DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

ON THE MEASUREMENT OF HARMONY IN NORMAL FORM GAMES

Daniel John Zizzo

Number 116

September 2002

http://www.economics.ox.ac.uk

Manor Road Building, Oxford OX1 3UQ

On the Measurement of Harmony in Normal Form Games

Daniel John Zizzo
University of Oxford

September 3, 2002

Abstract

Game harmony is a generic game property that describes how harmonious (non-conflictual) or disharmonious (conflictual) the interests of players are, as embodied in the payoffs. Pure coordination games are games of complete harmony, and constant-sum games are games of pure disharmony: the majority of games is somewhere in the middle. This paper provides measures of game harmony that can be used to classify normal form games, and analyzes their properties. Game harmony is positively associated with cooperation, and we review evidence that this is so. Framing effects increasing cooperation may work by increasing perceived game harmony.

JEL Classification Numbers: C72, H41.

Keywords: game harmony, game classification, framing effects, social dilemmas.

1. Introduction

Ever since Von Neumann and Morgenstern (1944), economists have known and discussed the special problems that are presented by zero-sum games, games where the gain of a player means an equivalent loss by the other player. In recent decades, they have also spent a considerable amount of effort trying to understand the principles behind play in pure coordination games (e.g., Bacharach 1993; Ochs 1995). These two sets of games make unlikely bedfellows: zero-sum games (and, more generally, constant-sum games) model situations of pure conflict, whereas pure coordination games call for unqualified cooperation even on the part of a most purely self-interested and callous *homo oeconomicus.* Indeed, one could make a stronger claim, and argue that two-player zero-sum games and pure coordination games are the opposite of one another. This is because in the first set of games the interests of the players are the opposite of one another, while in the second set of games they are perfectly aligned. For most games, intuitively, things are different, and somewhere in the middle, between these opposite scenarios. For example, in games such as Prisoner's

Dilemmas, Stag-Hunts, trust games or even ultimatum games, there is partial conflict of interest but also partial alignment: the extent to which there is conflict or harmony of interests, as they are embedded in the payoffs, will depend on the particular game structure. While this is intuitive, there has been no generalized attempt in game theory to classify games according to the degree to which the interests of the players are harmonious or conflictual. This paper is a preliminary attempt to try to fill this gap.

We define *game harmony* as a generic game feature that describes how harmonious (non-conflictual) or disharmonious (conflictual) the interests of the n players are, as embodied in the game payoffs. We try to identify a class of real-valued scalar measures of game harmony, capturing in some formal sense some aspect of how "cooperative" we should consider a game to be. We focus on a class of measures that is essentially based on the simple correlation coefficient between payoff pairs. This is not the only way of modelling game harmony, and Appendix A informally presents an alternative based on measures of income distribution and used in Zizzo and Sgroi (2000); still another one might be to follow a partial ordering approach (Epstein and Tanny 1980), although it is not intuitive how one would extend this to more than two players. More generally, correlation-based measures are easier to interpret, as correlation coefficients are definitionally a normalized function of covariance between payoffs, and covariance between payoffs is what game harmony is about. They are also normally easier to handle. Hence, the focus of the main text of this paper is on the correlation-based measures of game harmony. Also, we restrict our attention to games in their normal form presentation.

Game harmony measures are useful classification tools for economists, helping to order games that are neither zero-sum nor pure coordination along a scalar dimension. Does game harmony matter beyond this taxonomic goal? We show what we can label an *equivalence result* of game harmony: an agent who perceives a game as more harmonious is an agent whose behavior can be represented by more positive interdependent preferences and, thus, an agent who behaves more cooperatively in all games where having more positive interdependent preferences is sufficient to achieve this result. We review empirical evidence showing that game harmony can be used as an effective predictor of cooperation in a dataset of Prisoner's Dilemmas (taken from Rapaport and Chammah 1965) and among a wider set of 2×2 games (Zizzo and Tan 2002) and 3×3 games (Zizzo 2002). We also suggest that (one way in which) some framing effects are effective in changing behavior is that they shape the perceived harmony of the game. Psychologists sometimes make claims that, if people are made to feel all "on the same boat" (the so-called "common fate" framing manipulation), they identify with the group and, as a result, they cooperate more in social dilemmas (e.g., Rabbie and Horwitz 1969; Brewer and Miller 1996). We can make this claim more precise by identifying the effect of framing as the effect of inducing perception of greater game harmony and, therefore, by the equivalence result, more cooperative behavior. We argue that the existing evidence on common fate is consistent with our claim, which is based on properties of our game harmony measures that we prove in this paper.

Nevertheless, we are making no claim that game harmony is the only thing that matters in determining cooperativeness. It is straightforward to think of examples where the likelihood of achieving a cooperative outcome is affected by strategic elements that are not captured by game harmony measures (we mention one in section 5). These examples only show that game harmony is not the whole story, not that game harmony may not be part of the story. The rest of this paper is organized as follows. Section 2 describes the most basic measure of game harmony, and section 3 analyzes its properties. Sections 4 and 5 discuss a few extensions and applications, including the equivalence result. Section 6 concludes. Most of the proofs are in Appendix B, while Appendix C contains an additional example.

2. Unweighted Cardinal Game Harmony

Let Γ be a finite n-person game in normal form, and denote N the set of players such that $|N| = n$. Denote W_i the actions available to player i, so $W = W_1 \times W_2 \times ... \times W_n$ is the set of possible outcomes or states of the world, each of which we label by $w \in W$. There is no restriction, in principle, on the number of available actions. Payoffs are defined by $x_i : W \to \mathbb{R}$, a standard Von Neumann-Morgernstern utility function, and so x_{iw} is the payoff for player i ($i \in N$) in state of the world w.

We require that, for at least one player, payoffs do change at least once across states of the world in W. This is, intuitively, a minimum requirement for us to be able to talk meaningfully of a game as a situation where strategic interaction is involved: if every player obtains the same regardless of her actions and of those of the other players, then there is no issue of strategic play.

Condition 1 (Weak Variability) $\neg [\forall\ i \in N\ x_{iw} = k_i\ \forall\ w \in W]$

Now assume that Condition 1 holds but $\exists\ i : x_{iw} = k_i\ \forall\ w$: we define all i for which this property holds as $i \in K$,where obviously $K \subset N$. We shall, for now, assume that K is an empty set, that is Condition 2.

Condition 2 (Strong Variablity) $\forall\ i \in N\ \ \neg [k_i : x_{iw} = k_i\ \forall w \in W]$

Clearly Condition 2 implies Condition 1, but not vice versa. Condition 2 is not fulfilled whenever some player's payoffs are invariant across all states of the world. The scenario that may get closest to this may be one in which there is an indifferent third party whose action affects the outcome but who does not actually care about what the outcome may be (e.g., a bored and indifferent judge). We exclude this scenario for now, but address it later.

In deriving a measure of game harmony, we need to consider the n players who receive a payoff, since we are interested in the covariation between combinations - more specifically, pairs - of payoffs. For n players, the number of payoff pairs is:

$$C = \frac{1}{2}n(n-1) \tag{1}$$

Let us label the payoffs x_{iw} for each payoff pair c as a_{cw}, b_{cw} for $w \in W$. The means and variances of the marginal distributions are μ_a, μ_b and σ_a^2, σ_b^2. σ_{a+b}^2 is the variance of the sum of the payoffs in each pair. We can now state a further condition.

Condition 3 (Second Moments) $\exists\ Q \geq 0 : \sigma_a^2, \sigma_b^2, \sigma_{a+b}^2 < Q\ \forall\ i \in N$

As long as W is a finite set, this condition will always hold.

Definition 1 *The cardinal harmony G of game Γ is the arithmetic mean of the Pearson correlations between the C pairs of Γ:*

$$G(\Gamma) = G(x_i, W) = \frac{1}{C}\sum_{c=1}^{C} r_c(a_{cw}, b_{cw}) = \frac{1}{C}\sum_{c=1}^{C} \frac{Cov(a_{cw}, b_{cw})}{\sigma_a \sigma_b} \tag{2}$$

Let us say that W is *countable* if the game has a discrete action schedule; in this case (2) reduces to:

$$G(\Gamma) = \frac{1}{C}\sum_{c=1}^{C} \frac{|W|\sum_{w=1}^{|W|} a_{cw}b_{cw} - \sum_{w=1}^{|W|} a_{cw} \sum_{w=1}^{|W|} b_{cw}}{\sqrt[2]{\left[|W|\sum_{w=1}^{|W|} a_{cw}^2 - \left(\sum_{w=1}^{|W|} a_{cw}\right)^2\right]\left[|W|\sum_{w=1}^{|W|} b_{cw}^2 - \left(\sum_{w=1}^{|W|} b_{cw}\right)^2\right]}} \tag{3}$$

Let us say that W is *uncountable* if the game has a continuous action schedule; then it may be possible to define a density function $f(W)$ of states of the world on the real line, and $G(\Gamma)$ can be defined as:

$$G(\Gamma) = \frac{1}{C}\sum_{c=1}^{C} \frac{\int_W a_{cw}b_{cw}f(W)dW - \int_W a_{cw}f(W)dW \int_W b_{cw}f(W)dW}{\sqrt[2]{\left[\int_W a_{cw}^2 f(W)dW - \left(\int_W a_{cw}f(W)dW\right)^2\right]\left[\int_W b_{cw}^2 f(W)dW - \left(\int_W b_{cw}f(W)dW\right)^2\right]}} \tag{4}$$

Equations (3) and (4) are examples of ways of computing $G(\Gamma)$, depending on the nature of the game. In order for $G(\Gamma)$ to be defined, all we need is for Conditions 1 and 3 to hold (this will be relaxed later).

Proposition 1 $G(\Gamma)$ *is defined iff Conditions 1 and 3 hold.*

Proof. Condition 3 directly implies that the denominator of (2) is bounded, i.e. $\sigma_a\sigma_b \leq Q'$ for some finite constant Q'. Also, by Condition 2, $\sigma_a\sigma_b > 0$. Finally, we need to show that the numerator is also bounded: this follows by noting that $\sigma_{a+b}^2 = \sigma_a^2 + \sigma_b^2 + 2Cov(a_{cw}, b_{cw})$, so $Cov(a_{cw}, b_{cw})$ is a linear function of bounded variables (by Condition 3 again), implying that it is itself bounded. ■

Condition 3 always holds if $G(\Gamma)$ is countable, and needs to be postulated otherwise.

