rational players will follow through on their strategies for all states and in all periods.

Equation (12) describes the intertemporal trade-off of the manager: She is willing to incur effort disutility today if she expects to be rewarded for that in future periods. The left-hand side incorporates the payoff effects of a deviation from the equilibrium price path.

⁵To obtain her precommitment optimum, the period-1 principal chooses her preferred price $p^1$ and a wage $w^1$ that motivate the manager to choose the precommitment price $p^2$ in the following period, i.e. $w^1$ solves $f(p^1, w^1) = p^2$. Subsequent interactions will yield a price sequence that follows (11).
A marginal increase in $p_t$ will affect both current and future demand. Dynamic consistency and rational expectations imply that the resulting change in effort disutility can be broken down into a direct effect, $u_2^t + \delta u_4^{t+1}$, and an internal strategic effect, $f_1(s^{t+1})u_3^t$.

Furthermore, the deviation will influence future remuneration. This will affect managerial utility directly by $\delta g_1(s^{t+1})(u_4^{t+1} - \gamma \delta u_4^{t+2})$ and through the internal strategic effect by $g_1(s^{t+1})f_2(s^{t+2})u_3^{t+1}$.

Rational expectations imply that the manager will react concurrently to the anticipated wage adjustment. The resulting effort disutility effects are captured by the term $-\delta g_1(s^{t+1})g_2(s^{t+2})(u_2^t + f_1(s^{t+2})u_3^{t+1} + \delta u_1^{t+2})$.

Prices are determined optimally in equilibrium, so all payoff effects sum up to 0.

### 4.1 Linear-Quadratic Example

In this subsection, we compute an example of the delegation game studied above. We adopt a linear-quadratic payoff specification, which yields a PREE in linear strategies.

Assume that the utility of the representative consumer is:

$$
\phi(m^t, x^t, x_o^{t+1}) = m^t + (\mu_y x^t - (\eta_y/2)(x^t_y)^2) + \beta (\mu_o(x_o^{t+1} + \lambda x^t_y) - (\eta_o/2)(x_o^{t+1} + \lambda x^t_y)^2).
$$

This yields a linear instantaneous demand:

$$
x^t = A + B_1 p^{t-1} - B_2 p^t + B_3 p^{t+1},
$$

where $A = (1-\lambda)\mu_y/\eta_y + \mu_o/\eta_o$, $B_1 = \lambda/\eta_y$, $B_2 = (1+\beta\lambda^2)/\eta_y + 1/\eta_o$, $B_3 = \beta\lambda/\eta_y$. Assuming
a quadratic cost function, $C(x^t) = \psi(x^t)^2/2$, the monopolist’s instantaneous profit will be given by:

$$\pi^t = p^t (A + B_1 p^{t-1} - B_2 p^t + B_3 p^{t+1}) - \psi(A + B_1 p^{t-1} - B_2 p^t + B_3 p^{t+1})^2/2.$$  \hspace{1cm} (14)

Finally, suppose that managerial preferences are represented by the following utility function:

$$u^t = S_1(w^t - \gamma w^{t-1}) - (S_2/2)(w^t - \gamma w^{t-1})^2 - L_1 x^t - (L_2/2)(x^t)^2.$$  

Under this linear-quadratic specification, the benchmark precommitment and time-consistent Euler equations (4), (6) become respectively

$$B_3(p^t - \psi x^t) + \delta \left(x^{t+1} - B_2(p^{t+1} - \psi x^{t+1})\right) + \delta^2 B_1(p^{t+2} - \psi x^{t+2}) = 0$$ \hspace{1cm} (15)

and:

$$(x^t - B_2(p^t - \psi x^t)) + f_1(p^t) B_3(p^t - \psi x^t) + \delta B_1(p^{t+1} - \psi x^{t+1}) = 0.$$ \hspace{1cm} (16)

Now consider the PREE of the simultaneous-move costless delegation game. We conjecture that the equilibrium remuneration and pricing strategies are:

$$w^t = d + e_1 p^{t-1} + e_2 w^{t-1}, \hspace{0.5cm} p^t = a + b_1 p^{t-1} + b_2 w^{t-1}.$$  

