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Richard Mash

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Manor Road Building, Oxford OX1 3UQ

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Richard Mash

Department of Economics and New College
University of Oxford

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Abstract

We analyze the microfoundations of the Phillips curve and the close links between that relationship and results concerning optimal monetary policy, stabilisation bias and monetary policy delegation. Most recent literature has used a New Keynesian Phillips Curve based on Calvo pricing, often with an additional lagged inflation term motivated by rule-of-thumb behaviour. We develop a framework which encompasses this workhorse model while allowing for a richer time dependent pricing rule. This permits a more general analysis while showing that the standard model and policy conclusions derived from it are not robust to relatively minor changes in its microfoundations.

Key Words

Monetary Policy, New Keynesian Phillips Curve, Calvo Pricing, Rule of Thumb, Stabilisation Bias, Monetary Policy Delegation.

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Contact Details

Mailing address	Richard Mash, New College, Oxford, OX1 3BN, UK
Telephone	+44-1865-289195
Fax	+44-1865-279590
email	richard.mash@economics.ox.ac.uk.

Introduction

A central concern of macroeconomics is the development of well microfounded models suitable for policy analysis. In relation to monetary policy a key component of such a model is the Phillips curve or aggregate supply relationship linking inflation to output or unemployment which, combined with the actions of the policy maker, determine outcomes for key macroeconomic variables and the success or otherwise of monetary policy from a welfare perspective. The Phillips curve has been the subject of considerable controversy and while certain forms of it are widely used there is no clear consensus on the correct specification. McCallum (1999a) refers to aggregate supply as being the least well understood component of current monetary policy models. This uncertainty over the true model is clearly problematic for monetary policy in general, since optimal policy depends on the structure of the economy, and also for the analysis of monetary policy delegation to overcome stabilisation bias.

Given model uncertainty, considerable attention has been placed on robustness issues in policy design with the aim of finding policy rules or regimes which work well across a range of possible models of the underlying economy. For example the introduction and many of the contributions to the Taylor (1999a) volume on policy rules places great emphasis on robustness as does McCallum (1999a) and more recent prominent contributions such as Amato and Laubach (2003a), Jensen (2002), Levin and Williams (2003), Levin, Wieland and Williams (2003), Rudebusch (2002) and Walsh (2003a).

This paper re-examines the microfoundations underlying the Phillips curves commonly used in monetary policy analysis, in the first instance to assess their robustness to plausible changes in the microfoundations, while also analyzing the relationship between the more general Phillips curve derived and optimal monetary policy and delegation. More specifically, the paper is concerned with the microfoundations of price setting in New Keynesian staggered price or overlapping contracts models and the implications of different time dependent pricing rules for the Phillips curve and optimal policy. Most recent research in this area has used the comparatively simple Calvo (1983) pricing specification in which the probability of a firm changing its price is constant and thus independent of the time since the last price change. A parallel literature, discussed further below, has examined richer time dependent pricing rules, characterised by an increasing probability of price change, though without analyzing the implications for delegation and usually without explicit derivation of the Phillips curve. This paper aims to bridge the gap between these two strands of research by deriving an explicit Phillips curve for a simple increasing probability pricing rule that is used for a more general analysis of optimal delegation.

The policy context of this work is the rapid progress made in the recent monetary policy literature in analyzing the consequences for optimal policy of the presence of forward looking expectations in the Phillips curve. When commitment is feasible, optimal policy in response to a cost push or inflation shock adjusts future policy to improve current outcomes through the intertemporal link of expected future inflation in the Phillips curve.[1] From the point of view of the following periods taken in isolation such a policy is costly (and hence is not generally carried out under discretion, resulting in stabilisation bias) but optimal policy under commitment balances these future costs against current benefits. It may be noted that stabilisation bias is a form of time inconsistency separate from the classic inflation bias problem of Kydland and Prescott (1977) and Barro and Gordon (1983) which has been less of a concern in the recent literature.

Given the presence of stabilisation bias (and hence lower welfare) under discretion a growing literature assesses how improved outcomes may be achieved (assuming that intertemporal commitment is generally infeasible) by altering the loss function to be minimised under delegation by an independent central bank. The classic early case is Rogoff's (1985) conservative central banker, even though at that time the concern was with inflation bias rather than stabilisation bias. Notable early examples of delegation regimes to offset stabilisation bias are Woodford's (1999) demonstration of the potential benefits of interest rate smoothing and Clarida, Gali and Gertler's (1999) result that with a Calvo Phillips curve optimal policy under commitment results in a stable long run price level even though society cares about stability in inflation rather than the price level. This is suggestive of welfare gains when commitment is not possible from a delegated loss function specified in terms of the price level, a result shown by Vestin (2000). Other possible beneficial features of a delegated loss function are nominal income growth targeting (Jensen, 2002, Rudebusch, 2002), average inflation targeting (Nessen and Vestin, 2000, Nessen, 2002, and Batini and Yates, 2003), targeting the change in the output gap (Soderstrom, 2001, Walsh, 2003a), and allowing for changes in inflation expectations (Svensson and Woodford, 1999). A parallel literature examines the results that would obtain from implementation of different forms of the Taylor rule.

With few exceptions the recent literature makes use of New Keynesian Phillips curves based on Calvo pricing. The 'pure' form of the model is shown by (1) where π is the rate of inflation, β the (real) discount factor, x the driving variable (such as the output gap or marginal cost), k a constant and u a shock variable.

$$\pi_t = \beta E_t[\pi_{t+1}] + kx_t + u_t \tag{1}$$

Given the Calvo constant probability pricing rule, (1) reflects fully optimising behaviour but many authors have argued that it is not satisfactory since it does not result in the persistence in inflation (unless the process for u_t is strongly serially correlated) which seems to be a robust feature of actual data. Given this the Calvo Phillips curve (1) is often amended to the form (2) which includes lagged inflation and is much better able to account for inflation persistence.

$$\pi_t = \beta(1-\delta)E_t[\pi_{t+1}] + \delta\pi_{t-1} + kx_t + u_t \tag{2}$$

[1] See Clarida, Gali and Gertler (1999) and Woodford (2000) for summaries.

While empirically much more successful, (2) has given rise to two debates. The first is the size of δ, which indexes the relative importance of lagged inflation.[2] The second is the rationale for the inclusion of lagged inflation which, while empirically supported, gives rise to Lucas critique concerns unless it can be given a plausible microfoundation. Key contributions here are Gali and Gertler (1999) and Christiano, Eichenbaum and Evans (2001) which both use the Calvo pricing framework but assume that firms face information constraints and/or high decision making costs such that some price setting actions are based on rules of thumb rather than full (unconstrained/low decision cost) optimisation.[3] There is perhaps room for debate about the details of these mechanisms and how convincing they are as optimising microfoundations but for present purposes we highlight the feature that both the rule of thumb models use a Calvo pricing rule.

Hence, leaving aside the important empirical and microfoundations debates about (1) and (2), the widespread reliance on these specifications for monetary policy conclusions implies equally strong reliance on the Calvo (1983) mechanism as the underlying time dependent pricing rule.[4] Given the concern for robustness in this literature an important question is the robustness of conclusions based on (1) or (2) to variations away from Calvo pricing. This is in addition to the impact of different values of δ in (2) which has already been the focus of research.

Of course it might be the case that reliance on Calvo pricing is not a key concern, either because it is supported empirically or because it is a relatively innocuous simplifying assumption such that varying it makes little difference to policy analysis. From an empirical viewpoint, the microeconomic evidence on price setting frequency and price duration is not supportive of the Calvo framework, at least when applied to quarterly models which is the standard practice. Wolman (1999, 2000) reviews this literature and argues in favour of increasing rather than constant price change probabilities. In terms of price duration the Calvo model predicts that the most frequently observed duration will be a single quarter with frequency declining with duration thereafter. However, Taylor's (1999b) survey of the evidence suggests that frequency is 'hump-shaped' with duration such that relatively few prices remain fixed for only one or two or more than four or five quarters with 3-4

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[2] See Erceg and Levin (2003), Estrella and Fuhrer (2002), Gali and Gertler (1999), Gali, Gertler and Lopez-Salido (2001), Mankiw (2001), Nelson (1998), Roberts (1997, 2001), Rudd and Whelan (2001), Rudebusch (2002), Rudebusch and Svensson (1999, 2002), and Soderlind, Soderstrom and Vredin (2002).

[3] The two mechanisms differ in the application of rule of thumb price setting. In Gali and Gertler (1999) a proportion of firms use a rule of thumb rather than full optimisation when they receive a 'Calvo signal' that they may change price but all firms keep prices fixed between such signals. In Christiano, Eichenbaum and Evans (2001) all firms optimise fully when a Calvo signal arrives but apply a rule of thumb, effectively full indexing, between signals. Woodford (2003) develops the latter by allowing for partial indexing. Amato and Laubach (2003b), Calvo, Celasun and Kumhof (2001), Sbordone (2001) and Steinsson (2000) assume rule of thumb behaviour in their analyses. The Fuhrer and Moore (1995) model has also been used as a justification for (2) but the extent to which this model is consistent with optimising microfoundations has been strongly questioned (see Taylor, 1999b).

[4] The widespread use of Calvo pricing is documented by Rudebusch (2002). Influential examples include Clarida, Gali and Gertler (1999), McCallum and Nelson (1999, 2000) and Rotemberg and Woodford (1997, 1999).

quarters the peak of the distribution. It may also be noted that Calvo (1983) presents the constant probability of price change as a simplifying rather than fundamental assumption.[5]

Given the lack of microeconomic empirical support for Calvo pricing the question becomes whether different price changing assumptions nevertheless imply similar forms for the Phillips curve and thus in turn similar results for monetary policy. One alternative specification is Rotemberg's (1987) model of convex price adjustment costs which also leads to a Phillips curve of the Calvo form (1), though Rotemberg presents convex price adjustment costs as a simplifying rather than fundamental assumption also. A number of authors (for example Roberts, 1995, and Walsh, 2003b) have also pointed to similarities between the Calvo model and that of Taylor (1979, 1980) where price changes are also staggered but are simply fixed for two (or more) periods at a time. This is analyzed in full below (since the new framework we develop encompasses a two-period Taylor model) and confirms that the Taylor model differs significantly from Calvo.[6]

The contributions of Wolman (1999) and Kiley (2002) also show significant effects of moving away from Calvo pricing though without explicit derivation of non-Calvo Phillips curves or optimal policy. Dotsey (2002) presents an amended three period Taylor Phillips curve, highlighting its differences from the Calvo form (1) though without cost push shocks or policy analysis since its focus is on econometric issues. At a more general level, Dotsey, King and Wolman's (1999) analysis of state dependent pricing points the way to much richer models, both in terms of more general probabilities of price change as well as their state dependence though this contribution does not derive an explicit form for the Phillips curve or analyze the implications of the framework for stabilisation bias or policy delegation. In related work, Ascari (2000) and Ascari and Rankin (2002) derive microfounded Taylor models though without explicit comparison with Calvo or general policy analysis. Bakhshi et. al. (2003) show that the Calvo framework may break down under average inflation rates comparable to the historical experience of the last three decades.

Hence from the literature it can be concluded that there is reason to question the Calvo pricing formulation on the grounds of microeconomic realism and also reason to suppose that it is not a trivial simplifying assumption for monetary policy analysis. Clearly also the current paper is by no means the first to question the Calvo framework or develop alternatives but nevertheless the policy implications of moving away from Calvo pricing have not been analyzed thoroughly and the implications for monetary policy delegation not at all.

[5] Woodford (1995) for example refers to the Calvo specification as being, "a device that is of great convenience, even if not entirely realistic" (p. 1281) and the constant probability assumption as, "an obvious oversimplification" (p. 1282).

[6] The reasons for the difference between this conclusion and those of Roberts (1995) and Walsh (2003b) is that these authors either assume that there are no cost push or inflation shocks from price setting, or that part of the impact of such shocks may be represented in an expectational error term. While formally correct it may be argued that these presentations of the Taylor model are potentially misleading since for policy analysis such shocks are generally considered important and expectational error terms may obscure their significance. This is shown in Appendix C where we also highlight a similar implicit expectational error term in the standard presentation of the Fuhrer-Moore (1995) model.

We address this gap in our knowledge by analyzing the consequences of a time dependent pricing rule that remains fairly close to the Calvo form while reflecting in part the empirical evidence that the probability of price change tends to rise rather than remain constant with the time since the last price change. A special case of the new rule is Calvo pricing itself so the model encompasses the Calvo specification (1) above. We also retain the possibility of rule of thumb behaviour so that the model encompasses (2) as well as (1), thus nesting both the formulations in widespread use as special cases. Our motivation for including rule of thumb behaviour is pragmatic in that it is informative to have a model which encompasses existing specifications and as such we do not take any particular stand on the debate about whether rule of thumb behaviour is a convincing microfoundation for lagged inflation in (2). For this part of the model we build on the Christiano, Eichenbaum and Evans (2001) form of rule of thumb behaviour, generalising it somewhat to allow for only a proportion of firms rather than all of them to update their prices by rule of thumb between optimisations.[7]

The key innovation in the model is the time dependent pricing rule. One may think of such a rule as being a sequence of probabilities of price change indexed by the time since the last price change. If these are denoted q_0, q_1, q_2, q_3 etc. it is standard to set $q_0=0$ which simply implies that all prices are fixed for at least one period, following which different pricing rules adopt different values for the other probabilities. In the Calvo model these probabilities are all the same whereas in a two-period Taylor model, $q_1=0$ and $q_2=1$ so the price will remain fixed for sure for two periods and may then be reset with probability one. The pricing rule which we analyze is a simple variant on these two cases in that we allow q_1 to take any value (between zero and one) while assuming that q_2, q_3 etc. are all equal at a value denoted q. Hence the probability of price change may vary between the first period after a new price takes effect and the value it will then take for all remaining periods. A key parameter of the model is $q-q_1$, the extent of the increase in the probability of price change which we denote q^*. From the above discussion, $q^*=0$ implies the Calvo form while $q^*=1$ implies two-period Taylor.

From this description the strengths and weaknesses of our framework are immediately apparent. It is clear that the sequence of probabilities assumed of $(0, q_1, q, q, q, ...)$ is itself a less than fully general case since in principle all of these probabilities may take different values. Given that the

[7]This generalisation is discussed by Amato and Laubach (2003b) but not reported formally. In Christiano, Eichenbaum and Evans (2001) all firms practice rule of thumb updating between optimised price changes which fixes δ in (2) to a figure close to a half. For policy analysis it is useful to have a framework in which δ can vary continuously. This is achieved by having the proportion of rule of thumb firms as an independent parameter. The change also addresses a criticism of the model that the data strongly suggests that many prices remain fixed for several periods whereas if all firms update their prices each period this would not be the case. A variation in the model developed below is that we assume that all price setting is contemporaneous with full information at that time whereas Christiano et. al. assume that firms commit to new prices one period in advance (or contemporaneously but with a one period lagged information set). Rotemberg and Woodford (1997, 1999), and Woodford (2003) also incorporate this feature. Mankiw and Reis (2002) assume that different groups of firms have information sets lagged a different number of periods. Lagged information sets could be incorporated in the model of this paper but for simplicity we restrict its scope in this dimension.

results below show that the nature of the time dependent pricing rule is highly significant for monetary policy outcomes this possibility is almost certainly worth pursuing. Equally the sensitivity of outcomes to the timing of price changes also increases the potential value of moving to a state dependent pricing model where the timing of price changes is chosen by firms rather than exogenously imposed by the modeller. That having been said there are strong arguments for developing the current framework, albeit possibly as an interim step to more complex formulations.

First, while there are standard and strong arguments for the potential merits of state dependent pricing, models of that type are difficult to derive and analyze and while Dotsey, King and Wolman (1999) make significant progress in this direction, they nevertheless present only fairly basic policy experiments. Given those difficulties it is not surprising that virtually the whole monetary policy literature uses time dependent pricing rules and thus understanding their implications is an important task.[8]

Second, the literature has relied not on time dependent pricing in general but on Calvo pricing in particular. Hence while the analysis of more general time dependent pricing rules may yield useful insights, the fairly simple formulation used here is sufficient to examine the robustness of many current monetary policy results. Given that we find non-robustness with a small variation from Calvo pricing it is highly plausible that more general variations would strengthen that conclusion further.

Third, there is a value to understanding the properties of relatively simple models from the point of view of tractability and building intuition even if it is recognised that more complex formulations may also be informative. As noted above the assumed pricing rule is sufficiently general to encompass not only the Calvo model but the two-period Taylor model and cases between the two as well through the single parameter, q^*. We also include rule of thumb behaviour so lagged inflation may be made a prominent feature of the model or weakened or excluded altogether through the single additional parameter of the fraction of firms which index their prices. Hence varying these two parameters covers a wide range of possibilities for the Phillips curve in a fairly compact model.

We briefly summarise the main results of the paper and its contribution relative to the existing literature. These may usefully be grouped into the Phillips curve material followed by its implications for monetary policy. With respect to the former we derive a Phillips curve which encompasses (1) and (2) through the inclusion of some rule of thumb behaviour and a pricing rule which nests Calvo as a special case. The new feature is the increasing probability of price change rule described above which shows what happens to the Phillips curve as we move away from Calvo pricing while also encompassing a two period Taylor model as a further special case. Given the microeconomic evidence discussed above we argue that the increasing probability feature is a useful

[8] Woodford (2003) argues that in reasonably stable macroeconomic environments a move to state dependent pricing may not lead to major changes. This is supported by the analysis of Burstein (2003) who finds little difference between state and time dependent pricing at moderate inflation levels. Bhaskar (2002) derives microfoundations for staggered pricing when the timing of price changes is chosen by firms rather than imposed exogenously.

step towards realism. In passing we also clarify the differences between Calvo and Taylor, which have been subject to a degree of confusion as noted above, and we also clarify that the Taylor model rests on exactly the same microfoundations as Calvo apart from the time dependent pricing rule. We note this point since a view is sometimes expressed that Calvo has more explicit microfoundations in terms of optimising behaviour than Taylor. We also extend the rule of thumb framework of Christiano, Eichenbaum and Evans (2001) to allow only a fraction of firms to index their prices with the remainder keeping their prices constant between optimisations (which was touched on by Amato and Laubach, 2003b). It turns out that this changes the Phillips curve in a way that is observationally equivalent to Woodford's (2003) extension of partial indexing but the result is significant nevertheless since the Christiano, Eichenbaum and Evans (2001) model is otherwise open to criticism on microeconomic grounds that it implies that all firms change prices every period. The extension is also of value in that context if full rather than partial indexing is more plausible. Hence the new Phillips curve encompasses the existing formulations in widespread use while extending them in the twin directions of increasing probability of price change and non-universal indexing, both of which are empirically supported.