So far we analyzed the conditions for existence of $G(\Gamma)$, but the fact that $G(\Gamma)$ exists does not make it necessarily a good measure of game harmony. Denote $G^*(\Gamma)$ the "true" value of game harmony for game Γ. Ideally we would want $G(\Gamma) = f[G^*(\Gamma)]$ with f a positive strictly monotonic real function. One important way in which this may not hold is in the *aggregation* from payoff pairs (a_{cw}, b_{cw}) to $r_c(a_{cw}, b_{cw})$, and that from $r_c(a_{cw}, b_{cw})$ to $G(\Gamma)$: we are both assuming (a) that each payoff pair weights equally in the determination of $r_c(a_{cw}, b_{cw})$, and (b) that each $r_c(a_{cw}, b_{cw})$ weights equally in the determination of of $G(\Gamma)$. Obviously, neither of these assumptions is innocuous. For example, certain payoff cells may correspond to actions that are strictly dominated according to each player, and it is then unclear that they should have the same weight, when paired, as payoff pairs between undominated outcomes. Also, in games where $n > 2$, it is unclear that, from the perspective of the individual self-interested decision-maker, correlations between payoff pairs of other players should count as much as correlations involving one's own payoffs.

These assumptions will be discussed, and relaxed, later. For the time being, however, we shall add the following condition to rule out these aggregation problems, with $\alpha, \beta, \eta, \theta > 0$ arbitrary finite constants.

Condition 4 (Equal Weight) $\partial G^*(\Gamma)/\partial r_c(a_{cw}, b_{cw}) = \eta$ *for* $c = 1, ..., C$ & $\exists(a_{cw} = \alpha, b_{cw} = \beta)$: $\{\partial r_c(a_{cw}, b_{cw})/\partial w = \theta$ *for all* α, β *and for* $c = 1, ..., C\}$

One limitation of the equal weight condition is that it makes game harmony values sensitive to irrelevant duplication of strategies. Whenever this is considered a problem, it can be eliminated by transforming the normal form game to its *semi-reduced form* where strategically equivalent strategies get replaced by a single representative strategy (Ritzberger 2002).

Since we are assuming that the equal weight condition holds, we are giving the same weight to every state of the world, and so $f(W) = (\max|W| - \min|W|)^{-1}$ for the uncountable case of equation (4), as it follows a uniform distribution. $\max|W|$ is defined as $\max(a_{cw}b_{cw})$ in W for the integral term involving $a_{cw}b_{cw}$ in equation (4), and similarly $\max|W|$ is defined as $\min(a_{cw}b_{cw})$.

Since $G(\Gamma)$ is a positive function of the correlation among payoffs, higher $G(\Gamma)$ values imply higher correlation among payoffs and, therefore, higher game harmony. We can now ask ourselves what is the admissible set of $G(\Gamma)$ values. From our informal discussion, we postulate that pure coordination games are games of perfect game harmony and so with the highest values of game harmony, and zero-sum (or, more generally, constant-sum) games are games of perfect game disharmony and so with the lowest values of game harmony. Hence, finding values for these two classes of games will provide information on the Borel set B_G of admissible values of game harmony, and on how B_G relates to B, the Borel set defined over $[-1, 1]$.

Definition 2 *Let a pure coordination game be defined by* $\Gamma_d : a_{cw} = kb_{cw} + h$ *for all* c, w, *and a constant-sum game be defined by* $\Gamma_0 : \sum_{i=1}^{n} x_{iw} = q$ *for all* w, *with* k, h *and* q *finite constants and* $k > 0$.

Proposition 2 *In pure coordination games* $G(\Gamma_d) = 1$.

Proof. See Appendix B. ■

We turn to constant-sum games, of which zero-sum games are the most well-known subclass.

Proposition 3 *In constant-sum games and with* $n = 2$, $G(\Gamma_0) = -1$; *with* $n > 2$, $G(\Gamma) \in (-1, 1)$.

Proof. See Appendix B. ■

The interpretation of $G(\Gamma)$ is transparent with $n = 2$, with zero-sum games as perfectly disharmonious at - 1 and pure coordination games as perfectly harmonious at + 1. While the upper bound to $G(\Gamma)$ carries through for $n > 2$, this is not the case for the lower bound: zero-sum games will have $G(\Gamma) > -1$ for $n > 2$. This can be stated more formally in the following corollary.

Corollary 1 $B_G = [-1, 1]$ *iff* $n = 2$, *and* $B_G = [\lambda, 1]$ *iff* $n > 2$, *for some constant* $\lambda \in (-1, 1)$.

The intuition for this is that whenever there are three or more objects but a single dimension and so only two directions, it is not possible for each object to move in a different direction at the same time. This implies that a winner-take-all context is perceived as more harmonious with many rather than two players involved in the competition, because of the prospective harmony of interests among the losers. An objection to this may be that each player might perceive the game as one of himself vs. "all the others": this would call for a different way of conceiving game harmony, which will be explored later.

Since $\lambda < 1$, $G(\Gamma)$ anyway retains sensitivity to the usage of different games, since B_G never collapses to a single point. It would nevertheless be helpful to determine $\lambda_{\max}$, the upper bound to the lower bound of game harmony, and how this changes with n. We conjecture that $\lambda_{\max}$ can be found by looking at the constant-sum game Γ_{0d} which is identical to a Γ_d for all players but one.

Proposition 4 *If* $\Gamma_{0d} : G(\Gamma_{0d}) = \lambda_{\max}$, *then* $\lambda_{\max} < 1$, *and* $\triangle\lambda_{\max}/\triangle n > 0$.

Proof. See Appendix B. ■

3. Properties of Game Harmony

In this section we characterize some basic properties of $G(\Gamma)$. The first property mirrors Axiom 2 of Luce and Raiffa's (1957, p. 287) "desiderata" for decision-making rules.

Proposition 5 (Scale Independence) $G(\Gamma) = G(x_i, W) = G(k_i x_i + h_i, W)$ *for any finite* $k_i > 0$ *and* h_i.

Proof. See Appendix B ■

Now consider what happens if a game Ψ is formed in terms of $k \times W$ states of the world W of the parent game Γ $\forall$ $k \in \mathbb{N}^{++}$, for example if the Battle of the Sexes game matrix Γ

2,1	0,0
0,0	1,2

is duplicated to give Ψ

2,1	0,0	2,1	0,0
0,0	1,2	0,0	1,2

This is a form of relabeling and, as such, it should not matter: see Luce and Raiffa's (1957) Axiom 11, p. 295. A different and perhaps more interesting example of this is when identical firms compete in duplicate identical markets: when this is the case, one can show that indeed the optimal behavior of firms (whether to collude or not) is not affected by the duplication (Bernheim and Whinston 1990). Since the semi-reduced normal form of Ψ is the same as that of Γ, finding the game harmony of the semi-reduced game would be sufficient to make the relabeling irrelevant. It turns out, however, that working on the semi-reduced form is not needed to get this invariance result and that, further, the invariance result can be generalized by allowing rescaling within each replica.

Proposition 6 (Replication) *Denote the operator* $\psi : \Gamma(x_i, W) \to \Psi = \gamma\Phi = \Gamma[x_i, k_i(\Phi), h_i(\Phi), \gamma \times W] \forall \gamma \in \mathbb{N}^{++}$ *as the transformation of a parent game* Γ *into a game* Ψ *that is the aggregation of* γ*-replicas of* Γ*, where each replica* Φ *has payoffs* $k_i(\Phi)x_i + h_i(\Phi)$ *for any finite* $k_i > 0$ *and* h_i*. Then* $G(\Psi) = G(\Gamma) \ \forall \ \Psi$.

Proof. See Appendix B. ∎

The scale independence and replication properties are neutrality properties; the next property, instead, is one of non-neutrality to a given transformation of Γ (except if the game is a pure coordination game). Denote $X_i = \{x_{i\pi} : \pi \in W\}$ the set of all payoff values for player i in W.

Proposition 7 (Outcome Addition) *Denote the operator* $\vartheta : \Gamma(x_i, W) \to \Theta(x_i, W \cup \{\omega\})$ *where* ω *is an additional outcome with* $x_{i\pi} \neq x_{i\omega} \ \forall \ \pi \in W$ & $[(x_{i\omega} = \sup X_i) \vee \ (x_{i\omega} = \inf X_i)] \ \forall \ i$*. Then* $G(\Theta) > G(\Gamma)$ *unless* $\Gamma = \Gamma_d$*, in which case* $G(\Theta) = G(\Gamma) = 1$.

Proof. See Appendix B. ∎

If either a Paradise or a Doomsday scenario is introduced for all players, its effect is that of increasing game harmony in any game which has some element of conflict. This would be akin to a "common fate" treatment of the kind employed by social psychologists to induce group identity and cooperation in social dilemmas (see section 5). Another example of this, in relation to bargaining games, is presented in Appendix C.

The final property to be discussed in this section concerns the marginal effect of the addition of a player rather than of a state of the world to a game.

Proposition 8 (Player Addition) *Let there be* $(n-1)$ *players and let a single player be added to the game; denote by* $\mu_r = [1/(n-1)] \sum_{c=1}^{n-1} r_c^x(a_{cw}, b_{cw})$ *the mean of the* $(n-1)$ *correlation values* $r_c^x(a_{cw}, b_{cw})$ *between the payoffs of the nth player and those of the remaining* $(n-1)$ *human players. Then the change in game harmony* $\triangle G = G(\Gamma|n) - G[\Gamma|(n-1)]$ *is equal to* $(2/n)\{\mu_r - G[\Gamma|(n-1)]\}$.

Proof. See Appendix B. ∎

Three remarks can be made on this property. First, it is computationally convenient. Second, it implies $G(\Gamma|n) \lesseqgtr G[\Gamma|(n-1)] \leftrightarrow \mu_r \lesseqgtr G[\Gamma|(n-1)$, an intuitive result: game harmony increases, stays the same or decreases depending on whether the average correlation in relation to the new payoff pairs is higher, the same or lower than the original value of harmony, respectively. Third, it implies that the marginal effect of an extra player on game harmony is decreasing in the number of players (tending to 0 as $n \to \infty$), again an intuitive result.

4. Extensions

We now consider a few extensions giving rise to complementary or more general measures of game harmony. For every payoff value, we could replace the value by its rank among the payoff values for player i in W. If we do this, we get an ordinal measure of game harmony, $G_\rho(\Gamma)$.

Definition 3 *Let X_i be the set of all payoff values for player i in W, and let $x^\rho_{iw} = rank(x_{iw}|X_i)$, which can be mapped into rank payoff pairs a^ρ_{cw}, b^ρ_{cw}. Then:*

$$G_\rho(\Gamma) = G_\rho(x_i, W) = \frac{1}{C}\sum_{c=1}^{C} r_c(a^\rho_{cw}, b^\rho_{cw}) \tag{5}$$

The scale independence, replication and player addition properties of the previous section still hold. Cardinal game harmony can be quite sensitive to the parameter values adopted, more so perhaps than psychologically or normatively plausible. Ordinal game harmony remedies this by associating $G_\rho(\Gamma)$ to a particular ordinal preference ordering among outcomes, and among that only: to the extent that economists tend to refer to a "game" as being defined purely in terms of preference orderings (e.g., that is what differs between a Prisoner's Dilemma and a Stag-Hunt), ordinal game harmony is able to associate a single game harmony value to each game. In this sense, while one cannot adjudicate *a priori* whether $G(\Gamma)$ or $G_\rho(\Gamma)$ is better, there are some obvious advantages in using $G_\rho(\Gamma)$ for making "between-games" comparisons.