Substituting these conjectures and the payoff definitions in (12) yields:

$$(-B_2 + b_1 B_3)(-L_1 - L_2 x^t) + \delta ((B_1 + e_1 b_2 B_3) - e_2(-B_2 + b_1 B_3))(-L_1 - L_2 x^{t+1})$$ \hspace{1cm} (17)

$$-\delta^2 e_2 B_1(-L_1 - L_2 x^{t+2}) + \delta e_1(S_1 - S_2(w^{t+1} - \gamma w^t)) - \delta \gamma(S_1 - S_2(w^{t+2} - \gamma w^{t+1})) = 0.$$  

Applying the method of undetermined coefficients to (15) and (17) gives us equations for the coefficients of the PREE strategies. The parameter values of our example and the computed equilibrium strategies are provided in Table 1 and Table 2, respectively.

$$\begin{array}{cccccccccc}
\mu_y & \eta_y & \mu_o & \eta_o & \beta & \lambda & \delta & \psi & \gamma & L_1 & L_2 & S_1 & S_2 \\
0.7 & 0.1 & 1 & 0.1 & 0.6 & 0.7 & 0.8 & 0.1 & 0.1 & .05 & .01 & 50 & 1 \\
\end{array}$$

Table 1: Parameter Values of the Numerical Example

$$\begin{array}{cccccc}
a & b_1 & b_2 & d & e_1 & e_2 \\
.4670 & .2925 & .0006 & 46.09 & -.6985 & .1365 \\
\end{array}$$

Table 2: Equilibrium Strategies

Figure 1 presents the no-delegation benchmark plans for an initial condition \( p^0 = 1 \). As in Kahn (1986), precommitment prices are higher than time consistent prices without delegation in all periods. Figure 2 illustrates the wages that support the precommitment price plan in
a PREE. It also shows that an increase in $S_2$ reduces equilibrium wages: managers with a sensitive marginal utility of consumption are more responsive to monetary incentives.

5 Extensions

This section analyzes the robustness of our results to departures from the assumptions underlying the costless delegation game. In particular, we explore the impact of cost considerations, alternative timing and managerial consumption decisions on the commitment properties of delegation.

5.1 Costly Delegation

Suppose that the remuneration needed to motivate the manager to choose the precommitment price path is non-negligible relative to monopoly profits. Thus the principal has to weigh the possible commitment value of delegation against its cost. To study the effect of cost considerations on equilibrium prices, we assume that the principal’s objective is to maximize lifetime profit net of managerial wages:

$$\Pi = \sum_{t=\tau}^{\infty} \delta^{t-1} (\pi_t - w_t).$$

This formulation suggests that remuneration will affect the principal’s payoff directly, as well as through the intertemporal incentive effect on managerial decisions.

In the absence of delegation, wage costs are not offset by future benefits. Thus, if a monopolist chooses $\{w^t, p^t\}_{t=1}^{\infty}$ to maximize net profits under precommitment, she would set $w^t = 0, \forall t$ and the optimal price plan will still satisfy (3), (4).

Under costly delegation, the principal’s Bellman equation becomes:

$$\Pi(s^t) = \max_{w^t} \{ \pi (p^{t-1}, f(s^t), f(f(s^t), w^t)) - w^t + \delta \Pi(f(s^t), w^t) \}. \quad (18)$$
The manager’s objective is unchanged. Her equilibrium strategy solves (8).

A brief inspection of (18) shows that costly delegation still eliminates the links between the principal’s current payoffs and her future remuneration strategies, thus preserving its commitment properties. However, cost considerations will prevent the principal from precisely attaining the precommitment price path.

**Proposition 2** The PREE strategies of the costly delegation game satisfy (12) and

\[
- \frac{1}{f_2(s^t)} + \frac{\delta f_1(s^{t+1})}{f_2(s^{t+1})} + (\pi_{t-1}^{t-1} + \delta \pi_2 + \delta^2 \pi_{t+1}^{t+1}) = 0. \tag{19}
\]

**Proof.** See Appendix C. □

The new equilibrium condition (19) has an additional term \(- \frac{1}{f_2(s^t)} + \frac{\delta f_1(s^{t+1})}{f_2(s^{t+1})}\) which accounts for current and future cost considerations. It distorts prices away from the precommitment optimum.

The value of costly delegation as a commitment instrument will depend on managerial preferences. When the game fundamentals translate into PREE wages that are negligible relative to profits, the equilibrium price plan will be close to (11). Under the linear-quadratic specification, the distortion term will be small when the value of \(S_2\) is sufficiently high. This implies that the marginal utility of consumption is sensitive to changes in remuneration, so the principal can easily manipulate the manager’s intertemporal trade-off.

