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THE CAPITAL STOCK AND EQUILIBRIUM UNEMPLOYMENT: A NEW THEORETICAL PERSPECTIVE

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The Capital Stock and Equilibrium Unemployment: A New Theoretical Perspective

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Abstract

By assuming Cobb-Douglas production technology, many well-known imperfectly competitive macroeconomic models of the labour market (e.g. Layard, Nickell and Jackman, 1991) imply that equilibrium unemployment is independent of the capital stock. This paper introduces a new notion of capacity into the standard framework. Specifically, we adapt the Cobb-Douglas production function so that when the capital-labour ratio drops below a certain threshold, the returns to labour fall while the returns to capital increase. Using this assumption, we show that equilibrium unemployment depends on the capital stock over a certain range. We also briefly discuss the generalisation for an endogenous capital stock.

Keywords: Unemployment, Capital Stock, Investment, Capacity, Technology.

JEL Classification Numbers: E24, E22.

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"It takes capital and entrepreneurship to create new firms and jobs."

(Stiglitz, 2002, p. 59)

1. Overview

Although it is intuitively obvious for some economists that equilibrium employment and hence unemployment[2] should depend on investment and the level of the capital stock, many influential authors (e.g. Layard, Nickell and Jackman, 1991, henceforth LNJ) have argued theoretically that this is not the case. This result hinges on the assumption of Cobb-Douglas production technology: assuming CES production does break it down (Rowthorn, 1999).

However, although the notion of "capacity" seems as if it should be fundamental to the analysis of the relationship between the capital stock and equilibrium unemployment, neither of these approaches contain any real notion of the concept. Meanwhile, for reasons explained below, in (Cobb-Douglas) putty-clay and putty-semiputty models which do introduce meaningful capacity constraints, changes in the capital stock are not normally able to explain *permanent* changes in equilibrium unemployment (though they may explain persistence in unemployment).

In an attempt to offer a more convincing theoretical explanation of the relationship between the capital stock and equilibrium unemployment than currently exists, this paper therefore introduces a new production function in which the notion of capacity is meaningful. Specifically, we adapt the Cobb-Douglas production function so that when the capital-labour ratio drops below a certain threshold, the returns to labour fall discretely while the returns to capital increase discretely by the same amount. We introduce this type of capacity constraint into a standard imperfectly competitive macroeconomic model of unemployment of the type used by LNJ. Assuming that the capital stock is exogenous, we show that if the initial capital stock is within a certain range, increases in its level can permanently reduce equilibrium unemployment. We then illustrate how this result generalises for the case of an endogenous capital stock: in this case, equilibrium unemployment depends on the real user cost of capital over a

[2] In this paper, we follow many authors in ignoring potential changes in inactivity and assuming that the employment and unemployment rates are related by the identity $u \equiv 1 - e$. Therefore, when we talk about one of the concepts, the reverse statement will always apply to the other concept.

certain range. In addition, we explain how our model may be adapted so that it is consistent with the stylised fact that the unemployment rate is untrended in the very long-run while the capital-labour ratio has grown steadily since the Industrial Revolution. The policy implications of our results are clear. Policies to promote investment may help to tackle unemployment. By contrast, the current focus on labour market reforms may be overstated.

2. Introduction

Since the early 1970s, unemployment has risen substantially in most developed countries. Having said this, unemployment experiences have been diverse, especially over the past 15 years. In particular, there were several "success stories" in the 1990s: the equilibrium rate of unemployment fell substantially in the United Kingdom, the Netherlands, Ireland, Portugal and Denmark.

Current thinking (e.g. LNJ; OECD, 1994; Siebert, 1997; IMF, 2003; Nickell, Nunziata and Ochel, 2005) usually attributes unemployment to labour market inflexibility coupled with an inadequately skilled workforce. In recent years, much research on the topic has therefore focussed on which particular labour market institutions and rigidities are responsible for high levels of unemployment. A related research agenda (e.g. Blanchard and Wolfers, 2000) attempts to explain the evolution of unemployment across countries in terms of the interactions between shocks and institutions. The main argument here is that "flexible" institutions may allow economies to adapt more easily to adverse shocks. Proponents of both of these views often cite the United States as an example of a country where a "flexible" labour market has helped to keep the equilibrium unemployment rate at relatively low levels. In addition, the successes of the United Kingdom and the Netherlands in reducing unemployment are often attributed to their adoption of various labour market reforms.

However, the evidence linking labour market reforms to lower equilibrium unemployment is controversial. Baker, Glyn, Howell and Schmitt (2005) argue that, across a wide range of well-known papers, the empirical results supposedly supporting the conventional wisdom are not particularly robust, especially with respect to changes in variable specification and the time period under consideration. Moreover, Ball

(1999) raises three specific issues. Firstly, he argues that the British and Dutch reforms only moved these countries a small way towards the highly "flexible" American labour market.[3] More importantly, he argues that many countries, including Belgium, Canada and Spain, failed to reduce equilibrium unemployment significantly in spite of pursuing labour market reforms which were probably as large as (if not larger than) the British and Dutch reforms. Finally, he argues that conventional explanations cannot account for the successes of Portugal or Ireland since neither of these countries experienced major changes in their labour market institutions.[4] The comparative success of Portugal in relation to Spain (which had the worst unemployment record in the OECD during the 1980s and 1990s) is particularly striking since, as Blanchard and Jimeno (1995) point out, the countries have remarkably similar institutions.

These arguments should not necessarily be viewed as suggesting that labour market reforms are totally unimportant in reducing unemployment. However, they do seem to imply that their importance may be somewhat overstated. In addition, they suggest that we may need to look elsewhere to explain at least part of the falls in equilibrium unemployment rates witnessed in the "success stories" of the 1990s.

In particular, it is striking that high levels of investment were prevalent in many of the "success stories" just before their equilibrium unemployment rates started to fall significantly.[5] For example, Portugal and Ireland both experienced investment booms: the former following accession to the European Economic Community in 1986; the latter driven by a very large upswing in foreign direct investment during the mid-1990s. Meanwhile, it could be argued that the Netherlands benefited from slightly increased investment in the early 1990s following German reunification. The historical experience of the United Kingdom economy is also interesting. As argued by Kitson and Michie (1996), the British investment record between the 1960s and the mid-1990s was dismal. Over this period, the country's employment record was poor. By

[3] As pointed out by Schettkat (2003), the Dutch labour market is probably still less "flexible" than even the German labour market.

[4] Glyn (2005) discusses the Irish experience in more detail and compares it to the experience of New Zealand, where the equilibrium unemployment rate hardly changed during the 1990s despite major labour market reforms. Based on the evidence from these two countries, he concludes (p. 213) that "extensive labor market deregulation is neither a necessary nor a sufficient condition for a radical improvement in employment".

[5] It is also striking that at a broad cross-country level, equilibrium unemployment rose significantly following the worldwide post-1973 investment slowdown.

contrast, the United Kingdom's strong investment performance during the mid to late 1990s came at a time when equilibrium unemployment started to fall significantly.

Although there are likely to be some common causal factors which simultaneously drive investment and employment growth, we should note that the reduction in unemployment in all of these countries has been sustained long after investment rates have fallen. Therefore, despite the usual caveats relating to causation, these wide-ranging experiences do suggest that above trend increases in the capital stock may have some effect in reducing equilibrium unemployment. Moreover, several formal studies are quite supportive of this assertion (e.g. Rowthorn, 1995, using cross-sectional data for ten OECD countries over the period 1960-1992; Arestis and Biefang-Frisancho Mariscal, 2000, using time series data for the United Kingdom and Germany over the period 1966-1995; Miaouli, 2001, using time series data from the manufacturing sectors of France, Greece, Italy, Portugal and Spain over the period 1960-1997; and Alexiou and Pitelis, 2003, using panel data for twelve EU countries over the period 1961-1998). Despite this, promoting investment is rarely proposed as a policy to tackle unemployment. This seeming contradiction between theory and reality motivates our specific theoretical interest in the relationship between the capital stock and the equilibrium rate of unemployment.[6] However, before discussing our model, we briefly survey the contributions of other authors who have written on this topic.

Several authors, including Malinvaud (1980), Soskice and Carlin (1989), Bean (1989, 1994) and Rowthorn (1995), have discussed the potential impact of capacity utilisation on pricing decisions in order to show how falls in the capital stock may generate persistence in unemployment (though without affecting its long-run equilibrium rate). The key assumption of these models is that production technology is putty-clay or putty-semiputty so that ex-post, once capital has been installed, the elasticity of substitution between factors of production is low (or even zero). It is therefore argued that when capacity utilisation is high, firms are likely to increase their price mark-ups in order to choke off excess demand for their products and increase their profit margins. As a result, if an adverse shock erodes the capital stock, then when the shock is reversed, inflation will be generated at lower levels of output and higher levels of

[6] As Hahn (1995, p. 52) comments: "One can only be amazed at the neglect of investment and of the capital stock in theories of the natural rate."

unemployment than were previously the case. As a result, unemployment may remain persistently high until any capital shortfall has been eliminated. However, in these models, the capital stock must always eventually return to its old level. This is because, in the long-run, once the limited factor substitutability constraint has been relaxed, firms will be able to choose their capital and labour inputs optimally. Since a shock which is subsequently reversed does not change anything fundamental which affects this choice, firms will choose their capital and labour inputs in the same way after a shock as they did before it. Therefore, unemployment will always eventually return to its old level in these models, meaning that they cannot explain changes in the equilibrium rate of unemployment in the long-run.

Less theoretical research has been done on whether changes in the capital stock have direct and *permanent* effects on equilibrium unemployment. This is partly because Bean (1989), LNJ and others have argued that since the unemployment rate is untrended in the very long-run, it cannot be affected by trended variables such as the capital-labour ratio. But this neglects the possibility that a one-off permanent step change in the absolute *level* of the capital-labour ratio (i.e. relative to its long-run trend growth rate) could potentially permanently affect the equilibrium unemployment rate.

Moreover, using the competing claims imperfectly competitive macroeconomic model which has (at least in Europe) become one of the standard frameworks for analysing unemployment, LNJ (p. 107) have argued theoretically that the equilibrium rate of (un)employment does not depend on the capital stock when production is Cobb-Douglas. This result is illustrated in (log) real wage-employment space in Figure 1, which, for three different production functions, presents labour demand curves and indifference curves in the special case of a monopoly union.[7] Increasing the capital stock shifts the labour demand curve out in all the diagrams. However, as depicted in panel (a), there is no change in employment in the Cobb-Douglas case, though the real wage does increase.[8] Although LNJ acknowledge that their result hinges on the assumption of Cobb-Douglas production (for further intuition on this, see Rowthorn,

[7] Indifference curves are strictly convex in the monopoly union case. The more general case where firms and unions are assumed to bargain over wages is considered in the main body of the paper.

[8] When production is Cobb-Douglas, labour demand curves are linear in (log) real wage-employment space, with their slope depending on the factor shares (see the derivatives in Appendix B, which also apply in the aggregate case). Given these labour demand curves, the LNJ result implies that the equilibrium must move from (1) to (3) as the capital stock increases.

1999), they claim (p. 107) that it is “not a bad assumption” and adopt it in all of their analysis. However, as we shall argue below, the implication of this assumption that the shares of capital and labour in output are constant may not be appropriate if the capital-labour ratio fluctuates.