A second possible extension is to relax Condition 2 by replacing it with Condition 1:

Definition 4 *Let any correlation $r^k_c(a_{cw}, b_{cw})$ involving $x_{iw} = k_i$ for some perfectly impartial player i be set equal 0, and to the Pearson correlation $r_c(a_{cw}, b_{cw})$ otherwise. Then the cardinal harmony G_k of game Γ is the arithmetic mean of the $r^k_c(a_{cw}, b_{cw})$ correlations between the C pairs of Γ:*

$$G_k(\Gamma) = G_k(x_i, W) = \frac{1}{C}\sum_{c=1}^{C} r^k_c(a_{cw}, b_{cw}) \tag{6}$$

Now only Conditions 1 and 2 are required for existence. $G_k(\Gamma)$ has the same properties as $G(\Gamma)$, as analyzed in the previous section. An interesting application of the $r_c^k(a_{cw}, b_{cw})$ measure can be made when combined with the player addition property.

Proposition 9 *Let there be $(n-1)$ non perfectly impartial players and let a single player be added to the game, and let this player be perfectly impartial according to equation (6). Then $\triangle G_k \lesseqgtr 0 \leftrightarrow G_k[\Gamma|(n-1)] \gtreqless 0$, i.e. any non-zero game harmony decreases in absolute value.*

Proof. By direct application of the player addition property. We only need to note that here $\mu_r = 0$, so $\triangle G = (2/n)\{\mu_r - G[\Gamma|(n-1)]\} = -(2/n)G[\Gamma|(n-1)]\}$ ∎

An application of this proposition is to litigation settings which are highly conflictual (and so where $G(\Gamma) \ll 0$): the presence of an impartial third party, if it is perceived as such, can by itself help in raising the perceived harmony of the game. This result appears an interesting one when a relatively small number of perfectly impartial parties is added to the game. What happens, however, if a large number of perfectly impartial, indifferent players is added to the game?

Proposition 10 *Let there be $(a-1)$ non perfectly impartial players and let there be $n-(a-1)$ perfectly impartial players. Then $\lim_{n\to\infty} G_k(\Gamma) = 0$.*

Proof. See Appendix B. ∎

If almost everyone is indifferent, then the harmony of the game is close to 0, no matter how harmonious or disharmonious the game is between the players who care. If this result seems impalatable, it is because we usually think of a game as involving mainly, or exclusively, parties who have something at stake in the game outcomes: indifferent players are simply not taken to be part of the game. If an economist is interested in the bargaining process between two agents, and if the stakes involved are small enough that general equilibrium effects depending on the bargaining outcome are trivial, then the economist can ignore all but the two agents involved in the bargaining process. There may nevertheless be situations where it is psychologically plausible that the bargaining agents perceive third parties as part of the game, such as when an arbiter is involved. How players perceive the game will then determine whether the game has one, two or a large set of indifferent players or not. Still, even when perfectly impartial agents are plausibly cast as part of the game, we may not want to give them as much weight to the payoff pairs involving them as we do in relation to those in relation to the other pairs: since our measure of game harmony so far has been an equally weighted average of correlations between payoff pairs, this cannot be achieved without extending further our measure.

We now relax the equal weight condition relative to our original measure $G(\Gamma)$: it should be obvious however that the same extension could be made to $G_k(\Gamma)$, in order to meet the point raised in the previous paragraph. Relaxing the equal weight condition produces a class of measures of game harmony, $G_t(\Gamma)$, of which $G(\Gamma)$ is a special case where the equal weight condition holds (specifically, where the weights $\eta_w = 1$ $\forall\ w \in W$ and $\theta_c = 1\ \forall\ c = \overline{1,C}$).

Definition 5 *Let* $\eta_w, \theta_c \in \mathbb{R}^+ : \left\{ \sum_{w=1}^{|W|} \eta_w = W \ \&\ \sum_{c=1}^{|C|} \theta_c = C \right\}$ *and let* $W\natural \subset W\ \&\ C\natural \subset C$ *be the set of outcomes s.t.* $\eta_w > 0\ \forall w \in W\natural\ \&\ \theta_c > 0\ \forall c \in C\natural$*. Then:*

$$G_t(\Gamma) = G(x_i, \eta_w, \theta_c, W) = \frac{1}{|C\natural|}\sum_{c=1}^{|C\natural|} \theta_c r_c(\eta_w a_{cw}|W\natural, \eta_w b_{cw}|W\natural) = \frac{1}{|C\natural|}\sum_{c=1}^{|C\natural|} \frac{\theta_c Cov(\eta_w a_{cw}|W\natural, \eta_w b_{cw}|W\natural)}{\sigma_a(\eta_w|W\natural) \times \sigma_b(\eta_w|W\natural)} \tag{7}$$

$G_t(\Gamma)$ is the arithmetic average of the Pearson correlations computed over the set of strictly positive η_w and θ_c-weighted outcomes. The scale independence and replication properties still hold with $G_t(\Gamma)$. The zero payoff property carries through only if a non-zero weight is placed on the extra outcome ω, e.g. iff $\eta_\omega > 0$. The player addition property does not hold in general, although it does hold if there are unequal weights for payoff cells but not for Pearson correlations, i.e. if $\theta_c = 1\ \forall\ c = \overline{1,C}$.

This is a very general class of measures, and one can easily think of subclasses of $G_t(\Gamma)$ relaxing the equal weight condition only under one dimension, and so where either $\eta_w = 1\ \forall\ w \in W$ or $\theta_c = 1\ \forall\ c \in C$ hold. There are at least two ways in which one can interpret the outcome weights η_w. The first one is to treat η_w as reflecting the subjective probability assigned by a player to the likelihood of ending up in a particular outcome: more specifically, $\eta_w = \Pr(w|W)$. A scenario where every outcome is equiprobable ($\Pr(w) = 1/|W|$) is then one where an equal weight of 1 is assigned across outcomes, i.e. $\eta_w = 1$, as for the equal weight condition. The higher the subjective probability of an outcome w, the higher would be the weight assigned by a player to w. If the player believes impossible that a certain w is achieved, then that is entirely neglected. The way the probability profile is determined will depend on the algorithm ℓ employed by each player to choose an action, and by their beliefs on what algorithms are used by the other players. Hence, this probabilistic interpretation of $G_t(\Gamma)$ makes game harmony ℓ-dependent, i.e. dependent on such algorithms: for example, Nash or rationalizability. This has some obvious normative appeal: it implies that, if all agents use Nash, the addition of a strictly dominated strategy will not change the ℓ-probability weighted game harmony measure $G_{t|\ell}(\Gamma)$, a potentially desirable result that mirrors Axiom 6 of Luce and Raiffa (1957, p. 288). Nevertheless, it also leads to a paradox.

Proposition 11 (Uniqueness Paradox) *If* $\exists\ !\ \ell$*-solution to* Γ, $\nexists G_{t|\ell}(\Gamma)$.

Proof. If there exists a unique ℓ-solution to Γ, the agent will assign probability 1 to that solution and 0 to all other outcomes, implying that the correlation measure $r_c(\eta_w a_{cw}|W\natural, \eta_w b_{cw}|W\natural)$ should be computed on the basis of only one observation (the payoff pair in the ℓ-solution), which is impossible. ■

The uniqueness paradox implies two unpalatable results: first, ℓ-dependent game harmony cannot be constructed for the usually important class of games with a unique solution (such as the Prisoner's Dilemma according to Nash, L1, L2 or rationalizability); second, insofar as the success of an algorithm is measured in terms of its ability to spot a unique solution, the class of games for which a ℓ-dependent game harmony measure can be constructed is an *inverse* function of the success of the ℓ on the basis of which it is built. The paradox suggests that, while ℓ-dependent game harmony might provide valuable information in games with multiple solutions, it can hardly be used as the main measure to classify harmony of games.

The second interpretation of the outcome weights η_w is one based on the salience of particular outcomes, for example as determined by variable frame theory (Bacharach 1993). The relative salience of certain outcomes relative others may be due, say, to the graphical presentation of the game where particular objects get highlighted and others are not (Bacharach and Bernasconi 1997; Bacharach 1993). It may also be due to the way the instructions are formulated, for example if a comprehension task is phrased in terms of benefits to the group (the "we" frame) or to the individual (the "I" frame) in a social dilemma environment (Cookson 2000): the effect of this may be to highlight the cooperative outcome relative to the defection outcome, and hence to increase the $G_t(\Gamma)$ of Γ. If one were to believe that greater perceived game harmony is associated to more cooperative behavior, then the variation of $G_t(\Gamma)$ may explain the effectiveness of some framing effects in altering behavior. The uniqueness paradox can clearly be extended to cases where agents assign a saliency of 1 to a unique outcome and 0 to all other outcomes.

If one employs this measure of game harmony, whether in its probabilistic or salience interpretation, unless probabilities or saliencies are identical across subjects, one would not be able to associate a single value of game harmony to a game Γ, as this would change according to the subject. Of course, if one accepts a Bayesian equilibrium framework where agents have the same probability profile, the problem disappears in relation to probability weights; equally, it disappears if we believe that people have the same saliency profile, and the success of coordination when focal points are available (Bacharach and Bernasconi, 1997) suggests that, at least sometimes, this is realistic to assume.

A different way in which a measure of game harmony can be made subjective is by assigning different weights θ_c to payoff pairs involving oneself (i.e., player i, for $i \in N$) and payoff pairs involving other subjects. In the limit, one could care only about her individual payoffs a^i_{cw} relative to those of others b_{cw}.

Definition 6 *Let* $\theta_c = 1$ *iff* $c = (a^s_{cw}, b_{cw})$*; let* $\theta_c = 0$ *iff* $\neg c = (a^s_{cw}, b_{cw})$*. Then:*

$$G_{t|i}(\Gamma, i) = \frac{1}{n-1} \sum_{c=1}^{n-1} r_c(\eta_w a^i_{cw} | W\natural, \eta_w b_{cw} | W\natural) \tag{8}$$

There are n values of $G_{t|i}(\Gamma, i)$ for any given Γ, one for each player i (they will all be the same with perfectly symmetric games).