### 5.2 Alternative Timing

Next, we investigate the sensitivity of the equilibrium to changes in timing. In particular, we analyze a costless delegation game where in each period the principal chooses her strategy *before* the contemporary pricing decision. For simplicity assume that \(\gamma = 0\).
Sequential choice implies an asymmetry of the players’ perceptions regarding the current state. From the principal’s viewpoint, the period-$t$ state can be summarized only by the previous price $p^{t-1}$. Let her feedback Nash strategy be $w^t \equiv g(p^{t-1})$ alone. Since the manager is the second mover, she takes her current wage as given. Thus, her perceived state is $(p^{t-1}, w^t)$. Let her feedback Nash strategy be $p^t \equiv f(p^{t-1}, w^t)$.

The PREE of the sequential-move costless delegation game solves:

$$\Pi(p^{t-1}) = \max_{w^t} \left\{ \pi_t(p^{t-1}, f(p^{t-1}, w^t), f(f(p^{t-1}, w^t), g(f(p^{t-1}, w^t))) + \delta \Pi(f(p^{t-1}, w^t)) \right\} \quad (20)$$

$$V(p^{t-1}, w^t) = \max_{p^t} \left\{ u(p^{t-1}, p^t, f(p^t, g(p^t)), w^t) + \delta V(p^t, g(p^t)) \right\}, \quad (21)$$

where

$$g(p^{t-1}) = \arg \max_{w^t} \left\{ \pi_t(p^{t-1}, f(p^{t-1}, w^t), f(f(p^{t-1}, w^t), g(f(p^{t-1}, w^t))) + \delta \Pi(f(p^{t-1}, w^t)) \right\}$$

$$f(p^{t-1}, c^t) = \arg \max_{p^t} \left\{ u(p^{t-1}, p^t, f(p^t, g(p^t)), w^t) + \delta V(p^t, g(p^t)) \right\}.$$

Bellman equation (20) shows that sequential-move delegation fails to sever the link between the current profit $\pi^t$ and the principal’s future strategy $w^{t+1} \equiv g(p^t)$. For the period-$t+1$ manager, the period-$t+1$ wage becomes an element of her state. Thus, $w^{t+1}$ affects the period-$t+1$ price $p^{t+1} \equiv f(p^t, w^{t+1})$, and consequently period-$t$ profits. This implies that sequential-move costless delegation will not resolve the principal’s time inconsistency.

**Proposition 3** The PREE strategies of the sequential-move costless delegation game satisfy:

$$\pi_2^t + (f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g'(p^t)) \pi_3^t + \delta \pi_1^{t+1} = 0 \quad (22)$$
\[ u_2^t + f_1(p^t, w^{t+1})u_3^t + \delta u_1^{t+1} + g'(p^t)(f_2(p^t, w^{t+1})u_3^t + \delta u_4^{t+1}) = 0. \] (23)

**Proof.** See Appendix C. □

It is easy to demonstrate that sequential-move costless delegation has no commitment power: it generates a price sequence identical to the time-consistent plan of a durable goods monopolist who does not resort to delegation. Consider the principal’s necessary condition (22). Let \( p^{t+1} = \hat{f}(p^t) \) denote the equilibrium law-of-motion of prices under sequential-move delegation. Since the PREE pricing and remuneration strategies are respectively \( f(p, w) \) and \( g(p) \), this implies that \( \hat{f}(p) = f(p, g(p)) \). Thus, we can rewrite (22) as:

\[ \pi_2^t + \hat{f}(p^t)\pi_3^t + \delta \pi_1^{t+1} = 0. \] (24)

Any law-of-motion function \( \hat{f}(p) \) that solves (24) would also solve (6). Similarly, if \( f(p) \) solves (6), it would also solve (24).