In relation to the monetary policy results, the new model permits a more general analysis of optimal monetary policy, stabilisation bias and monetary policy delegation. We replicate results in the existing literature based on (2) while examining the implications of moving away from Calvo pricing. We find that this change in the pricing rule significantly alters the optimal responses to cost push shocks under both commitment and discretion, primarily because non-Calvo pricing implies that both the current shock and the previous period's shock appear in the Phillips curve. The same factor also induces positive serial correlation in inflation and the output gap even if rule of thumb behaviour is excluded. This is of interest since the rule of thumb models were developed in response to the lack of persistence in outcomes predicted by the pure Calvo Phillips curve (1). A further result relates to the stationary price level finding of Clarida, Gali and Gertler (1999) with a Calvo Phillips curve. Jensen (2002) has shown that this no longer holds for the rule of thumb Phillips curve (2) and we find that it no longer holds once we move away from Calvo pricing even if rule of thumb behaviour is excluded. This is significant because that result has stimulated discussion of a shift from inflation targeting to price level targeting. This is considered at a general level by King and Wolman (1996), Dittmar and Gavin (2000), King (1999) and Goodfriend and King (2001). The debate is also directly relevant to the appropriate interpretation of the price stability mandate of the European Central Bank (see Alesina et. al., 2001, and Rudebusch and Svensson, 2002) as well as the possibility of an explicit inflation or price stability objective for the Federal Reserve (Goodfriend, 2003).

We also find that the new pricing rule increases the size of stabilisation bias in the sense that the ratio of expected loss under discretion to expected loss under commitment increases. This raises the potential gain from successful delegation for which we analyze five regimes prominent in the recent literature. In the first instance we follow the standard assumption that the delegation of an amended loss function takes place with full information about the structure of the economy. Initially we consider the special case of the model with Calvo pricing and rule of thumb behaviour which corresponds to the current literature. We replicate existing results but also provide a more extensive sensitivity analysis of the gains from the delegation regimes across different parameter values. Optimal delegation remains superior to discretion in most cases but the exercise reveals significant non-robustness of both the gains from delegation and the welfare ranking of the regimes. Introducing the new pricing rule, we find further sizeable sensitivity of the gains from delegation and the ranking of the regimes to the q* parameter so the possibility of non-Calvo pricing cannot safely be ignored in the analysis of delegation.

Finally we allow for delegation to be based on incorrect information about the structure of the economy. This is discussed briefly by Jensen (2002) for nominal income growth targeting but has not otherwise been a focus of research. This reduces the gains from delegation since the coefficients in the delegated loss function are optimal given beliefs at the time of delegation but not optimal for the actual economy to which that loss function is applied. Clearly this potential problem applies with or without Calvo pricing but allowing for a more general time dependent pricing rule increases the potential cost of incorrect information at the time of delegation.

Given these results our overall conclusion is simply that varying the time dependent pricing rule away from the Calvo assumption of a constant probability of price change has significant implications for both the Phillips curve and a wide range of monetary policy results. Given that the microeconomic evidence on pricing behaviour questions the realism of the Calvo assumption this is a challenging conclusion for much of the literature.

The remainder of the paper is structured as follows. Section 1 summarises the derivation of the new generalised Phillips curve model. This is a little more complex than the derivation of existing models and we present more detailed material in Section A (optimal prices in the absence of staggering constraints) and Section B (the implications of staggering constraints for optimal price setting and the Phillips curve). Appendix C compares the presentation of the Phillips curve here with the usual form of presenting the Taylor Phillips curve as noted above. Section 2 analyses stabilisation bias, comparing optimal policy under commitment and discretion in the new model while Section 3 presents the delegation results. The detailed derivation of optimal policy in the different regimes is given in Appendix D. Section 4 concludes.

1. The Calvo-Taylor-Rule-of-Thumb Phillips Curve

We summarise the derivation of the generalised Phillips curve, present its final form and assess its characteristics before analysing its policy implications in the two sections which follow. Appendix A presents the optimal log single period flexible price, denoted p_t^* for period t, which is used in the derivation and Appendix B provides further detail on the derivation itself.

We assume that there are two types of firm. One type keeps its price fixed between price optimisations (as in the standard Calvo framework), the other indexes its price by a rule of thumb between optimisations following Christiano, Eichenbaum and Evans (2001). For the latter group of firms the time dependent pricing rule is interpreted as a time dependent price optimisation rule with infrequent price optimisations but regular price changes reflecting decision costs rather than menu costs. From the earlier discussion we assume the following time dependent pricing/price optimisation rule. After setting a new optimal price the firm will i) keep that price fixed for one period, ii) have a probability q_1 of changing/optimising the price the following period, and iii) have a probability of changing/optimising the price, if it has not already been changed/optimised, of q each period thereafter. We denote $q-q_1$ by q^* and for simplicity assume that the probability structure is the same for both types of firms.

We follow the standard discrete time solution procedure for the Calvo model (as in Rotemberg, 1987 and summarised in Walsh, 2003b) amended to reflect the different probability structure and allowing for indexing behaviour by the rule of thumb firms. Based on a second order Taylor series for profits as a function of price this approximates the firm's optimisation by the minimisation of a per period loss function that is quadratic in the difference between the logs of the firm's price and the ideal single period flexible price, p*. The quadratic term for each period is discounted by the (real) per period discount factor, β, and weighted by the probability of the price set at some time t still being in place in each subsequent period, t+j. This probability is simply $(1-q_1)(1-q)^{j-1}$ for $j \geq 1$ and unity for j=0. The optimisation need not consider what happens after the firm has been able to reset or re-optimise its price since the initial choice of price does not constrain that subsequent optimisation. Hence the choice problem for both types of firms may be expressed by (3) where $L^f_{i,t}$ is the total loss function for firm i from time t onwards. The new price it sets is $x_{i,t}$ with the level of that price in future periods before the next optimisation being $x_{i,t+j}$.

$$Min_{x_{i,t}} \; L^f_{i,t} = E_t \left[(x_{i,t}-p^*_t)^2 + \Sigma^{\infty}_{j=1}\beta^j(1-q_1)(1-q)^{j-1} (x_{i,t+j}-p^*_{t+j})^2 \right] \tag{3}$$

The two types of firm differ in the nature of $x_{i,t+j}$ on the right hand side. For standard firms this is simply $x_{i,t}$ since their price is unchanged between optimisations whereas for rule of thumb firms this price is the original price set at t plus the effect of subsequent indexing. This difference means that the two types will set different initial prices which we denote x^s and x^r for the standard and rule of thumb firms respectively. For the latter it is assumed that x^r is indexed by the previous period's inflation rate (as in Christiano, Eichenbaum and Evans, 2001) times γ to allow for partial indexing (the addition to the Christiano et. al. model suggested by Woodford, 2003). With inflation at time t denoted π_t ($\pi_t=p_t-p_{t-1}$ with p as the log price level) these assumptions give (4).

$$x^r_{i,t+j} = x^r_{i,t} + \gamma \, (p_{t+j-1}-p_{t-1}) \tag{4}$$

With the amendment to (3) for the rule of thumb firms given by (4), the first order conditions for the two types of firm are given by (5) and (6).

$$x^s_{i,t} = \left[\frac{1-\beta(1-q)}{1+\beta q*}\right]\left[p^*_t + \frac{(1-q_1)}{(1-q)}E_t[\Sigma^{\infty}_{j=1}\beta^j(1-q)^j p^*_{t+j}] \right] \tag{5}$$

$$x^r_{i,t} = \left[\frac{1-\beta(1-q)}{1+\beta q*}\right]\left[p^*_t + \frac{(1-q_1)}{(1-q)}E_t[\Sigma^{\infty}_{j=1}\beta^j(1-q)^j[p^*_{t+j}-\gamma(p_{t+j-1}-p_{t-1})]] \right] \tag{6}$$

In both (5) and (6) the optimal price set by a firm depends on the current and expected future optimal single period prices, appropriately weighted by the discount factor and the probability of no further optimisation having taken place. Equation (6) differs from (5) for the periods t+1 onwards to reflect the future indexing effect on x^r. Since all firms within each type set the same new prices we may drop the i subscript from this point.

These first order conditions involve expectations of all future values of the ideal single period prices but these may conveniently be substituted out by leading (5) and (6) one period, taking expectations at t and substituting back into (5) and (6) to give (7) and (8). These express the new prices set at time t as functions of the expected new prices at time t+1. Comparing the expressions, the new price set by rule of thumb firms is lower relative to next period's expected price since these firms know that their price next period will be indexed by $\gamma\pi_t$ if it is not immediately re-optimised.

$$x_t^s = \beta(1-q)E_t[x_{t+1}^s] + [\frac{1-\beta(1-q)}{1+\beta q^*}](p_t^* + \beta q^* E_t[p_{t+1}^*]) \tag{7}$$

$$x_t^r = \beta(1-q)E_t[x_{t+1}^r] + [\frac{1-\beta(1-q)}{1+\beta q^*}](p_t^* + \beta q^* E_t[p_{t+1}^*]) - \frac{\beta(1-q_1)\gamma\pi_t}{1+\beta q^*} \tag{8}$$

The Phillips curve derivation next makes use of the appropriate expression for the price level in terms of its own past level and the new prices set by firms. From Appendix B this is given by (9) where r is the proportion of rule of thumb firms and hence (1-r) the proportion of standard or non-indexing firms.

$$p_t = \frac{q}{1+q^*}[(1-r)x_t^s + rx_t^r + q^*[(1-r)x_{t-1}^s + rx_{t-1}^r]] + (1-q)p_{t-1} + \frac{(1-q_1)}{1+q^*}r\gamma\pi_{t-1} \tag{9}$$

A key feature of (9), the reason why the Phillips curve below differs from the Calvo or Calvo-rule of thumb hybrid, is that if q* is non-zero the new prices that were set at t-1 matter for the current price level and hence inflation. In turn that implies that the previous period's cost push shock, output gap and expectations of the future as seen from the previous period will all appear in the Phillips curve. With Calvo probabilities, q*=0 and last period's new prices are immaterial.

The Phillips curve derivation is completed (see Appendix B) by leading (9) one period to give p_{t+1} as a function of the new prices set at t+1 which may then be substituted out of (7) and (8) as they (and the same expressions lagged one period for t-1 new prices) are substituted into (9). We also substitute for the ideal single period prices using (10) where ε_t is the time t cost push shock and κ is the sensitivity of the single period flexible price to the output gap, y_t. This expression, including the microfoundation for κ, is standard but for completeness Appendix A shows its derivation.

$$p_t^* = p_t + \kappa y_t + \varepsilon_t \tag{10}$$

Consolidating the result of those substitutions give the new generalised Phillips curve in (11) where the constant, c, is shown by (12).

$$\pi_t = \frac{\beta E_t[\pi_{t+1}] + \beta q^* E_{t-1}[\pi_t] + r\gamma\pi_{t-1}}{1 + \beta q^* + \beta r\gamma} + c(\kappa y_t + \varepsilon_t + \beta q^* E_t[\kappa y_{t+1} + \varepsilon_{t+1}]) + cq^*(\kappa y_{t-1} + \varepsilon_{t-1} + \beta q^* E_{t-1}[\kappa y_t + \varepsilon_t]) \tag{11}$$

$$c = \frac{q[1-\beta(1-q)]}{(1-q_1)[1+\beta q^*(1-q)][1+\beta q^* + \beta r\gamma]} \tag{12}$$

The Phillips curve (11) is the key result of this section and we briefly outline its properties. Firstly it may readily be reduced to the special cases in the existing literature if $q^*=0$ so Calvo probabilities apply. In this case only the current period output gap and cost push shock remain from the second and third lines. In relation to the expected/lagged inflation terms of the top line; i) if $r\gamma$ is also zero we simply have the standard Calvo form (1), ii) if $r\gamma=1$ we have the Christiano, Eichenbaum and Evans (2001) Phillips curve (allowing for contemporaneous information) with coefficients on expected future and lagged inflation of close to a half each (given that β is close to unity), iii) if $r=1$ but γ may vary we have the Woodford (2003) partial indexing form which supports the widely used Phillips curve (2) though with a maximum value of δ just under a half.[9] Lastly, iv) still with $q^*=0$, allowing r (the proportion of rule of thumb firms) to vary below unity is a new formal feature of this model[10] and reduces the significance of lagged inflation. This may be important in that even if rule of thumb behaviour is present, not all firms will have low menu costs (given the empirical evidence that many prices remain fixed for several quarters) and hence not all will index their prices between optimisations. The proportion of indexing firms, r, appears only in the product $r\gamma$ in (11) so its impact on the Phillips curve coefficients is the same as γ.

Secondly, the main innovation of this model is the variation away from Calvo probabilities with $q^*>0$ and the 'headline' result from (11) is simply that this matters for the Phillips curve, with new terms appearing and the coefficients on existing terms changing via the denominators. From the discussion following (9) the key implication of $q^*>0$ is that the new prices set at t-1 now matter for p_t and thus π_t. This has the effect of introducing $E_{t-1}[\pi_t]$ into the top line of (11) as well as the whole of the third line from which the t-1 output gap and cost push shock matter for current inflation. In addition, $q^*>0$ implies the presence of $E_t[y_{t+1}]$ and $E_{t-1}[y_t]$ in the second and third lines of (11),

---

[9] The expected future/lagged inflation coefficients in (11) are also close to but not exactly equal to the simple $(1-\delta),\delta$ linear form in (2) unless $\beta=1$.

[10] Though see Amato and Laubach (2003b) for a discussion and numerical examples.

together with the equivalent terms for the shock variable though these disappear if it is iid which is often assumed. Lastly, if q*=1 we obtain the two-period Taylor model (see Appendix C).

2. Stabilisation Bias

This section compares optimal monetary policy under commitment and discretion, leaving to the following section the analysis of delegation. We discuss the loss function used in the policy derivations and the solution method employed before turning to the results. Appendix D documents the derivation of the results in detail.

In common with most recent literature we assume that the policy maker targets the natural rate of output and hence there is no steady state inflation bias in the standard sense of Barro and Gordon (1983). We also assume that there is no interest rate smoothing objective in the social loss function which means that, for standard reasons (see McCallum and Nelson, 2000, for example), we do not include an IS curve in the model. In turn this means that we think of the policy maker choosing output and inflation directly, subject to the Phillips curve constraint, without explicit modelling of the policy instrument.

In particular we assume a social loss function of the form given in (13) where λ is the relative weight on output gap deviations and for simplicity we normalise the inflation target to zero.

$$L_t = E_t[\Sigma_{t=0}^{\infty}\beta^t(\pi_t^2 + \lambda y_t^2)] \tag{13}$$

The loss function (13) is very much a standard form in monetary policy research, including much of the delegation literature discussed in the introduction, but nevertheless appropriate practical policy conclusions depend on it being a good approximation to the true social loss function. Woodford (2002) finds that (13) is appropriate for the Calvo Phillips curve (1) but equally that changes in the Phillips curve generally imply changes in the form of the microfounded loss function. Amato and Laubach (2003b) and Steinsson (2000) also derive microfounded loss functions when the Phillips curve takes the Calvo-rule of thumb form (2). These differ from (13) though they find that results based on them are close to those from using the traditional form (13). The two most prominent contributions to the delegation literature with which we compare our results, Jensen (2002) and Walsh (2003a), use (13). We also choose to work with (13) but it is clear that deriving the microfounded loss function for the model above and tracing its implications would be a useful exercise. This choice is motivated partly on the pragmatic grounds of comparability with the results of Jensen and Walsh (which is useful for exploring robustness) and partly from the analytical perspective that it is informative to focus on the direct implications of the Phillips curve for delegation.

The solution method employed is the MSV approach set out in McCallum (1983, 1999b, 2003) and used in the examples considered by McCallum and Nelson (2000). We choose a simple application of the MSV approach rather than the methods outlined in Soderlind (1999) based on the state-space form of the model partly for simplicity but also because it makes the choice of optimal delegation parameters very straightforward. The latter point is discussed in general terms in Mash (2003) and in the context of this model in the following section but in essence the MSV approach leads to a set of simultaneous equations involving the parameters in the Phillips curve and those in the loss function. The solution to these with given loss function parameters is the solution for optimal policy and this is readily solved using a standard spreadsheet. In addition, it is very easy to derive delegation results which amount to finding the loss function parameters which minimise loss given the constraint of the simultaneous equations which characterise the discretion solution. This feature is convenient since it avoids the need for the grid search procedures required by the Soderlind (1999) method reported by Jensen (2002), Soderlind, Soderstrom and Vredin (2002) and Walsh (2003a) amongst others.

Under discretion and delegation the derivation follows McCallum and Nelson (2000) very closely, the only difference being the addition of extra state variables. The key to the approach is recognition that while the discretionary policy maker cannot influence expectations that have already been formed there are endogenous state variables in (11) and hence optimal discretionary policy allows for current choices to affect optimal subsequent outcomes which will in turn affect expectations of the future. For example, given y_{t-1} in the Phillips curve (11), a choice of output in, say, period s, y_s, will determine "y_{t-1}" at time s+1 and thus affect optimal choices at that time. In turn that means that the choice of y_s will influence $E_s[y_{s+1}]$ and $E_s[\pi_{s+1}]$ so there is an indirect influence of policy on expectations even though the policy maker cannot make binding commitments. In practice (see McCallum and Nelson, 2000) this channel is dealt with by assuming MSV reduced forms for the variables of interest and deriving policy choices with the policy maker aware of, but not able to directly optimise, the coefficients of the reduced form. From (11) and (13) we conclude that the state variables are ε_t, ε_{t-1}, π_{t-1} and y_{t-1} and hence we assume the reduced forms (14) and (15) where the l and m coefficients are initially unknown. These are used to substitute out the expectational variables in the Phillips curve after which a standard Lagrangean may be used with the first order conditions treating the l and m coefficients as constants. Appendix D gives further detail on this procedure.

$$\pi_t = l_0\varepsilon_t + l_1\varepsilon_{t-1} + l_2 y_{t-1} + l_3\pi_{t-1} \tag{14}$$

$$y_t = m_0\varepsilon_t + m_1\varepsilon_{t-1} + m_2 y_{t-1} + m_3\pi_{t-1} \tag{15}$$

Under commitment an issue arises since the Phillips curve (11) contains both the actual values of inflation and output at time t and their expectation at time t-1 and thus implicit expectations errors are present. This means that one cannot derive simple first order conditions for the variables of interest, as seen from the time a commitment rule is adopted, because the expectational errors would be set to zero when taking expectations of the first order conditions in the initial period.