We next consider a different kind of aggregation relative to $G(\Gamma)$. We assume that players are bounded-rational in the following respect: when n is large, they lump together groups of agents and estimate game harmony purely on the basis of inter-group comparisons (e.g., "Americans" vs. "Al-Qaeda terrorists"). This technique can also be useful to the economist who may want to estimate the game harmony of games with a large number of players, but only a limited number of roles, and is interested in how payoffs covary across types (e.g., insurance companies and prospective customers in a standard screening model).

Definition 7 *Let there be* $n_G \in [2, n]$ *non-empty groups of players and let* $x^G_{iw} = E(x_i|G)$ *be the average payoff in each group* G *for each outcome* w*. Let* $r_c(a^G_{cw}, b^G_{cw})$ *be the Pearson correlation between each pair of* x^G_{iw} *payoffs. Denote* $C_G = (1/2)n_G(n_G - 1)$ *as the number of* $r_c(a^G_{cw}, b^G_{cw})$ *combinations. Then:*

$$G_G(\Gamma, n_G) = \frac{1}{C_G} \sum_{c=1}^{C_G} r_c(a^G_{cw}, b^G_{cw}) \tag{9}$$

There can be singleton groups: indeed $G(\Gamma)$ is the special case of $G(\Gamma, n_G)$ where every group is a singleton. A special case would be with oneself as a singleton group and everyone else as a single different group. This measure, and any other where $n_G = 2$, have game harmony values of -1 for any constant-sum game, no matter the number of players.

Proposition 12 $G_G(\Gamma_0, 2) = -1 \; \forall \; n$.

Proposition 13 $G_G(\Gamma, 2) : \{B_F | G_G(\Gamma, 2) = [-1, 1] \forall n\}$.

Proof. See Appendix B. ■

5. Game Harmony and Cooperative Behavior

This section discusses some theoretical and empirical results on the relationship between game harmony and cooperation. The harmony of game Γ might matter for cooperation in two ways: it may affect the cooperation of Γ and, by changing the perception of harmony of later games, it may affect the cooperation of them as well. An *equivalence result* may underlie the first route, while an additional hypothesis on how framing effects operate is required to justify the second route.

THE EQUIVALENCE RESULT. Consider the relationship between game harmony and interdependent preferences. Take a player i whose preferences are dependent not only on one's own material payoff $u(x_i)$ but also on some β-weighted function v of the payoffs of the other players j, with $\beta_{iw} \in [-1, 1]$ and $dv/dx_j > 0$. We have pure altruism if $\beta_{iw} = \beta_i > 0$, pure envy if $\beta_{iw} = \beta_i < 0$, and more complex transformations if β_{wi} is positive or negative depending on the outcome (for example, because of inequality aversion). We are back to self-interest if $\beta_{iw} = 0 \ \forall i, w$. We consider the case of a simple payoff transformation.

Definition 8 *A player has interdependent preferences if*

$$V_{iw} = x_{iw} + \sum_{j \neq i}^{n} \beta_{iw} v(x_{jw}) \tag{10}$$

where $v(x_{jw})$ is a positive monotonic transformation of x_{jw}. Denote Γ^u the untransformed payoff matrix and Γ^v the payoff matrix transformed according to V.

Proposition 14 (Equivalence Result) *Assume that the material payoffs a_{cw} and b_{cw} are given. Then $G(\Gamma^v) \gtreqless G(\Gamma^u) \leftrightarrow \beta_i \gtreqless 0 \ \forall \ i$, and $\partial \beta_i / \partial G(\Gamma^v) > 0$.*

Proof. Since $G(\Gamma^v)$ is

$$G(\Gamma^v) = \frac{1}{C} \sum_{c=1}^{C} r_c[a_{cw} + \beta_a v(b_{cw}), b_{cw} + \beta_b v(a_{cw})] \tag{11}$$

and $dv/da_{cw} > 0$, $dv/db_{cw} > 0$, then $\partial G(\Gamma^v)/\partial \beta_i > 0$. Also, it is legitimate to apply the implicit function theorem by rewriting (11) as $G(\Gamma^v) - (1/C) \sum_{c=1}^{C} r_c[a_{cw} + \beta_a v(b_{cw}), b_{cw} + \beta_b v(a_{cw})] = 0$ and noting that $\partial \beta_i / \partial r_c > 0$ is well-defined, monotonic and continuous, so $\partial \beta_i / \partial G(\Gamma^v) > 0$. $G(\Gamma^v) \gtreqless G(\Gamma^u) \leftrightarrow \beta_i \gtreqless 0 \ \forall \ i$ follows. ■

The equivalence result stresses the correspondence between simple payoff transformation and perception of the game as more or less harmonious. It implies that, if for whatever reason (e.g., framing) the agent perceives the game as more harmonious, she will behave as if she had more positive, or less envious, preferences. One might conjecture that this willl lead to more cooperative behavior in games where simple payoff transformation will do, such as in relation to games that are Prisoner's Dilemmas in their material payoffs. This is indeed the case.

Definition 9 *Define a cooperative outcome payoff for agent* i *as the utilitarian outcome conditional on no one else being damaged, i.e.* $V_{i,coop} : \left\{ \sum_{x=1}^{n} V_{i,coop} = \max_{w \in W} \left(\sum_{x=1}^{n} V_{iw} \right) | x_{j,coop} \geq x_{jw} \forall j \neq i \right\}$. *Define the strategy profile* W^w *producing a given payoff outcome* $x_{iw}(W_w)$ *for player* i *as* $W^w : \{W_1, W_2, ..., W_n\} \rightarrow x_{iw}(W_w)$. *Denote* W^{coop} *as the strategy profile resulting in the occurrence of* $v(x_{jw}^{coop})$, *i.e.* $W^{coop} : \{W_1, W_2, ..., W_n\} \rightarrow v(x_{jw}^{coop})$. *Define* $W_i^{\widetilde{coop}}$ *as some strategy profile if player* i *deviates to action* D_i, *so* $W_i^{\widetilde{coop}} : \{W_1, W_2, ..., D_i, ..., W_n\} \rightarrow v(x_{iw}^{\widetilde{coop}})$.

Definition 10 *Define* $W_i^{\widetilde{w}}$ *as some strategy profile if player* i *deviates to action* D_i, *so* $W_i^{\widetilde{w}} : \{W_1, W_2, ..., D_i, ..., W_n\} \rightarrow w_i(W_i^{\widetilde{w}})$. *Then* W^w *is a Nash equilibrium in pure strategies* W^* *iff* $V_{iw}(W_i^*) \geq V_{iw}(W_i^{\widetilde{w}}) \ \forall \ i$.

Proposition 15 *Assume that* $\exists W^w, W^*$ *and that agents have interdependent preferences as specified by definition (10). Then* $\Delta Prob(W^{coop} = W^*)/\Delta\beta_i \geqslant 0$. *Also, if we define* $\Delta_\beta G(\Gamma^v)$ *as a change in game harmony occurring exclusively through a change in* β_i, $\Delta Prob(W^{coop} = W^*)/\Delta_\beta G(\Gamma^v) \geqslant 0$.

Proof. See Appendix B. ■

Corollary 2 *Let* $V_{i,par}$ *be a unique Pareto dominant Nash equilibrium corresponding to the strategy profile* W^{par}. *Then, conditionally on the existence of* $V_{i,par}$, $\Delta Prob(W^{par} = W^*)/\Delta G(\Gamma^v) \geqslant 0$.

Proof. Trivial by noting that, if defined, $W^{par} = W^{coop}$ by the definition of $V_{i,par}$, hence one can apply Proposition 15. ■

The greater the game harmony, the greater the corresponding β_i is, and the higher (or at least as high) the likelihood that the cooperative outcome will correspond to a Nash equilibrium. When a unique Pareto dominant Nash equilibrium exists, the likelihood of Nash players going for it will increase in the harmony of the game. Higher perceived game harmony can be mapped into higher β_i values and a tendentially greater likelihood of play of the cooperative outcome by Nash players. A qualification is required because simple payoff transformation is not sufficient to achieve more cooperative behavior in some cases: this may be because agents are not Nash players after all; it may also be because there may be a problem of coordination when there is more than one Nash equilibrium (e.g., Sugden 1993). But it is also quite possible that game harmony is associated to cooperation by other routes as well, not requiring Nash playing.

Framing effects And Common Fate. As they stand, these results show a connection between $G(\Gamma)$ or $G_\rho(\Gamma)$ and cooperation in game Γ. This psychological interpretation of game harmony can help explaining why, for example, having a perfectly impartial third party can be helpful in reducing conflictuality in bargaining and contextual contexts.

A stronger hypothesis is that, if subjects play Γ or a "cooperative frame" is otherwise elicited, this may change the perception of harmony of later games that are played and thus, by the equivalence result, change cooperation in later games that are played. This is a empirical hypothesis on how framing effects operate in relation to cooperation in games: a hypothesis based on the effect that the frame inducement has on later game perception.

This hypothesis can explain why, if play in a social dilemma is preceded by a comprehension task phrased in terms of benefits to the group (the "we" frame) rather than to the individual (the "I" frame), more cooperation follows (Cookson 2000). Similarly, if subjects first read some news briefs on cooperatives business strategies (emphasizing teamwork and group achievement) and have to answer a few questions on this topic, they contribute more in the follow-up play of a public good contribution game (Elliott et al. 1998). We now discuss a specific application of this hypothesis to explain some puzzling results in relation to a specific framing effect.

People cooperate more within a group if they identify with the group (e.g., Brewer and Miller 1996). A powerful determinant of why people feel to be a group is if they feel that "they are all in the same boat": Rabbie and Horwitz (1969) and Rabbie et al. (1989) have claimed that this feeling of positive interdependence would make subjects more cooperative to one another exactly because of the feeling that their outcomes are related to one another. In one of the conditions of Rabbie and Horwitz (1969), subjects were arbitrarily divided in Greens and Blues; the experimenter announced that a radio would be given to each Green member *or* to each Blue member depending on the outcome of a roll of a die. This was sufficient to induce an ingroup bias as measured by some questionnaire ratings. Wit and Wilke (1992) found that a common fate manipulation increased cooperation in later game play. They used three social dilemma variations, based on the PD, the Chicken, and the Stag-Hunt game. They induced a common fate manipulation by assigning endowments by random draws that were identical within each group (in the common fate condition) or entirely independent between-subjects (in the control condition). However Bouas and Komorita (1996) found that common fate played no role in either triggering feelings of group identity or in increasing cooperation in a public goods contribution game. Subjects earned experimental points, that were convertible in lottery tickets at the end of the experiment. In the common fate condition, they told subjects that a single random draw would be used to determine whether the lottery prizes were large (U.S. \$ 50) or small (U.S. \$ 5).