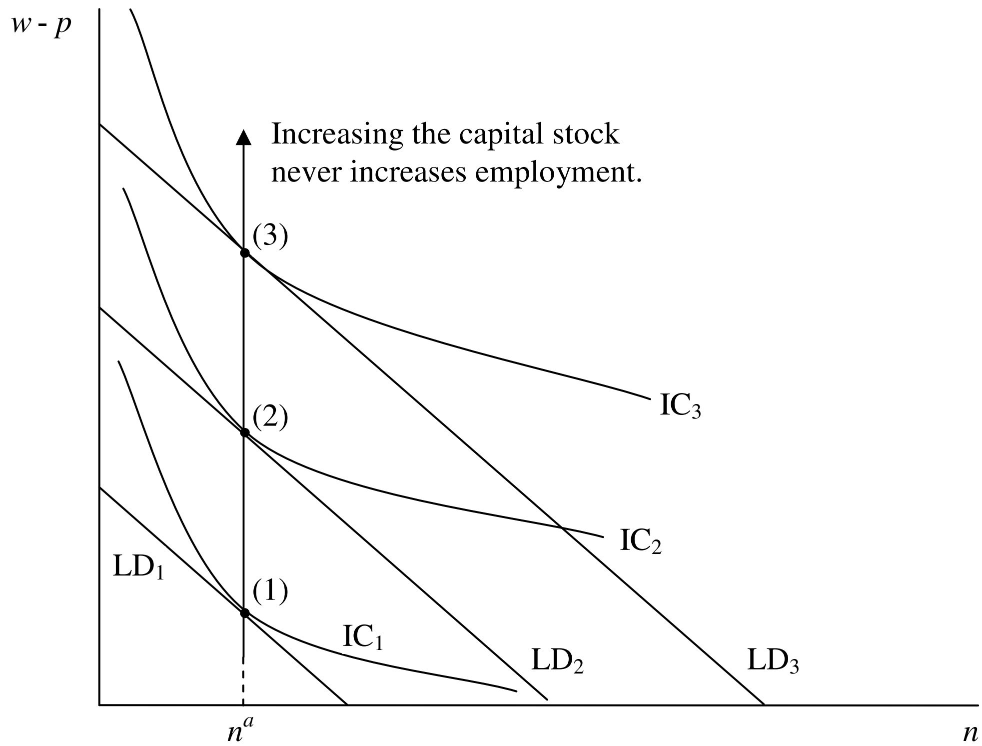

Figure 1(a): Cobb-Douglas

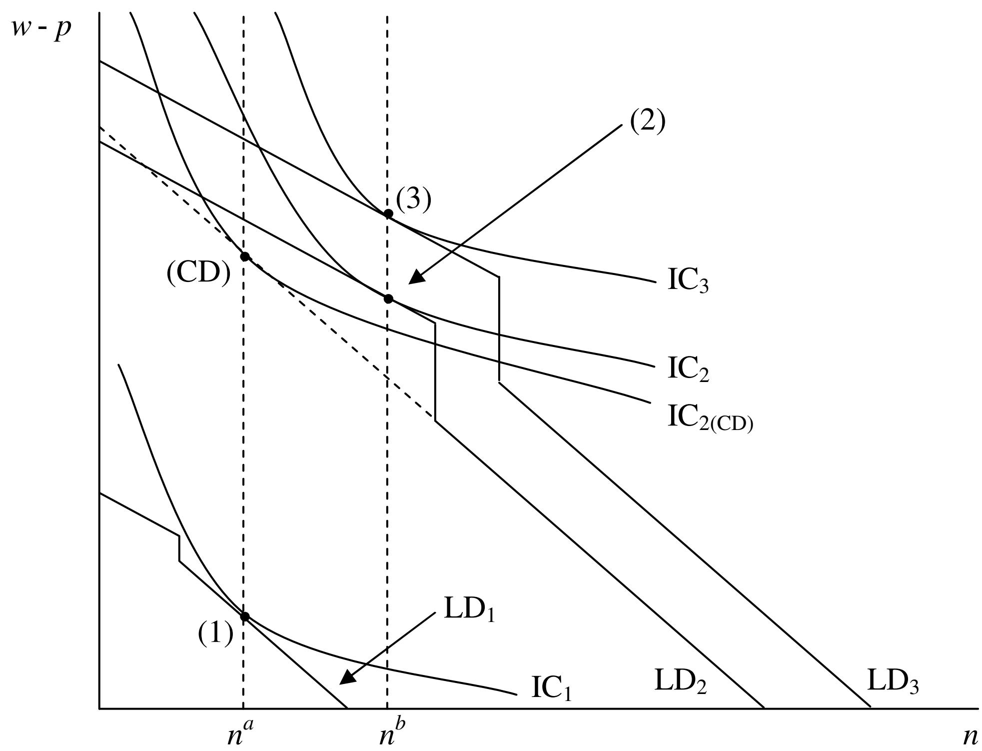

Figure 1(b): KC (with Leontief as a special case)

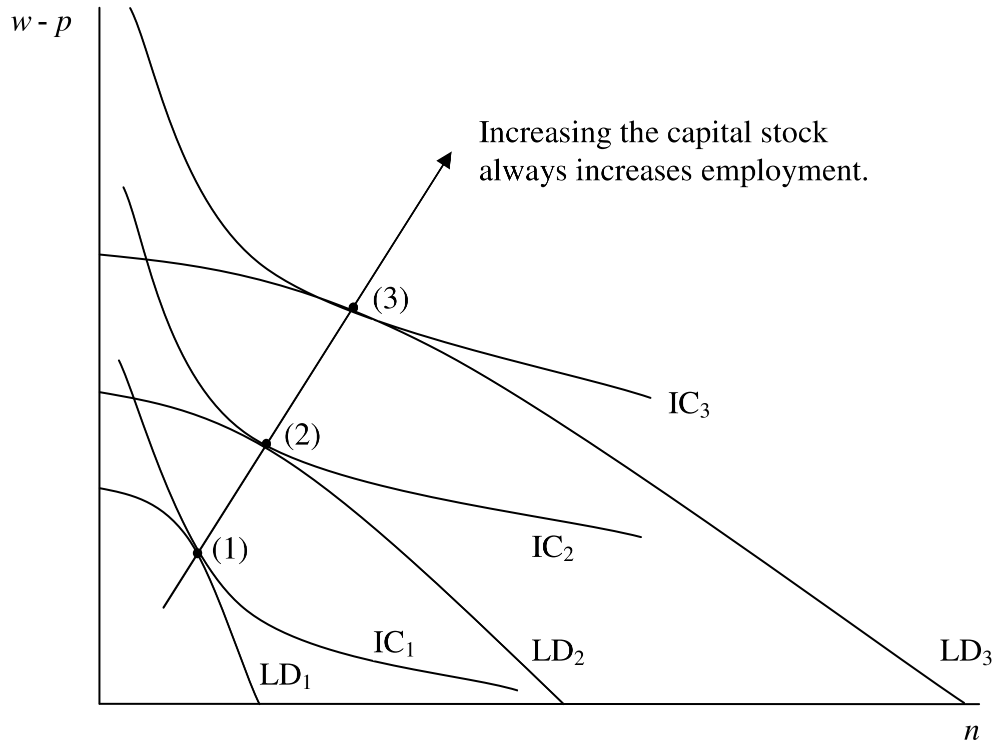

Figure 1(c): CES

Since authors working in this area usually adopt a competing claims framework similar to that used by LNJ, we can therefore see how most of the existing literature has implicitly ignored any potential direct and permanent relationship between the capital stock and equilibrium unemployment. The one major exception is Rowthorn (1999). He argues that the elasticity of substitution between capital and labour is considerably less than one and therefore introduces a CES production function into the LNJ model. Based on this assumption, he shows that increasing the capital stock can theoretically reduce equilibrium unemployment. However, as illustrated in Figure 1(c), in his model, this conclusion holds regardless of how much capital firms already have.[9] This may be viewed as being slightly unrealistic since we might not expect investment to have much impact on equilibrium (un)employment if firms already have a very high capital stock. More fundamentally, like the Cobb-Douglas production function, the CES function does not really contain any notion of capacity and it therefore seems slightly odd to use it for the specific purpose of analysing the relationship between the capital stock and equilibrium unemployment.

[9] Under CES production, labour demand curves are not linear in (log) real wage-employment space. An appendix which shows this and justifies their shape is available on request from the author.

By contrast, our model introduces a new production function, referred to as the ***capital constrained (KC) production function***, in which the notion of capacity is meaningful. This production function (which is presented formally alongside the other components of our model in section 3) is a more general version of the Cobb-Douglas production function which also nests Leontief production as a special case.

We introduce the KC production function into a standard imperfectly competitive macroeconomic model of the type used by LNJ, initially assuming that the capital stock is exogenous. Solving this model (section 4), we show that the incorporation of our notion of capacity generates kinked labour demand curves and implies that changes in the capital stock affect equilibrium (un)employment over a certain range. These results are illustrated in Figure 1(b). From this diagram (which is justified in the main body of the paper), it is clear that when the capital stock is low, investment increases employment from n^a to n^b by changing the equilibrium from (1) to (2) (contrast with the movement to (CD) which would occur under Cobb-Douglas production and note that the difference arises solely due to the change in the shape of the labour demand curves). However, further investment beyond this level does not increase employment any further: only the real wage changes as the equilibrium moves from (2) to (3). This clearly contrasts with the results of both LNJ and Rowthorn (1999). In addition, by showing that an alternative assumption to CES production can generate the result that equilibrium unemployment depends on the capital stock, our model adds further weight to the view of those economists who believe that promoting investment is important in tackling unemployment.

We then consider the case of an endogenous capital stock in section 5, explaining intuitively how equilibrium employment depends negatively on the real user cost of capital (and hence on the real interest rate) over a certain range in this long-run analysis (the formal justification for this result is contained in Kapadia, 2003). That unemployment should be affected adversely by increases in the real interest rate is not a new idea. In particular, a number of potential causal links (of which the effect via capital accumulation is only one) have been discussed by Fitoussi and Phelps (1988) and Phelps (1994). However, in the existing theoretical literature, shifts in the real interest rate are not normally viewed as having a long-run impact on the equilibrium unemployment rate, despite some empirical evidence to the contrary (e.g. Logeay and

Tober, 2003). For example, asking themselves whether changes in the real interest rate are likely to have permanent effects on unemployment, Blanchard and Wolfers (2000, p. C6) answer that "theory is largely agnostic here". By contrast, our model provides a concrete theoretical explanation of why a change in the real interest rate can permanently affect the equilibrium rate of unemployment.

In section 6, we introduce labour-augmenting technical progress into our model to show that it is consistent with the stylised fact that the unemployment rate is untrended in the very long-run while the capital-labour ratio has grown steadily since the Industrial Revolution. We also note that if technology varies across countries, our model need not predict that countries with low capital-labour ratios should have higher equilibrium unemployment rates than countries with high capital-labour ratios. Finally, section 7 concludes and considers the policy implications of our model.

3. Introducing the Model

The broad approach taken is to adopt an imperfectly competitive macroeconomic model of the type first used by Rowthorn (1977). The product market is characterised by monopolistic competition while wages are the outcome of a bargain between firms and trade unions. Perhaps the most well-known application of this type of model is contained in LNJ and the model presented below is essentially a development of their work. It also has close similarities to the models in Manning (1992) and, to a lesser extent, Blanchard and Kiyotaki (1987).

3.1 The Basic Structure

The economy is closed. We assume that it is composed of F identical imperfectly competitive firms, all of whom are assumed to maximise profits in the standard sense. In addition, we assume that all firms are small relative to the aggregate economy. As a result, these firms do not consider the effect of their individual actions on aggregate variables. Finally, in all of what follows, we treat the number of firms as fixed.

3.1.1 The KC Production Function

We assume that the output, Y_i, of firm i is given by:

$$Y_i = A_i \left(\frac{K_i}{N_i C} \right)^{\beta} N_i^{\alpha} K_i^{1-\alpha} = A_i \left(\frac{1}{C} \right)^{\beta} N_i^{\alpha-\beta} K_i^{1-\alpha+\beta} \tag{3.1}$$

where:

$$\begin{aligned} &\beta > 0 \ \text{ for } \ \frac{K_i}{N_i} < C \ \text{ ("full capacity")} \\ &\beta = 0 \ \text{ for } \ \frac{K_i}{N_i} > C \ \text{ ("spare capacity")} \end{aligned} \tag{3.2}$$

$$0 \leq \beta < \alpha < 1$$

and where N_i is employment, K_i is capital and A_i (which is assumed to be exogenous) captures the effects of other inputs and technological progress. Meanwhile, C is a constant reflecting the threshold capital-labour ratio at which "full capacity" is reached. Since this threshold relates to the capital-labour ratio, the capacity constraint described is independent of the scale of the firm. Note also that from (3.1), it is clear that α must be greater than β (otherwise the returns to employment would be negative in the $\beta > 0$ case) but less than one (otherwise the returns to capital would be negative in the $\beta = 0$ case). These restrictions rule out the possibility of increasing returns, a feature which Manning (1990, 1992) has shown may generate multiple equilibria.