This situation is analysed by Sims (2000) in relation to state-space methods but appears to have received less attention from an MSV perspective. The problem and the method adopted below is discussed in more detail in Mash (2003) and is based on the policy maker optimising separately over the contemporaneous response of the variables to shocks and their expected values one period before. Given rational expectations and iid shocks we may express the time series processes for inflation and output by (16) where a_0, b_0 and the processes for $E_{t-1}[\pi_t]$ and $E_{t-1}[y_t]$ have yet to be determined. We re-express the Phillips curve and loss function using these reduced forms, so the whole model is written in terms of the four components in (16), and derive separate first order conditions for each of them, thus respecting the difference between actual and prior expected values in the Phillips curve. This procedure is documented in Appendix D.

$$\pi_t = a_0 \varepsilon_t + E_{t-1}[\pi_t] \qquad y_t = b_0 \varepsilon_t + E_{t-1}[y_t] \tag{16}$$

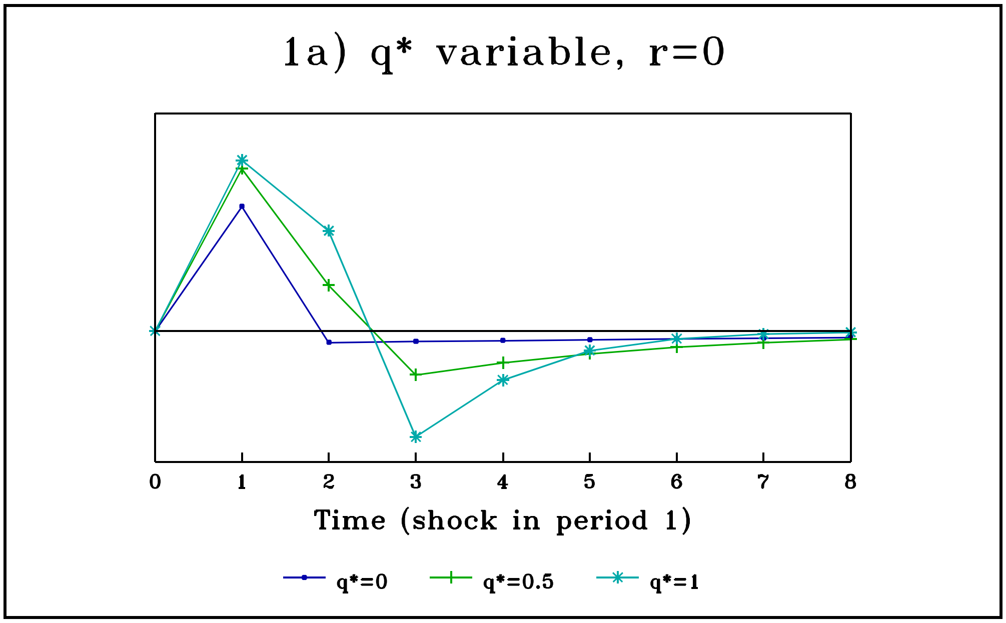
Turning to the results we show the implications of the generalised model for optimal policy under commitment and discretion (and hence stabilisation bias) in Figures 1-7. Assumed parameter values for a baseline case are β=0.99, $c\kappa$=0.05[11] and λ=0.25 while q* and r are varied to explore their influence on outcomes. The overall pattern of the results is robust to changing these parameter assumptions but for brevity we do not report multiple cases. We also assume that γ=1 in (11) so results for different values of r implicitly refer to rγ. Varying the size of the shock, ε, has a multiplicative effect on all the variables in all time periods so does not affect the comparisons between them shown by the impulse response functions. For this reason we do not show a numerical scale on the vertical axes of these charts while keeping a common underlying scale for each variable of interest to facilitate comparisons. We also express the expected level of loss in a way that does not require an explicit assumption about the variance of the shock process.

We first examine the properties of the model under commitment. Figures 1a and 1b show the optimal response of inflation to a single cost push shock in period 1. Figure 1a fixes r at zero to focus on the effects of q* in isolation. The q*=0 case simply reproduces the well known inflation profile for the Calvo Phillips curve. The key feature is that inflation in period 2 is below zero (more generally below target) which we refer to as the 'crossing property'. This is optimal for the well understood reason that while it is sub-optimal for period 2 taken on its own, this cost is more than outweighed by the effect it has on the period 1 Phillips curve via the forward looking inflation expectation in (1). Loss is convex in the deviations from target of inflation and output so the comparatively small deviations in period 2 imply a relatively large benefit in period 1 when the shock impacts on the system and deviations will tend to be large.

Once we allow for q*>0 the period 1 shock also impacts on the period 2 Phillips curve through the ε_{t-1} term in (11). Figure 1a shows that optimal inflation is above target in period 2 in response to this but the crossing property is present in period 3 when the shock no longer has a direct impact. Period 3 inflation is further below target the larger is q* which reflects a larger benefit from influencing the period 2 Phillips curve and, through that, the period 1 Phillips curve also.

[11]In line with Jensen (2002) and Walsh (2003a) we keep this value constant as we change other parameters. The baseline values coincide with those in Walsh.

FIGURE 1: INFLATION UNDER COMMITMENT



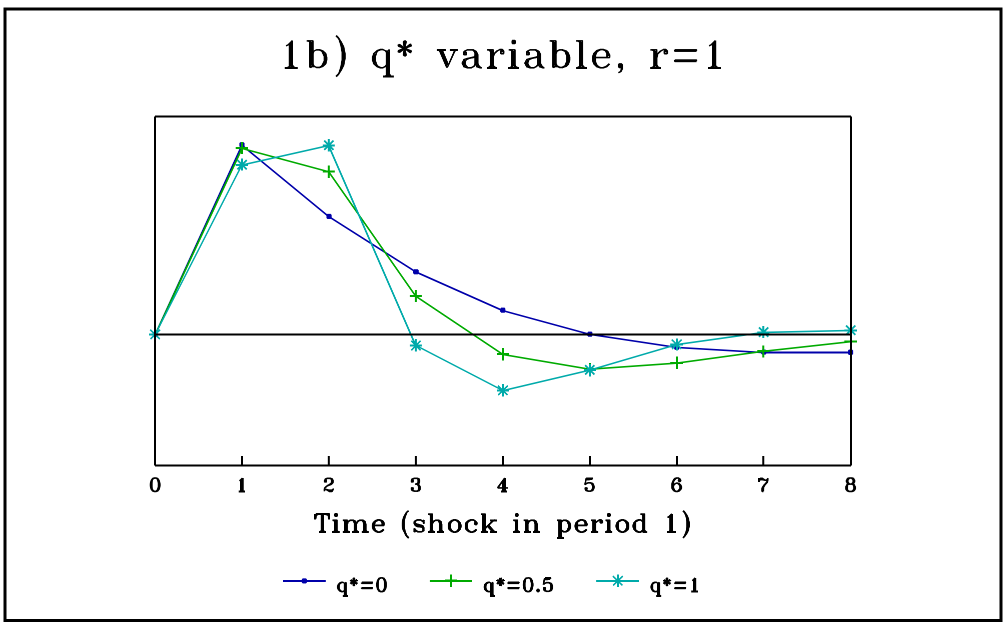
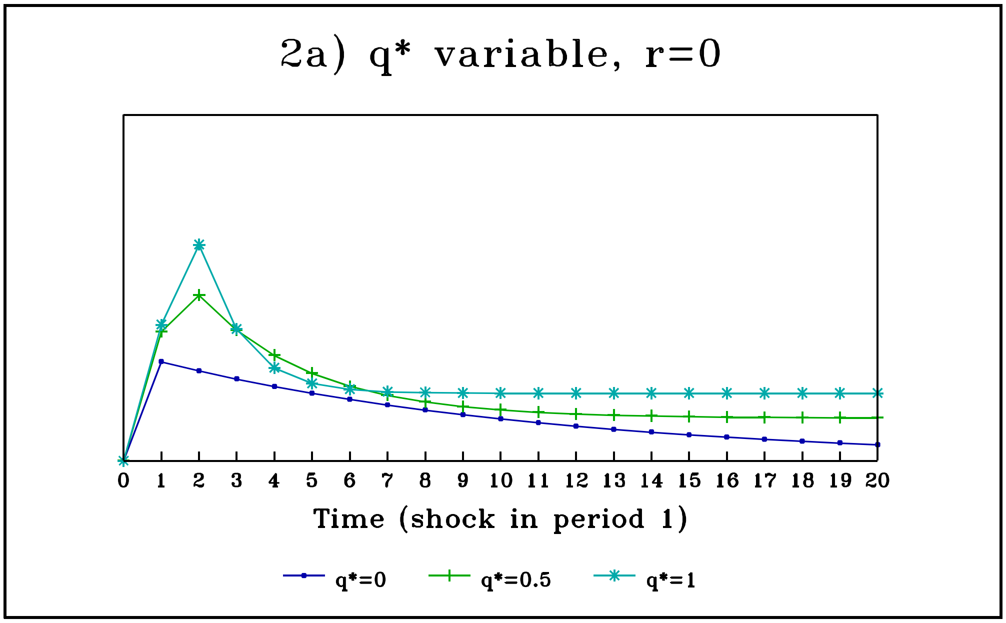


Figure 1b shows optimal inflation for the different q* cases when r, the proportion of rule of thumb firms, is unity. For q*=0 this reproduces optimal inflation for the Calvo-rule of thumb hybrid Phillips curve (2) with approximately equal coefficients on lagged inflation and its forward looking expectation. This case is well understood from the literature, corresponds to the charts in Steinsson (2000), and also features the crossing property, albeit delayed due to the inertia in inflation. An interpretation is that below target inflation after crossing (period 6 in this simulation) improves the Phillips curve tradeoff for the previous period (period 5), thus facilitating lower inflation at that time which in turn improves the tradeoff for the previous period and so on all the way back to the time of the shock. Retaining r=1 but allowing for q*>0 increases inflation in period 2 for the same reasons as above but it also shortens the optimal time before inflation crosses its target value. This is consistent with a greater benefit from earlier crossing due to the higher overall impact of the shock once ε_{t-1} is present in the Phillips curve.

Figures 2a and 2b show the same cases but for the price level rather than inflation. Given that the price level reflects cumulative inflation there is no new information here but the figures are nevertheless informative. It may be recalled that Clarida, Gali and Gertler (1999) showed that the long run price level is stationary with a Calvo Phillips curve, thus leading to suggestions (confirmed by Vestin, 2000) that price level targeting may be desirable even if society's concern is ultimately with inflation and not the price level. The q*=0 case in Figure 2a shows this result with the price level gradually returning towards its initial value. Jensen (2002) showed that this no longer holds once lagged inflation is present in the Phillips curve. This is illustrated by the q*=0 case in Figure 2b. A new result is that Figure 2a shows that the Clarida, Gali and Gertler result no longer holds with q*>0 even if r=0, thus providing a further counterweight to the suggestion that price level targeting may be first best. Figure 2b shows that q*>0 reduces the long run optimal price increase following the shock (due to the earlier crossing noted above) but not by enough to reverse the Jensen result.

Having examined the properties of the new model under commitment we turn to the comparison between commitment and discretion and thus stabilisation bias. Figures 3 and 4 present optimal inflation and output respectively for the six cases generated by allowing each of the key parameters q* and r to take the values 0, 0.5, 1 while the other is either zero or unity. Across the figures as a whole the key difference between commitment and discretion in Figure 3 is the failure of the latter to return inflation towards its target quickly enough (for r>0) and then achieve the crossing property for inflation (for all parameter combinations) to any significant extent. Corresponding to this, Figure 4 shows that the output gap returns to zero too quickly under discretion when a more sustained negative output gap is required to reduce inflation more quickly and to achieve crossing. These patterns reflect the simple and well understood 'bygones are bygones' property of discretion such that optimal discretion in the period after the direct impact of the shock has passed cannot take advantage of the benefit under commitment of adjusting outcomes at that time to influence earlier periods. Furthermore the scale of stabilisation bias is increasing in both r (comparing from left to right) and q* (comparing from top to bottom).

FIGURE 2: PRICE LEVEL UNDER COMMITMENT



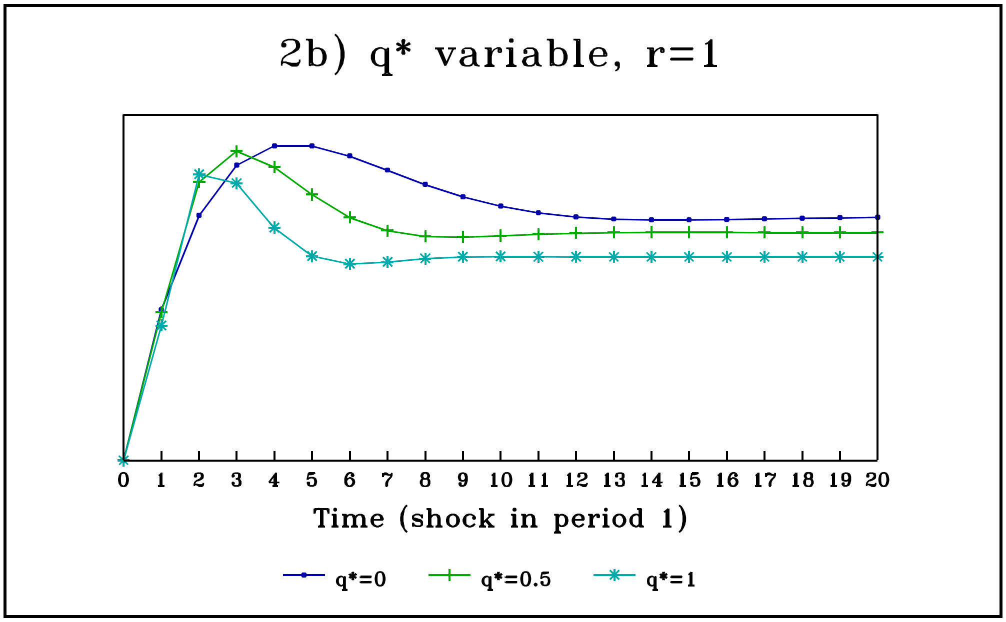


FIGURE 3: INFLATION UNDER COMMITMENT AND DISCRETION

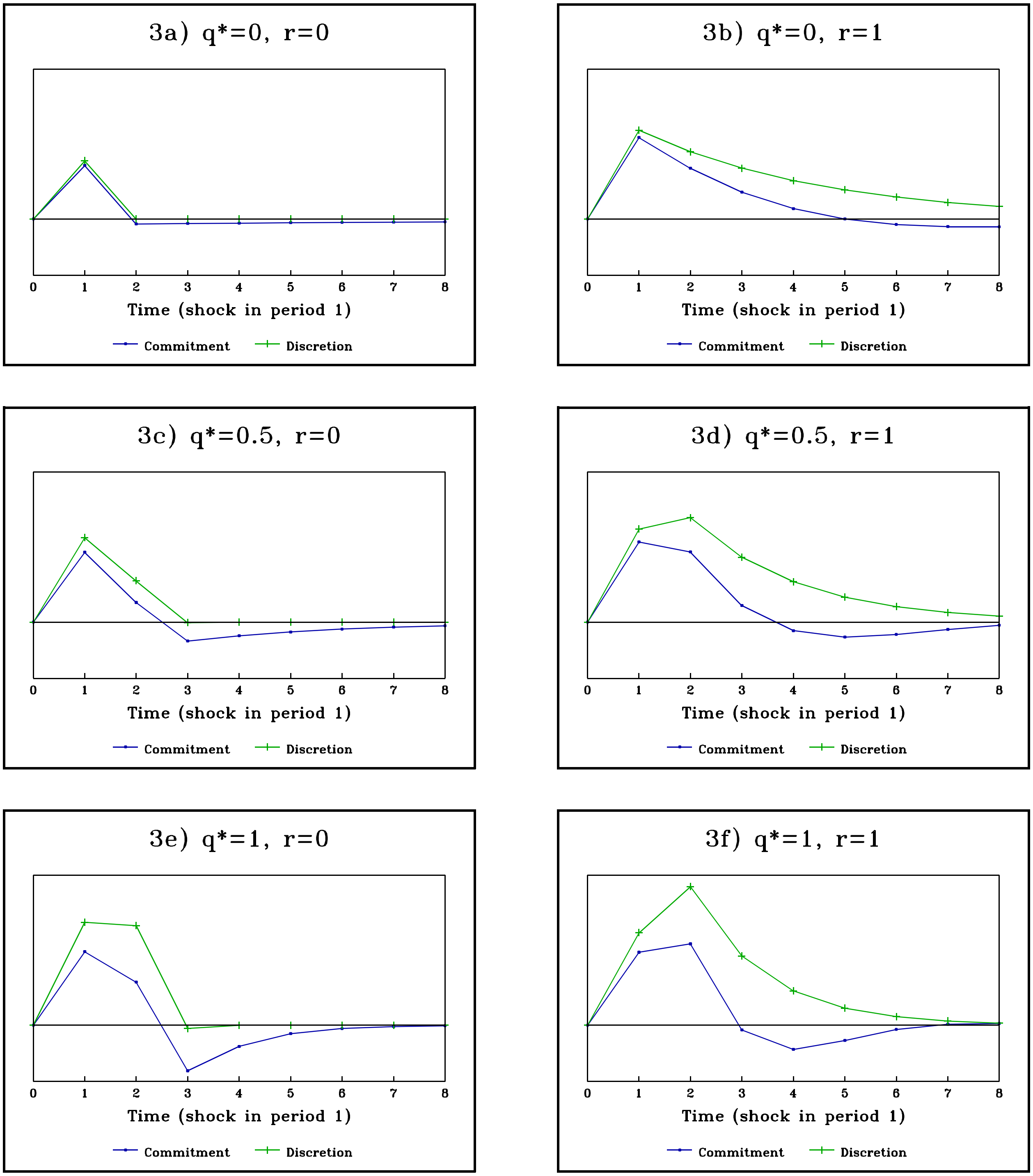
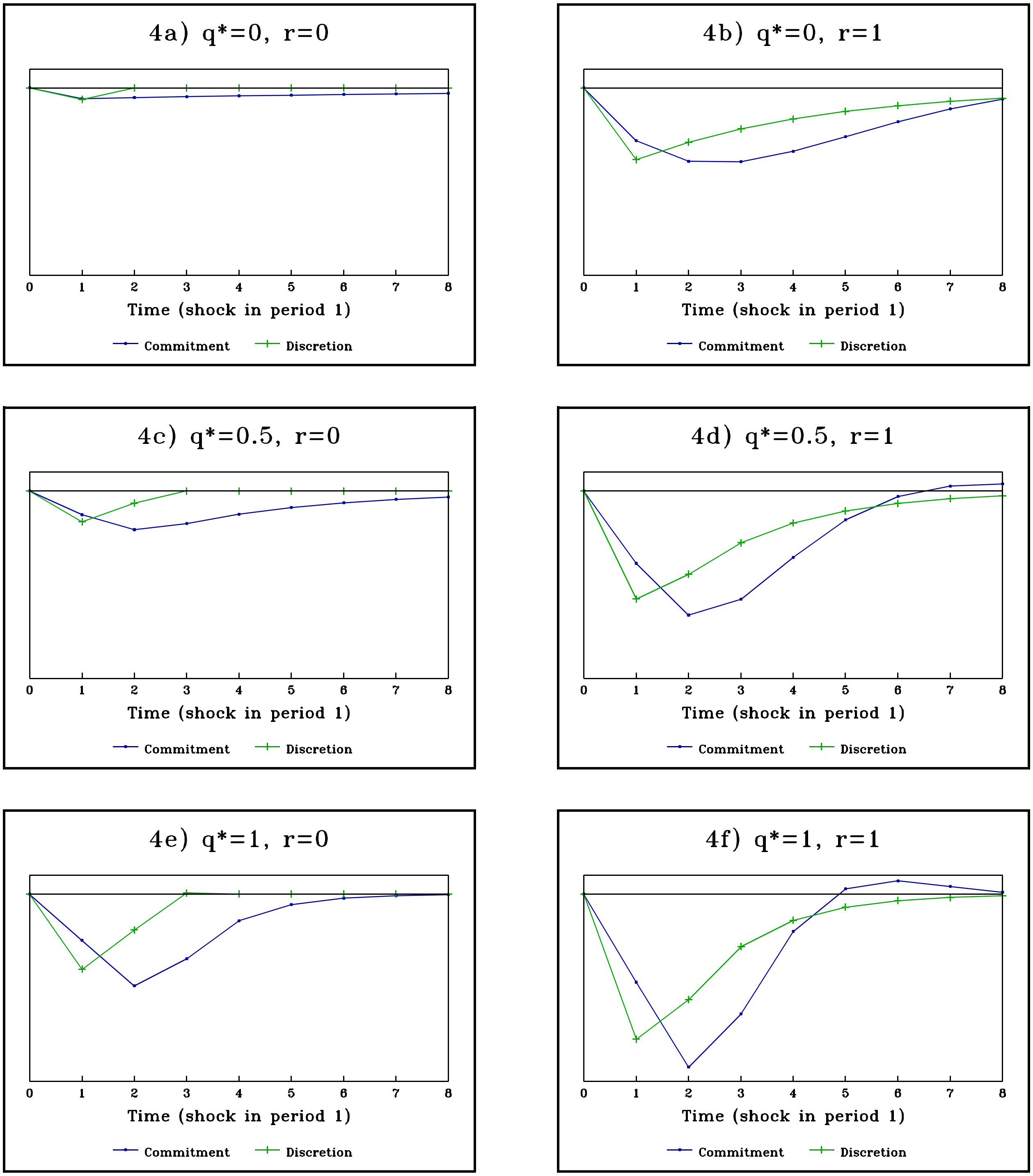


