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**MONETARY POLICY WITH AN ENDOGENOUS CAPITAL STOCK  
WHEN INFLATION IS PERSISTENT**

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Monetary Policy with an Endogenous Capital  
Stock When Inflation is Persistent

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## Abstract

The paper presents a monetary policy model with an endogenous capital stock when a backward looking element in wage setting causes inflation persistence. We analyse how the endogeneity of the capital stock changes the macroeconomic dynamics with which policy interacts and its implications for optimal policy and time inconsistency. Capital stock endogeneity makes inflation more persistent in reduced form. This makes the optimal contemporaneous policy response to shocks more vigorous but the subsequent return to steady state more gradual. Observed output becomes more serially correlated. Capital endogeneity can also give rise to *disinflation bias* under discretion for some parameter values.

## Key Words

Monetary policy, time inconsistency, inflation persistence, investment, capital stock.

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## Introduction

The monetary policy literature has often used models in which the capital stock is exogenous and constant. This assumption is innocuous in the standard models following Barro and Gordon (1983) and Rogoff (1985) and the New Keynesian Phillips curve under discretion (see Clarida, Gali and Gertler, 1999) since they contain no source of persistence. As a result time periods are usually identical ex ante so if the capital stock is endogenous, but has a realistic delivery or time to build lag so it must be set in advance, it would always be set at the same level. More recent literature has incorporated persistence in either output or inflation while retaining the fixed capital stock assumption.<sup>1</sup> In these models different time periods are not identical ex ante since a movement away from the steady state tends to persist in which case an endogenously determined capital stock will vary period by period. This affects firms' pricing decisions and the set of possible outcomes available to the policy maker and hence assuming that the capital stock is fixed is much harder to defend.<sup>2</sup>

This paper contributes to filling this gap by endogenising the capital stock for the case of

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<sup>1</sup> Endogeneity of the capital stock is common in business cycle models but its implications for the Phillips curve and monetary policy have not generally been analysed. On output persistence see Goodhart and Huang (1998), Lockwood (1997), Lockwood, Miller and Zhang (1998) and Svensson (1997); and on inflation persistence see Bean (1998), Clark, Goodhart and Huang (1999), Mash (2000) and Steinsson (2000).

<sup>2</sup> King and Wolman (1996), Yun (1996), Fuhrer (1997a), Casares and McCallum (2000), Edge (2000a, 2000b), Erceg and Levin (2000), Woodford (2000) and Christiano et. al. (2001) include an endogenous capital stock in a monetary policy context but do not directly analyze its effect on optimal policy and time inconsistency problems. Woodford (2000, pp.66-67) remarks that "...accurate quantitative conclusions about the nature of optimal monetary policy are likely to require explicit allowance for the dynamics of the capital stock". This paper supports that conjecture while also showing that capital stock dynamics can change the sign of time inconsistency biases also.

inflation persistence, showing that an endogenous capital stock has important implications for the Phillips curve and optimal policy. While the monetary policy and investment literatures are both extensive this paper appears to be the first to directly link the two in relation to inflation/output dynamics, optimal policy and time inconsistency problems.

Three questions are analysed; firstly what is the effect of an endogenous capital stock on the strength of inflation persistence and the way in which the economy responds to policy? Secondly, how does an endogenous capital stock affect optimal policy? Thirdly, how does an endogenous capital stock interact with time inconsistency problems if the policy maker cannot commit to a rule (or cannot use reputation effects to the same end)? In a model with persistence, time inconsistency problems are of heightened interest because they are present when the system is away from its steady state (Svensson, 1997, Clark, Goodhart and Huang, 1999), irrespective of whether the policy maker has a target output level higher than the natural rate which determines whether steady state time inconsistency occurs. These questions are addressed in a model which introduces inflation persistence by assuming that a fraction of wage setters behave in a backward looking rule of thumb manner. The other wage setters and firms setting the capital stock are fully forward looking, the former as in Rogoff (1985).<sup>3</sup>

The paper shows that an endogenous capital stock worsens the responsiveness of the economy to monetary policy (in the sense that the minimised value of the social loss function is higher

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<sup>3</sup> We leave to future research the extension of the analysis to a New Keynesian setting with forward looking expectations. It may be noted that both approaches assume optimising wage or price setting, the difference being the assumptions imposed on the staggering structure. In addition to Rogoff (1985) the model follows Svensson (1997) and Clark, Goodhart and Huang (1999) amongst others in assuming that there is no staggering but wages are set in advance of the time period in which they will apply.

with endogenous capital). For example, if inflation is high (and persistent) so policy will tend to be tight, the anticipation of tight policy (and hence a lower return to capital) by firms will lead to a lower capital stock which in turn will give rise to higher prices than would have been the case for the same policy but a fixed capital stock. Higher prices imply a smaller reduction in inflation and thus the effect of an endogenous capital stock is to act against the direction of policy and hence increase the output costs of disinflation and the degree of 'backward lookingness' in the reduced form Phillips curve. Thus capital stock setting by fully forward looking firms increases the degree of inflation inertia that originates with backward looking wage setting by some workers. This has important implications for the interpretation of empirical results on the dynamics of inflation since high apparent backward lookingness in reduced form may result from a relatively small amount of underlying backward looking behaviour. The paper thus contributes to our understanding of the microfoundations of inflation persistence but without claiming to resolve this issue fully since if all wage setting is forward looking the economy has no source of inertia and endogeneity of the capital stock does not of itself introduce persistence.

These changes in the supply side dynamics of the economy mean that optimal policy, with or without the ability to commit to a rule, is changed by the endogeneity of the capital stock which should thus be taken into account when policy decisions are made. In particular the output costs of returning inflation to its steady state after a shock are higher with an endogenous capital stock, the optimal contemporaneous response to shocks is more vigorous but the subsequent return to steady state is more gradual.