The above expressions characterise the capital constrained (KC) production function. This clearly differs from the standard Cobb-Douglas formulation which is used by LNJ. However, we can see that when $\beta = 0$ (i.e. when there is "spare capacity"), the KC production function reduces to the Cobb-Douglas case. Since the remaining parts of our model are standard, we can therefore see that if there is always "spare capacity", the result shown by LNJ (p. 107) that capital accumulation does not reduce equilibrium unemployment will also hold in our model. So, we can see that our model encompasses the standard framework.

The interesting case occurs when the capital-labour ratio drops below the threshold capacity constraint at C (i.e. when the firm reaches "full capacity") and β increases discretely from zero to a constant positive value, causing the coefficients on capital and labour in the production function to change discretely. When this happens, it is clear from (3.1) that the returns to labour fall discretely while the returns to capital increase discretely by the same amount. In other words, the share of labour in output (and hence the wage share) falls discretely while the share of capital in output (and hence the profit share) increases discretely.[10] Moreover, note that if we had $\alpha = 1$, then crossing the threshold would entail moving from a situation resembling Leontief production under "spare capacity" to Cobb-Douglas production under "full capacity". In this scenario, we would have constant returns to labour and zero returns to capital under "spare capacity", with diminishing returns to labour and a positive return to capital only being present under "full capacity" (note that by additionally setting $\beta = 1$, we could also capture the possibility that there may be zero returns to labour under "full capacity"). Therefore, in some sense, the KC production function may be viewed as nesting the Leontief production function.

To further clarify what happens at the threshold, we sketch the KC production function for a fixed capital stock in (y_i, n_i) space in Figure 2 (lower case letters denote logs of capital letters, as they will throughout this paper). This diagram clearly illustrates the difference between the Cobb-Douglas and KC production functions. In the former case, the production function is linear in log space; in the latter case, it is kinked at the threshold capacity constraint, with the reduction in its slope reflecting the discrete fall in the returns to labour at the threshold.

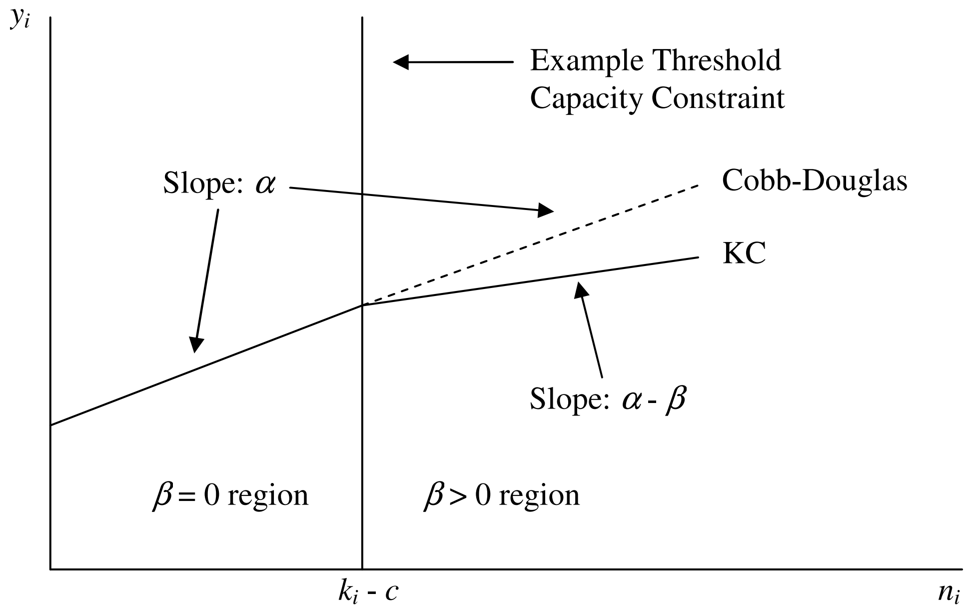

Figure 2

[10] Note also that since labour productivity falls discretely at the threshold, it is clear that for a fixed capital stock and real wage, marginal costs must increase discretely as the threshold is crossed.

3.1.2 Motivating the KC Production Function

The assumption of KC production is critical: it drives all of the new results in our model. The key novel aspect of this production function is its notion of "capacity". In the existing literature, the idea of "capacity" is often mentioned but the term is not usually precisely defined. However, it seems plausible to suggest that it represents a region where diminishing returns to labour really kick in because workers do not have enough capital to work with.

For example, suppose that we have a production process in which workers would ideally each have their own machine. Suppose also that the number of machines is fixed at five. Now, as the number of workers increases, there may be diminishing returns to labour for standard reasons. However, it seems likely that diminishing returns will be much more severe when we move from five to six workers (and workers are first forced to share machines) than when we move from either four to five workers or from six to seven workers. Moreover, it seems likely that the returns to capital will increase discretely as soon as workers are forced to share machines.

Since the Cobb-Douglas production function is unable to capture these effects, it is unlikely to be appropriate if the capital-labour ratio fluctuates. By contrast, the KC production function can capture these effects and may therefore be a superior alternative. Specifically, the threshold can act as a metaphor for a region where diminishing returns to labour really kick in (or, in the extreme Leontief case, first start to be present), because when the capital-labour ratio drops below a certain level and the threshold is crossed, the returns to labour fall sharply. Moreover, the KC production function is consistent with the stylised fact that during booms, when we would expect some firms to be capacity constrained, the share of labour in output falls.

3.1.3 The Remaining Components of the Model

Having introduced the KC production function, we briefly describe the remaining components of our model. These are standard and closely follow Manning (1992). We assume that demand for the output of firm i is given by:

$$Y_i = \left(\frac{1}{F}\right)\left(\frac{P_i}{P}\right)^{-\theta} D(P, \mathbf{X}) \tag{3.3}$$

where P_i is the firm's output price, P is the aggregate price level and D (P, $\mathbf{X}$) is an index of aggregate demand facing the firm with $\mathbf{X}$ being a vector of exogenous variables affecting this. This demand function is derived formally by Blanchard and Kiyotaki (1987, p. 664) using the assumptions (p. 649) that θ is greater than one and that households have CES preferences. Note that θ (which is technically the elasticity of substitution between goods in household utility) may be taken to represent the degree of product market competitiveness, with $\theta = \infty$ being perfect competition.

The real profits, Π_i, of firm i are given by:

$$\Pi_i = \left(\frac{P_i}{P}\right) Y_i - \left(\frac{W_i}{P}\right) N_i - RK_i \tag{3.4}$$

where R is the real user cost of capital (assumed to be exogenous) and W_i is the nominal wage rate. Substituting (3.3) into (3.4) gives:

$$\Pi_i = \left(\frac{D(P, \mathbf{X})}{F}\right)\left(\frac{P_i}{P}\right)^{1-\theta} - \left(\frac{W_i}{P}\right) N_i - RK_i \tag{3.5}$$

Each firm has a corresponding (risk-neutral) trade union with which it bargains. We assume that the utility of trade union i is:

$$U_i\left(\frac{W_i}{P}, N_i\right) = N_i^{\gamma}\left(\frac{W_i}{P} - \overline{V}\right)^{1-\gamma} \tag{3.6}$$

where $\overline{V}$ is the alternative wage available to a worker who loses his job with the firm. This is treated as exogenous when we examine the partial equilibrium bargain between an individual firm and its union but is endogenised when we consider the general equilibrium solution. Meanwhile γ (which is constrained to lie between zero and one) reflects the relative weighting of employment and relative real wages in the union's utility function. Equation (3.6) is quite a general specification. It covers both the utilitarian ($\gamma = ½$) and seniority ($\gamma = 0$) models of Oswald (1982, 1993 respectively).

Since the nominal wage in each firm is the outcome of a bargain between the firm and its union, we assume that it is chosen to maximise the Nash product:

$$\Omega = \left[U_i \left(\frac{W_i}{P}, N_i \right) \right]^{\lambda} \left[\Pi_i - \left(-RK_i \right) \right]^{1-\lambda} \tag{3.7}$$

or alternatively, using (3.6):

$$\Omega = \left[N_i^{\gamma} \left(\frac{W_i}{P} - \overline{V} \right)^{1-\gamma} \right]^{\lambda} \left(\Pi_i + RK_i \right)^{1-\lambda} \tag{3.8}$$

where λ (which is constrained to lie between zero and one) represents the bargaining power of the union. If $\lambda = 1$, we are just maximising union utility, while if $\lambda = 0$, union utility plays no role and we will obtain the competitive outcome for the real wage. This Nash bargaining solution can be derived as the subgame perfect equilibrium of a formal bargaining game (Binmore, Rubinstein and Wolinsky, 1986). Note that since the capital stock is already determined when the Nash bargain takes place (see the timeline below), the fallback level of profits for the firm if it does not reach agreement with the union is equal to $-RK_i$. This relates to the notion of "bygones being bygones" when the bargain takes place. It explains why we use operating profits (i.e. gross of capital costs) rather than total profits in (3.7) and (3.8). Finally, note that by taking logs, we may rewrite (3.8) as:

$$\ln \Omega = \lambda \left[\gamma \ln N_i + (1-\gamma) \ln \left(\frac{W_i}{P} - \overline{V} \right) \right] + (1-\lambda) \ln \left(\Pi_i + RK_i \right) \tag{3.9}$$

3.2 Timeline of Decisions

We assume that decisions are made in the order depicted in the diagram below:

K_i	W_i	N_i and P_i
Period t-1	Start of Period t	End of Period t

This is a conventional sequence. The capital stock is determined in the period prior to the determination of wages, employment and prices. This is our justification for treating the capital stock as exogenous in our short-run analysis in section 4. Given the capital stock, the nominal wage is determined according to the Nash bargain (characterised by (3.9)) between the firm and the union at the start of period t. Finally, at the end of period t, the firm chooses employment and the output price to maximise its profits, taking both the capital stock and the nominal wage level as predetermined.

4. Short-Run Analysis: Exogenous Capital Stock

In this section, we solve our model assuming an exogenous capital stock, considering only decisions made in period *t*. After giving a brief intuitive discussion of our main results, we focus on the partial equilibrium solution at the level of the individual firm. Our analysis discusses both the end of period pricing and employment decisions (this entails deriving the firm's labour demand curve) and start of period wage-setting. We then consider the general equilibrium solution, showing that, over a certain range, aggregate equilibrium employment depends on the level of the capital stock.

4.1 Intuitive Discussion of the Main Results

The nature of the KC production function means that it is effectively composed of two separate Cobb-Douglas "production functions", each with different factor shares, with the relevant one depending on whether the capital-labour ratio is above or below the specified threshold. Therefore, when we solve the short-run model, we obtain a solution for aggregate employment corresponding to each "production function".[11] (We also obtain a corner solution, the details of which are discussed in our formal analysis.) As in the analysis of LNJ, at each of these two solutions, employment does not depend on the capital stock. This is as we would expect from the Cobb-Douglas nature of each of our "production functions". However, the key point is that *the solution at which the economy ends up* will depend on the level of the capital stock, as this determines the capital-labour ratio and hence the relevant "production function". Moreover, aggregate employment is different in each of the two solutions. In particular, it is higher in the solution for which the capital-labour ratio is above the threshold (i.e. when the capital stock is high). This is because, in this solution, the share of labour in output (and hence the wage share) is higher. Therefore less unemployment is required to keep union wage demands in check. Alternatively, we could view the result as stemming from the fact that when the capital stock is high, the productivity of workers is high, meaning that workers are effectively "cheap" and marginal costs are low compared to the case where labour productivity is low. As a result, firms will employ more workers in this case. Overall then, we can start to see

[11] To see why this must be the case, consider what would happen if we treated our model as two separate models in isolation, with only the coefficients on capital and labour in the production function differing between them.

how equilibrium employment will depend positively on the capital stock over a certain range in our model. However, it is clear that if the capital stock is already high, increases in its level will not have any impact on equilibrium employment.