FIGURE 4: OUTPUT GAP UNDER COMMITMENT AND DISCRETION



The upper pair of charts in Figures 3 and 4 reproduces well known results for the Calvo model and the Calvo-rule of thumb hybrid shown in Steinsson (2000).[12] The q*>0 cases are new to the literature and show significant changes from generalising the Calvo constant probability of price change assumption, with or without rule of thumb behaviour in the model. As above q*>0 increases the scale of stabilisation bias but it also changes the optimal inflation and output dynamics following shocks in both policy regimes. We highlight two aspects of this, both shown in more detail below. Firstly the changed dynamics show hump-shaped responses that imply endogenous persistence in inflation and output, even if r=0, which is not present under the Calvo Phillips curve. Secondly the changed dynamics question whether existing delegation results based on q*=0 will be robust to q*>0.

Figures 5 and 6 show results for expected loss as a function of q* and r. Figure 5 presents absolute loss (indexed to expected loss for the Calvo case) while Figure 6 gives results for relative loss, comparing discretion with commitment, and thus stabilisation bias. From Figure 5 it may be seen that absolute expected loss is increasing in q* and r. This is to be expected since q* effectively raises the impact of shocks through the lagged shock term in (11) and r increases the output gap costs of bringing inflation back to target. The gap between commitment and discretion also widens in absolute terms as these parameters increase. In relative terms, Figure 6 shows that the relative loss is increasing in q* and also broadly increasing in r unless q* is high (the upper line in Figure 6a). McCallum and Nelson (2000) derive similar results to the q*=0 cases (albeit strictly speaking they compare timeless perspective rather than commitment outcomes (which may differ slightly) with discretion. Their Table 1 with $\alpha=0.05$, $\omega=0.25$ ($c\kappa$ and λ respectively in the notation here) gives a relative loss close to 9% which corresponds to the q*=r=0 point on the lower line of Figure 6a. With r approximately one ($\theta=0.5$ in their notation) the relative loss climbs to around 30% (their Table 3, same α and ω values) corresponding to the lower line in Figure 6a rising with r.

Figure 7 returns to the persistence properties of inflation and output under discretion implicit in Figures 3 and 4. Figure 7a simply reproduces the standard result that rule of thumb behaviour (r>0) results in endogenous persistence though showing this effect for continuous values of r is new. It may be recalled that this was the key motivation in the literature for introducing rule of thumb behaviour. Figure 7b confirms the conjecture from Figures 3 and 4 that there is positive serial correlation in inflation and the output gap under discretion even when r=0. This result is notable since it suggests that moving away from Calvo pricing can generate fairly high levels of persistence either on its own (Figure 7b) or at lower levels of r than otherwise (Figure 7c), even if q*>0 is not required if a high level of r is considered plausible (Figure 7d).

Overall we conclude that varying the Calvo pricing assumption (q*>0) significantly affects optimal policy under commitment and discretion, expected absolute loss, the scale of stabilisation bias, and the dynamics and persistence properties of the model. The following section extends the analysis to the impact on monetary policy delegation.

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[12] The quicker return to target of inflation under commitment in Figure 3b also arises in the model of Clark, Goodhart and Huang (1999), also analyzed in Mash (2002), where the Phillips curve takes the form (2) but with $E_t[\pi_{t+1}]$ replaced by $E_{t-1}[\pi_t]$. In this case there is no optimal crossing property.

FIGURE 5: EXPECTED LOSS
(Index: Calvo model (q*=r=0) loss = 100)

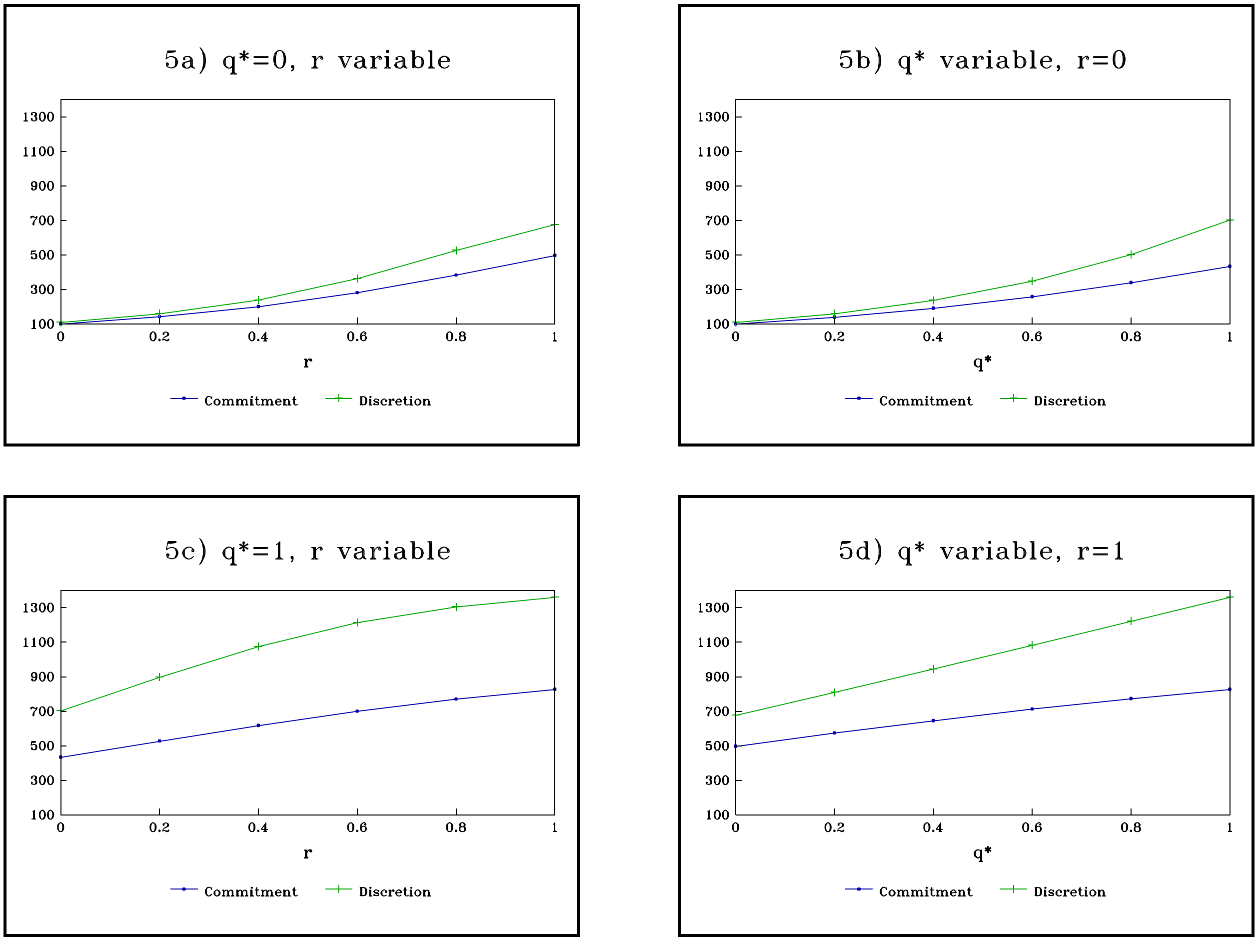


FIGURE 6: RELATIVE EXPECTED LOSS (DISCRETION/COMMITMENT)

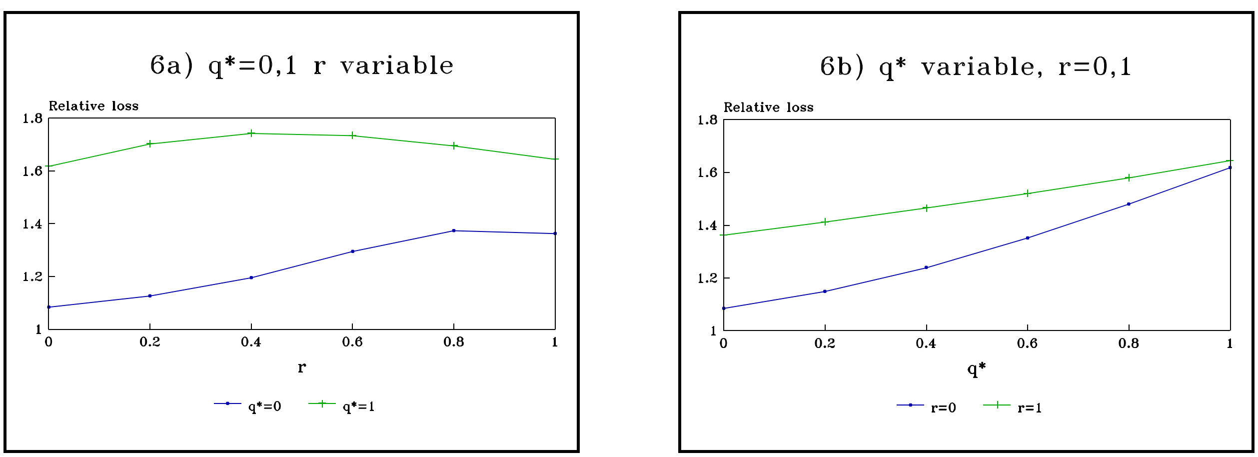
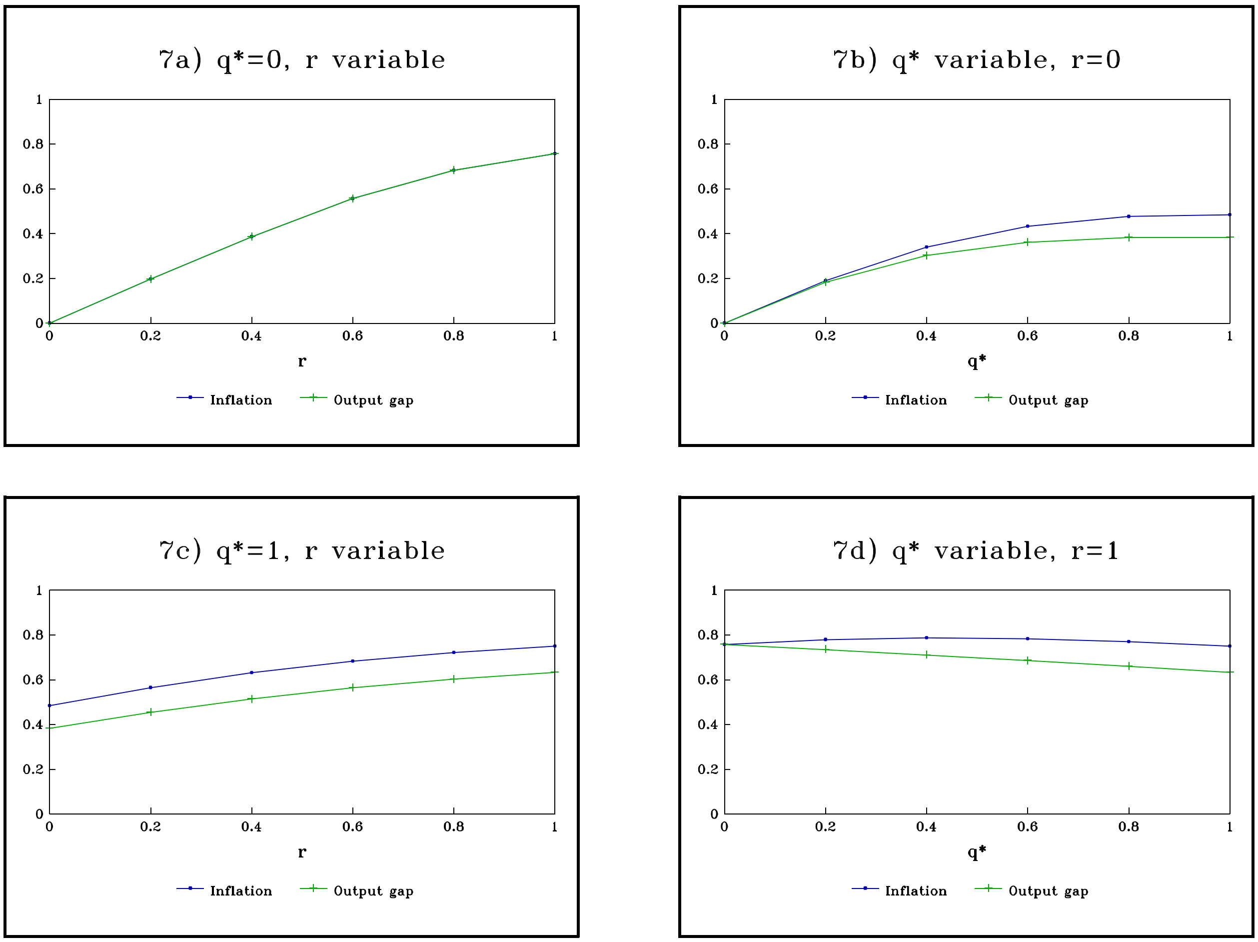


FIGURE 7: INFLATION AND OUTPUT PERSISTENCE UNDER DISCRETION
(First order autocorrelation coefficient)



3. Monetary Policy Delegation

We turn to the analysis of optimal monetary policy delegation whereby a delegated policy maker is given a loss function that differs from the social loss function. The delegated loss function is designed such that discretionary policy given this 'false' loss function minimises social loss evaluated using the true loss function. A parallel literature which we do not consider is the possibility of delegating an optimised Taylor rule for setting interest rates to minimise social loss with that Taylor rule differing from the Taylor rule representation of optimal discretionary policy. In both cases the maintained assumption is that the delegated policy maker will minimise/implement the delegated loss function/Taylor rule.

Key questions to be addressed are the extent to which delegation can offset stabilisation bias and which form of delegation is most successful at doing this. We explore these initially using the common assumption that the structure and coefficients of the Phillips curve are known with certainty. This establishes benchmark results but in practice an important issue will be the performance of different delegation regimes when the true model is not known. It is possible that a delegated loss function (or Taylor rule) will lead to significant welfare gains given a particular form of the model (chiefly the Phillips curve unless interest rate smoothing is important) but much less well if the structure of the model is different in practice. If this effect is strong, delegation may result in higher loss than pure discretion. We pursue this by considering the robustness of the delegation regimes to initial uncertainty about the structure of the model in the form of the q* and r parameters which determine the significance of non-Calvo price change probabilities and rule of thumb behaviour respectively. It may be noted that the uncertainty about the model in this type of exercise refers to uncertainty when the delegation regime is chosen rather than uncertainty once policy choices are being made[13] and private sector expectations formed. It is plausible of course that there is ongoing uncertainty about the structure of the economy with learning behaviour by the policy maker and the private sector. While this is an important topic, from the point of view of delegation the question of the robustness of delegation regimes to uncertainty about the model at the time of delegation, abstracting from subsequent uncertainty, remains important.