In relation to time inconsistency problems it is shown that an endogenous capital stock can give rise to a *disinflation* bias in policy for some parameter values. When the capital stock is

exogenous and constant, wage setting in advance of policy decisions gives rise to inflation bias. This occurs in the steady state if the output target is above the natural rate but also in the transition back to steady state for any output target if inflation is persistent (Clark, Goodhart and Huang, 1999). Wage setting amounts to setting a price and with this fixed the quantity variables of employment and output are more flexible ex post than they would be if wages were variable, giving rise to an expansionary or inflationary bias in discretionary policy making. Keeping wage setting to one side for the time being, if the capital stock is endogenous but must be set in advance of policy choices (due to a plausible time to build or delivery lag) the direction of bias reverses. With capital it is a quantity that is fixed and this makes prices and thus inflation more flexible ex post than they would be if capital could be chosen contemporaneously. For example if current inflation is high a discretionary policy maker has an incentive to reduce it quickly while the capital stock remains inflexible since this implies smaller output costs than waiting until the next period by which time the capital stock would have fallen and output costs risen. While optimal ex post for a discretionary policy maker, the anticipation of this incentive leads to a lower capital stock than would be the case under a rule when the policy maker can precommit not to take advantage of the short run fixity of the capital stock. It should be emphasised that the discretionary policy maker does not want a lower capital stock but, following standard time inconsistency arguments, cannot prevent this outcome due to the ex post incentive to reduce inflation quickly. If the inflation bias from ex ante wage setting is combined with the disinflation bias from ex ante capital stock setting the net effect is ambiguous and depends on parameter values though the endogeneity of the capital stock matters for optimal policy in either event.

Two caveats are in order. The first is that inflation persistence, while strongly supported empirically and taken to be necessary for realism by numerous authors, does not as yet have a

widely agreed microfoundation consistent with optimising behaviour, though Woodford (2001) extends recent work on rational indexing which is highly promising in this regard.<sup>4</sup> As a result the model in this paper, which is concerned with the consequences of inflation persistence with an endogenous capital stock rather than its origins, introduces persistence by the simple assumption that a proportion of wage setters behave in a backward looking manner according to a simple rule of thumb. Introducing inflation persistence through wage setting behaviour is consistent with the recent empirical work of Gali, Gertler and Lopez-Salido (2001) who find that marginal cost, which depends on wages, is strongly persistent. They remark that, "In part, our results push the mystery of inflation back to understanding the factors that underlie inertia in the real marginal cost" (p.1239). The empirical evidence of Roberts (1997, 1998) is suggestive of a backward looking element in expectations formation. Clark, Goodhart and Huang (1999), appealing to realism, simply impose a backward looking component in the reduced form Phillips curve. A Fuhrer-Moore (1995) real contracting structure is a possible microfoundation but the consistency of this model with optimising behaviour has been strongly questioned (Taylor, 1999; Rotemberg 1997; Roberts, 1998). Thus we follow earlier papers with some wage setters following backward looking rules of thumb rather than fully optimising (though such behaviour may be optimal given unobserved constraints and/or a microfoundation for rational indexing). This has obvious disadvantages but the assumptions made may correspond to microfoundations

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<sup>4</sup> See also Calvo, Celasun and Kumhof (2001), Christiano, Eichenbaum and Evans (2001), Sbordone (2001), Walsh (2001) and Rudebusch (2000) who also provides an excellent summary of the empirical evidence. Also Ball (1991), Batini and Haldane (1999), Bean (1998), Blake and Westaway (1996), Clarida, Gali and Gertler (1999), Estrella and Fuhrer (1998), Fuhrer (1997b), Fuhrer and Moore (1995), Gali and Gertler (1999), Gali, Gertler and Lopez-Salido (2001), Mankiw (2001), Mankiw and Reiss (2001), Nelson (1998), Roberts (1997, 1998), Rudd & Whelan (2001) and Steinsson (2000). Rotemberg (1997) suggests correlated supply shocks as an alternative basis for observed inflation persistence while Erceg and Levin (2000) point to imperfect credibility.

for backwards looking wage setting which are not yet well understood. The absence of fully specified optimising microfoundations means that we treat the proportion of backward looking wage setters as an exogenous constant while noting that in practice it is possible (but not certain) that it may be regime dependent. This point qualifies the results which compare different regimes though in practice these are likely to be robust to at least small changes in this proportion. All other agents optimise fully and hence their response to different policy regimes is allowed for.<sup>5</sup>

We also make use of a comparatively simple model of capital stock setting. The approach taken has the virtue of analysing the issues in a tractable and transparent way, and the most important aspects of inflation persistence and monetary policy-capital stock linkages are likely to be captured, but the model could undoubtedly be enriched in this dimension.<sup>6</sup> Clearly the applicability of the paper also depends on there being sufficient flexibility or variation in the capital stock for its endogeneity to matter. McCallum and Nelson (1999) suggest from graphical evidence that measures of the capital stock vary too little in the short run to matter much for monetary policy. While ultimately this is an empirical issue, and we cite detailed evidence below which is supportive of flexibility in at least part of the capital stock, two analytic points may be

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<sup>5</sup> Clarida, Gali and Gertler (1999), Clark, Goodhart and Huang (1999), Jensen (1999), McCallum and Nelson (2000), Walsh (2001) and Steinsson (2000) also take the degree of inflation persistence as fixed while studying its consequences. This receives some support from Rudebusch and Svensson's (2002) finding of sub-sample stability of their estimates of the reduced form Phillips curve and Roberts' (1998) finding of sub-sample stability in estimates of the proportion of adaptive expectations agents from survey data.

<sup>6</sup>See Bernanke, Gertler and Gilchrist (1999) and Caballero (1999). Blanchard (1998), Hall and Nixon (2000) and Rowthorn (1995) consider the possible interaction between the capital stock and the natural rate of unemployment which complements the focus below on the dynamics of the capital stock around a given steady state in relation to the dynamics of inflation and output.

made in defence of the approach taken. First, once there is persistence in the model the economy may be away from its steady state for sustained periods in which case the time horizon over which monetary policy will have a predictable component (to which the capital stock responds) is longer than for static models. Second, caution is required in the interpretation of aggregate measures of the capital stock since what matters is the productive capital stock and this may correlate imperfectly with aggregate measures.

The paper is structured as follows. Section 1 develops the microfoundations of wage and capital stock setting in an environment with persistent inflation, highlighting the issues raised above. Section 2 analyses the implications of these decisions for the Phillips curve faced by policy makers, including a numerical demonstration of significant effects from capital endogeneity. Section 3 models policy and time inconsistency issues, the main arguments being presented using diagrams prior to a more rigorous dynamic programming analysis. Section 4 states the key results of how the endogeneity of the capital stock changes the aggregate supply dynamics of the economy, the way in which this changes optimal monetary policy, and when inflation or disinflation bias will occur under discretion, both at and away from the steady state. Section 5 concludes.