4.2 Partial Equilibrium Labour Demand Curve

We start our formal analysis by deriving the firm's labour demand curve for a given capital stock and nominal wage level (i.e. we start by considering the firm's decision at the end of period *t*). This also allows us to justify the kinked labour demand curves depicted in Figure 1(b). For the derivation, we first eliminate the price term from (3.5) to enable us to maximise the profit function with respect to N_i. Substituting (3.1) into (3.3) and rearranging gives:

$$\left(\frac{P_i}{P}\right) = \left[\left(\frac{D(P,\mathbf{X})}{F}\right)^{-1} A_i \left(\frac{1}{C}\right)^{\beta} N_i^{\alpha-\beta} K_i^{1-\alpha+\beta}\right]^{-\frac{1}{\theta}} \tag{4.1}$$

Since (4.1) uniquely determines the firm's output price for a given level of employment, we can see that by considering the firm's employment decision at the end of period *t* (as we do below), we are implicitly taking into account their contemporaneous pricing decision. Substituting (4.1) into (3.5) and simplifying gives:

$$\Pi_i = \left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} N_i^{\frac{(\alpha-\beta)(\theta-1)}{\theta}} K_i^{\frac{(1-\alpha+\beta)(\theta-1)}{\theta}} - \left(\frac{W_i}{P}\right) N_i - RK_i \tag{4.2}$$

which may be rewritten as:

$$\Pi_i = \left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta}} N_i^{\alpha'} - \left(\frac{W_i}{P}\right) N_i - RK_i \tag{4.3}$$

where:

$$\alpha' = \frac{(\alpha-\beta)(\theta-1)}{\theta} \tag{4.4}$$

Note that the assumptions on the parameters ($0 \le \beta < \alpha < 1$ and $\theta > 1$) imply that $0 < \alpha' < 1$.

The firm wants to choose employment, N_i, to maximise its real profits (4.3) for the cases $\beta = 0$ and $\beta > 0$ subject to (3.2) (i.e. subject to $N_i \le (K_i/C)$ and $N_i \ge (K_i/C)$ respectively). We proceed by solving these two constrained optimisation problems alongside each other using Kuhn-Tucker theory. If Π_i^0 corresponds to real profits given by (4.3) when β = 0, then using (3.2), we can see that the complementary slackness conditions for the $\beta = 0$ problem are:

$$\frac{d\Pi_i^0}{dN_i} = 0 \ ; \quad N_i \leq \frac{K_i}{C} \tag{4.5}$$

$$\frac{d\Pi_i^0}{dN_i} > 0 \ ; \quad N_i = \frac{K_i}{C} \tag{4.6}$$

For the $\beta > 0$ problem, if Π_i^+ corresponds to real profits given by (4.3) when $\beta > 0$, the complementary slackness conditions are:

$$\frac{d\Pi_i^+}{dN_i} = 0 \ ; \quad N_i \geq \frac{K_i}{C} \tag{4.7}$$

$$\frac{d\Pi_i^+}{dN_i} < 0 \ ; \quad N_i = \frac{K_i}{C} \tag{4.8}$$

We start by considering the corner solutions (4.6) and (4.8). Employment at both corner solutions is given by:

$$N_i = \frac{K_i}{C} \tag{4.9}$$

Moreover, both corner solutions yield the same level of real profits to the firm. To see this, substitute (4.9) into (4.2) to get:

$$\Pi_i = \left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} \left(\frac{K_i}{C}\right)^{\frac{(\alpha-\beta)(\theta-1)}{\theta}} K_i^{\frac{(1-\alpha+\beta)(\theta-1)}{\theta}} - \left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right) - RK_i$$

This simplifies to:

$$\Pi_i = \left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} K_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C}\right)^{\frac{\alpha(\theta-1)}{\theta}} - \left(\frac{W_i}{P}\right)\left(\frac{K_i}{C}\right) - RK_i \tag{4.10}$$

which is independent of β. Since β is the only variable that differs between (4.6) and (4.8), we can see that, for a given wage, profits are the same at both corner solutions. Therefore, in terms of the key variables of interest, the two corner solutions are effectively the same, meaning that we do not need to consider them separately in what follows.

We now consider the interior solutions (4.5) and (4.7), deriving the labour demand curves which we would get if we always had $\beta = 0$ or $\beta > 0$ respectively. As all of the firms are small relative to the aggregate economy, we assume that the aggregate price level, P, is fixed when firms maximise their profits (since $\mathbf{X}$ is a vector of exogenous variables, this implies that we can also treat D (P, $\mathbf{X}$) as fixed). Moreover, as our timeline shows, the nominal wage and capital stock are already determined when the firm makes its profit-maximising decision at the end of period t. Finally, recall that F, C and R are all fixed. Therefore, we can proceed by differentiating the profit function given by (4.3) with respect to N_i. The first order condition is:

$$\left(\frac{W_i}{P}\right) = \alpha'\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta}} N_i^{\alpha'-1} \qquad (4.11)$$

Since (4.3) is concave in N_i (this follows from the fact that $\alpha' < 1$), this solution is a global maximum. Rearranging (4.11) to make N_i the subject, we get:

$$N_i = \left[\alpha'\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta}} \left(\frac{W_i}{P}\right)^{-1}\right]^{\frac{1}{1-\alpha'}} \qquad (4.12)$$

At interior solutions, (4.12) is the firm's labour demand curve: it applies for both $\beta = 0$ and $\beta > 0$ and gives the firm's optimal employment choice for a given wage and capital stock. As noted above, it also determines the firm's output price via (4.1). In Appendix A, we show that the firm's real operating profits in terms of the wage at optimal (interior) solutions for employment are given by:

$$\Pi_i + RK = (1-\alpha')\left[(\alpha')^{\alpha'}\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{\theta-1}{\theta}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta}} \left(\frac{W_i}{P}\right)^{-\alpha'}\right]^{\frac{1}{1-\alpha'}} \qquad (4.13)$$

We are now able to illustrate the firm's overall labour demand curve by constructing a diagram (Figure 3) in ($w_i - p$, n_i) space. This consists of the firm's labour demand curves for both $\beta = 0$ and $\beta > 0$ (sketched using the log version of (4.11)) and an example threshold capacity constraint. The justification for the relative slopes of the curves is contained in Appendix B. We also show in this appendix that the intersection of these two curves at (X) will always occur at a level of employment below the threshold capacity constraint.

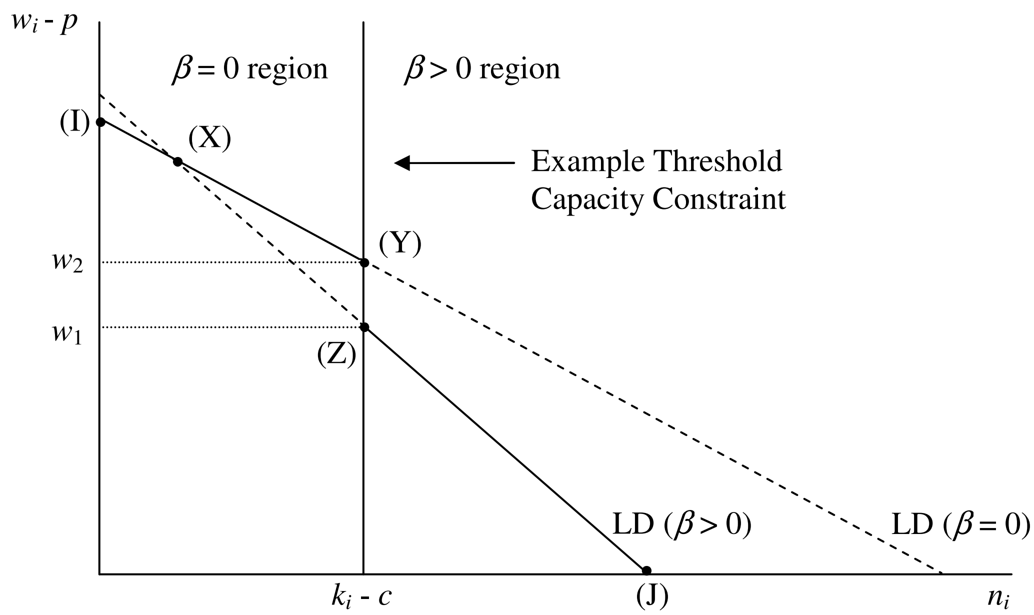

Figure 3

Since the threshold capacity constraint is to the right of (X), it is immediately clear from Figure 3 that for any given wage and threshold, there is only a single applicable labour demand curve (with an invariant section at the threshold capacity constraint). For wages between w_1 and w_2, neither interior solution is "feasible" (i.e. consistent with (3.2)). As a result, the corner solution (which coincides with the threshold capacity constraint) must be chosen in this wage range. By contrast, outside of this range, the interior solution on the applicable labour demand curve will be chosen by the firm (the corner solution is clearly inferior, regardless of whether $\beta = 0$ or $\beta > 0$ at it). Specifically, if the wage is greater than w_2, the interior solution corresponding to the $\beta = 0$ labour demand curve will be chosen, while if the wage is less than w_1, the interior solution corresponding to the $\beta > 0$ labour demand curve will be chosen. Therefore the firm's overall labour demand curve is IYZJ. Finally, from (4.9) and (4.11), it is clear that if the capital stock increases, this labour demand curve will shift out. Together, these results justify the labour demand curves depicted in Figure 1(b).

4.3 Partial Equilibrium Wage-Setting

We now move on to discuss the decisions made at the previous stage in our timeline (i.e. at the start of period *t*). At this point, the firm and its union bargain over the nominal wage, taking the capital stock as given and assuming that they are too small to affect the aggregate price level. We assume that both parties are also fully aware of the employment and pricing decisions that the firm will make for a given wage at the end of period t and that this information is used when bargaining. Therefore, we may view the wage determination problem as requiring us to choose the nominal wage to maximise the Nash product given by (3.8) (or, in log form, (3.9)) subject to a "budget constraint" which, in this context, is the (kinked) labour demand curve derived above.

4.3.1 Illustrative Solution in the Special Monopoly Union Case

The general solution to this problem is quite complicated. Therefore, we start by illustrating the solution in the special monopoly union case, in which it is assumed that λ = 1 in (3.9), before explaining how the results generalise. When λ = 1, it is straightforward to show from (3.9) that the (societal) indifference curves are strictly convex in (log) real wage-employment space, and that their slope does not depend on β, implying that they are not kinked at the threshold. In Figure 4, we add some illustrative indifference curves of this type onto Figure 3.

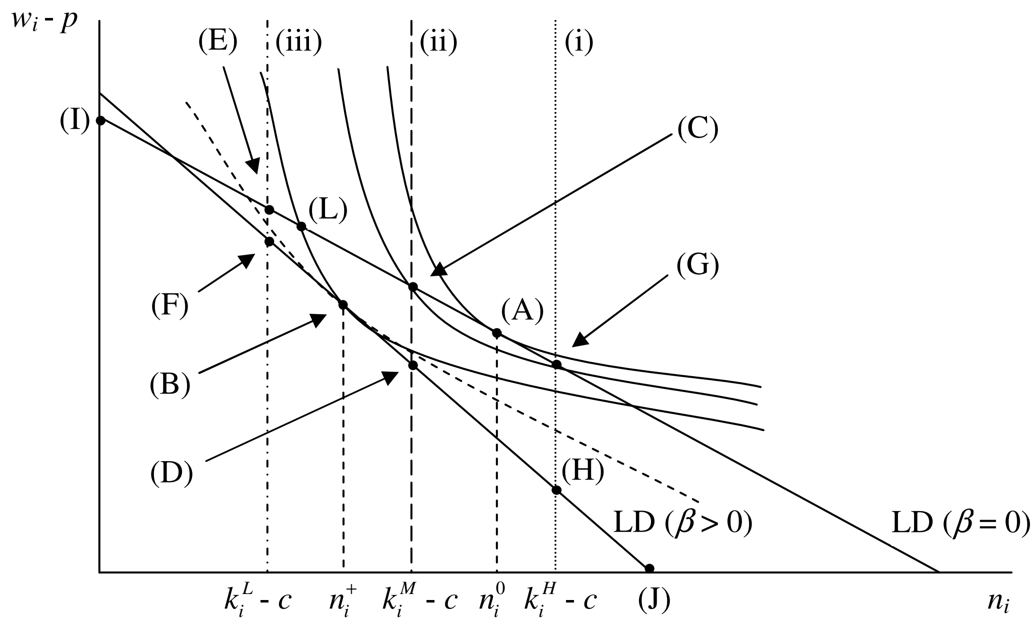

Figure 4

Figure 4 also includes example threshold capacity constraints representing each of the following three cases (the type of line used in Figure 4 is described in brackets):[12]

[12] Since the level of the capital stock affects the position of the labour demand curves, we should technically draw three different diagrams to represent the three different cases. However, since we are

(i) "Capacity Unconstrained" (dots)

$$K_i = K_i^H \quad \text{where} \quad N_i^+ < N_i^0 < \frac{K_i^H}{C} \tag{4.14}$$

(ii) "Moderately" Capacity Constrained (long dashes)

$$K_i = K_i^M \quad \text{where} \quad N_i^+ < \frac{K_i^M}{C} < N_i^0 \tag{4.15}$$

(iii) "Severely" Capacity Constrained (dots and dashes)

$$K_i = K_i^L \quad \text{where} \quad \frac{K_i^L}{C} < N_i^+ < N_i^0 \tag{4.16}$$

For the solid indifference curves, the partial equilibrium outcomes for wages and employment in each of the three cases can clearly be seen from Figure 4. Since $\beta = 0$ at levels of employment lower than the threshold (i.e. to the left of the threshold) and since $\beta > 0$ at levels of employment higher than the threshold, the overall labour demand "constraint" in case (i) is given by IGHJ. For this labour demand curve, it is clear that the highest possible indifference curve is attained at (A). The union will therefore choose the wage corresponding to this point at the start of period *t* and, as a result, the firm will choose a level of employment equal to N_i^0 (i.e. n_i^0 in log terms) at the end of period *t*. By contrast, in case (ii), the overall labour demand curve is ICDJ and (A) is not feasible. However, it is clear from the indifference curves that the optimal outcome will be at the corner solution (C), meaning that employment will be given by $N_i = K_i^M / C$, and will depend directly on the (exogenous) capital stock.