With Propositions 14 and 15, game harmony provides one way of formalizing the idea that being on the same boat matters, as this corresponds to a greater perceived harmony of the social dilemma. The Rabbie and Horwitz (1969) and Wit and Wilke (1992) designs change the perception of subjects in the later task by having a game against nature (or the experimenter) at the start, a game with perfect game harmony among players. The outcome addition property predicts, and the 2×2 games example illustrates, that the introduction of a chance Doomsday scenario in which everyone loses their earnings would also be effective in raising game harmony. Conversely, the Bouas and Komorita manipulation is not effective, nor would we predict it to be. The random draw can be interpreted as a move by Nature and the common fate treatment only works as a ψ manipulation transforming the parent game Γ into a game Ψ that is the aggregation of two replicas of Γ, where the first replica is identical to the original game without the manipulation (which always had lottery tickets of \$ 50) and the second replica is identical to the original game, but with all the expected gains divided by a factor of 10. By the replication property, we know that ψ does not change game harmony and, thus, does not induce group identity and positively interdependent preferences.

This evidence is indirect for two reasons. First, it is drawn from different sets of experiments, and so it is not conclusive without a specific experimental test. Second, in the successful instances of induced cooperation, $G(\Gamma)$ is hypothesized to operate through a framing effect on some different and later game, rather than directly on the cooperation of Γ. The next two pieces of evidence are more direct on both counts.

Prisoner's Dilemmas. Rapaport and Chammah (1965, ch. 1) elaborated two cooperation indices to predict cooperation in 2×2 Prisoner's Dilemmas (PD). Consider the following generic PD:

PD	
R, R	S, T
T, S	P, P

where $S < P < R < T$ and $2R > S + T$. Rapaport and Chammah defined two indices, $RA_1 = (R - P)/(T - S)$ and $RA_2 = (R - S)/(T - S)$, to summarize the extent of cooperation we would expect in different parametrizations of the PD. They described three experimental conditions where subjects played a variety of fully displayed PDs in different combinations, the cleanest test of a correlation between their indices and mean cooperation rates by game. Direct estimation from their data shows a Pearson (Spearman) correlation of 0.948 (0.954) between mean cooperation rate and RA_1 ($P < 0.01$), and of 0.624 (0.767) between mean cooperation rate and RA_2 ($P < 0.05$ only for the Spearman correlation). But while RA_1 performs well, its usefulness has been limited: it is a very *ad hoc* measure, and one that cannot be generalized to other games, or indeed even to PD-like games of different dimensionality.

We cannot apply $G_\rho(\Gamma)$ to this problem since it has always the same value (-0.8) for different parametrizations of the PD, because of its ordinal nature. We can apply $G(\Gamma)$, though. The Pearson (Spearman) correlation between mean cooperation rate and $G(\Gamma)$ is 0.895 (0.982), with $P < 0.01$, slightly worse for the Pearson measure but almost perfect for the Spearman measure. Revealingly, the Pearson (Spearman) correlation of $G(\Gamma)$ with RA_1 is 0.975 (0.972), whereas it is quite lower, 0.594 (0.657), with RA_2.

While more general and less *ad hoc*, our simplest game harmony measure performs virtually as well as Rapaport and Chammah's best cooperation index in 2×2 PDs. Indeed, Rapaport and Chammah's RA_1 measure may have been largely successful because it works as a proxy for a more general measure of game harmony in 2×2 PDs.

2x2 And 3x3 Games. Zizzo and Tan (2002) present the results of an experiment where, after some practice, subjects played payoff-perturbed versions of well-known 2×2 games such as the PD, the Stag-Hunt, the Chicken, a coordination game and three variants of trust games. They played both as row players and as column players. Tables 1 and 2 contain the game matrices and the $G(\Gamma)$, $G_\rho(\Gamma)$ and mean cooperation rates by game (where defined): see Zizzo and Tan (2002) for details.

(*Insert Tables 1 and 2 about here*).

The Pearson (Spearman) correlation between $G(\Gamma)$ and mean cooperation rate by game is 0.921 (0.962); the corresponding one replacing $G(\Gamma)$ with $G_\rho(\Gamma)$ is 0.923 (0.911). Zizzo (2002) contains similar experimental conditions with 3×3 games, eight out of ten of which were chosen drawing payoff values randomly from a uniform distribution (the remaining two were 3×3 variants of the PD). It is harder to define a cooperative outcome in generic games, and the possibility of partial cooperation makes the choice of a fully cooperative outcome a less than perfect summary statistic of cooperation in the context of a 3×3 game than it is in the context of a 2×2 game. Zizzo (2002) still finds Pearson and Spearman correlation coefficients between our simple game harmony measures and the mean cooperation rate by game in the 0.579-0.627 range ($P < 0.05$). Figure 1 illustrates the correlation between cooperation and game harmony measures using both 2×2 and 3×3 games.

(*Insert Figure 1 about here).*

Alternative measures of mean cooperation rates, by role (i.e., row player in a trust game, column player in the same trust game, and so on) rather than by game, give correlation values with $G(\Gamma)$ and $G_\rho(\Gamma)$ lower by 0.1-0.15 for both 2×2 and 3×3 games, but still significant at $P < 0.05$ or better.

Zizzo and Tan (2002) and Zizzo (2002) varied the game harmony of the practice stage games: in one condition during the practice subjects would play games that were "close", in terms of $G(\Gamma)$ (and $G_\rho(\Gamma)$), to pure coordination games; in another condition subjects would play games "close" to zero-sum games; in a third condition, subjects would play games of intermediate game harmony. This simple framing manipulation was ineffective in altering the extent to which subjects cooperated in later game play with the fully displayed 2×2 games. It was somewhat effective with 3×3 games, particularly because of further experimental conditions in which, after having done the practice, subjects had to play ambiguous decision problems, where ambiguity was triggered by having payoff matrices that were not fully displayed.

A Counterexample. While game harmony matters for cooperation, Figure 1 makes clear that it is not the only thing that matters. Most importantly, there are strategic aspects of the games that cannot be captured by a measure of correlation between payoffs but which may matter for cooperation. Take the following example:

2,2	0,3	1,1	0,3
3,0	1,1	3,0	2,2

where we have a PD and a game that is obtained by swapping the (cooperation, cooperation) and (defection, defection) cells. While the likelihood of achieving the cooperative outcome (2, 2) is very different in the two games, they two games have the same $G(\Gamma)$ and $G_\rho(\Gamma)$ values. Game harmony is not the only thing that matters in determining cooperation. This does not mean, however, that it may not be one possible factor.

6. Conclusions

Game harmony is a generic game feature that describes how harmonious (non-conflictual) or disharmonious (conflictual) the interests of the n players are, as embodied in the game payoffs. It is one (but of course only one) of the factors that may determine the extent of cooperation in games. This paper presents ways in which we may try to measure the degree of harmony on a real-valued scale which has pure coordination and constant-sum games at its polar opposites. In particular, we focus on correlation-based measures of game harmony and we analyze their properties such as scale independence. We find that the introduction of Doomsday or Paradise outcomes that are common across players increases game harmony, and so does the introduction of a totally impartial third party in a conflictual situation. Game harmony can be useful as a classification tool; while cardinal measures of game harmony are useful to compare game harmony of games with the same payoff structure, rank measures can be more useful when the comparison is between games with different payoff structures, as they are less sensitive to small changes in the payoff values.

An equivalence holds between perception of game harmony and the weight assigned to the payoff of another agent in one's own utility function. An agent who perceives a game as more harmonious will behave as if she has more positive interdependent preferences. Therefore, perception of greater game harmony will lead to more cooperative behavior in any game where simple payoff transformation can obtain it. Framing effects may be effective in changing the extent of observed cooperative behavior, for example in social dilemmas, insofar as they change the perceived game harmony of the game. We discuss in some detail the "common fate" manipulation used by researchers with mixed success: scale independence and the Doomsday scenario introduction properties explain why cooperation rates can be more successfully raised in some instances than in others.

The empirical evidence shows that simple game harmony measures can be used as effective, if partial, predictors of cooperation in Prisoner's Dilemmas (by calibration on Rapaport and Chammah's 1965 data), and more generally among 2×2 and 3×3 normal form games (Zizzo and Tan 2002; Zizzo 2002).

Appendix A. IDM Measures of Game Harmony

IDM measures of game harmony are an alternative class of measures based on traditional measures of income distribution. Measures of income distribution can be useful to summarize information on the payoff distribution for each payoff outcome, with no general restriction on the number of players. IDM measures can be computed in three steps.

The first step is *ratio-normalization* of payoffs, by finding, for each payoff value,

$$x^*_{iw} = \frac{x_{iw}}{\sum\limits_{w=1}^{|W|} x_{iw}} \tag{12}$$

i.e., by dividing each payoff by the sum of all possible payoffs for the player. Ratio-normalization is essential because, otherwise, the IDM measure would mirror the average payoff distribution of the game, but not the extent to which the players' interests in achieving one or another game outcome are the same or are in conflict.

The second step is to compute the payoff distribution index for each $w \in W$, using the algorithm of one of the standard indexes of income distribution, such as Gini or Theil (see Cowell 1978 for a review). We can define the IDM game harmony index as:

$$GH_{IDM} = \frac{\sum\limits_{w=1}^{|W|} I_w}{|W|} \tag{13}$$

One general feature of IDM game harmony indices is that a greater value corresponds to less, and not more, game harmony, unlike the correlation-based indices. Suitable reparametrizations can sidestep this potential source of confusion.

What index of income distribution might be potentially good candidates to measure game harmony? One restriction is imposed by the constraint that many games of economic interest have few players, even just two players: the index must be defined and retain desirable features for $n = 2$. For example, the relative mean deviation criterion is not applicable in this case; while the value of the Gini index is rather sensitive to the value of n for low n values, and has a maximum value of $1/2$ rather than 1 for $n = 2$. To avoid the latter problem, one can normalize the Gini index by multiplying its value by $n/(n-1)$: labeling this normalized index as I_w^G and ordering the n players from "poorest" (i.e., that with the lowest payoff) to "wealthiest" (i.e., that with the highest payoff) for some $w \in W$, we can define I_w^G as:

$$I_w^G = \frac{n}{n-1} \frac{2}{n \sum\limits_{i=1}^{n} x_{iw}^*} \sum_{i=1}^{n} i \left(x_{iw}^* - \frac{\sum\limits_{i=1}^{n} x_{iw}^*}{n} \right) = \frac{1}{n-1} \frac{2}{\sum\limits_{i=1}^{n} x_{iw}^*} \sum_{i=1}^{n} i \left(x_{iw}^* - \frac{\sum\limits_{i=1}^{n} x_{iw}^*}{n} \right) \tag{14}$$

One can then find the normalised Gini-based game harmony index GH_G:

$$GH_G = \frac{\sum\limits_{w=1}^{|W|} I_w^G}{|W|} \tag{15}$$

One possible criticism of the normalised Gini criterion is that it loses its boundedness between 0 and 1 if there are negative payoffs. Chen et al. (1982) suggest a way out of this problem.