## **1. Microfoundations**

We assume that there are a large number of identical firms, indexed by  $i$ , each with the Cobb-Douglas production function (1) where  $A$  is a constant (normalised to unity),  $Y_i$ ,  $K_i$  and  $L_i$  each firm's output, capital stock and labour force respectively. We treat the capital stock as a single variable for the time being, without loss of generality, but later disaggregate it into capital that

may be varied period by period (subject to a standard one period lag) and capital that is fixed, at least beyond the time horizon of the dynamics resulting from monetary policy. Serially uncorrelated supply shocks are introduced through price setting below.

$$Y_i = AK_i^{1-\alpha} L_i^\alpha \quad (1)$$

The demand curve facing each firm is standard and shown by (2) where  $Y_i$  is the quantity demanded,  $P_i$  the price of firm  $i$ 's output,  $P$  the general price level,  $\epsilon$  the price elasticity of demand (defined such that  $\epsilon > 0$ ), and  $Y_{di}$  an index of aggregate demand per firm. With constant returns in the production function (1) we also assume that  $\epsilon$  is finite in order to bound the size of each firm.

$$Y_i = \left(\frac{P_i}{P}\right)^{-\epsilon} Y_{di} \quad (2)$$

When analysing optimising choices at the firm level we treat  $P$  as exogenous but having derived the optimal choice for a representative firm we impose symmetry to derive the economy-wide equilibrium such that  $P_i = P$  (and hence  $Y_i = Y_{di}$ ). Variables without the  $i$  subscript refer to aggregate values across all firms. It should be noted that in common with standard practice we assume that all (forward looking) agents know the parameters of the model, implicitly assuming that any necessary learning processes have already taken place.

The time structure of the model follows Barro and Gordon (1983) and Rogoff (1985) in that nominal wages are set one period in advance of the price setting shock being realised and policy choices made.<sup>7</sup> We assume a standard (see Edge, 2000a, 2000b; Cooper and Haltiwanger, 2000)

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<sup>7</sup>Goodhart and Huang (1998) discuss the implications of varying the timing assumption.

delivery, installation or 'time to build' lag such that the stock that will be operational in a given period must be committed to in the previous period. The model is in discrete time and it would be usual to consider the length of a period to be about a year.

Hence at some time  $t$ , with the nominal wage,  $W$ , and capital stock pre-determined from  $t-1$ , the shock variable is realised and the policy choice made. The latter amounts to setting aggregate demand for time  $t$ , and hence  $Y_{dt}$  in (2), after which firms maximise profits for given  $W$ ,  $K_t$ , and  $Y_{dt}$ .<sup>8</sup> We adopt the usual practice of assuming that labour is supplied to meet demand at the fixed nominal wage each period. In the previous period,  $t-1$ , firms set  $K_{it}$  and workers set the nominal wage  $W_t$ , choices which are analysed in detail below. We assume that the capital stock is set to maximise expected profits in a fully forward looking manner whereas wage setting combines forward and backward looking components. The forward looking component is modelled as in Rogoff (1985), the backwards looking part being exogenously imposed (and important since it gives rise to inflation persistence). Three further assumptions are made about the choices of  $W$  and  $K$ . Firstly that firms and groups of workers are sufficiently small that their choice is made ignoring any possible feedback effects on the aggregate outcome the following period. Secondly that their choices are independent in the sense that we solve for the Nash equilibrium in  $W$  and  $K$  setting and thus leave to one side the possibility that there may be gains from coordination in the setting of these variables. Thirdly we assume that no new information becomes available to

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<sup>8</sup> We follow Svensson (1997), Clarida, Gali and Gertler (1999) and McCallum and Nelson (2000) amongst others (see also McCallum, 2001) and adopt the convenient simplification of thinking of the policy maker setting output (and hence inflation) directly without modelling the transmission mechanism explicitly. It should be emphasised that the results arise from the link between optimal price setting and the capital stock. The flow of investment expenditure does not play an independent role because the setting of the (implicit) policy instrument allows for the effect of investment on demand.

either party between their decisions such that one may think of W and K setting as being either simultaneous or sequential (in either order) but with the later decision being fully anticipated when the earlier decision is made.

### Price setting for given wages and capital stock

We first analyse firms' choices in period t in response to  $Y_{dit}$  for given W and  $K_i$  before analysing the choice of W and  $K_i$  at t-1 given expectations of time t outcomes. The representative firm's profits,  $R_i$ , are given by (3) where  $c_K K_i$  is the total cost of capital.

$$R_i = P_i Y_i - W L_i - c_K K_i \quad (3)$$

With W and  $K_i$  predetermined the firm faces a single choice which may be expressed in terms of  $P_i$ ,  $Y_i$  or  $L_i$  (each of which implies the other two). We work in terms of  $L_i$  and use (2) to substitute for  $P_i$  and (1) to substitute for  $Y_i$  which gives (4).

$$R_i = Y_{di}^{\frac{1}{\epsilon}} P K_i^{(1-\alpha)(1-\frac{1}{\epsilon})} L_i^{\alpha(1-\frac{1}{\epsilon})} - W L_i - c_K K_i \quad (4)$$

Of the variables on the right hand side of (4), only  $L_i$  is a choice variable for the firm ex post and differentiation yields the first order condition (5). It is straightforward to show that the second order condition is satisfied (both here and for the other optimisations below).

$$L_i = \left[ \frac{Y_{di}^{\frac{1}{\epsilon}} P K_i^{(1-\alpha)(1-\frac{1}{\epsilon})} \alpha(1-\frac{1}{\epsilon})}{W} \right]^{\frac{1}{1-\alpha(1-\frac{1}{\epsilon})}} \quad (5)$$

From (5), (1) and (2), the implied optimal choices of output and  $P_i$  are shown by (6) and (7).

$$Y_i = K_i^{\frac{1-\alpha}{1-\alpha(1-\frac{1}{\epsilon})}} \left[ \frac{\alpha(1-\frac{1}{\epsilon})PY_{di}^{\frac{1}{\epsilon}}}{W} \right]^{\frac{\alpha}{1-\alpha(1-\frac{1}{\epsilon})}} \quad (6)$$

$$P_i = \left[ (PY_{di}^{\frac{1}{\epsilon}})^{1-\alpha} K_i^{-\frac{1}{\epsilon}(1-\alpha)} \left( \frac{\alpha(1-\frac{1}{\epsilon})}{W} \right)^{-\frac{\alpha}{\epsilon}} \right]^{\frac{1}{1-\alpha(1-\frac{1}{\epsilon})}} \quad (7)$$