The literature has examined many different forms of delegation including a conservative central banker (Rogoff, 1985), interest rate smoothing (Woodford, 1999), price level targeting (Vestin, 2000), nominal income growth targeting (Jensen, 2002, Rudebusch, 2002), average inflation targeting (Nessen and Vestin, 2000, Nessen, 2002, and Batini and Yates, 2003), and targeting the change in the output gap (Soderstrom, 2001, Walsh, 2003a). Our main purpose is to examine the robustness of delegation to changes away from Calvo pricing, the new feature of the model above, and for this exercise we choose a subset of these regimes. In particular we consider a conservative central banker, output gap growth targeting (or speed limit targeting in Walsh, 2003a) and three forms of nominal income growth targeting. We do not consider price level targeting explicitly but note that from the behaviour of the price level under commitment shown in Figure 2 above this is likely to be a less successful regime with q*>0 rather than q*=0.

The regimes may be clarified by considering (17), which is the generalised delegated loss function that we consider, in tandem with Table 1 which documents the values of the parameters in (17) assumed under each regime.

$$L_0 = E_0 \Sigma_{t=0}^{\infty} \beta^t [\lambda_\pi \pi_t^2 + \lambda_y y_t^2 + \lambda_s (y_t - y_{t-1})^2 + \lambda_n (\pi_t + y_t - y_{t-1})^2] \tag{17}$$

[13] Assuming that delegation takes the form of amended loss function minimisation as here rather than the simple implementation of a Taylor rule.

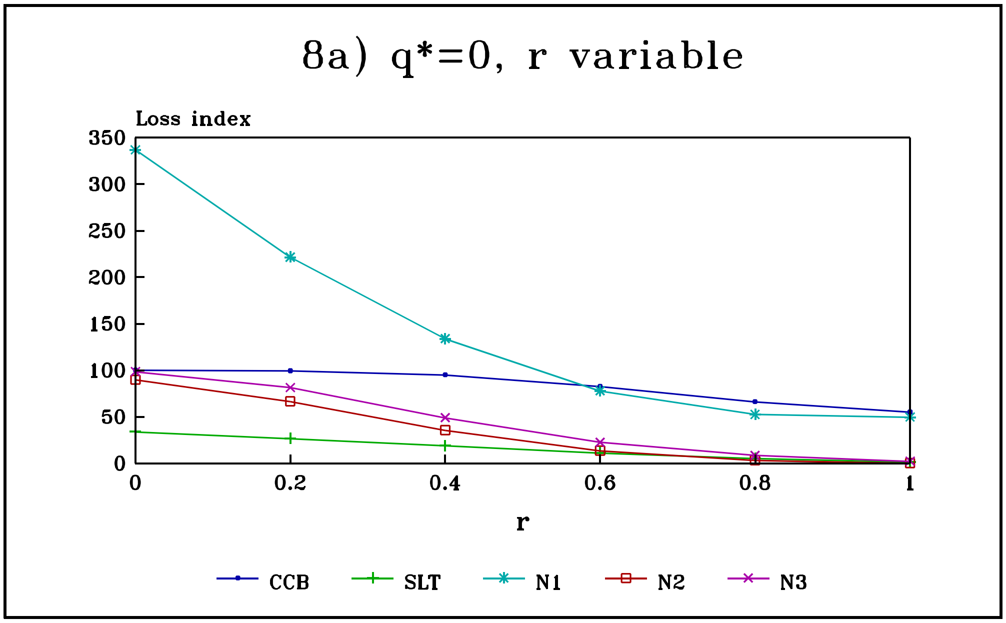
Table 1: Policy Regimes and Loss Function Parameters

Regime: / Parameter:	λ_π	λ_y	λ_s	λ_n	Delegation choice
Discretion (DIS)	1	λ	0	0	-
Conservative central banker (CCB)	1	λ_y	0	0	λ_y
Speed limit targeting (SLT)	1	0	λ_s	0	λ_s
Nominal income growth targeting 1 (N1)	1	0	0	λ_{n1}	λ_{n1}
Nominal income growth targeting 2 (N2)	0	λ	0	λ_{n2}	λ_{n2}
Nominal income growth targeting 3 (N3)	1	λ	0	λ_{n3}	λ_{n3}

Taking these in order, we include discretion (DIS) for completeness to confirm that in this case (17) is the same as the social loss function (13). The conservative central banker (CCB) simply has a different (lower) relative weight on output gap fluctuations. Speed limit targeting (SLT) involves the change rather than the level of the output gap. The three forms of nominal income growth targeting all have nominal income growth in the loss function (via λ_n) but differ in relation to the inflation and output gap terms. N1 is considered by Walsh (2003a) and allows for an optimal mixture of inflation variability and nominal income growth variability. N2 is the flexible nominal income growth targeting of Jensen (2002) which allows an optimal mixture of output gap and nominal income growth variability. N3 is essentially an optimal mixture of the discretion loss function and pure nominal income growth targeting. This is presented by Soderstrom (2001). The final column reports the parameters to be optimised at the time of delegation, these being set so social loss is minimised by the subsequent discretionary behaviour of the policy maker minimising the delegated loss function. As a final note it is clear that combination regimes may also be considered but we focus on the individual ones above. These have been proposed in the literature and our chief concern is to examine their robustness to moving away from Calvo pricing rather than making concrete proposals for new forms of delegation.

We present the results for full information delegation in Figures 8a-8d and Tables 2a-2b. The figures provide a visual summary of the properties of the delegation regimes as the key parameters of q* and r are varied (with the same baseline values for the other parameters as above) while the tables give more detailed information and a robustness analysis. For ease of comparison and to reflect the motivation for delegation being the reduction of stabilisation bias we express the expected loss with a simple index which is zero for the expected loss under commitment and 100 for the loss under discretion.

FIGURE 8: EXPECTED LOSS UNDER DELEGATION
(Index: Commitment=0, Discretion=100)



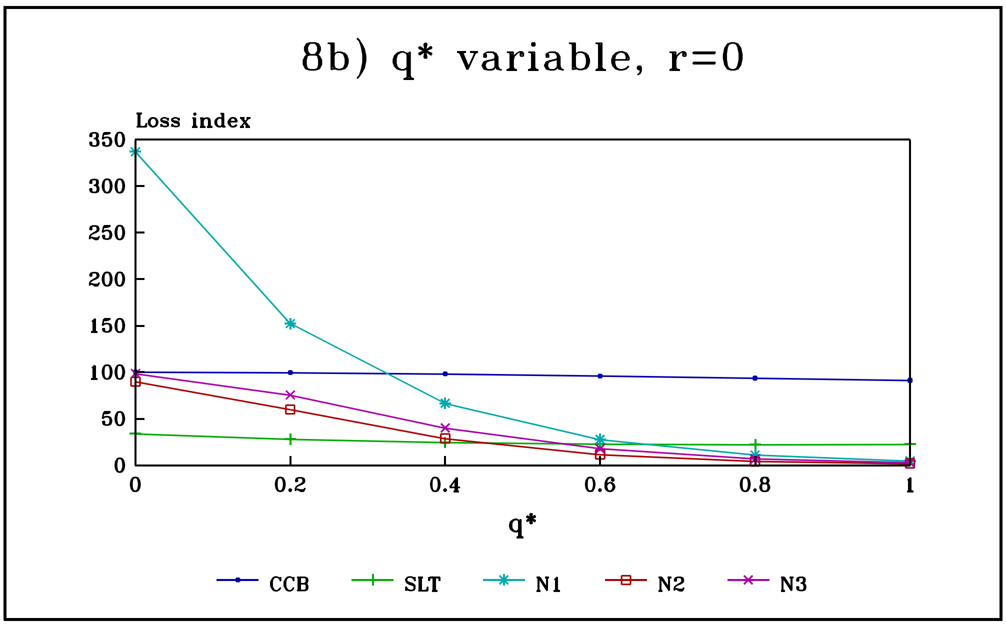
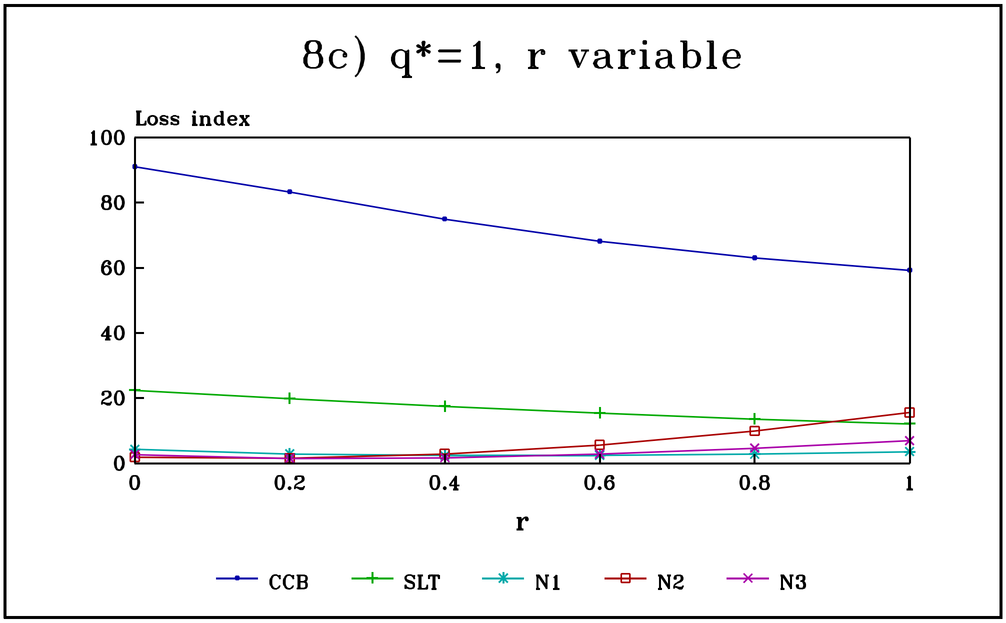


FIGURE 8 continued



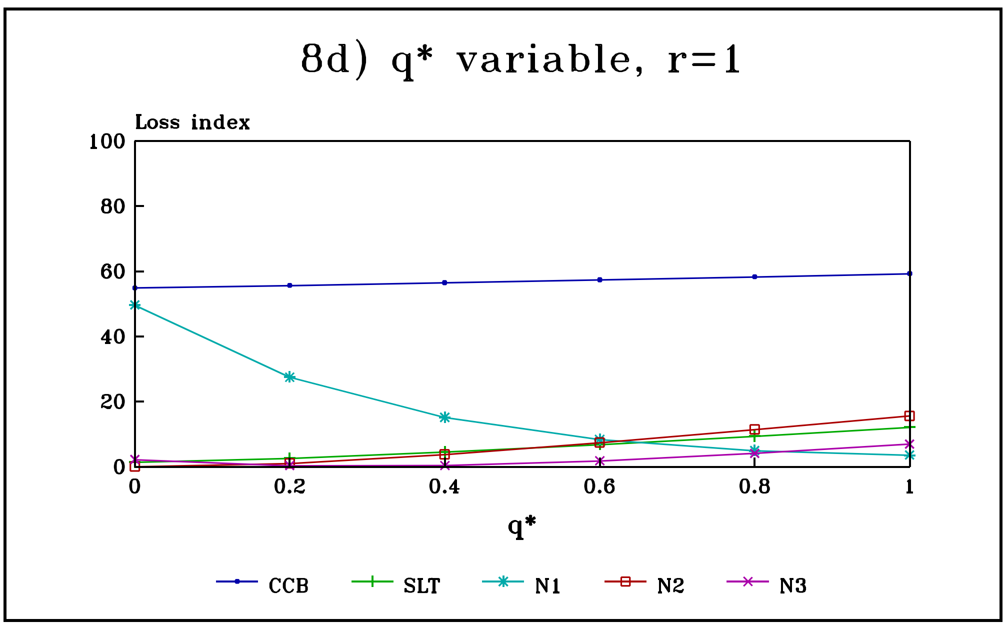


Figure 8a fixes q* at zero while allowing r to vary and corresponds to the scope of the existing literature. As a consequence these results are not new but it is useful to review them to establish a benchmark for comparison once q*>0, the new feature of this paper, is introduced.[14] Taking the regimes in order, the conservative central banker (CCB) regime reproduces the discretion outcome if r=0, the Calvo case, while improving on discretion if r>0. This is discussed in Clarida, Gali and Gertler (1999) and shown in Jensen (2002, Tables 3 and 4 compared with Table 2, where CCB here is IT in Jensen's terminology) and Soderstrom (2001, optimized discretion, Table 3 or 4). With the baseline values used here, the loss index for CCB is 55 once r=1 so a little less than half of the stabilisation bias under discretion is offset.

Speed limit targeting (SLT) offsets more than half the stabilisation bias in the Calvo case and becomes more powerful as r increases with the outcome being very close to commitment when r=1. This was shown by Walsh (2003a) whose baseline assumption for the coefficient on lagged inflation in (2) of 0.5 corresponds to r very close to unity in the notation of this paper. Walsh's Figure 3 shows SLT improving on CCB (IT in Walsh's terminology) as that coefficient increases from zero to around 0.5 (r increases from zero to unity) as in Figure 8a.

Walsh also considers the N1 regime (NIT1 in Walsh) finding it to be very close to CCB (IT) at his baseline (r close to unity). Figure 8a reproduces this while also showing that the N1 regime does much worse if r is smaller (lagged inflation is less important) with expected loss almost three and a half times worse than discretion for the Calvo case.

The N2 and N3 regimes have similar properties and are close to discretion with low r but improve to almost reproduce the commitment outcome when r=1. Regime N2 is considered in Jensen (2002, regime NIGT, Tables 2-4) and Walsh (2003, regime NIT2, Table 4 and Figure 3) while N3 is analysed by Soderstrom (2001, nominal income target, Tables 3 and 4) with similar results.

Hence from Figure 8, which uses the baseline parameter values, an interim conclusion is that CCB is weak, SLT powerful and the superior regime until r becomes large, N1 worse than discretion over a significant range of r and at best only a little better than CCB, and N2 and N3 are fairly powerful but dominated by SLT until r is large. The effect of changes in the parameter values (while still keeping q* at zero) are shown in columns (1)-(3) of Table 2a (which reports the loss index portrayed in Figure 8) and Table 2b (which reports the ranking of the regimes). These figures reveal a fairly high degree of sensitivity to the assumptions made about $c\kappa$, which determines the sensitivity of inflation to the output gap, and λ which is the relative weight on output gap variability in the social loss function.

[14] We compare our results mainly with Jensen (2002), Walsh (2003a) and Soderstrom (2001). When q*=0 the results are very close to those in these contributions. There are two sources of variation. First we assume that the natural rate of output is known with certainty and thus exclude a potential source of additional loss from the nominal income regimes from a fluctuating natural rate; this is discussed in Jensen and Walsh. Second our baseline parameter assumptions coincide with Walsh but differ slightly from Jensen and Soderstrom. Rudebusch (2002) gives results for nominal income growth targeting but his model has a different lag structure to that used here and the other references above.

TABLE 2a: Full Information Delegation
(Indexed loss; Commitment=0, Discretion=100)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
q*	0	0	0	0.5	0.5	0.5	1	1	1	Averages over:						
r	0	0.5	1	0	0.5	1	0	0.5	1	All	q*=0	q*=0.5	q*=1	r=0	r=0.5	r=1
Baseline (cκ=0.05, λ=0.25)																
CCB	100	90	55	97	76	57	91	71	59	77	82	77	74	96	79	57
SLT	34	15	1	23	13	6	22	16	12	16	17	14	17	26	15	6
N1	336	102	50	43	13	11	4	2	4	63	162	22	3	128	39	21
N2	90	23	0	19	1	6	2	4	16	18	38	8	7	37	9	7
N3	98	34	2	27	3	1	3	2	7	20	45	10	4	43	13	3
a) cκ=0.2																
CCB	100	91	82	93	83	77	87	80	76	86	91	85	81	94	85	78
SLT	21	10	2	22	19	15	32	31	29	20	11	18	31	25	20	15
N1	3	2	10	4	4	4	11	11	10	6	5	4	11	6	5	8
N2	2	5	45	8	29	71	28	54	84	36	17	36	55	13	29	67
N3	3	1	12	6	16	27	26	32	39	18	5	16	32	11	16	26
b) cκ=0.01																
CCB	100	97	32	99	90	36	98	80	39	75	77	75	72	99	89	36
SLT	47	27	1	35	19	2	30	15	3	20	25	18	16	37	20	2
N1	7552	2055	119	1325	293	49	259	61	20	1304	3242	556	113	3045	803	63
N2	100	91	6	93	43	1	62	10	0	45	66	46	24	85	48	2
N3	100	94	8	96	52	3	72	16	0	49	67	50	30	90	54	4
c) λ=0.5																
CCB	100	92	49	98	78	52	93	71	55	76	80	76	73	97	80	52
SLT	37	17	1	26	13	4	23	15	9	16	18	14	16	28	15	5
N1	1262	358	118	201	60	39	32	13	13	233	579	100	19	498	144	57
N2	98	56	5	57	10	0	12	1	2	27	53	23	5	56	22	2
N3	100	67	11	72	18	2	19	3	1	32	59	31	8	64	29	4
d) λ=0.05																
CCB	100	88	70	94	79	69	87	76	69	81	86	81	77	94	81	69
SLT	26	11	2	20	15	10	26	23	21	17	13	15	23	24	17	11
N1	0	1	0	7	6	4	14	13	12	6	0	6	13	7	7	5
N2	0	11	78	15	48	129	42	89	162	64	30	64	98	19	50	123
N3	0	7	37	14	34	53	36	51	59	32	15	34	49	17	31	50
Average over all parameter values																
CCB	100	92	58	96	81	58	91	76	60	79	83	79	76	96	83	58
SLT	33	16	1	25	16	7	27	20	15	18	17	16	20	28	17	8
N1	1831	504	59	316	75	22	64	20	12	322	798	138	32	737	200	31
N2	58	37	27	38	26	41	29	32	53	38	41	35	38	42	32	40
N3	60	41	14	43	25	17	31	21	21	30	38	28	24	45	29	17
Average over all regimes																
	416	138	32	104	45	29	49	34	32	97	195	59	38	190	72	31