The Theil and the Herfindahl indexes are other possible candidates for I_w. Instead income distribution criteria that overweigh low relative to high payoffs (such as logarithmic variance: Cowell 1978) do not appear useful, since it is not clear how the overweighting may be justified, in relation to game harmony. Criteria stating what fraction of income/payoff is below some target level (e.g., the minimal majority criterion) also do not appear to have any obvious justification in relation to game harmony. A similar point can be made in relation to criteria of income distribution based on social welfare functions.

IDM measures have some more general limitations. First, they may be analytically unwieldy. Second, they may be computationally more tractable than the correlation-based measures only when n is large. Third, correlation-based measures are more transparently interpretable than IDM measures as measures of game harmony, and it is unclear why we should prefer the latter to the former whenever there is a discrepancy in prediction.

Appendix B. Proposition Proofs

Proof of Proposition 2. First, note that, since $a_{cw} = kb_{cw} + h$, $\sigma_a^2 = Var(kb_{cw} + h) = k^2\sigma_b^2$, so

$\sigma_a = k\sigma_b$. Second, note that $Cov(a_{cw}, b_{cw}) = Cov(kb_{cw} + h, b_{cw}) = Cov(kb_{cw}, b_{cw}) = k\sigma_b^2$. It follows that

$$G(\Gamma_d) = \frac{1}{C}\sum_{c=1}^{C}\frac{Cov(a_{cw}, b_{cw})}{\sigma_a\sigma_b} = \frac{1}{C}\sum_{c=1}^{C}\frac{k\sigma_b^2}{k\sigma_b\sigma_b} = 1 \tag{16}$$

■

Proof of Proposition 3. Consider first the case where $n = 2$. Then $x_{1w} + x_{2w} = q$, so $x_{1w} = q - x_{2w}$: since $C = 1$, we can re-write this simply as $a_{cw} = q - b_{cw}$. Now note that $\sigma_a^2 = Var(q - b_{cw}) = \sigma_b^2 \rightarrow \sigma_a\sigma_b = \sigma_b^2$ and that $Cov(a_{cw}, b_{cw}) = Cov(q - b_{cw}, b_{cw}) = -\sigma_b^2$. Thus $G(\Gamma_0) = -1$. Consider now $n > 2$ players; then, for some i and two states of the world 1 and 2, Condition 2 implies that $x_{i1} > x_{i2}$, i.e. $\triangle x_+ = x_{i1} - x_{i2} > 0$. Because of the definition of $G(\Gamma_0)$, then $\sum_{j\neq i}^{n} \triangle x_j < 0$. Hence, $\exists\ \triangle x_- < 0$ in J. Now consider $\triangle x_m | m \neq i, j$. Either $\triangle x_m > 0$, or $\triangle x_m = 0$, or $\triangle x_m < 0$. If $\triangle x_m > 0$, then $sgn(\triangle x_m) = sgn(\triangle x_+) \rightarrow r_c(x_{iw}, x_{+w}) > -1$. If $\triangle x_m < 0$, then $sgn(\triangle x_m) = sgn(\triangle x_-) \rightarrow r_c(x_{iw}, x_{-w}) > -1$. Finally, if $\triangle x_m = 0, x_{mw}$ will be imperfectly negatively correlated with both x_{+w} and x_{-w}, so again $r_c(x_{iw}, x_{-w}) > -1$. Hence $\exists c : \{r_c(x_{iw}, x_{+w}) > -1 \rightarrow G(\Gamma_0) > -1\}$. A parallel argument can be made to show that $G(\Gamma_0) < 1$ (e.g., if $\triangle x_m > 0$, then $sgn(\triangle x_m) \neq sgn(\triangle x_-) \rightarrow r_c(x_{iw}, x_{-w}) < 1$). ■

Proof of Proposition 4. Consider the constant-sum game where $(n-1)$ players have identical payoffs m_{cw} and the nth player has payoffs defined by $q - (n-1)m_{cw}$. Then we have:

$$\begin{aligned} G(\Gamma_{0d}) &= \frac{1}{C}\sum_{c=1}^{C} r_c(a_{cw}, b_{cw}) = \frac{1}{C}\sum_{c=1}^{C-(n-1)} r_c(m_{cw}, m_{cw}) + \sum_{c=1}^{n-1} r_c[m_{cw}, q-(n-1)m_{cw}] = \\ &= \frac{1}{C}[C-(n-1)-(n-1)] = \frac{C-2(n-1)}{C} = 1 - 4\frac{(n-1)!}{n!} = 1 - \frac{4}{n} \end{aligned} \tag{17}$$

so $G(\Gamma_{0d}) < 1$ (since $n < \infty$) and $\triangle G(\Gamma_{0d})/\triangle n > 0$, hence, if $G(\Gamma_{0d}) = \lambda_{\max}$, the proposition is proven. ■

Proof of Proposition 5 (Scale Independence). Consider $G(k_i x_i + h_i, W)$. Let k_a, h_a and k_b, h_b be the unit and additive constants, respectively, for the two players of each payoff pair $c \in C$.

$$\begin{aligned} G(k_i x_i + h_i, W) &= \frac{1}{C}\sum_{c=1}^{C} r_c(k_a a_{cw} + h_a, k_b b_{cw} + h_b) = \frac{1}{C}\sum_{c=1}^{C}\frac{Cov(k_a a_{cw} + h_a, k_b b_{cw} + h_b)}{k_a k_b \sigma_a \sigma_b} = \\ &= \frac{1}{C}\sum_{c=1}^{C}\frac{k_a k_b Cov(a_{cw}, b_{cw})}{k_a k_b \sigma_a \sigma_b} = G(x_i, W) \end{aligned} \tag{18}$$

■

Proof of Proposition 6 (Replication). Consider each original payoff pair $a_{cw}, b_{cw} \rightarrow r(a_{cw}, b_{cw})$ in the parent game Γ: with γ replicas, there are now $\gamma \times (a_{cw}, b_{cw}) \rightarrow \gamma r(a_{cw}, b_{cw})$ for every $(a_{cw}, b_{cw}) \subset \Gamma$, since $r_c(k_a a_{cw} + h_a, k_b b_{cw} + h_b) = r_c(a_{cw}, b_{cw})$ (by the proof of the scale independence property). Since, in Γ and each replica Φ, $r(a_{cw}, b_{cw})$ is the same, $r(a_{cw}, b_{cw}) = r(a_{c,\gamma w}, b_{c,\gamma w})$. Thus:

$$G(\Psi) = \frac{1}{C}\sum_{c=1}^{C} r_c(a_{c,\gamma w}, b_{c,\gamma w}) = \frac{1}{C}\sum_{c=1}^{C} r_c(a_{cw}, b_{cw}) = G(\Gamma) \tag{19}$$

■

Proof of Proposition 7 (Outcome Addition). We present the proof for the case where W is countable: the proof for the uncountable W case is identical by replacing the sum signs over $|W|$ with the corresponding integrals. If $(x_{i\omega} = \sup X_i) \vee$ $(x_{i\omega} = \inf X_i)$ $\forall$ i, $\exists$ $h_i : \{(x_{i\omega} + h_{i\omega} = 0)$ & $[(x_{i\pi} + h_{i\pi} > 0 \; \forall \; \pi \neq \omega) \vee (x_{i\pi} + h_{i\pi} < 0 \; \forall \; \pi \neq \omega)]\}$. Since by scale independence $G(\Gamma) = G(x_i, W) = G(x_i + h_i, W)$, we can consider the game defined by adding such h to all payoffs knowing that by doing so $G(\Gamma)$ does not change. First analyze how $G(\Gamma)$ depends on w in this rescaled game:

$$G(\Gamma) = \frac{1}{C}\sum_{c=1}^{C} r_c(a_c, b_{cw}) = \frac{1}{C}\sum_{c=1}^{C} \frac{|W|\sum_{w=1}^{|W|} a_{cw}b_{cw} - \sum_{w=1}^{|W|} a_{cw}\sum_{w=1}^{|W|} b_{cw}}{\sqrt[2]{\left[|W|\sum_{w=1}^{|W|} a_{cw}^2 - \left(\sum_{w=1}^{|W|} a_{cw}\right)^2\right]\left[|W|\sum_{w=1}^{|W|} b_{cw}^2 - \left(\sum_{w=1}^{|W|} b_{cw}\right)^2\right]}} \tag{20}$$

If $\Gamma = \Gamma_d$, this collapses to $G(\Gamma) = 1$ as we would expect, so $G(\Gamma)$ is independent of w and $G(\Theta) = G(\Gamma) = 1$. Now assume $\Gamma \neq \Gamma_d$. For tractability, we employ a continuous approximation to see the effect of a change in w on game harmony. Denote $\tau_a = |W|\sum_{w=1}^{|W|} a_{cw}^2 - \left(\sum_{w=1}^{|W|} a_{cw}\right)^2 > 0, \tau_b = |W|\sum_{w=1}^{|W|} a_{cw}^2 - \left(\sum_{w=1}^{|W|} a_{cw}\right)^2 > 0$, $A = \left(|W|^2\sum_{w=1}^{W} a_{cw}b_{cw}\right) / \sqrt[2]{\tau_a\tau_b}$ and $B = \left(-\sum_{w=1}^{|W|} a_{cw}\sum_{w=1}^{|W|} b_{cw}\right) / \sqrt[2]{\tau_a\tau_b}$. We need not worry about the effect of a marginal increase in w on $|W|$ in the $\sum_{w=1}^{|W|}$ terms, since, by our rescaling, $\sum_{w=1}^{|W+1|} a_{cw}b_{cw} = \sum_{w=1}^{|W|} a_{cw}b_{cw}$, and similarly for the other summation terms. Since τ_a and τ_b are positive monotonic transformations of σ_a and σ_b, who are themselves positive, $\vartheta G/\vartheta A > 0$ and $\vartheta G/\vartheta B > 0$. Also, $\sum_{w=1}^{|W|} a_{cw}\sum_{w=1}^{|W|} b_{cw} > 0$ by our rescaling, so, as τ_a & τ_b $\uparrow$ if w $\uparrow$, $\vartheta B/\vartheta w > 0$ monotonically for any value of w. Now consider:

$$\frac{\vartheta A}{\vartheta w}=\frac{1}{C}\sum_{c=1}^{C}\frac{|W|\,\tau_a\tau_b-\frac{\tau_b}{2}\sum_{w=1}^{|W|}a_{cw}b_{cw}\sum_{w=1}^{|W|}a_{cw}\sum_{w=1}^{|W|}b_{cw}\sum_{w=1}^{|W|}a_{cw}^2-\frac{\tau_a}{2}\sum_{w=1}^{|W|}a_{cw}b_{cw}\sum_{w=1}^{|W|}a_{cw}\sum_{w=1}^{|W|}b_{cw}\sum_{w=1}^{|W|}a_{cw}^2}{\sqrt[2]{\tau_a\tau_b}} \tag{21}$$