It may be noted that the sign of the responses of  $L_i$ ,  $Y_i$  and  $P_i$  in (5)-(7) to changes in the exogenous variables are standard and intuitive. Each is increasing in the demand side variables,  $P$  and  $Y_{di}$ , whereas  $P_i$  changes in the opposite direction to  $L_i$  and  $Y_i$  in response to the supply side variables,  $W$  and  $K_i$  (with  $P_i$  increasing in  $W$  and decreasing in  $K_i$ ). The inverse link in (7) between the optimal  $P_i$  set by the firm and its previous choice of  $K_i$  is the key mechanism by which the endogeneity of capital matters for price and inflation outcomes. We add a multiplicative shock (common across all firms) to the right hand side of (7) and derive the aggregate outcome by imposing symmetry across all firms so  $P_i=P$  to give (8) where  $e^\theta$  is the impact of the price setting shock where  $\theta$  has the properties  $E[\theta]=0$ ,  $E[\theta_i\theta_{i-1}]=0$  and  $E[\theta^2]=\sigma^2$ . From (8) it may be noted that a reduction in the capital stock (in response to an expectation of tight policy for example) has a similar effect on the price-output relationship as a negative supply shock (positive  $\theta$ ).<sup>9</sup>

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<sup>9</sup>This is reminiscent of the analysis of Barth and Ramey (2001) on the 'price puzzle' of monetary policy though the mechanism here is weaker (prices do not rise with tight policy) and arises from the impact of capital setting (in response to anticipated policy) on optimal price setting rather than a cost channel effect. I am grateful to a referee for this point.

$$P_t = \frac{Y_{dit}^{\frac{1-\alpha}{\alpha}} W_t e^{\theta_t}}{\alpha(1-\frac{1}{\epsilon})K_{it}^{\frac{1-\alpha}{\alpha}}} \quad (8)$$

Taking logs of (8) and subtracting  $\ln P_{t-1}$  gives (9) which is the ex post Phillips curve in inflation-output space. This makes use of  $Y_{dit}=Y_i$  (which follows from (2) given  $P_i=P$ ), and drops the  $i$  subscript such that (9) refers to aggregate outcomes. Lower case letters refer to natural logarithms of the corresponding upper case and  $\pi_t$  is the rate of inflation.

$$\pi_t = (\frac{1-\alpha}{\alpha})y_t + w_t - p_{t-1} - (\frac{1-\alpha}{\alpha})k_t - \ln[\alpha(1-\frac{1}{\epsilon})] + \theta_t \quad (9)$$

Thus (9) shows the combinations of inflation and output available to the policy maker for given  $w_t$ ,  $p_{t-1}$  and  $k_t$ . The remainder of this section endogenises the nominal wage and the capital stock, these being substituted into (9) to show their implications for the policy set.

### Wage Setting

The nominal wage is set one period in advance. We think of it as being set by groups of workers who take into account the effects of the wage they set on their own employment but are small enough to take aggregate outcomes and the capital stock the following period as independent of their decision. By symmetry each group will set the same wage and for brevity we derive the common  $W$  directly while allowing for this independence. Wage setting is assumed to consist of forward and backward looking components. Denoting the forward looking part  $W_f$  and the backward looking part  $W_b$ , the actual log wage is assumed to be given by  $(1-\lambda)w_f + \lambda w_b$  where  $\lambda$ , which lies between 0 and 1 and assumed to be constant, indexes the importance of the backward looking part. With respect to the forward looking component,  $W_f$ , we follow Rogoff (1985) in

assuming that workers set  $W_f$  to minimise the expected value of the square of actual employment the following period minus what employment would be if it was possible to set wages ex post. The first part of this is given by (5) in aggregate form. For the second we require an expression for labour supply which is assumed to take the form given by (10) which may be thought of as reflecting workers true consumption-leisure preferences and/or the influence of trade unions on wage setting.  $W/P$  is the real wage and  $\beta$  the elasticity of labour supply with respect to the real wage. Without loss of generality we normalise a multiplicative constant that may be present in (10) to unity.

$$L^s = \left(\frac{W}{P}\right)^\beta \quad (10)$$

If it was possible for wages to be set ex post, their level would equate labour supply in (10) with labour demand from (5) in aggregate form which gives (11) for the real wage that would prevail and hence (12) for employment from substituting (11) into either (5) or (10).

$$\frac{W}{P} = [\alpha(1-\frac{1}{\epsilon})Y^\frac{1}{\epsilon}K^{(1-\alpha)(1-\frac{1}{\epsilon})}]^{\frac{1}{1+\beta[1-\alpha(1-\frac{1}{\epsilon})]}} \quad (11)$$

$$L|_{ex\ post\ w} = [\alpha(1-\frac{1}{\epsilon})Y^\frac{1}{\epsilon}K^{(1-\alpha)(1-\frac{1}{\epsilon})}]^{\frac{\beta}{1+\beta[1-\alpha(1-\frac{1}{\epsilon})]}} \quad (12)$$

The Rogoff criterion stated above is thus equivalent to setting  $W_f$  to minimise the expected squared difference between (5) in aggregate form and (12), the first order condition for which is that  $W_f$  should be set to equate the expected value of (5) with  $W$  equal to  $W_f$  with the expected

value of (12).<sup>10</sup> This amounts to  $W_f$  being set to satisfy (11) in expectation, the outcome of which is (13) where  $\ln P_{t-1}$  has been subtracted from both sides.

$$w_f - p_{t-1} = E_{t-1}[\pi_t] + \frac{\frac{1}{\epsilon} E_{t-1}[y_t] + (1 - \frac{1}{\epsilon})(1 - \alpha)k_t + \ln[\alpha(1 - \frac{1}{\epsilon})]}{1 + \beta[1 - \alpha(1 - \frac{1}{\epsilon})]} \quad (13)$$

The second term on the right hand side of (13) is the target real wage and  $w_f$  is set to achieve this target in an expectational sense for given  $p_{t-1}$  and expected inflation. If there was no persistence in the model  $E_{t-1}[y_t]$  and  $k_t$  would be constants in which case  $w_f$  would be solely determined by expected inflation plus a constant. Once there is persistence in the model these terms vary and the forward looking part of wage setting will be responsive to expected quantities unless  $\beta$  is large.