Case (iii), for which the overall labour demand curve is IEFJ, is slightly more tricky. For the solid indifference curves, the solution will be at (B), implying that employment will be given by N_i^+. However, for flatter indifference curves (e.g. as illustrated by the dashed indifference curve), it is possible that (E) might be preferred to (B), leading to an employment level of $N_i = \left(K_i^L / C\right)$. Therefore, in case (iii), the outcome is ambiguous and will depend on both the level of the (exogenous) capital stock (through its impact on the position of the threshold) and the parameter values. Having said this, it is clear that the *maximum* level of employment in case (iii) is N_i^+.

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only ever considering one of the thresholds at any given time in the analysis, we can abstract from this point and do so in order to illustrate the differences between the cases more clearly.

This is because if employment at an arbitrary corner solution like (E) were greater than this, then the threshold would be to the right of (B) and we would be in case (ii).

From the indifference curves, it is also clear that for given parameters, there will be a particular level of the capital stock in case (iii) for which (B) and some arbitrary corner solution represented by (E) are indifferent (for the solid indifference curves, this will be at point (L)). Let us define this particular level of the capital stock as K_i^1. As the capital stock falls below K_i^1, it is clear that the solution switches from a point like (L) to (B), with employment increasing discretely. (Note that this implies that the *minimum* level of employment in case (iii) is K_i^1/C.) This rise in employment for a fall in the capital stock in the vicinity of K_i^1 seems quite surprising at first, especially since everywhere else in the monopoly union case, a rising capital stock either increases employment or leaves it unchanged. The result is essentially driven by the simplifying assumption of a two-regime model which results in switching at a specific point. Intuitively, we may explain what happens as follows. The (monopoly) union generally prefers to be on the $\beta = 0$ labour demand curve rather than the $\beta > 0$ labour demand curve if possible. This is because the returns to labour and hence the overall wage share are greater when $\beta = 0$ than when $\beta > 0$. Therefore, starting at the boundary between cases (ii) and (iii), as the threshold moves to the left within case (iii) (due to falls in the capital stock), the union initially wants to maintain the value of β at zero even though it must sacrifice some employment to achieve this. However, as the threshold continues to move to the left, the cost in terms of lost employment increases and eventually (when the capital stock reaches K_i^1), the union allows β to take a positive value. This causes a discrete fall in the wage share (and the wage level) but results in a discrete increase in employment because the threshold constraint is released.

Summarising this section, we can therefore see that if the firm is "capacity unconstrained", employment will be N_i^0. Meanwhile, if the firm is "severely" capacity constrained, the maximum level of employment will be N_i^+, which is less than N_i^0, and the minimum level of employment will be K_i^1/C. Finally, if the firm is "moderately" capacity constrained, we will be at the corner solution and employment will be K_i/C, which depends directly on the level of the exogenous capital stock. Overall then, we have a partial equilibrium range of employment between K_i^1/C and N_i^0 in the monopoly union model. The exact employment outcome within this range will depend on the level of the capital stock. Moreover, we can see that, with a minor exception in the vicinity of K_i^1, increases in the capital stock increase employment over this range. However, outside of this range, changes in the capital stock will not affect employment (see also Figure 1(b)).

4.3.2 General Solution

By continuity, it is clear that if λ is sufficiently close to one, the results and discussion from the monopoly union case will continue to apply. More precisely, Kapadia (2003) proves that the discussion in the previous section carries over to the general case provided that:

$$\lambda > \frac{\alpha(\theta-1)}{\theta(1-\gamma)+\gamma\alpha(\theta-1)} \tag{4.17}$$

As we would expect, (4.17) will be satisfied if the union's bargaining power is sufficiently high. It is also more likely to be satisfied if γ is low (i.e. if the union attaches a relatively low weight to employment in its utility function).[13]

In what follows, we assume that (4.17) is always satisfied. This may be justified on the grounds that the whole imperfectly competitive approach to the determination of unemployment is only interesting if the union has a reasonable amount of bargaining power (i.e. λ is significantly greater than zero) and cares to a certain degree about relative real wages (i.e. γ is significantly less than one). (As noted in section 3, if the union had no bargaining power or if it only cared about employment, we would obtain the perfectly competitive outcome for the real wage.) We also make this assumption because it shortens and simplifies the analysis. Although we could consider the other

[13] To shed further light on this condition, we calibrate the parameters. We adopt the standard assumption that α = 0.66. Estimates of θ vary widely: surveying the empirical literature on markups (i.e. $\theta/(\theta-1)$) in the United States, Rotemberg and Woodford (1995) find estimates ranging from 1.2 to 2, implying values of θ ranging from 2 to 6. Assuming θ = 6 implies that (4.17) will be satisfied for λ > 0.71 when γ = 0.5 (i.e. when the union attaches an equal weight to employment and relative real wages in its utility function) and for λ > 0.55 when γ = 0 (i.e. when the union only cares about relative real wages); assuming θ = 2 (which may be closer to the true value in Europe where markups tend to be higher) implies that the condition will be satisfied for λ > 0.5 when γ = 0.5 and for λ > 0.33 when γ = 0.

cases, we would not gain any further major insights: the details would be different but the overall conclusion about equilibrium employment depending on the capital stock over some range would be unaffected.[14]

4.3.3 Algebraic Expressions for Wages and Employment: Interior Solutions

At interior solutions, the partial equilibrium wage will be given by the value of W_i which maximises the Nash product (3.9) for the relevant labour demand curve. To determine this, we differentiate (3.9) with respect to ln W_i (note that the chain rule implies that this will generate the same solution as differentiating with respect to W_i). Recalling our assumption that the alternative wage, $\bar{V}$, is exogenous in the partial equilibrium analysis, we can see that the first order condition is:

$$\lambda\left[\gamma\frac{d\ln N_i}{d\ln W_i}+(1-\gamma)\frac{d\ln\left[(W_i/P)-\bar{V}\right]}{d\ln W_i}\right]+(1-\lambda)\frac{d\ln(\Pi_i+RK_i)}{d\ln W_i}=0 \quad (4.18)$$

In Appendix C, we show that at interior solutions this expression reduces to:

$$\frac{W_i}{P}=\psi\bar{V} \quad (4.19)$$

where:

$$\psi=\frac{\lambda\gamma+(1-\lambda)\alpha'}{\lambda\gamma+(1-\lambda)\alpha'-(1-\alpha')\lambda(1-\gamma)} \quad (4.20)$$

In other words, the wage is simply marked-up over the alternative wage by a factor of ψ. However, the value of ψ depends on α' and therefore on β. We also show in Appendix C that ψ depends positively on β. Therefore, if we let ψ^0 correspond to the case $\beta = 0$ in (4.20) and ψ^+ correspond to the case $\beta > 0$, then we have $\psi^+ > \psi^0$. So, if the firm is operating under "full capacity", the mark-up and hence wage is higher (but employment is lower: to see this consider adding wage-setting curves onto Figure 3). Intuitively, this is because when $\beta > 0$, the returns to labour are lower and the employment gain from accepting a lower wage is therefore lower.

[14] To give a flavour of the other cases, consider the situation in which the firm is relatively strong. In this scenario, (D) would clearly be preferred to (C) in case (ii) in Figure 4 since the firm would rather pay a lower wage for the fixed level of employment. In case (iii), the interior solution (B) would be unambiguously preferred to the arbitrary corner solution (E) (since (B) would be superior to (F), which in turn would be superior to (E)), but in case (i), the choice between (A) and (H) would be ambiguous. So, we can see that the broad result would be the same: equilibrium employment would still depend on the capital stock over a certain range, though the precise range would be slightly different.

To derive analytical expressions for interior solution employment when $\beta = 0$ (N_i^0) and $\beta > 0$ (N_i^+), we substitute (4.19) into (4.12) and take $\beta = 0$ or $\beta > 0$ as appropriate. This gives:

$$N_i^0 = \left[\frac{\alpha(\theta-1)}{\theta} \left(\frac{D(P,\mathbf{X})}{F} \right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} K_i^{\frac{(1-\alpha)(\theta-1)}{\theta}} \left(\psi^0 \bar{V} \right)^{-1} \right]^{\frac{\theta}{\theta-\alpha(\theta-1)}}$$

$$N_i^+ = \left[(\alpha') \left(\frac{D(P,\mathbf{X})}{F} \right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C} \right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta}} \left(\psi^+ \bar{V} \right)^{-1} \right]^{\frac{1}{1-\alpha'}}$$

4.3.4 Algebraic Expressions for Wages and Employment: Corner Solution

Employment at the corner solution is given by (4.9): $N_i = (K_i/C)$. In Kapadia (2003), it is proved that provided condition (4.17) is satisfied, the wage at this corner solution will correspond to the wage at the intersection of the threshold capacity constraint and the $\beta = 0$ labour demand curve. To obtain an expression for the partial equilibrium real wage at the corner solution, we therefore substitute (4.9) into (4.11) and set $\beta = 0$. This gives:

$$\left(\frac{W_i}{P} \right) = \left[\frac{\alpha(\theta-1)}{\theta} \right] \left(\frac{D(P,\mathbf{X})}{F} \right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C} \right)^{\frac{\alpha(\theta-1)}{\theta}-1} K_i^{\frac{-1}{\theta}} \tag{4.21}$$

4.4 General Equilibrium Solution

We now analyse our model in a general equilibrium context. In aggregate, since all firms are identical, we have:

$$Y_i = \frac{Y}{F}; \quad N_i = \frac{N}{F}; \quad K_i = \frac{K}{F}; \quad A_i = A; \quad P_i = P; \quad W_i = W \tag{4.22}$$

where the absence of a subscript denotes an economy-wide variable.

4.4.1 Labour Demand Curve

We start by using the relationships in (4.22) to derive the aggregate labour demand curve. At the corner solution, this immediately follows from (4.9):

$$N = \frac{K}{C} \tag{4.23}$$

At the interior solutions, we show in Appendix D that it is given by:

$$\left(\frac{N}{K}\right) = \left[A\left(\frac{1}{C}\right)^{\beta}\right]^{\frac{1}{1-\alpha+\beta}} \left[\frac{\theta W}{(\alpha-\beta)(\theta-1)P}\right]^{\frac{-1}{1-\alpha+\beta}} \tag{4.24}$$

If we set $\beta = 0$ and $A = 1$, (4.24) reduces to:

$$\left(\frac{N}{K}\right) = \left[\frac{\theta W}{\alpha(\theta-1)P}\right]^{\frac{-1}{1-\alpha}} \tag{4.25}$$

This is identical to equation (14) on page 105 of LNJ.[15] So, if β always equals zero in our model (i.e. there is always "spare capacity"), then we are back to standard theory and all of the results shown by LNJ continue to apply.