Note that, since $x_{i\pi}\neq x_{i\omega}$ and $[(x_{i\omega}=\sup W)\vee(x_{i\omega}=\inf W)]\,\forall\, i$, $\sum_{w=1}^{|W|}a_{cw}b_{cw}>0$ and $\sum_{w=1}^{|W|}a_{cw}\sum_{w=1}^{|W|}a_{cw}b_{cw}>0$, hence the second and third term in the numerator are positive. Since τ_a and τ_b are themselves positive, the first term of the numerator and the denominator of $\vartheta A/\vartheta w$ are also positive. It follows that $\vartheta A/\vartheta w>0$ monotonically for any value of w and, since $\vartheta B/\vartheta w>0, \vartheta G/\vartheta A>0, \vartheta G/\vartheta B>0$, we can conclude that $\vartheta G/\vartheta w>0$. Further, since $\vartheta G/\vartheta w$ is continuous and positive for any W, G is monotonically increasing in w, and so $sgn(\triangle G/\triangle w)=sgn(\vartheta G/\vartheta w)$. Thus, $G(\Theta)>G(\Gamma)$. ■

Proof of Proposition 8 (Player Addition). Note that:

$$G[\Gamma|(n-1)]=\frac{1}{C-n+1}\sum_{c=1}^{C-n+1}r_c(a_{cw},b_{cw})\leftrightarrow\sum_{c=1}^{C-n+1}r_c(a_{cw},b_{cw})=(C-n+1)G[\Gamma|(n-1)] \tag{22}$$

so

$$\begin{aligned}
G[\Gamma|n] &= \frac{1}{C}\sum_{c=1}^{C-n+1}r_c(a_{cw},b_{cw})+\frac{1}{C}\sum_{c=1}^{n-1}r_c^x(a_{cw},b_{cw})=\frac{(C-n+1)}{C}G[\Gamma|(n-1)]+\frac{(n-1)}{C}\mu_r &(23)\\
\triangle G &= \frac{1}{C}\{(C-n+1-C)G[\Gamma|(n-1)]+(n-1)\mu_r\}=\frac{2}{n}\{\mu_r-G[\Gamma|(n-1)]\}= &(24)\\
&= \frac{2}{n}\{\mu_r-G[\Gamma|(n-1)]\}
\end{aligned}$$

■

Proof of Proposition 10. We can decompose C in terms of the $C(a-1)$ payoff pairs between non perfectly impartial players, $C(n-a+1)$ pairs between perfectly impartial players and $(a-1)(n-a+1)$ pairs between a perfectly impartial and a non perfectly impartial player: $C=C(a-1)+C(n-a+1)+$

$(a-1)(n-a+1)$. We have

$$\begin{aligned} G_k(\Gamma) &= \frac{C(a-1)}{C(a-1)+C(n-a+1)+(a-1)(n-a+1)} G[\Gamma|(a-1)] + \quad (25) \\ &\quad + \left[\frac{C(n-a+1)+(a-1)(n-a+1)}{C(a-1)+C(n-a+1)+(a-1)(n-a+1)}\right]\mu_r \\ &= \frac{C(a-1)}{C(a-1)+C(n-a+1)+(a-1)(n-a+1)} G[\Gamma|(a-1)] \quad (26) \end{aligned}$$

Thus, $\lim_{n\to\infty} G_k(\Gamma) = 0$. ■

Proof of Proposition 13. The proof that the Borel set of admissible value of $G_G(\Gamma, 2)$ has an upper bound of 1 is the same as the proof that in pure coordination games Γ_d, $G(\Gamma_d) = 1\ \forall\ n$. Since there are only two groups, in the constant-sum game $G_G(\Gamma_0, 2)$, $a^G_{cw} + b^G_{cw} = q$ and so $G_G(\Gamma_0, 2) = r_c(a^G_{cw}, b^G_{cw}) = r_c(a^G_{cw}, q - a^G_{cw}) = -1$. ■

Proof of Proposition 15. For $W^w = W^*$, it must be true that $x^{coop}_{iw} + \beta_i \sum^n_{j\neq i} v(x^{coop}_{jw}) \geq x^{\widetilde{coop}}_{iw} + \beta_i \sum^n_{j\neq i} v(x^{\widetilde{coop}}_{jw})$, and so the key condition can be written as:

$$x^{coop}_{iw} + \beta_i \left[\sum^n_{j\neq i} v(x^{coop}_{jw}) - \sum^n_{j\neq i} v(x^{\widetilde{coop}}_{jw})\right] \geq x^{\widetilde{coop}}_{iw} \quad (27)$$

$x^{coop}_{jw} \geq x^{\widetilde{coop}}_{jw} \forall j$ by the definition of cooperative outcome, so $v(x^{coop}_{jw}) \geq v(x^{\widetilde{coop}}_{jw}) \forall j \to \sum^n_{j\neq i} v(x^{coop}_{jw}) - \sum^n_{j\neq i} v(x^{\widetilde{coop}}_{jw}) \geq 0$. It follows that the probability that the key condition holds is non-decreasing in β_i, i.e. $\Delta Prob(W^{coop} = W^*)/\Delta\beta \geqslant 0$. Also, since $\partial G(\Gamma^v)/\partial\beta_i > 0$ monotonically for any admissible β, $\Delta\beta_i/\Delta_\beta G(\Gamma^v) > 0$, and so $\Delta Prob(W^{coop} = W^*)/\Delta_\beta G(\Gamma^v) \geqslant 0$. ■

Appendix C. An Example with Bargaining Games.

There are two players who need to split a bargaining pie of size 1: any outcome w_i determines a payoff $x_{1w} = w$ and $x_{2w} = 1 - x_{1w} = 1 - w$ for players 1 and 2, respectively. If no exit is possible, this is a constant-sum game as $x_{1w} + x_{2w} = 1$, implying that game harmony is equal to -1 by Proposition 3 as $n = 2$. Assume now instead that we have an *ultimatum game*, with player 1 making an offer between 0 and 1, and player 2 accepting or rejecting the offer; if the offer is rejected, $x_{1w} = 0$ and $x_{2w} = 0$. This is a case in which there is a combination of a continuous action schedule (on the part of the proposer) and a discrete action schedule (on the part of the receiver). Let us say that W is *quasi-uncountable* in this case, and let, in general for quasi-uncountable cases, $W^1 ... W^P$ for $P \in \mathbb{N}^{++}$ be the discrete partitions W is divided in (we are assuming that the number of partitions is finite), with $f(W^p)$ being the corresponding densities; by using the equal weights condition, we can then write down $G(\Gamma)$ as:

$$\mathrm{G}(\Gamma)=\frac{1}{C}\sum_{c=1}^{C}\frac{P\sum_{p=1}^{P}\int_{W^P}a_{cw}b_{cw}f(W^p)dW^p-\sum_{p=1}^{P}\int_{W^P}a_{cw}f(W^p)dW^p\sum_{p=1}^{P}\int_{W^P}b_{cw}f(W^p)dW^p}{\sqrt[2]{\left[P\sum_{p=1}^{P}\int_{W^P}a_{cw}^2f(W^p)dW^p-\left(\sum_{p=1}^{P}\int_{W^P}a_{cw}f(W^p)dW^p\right)^2\right]\left[P\sum_{p=1}^{P}\int_{W^P}b_{cw}^2f(W^p)dW^p-\left(\sum_{p=1}^{P}\int_{W^P}b_{cw}f(W^p)dW^p\right)^2\right]}} \tag{28}$$

where $f(W^p) = (\max|W^p| - \min|W^p|)^{-1}$. In the ultimatum game $P = 2$, and we can label W^1 for when the offer is accepted (with $f(W^1) = (\max|W^1| - \min|W^1|)^{-1} = 1$) and W^2 for when the offer is rejected. Note that $\mu_1 = \mu_2 = 1/4$, $\mu_1\mu_2 = 1/16$ and that

$$E(x_1, x_2) = \frac{1}{2}\int_0^1 w(1-w)dw + \frac{1}{2}\int_0^1 0dw = \frac{1}{2}\left[\frac{1}{2}w^2 - \frac{1}{3}w^3\right]_0^1 = \frac{1}{12} \tag{29}$$

$$E(x_1^2) = \frac{1}{2}\int_0^1 w^2dw + \frac{1}{2}(0) = \frac{1}{2}\left[\frac{1}{3}w^3\right]_0^1 = \frac{1}{6} \tag{30}$$

so, as $C = 1$ (since $n = 2$) and $\sigma_a = \sigma_b$ (since payoffs are symmetric):

$$G(\Gamma) = \frac{Cov(a_{cw}, b_{cw})}{\sigma_a\sigma_b} = \frac{E(x_1, x_2) - \mu_1\mu_2}{E(x_1^2) - \mu_1^2} = \frac{\frac{1}{12} - \frac{1}{16}}{\frac{1}{6} - \frac{1}{16}} = 0.2 \tag{31}$$

hence the existence of a Doomsday threat point has the effect of increasing game harmony considerably, as for the common outcome property.

Acknowledgements

Thanks to Michael Bacharach, Oliver Board, Daniel Sgroi, Michael Vaughan-Lee and participants at a Nuffield College (Oxford) seminar for advice, and to Lucia Buenrostro for research assistance. The usual disclaimer applies. This paper is dedicated to the memory of Michael Bacharach.

References

Bacharach, Michael (1993), Variable Universe Games, in Ken Binmore, Alan Kirman and Piero Tani (eds.), Frontiers of Game Theory, Cambridge (Mass.) and London: MIT Press, pp. 255-275.

Bacharach, Michael and Michele Bernasconi (1997), The Variable Frame Theory of Focal Points: An Experimental Study, Games and Economic Behavior 19 (1), April, pp. 1-45.

Bacharach, Michael, Gerardo Guerra and Daniel J. Zizzo (2001), "Is Trust Self-Fulfilling? An Experimental Study", Oxford University Department of Economics Discussion Paper n. 76, October.

Bernheim, B. Douglas and Michael D. Whinston (1990), Multimarket Contact and Collusive Behavior, Rand Journal of Economics 21 (1), Spring, pp. 1-26.

Bouas, Kelly S. and Samuel S. Komorita (1996), Group Discussion and Cooperation in Social Dilemmas, Personality and Social Psychology Bulletin 22 (11), November, pp. 1144-1150.