### 5.3 Access to Credit Markets

Finally, consider a simultaneous-move costless delegation game where the manager has access to savings and her consumption decisions are endogenous. That is, in each period she chooses her current consumption \( c^t \) as well as the market price \( p^t \). Her wealth evolves according to \( \omega^t = R\omega^{t-1} + w^t - qc^t \), where \( R \) is the gross interest rate and \( q \) is the price of the consumption good. For simplicity assume that \( \gamma = 0 \). In any period \( t \geq 2 \), the state is given by \( s^t = (p^{t-1}, \omega^{t-1}) \). Let \( w^t = g(s^t), p^t = f(s^t) \) and \( c^t = h(s^t) \) be the period-\( t \) remuneration,
pricing and consumption strategies. In equilibrium they solve:

\[
\Pi(s^t) = \max_{\omega^t} \left\{ \pi \left( p^{t-1}, f(s^t), f(f(s^t), R\omega^{t-1} + w^t - qh(s^t)) \right) + \delta \Pi(f(s^t), R\omega^{t-1} + w^t - qh(s^t)) \right\}
\]

(25)

\[
V(s^t) = \max_{p^t, c^t} \left\{ u \left( p^{t-1}, p^t, f(f(s^t), R\omega^{t-1} + g(s^t) - qe^t), c^t \right) + \delta V \left( p^t, R\omega^{t-1} + g(s^t) - qe^t \right) \right\},
\]

(26)

where:

\[
g(s^t) = \arg \max_{\omega^t} \left\{ \pi \left( p^{t-1}, f(s^t), f(f(s^t), R\omega^{t-1} + w^t - qh(s^t)) \right) + \delta \Pi(f(s^t), R\omega^{t-1} + w^t - qh(s^t)) \right\}
\]

\[
f(s^t) = \arg \max_{p^t} \left\{ u \left( p^{t-1}, p^t, f(f(s^t), R\omega^{t-1} + g(s^t) - qe^t), c^t \right) + \delta V \left( p^t, R\omega^{t-1} + g(s^t) - qe^t \right) \right\}
\]

\[
h(s^t) = \arg \max_{c^t} \left\{ u \left( p^{t-1}, p^t, f(f(s^t), R\omega^{t-1} + g(s^t) - qe^t), c^t \right) + \delta V \left( p^t, R\omega^{t-1} + g(s^t) - qe^t \right) \right\}.
\]

As Bellman equation (25) demonstrates, endogenizing managerial consumption does not interfere with the commitment properties of delegation: the principal’s instantaneous payoff \(\pi^t\) is not affected by her future remuneration strategy \(w^{t+1}\). Thus, the precommitment and the PREE price plans will be identical. This observation is summarized in Proposition 4.

**Proposition 4** The PREE of the game with endogenous consumption solves (11) and

\[
u^t_1 + f_1(s^{t+1})u^t_3 + \delta u^{t+1}_1 + \delta g_1(s^{t+1})u^{t+1}_4 / q = 0 \quad (27)
\]

\[
u^t_4 - qf_2(s^{t+1})u^t_3 - \delta \left( R + g_2(s^{t+1}) \right) u^{t+1}_4 = 0. \quad (28)
\]

**Proof.** See Appendix C
6 Conclusion

This paper studies intertemporal commitment through delegation of management in a durable goods monopoly. We explore a simultaneous-move infinite-horizon game in which the principal entrusts pricing decisions to a manager who dislikes production effort but enjoys consuming her monetary rewards. The separation of ownership and management eliminates the link between current profits and the principal’s future policies, which alleviates dynamic inconsistencies.

The analysis demonstrates that when delegation costs are low relative to instantaneous profits, the principal can attain the optimal precommitment price plan in a perfect rational expectations equilibrium. The management contract is self-enforcing: no player has an incentive to deviate from her strategy in any period and for all states. In the case of linear quadratic payoffs, we provide a numerical characterization of the delegation equilibrium.

We also explore the sensitivity of our result to changes in payoff specification and the timing of activities: i) costly delegation has commitment properties, but cost considerations distort the price path away from the precommitment optimum; and ii) sequential-move delegation has no commitment power and yields the time-consistent price path that would occur without delegation.
Appendix A. Pricing Without Delegation

Suppose that in period 1 the monopolist can precommit to an lifetime sequence \( \{p^t\}_{t=1}^{\infty} \) of market prices. The decision maker solves \( \max_{\{p^t\}_{t=1}^{\infty}} \Pi^1 = \sum_{t=1}^{\infty} \delta^{t-1} \pi^t(p^{t-1}, p^t, p^{t+1}) \). Differentiation with respect to \( p^1 \) yields the first-order condition (3). Similarly, differentiation with respect to an arbitrary \( p^t \) (where \( t \geq 2 \)) gives us condition (4).

Now consider the case with no intertemporal precommitment. Suppose that the PREE strategy is \( p^t = f(p^{t-1}), \forall t \). The assumptions on \( v \) and \( \tilde{v} \) guarantee the concavity of \( \pi^t \) in the current price \( p^t \).