We turn to the backward looking component of wage setting. While the Rogoff criterion for the forward looking part is plausible and gives intuitive results, no equivalent uncontroversial way of modelling the backward looking part is available, despite its significance in generating empirically realistic inflation persistence. Two assumptions are made and may be considered in relation to (13), the forward looking wage, with the outcome stated in (14). First we assume that expectations of inflation for this group of workers are adaptive (static) and impose  $E_{t-1}[\pi_t] = \pi_{t-1}$ . The second assumption relates to the real variables on the right hand side that represent the target real wage. One possibility would be to impose backward looking values for  $y$  and  $k$  (whose values at  $t-1$  would be used) and a second to assume a constant target real wage. Both of these

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<sup>10</sup>The shock variable in (8) is a geometric mean preserving spread in levels and thus an arithmetic mean preserving spread in logs. Combined with the variables in the system having a log-linear relationship with each other as shown by (9) this means that in this model the log of an expectation is equal to the expectation of a log.

may be plausible but we adopt the second and impose values for  $y$  and  $k$  equal to their steady state levels. These are denoted  $y^*$  and  $k^*$  and derived in the appendix (A1-A3). With this assumption the target real wage will be achieved on average and exactly if the system is at its steady state.

$$w_b^{-p_{t-1}} = \pi_{t-1} + \frac{\frac{1}{\epsilon}y^* + (1 - \frac{1}{\epsilon})(1 - \alpha)k^* + Ln[\alpha(1 - \frac{1}{\epsilon})]}{1 + \beta[1 - \alpha(1 - \frac{1}{\epsilon})]} \quad (14)$$

Using  $y^*$  from (A1) and combining (13) and (14) the overall wage may be expressed by (15).

$$w_t^{-p_{t-1}} = \frac{(1 - \lambda)E_{t-1}[\pi_t] + \lambda\pi_{t-1} + \frac{k^*(1 - \alpha) + Ln[\alpha(1 - \frac{1}{\epsilon})]}{1 + \beta(1 - \alpha)}}{(1 - \lambda)[\frac{1}{\epsilon}(E_{t-1}[y_t] - y^*) + (1 - \frac{1}{\epsilon})(1 - \alpha)(k_t - k^*)]} \quad (15)$$

### Choice of Capital Stock

We turn to the choice of capital stock made by firms. We disaggregate the capital stock of each firm,  $K_i$ , into "fixed" ( $K_{fi}$ ) and "variable" ( $K_{vi}$ ) capital. Fixed capital remains constant while variable capital is flexible subject to i) a one period time to build or delivery lag, and ii) complete irreversibility (apart from subsequent depreciation) of capital goods that have already been ordered.<sup>11</sup> Despite the irreversibility assumption this form of capital remains flexible in the medium run (apart from being set one period in advance) because we assume for simplicity that

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<sup>11</sup> In general capital may be irreversible for technological reasons (such as the costs of de-installation) or due to conditions in second hand capital goods markets which may be subject to lemons problems or have prices which move adversely when firms wish to buy or sell.

it has a sufficiently high rate of depreciation for the firm to always have positive gross investment in variable capital, even if net investment is negative and the capital stock is shrinking.<sup>12</sup> We assume that the firm level combined capital measure is a simple Cobb-Douglas aggregate of these two such that  $K=K_v^V K_f^{1-V}$ . This disaggregation is done for two reasons. The first is realism, since some forms of capital have long delivery or time to build lags and low rates of depreciation so they will vary little in the short run whereas other forms of capital are more flexible (see Edge, 2000a, 2000b; Cooper and Haltiwanger, 2000). The second is analytical convenience since  $V$  indexes the flexibility of the overall capital stock between  $V=0$  (capital fixed) and  $V=1$  (upper limit on flexibility).<sup>13</sup>

It is assumed that firms are risk neutral and set their variable capital stock,  $K_{vit}$ , at time  $t-1$  to maximise expected profits at time  $t$  when that capital stock is operational. Hence firms will anticipate the choice of  $L_{it}$ ,  $Y_{it}$  and  $P_{it}$  that they may make at time  $t$  so we may substitute (5)-(7) into (4) to give profits,  $R_{it}$ , in terms of  $K_{vit}$  and exogenous parameters by (16). Since  $K_f$  is fixed throughout we normalise it to unity to avoid unnecessary constants.

$$R_{it} = K_{vit}^{\frac{V(1-\alpha)(1-\frac{1}{\epsilon})}{1-\alpha(1-\frac{1}{\epsilon})}} [1-\alpha(1-\frac{1}{\epsilon})] [Y_{dit}^{\frac{1}{\epsilon}} P_t (\frac{\alpha(1-\frac{1}{\epsilon})}{W_t})^{\alpha(1-\frac{1}{\epsilon})}]^{\frac{1}{1-\alpha(1-\frac{1}{\epsilon})}} - c_{K_v} K_{vit} - c_{K_f} \quad (16)$$

In (16) the constants,  $c_{K_v}$  and  $c_{K_f}$  represent the unit costs of variable and fixed capital respectively.

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<sup>12</sup> Varying this assumption would imply that the firm's capital stock decision would become multi-period in nature and considerably more complex (see Caballero, 1999; Mash, 1999).

<sup>13</sup> An alternative specification to the Cobb-Douglas aggregate would be to assume that it takes a CES form in which case the elasticity of substitution would become an important parameter in determining the flexibility of the productive capital stock. From an analytic perspective this flexibility is already captured by the parameter,  $V$ . If the elasticity of substitution was less than unity it is likely that the effective value of  $V$  would increase.