Brewer, Marilynn B. and Norman Miller (1996), Intergroup Relations, Buckingam: Open University Press.

Chen, Chau Nan, Tien Wang Tsaur and Tong Shieng Rhai (1982), The Gini Coefficient and Negative Income, Oxford Economic Papers 34 (3), November, pp. 473-478.

Cookson, Richard (2000), Framing Effects in Public Goods Experiments, Experimental Economics 3 (1), June, pp. 55-79.

Cowell, Frank A. (1978), Measuring Inequality, Oxford: Philip Allan.

Elliott, Catherine S., Donald M. Hayward and Sebastian Canon (1998), Institutional Framing: Some Experimental Evidence, Journal of Economic Behavior and Organization 35 (4), May, pp. 455-464.

Epstein, Larry G. and Stephen M. Tanny (1980), Increasing Generalized Correlation: A Definition and Some Economic Consequences, Canadian Journal of Economics 13 (1), February, pp. 16-34.

Luce, R. Duncan and Howard Raiffa (1957), Games and Decisions, New York: Wiley.

Ochs, Jack (1995), Coordination Problems, in John H. Kagel and Alvin E. Roth (eds.), The Handbook of Experimental Economics, Princeton: Princeton University Press, pp. 195-251.

Rabbie, Jacob M. and Murray Horwitz (1969), Arousal of Ingroup-Outgroup Bias by a Chance Win or Loss, Journal of Personality and Social Psychology 13 (3), November, pp. 269-277.

Rabbie, Jacob M., Jan C. Schot and Lieuwe Visser (1989), Social Identity Theory: A Conceptual and Empirical Critique from the Perspective of a Behavioural Interaction Model, European Journal of Social Psychology 19 (3), May-June, pp. 171-202.

Ritzberger, Klaus (2002), Foundations of Non-Cooperative Game Theory, Oxford: Oxford University Press.

Sugden, Robert (1993), Thinking as a Team: Towards an Explanation of Nonselfish Behavior, in Ellen Frankel Paul, Fred D. Miller, Jr., and Jeffrey Paul (eds.), Altruism, Cambridge: Cambridge University Press, pp. 69-89.

Von Neumann, John and Oskar Morgenstern (1944), Theory of Games and Economic Behavior, Princeton: Princeton University Press.

Wit, Arjaan P. and Henk A.M. Wilke (1992), The Effect of Social Categorization on Cooperation in Three Types of Social Dilemmas, Journal of Economic Psychology 13 (1), March, pp. 135-151.

Zizzo, Daniel J. and Daniel Sgroi (2000), "Bounded-Rational Behavior by Neural Networks in Normal Form Games", Nuffield College Oxford Economics Discussion Paper No. 2000-W30, November.

Zizzo, Daniel J. and Jonathan H.W. Tan (2002), "Game Harmony as a Predictor of Cooperation in 2×2 Games", Oxford University Department of Economics Discussion Paper 117, September.

Zizzo, Daniel J. (2002), "Game Perception and Harmony in 3×3 Games: An Experiment in Cognitive Game Theory", Oxford University Department of Economics Discussion Paper 118, September.

TABLE 1.
2×2 GAMES USED BY ZIZZO AND TAN (2002).

Prisoner's Dilemma		"Envy" Game		"Altruism" Game	
92, 11	38, 37	61, 72	59, 73	59, 28	61, 29
64, 63	10, 93	50, 28	48, 29	48, 72	50, 73

Stag-Hunt		Chicken	
10, 51	92, 93	92, 36	10, 11
52, 53	52, 9	63, 62	38, 94

Trust Game 1		Trust Game 2		Trust Game 3	
33, 34	34, 35	52, 3	53, 4	3, 34	4, 35
81, 82	14, 100	100, 51	33, 69	81, 82	14, 100

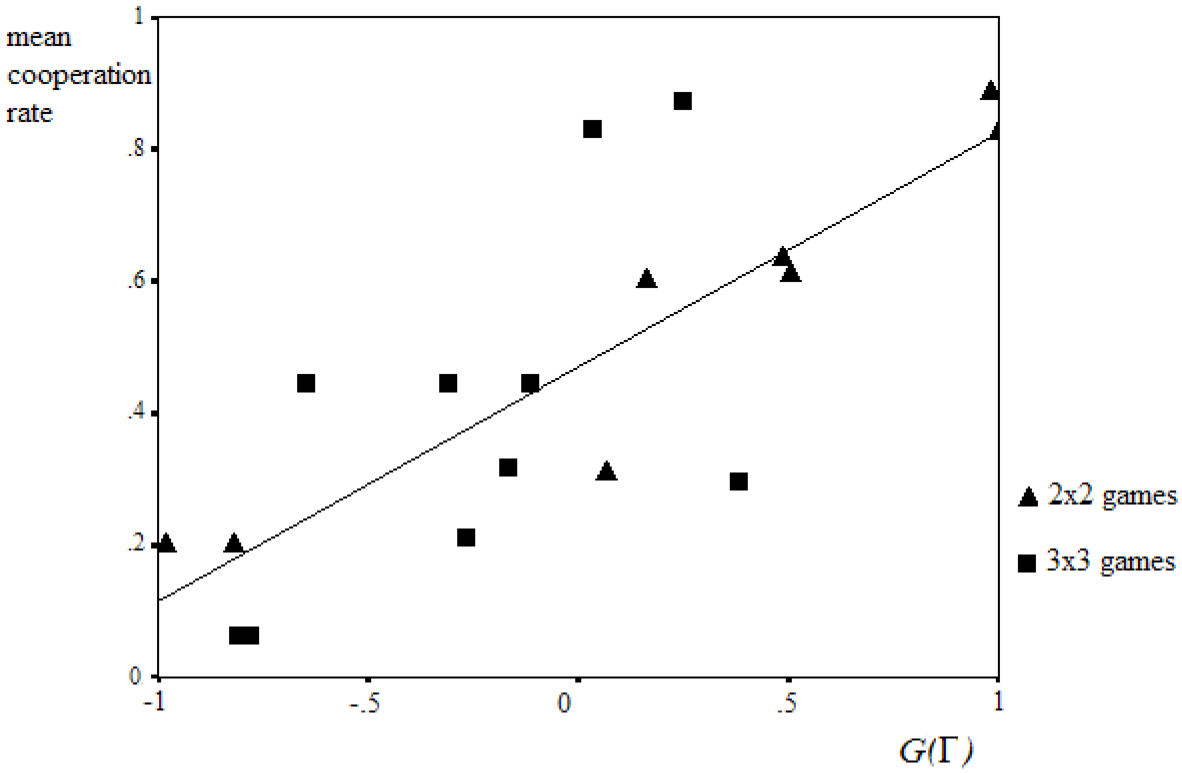
Constant-Sum Game		Coordination Game	
71, 31	18, 84	74, 75	32, 31
26, 76	89, 13	13, 12	85, 86

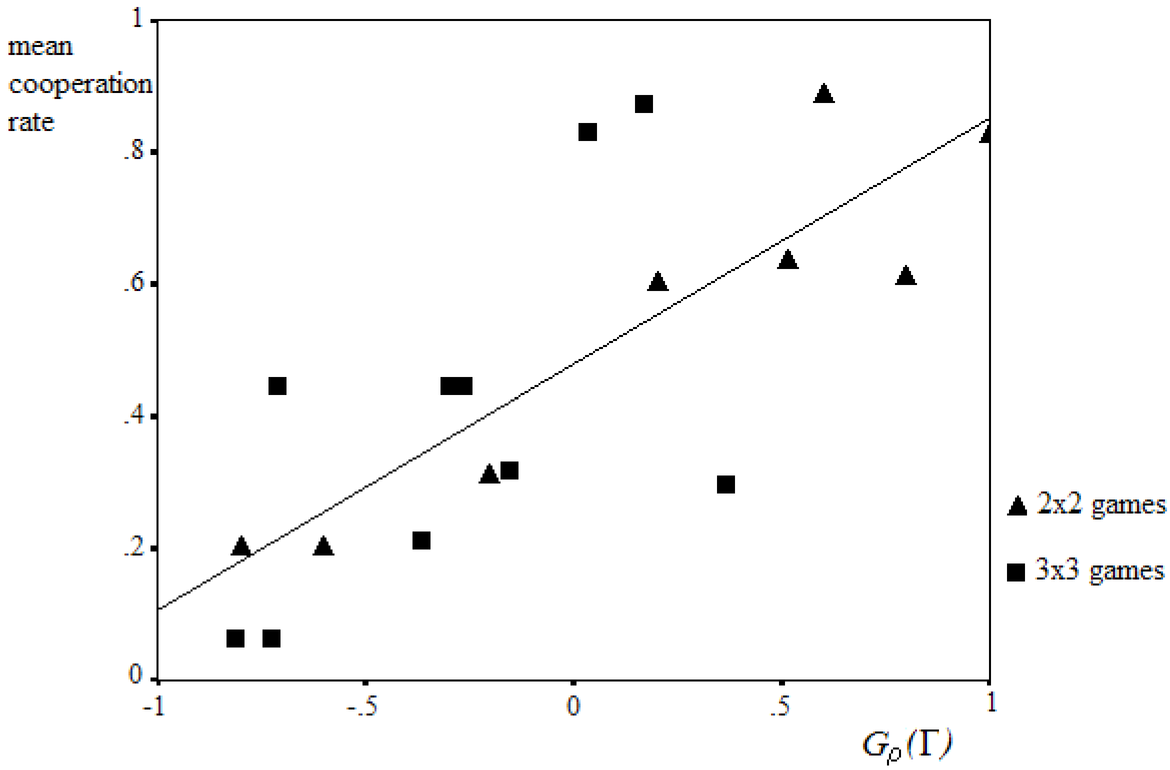
Trust Game 1, 2 and 3 correspond to the "Kind", "Inequitable" and "Needy Trust Game" as defined by Zizzo and Tan (2002).

TABLE 2.
GAME HARMONY VALUES AND MEAN COOPERATION RATES BY GAME IN 2×2 GAMES.

Game (Γ)	G(Γ)	Gρ(Γ)	mean cooperation rate
Prisoner's Dilemma	-0.817	-0.8	0.2
Coordination Game	1	1	0.825
Envy Game	0.98	0.6	0.883
Altruism Game	-0.98	-0.6	0.2
Constant-Sum Game	-1	-1	
Trust Game 1	0.066	-0.2	0.308
Trust Game 2	0.066	-0.2	0.308
Trust Game 3	0.503	0.8	0.608
Stag-Hunt	0.488	0.513	0.633
Chicken	0.16	0.2	0.6

FIGURE 1. SCATTERPLOTS OF GAME HARMONY VS. MEAN COOPERATION RATE BY GAME.





The data is taken from Zizzo and Tan (2002) and Zizzo (2002).