Differentiating Bellman equation (5) with respect to \( p^t \) and \( p^{t-1} \) yields a first-order condition and an envelope condition:

\[
\pi^t_2 + f'(p^t)\pi^t_3 + \delta V'(p^t) = 0
\]
\[
V'(p^{t-1}) = \pi^t_1 + f'(p^t)f'(p^{t-1})\pi^t_3 + \delta f'(p^{t-1})V_1(p^t).
\]

Substituting \( V'(p) \) from (29) into (30) gives us (6).

Appendix B. The Delegation Game

Suppose that the PREE strategies of the principal and the manager are respectively \( w^t = g(p^{t-1}, w^{t-1}) \) and \( p^t = f(w^{t-1}, p^{t-1}) \).

First consider the principal’s Bellman equation (7). The assumptions on \( v \), \( \tilde{v} \) and \( C \) ensure that \( \pi^t \) is concave in \( w^t \). Differentiation yields the first-order condition:

\[
f_2(p^t, w^t)\pi^t_3 + \delta \Pi_2(p^t, w^t) = 0.
\]
Differentiating (7) with respect to \( w_{t-1} \), we obtain the envelope condition:

\[
\Pi_2(p_{t-1}, w_{t-1}) = f_2(p_{t-1}, w_{t-1}) \left( f_1(p_t, w_t)\pi_3^t + \pi_2^t + \delta \Pi_1(p_t, w_t) \right).
\] (32)

Substitution of \( \Pi_2(p, w) \) from (31) into (32) gives us an expression for \( \Pi_1(p, w) \):

\[
\Pi_1(p^t, w^t) = -\frac{\pi_3^{t-1}}{\delta^2} - \frac{\pi_2^t}{\delta} - \frac{f_1(p^t, w^t)\pi_3^t}{\delta}.
\] (33)

Similarly, differentiating (7) with respect to \( p_{t-1} \) yields the envelope condition:

\[
\Pi_1(p_{t-1}, w_{t-1}) = \pi_1^t + f_1(p_{t-1}, w_{t-1}) \left( \pi_2^t + f_1(p_t, w_t)\pi_3^t + \delta \Pi_1(p_t, w_t) \right).
\] (34)

After substituting \( \Pi_1(p, w) \) from (33) into (34) we obtain (11).

Now consider the problem of the manager. Differentiating Bellman equation (8) yields the first-order condition:

\[
u_2^t + f_1(p^t, w^t)u_3^t + \delta V_1(p^t, w^t) = 0.
\] (35)

After differentiating (8) with respect to \( p_{t-1} \) we get:

\[
V_1(p_{t-1}, w_{t-1}) = u_1^t + g_1(p_{t-1}, w_{t-1}) \left( f_2(p^t, w^t)u_3^t + u_4^t + \delta V_2(p^t, w^t) \right).
\] (36)

Substitution of \( V_1(p, w) \) from (35) in (36) gives us an equation for \( V_2(p, w) \):

\[
V_2(p^t, w^t) = -\frac{u_2^{t-1} + f_1(p_{t-1}, w_{t-1})u_3^{t-1} + \delta u_1^t}{\delta^2 g_1(p_{t-1}, w_{t-1})} - \frac{f_2(p^t, w^t)u_3^t + u_4^t}{\delta}.
\] (37)
Differentiating (8) with respect to $w^{t-1}$ yields:

$$V_2(p^{t-1}, w^{t-1}) = -\gamma u_4^t + g_2(p^{t-1}, w^{t-1}) \left( f_2(p', w') u_3^t + u_4^t + \delta V_2(p', w') \right).$$

(38)

Finally, after substituting $V_2(p, w)$ from (37) in (38) we obtain (12).

**Appendix C. Extensions**

1. The Costly Delegation Game

First, consider the principal’s problem. Differentiating Bellman equation (18) with respect to $w_t$ yields the first-order condition

$$f_2(p', w') \pi_3^t - 1 + \delta \Pi_2(p', w') = 0.$$  

(39)

Differentiating (18) with respect to $p^{t-1}$ yields

$$\Pi_2(p^{t-1}, w^{t-1}) = f_2(p^{t-1}, w^{t-1}) \left( \pi_2^t + f_1(p', w') \pi_3^t + \delta \Pi_1(p', w') \right).$$  

(40)