TABLE 2b: Full Information Delegation
(Regime ranking: 1=lowest loss, 5=highest loss)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
q*	0	0	0	0.5	0.5	0.5	1	1	1	Averages over:						
r	0	0.5	1	0	0.5	1	0	0.5	1	All	q*=0	q*=0.5	q*=1	r=0	r=0.5	r=1
Baseline (cκ=0.05, λ=0.25)																
CCB	4	4	5	5	5	5	5	5	5	4.8	4.3	5.0	5.0	4.7	4.7	5.0
SLT	1	1	2	2	3	3	4	4	3	2.6	1.3	2.7	3.7	2.3	2.7	2.7
N1	5	5	4	4	4	4	3	2	1	3.6	4.7	4.0	2.0	4.0	3.7	3.0
N2	2	2	1	1	1	2	1	3	4	1.9	1.7	1.3	2.7	1.3	2.0	2.3
N3	3	3	3	3	2	1	2	1	2	2.2	3.0	2.0	1.7	2.7	2.0	2.0
a) cκ=0.2																
CCB	5	5	5	5	5	5	5	5	4	4.9	5.0	5.0	4.7	5.0	5.0	4.7
SLT	4	4	1	4	3	2	4	2	2	2.9	3.0	3.0	2.7	4.0	3.0	1.7
N1	3	2	2	1	1	1	1	1	1	1.4	2.3	1.0	1.0	1.7	1.3	1.3
N2	1	3	4	3	4	4	3	4	5	3.4	2.7	3.7	4.0	2.3	3.7	4.3
N3	2	1	3	2	2	3	2	3	3	2.3	2.0	2.3	2.7	2.0	2.0	3.0
b) cκ=0.01																
CCB	3	4	4	4	4	4	4	5	5	4.1	3.7	4.0	4.7	3.7	4.3	4.3
SLT	1	1	1	1	1	2	1	2	3	1.4	1.0	1.3	2.0	1.0	1.3	2.0
N1	5	5	5	5	5	5	5	4	4	4.8	5.0	5.0	4.3	5.0	4.7	4.7
N2	2	2	2	2	2	1	2	1	1	1.7	2.0	1.7	1.3	2.0	1.7	1.3
N3	4	3	3	3	3	3	3	3	2	3.0	3.3	3.0	2.7	3.3	3.0	2.7
c) λ=0.5																
CCB	4	4	4	4	5	5	5	5	5	4.6	4.0	4.7	5.0	4.3	4.7	4.7
SLT	1	1	1	1	2	3	3	4	3	2.1	1.0	2.0	3.3	1.7	2.3	2.3
N1	5	5	5	5	4	4	4	3	4	4.3	5.0	4.3	3.7	4.7	4.0	4.3
N2	2	2	2	2	1	1	1	1	2	1.6	2.0	1.3	1.3	1.7	1.3	1.7
N3	3	3	3	3	3	2	2	2	1	2.4	3.0	2.7	1.7	2.7	2.7	2.0
d) λ=0.05																
CCB	5	5	4	5	5	4	5	4	4	4.6	4.7	4.7	4.3	5.0	4.7	4.0
SLT	4	4	2	4	2	2	2	2	2	2.7	3.3	2.7	2.0	3.3	2.7	2.0
N1	1	1	1	1	1	1	1	1	1	1.0	1.0	1.0	1.0	1.0	1.0	1.0
N2	1	3	5	3	4	5	4	5	5	3.9	3.0	4.0	4.7	2.7	4.0	5.0
N3	1	2	3	2	3	3	3	3	3	2.6	2.0	2.7	3.0	2.0	2.7	3.0
Average over all parameter values																
CCB	4.2	4.4	4.4	4.6	4.8	4.6	4.8	4.8	4.6	4.6	4.3	4.7	4.7	4.5	4.7	4.5
SLT	2.2	2.2	1.4	2.4	2.2	2.4	2.8	2.8	2.6	2.3	1.9	2.3	2.7	2.5	2.4	2.1
N1	3.8	3.6	3.4	3.2	3.0	3.0	2.8	2.2	2.2	3.0	3.6	3.1	2.4	3.3	2.9	2.9
N2	1.6	2.4	2.8	2.2	2.4	2.6	2.2	2.8	3.4	2.5	2.3	2.4	2.8	2.0	2.5	2.9
N3	2.6	2.4	3.0	2.6	2.6	2.4	2.4	2.4	2.2	2.5	2.7	2.5	2.3	2.5	2.5	2.5

The most striking example is the N1 regime which was weak in the baseline case shown in Figure 8a and the top block in the Tables. If cκ is low or λ high this regime deteriorates considerably but for high cκ or low λ its performance dramatically improves. Regimes N2 and N3 show strong parameter sensitivity both in absolute terms, as their loss indices vary considerably with the different parameter combinations, and also whether those indices are increasing or decreasing in r. Overall the regimes other than N1 remain at least as good as discretion across all the q*=0 cases. In that sense the overall potential of delegation to offset stabilisation bias is broadly unchanged but the rankings in columns (1)-(3) of Table 2b in particular (where only CCB fails to rank first at least once) show that the optimal choice of delegation regime is sensitive to parameter values, even while assuming that q*=0 so the Phillips curve (2) applies.

Clearly if (2) was a consensus model it would be appropriate to consider the parameter sensitivity of these regimes in more detail but the Phillips curve derivation in Section 1 showed that (2) is a special case of (11) and hence we move on to considering the impact of the shift away from Calvo pricing generated by q*>0. These cases are shown in Figures 8b-8d and columns (4)-(9) of Tables 2a-2b. Figure 8b fixes r at zero and allows q* to increase from zero to unity. As a result the points on the vertical axis remain unchanged from Figure 8a. In this case CCB improves only marginally on discretion as q* increases while the index for SLT remains fairly stable. N1 shows a dramatic improvement as q* increases such that for q*=1 it offsets virtually all the stabilisation bias. N2 and N3 show a similar pattern to Figure 8a, improving as q* increases as they did when r increased. Hence the major changes from Figure 8a are a worsening of the performance of CCB and SLT and an improvement in N1 at high values of q*. Figures 8c and 8d allow for both q* and r to be positive. A broadly similar pattern emerges. CCB is favoured by high r but is largely unaffected by q*. The performance of SLT is better when q* is low but is always good. N1 is also weak at low q* but much less so than when r was also low. N2 and N3 do well when either q* or r are high but deteriorate if both are high.

The main impact of these results taken together, comparing Figure 8a with Figures 8b-8d, is that the shift away from Calvo pricing represented by q*>0 leads to a significant improvement in the nominal income growth regimes relative to SLT such that the latter is much less often the regime of choice. We know already from columns (1)-(3) of Tables 2a-2b that the general superiority of SLT in Figure 8a is contingent on parameter values even if q*=0 is imposed. From Figures 8b-8d that superiority disappears even with the baseline assumptions unless r is large (most readily seen by comparing Figure 8a with Figure 8c). Columns (4)-(9) of Tables 2a-2b allow for changes in the baseline values and reinforce this conclusion with SLT frequently being outranked by at least one of the nominal income growth regimes. From the averages at the base of columns 10-16 it is clear, however, that SLT remains potent overall.

From these results it may be concluded that appropriate delegation is potentially very powerful in offsetting stabilisation bias, but the identity of the best regime is strongly parameter dependent. These conclusions follow from the evidence for the q*=0 cases taken alone but are strongly reinforced by allowing for q*>0. In turn it is clear that none of the regimes (apart from CCB) should be ruled out if delegation is being considered.

The results presented so far assume full information about the economy at the time of delegation. If this was fully realistic, optimal delegation would be a fairly simple matter of comparing minimised loss under all possible delegation regimes given the known model. Unfortunately that is unlikely to be the case and hence an important issue is the robustness of regime performance under imperfect information. It should be emphasised that the results above are not sufficient in themselves to address this, even if one had strong priors about the probability distributions of the parameter values or took a robust control approach. This is because in each case delegation was based on optimised values for the loss function parameters listed in the last column of Table 1, with those values changing each time the parameter assumptions were varied.

A full investigation of imperfect information delegation is beyond the scope of this paper but we present some preliminary evidence and highlight the impact of moving away from Calvo pricing. In particular we examine the consequences of delegation assuming that q* and r take particular values (implicitly believed with certainty by the delegator) whereas in practice they take different values. For simplicity this exercise is conducted using the baseline value for $c\kappa$ (which is assumed to be known with certainty) so we focus on incorrect beliefs about q* and r. We assume known values for β and λ since they appear in the social loss function.

The results for expected loss are reported in Table 3a using the same index as above. The upper block of figures reproduces the full information baseline case from the top of Table 2a for comparison. The figures beneath are the loss index given the assumed values for q* and r shown while their actual values are those given at the top of the columns. Figure 3b reports the same underlying information but in the form of the change in the loss index relative to the full information case.

The figures in Table 3a are informative about the likelihood that delegation may lead to a worse outcome than discretion. With full information only two cases of this arise, both with the N1 regime and when (q*,r) is (0,0) or (0,0.5). Since imperfect information cannot improve the outcome these cases repeat across all the imperfect information possibilities. Further regime and parameter combinations when this occurs are: i) CCB when (q*,r) is (0,0) or (0.5,0) for all the imperfect information cases except when the values (0,0) were believed in which case this regime effectively disappears since the optimal delegated value of λ is simply the social value; ii) N2 when r=1 (for any q*) but the belief was that (q*,r) was (0,0); and iii) N2 and N3 when (q*,r) is (0,0) but other values were assumed. These cases are a small proportion of all those considered. SLT is robust in the sense that its outcome is never worse than discretion, and under N1 the outcome is worse than discretion only for those cases when it was already inferior under full information.

Table 3b gives a clearer picture of the robustness of the regimes. We first suppose that q*=0 and this is known with certainty so only columns (1)-(3) of the first two blocks of figures apply. Here SLT is clearly very robust, its loss index deteriorating by a maximum of 2, whereas the nominal income growth regimes are much less robust with N2 and N3 recording their maximum deteriorations in the whole table when (q*,r) is (0,1) but was believed to be (0,0).

TABLE 3a: Imperfect Information Delegation (baseline parameter values)
(Indexed loss: Commitment=0, Discretion=100)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
q*	0	0	0	0.5	0.5	0.5	1	1	1	Averages over:						
r	0	0.5	1	0	0.5	1	0	0.5	1	All	q*=0	q*=0.5	q*=1	r=0	r=0.5	r=1
Full information																
CCB	100	90	55	97	76	57	91	71	59	77	82	77	74	96	79	57
SLT	34	15	1	23	13	6	22	16	12	16	17	14	17	26	15	6
N1	336	102	50	43	13	11	4	2	4	63	162	22	3	128	39	21
N2	90	23	0	19	1	6	2	4	16	18	38	8	7	37	9	7
N3	98	34	2	27	3	1	3	2	7	20	45	10	4	43	13	3
Delegation assumes q*=0, r=0																
CCB	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
SLT	34	15	3	26	19	17	30	31	33	23	17	21	31	30	22	17
N1	336	103	72	43	15	31	5	7	24	71	170	30	12	128	42	42
N2	90	84	219	72	91	180	68	94	154	117	131	114	105	77	89	184
N3	98	87	79	87	77	74	77	71	70	80	88	79	73	88	78	74
Delegation assumes q*=0, r=1																
CCB	123	94	55	105	77	57	95	71	60	82	91	80	75	108	81	57
SLT	36	15	1	24	15	10	26	24	24	19	17	16	25	28	18	12
N1	382	110	50	56	14	12	7	2	6	71	181	27	5	148	42	22
N2	131	24	0	19	1	6	4	5	16	23	52	8	8	51	10	7
N3	148	35	2	29	4	1	7	4	7	26	62	11	6	61	14	3
Delegation assumes q*=0.5, r=0.5																
CCB	117	92	55	102	76	58	93	72	61	81	88	79	75	104	80	58
SLT	47	21	4	25	13	6	23	18	16	19	24	14	19	31	17	9
N1	354	103	54	48	13	16	5	3	11	67	170	26	6	135	40	27
N2	139	26	1	19	1	6	3	5	16	24	55	8	8	53	11	7
N3	171	40	5	27	3	2	4	2	7	29	72	11	5	67	15	5
Delegation assumes q*=1, r=1																
CCB	132	98	56	110	78	57	98	72	59	84	95	82	76	113	83	57
SLT	86	42	16	37	17	8	25	17	12	29	48	21	18	49	25	12
N1	439	130	54	73	21	12	13	4	4	83	208	35	7	175	52	23
N2	146	29	2	19	1	6	2	4	16	25	59	9	7	56	11	8
N3	155	36	3	28	3	1	6	3	7	27	65	11	5	63	14	3
Average over incomplete information cases																
CCB	118	96	66	104	83	68	96	79	70	87	93	85	82	106	86	68
SLT	51	23	6	28	16	10	26	22	21	23	27	18	23	35	21	12
N1	378	112	57	55	16	18	7	4	11	73	182	30	8	147	44	29
N2	126	41	55	32	23	49	19	27	50	47	74	35	32	59	30	52
N3	143	49	22	43	22	19	24	20	23	41	72	28	22	70	30	21
Average over incomplete information cases and all regimes																
	163	64	41	52	32	33	34	30	35	54	90	39	33	83	42	36

TABLE 3b: Imperfect Information Delegation (baseline parameter values)
(Increase in indexed loss due to imperfect information)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
q*	0	0	0	0.5	0.5	0.5	1	1	1	Averages over:						
r	0	0.5	1	0	0.5	1	0	0.5	1	All	q*=0	q*=0.5	q*=1	r=0	r=0.5	r=1
Delegation assumes q*=0, r=0																
CCB	0	10	45	3	24	43	9	29	41	23	18	23	26	4	21	43
SLT	0	0	1	3	6	11	8	14	21	7	1	7	14	3	7	11
N1	0	1	22	0	3	19	0	5	20	8	8	7	9	0	3	21
N2	0	61	219	54	90	175	66	90	138	99	93	106	98	40	80	177
N3	0	53	77	60	74	73	75	69	64	60	43	69	69	45	65	71
Delegation assumes q*=0, r=1																
CCB	23	4	0	8	0	0	4	0	1	4	9	3	1	12	2	0
SLT	2	0	0	0	2	5	3	8	12	4	1	2	8	2	3	6
N1	46	8	0	13	2	1	3	0	2	8	18	5	2	21	3	1
N2	41	1	0	0	0	0	2	1	1	5	14	0	1	14	1	0
N3	50	1	0	2	1	0	4	2	0	7	17	1	2	18	1	0
Delegation assumes q*=0.5, r=0.5																
CCB	17	2	0	5	0	1	2	0	2	3	6	2	1	8	1	1
SLT	13	6	3	1	0	0	0	2	4	3	7	1	2	5	3	2
N1	18	1	4	4	0	5	1	1	8	5	8	3	3	8	1	6
N2	49	3	1	0	0	0	1	1	0	6	18	0	1	17	1	0
N3	73	6	3	0	0	1	1	0	1	9	27	0	1	25	2	1
Delegation assumes q*=1, r=1																
CCB	32	8	1	13	2	0	7	1	0	7	14	5	2	17	4	0
SLT	52	27	15	14	4	2	2	0	0	13	31	7	1	23	11	6
N1	103	28	5	30	8	1	9	2	0	21	45	13	3	47	13	2
N2	56	6	2	0	0	1	1	0	0	7	21	0	0	19	2	1
N3	57	2	0	1	1	0	3	1	0	7	20	0	1	20	1	0
Average over incomplete information cases																
CCB	18	6	12	7	6	11	5	7	11	9	12	8	8	10	7	11
SLT	17	9	5	5	3	5	3	6	9	7	10	4	6	8	6	6
N1	42	10	8	12	3	7	3	2	8	10	20	7	4	19	5	7
N2	37	18	55	14	23	44	17	23	35	29	37	27	25	23	21	45
N3	45	15	20	16	19	18	21	18	16	21	27	18	18	27	17	18
Average over incomplete information cases and all regimes																
	32	12	20	11	11	17	10	11	16	15	21	13	12	17	11	17

Allowing for q*>0 and hence considering columns (1)-(9) for all the imperfect information cases leads to a similar conclusion about the relative robustness of the regimes but with a narrowing of the gap between SLT and N2/N3 since SLT is less robust to incorrect assumptions about q* than r. The overall average deterioration (reported at the base of column 10) is 3-4 times as high for N2 and N3 as for SLT. The average deterioration for CCB and N1 is only a little higher than that for SLT but this is perhaps of less interest since CCB and N1 have high average losses in Table 2a, and the relative performance of the N1 regime is highly model specific even with full information. Hence consideration of imperfect information delegation appears to strengthen the case for the SLT regime over the nominal income growth based regimes though a full analysis of the preferred regime would require a much more detailed assessment of possible priors or feasible intervals for the different parameters, including cκ which we have kept fixed in Tables 3a-3b, as well as translation of the loss indices into actual loss values using the results underpinning Figure 5. That exercise is not an objective of this paper since we are concerned with the analytic question of the consequences of moving away from Calvo pricing. From that perspective we conclude both that the q* parameter matters for the optimal choice of delegation regime under full information, and that uncertainty about the true value of q* would be a potentially important component of robust regime choice under imperfect information.

4. Conclusion

The paper was motivated by the tension between the near-universal reliance on the Calvo pricing model for monetary policy research and the combination of microeconomic evidence challenging its realism and a parallel literature which suggests non-robustness to changing the time dependent pricing rule. The latter literature had not yet fully analysed the implications of varying the Calvo constant probability of price change assumption for the Phillips curve and monetary policy.

A fairly simple variant on the constant probability Calvo pricing rule was developed with the probability of price change permitted to increase between one and two periods after a price change, after which it remained constant. The model also allowed for a proportion of firms to follow rules of thumb, a standard way of introducing lagged inflation into the Phillips curve. The new pricing rule is itself open to question for being less than fully general but nevertheless the model derived from it is more general than virtually all existing formulations used for monetary policy analysis. As a consequence the paper was able to systematically explore the robustness of the Phillips curve and the monetary policy conclusions that follow from it to varying the Calvo assumption.

The results were described in some detail in the introduction so we summarise the key findings and implications here. The consistent theme is that varying the Calvo pricing rule leads to significant changes and thus ignoring the possibility of non-Calvo microfoundations for monetary policy models is hazardous. We found that varying the time dependent pricing rule gives rise to a very different form of the Phillips curve with much richer endogenous dynamics. In turn optimal policy under both commitment and discretion is sensitive to the pricing rule microfoundation with the size and nature of stabilisation bias also affected. The analysis of optimal delegation showed that the best choice of delegation regime depends on the underlying pricing rule as well as other parameters and that the possibility of non-Calvo pricing would also be important if delegation were to take place with imperfect information about the structure of the economy.

Taken together, these results strongly challenge the reliance on Calvo pricing in much of the monetary policy literature. More positively we have shown that the derivation of Phillips curves based on more general pricing rules is feasible, that rule of thumb behaviour may readily be incorporated if desired, and that policy results are fairly easily obtainable in such a model. The stronger endogenous dynamics from a more general pricing rule are also of interest and open up the possibility of a model that is closer to both the microeconomic and macroeconomic evidence while retaining fully optimising microfoundations.