4.4.2 Wage-setting

We now consider wage-setting - this will enable us to derive an expression for aggregate unemployment in the (short-run) model. Here, the key difference between the partial and general equilibrium cases is that in the general equilibrium case, we can no longer treat the alternative wage, $\overline{V}$, as exogenous. Instead, following standard convention (and implicitly assuming that workers are risk-neutral), we model it by:

$$\overline{V} = \frac{N}{L}\left(\frac{W}{P}\right) + \left(1-\frac{N}{L}\right)\left(\frac{B}{P}\right) \tag{4.26}$$

where L is the size of the labour force, implying that:

$$\frac{N}{L} = e \tag{4.27}$$

is the (endogenous) probability of being employed while

$$\left(1-\frac{N}{L}\right) = u \tag{4.28}$$

[15] Their W denotes the real wage while ours denotes the nominal wage; their κ is elsewhere defined by $\kappa = (\theta-1)/\theta$

is the probability of being unemployed and receiving the nominal unemployment benefit, *B*. Note that by our definition, we have assumed that the employment rate, *e*, and the unemployment rate, *u*, are related by the identity $u \equiv 1 - e$.

At the interior solutions, the partial equilibrium wage is given by (4.19). Substituting (4.26) into this, aggregating using (4.22) and rearranging gives:

$$\frac{W}{P} = \psi\left(\frac{L-N}{L-\psi N}\right)\left(\frac{B}{P}\right) \tag{4.29}$$

Following LNJ (p. 107), we assume that the government indexes benefits to wages so that the benefit replacement ratio

$$\frac{(B/P)}{(W/P)} = \frac{B}{W} = b \quad \text{where} \quad 0 < b < 1 \tag{4.30}$$

is fixed. Aggregate equilibrium employment at the interior solutions then follows directly from (4.29) and (4.30):

$$\frac{1}{b} = \psi\left(\frac{L-N}{L-\psi N}\right)$$

$$N = \frac{L(1-b\psi)}{\psi(1-b)} \tag{4.31}$$

Since aggregate employment cannot be negative, (4.31) is only valid if $b\psi < 1$. If this condition is violated, then $N = 0$. Intuitively, this means that if both the mark-up and the benefit replacement ratio are very high, then, as we might expect, we will have no employment.

Substituting (4.31) into (4.24), we can derive an expression for the aggregate real wage at the interior solutions:

$$\left(\frac{W}{P}\right) = A\left(\frac{1}{C}\right)^{\beta}\left[\frac{K\psi(1-b)}{L(1-b\psi)}\right]^{1-\alpha+\beta}\left[\frac{(\alpha-\beta)(\theta-1)}{\theta}\right] \tag{4.32}$$

We can also use (4.27), (4.28) and (4.31) to derive expressions for the aggregate employment and unemployment rates:

$$e = \frac{1-b\psi}{\psi(1-b)} \quad \text{for } b\psi < 1;\quad e = 0 \text{ otherwise} \tag{4.33}$$

$$u = \frac{\psi - 1}{\psi(1-b)} \quad \text{for } b\psi < 1;\quad u = 1 \text{ otherwise} \tag{4.34}$$

All these expressions depend on the value of ψ. Returning to (4.31), we can see that:

$$\frac{dN}{d\psi}=\frac{\psi(1-b)(-bL)-L(1-b\psi)(1-b)}{\psi^2(1-b)^2}=\frac{-L}{\psi^2(1-b)}<0 \text{ since } b<1 \qquad (4.35)$$

As shown in Appendix C, ψ depends positively on β. Therefore, it follows from (4.35) that if β takes a constant positive value, aggregate equilibrium employment will be lower than if $\beta = 0$. So, a rise in β (which could be induced by a fall in the exogenous capital stock) will lead to a fall in aggregate equilibrium employment. In other words, if, as defined above, we let ψ^0 correspond to the case $\beta = 0$ in (4.20) and ψ^+ correspond to the case $\beta > 0$, then from (4.31):

$$N^0=\frac{L(1-b\psi^0)}{\psi^0(1-b)} \text{ and } N^+=\frac{L(1-b\psi^+)}{\psi^+(1-b)}$$

with $N^0 > N^+$.

As in the analysis of LNJ, it is clear from these expressions that aggregate equilibrium employment is independent of the level of the capital stock within each regime. However, its actual level does depend on the capital stock since this determines the regime which applies. Specifically, if the (exogenous) capital stock is sufficiently high (i.e. condition (4.14) is satisfied), the partial equilibrium solution will have $\beta = 0$ and aggregate employment will be N^0. By contrast, if the capital stock is very low (i.e. condition (4.16) is satisfied), then we know from our discussion above that the *maximum* level of aggregate employment will be N^+, which is lower than N^0. (More specifically, if $K < K^1$, then employment will definitely be N^+.) Therefore, over a certain range, increases in the (exogenous) capital stock may lower aggregate equilibrium unemployment and we have broken down the LNJ result (p. 107) that "unemployment…is independent of capital accumulation". This follows solely from using the alternative KC production function presented above: indeed it is interesting to note how sensitive the LNJ result is to such a small change of assumption.

We may illustrate our result even more starkly by considering the corner solution. In this case, assuming that condition (4.17) holds, the partial equilibrium wage is given by (4.21):

$$\left(\frac{W_i}{P}\right)=\left[\frac{\alpha(\theta-1)}{\theta}\right]\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{\theta-1}{\theta}} K_i^{\frac{-1}{\theta}}\left(\frac{1}{C}\right)^{\frac{\alpha(\theta-1)}{\theta}-1} \qquad (4.21)$$

If this wage is set in all firms, then all firms will choose the corner solution for employment given by (4.9). However, since (4.21) is independent of the alternative wage, $\overline{V}$, the employment choices of individual firms will have no bearing on the aggregate real wage. As a result, the aggregate real wage at the corner solution will simply be given by the aggregate version of (4.21):

$$\left(\frac{W}{P}\right)=\left[\frac{\alpha(\theta-1)}{\theta}\right]\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}}A^{\frac{\theta-1}{\theta}}\left(\frac{K}{F}\right)^{\frac{-1}{\theta}}\left(\frac{1}{C}\right)^{\frac{\alpha(\theta-1)}{\theta}-1} \tag{4.36}$$

Meanwhile, economy-wide employment will be given by aggregating (4.9) which gives (4.23): $N=K/C$. Therefore, we can see that in this case, aggregate equilibrium employment depends directly and continuously on the (exogenous) capital stock. In Appendix D, we eliminate $D\,(P,\,\mathbf{X})$ from (4.36) to derive an expression for the corner solution aggregate real wage solely in terms of exogenous variables and parameters:

$$\left(\frac{W}{P}\right)=\left[\frac{\alpha(\theta-1)}{\theta}\right]A\left(\frac{1}{C}\right)^{\alpha-1} \tag{4.37}$$

4.4.3 Summary of the General Equilibrium Solution

Equilibrium outcomes for aggregate employment in the short-run model are given by:

$$N=\begin{Bmatrix} N^{+}=\dfrac{L(1-b\psi^{+})}{\psi^{+}(1-b)} \\ \\ \dfrac{K}{C} \\ \\ N^{0}=\dfrac{L(1-b\psi^{0})}{\psi^{0}(1-b)} \end{Bmatrix} \quad \text{for} \quad \begin{Bmatrix} K<K^{1} \\ \\ K^{1}\le K\le N^{0}C \\ \\ K>N^{0}C \end{Bmatrix} \tag{4.38}$$

Aggregate employment may lie anywhere in the range $\left[K^{1}/C,\,N^{0}\right]$. The exact employment outcome within this range (and hence the equilibrium employment and unemployment rates) will depend on the level of the (exogenous) aggregate capital stock. If the capital stock is high, employment is likely to be high; if it is low, employment is likely to be low. However, outside of the range $\left[K^{1},\,N^{0}C\right]$, changes in the capital stock will not affect employment. In particular, if the initial capital stock is fairly high, increases in its level will not be able to increase employment. As Figure 1 illustrates, this contrasts with the results derived by both LNJ using Cobb-Douglas production technology and Rowthorn (1999) using CES production technology.

5. Endogenising the Capital Stock

The results presented above were derived under the assumption that the capital stock was exogenous. In Kapadia (2003), we endogenise the capital stock (but continue to take the real user cost of capital as exogenous). In other words, referring back to our timeline, we consider the firm's optimal decision in period t-1. We show that even with the capital stock endogenous, there is no guarantee that aggregate employment will always be at the high equilibrium level. Instead, it will be affected by the real user cost of capital (and hence by the real interest rate) over a certain range.

Since the derivation of these results is quite long and since the results do not change the essence of our short-run conclusions in any case, we do not present the formal argument here. However, the intuition behind the results is fairly easy to see. If the real user cost of capital is relatively high, it is clear that the firm will choose a low capital stock, meaning that the capital-labour ratio will be below its threshold and the low equilibrium employment outcome (which is associated with the low wage share and high profit share) will result. It is not quite so obvious that the firm will choose a high capital stock (which generates the high employment equilibrium) if the real user cost of capital is low because, with the capital-labour ratio above its threshold, the profit share will take its lower value. Nevertheless, if the real user cost of capital is sufficiently low, it is clear that the firm will indeed choose a high capital stock. This is because the direct loss from deliberately choosing a lower capital stock just to maintain a high profit share must eventually outweigh any potential gain. To see this, consider what happens in the limit as the real user cost of capital approaches zero. In this scenario, the firm can earn infinite profits by choosing an infinite capital stock. It is clearly not going to choose a finite capital stock just to increase its profit share.

From this, we can therefore see how if the capital stock is endogenous in our model, equilibrium employment (and hence unemployment) will depend on the real user cost of capital (and therefore on the real interest rate) over a certain range. Specifically, if the real interest rate is high, equilibrium unemployment is likely to be high; if the real interest rate is low, equilibrium unemployment is likely to be low.[16] Moreover, if investment (and hence the aggregate capital stock) is affected by factors other than the real user cost of capital (e.g. by taxes, the level of current and expected future profitability, corporate governance structure, or the ease of access to credit), then these factors will also have an impact on long-run equilibrium unemployment.

6. The Equilibrium Employment Rate Over Time

In this section, we consider the evolution of the equilibrium employment rate over time. In particular, we adapt an argument developed by Rowthorn (1999, pp. 421-423) to show how our model can be made consistent with the stylised fact, mentioned by many authors (e.g. Bean (1989); LNJ), that the (un)employment rate is untrended in the very long-run while the capital-labour ratio has grown steadily over time.

As our model currently stands, there is no mechanism which generates a constantly growing capital-labour ratio over time. Moreover, we cannot simply postulate that the capital-labour ratio grows at some exogenous rate, since the model would then imply that the economy would always eventually end up with a sufficiently high capital-labour ratio to ensure that the high aggregate equilibrium employment rate results.

However, suppose that all technical progress is labour-augmenting. (Our results would not be affected if we used the weaker assumption that technical progress has a labour-augmenting bias – see Rowthorn (1999)). In this case, to maintain the status quo of our model (i.e. to keep the relative threshold unchanged), the capital-labour ratio must grow at the same rate as labour productivity. Intuitively, this is because if labour productivity increases, each unit of labour "requires" a greater level of capital to work on for the returns to labour not to fall.

To see this more formally, let us adapt the threshold capacity constraint (3.2) to be:

[16] Note that changes in the real interest rate may partially be driven by changes in both output and inflation volatility. This then raises the interesting point that changes in volatility could potentially affect equilibrium unemployment. In other words, the long-run steady state could be influenced by the degree of short-term volatility, thus breaking down the dichotomy between the short-run and the long-run which is sometimes assumed in macroeconomic models.

$$\begin{aligned} &\beta > 0 \text{ for } \frac{K_i}{\Lambda_N N_i} < C \text{ ("full capacity")} \\ &\beta = 0 \text{ for } \frac{K_i}{\Lambda_N N_i} > C \text{ ("spare capacity")} \end{aligned} \tag{6.1}$$

where Λ_N is an index of the productive efficiency of labour (assumed to be the same across all firms).[17] Labour-augmenting technical progress is indicated by an increase in Λ_N. The rate of growth of Λ_N is assumed to be exogenous.

With this new condition, the employment level at which the threshold is crossed is:

$$N_i = \frac{K_i}{\Lambda_N C} \tag{6.2}$$

It is clear that if Λ_N and (K_i/N_i) grow at the same rate, then the model we have developed will apply in every time period (i.e. the status quo of our model will always apply). By contrast, if (K_i/N_i) grows more slowly than Λ_N, then the threshold will be hit at lower and lower levels of employment as time progresses and it will become increasingly hard to sustain the high aggregate equilibrium employment rate.