Substituting $\Pi_2(p, w)$ from (39) into (40) gives us an equation for $\Pi_1(p, w)$:

$$\Pi_1(p', w') = -\frac{\pi_3^{t-1}}{\delta^2} - \frac{\pi_2^t}{\delta} - \frac{1}{\delta} f_2(p^{t-1}, w^{t-1}).$$  

(41)

Next, differentiating (18) with respect to $p^{t-1}$ yields

$$\Pi_1(p^{t-1}, w^{t-1}) = \pi_1^t + f_1(p^{t-1}, w^{t-1}) \left( f_1(p', w') \pi_3^t + \pi_2^t + \delta \Pi_1(p', w') \right).$$  

(42)
Substituting $\Pi_1(p, w)$ from (41) into (42) obtains (19). The manager’s Bellman equation remains unchanged: her equilibrium necessary condition is still given by (12).

2. Sequential-Move Delegation

Differentiating the principal’s Bellman equation (20) yields the first-order condition

$$\pi'_2 + \left( f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g'(w^{t+1}) \right) \pi_1^3 + \delta \Pi_1(p^t) = 0. \quad (43)$$

Differentiating (20) with respect to $p^{t-1}$ gives us the envelope condition

$$\Pi_1(p^{t-1}) = \pi'_1 + f_1(p^{t-1}, u')\left( (f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g'(p^t) \right) \pi'_3 + \pi'_2 + \delta \Pi_1(p^t). \quad (44)$$

Substituting (43) into (44) obtains (22).

Now consider the manager. Her Bellman equation (21) yields the first-order condition

$$u'_2 + \left( f_1(p^t, w^{t+1}) + f_2(p^t, w^{t+1})g'(p^t) \right) u'_3 + \delta (V_1(p^t, w^{t+1}) + g'(p^t)V_2(p^t, w^{t+1})) = 0. \quad (45)$$

Differentiating (21) with respect to $p^{t-1}$ and $w'$ yield the envelope conditions

$$V_1(p^{t-1}, w^t) = u'_1, \quad V_2(p^{t-1}, w^t) = u'_4. \quad (46)$$

Substituting $V_1(p^{t-1}, w^t)$ and $V_2(p^{t-1}, w^t)$ into the first-order condition (45) gives us (23).
3. Access to Credit Markets

First consider the principal’s Bellman equation (25). He first-order condition is:

$$f_2(p^t, \omega^t)\pi^t_3 + \delta\Pi_2(p^t, \omega^t) = 0. \quad (47)$$

Differentiating (25) with respect to $\omega^{t-1}$, we obtain the envelope condition

$$\Pi_2(p^{t-1}, \omega^{t-1}) = f_2(p^{t-1}, \omega^{t-1})(\pi^{t-1}_2 + f_1(p^t, \omega^t)\pi^t_3 + \delta\Pi_1(p^t, \omega^t)). \quad (48)$$

Substitute $\Pi_2(p^t, \omega^t)$ from (47) in (48) to obtain

$$\Pi_1(p^{t-1}, \omega^{t-1}) = \pi^{t-1}_1 + f_1(p^{t-1}, \omega^{t-1}) (\pi^{t-1}_2 + f_1(p^t, \omega^t)\pi^t_3 + \delta\Pi_1(p^t, \omega^t)). \quad (49)$$

After substituting $\Pi_1(p, \omega)$ into (49) we obtain (11).

Now consider the manager. Bellman equation (26) gives us the first-order conditions:

$$u_2^t + f_1(p^t, \omega^t)u_3^t + \delta V_1(p^t, \omega^t) = 0, \quad -qf_2(p^t, \omega^t)u_3^t + u_4^t - \delta qV_2(p^t, \omega^t) = 0. \quad (50)$$

After differentiating (26) with respect to $p^{t-1}$ and $\omega^{t-1}$, we get the envelope conditions:

$$V_1(p^{t-1}, \omega^{t-1}) = u_1^t + g_1(p^{t-1}, \omega^{t-1}) (f_2(p^t, \omega^t)u_3^t + \delta V_2(p^t, \omega^t)) \quad (51)$$

$$V_2(p^{t-1}, \omega^{t-1}) = (R + g_2(p^{t-1}, \omega^{t-1})) (f_2(p^t, \omega^t)u_3^t + \delta V_2(p^t, \omega^t)). \quad (52)$$

Substituting $V_1(p, \omega)$ and $V_2(p, \omega)$ from (50) in (51), (52), we obtain (27) and (28).
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