Both are assumed to be exogenous to the firm (so  $c_{Kf}$  becomes irrelevant to the firm's optimisation) and the appendix shows that  $c_{Kv}$  is given by (17). Substituting (17) into (16), rearranging and taking expectations gives (18).

$$c_{Kv} = c_1 E_{t-1}[P_t] \quad ; \quad c_1 = \left(\frac{P_K}{P}\right)(r+\delta) \quad (17)$$

$$E[R_{it}] = E\left[ P_t K_{vit}^{\frac{V(1-\alpha)(1-\frac{1}{\epsilon})}{1-\alpha(1-\frac{1}{\epsilon})}} \left[1-\alpha\left(1-\frac{1}{\epsilon}\right)\right] \left[ Y_{dit}^{\frac{1}{\epsilon}} \left(\frac{\alpha(1-\frac{1}{\epsilon})}{\frac{W_t}{P_t}}\right)^{\alpha(1-\frac{1}{\epsilon})} \right]^{\frac{1}{1-\alpha(1-\frac{1}{\epsilon})}} \right] \quad (18)$$

$$- P_t c_1 K_{vit} - c_{Kf}$$

Differentiating (18) with respect to  $K_{vi}$ , noting that all other variables including  $P$ , the general price level, are exogenous to  $K_{vi}$  yields the first order condition (19).

$$K_{vit} = \left[ \frac{V(1-\alpha)(1-\frac{1}{\epsilon})}{c_1 E[P_t]} E_{t-1}[P_t] \left[ Y_{dit}^{\frac{1}{\epsilon}} \left(\frac{\alpha(1-\frac{1}{\epsilon})}{\frac{W_t}{P_t}}\right)^{\alpha(1-\frac{1}{\epsilon})} \right]^{\frac{1}{1-\alpha(1-\frac{1}{\epsilon})}} \right]^{\frac{1-\alpha(1-\frac{1}{\epsilon})}{1-\alpha(1-\frac{1}{\epsilon})-V(1-\alpha)(1-\frac{1}{\epsilon})}} \quad (19)$$

As before the firm's choice problem at  $t-1$  concerns time  $t$  only because, while the variable capital stock is irreversible for one period, there are no multi-period irreversibility effects. Key additional simplifying assumptions underlying (19) are that we do not consider the effects of i) short run real interest rates on the cost of capital (when in practice interest rates are likely to be the policy instrument), ii) credit or financial constraints on firms' investment, and iii) convex adjustment costs. These assumptions are made for tractability and relaxing them would enhance the realism of the model. It may be noted that the first two would tend to reinforce the effect of

expected output in (19). For example real interest rates would tend to be high at a time of low output when financial constraints would also be more than usually severe, whereas adjustment costs would slow down the change in capital stock. The empirical success of output as an explanatory variable for investment relative to the cost of capital (Caballero, 1999) and Cooper and Haltiwanger's (2000) finding that the literature has overstated the importance of convex adjustment costs, also suggests that (19) may roughly approximate the capital stock dynamics that we wish to capture. That having been said a much more sophisticated analysis would be required to calibrate  $V$  (or ideally to disaggregate capital further), though we cite evidence below that suggests that  $V$  is likely to be significantly above zero.

With  $K_f$  normalised to unity the aggregate firm level capital stock is given by (19) raised to the power  $V$  from which, taking logs and dropping the  $i$  subscript to move to an aggregate expression across all firms, gives the log desired capital stock in (20) where the constants  $c_2$  and  $c_3$  are given by (A4) and (A5) in the Appendix.

$$k_t = c_2 + c_3 [ E_{t-1}[Y_{dit}] - \alpha(\epsilon-1)(w_t - E_{t-1}[p_t]) ] \quad (20)$$

Equation (20) shows that the capital stock depends positively on expected output and negatively on the expected real wage, the signs of these relationships following the effect of these variables on profitability. Since expected output depends on the expected policy stance, (20) shows the connection between policy expectations and the level of the capital stock which in turn affects the policy set through (9). The coefficient  $c_3$  equals unity if  $V=1$  (all capital is variable) and tends to zero if  $V$  tends to zero and the capital stock becomes fixed. The steady state level of the capital stock,  $k^*$ , may readily be derived from (20) having substituted for  $y^*$  from (A1) and the steady state real wage from (11) with  $K$  and  $Y$  at their steady state values, the outcome being

given by (A2) in the appendix.

## 2. The Phillips Curve/Aggregate Supply With Endogenous Wages and Capital Stock

We now derive the Phillips curve with both wages and capital endogenous. Substituting (15), (20), (A2) and (A3) into (9) gives (21) where  $\gamma=(1-\alpha)/\alpha$  and  $c_4$  is given by (22).

$$\pi_t = \begin{aligned} & (1-\lambda)E_{t-1}[\pi_t] + \lambda\pi_{t-1} + \theta_t \\ & + \gamma[y_t - y^* - V(E_{t-1}[y_t] - y^*)] + (1-\lambda)c_4(E_{t-1}[y_t] - y^*) \end{aligned} \quad (21)$$

$$c_4 = \frac{\frac{1}{\epsilon} + V(1-\alpha)(1 - \frac{1}{\epsilon})}{1 + \beta[1 - \alpha(1 - \frac{1}{\epsilon})]} \quad (22)$$

The first two terms in (21) arise from wage setting (forwards and backwards components respectively), which may be seen from (15) and (9), and the shock term is carried through from (9). On the second line the first term combines a term in  $(y_t - y^*)$ , which arises directly from the original ex post Phillips curve (9), and the term  $-V(E_{t-1}[y_t] - y^*)$  which arises from the endogeneity of the capital stock, showing that an expectation of output below its natural rate will, ceteris paribus, raise inflation. The mechanism for this is that low expected output implies low expected profitability and, from (20), a low capital stock in turn gives rise to higher prices from (7) and hence higher inflation from (9), than otherwise. Hence the endogeneity of the capital stock works against the policy maker by reducing the effectiveness of anticipated policy.