Now suppose that it is indeed the case that Λ_N and (K_i/N_i) grow at the same rate on the trend equilibrium path. This assumption has been justified by Rowthorn (1999), but may also be justified by appealing to the Solow growth model (consider what happens on the balanced growth path). In this case, only above trend or below trend increases in the capital-labour ratio can affect the equilibrium rate of employment, with above trend increases having a positive impact and below trend increases having a negative impact. Therefore, our model is able to simultaneously generate a constantly growing capital-labour ratio and an untrended (un)employment rate.

Moreover, any policy change which results in a shift in the real user cost of capital, *R*, will affect the absolute level of the capital-labour ratio and will therefore (over a certain range) have a one-off (i.e. level) effect on employment. However, it will have no effect on the long-run growth rate of the capital-labour ratio since this is

[17] Note that Λ_N could be viewed as varying across countries. Since Λ_N is likely to be higher in rich countries, we can see from the modified threshold capacity constraint that rich countries will need a higher capital-labour ratio to be operating under "spare capacity" than poorer countries. Therefore, even though capital-labour ratios are likely to be lower in poor countries, we can see that our model does not necessarily imply that equilibrium unemployment will be higher in these countries.

determined by the rate of growth of Λ_N. Therefore, we can see that in our model, it is really the real user cost of capital (and hence the real interest rate) which determines the equilibrium employment rate (though the channel is through a one-off change in the level of the capital stock). Since the real interest rate is untrended over time, our model therefore does not succumb to the argument of Bean (1989), LNJ and others that the (un)employment rate cannot be related to trended variables in the very long-run. Finally, we should also note that the other determinants of investment, which we cited above as potentially being able to affect the equilibrium unemployment rate, are all untrended over time as well.

7. Conclusion

7.1 Summary of the Paper and its Main Results

The main objective of this paper was to introduce a new notion of "capacity" in order to theoretically investigate the relationship between the capital stock and the equilibrium rate of unemployment. Our approach involved using a new production function, referred to as the KC production function. This was designed to incorporate meaningful capacity effects not captured by either the Cobb-Douglas or CES production functions. We introduced the KC production function into an otherwise standard imperfectly competitive macroeconomic model of unemployment. Solving our short-run model, we showed that equilibrium unemployment depends on the level of the capital stock over a certain range. We also explained intuitively how endogenising the capital stock implies that it is the real user cost of capital which affects equilibrium unemployment instead. Finally, we showed how our model could be adapted to make it consistent with the stylised fact that the unemployment rate is untrended in the very long-run while the capital-labour ratio has grown steadily since the Industrial Revolution.

7.2 Possible Extensions

In this paper, we assumed that all firms were identical. However, with heterogeneous firms, it is possible for some firms to be operating under "spare capacity" while other firms are operating under "full capacity". Considering the implications of this on aggregate outcomes could be a possible extension. However, intuitively it does not seem that our broad conclusions would be affected: even with heterogeneous firms, provided that the real user cost of capital were sufficiently low (high), we would still expect most firms to be operating under "spare capacity" ("full capacity"), thus continuing to generate the high (low) aggregate equilibrium employment rate.

We also only searched for equilibrium solutions for employment and other variables. Developing the model in a dynamic context to consider what might happen during the transition to equilibrium following a shock is clearly another possible extension. For example, we could assume that adjustment to a new equilibrium capital stock following a change in the real interest rate takes time rather than being instantaneous. This could generate the potential for persistence in unemployment following a rise in the real interest rate which is then reversed. Perhaps a more interesting alternative would be to assume, in the spirit of Keynes, that the investment rate is influenced by the level of aggregate demand via its impact on current and hence expected future profitability. This then opens up the possibility of an interesting hysteresis mechanism whereby a gradual expansion in demand might lower the equilibrium rate of unemployment (through capital stock effects) before significant inflation is generated. If so, the effects of the demand expansion would be permanent with the economy left in a new equilibrium with a higher level of both capital and employment.

Finally, the notion of "capacity" introduced in this paper could be embedded into other production functions (e.g. CES). These augmented production functions (along with the KC production function itself) could be useful for modelling capacity in other areas of economics.

7.3 Policy Implications

The policy implications of our results are clear. They are fairly similar to those discussed by Rowthorn (1995, 1999). In particular, to tackle unemployment, promoting investment may often be a superior alternative to pursuing labour market reforms. The case for this is made even stronger when we consider that high levels of investment are generally seen as beneficial to the economy as a whole, while some labour market reforms are associated with adverse effects on other aspects of welfare.

There is much debate concerning how best to encourage investment (see, for example, the discussion in Bond and Jenkinson, 2000). However, possible policies include making the tax regime more favourable for investment, encouraging savings, and trying to encourage equity market investors to have longer time horizons (e.g. by reducing capital gains tax on long-term equity holdings). Meanwhile, if there is scope for positive hysteresis along the lines discussed above, then gradual demand expansions may also be effective. Finally, it has been argued (e.g. McKibbin and Vines, 2000) that increases in real interest rates were caused by the large fiscal deficits and restrictive monetary policy associated with both "Reaganomics" and German reunification. Since high real interest rates are bad for investment, this suggests that it might be beneficial for countries to pursue the reverse of this combination of policies: namely a reduction in fiscal deficits coupled with an expansionary monetary policy.[18] Obviously, this is quite a sweeping statement which neglects both the probable desirability of expansionary fiscal policy during recessions and the fact that restrictive monetary policy may sometimes be necessary to counter inflation. Nevertheless, it sheds interesting light on the large tax cuts recently made by the Bush administration in the United States: the associated fiscal deficits could cause world real interest rates to rise, something which may possibly have adverse effects on equilibrium unemployment rates. It also suggests that the current hawkish attitude of the European Central Bank coupled with the rising fiscal deficits in many Eurozone countries may mean that the outlook for employment in parts of Europe is not particularly good.

Appendix A

We wish to show how (4.13) is derived. From (4.12):

$$N_i^{\alpha'} = \left[\alpha' \left(\frac{D(P,\mathbf{X})}{F} \right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C} \right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta}} \left(\frac{W_i}{P} \right)^{-1} \right]^{\frac{\alpha'}{1-\alpha'}} \tag{A.1}$$

Substituting (4.12) and (A.1) into (4.3) and dropping the arguments of D (P, $\mathbf{X}$) gives:

[18] Solow (2000) also proposes this policy mix for tackling unemployment in some European countries.

$$\Pi_i = \left(\frac{D}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta}} (\alpha')^{\frac{\alpha'}{(1-\alpha')}} \left(\frac{D}{F}\right)^{\frac{\alpha'}{\theta(1-\alpha')}} A_i^{\frac{\alpha'(\theta-1)}{\theta(1-\alpha')}} \left(\frac{1}{C}\right)^{\frac{\alpha'\beta(\theta-1)}{\theta(1-\alpha')}} K_i^{\frac{\alpha'(\theta-1-\theta\alpha')}{\theta(1-\alpha')}} \left(\frac{W_i}{P}\right)^{\frac{-\alpha'}{(1-\alpha')}}$$

$$-\left(\frac{W_i}{P}\right)(\alpha')^{\frac{1}{(1-\alpha')}} \left(\frac{D}{F}\right)^{\frac{1}{\theta(1-\alpha')}} A_i^{\frac{(\theta-1)}{\theta(1-\alpha')}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta(1-\alpha')}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta(1-\alpha')}} \left(\frac{W_i}{P}\right)^{\frac{-1}{(1-\alpha')}} - RK_i$$

$$= (\alpha')^{\frac{\alpha'}{1-\alpha'}} \left(\frac{D}{F}\right)^{\frac{1}{\theta(1-\alpha')}} A_i^{\frac{\theta-1}{\theta(1-\alpha')}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta(1-\alpha')}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta(1-\alpha')}} \left(\frac{W_i}{P}\right)^{\frac{-\alpha'}{1-\alpha'}}$$

$$-(\alpha')^{\frac{1}{1-\alpha'}} \left(\frac{D}{F}\right)^{\frac{1}{\theta(1-\alpha')}} A_i^{\frac{\theta-1}{\theta(1-\alpha')}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta(1-\alpha')}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta(1-\alpha')}} \left(\frac{W_i}{P}\right)^{\frac{-\alpha'}{1-\alpha'}} - RK_i$$

Therefore, as required:

$$\Pi_i + RK_i = (1-\alpha')\left[(\alpha')^{\alpha'} \left(\frac{D}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{\theta-1}{\theta}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta}} \left(\frac{W_i}{P}\right)^{-\alpha'}\right]^{\frac{1}{1-\alpha'}} \qquad (4.13)$$

Appendix B

Relative Slopes of the Labour Demand Curves

We wish to justify the relative slopes of the labour demand curves in Figure 3. These curves are sketched using the log version of (4.12) for the cases $\beta = 0$ and $\beta > 0$:

$$n_i = \frac{1}{(1-\alpha')}\left[\ln(\alpha') + \frac{1}{\theta}(d-f) + \frac{(\theta-1)}{\theta}a_i - \frac{\beta(\theta-1)}{\theta}c + \frac{(\theta-1-\theta\alpha')}{\theta}k_i - (w_i - p)\right]$$

(B.1)

Differentiating (B.1) with respect to w_i and then inverting gives:

$$\frac{dw_i}{dn_i} = -(1-\alpha') \qquad (B.2)$$

Since α', defined by

$$\alpha' = \frac{(\alpha-\beta)(\theta-1)}{\theta} \qquad (4.4)$$

is less than one (recall that $0 \le \beta < \alpha < 1$ and $\theta > 1$), both curves will be linear and downward sloping. Moreover, substituting (4.4) into (B.2) gives:

$$\frac{dw_i}{dn_i} = -\left[1 - \frac{\alpha(\theta-1)}{\theta} + \frac{\beta(\theta-1)}{\theta}\right] \tag{B.3}$$

From (B.3), we can clearly see that the $\beta > 0$ labour demand curve will be more steeply downward sloping than the $\beta = 0$ labour demand curve. This is as depicted in Figure 3.

Intersection of the Labour Demand Curves

We wish to show that the point of intersection (X) of the $\beta = 0$ and $\beta > 0$ labour demand curves will always occur at a level of employment below the threshold capacity constraint. We start by noting that the inverse labour demand curve is given by:

$$\left(\frac{W_i}{P}\right) = \alpha'\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{\theta-1-\theta\alpha'}{\theta}} N_i^{\alpha'-1} \tag{4.11}$$

which may be rewritten as:

$$\left(\frac{W_i}{P}\right) = \left[\frac{(\alpha-\beta)(\theta-1)}{\theta}\right]\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} \left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{(1-\alpha+\beta)(\theta-1)}{\theta}} N_i^{\frac{(\alpha-\beta)(\theta-1)}{\theta}-1} \tag{B.4}$$

If $\beta > 0$, the inverse labour demand curve is simply (B.4). Meanwhile, when $\beta = 0$, the inverse labour demand curve is given by setting $\beta = 0$ in (B.4):

$$\left(\frac{W_i}{P}\right) = \frac{\alpha(\theta-1)}{\theta}\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}} A_i^{\frac{(\theta-1)}{\theta}} K_i^{\frac{(1-\alpha)(\theta-1)}{\theta}} N_i^{\frac{\alpha(\theta-1)}{\theta}-1} \tag{B.5}$$

If we solve (B.4) and (B.5) as a pair of simultaneous equations, we will obtain the point of intersection of the $\beta = 0$ and $\beta > 0$ labour demand curves. Therefore, we set the right-hand sides of these two equations equal to each other. Doing this, cancelling several terms, and simplifying gives:

$$(\alpha-\beta)\left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{(1-\alpha+\beta)(\theta-1)}{\theta}} N_i^{X\frac{(\alpha-\beta)(\theta-1)}{\theta}-1} = \alpha K_i^{\frac{(1-\alpha)(\theta-1)}{\theta}} N_i^{X\frac{\alpha(\theta-1)}{\theta}-1}$$

$$(\alpha-\beta)\left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}} K_i^{\frac{\beta(\theta-1)}{\theta}} N_i^{X\frac{-\beta(\theta-1)}{\theta}} = \alpha$$

$$\left(\frac{K_i}{N_i^X C}\right)^{\frac{\beta(\theta-1)}{\theta}} = \frac{\alpha}{\alpha-\beta}$$

$$N_i^X = \left(\frac{K_i}{C}\right)\left(\frac{\alpha-\beta}{\alpha}\right)^{\frac{\theta}{\beta(\theta-1)}} \quad \text{(B.6)}$$

In (B.6), N_i^X is the level of employment at the intersection of the two curves. Since $0 \le \beta < \alpha < 1$, we have $\left[(\alpha-\beta)/\alpha\right] < 1$. Therefore:

$$N_i^X < \frac{K_i}{C} \quad \text{(B.7)}$$

This establishes the result.