Appendix A
Microfoundations of Optimal Price and Wage Setting

We derive the price that would be set each period in a flexible price environment in order to generate the 'ideal' price which forms part of the derivation of optimal price setting once staggering constraints are imposed. The microfoundations of these choices are standard. We consider a large number of symmetric, monopolistically competitive firms, indexed by i, each with production function (A1), where Y_i is firm output, K_i the firm's capital stock which we hold constant and L_i firm level employment. Any multiplicative constant that may be present in (A1) is normalised to unity for convenience and without loss of generality. All expressions in this appendix refer to a single period and hence for simplicity we do not include time subscripts.

$$Y_i = K_i^{1-\alpha} L_i^{\alpha} \qquad \text{(A1)}$$

Each firm also faces the demand curve (A2) where P_i is the firm's price, P the general price level (defined as the weighted geometric mean of firm prices with weights summing to unity and equal to the proportion of all firms with each particular price), η the common price elasticity of demand (defined such that $\eta>0$) and Y_{di} an index of aggregate demand per firm.

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y_{di} \qquad \text{(A2)}$$

We also make use of the notation W for the common nominal wage (labour is assumed mobile between firms) which is exogenous to firms individually, c_K for the per period cost of capital (which plays no part in the analysis since capital is fixed), and R_i for firm profits. Profits are given by $R_i=P_iY_i-WL_i-c_KK_i$ in terms of the three variables endogenous to the firm, P_i, Y_i and L_i. For given Y_{di} and P the choice of any of these three implies the other two through (A1) and (A2) and we substitute from those expressions for Y_i and L_i to give profits in terms of P_i by (A3).

$$R_i = P\left[\left(\frac{P_i}{P}\right)^{1-\eta} Y_{di} - K_i^{-\frac{(1-\alpha)}{\alpha}} \left(\frac{W}{P}\right)\left(\frac{P_i}{P}\right)^{-\frac{\eta}{\alpha}} Y_{di}^{\frac{1}{\alpha}}\right] - c_K K_i \qquad \text{(A3)}$$

Differentiating (A3) with respect to P_i gives the first order condition for price, (A4), which may be substituted into (A1) and (A2) to give optimal employment by (A5) and output by (A6). The second order condition for profit maximisation may readily be shown to be satisfied.

$$P_i = P\left[\frac{Y_{di}^{1-\alpha}\left(\frac{W}{P}\right)^{\alpha}}{K_i^{1-\alpha}\left[\alpha\left(1-\frac{1}{\eta}\right)\right]^{\alpha}}\right]^{\frac{1}{\eta\left[1-\alpha\left(1-\frac{1}{\eta}\right)\right]}} \qquad \text{(A4)}$$

$$L_i = \left[\frac{\alpha(1-\frac{1}{\eta})K_i^{(1-\alpha)(1-\frac{1}{\eta})} Y_{di}^{\frac{1}{\eta}}}{\frac{W}{P}}\right]^{\frac{1}{1-\alpha(1-\frac{1}{\eta})}} \tag{A5}$$

$$Y_i = \left[\frac{\alpha(1-\frac{1}{\eta})K_i^{\frac{1-\alpha}{\alpha}} Y_{di}^{\frac{1}{\eta}}}{\frac{W}{P}}\right]^{\frac{\alpha}{1-\alpha(1-\frac{1}{\eta})}} \tag{A6}$$

In (A4)-(A6) the powers on the firm level quantity variables, L_i, K_i and Y_{di} are such that we may multiply each of these by the number of firms which then cancels such that the P_i in (A4) may be expressed as a function of aggregate demand, Y_d, and the aggregate capital stock and (A5) and (A6) give aggregate employment and output simply by dropping the i subscripts. From this point K_i may be normalised to unity. P_i in (A4) may also be shown to be equal to nominal marginal cost divided by $(1-1/\eta)$, a standard feature. The output gap is proportional to marginal cost in this model so we may express the Phillips curve in terms of either of them. We choose the output gap since it has a more direct intuitive link with policy actions.

We turn to wage setting behaviour, assuming that wages are set competitively period by period by many small groups of workers (who by symmetry set the same wage) whose preferences may be summarised by the aggregate labour supply curve $L_s=(W/P)^\theta$ where without loss of generality a possible multiplicative constant is normalised to unity. Equating labour supply with aggregate labour demand from (A5) without i subscripts gives the equilibrium real wage by (A7). While we think of workers setting the nominal wage we express outcomes in terms of the real wage for convenience, noting that with contemporaneous wage setting rational workers will have full information about the real wage that will result from any given nominal wage. Using the labour supply curve and (A1) equilibrium employment and output are given by (A8) and (A9).

$$\frac{W}{P} = \left[\alpha(1-\frac{1}{\eta})Y^{\frac{1}{\eta}}\right]^{\frac{1}{1+\theta[1-\alpha(1-\frac{1}{\eta})]}} \tag{A7}$$

$$L = \left[\alpha(1-\frac{1}{\eta})\right]^{\frac{\theta}{1+\theta(1-\alpha)}} \tag{A8}$$

$$Y = [\alpha(1-\frac{1}{\eta})]^{\frac{\alpha\theta}{1+\theta(1-\alpha)}} \qquad (A9)$$

Given that we have assumed complete within period price and wage flexibility and not yet introduced shocks (A7)-(A9) may be interpreted as flexible wage-price natural rates. Denoting these with a "*" we may re-express (A4) and (A7) in terms of deviations from natural rates by (A10) and (A11).

$$P_i = P[(\frac{Y}{Y^*})^{1-\alpha}(\frac{\frac{W}{P}}{(\frac{W}{P})^*})^{\alpha}]^{\frac{1}{\eta[1-\alpha(1-\frac{1}{\eta})]}} \qquad (A10)$$

$$\frac{W}{P} = (\frac{W}{P})^* \; (\frac{Y}{Y^*})^{\frac{1}{\eta[1+\theta[1-\alpha(1-\frac{1}{\eta})]}} \qquad (A11)$$

The model assumes complete wage flexibility while prices are staggered so (A11) may simply be substituted into (A10) which gives (A12) as the ideal single period price which would be set by each firm in the absence of staggering constraints.

$$P_i = P(\frac{Y}{Y^*})^{\frac{1+\theta(1-\alpha)}{\eta[1+\theta[1-\alpha(1-\frac{1}{\eta})]}} \qquad (A12)$$

As a final step (see Walsh, 2003b) we take logs of (A12) and assume a log linear shock to price setting (ie. a multiplicative geometric mean preserving spread to the right hand side of (A12), ε, to give (10) in the text as the single period ideal price in logs which we denote p* (without an i subscript since it is symmetric across all firms). We also add time subscripts and denote the log of (Y/Y*) by y, the output gap. The parameter, κ, in (10) is given from (A12) by (A13).

$$\kappa = \frac{1+\theta(1-\alpha)}{\eta[1+\theta[1-\alpha(1-\frac{1}{\eta})]} \qquad (A13)$$

Appendix B
Phillips Curve Derivation

We present some of the more detailed steps in the derivation of the Phillips curve in Section 1. Equations (3)-(8) are largely self-contained, apart from algebraic steps, so we focus on deriving (9) for the price level and summarising the steps between (7)-(9) and the Phillips curve (11).

Following standard assumptions (Dixit and Stiglitz, 1977) we define the price level as the weighted geometric mean of the individual prices that exist with the proportion of each price in all prices as weights. It is convenient to work with the log of the price level, p, (since we use the approximation p_t-p_{t-1} for inflation) which is thus the weighted arithmetic mean of all the log prices with the same weights as before. The additive structure is helpful for combining the prices set by the two types of firms as well as combining the prices set in different time periods which still exist.

We start by deriving the price level for the 'standard' firms which do not index between price optimisations. Denoting this by p^s and with ω_i the (as yet unknown) weights within the price level for standard firms only we have (B1) for p^s at time t and t-1.

$$p_t^s = \Sigma_{i=0}^{\infty}\omega_i x_{t-i}^s \qquad p_{t-1}^s = \Sigma_{i=0}^{\infty}\omega_i x_{t-1-i}^s \tag{B1}$$

The weights clearly depend on the probabilities of price change and we assume that the total number of firms is large so the probabilities translate directly to proportions of firms. This justifies the common weights in the price level for each time period and allows us to solve for them. As a first step, ω_0 is given by the sum of $q_1\omega_0$ (since a proportion q_1 of the newly set prices at t-1 are set again at t) plus $q\omega_1$+$q\omega_2$+$q\omega_3$+... since a proportion q of all those earlier prices are newly set at t. This gives (B2) for ω_0 which makes use of $\sum\omega=1$.

$$\omega_0 = q_1\omega_o + q\Sigma_{i=1}^{\infty}\omega_i = q_1\omega_0 + q(1-\omega_0) \quad\Rightarrow\quad \omega_0 = \frac{q}{1+q^*} \tag{B2}$$

Similar reasoning using (B1) means that $\omega_1=(1-q_1)\omega_0$ since a proportion $(1-q_1)$ of last period's new prices at t-1 remain unchanged and thus become one period old prices at t. For the older prices in (B1) $\omega_k=(1-q)\omega_{k-1}$. Combining these expressions gives $\omega_j=(1-q_1)(1-q)^{j-1}\omega_0$ for j>0 and thus the price level for the non-indexing firms in (B3).

$$p_t^s = \frac{q}{1+q^*}[x_t^s + (1-q_1)\Sigma_{j=1}^{\infty}(1-q)^{j-1}x_{t-j}^s] \tag{B3}$$

Next we lag (B3) one period and substitute the result into (B3) to give the more convenient expression (B4).

$$p_t^s = (1-q)p_{t-1}^s + \frac{q}{1+q^*}[x_t^s + q^* x_{t-1}^s] \tag{B4}$$

The derivation of the equivalent price level for the rule of thumb firms, p^r, is similar and involves the same weights since the probabilities of price change/optimisation are assumed common to the two types of firms. The difference is that all the prices in p^r at t-1 that are not re-optimised at t are indexed by $\gamma\pi_{t-1}$ which leads to (B5).

$$p_t^r = (1-q)p_{t-1}^r + \frac{q}{1+q^*}[x_t^r + q^* x_{t-1}^r] + \frac{(1-q_1)}{(1+q^*)}\gamma\pi_{t-1} \tag{B5}$$

Having derived the price levels for the two types of firms treated as separate groups the aggregate price level (9) follows directly from $p=(1-r)p^s+rp^r$.

Following (9) an interim step to (11) is to derive (B6) where the left hand side is the sum of the new prices set at time t as they appear in (9) and the right hand side follows directly from (7) and (8).

$$(1-r)x_t^s + rx_t^r = \begin{array}{l} \beta(1-q)[(1-r)E_t[x_{t+1}^s] + rE_t[x_{t+1}^r]] \\ + \dfrac{[1-\beta(1-q)]}{1+\beta q^*}(p_t^* + \beta q^* E_t[p_{t+1}^*]) - \dfrac{\beta(1-q_1)}{1+\beta q^*} r\gamma\pi_t \end{array} \tag{B6}$$

Leading (9) one period and taking expectations at t gives $E_t[p_{t+1}]$ as a function of the new prices set at t+1 so we may use that expression to substitute for the first line of (B6) from which, also using (10), we have (B7).

$$(1-r)x_t^s + rx_t^r = \begin{array}{l} p_t + \dfrac{\beta(1-q_1)}{q(1+\beta q^*)}[E_t[\pi_{t+1}] - r\gamma\pi_t] \\ + \dfrac{[1-\beta(1-q)]}{(1+\beta q^*)[1+\beta q^*(1-q)]}(\kappa y_t + \varepsilon_t + \beta q^* E_t[\kappa y_{t+1} + \varepsilon_{t+1}]) \end{array} \tag{B7}$$

Substituting (B7) and the same expression lagged one period into (9) and rearranging gives (11).

Appendix C
The Taylor and Fuhrer-Moore Phillips Curves With Price or Wage Setting Shocks

This appendix notes the effect on a two-period Taylor Phillips curve (and also that in the Fuhrer and Moore (1995) model) of the use of either perfect foresight or expectational error terms. We highlight this point since a commonly expressed view is that the Calvo form of the Phillips curve (1) also applies to the Taylor model, at least with respect to the inflation expectations terms. For examples see Roberts (1995, equation 8), Walsh (2003b, equation 5.36), and more recently the introduction to Amato and Laubach (2003b) amongst many others.

Given the time dependent pricing/price optimisation rule, (11) with $q^*=1$ (and also iid shocks for simplicity) gives the Phillips curve for a two-period Taylor model by (C1).

$$\pi_t = \frac{\beta}{1+\beta}(E_t[\pi_{t+1}] + E_{t-1}[\pi_t]) + c(\kappa y_t + \varepsilon_t + \beta E_t[\kappa y_{t+1}] + \kappa y_{t-1} + \varepsilon_{t-1} + \beta E_{t-1}[\kappa y_t]) \qquad \text{(C1)}$$

This differs from the forms typically reported in the literature which derive the Taylor Phillips curve under perfect foresight or make use of expectational error terms. The difference may be seen by defining the expectational error $\eta_t = E_{t-1}[\pi_t] - \pi_t$ (in Walsh's notation). Substituting this into (C1) gives (C2).

$$\pi_t = \beta E_t[\pi_{t+1}] + (1+\beta)c(\kappa y_t + \varepsilon_t + \beta E_t[\kappa y_{t+1}] + \kappa y_{t-1} + \varepsilon_{t-1} + \beta E_{t-1}[\kappa y_t]) + \beta\eta_t \qquad \text{(C2)}$$

In (C2) the term $E_{t-1}[\pi_t]$ from (C1) is now implicit in the final term which, observationally (since expectations errors are white noise under rational expectations), can be combined with ε_t to form a composite error term.[15] We emphasise that formally there is no error involved in (C2) or the equivalent expressions in the literature, and from an empirical perspective the expectations error would not be readily identifiable[16], but from a *policy* perspective (C2) may be seriously misleading. The reason is that $E_{t-1}[\pi_t]$ is fixed when policy choices are made at time t but π_t is not. Hence η_t is *not* exogenous to policy at time t but if it is treated as an error term (or part of a composite error) an implicit assumption is being made that it is exogenous. Hence for policy analysis it is important that (C1) is used and not (C2) unless we are content to assume perfect foresight which is unlikely since most interesting policy issues, including stabilisation bias, are concerned with responses to shocks.

We also briefly note the same issue in relation to the Fuhrer and Moore (1995) Phillips curve. This model is used relatively little in the most recent literature for reasons noted earlier but it is sometimes referred to as supportive of the Phillips curve (2) which we show not to be the case, except under

[15] As can the term $E_{t-1}[y_t]$.

[16] Though the changed coefficients on $E_t[\pi_{t+1}]$ and the output gap terms in (C2) relative to (C1) for the same underlying model may be noted. This gives rise to an issue of interpreting estimates of these coefficients if (C2) is estimated on the assumption that it is structural (with η_t assumed exogenous) whereas in fact (C1) is the true model.

perfect foresight. The Phillips curve for this model is usually presented in a form such as (C3) which reproduces Walsh (2003b, equation 5.45) in the same notation.[17] Here β=1, there are no cost push shocks to wage setting and η_t has the same definition as above. This expression is standard in the literature.[18]

$$\pi_t = \frac{1}{2}(\pi_{t-1} + E_t[\pi_{t+1}]) + 2k(y_t + y_{t-1}) + \frac{1}{2}\eta_t \tag{C3}$$

The same concern about using (C2) for policy analysis applies to (C3). If we substitute in for η_t we obtain (C4) where $E_{t-1}[\pi_t]$ is now explicit and the coefficients on the other terms are reduced by two thirds.

$$\pi_t = \frac{1}{3}(\pi_{t-1} + E_{t-1}[\pi_t] + E_t[\pi_{t+1}]) + \frac{4}{3}k(y_t + y_{t-1}) \tag{C4}$$

Clearly the expected inflation terms in (C4) are inconsistent with (2) except under perfect foresight when η_t=0 and the same remarks as above apply.

[17] The coefficient of a half on the last term in (C3) is missing from Walsh's 5.45 which, given the derivation in his footnote 44, appears to be a typographical error.

[18] See Batini and Haldane (1999), for example, including the definition of ε_{5t} in eqn. (8).