The last term in (21) captures the impact of expected quantities on the forward looking part of

wage setting. If workers care about employment as well as real wages (finite  $\beta$  so  $c_4 > 0$ ), expectations of future output matter for wage setting because output will affect labour demand and employment. From (22) this effect is stronger the more variable is the capital stock (larger  $V$ ) since labour demand will vary more with expected output if the capital stock is also changing but is still present if  $V=0$ . Most generally the second line of (21) shows that quantity as well as inflation expectations influence the position of the short run Phillips curve when inflation is persistent.<sup>14</sup>

Having analysed the behavioural basis for the terms in (21) it is convenient to express it more compactly. We take expectations of (21) to derive  $E_{t-1}[y_t] - y^*$  which is shown in (23) and combines with (21) to give (24) where  $\lambda_n$  in (23) and (24) is defined in (25).

$$E_{t-1}[\pi_t] - \pi_{t-1} = \frac{\gamma}{\lambda_n} (E_{t-1}[y_t] - y^*) \quad (23)$$

$$\pi_t = (1 - \lambda_n) E_{t-1}[\pi_t] + \lambda_n \pi_{t-1} + \gamma (y_t - y^*) + \theta_t \quad (24)$$

$$\lambda_n = \frac{\gamma \lambda}{(1 - \lambda) c_4 + \gamma (1 - V)} \quad (25)$$

From (24) it may be seen that  $\lambda_n$  is the key parameter determining the dynamics of inflation and the importance of expectations. From (25) with (22) it is straightforward to derive the following properties for  $\lambda_n$ :  $\lambda_n = 0$  if  $\lambda = 0$ ,  $\lambda_n > 0$  if  $\lambda > 0$ ,  $d\lambda_n/d\lambda > 0$ ,  $d\lambda_n/dV > 0$  and  $d\lambda_n/d\beta > 0$ . As a base case

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<sup>14</sup>If the backward looking part of wage setting responded to lagged output and employment (a possibility ruled out above) there would also be terms in  $y_{t-1}$  in (21). Hence a low value of output at  $t-1$  would lower wages if  $y_{t-1}$  proxied for expected  $y_t$  but the opposite may occur if there were insider-outsider or more general hysteresis forces at work.

it may be noted that if the capital stock is fixed ( $V=0$ ) and forward looking wage setting is insensitive to expected labour demand ( $c_4=0$ ),  $\lambda_n$  is simply equal to  $\lambda$ . This corresponds to the case analysed by Clark et. al. (1999) and Mash (2000). It is convenient that these papers have already analysed the implications of the same reduced form as (24) since the endogeneity of the capital stock (through the parameter  $V$ ) affects the reduced form system only through  $\lambda_n$  with  $d\lambda_n/dV>0$  such that we can sign the effect of an increase in  $V$  (and hence endogenous capital stock effects) by making use of earlier results where relevant on the effect of an increase in  $\lambda_n$  (which simply equals  $\lambda$  in the earlier papers).

If we keep the capital stock fixed ( $V=0$ ) but allow for wage setting to respond to expected labour demand ( $c_4>0$ ),  $\lambda_n$  decreases and the degree of 'backward-lookingness' in (24) falls. If we now allow for an endogenous capital stock ( $V>0$ ),  $\lambda_n$  increases and the system becomes more backward looking in its reduced form (despite capital stock setting being entirely forward looking). A particular special case arises when the parameter values are such that  $\lambda_n=1$  when the reduced form Phillips curve becomes entirely backward looking. The following section shows that  $\lambda_n>1$  implies disinflation bias rather than the standard result of inflation bias (when  $\lambda_n<1$ ) and hence the possibility of  $\lambda_n>1$  is of interest. More generally the increase in  $\lambda_n$  from capital endogeneity contributes to our understanding of inflation persistence. Figure 1 plots (25) for different values of  $V$  with parameter assumptions  $\alpha=0.65$ ,  $\beta=0.5$ ,  $\epsilon=5$  (the outcome is not particularly sensitive to these values). The figure shows that, i) for any value of  $V$   $\lambda_n=0$  if  $\lambda=0$  (capital endogeneity does not create inertia in the Phillips curve if wage setting is fully forward looking), ii) with a fixed capital stock the degree of persistence is reduced via the sensitivity of forward looking wages to expected quantities (finite  $\beta$  in (22)) in that  $\lambda_n<\lambda$  if  $V=0$ ; but iii) the effect is more than offset if  $V=0.33$  which is not especially large, and iv) in general, flexibility

in the capital stock has quantitatively significant effects on  $\lambda_n$ , the degree of inflation persistence in the reduced form Phillips curve.

In relation to empirical evidence, Gali, Gertler and Lopez-Salido (2001), Rudd and Whelan (2000), Rudebusch and Svensson (1999, 2002) and Fuhrer (1997b), while sometimes preferring other specifications, report reduced form results consistent with  $\lambda_n$  in the neighbourhood of unity. Rudebusch (2000) discusses the empirical literature in detail and suggest an approximate 95% confidence interval for the backward looking part of inflation (corresponding to  $\lambda_n$ ) to be between 0.4 and 1. His own preferred estimate is 0.7 while many others are in the upper part of that range.<sup>15</sup>

There is rather less evidence on possible values of  $V$ , the proportion of the capital stock which is flexible and endogenous to the monetary policy stance. Edge (2000a, 2000b) presents data showing that for the US (excluding residential investment, which is not relevant here) approximately 45% of investment is completed within 6 months and 60% within 12 months. This suggests that in terms of lags, a significant proportion of the capital stock is potentially variable within the time horizon of the effects of monetary policy. Investment with fairly short lead times also tends to be for equipment which in turn tends to have higher rates of depreciation than other forms such as structures, thus making this part of the capital stock more flexible downwards beyond its lead time also. Cooper and Haltiwanger (2000) present US plant level data showing that approximately 50% of manufacturing investment is accounted for by 'spikes' when investment is more than 20% of existing capital. Periods of zero investment are also

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<sup>15</sup>These figures should be interpreted with caution in the context of the model since the empirical estimates typically estimate the Phillips curve with a term in  $E_t[\pi_{t+1}]$  rather than  $E_{t-1}[\pi_t]$ .

common which, with a combined depreciation and capital retirement rate of around 10%, suggests significant downward flexibility in capital on a one year time horizon also. Thus while it is difficult to favour any particular value or narrow range for  $V$ , it is plausible that  $V$  is significantly above zero in which case, if  $\lambda$  is also above zero, Figure 1 shows that capital endogeneity would have a significant impact on the degree of inflation persistence.

This has important implications for the interpretation of reduced form evidence on the Phillips curve when estimates of inflation on its own lags (and a driving variable) typically have a set of coefficients on the lags which sum to unity or close to it. Given that an endogenous capital stock raises  $\lambda_n$  for a given  $\lambda$  from (25), the reduced form evidence can suggest values of  $\lambda_n$  towards unity without wage setting being completely backward looking. For example, from Figure 1 a value of  $\lambda_n$  of 0.7 (Rudebusch's preferred estimate) requires approximate values of  $\lambda$ , the fraction of backward looking wage setters, of 0.75 if capital is fixed but only a little over 0.5 if  $V=0.5$  and 0.35 if  $V=1$ . Hence the empirical results may be much more consistent with forward looking behaviour than they would otherwise appear, though this still requires the backward looking fraction of wage setters to be significantly above zero.