Appendix C

We first wish to show how (4.19) is derived. From the main text, the first order condition is:

$$\lambda\left[\gamma\frac{d\ln N_i}{d\ln W_i} + (1-\gamma)\frac{d\ln\left[(W_i/P)-\overline{V}\right]}{d\ln W_i}\right] + (1-\lambda)\frac{d\ln(\Pi_i + RK_i)}{d\ln W_i} = 0 \quad \text{(4.18)}$$

At interior solutions, log labour demand is given by (B.1) and real operating profits are given by (4.13). Taking logs of (4.13), we get:

$$\ln(\Pi_i + RK_i) = \ln(1-\alpha') + \frac{1}{1-\alpha'}\left[\alpha'\ln(\alpha') + \frac{1}{\theta}(d-f) + \frac{\theta-1}{\theta}a_i - \frac{\beta(\theta-1)}{\theta}c + \frac{\theta-1-\theta\alpha'}{\theta}k_i - \alpha'(w_i-p)\right] \quad \text{(C.1)}$$

From (B.1):

$$\frac{d\ln N_i}{d\ln W_i} = \frac{dn_i}{dw_i} = \frac{-1}{1-\alpha'} \quad \text{(C.2)}$$

From (C.1):

$$\frac{d\ln(\Pi_i + RK_i)}{d\ln W_i} = \frac{d\ln(\Pi_i + RK_i)}{dw_i} = \frac{-\alpha'}{1-\alpha'} \quad \text{(C.3)}$$

To calculate, $\dfrac{d\ln\left[(W_i/P)-\overline{V}\right]}{d\ln W_i}$, we set:

$$s = \ln W_i \Rightarrow W_i = e^s \quad \text{(C.4)}$$

Now, using the chain rule:

$$\frac{d\ln\left[(W_i/P)-\overline{V}\right]}{d\ln W_i} = \frac{d\ln\left[(W_i/P)-\overline{V}\right]}{ds}\frac{ds}{d\ln W_i} = \frac{d\ln\left[(W_i/P)-\overline{V}\right]}{ds} \quad \text{(C.5)}$$

since $\dfrac{ds}{d\ln W_i} = \dfrac{d\ln W_i}{d\ln W_i} = 1$.

Therefore, using (C.4) and (C.5), we can see that:

$$\frac{d\ln\left[(W_i/P)-\overline{V}\right]}{d\ln W_i} = \frac{d\ln\left[(e^s/P)-\overline{V}\right]}{ds} = \frac{(e^s/P)}{\left[(e^s/P)-\overline{V}\right]} \tag{C.6}$$

Substituting (C.4) into (C.6), we get:

$$\frac{d\ln\left[(W_i/P)-\overline{V}\right]}{d\ln W_i} = \frac{(W_i/P)}{\left[(W_i/P)-\overline{V}\right]} \tag{C.7}$$

We now return to the first order condition. Substituting (C.2), (C.3) and (C.7) into (4.18), we get:

$$\lambda\left[\frac{-\gamma}{1-\alpha'} + \frac{(1-\gamma)(W_i/P)}{\left[(W_i/P)-\overline{V}\right]}\right] + (1-\lambda)\left(\frac{-\alpha'}{1-\alpha'}\right) = 0 \tag{C.8}$$

Rearranging to make the real wage the subject of (C.8) gives:

$$\frac{(1-\gamma)(W_i/P)}{\left[(W_i/P)-\overline{V}\right]} = \frac{\gamma}{1-\alpha'} + \frac{(1-\lambda)}{\lambda}\left(\frac{\alpha'}{1-\alpha'}\right)$$

$$\frac{(W_i/P)}{(W_i/P)-\overline{V}} = \frac{\lambda\gamma+(1-\lambda)\alpha'}{\lambda(1-\alpha')(1-\gamma)}$$

$$1 - \frac{\overline{V}}{(W_i/P)} = \frac{\lambda(1-\alpha')(1-\gamma)}{\lambda\gamma+(1-\lambda)\alpha'}$$

$$\frac{\overline{V}}{(W_i/P)} = \frac{\lambda\gamma+(1-\lambda)\alpha' - \lambda(1-\alpha')(1-\gamma)}{\lambda\gamma+(1-\lambda)\alpha'}$$

$$\frac{W_i}{P} = \psi\overline{V} \tag{4.19}$$

where:

$$\psi = \frac{\lambda\gamma+(1-\lambda)\alpha'}{\lambda\gamma+(1-\lambda)\alpha' - (1-\alpha')\lambda(1-\gamma)} \tag{4.20}$$

which is what we have in the main text.

We also wish to determine how the value of ψ depends on α' and therefore on β. From the main text, we have:

$$\alpha' = \frac{(\alpha-\beta)(\theta-1)}{\theta} \tag{4.4}$$

Differentiating (4.20) with respect to α' using the quotient rule gives:

$$\frac{d\psi}{d\alpha'}=\frac{\left[\lambda\gamma+(1-\lambda)\alpha'-(1-\alpha')\lambda(1-\gamma)\right](1-\lambda)-\left[\lambda\gamma+(1-\lambda)\alpha'\right]\left[(1-\lambda)+\lambda(1-\gamma)\right]}{\left[\lambda\gamma+(1-\lambda)\alpha'-(1-\alpha')\lambda(1-\gamma)\right]^2}$$

$$=\frac{\left[-(1-\alpha')\lambda(1-\gamma)\right](1-\lambda)-\left[\lambda\gamma+(1-\lambda)\alpha'\right]\left[\lambda(1-\gamma)\right]}{\left[\lambda\gamma+(1-\lambda)\alpha'-(1-\alpha')\lambda(1-\gamma)\right]^2}$$

$$=\frac{\alpha'\lambda(1-\gamma)(1-\lambda)-\lambda(1-\gamma)(1-\lambda)-\lambda^2\gamma(1-\gamma)-\alpha'\lambda(1-\gamma)(1-\lambda)}{\left[\lambda\gamma+(1-\lambda)\alpha'-(1-\alpha')\lambda(1-\gamma)\right]^2}$$

$$=\frac{-(1-\gamma)\left[\lambda(1-\lambda)+\lambda^2\gamma\right]}{\left[\lambda\gamma+(1-\lambda)\alpha'-(1-\alpha')\lambda(1-\gamma)\right]^2}$$

Since γ and λ are both constrained to lie between zero and one, this implies that:

$$\frac{d\psi}{d\alpha'}<0 \tag{C.9}$$

Moreover, since $\theta > 1$, it is clear from (4.4) that:

$$\frac{d\alpha'}{d\beta}=-\frac{(\theta-1)}{\theta}<0 \tag{C.10}$$

Putting (C.9) and (C.10) together gives:

$$\frac{d\psi}{d\beta}>0 \tag{C.11}$$

Therefore ψ depends positively on β.

Appendix D

We wish to show how (4.24) is derived. The relationships in (4.22) may be substituted into (4.11) to obtain:

$$\left(\frac{W}{P}\right)=\alpha'\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}}A^{\frac{(\theta-1)}{\theta}}\left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}}\left(\frac{K}{F}\right)^{\frac{(1-\alpha+\beta)(\theta-1)}{\theta}}\left(\frac{N}{F}\right)^{\frac{(\alpha-\beta)(\theta-1)-\theta}{\theta}} \tag{D.1}$$

Aggregating (3.1) gives:

$$\frac{Y}{F}=A\left(\frac{1}{C}\right)^{\beta}\left(\frac{N}{F}\right)^{\alpha-\beta}\left(\frac{K}{F}\right)^{1-\alpha+\beta} \tag{D.2}$$

while aggregating (3.3) gives:

$$\frac{Y}{F}=\frac{D(P,\mathbf{X})}{F} \tag{D.3}$$

From (D.2) and (D.3), we have:

$$\left(\frac{D(P,\mathbf{X})}{F}\right)^{\frac{1}{\theta}}=\left[A\left(\frac{1}{C}\right)^{\beta}\left(\frac{N}{F}\right)^{\alpha-\beta}\left(\frac{K}{F}\right)^{1-\alpha+\beta}\right]^{\frac{1}{\theta}} \tag{D.4}$$

which may be substituted into (D.1) to get:

$$\left(\frac{W}{P}\right)=\alpha'\left[A\left(\frac{1}{C}\right)^{\beta}\left(\frac{N}{F}\right)^{\alpha-\beta}\left(\frac{K}{F}\right)^{1-\alpha+\beta}\right]^{\frac{1}{\theta}}A^{\frac{(\theta-1)}{\theta}}\left(\frac{1}{C}\right)^{\frac{\beta(\theta-1)}{\theta}}\left(\frac{K}{F}\right)^{\frac{(1-\alpha+\beta)(\theta-1)}{\theta}}\left(\frac{N}{F}\right)^{\frac{(\alpha-\beta)(\theta-1)-\theta}{\theta}}$$

$$=\alpha' A\left(\frac{1}{C}\right)^{\beta}\left(\frac{N}{F}\right)^{\alpha-\beta-1}\left(\frac{K}{F}\right)^{1-\alpha+\beta}$$

Therefore, using (4.4):

$$\left(\frac{W}{P}\right)=A\left(\frac{1}{C}\right)^{\beta}\left(\frac{K}{N}\right)^{1-\alpha+\beta}\left[\frac{(\alpha-\beta)(\theta-1)}{\theta}\right] \tag{D.5}$$

Inverting (D.5) gives the aggregate labour demand curve at interior solutions for both $\beta=0$ and $\beta>0$:

$$\left(\frac{N}{K}\right)=\left[A\left(\frac{1}{C}\right)^{\beta}\right]^{\frac{1}{1-\alpha+\beta}}\left[\frac{\theta W}{(\alpha-\beta)(\theta-1)P}\right]^{\frac{-1}{1-\alpha+\beta}} \tag{4.24}$$

This is what we have in the main text.

To derive (4.37), we use (4.23) and (D.4) to eliminate $D\,(P,\,\mathbf{X})$ from (4.36):

$$\left(\frac{W}{P}\right)=\left[\frac{\alpha(\theta-1)}{\theta}\right]\left[A\left(\frac{1}{C}\right)^{\beta}\left(\frac{N}{F}\right)^{\alpha-\beta}\left(\frac{K}{F}\right)^{1-\alpha+\beta}\right]^{\frac{1}{\theta}}A^{\frac{\theta-1}{\theta}}\left(\frac{K}{F}\right)^{\frac{-1}{\theta}}\left(\frac{1}{C}\right)^{\frac{\alpha(\theta-1)}{\theta}-1}$$

$$=\left[\frac{\alpha(\theta-1)}{\theta}\right]\left[A\left(\frac{1}{C}\right)^{\beta}\left(\frac{K}{FC}\right)^{\alpha-\beta}\left(\frac{K}{F}\right)^{1-\alpha+\beta}\right]^{\frac{1}{\theta}}A^{\frac{\theta-1}{\theta}}\left(\frac{K}{F}\right)^{\frac{-1}{\theta}}\left(\frac{1}{C}\right)^{\frac{\alpha(\theta-1)}{\theta}-1}$$

$$=\left[\frac{\alpha(\theta-1)}{\theta}\right]\left[A\left(\frac{1}{C}\right)^{\alpha}\left(\frac{K}{F}\right)\right]^{\frac{1}{\theta}}A^{\frac{\theta-1}{\theta}}\left(\frac{K}{F}\right)^{\frac{-1}{\theta}}\left(\frac{1}{C}\right)^{\frac{\alpha(\theta-1)}{\theta}-1}$$

$$=\left[\frac{\alpha(\theta-1)}{\theta}\right]A\left(\frac{1}{C}\right)^{\alpha-1} \tag{4.37}$$

which is what we have in the main text.

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