Appendix D
Optimal policy: Commitment, Discretion and Delegation

We outline the detailed steps involved in the policy material of Sections 2 and 3, covering commitment followed by discretion and delegation taken together. It is convenient to re-express the Phillips curve (11) in the more compact form (D1) where the α, δ and χ summary coefficients follow directly from (11). The δ notation in this appendix is different from its use in equation (2).

$$\pi_t = \alpha_0 E_t[\pi_{t+1}] + \alpha_1 E_{t-1}[\pi_t] + \alpha_2 \pi_{t-1} + \delta_0 E_t[y_{t+1}] + \delta y_t + \delta_1 E_{t-1}[y_t] + \delta_2 y_{t-1} + \chi_0 \varepsilon_t + \chi_1 \varepsilon_{t-1} \qquad \text{(D1)}$$

Commitment

We solve for the choice of time series processes for inflation and output that minimises (13) subject to (D1) allowing the policy maker to commit to those processes and thus directly influence expectations. The approach is largely standard though as discussed in Section 2 the presence of $E_{t-1}[\pi_t]$ and $E_{t-1}[y_t]$ in (11) as well as π_t and y_t means that we make use of the reduced forms (16) to properly distinguish between inflation and output and their expected values one period before. Those reduced forms are supplemented by (D2) in the solution below where 2φ is the Lagrange multiplier on the Phillips curve constraint.

$$\varphi_t = \Sigma_{i=0}^{\infty} c_i \varepsilon_{t-i} = c_0 \varepsilon_t + E_{t-1}[\varphi_t] = c_0 \varepsilon_t + c_1 \varepsilon_{t-1} + E_{t-2}[\varphi_t] \qquad \text{(D2)}$$

Using (16) substituted into (D1) we may re-express (D1) by (D3).

$$0 = \begin{array}{l} \alpha_0 E_t[\pi_{t+1}] + (\alpha_1 - 1) E_{t-1}[\pi_t] + \alpha_2 E_{t-2}[\pi_{t-1}] \\ + \delta_0 E_t[y_{t+1}] + (\delta + \delta_1) E_{t-1}[y_t] + \delta_2 E_{t-2}[y_{t-1}] \\ + (\chi_0 - a_0 + \delta b_0) \varepsilon_t + (\chi_1 + \alpha_2 a_0 + \delta_2 b_0) \varepsilon_{t-1} \end{array} \qquad \text{(D3)}$$

Making use of (D3) and (16) we form the Lagrangean (D4) where for notational simplicity we assume that the policy solutions are to be chosen in period 0.

$$L_0 = E_0 \Sigma_{t=0}^{\infty} \beta^t \left[\begin{array}{c} (a_0 \varepsilon_t + E_{t-1}[\pi_t])^2 \\ + \lambda (b_0 \varepsilon_t + E_{t-1}[y_t])^2 \end{array} + 2\varphi_t \left[\begin{array}{l} \alpha_0 E_t[\pi_{t+1}] + (\alpha_1 - 1) E_{t-1}[\pi_t] + \alpha_2 E_{t-2}[\pi_{t-1}] \\ + \delta_0 E_t[y_{t+1}] + (\delta + \delta_1) E_{t-1}[y_t] + \delta_2 E_{t-2}[y_{t-1}] \\ + (\chi_0 - a_0 + \delta b_0) \varepsilon_t + (\chi_1 + \alpha_2 a_0 + \delta_2 b_0) \varepsilon_{t-1} \end{array} \right] \right] \qquad \text{(D4)}$$

We next find the first order conditions for the maximisation of (D4) with respect to a_0, b_0, $E_{t-1}[\pi_t]$ and $E_{t-1}[y_t]$. For the first pair of these, the contemporaneous shock responses, (D4) may be differentiated allowing for the responses to all the shocks that may occur but, through symmetry, we may more simply consider the responses to a single shock which we denote ε_s. In a similar fashion we evaluate the derivatives for the second pair for the expectation of inflation and output at some time s. The relevant partial derivatives are shown in (D5)-(D8) with the notation in the first pair indicating partial derivatives considering solely the shock, ε_s.

$$\frac{\partial L_0}{\partial a_0}\Big|_{\varepsilon_s} = 2E_0[\beta^s[(a_0\varepsilon_s + E_{s-1}[\pi_s])\varepsilon_s - \varphi_s\varepsilon_s] + \beta^{s+1}\varphi_{s+1}\varepsilon_s\alpha_2] \tag{D5}$$

$$\frac{\partial L_0}{\partial b_0}\Big|_{\varepsilon_s} = 2E_0[\beta^s[\lambda(b_0\varepsilon_s + E_{s-1}[y_s])\varepsilon_s + \varphi_s\varepsilon_s\delta] + \beta^{s+1}\varphi_{s+1}\varepsilon_s\delta_2] \tag{D6}$$

$$\frac{\partial L_0}{\partial E_{s-1}[\pi_s]} = 2E_0[\beta^s[a_0\varepsilon_s + E_{s-1}[\pi_s] + \varphi_s(\alpha_1 - 1)] + \beta^{s-1}\varphi_{s-1}\alpha_0 + \beta^{s+1}\varphi_{s+1}\alpha_2] \tag{D7}$$

$$\frac{\partial L_0}{\partial E_{s-1}[y_s]} = 2E_0[\beta^s[\lambda(b_0\varepsilon_s + E_{s-1}[y_s]) + \varphi_s(\delta + \delta_1)] + \beta^{s-1}\varphi_{s-1}\delta_0 + \beta^{s+1}\varphi_{s+1}\delta_2] \tag{D8}$$

We use (16) and (D2) together with E[ε]=0 and the assumption that the shocks are iid so $E[\varepsilon_i\varepsilon_j]=0$ ($i\neq j$) to simplify the expectational terms in (D5)-(D8). We also roll forward the timing of the expectation in (D7) and (D8) from $E_0[.]$ to $E_{s-1}[.]$ since the policy maker implements the first order conditions that follow from (D7) and (D8) at time s-1. Hence we have the first order conditions (D9)-(D12) where we change the time subscripts from s back to t.

$$0 = a_0 - c_0 + \beta\alpha_2 c_1 \tag{D9}$$

$$0 = \lambda b_0 + \delta c_0 + \beta\delta_2 c_1 \tag{D10}$$

$$0 = \beta E_{t-1}[\pi_t] + \beta(\alpha_1 - 1)E_{t-1}[\varphi_t] + \alpha_0\varphi_{t-1} + \beta^2\alpha_2 E_{t-1}[\varphi_{t+1}] \tag{D11}$$

$$0 = \beta\lambda E_{t-1}[y_t] + \beta(\delta + \delta_1)E_{t-1}[\varphi_t] + \delta_0\varphi_{t-1} + \beta^2\delta_2 E_{t-1}[\varphi_{t+1}] \tag{D12}$$

The commitment solution thus comprises (D9)-(D12) plus the Phillips curve constraint (D3). From these the relevant state variables are the current and once lagged shock terms together with once lagged inflation, output and the multiplier. Hence we conjecture the MSV solutions (D13)-(D15) where the coefficients on the contemporaneous shock are the same as (16) and (D2) but the other coefficients differ (since the last three variables in each expression will also respond to ε_{t-1} for example).

$$\pi_t = a_0\varepsilon_t + d_1\varepsilon_{t-1} + d_2 y_{t-1} + d_3\pi_{t-1} + d_4\varphi_{t-1} \tag{D13}$$

$$y_t = b_0\varepsilon_t + e_1\varepsilon_{t-1} + e_2 y_{t-1} + e_3\pi_{t-1} + e_4\varphi_{t-1} \tag{D14}$$

$$\varphi_t = c_0\varepsilon_t + f_1\varepsilon_{t-1} + f_2 y_{t-1} + f_3\pi_{t-1} + f_4\varphi_{t-1} \tag{D15}$$

To complete the solution requires two steps. First we substitute (D13)-(D14) into (D15) to derive c_1 in (D9)-(D10) as $c_1=f_1+f_2b_0+f_3a_0+f_4c_0$ and substitute this into the latter to give (D9)' and (D10)'.

$$0 = a_0 - c_0 + \beta\alpha_2(f_1 + f_2 b_0 + f_3 a_0 + f_4 c_0) \tag{D9)'}$$

$$0 = \lambda b_0 + \delta c_0 + \beta\delta_2(f_1 + f_2 b_0 + f_3 a_0 + f_4 c_0) \tag{D10)'}$$

Second we substitute (D13)-(D15) into (D3) and (D11)-(D12) until those equations are expressed in terms of the state variables identified in (D13)-(D15). This gives (D3)', (D11)' and (D12)'.

$$0 = \begin{aligned}&\varepsilon_t[\chi_0 + a_0(-1 + C_2) + b_0(\delta + C_1) + \alpha_0 d_1 + \delta_0 e_1 + c_0 C_3]\\ &+\varepsilon_{t-1}[\chi_1 + d_1(\alpha_1 - 1 + C_2) + e_1(\delta + \delta_1 + C_1) + f_1 C_3]\\ &+y_{t-1}[\delta_2 + d_2(\alpha_1 - 1 + C_2) + e_2(\delta + \delta_1 + C_1) + f_2 C_3]\\ &+\pi_{t-1}[\alpha_2 + d_3(\alpha_1 - 1 + C_2) + e_3(\delta + \delta_1 + C_1) + f_3 C_3]\\ &+\varphi_{t-1}[\quad d_4(\alpha_1 - 1 + C_2) + e_4(\delta + \delta_1 + C_1) + f_4 C_3]\end{aligned} \tag{D3)'}$$

$$0 = \begin{aligned}&\varepsilon_{t-1}[d_1 C_4 + f_1 C_5 + e_1 C_6]\\ &y_{t-1}[d_2 C_4 + f_2 C_5 + e_2 C_6]\\ &+\pi_{t-1}[d_3 C_4 + f_3 C_5 + e_3 C_6]\\ &+\varphi_{t-1}[\alpha_0 + d_4 C_4 + f_4 C_5 + e_4 C_6]\end{aligned} \tag{D11)'}$$

$$0 = \begin{array}{l} \varepsilon_{t-1}[d_1 C_9 + f_1 C_8 + e_1 C_7] \\ y_{t-1}[d_2 C_9 + f_2 C_8 + e_2 C_7] \\ +\pi_{t-1}[d_3 C_9 + f_3 C_8 + e_3 C_7] \\ +\varphi_{t-1}[\delta_0 + d_4 C_9 + f_4 C_8 + e_4 C_7] \end{array} \qquad \text{(D12)'}$$

Where

$$C_1 = \alpha_0 d_2 + \delta_0 e_2 \quad C_2 = \alpha_0 d_3 + \delta_0 e_3 \quad C_3 = \alpha_0 d_4 + \delta_0 e_4$$

$$C_4 = \beta(1 + \beta\alpha_2 f_3) \quad C_5 = \beta(\alpha_1 - 1 + \beta\alpha_2 f_4) \quad C_6 = \beta^2 \alpha_2 f_2$$

$$C_7 = \beta(\lambda + \beta\delta_2 f_2) \quad C_8 = \beta(\delta + \delta_1 + \beta\delta_2 f_4) \quad C_9 = \beta^2 \delta_2 f_3$$

Given that the state variables may take any value, (D3)' and (D11)'-(D12)' are satisfied by each of the 13 square bracketed terms following the state variables being set to zero. Hence they become 13 simultaneous equations which, together with (D9)'-(D10)', are solved to generate the 15 coefficients in (D13)-(D15).

It may be noted that from (D3)', (D11)'and (D12)' the simultaneous equations permit (and the MSV solution requires) i) if $\alpha_0=\delta_0=0$ so there are no forward looking expectations in (D1), $d_4=e_4=f_4=0$ so φ_{t-1} is no longer a state variable, ii) if $\chi_1=0$, $d_1=e_1=f_1=0$ and ε_{t-1} is no longer a state variable, iii) if $\delta_2=0$, $d_2=e_2=f_2=0$ and y_{t-1} is no longer a state variable, and iv) if $\alpha_2=0$, $d_3=e_3=f_3=0$ and π_{t-1} is no longer a state variable. More demandingly, the MSV solution requires the selection of roots for the MSV coefficients that are continuous with those that obtain as the parameters in these cases rise above zero (see McCallum, 2003). While we offer no general analysis of this multiplicity issue it appears that the numerical solution method (discussed below) achieves this since it looks for solutions near the initial trial values for the MSV coefficients and we build up the solutions sequentially starting from $q^*=0$, $r=0$ which removes the lagged state variables. In addition the impulse responses are checked at each step and show no evidence of the sharp discontinuities that would be expected if the solution method moved from one root to another.

The simultaneous equations are solved numerically (see software note below) and we simulate (D13)-(D15) in response to a single unit shock which gives us i) impulse responses for inflation and output, and ii) permits numerical calculation of the coefficients of (16) and (D2) in infinite moving average form which are simply the value of the impulse response in each period following the unit shock. It is then simple to calculate moments of interest such as the unconditional variances required for evaluation of the loss function (13). With iid shocks these are simply the sum of the squares of the infinite moving average coefficients multiplied by the variance of the shock process. The same procedure is adopted for discretion/delegation below so we may calculate relative expected loss without making any assumption about the variance of the shock process.

Discretion and Delegation

From the discussion in Section 2 we assume the MSV forms (14) and (15) plus the equivalent for the Phillips curve constraint multiplier, μ, introduced shortly given by (D16).

$$\mu_t = n_0\varepsilon_t + n_1\varepsilon_{t-1} + n_2 y_{t-1} + n_3\pi_{t-1} \qquad \text{(D16)}$$

Again following the earlier discussion we substitute (14)-(15) and (D16) into the Phillips curve (D1) to give (D17).

$$0 = D_1\pi_t + D_2 y_t + D_3\pi_{t-1} + D_4 y_{t-1} + D_5\varepsilon_t + D_6\varepsilon_{t-1} \qquad \text{(D17)}$$

Where

$$\begin{array}{lll} D_1 = -1 + \alpha_0 l_3 + \delta_0 m_3 & D_2 = \delta + \alpha_0 l_2 + \delta_0 m_2 & D_3 = \alpha_2 + \alpha_1 l_3 + \delta_1 m_3 \\ D_4 = \delta_2 + \alpha_1 l_2 + \delta_1 m_2 & D_5 = \chi_0 + \alpha_0 l_1 + \delta_0 m_1 & D_6 = \chi_1 + \alpha_1 l_1 + \delta_1 m_1 \end{array} \qquad \text{(D18)}$$

From which we form the Lagrangean (D19) which for simplicity is expressed from period 0. This solves for all the delegation regimes combined even though in practice we set at least one of λ_π, λ_y, λ_s and λ_n to zero according to the delegation regime under consideration (see Table 1).

$$L_0 = E_0\Sigma_{t=0}^{\infty}\beta^t\left[\begin{array}{c} \lambda_\pi\pi_t^2 + \lambda_y y_t^2 + \lambda_s(y_t - y_{t-1})^2 + \lambda_n(\pi_t + y_t - y_{t-1})^2 \\ +2\mu_t(D_1\pi_t + D_2 y_t + D_3\pi_{t-1} + D_4 y_{t-1} + D_5\varepsilon_t + D_6\varepsilon_{t-1}) \end{array}\right] \qquad \text{(D19)}$$

This gives first order conditions for inflation and output (D20)-(D21) with expectations taken at time t when the policy decision is made.

$$0 = \lambda_\pi\pi_t + \lambda_n(\pi_t + y_t - y_{t-1}) + D_1\mu_t + \beta D_3 E_t[\mu_{t+1}] \qquad \text{(D20)}$$

$$0 = \begin{array}{c} \lambda_n\pi_t + [(1+\beta)(\lambda_n + \lambda_s) + \lambda_y]y_t - (\lambda_s + \lambda_n)y_{t-1} + D_2\mu_t \\ -\beta(\lambda_s + \lambda_n)E_t[y_{t+1}] - \beta\lambda_n E_t[\pi_{t+1}] + \beta D_4 E_t[\mu_{t+1}] \end{array} \qquad \text{(D21)}$$

Lastly we substitute (14)-(15) and (D16) into (D17) and (D20)-(D21) in a similar way to the commitment case to give (D17)' and (D20)'-(D21)'.

$$0 = \begin{array}{l} \varepsilon_t[D_5 + D_1 l_0 + D_2 m_0] \\ +\varepsilon_{t-1}[D_6 + D_1 l_1 + D_2 m_1] \\ +y_{t-1}[D_4 + D_1 l_2 + D_2 m_2] \\ +\pi_{t-1}[D_3 + D_1 l_3 + D_2 m_3] \end{array} \tag{D17'}$$

$$0 = \begin{array}{l} \varepsilon_t[\beta D_3 n_1 + D_1 n_0 + D_7 l_0 + D_8 m_0] \\ +\varepsilon_{t-1}[D_1 n_1 + D_7 l_1 + D_8 m_1] \\ +y_{t-1}[-\lambda_n + D_1 n_2 + D_7 l_2 + D_8 m_2] \\ +\pi_{t-1}[D_1 n_3 + D_7 l_3 + D_8 m_3] \end{array} \tag{D20'}$$

$$0 = \begin{array}{l} \varepsilon_t[-\beta(\lambda_n + \lambda_s) m_1 - \beta\lambda_n l_1 + \beta D_4 n_1 + D_2 n_0 + D_9 l_0 + D_{10} m_0] \\ +\varepsilon_{t-1}[D_2 n_1 + D_9 l_1 + D_{10} m_1] \\ +y_{t-1}[-(\lambda_n + \lambda_s) + D_2 n_2 + D_9 l_2 + D_{10} m_2] \\ +\pi_{t-1}[D_2 n_3 + D_9 l_3 + D_{10} m_3] \end{array} \tag{D21'}$$

Where D_1-D_6 are given in (D18) and D_7-D_{10} are as follows.

$$D_7 = \lambda_\pi + \lambda_n + \beta D_3 n_3 \qquad D_8 = \lambda_n + \beta D_3 n_2$$

$$D_9 = \lambda_n + \beta D_4 n_3 - \beta(\lambda_n + \lambda_s) m_3 - \beta\lambda_n l_3$$

$$D_{10} = (1+\beta)(\lambda_n + \lambda_s) + \lambda_y + \beta D_4 n_2 - \beta(\lambda_n + \lambda_s) m_2 - \beta\lambda_n l_2$$

Following the same reasoning as under commitment, each of the twelve square bracketed expressions in (D17)' and (D20)'-(D21)' are set to zero and the resulting simultaneous equations are solved numerically for the twelve MSV coefficients in (14)-(15) and (D16). These may then be simulated with a unit shock to generate impulse responses and second moments as under commitment above. Those twelve equations characterise the discretion and delegation solutions for given parameters in the loss function (17). For discretion that comprises the full solution given appropriate choice of the loss function so (17) corresponds to the social loss function (13). For the delegation regimes the twelve equations characterise optimal choices by the delegated policy maker but there is a prior choice of the loss function to be delegated. Hence for these cases the solution amounts to a choice of the parameters in Table 1 for each regime to minimise loss subject to the twelve equations being satisfied.

A Note on Software

The commitment and discretion cases requires the solution of the relevant simultaneous equations. The delegation regimes are an optimisation exercise subject to the constraint of a set of simultaneous equations being satisfied.

We solve for optimal policy under these regimes using the Solver command in a standard Excel spreadsheet. The model is set up on the spreadsheet and the simultaneous equations specified using the relevant Phillips curve and loss function parameters. The Solver command solves for a specific cell to be maximised, minimised or set to a specified value subject to a set of constraints and with a specified set of adjustable cells. For commitment and discretion the adjustable cells are simply the MSV coefficients in the relevant reduced forms. For solving the relevant set of simultaneous equations the command is set up to set one of them to zero subject to the constraint that the others are also zero. For optimal delegation, the command is set up so loss is minimised with the relevant loss function parameter from Table 1 added to the list of adjustable cells and all the simultaneous equations specified as constraints.

The MSV approach used above is less elegant than a state space method, since it requires the material above to be derived rather than simply putting the model into solution code, though it also avoids the work required to change the model equations into state space form. A major advantage of the MSV method, however, is the relative ease of using a standard spreadsheet command to solve for optimal delegation rather than more cumbersome grid search procedures.

For ease of replication and as a demonstration of this method we provide the solution spreadsheet for this paper (in Excel) and a "Readme" file (in PDF) with the discussion paper version of this paper available by following links from www.economics.ox.ac.uk/Research/WorkPapers.asp to the Department of Economics Discussion Paper Series.

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