### **3. Policy Choices**

We turn to an analysis of monetary policy in the model above. This is done for its intrinsic importance, to demonstrate that capital endogeneity matters for policy, and to show that the structural inflation persistence in (24) implies observed persistence in both inflation and output. The policy maker's loss function is standard and given by (26) which is shown for period zero

onwards.<sup>16</sup> In (26)  $d$  is the discount factor ( $d < 1$ ),  $\phi$  the relative aversion to inflation, and  $\mu$  the parameter which determines the target level of output relative to its steady state or natural rate level. If  $\mu > 1$ , target output is greater than the natural rate and inflation bias is the standard result with a fixed capital stock.<sup>17</sup> We leave  $\mu$  general rather than imposing a fixed value and report results for  $\mu = 1$  and  $\mu > 1$ . A feature of the model is that even if  $\mu = 1$  so there is no inflation or disinflation bias in the steady state there may nevertheless be a time inconsistency in the transition back to the steady state.<sup>18</sup>

$$L_0 = \sum_{t=0}^{\infty} d^t [\phi \pi_t^2 + (y_t - \mu y^*)^2] \quad (26)$$

We consider outcomes under an optimal policy rule and optimal discretionary policy (defined in a standard way) when the policy maker has the same loss function preference parameters as society. Since the formal policy analysis is complex we briefly show how the key results arise using Figures 2-3. The vertical lines correspond to the natural rate of output and it is assumed that the past history of shocks and policy gives rise to a level of inflation in the previous period where the "Expectations fulfilled" lines cross the vertical Phillips curve. For simplicity it is assumed that there is no further shock in the time period illustrated. Starting with the general

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<sup>16</sup> In Steinsson (2000) where inflation persistence is introduced through backward looking 'rule of thumb' behaviour as here, the model based loss function is shown to include a term in the change in inflation. This is because larger changes in inflation will induce larger errors (and thus larger utility losses) by backward looking agents. We retain the usual loss function (26) to show the consequences of capital stock endogeneity with inflation persistence in an otherwise standard setting.

<sup>17</sup> See Bean (1998), Blinder (1997) and McCallum (1997) for arguments in favour of  $\mu = 1$  rather than the otherwise standard  $\mu > 1$ , at least with an independent central bank.

<sup>18</sup> This was shown by Clark et. al. (1999) and elaborated in Mash (2000) for inflation persistence with a fixed capital stock and occurs in this model also, the difference being that with an endogenous capital stock the bias may be disinflationary (too rapid return to steady state) rather than inflationary (return to steady state too slow).

issue of how an endogenous capital stock affects optimal policy and outcomes, from (23) it is clear that an increase in  $V$  (more flexibility in the capital stock) increases the output costs of anticipated changes in inflation by increasing  $\lambda_n$ . This is shown by the "Expectations fulfilled" lines which plot (23) and have gradient  $\gamma/\lambda_n$  and thus the steeper line in Figure 2 compared with Figure 3 corresponds to a lower value of  $\lambda_n$ . These lines would become vertical if  $\lambda_n=0$  and there was no inflation persistence. If  $\lambda_n$  increases this line becomes flatter (Figure 3) and the set of inflation-output combinations available deteriorates. If the policy maker can commit to a rule, and thereby influence expectations in advance, the optimal policy is where the best available contour of the loss function (26) is a tangent to the lines representing (23) as shown by the points marked R. Hence it may be seen that as  $\lambda_n$  increases and the line becomes flatter the optimal policy choice involves higher loss and a smaller reduction in inflation (and hence higher output). Thus outcomes after a shock away from steady state worsen with an endogenous capital stock, and the more so the more flexible it is since  $d\lambda_n/dV > 0$ , and the return of inflation to its steady state value will tend to be more gradual. A further implication, not readily shown by the figures, is that the optimal contemporaneous response to a shock will be more vigorous. This is because higher immediate output losses to offset the effect of a shock are warranted given the higher total losses that will be sustained along the subsequent path to steady state when capital is endogenous.

Figures 2-3 also show outcomes under discretion and hence the time inconsistency results. Under discretion (with no further shock as before) the outcome also lies on the "Expectations fulfilled" lines but the marginal condition is that the point chosen will be where the ex post Phillips curve, which has gradient  $\gamma$  from (9), is a tangent to the best available contour of the loss function. These points are marked 'D' in each figure. The time inconsistency results follow

directly from the relative gradients of the 'Discretion' and 'Expectations Fulfilled' (Rule) Phillips curves which are  $\gamma$  and  $\gamma/\lambda_n$  respectively and hence their relative magnitude depends on whether  $\lambda_n$  is greater than or less than unity. In Figure 2,  $\lambda_n < 1$  so the discretion line is flatter, point D is above R and there is an inflation bias during the transition back to steady state (the result first shown by Clark, Goodhart and Huang, 1999). Mash (2000) shows that this bias is present in the steady state also if  $\mu > 1$  in the loss function. In Figure 3  $\lambda_n > 1$ , the discretion line is steeper than the rule line, D is below R and there is a *disinflation* bias under discretion. As before these results carry over to the steady state if  $\mu_n > 1$  but apply in the transition to steady state for any value of  $\mu$ . We make no specific claim that  $\lambda_n > 1$  and thus disinflation bias the most likely outcome but demonstrate analytically that it is possible.

We turn to a formal analysis of optimal policy choices under a rule and discretion using dynamic programming. The policy maker in both cases minimises the value of the loss function (26) subject to the constraint of the reduced form Phillips curve (24). We present the outcome in summary form for each regime with detailed derivations given in the appendix.

With a commitment technology assumed to be available the optimal policy rule is given by (27) where  $\pi_s^R$  is the steady state or long run average inflation rate shown in (28). In these expressions  $\rho^R$  is the positive root of the quadratic (29).

$$\pi_t = \frac{\rho^R}{\lambda_n^2 + \rho^R} \pi_s^R + \frac{\lambda_n^2}{\lambda_n^2 + \rho^R} \pi_{t-1} + \frac{1}{1 + \rho^R} \theta_t \quad (27)$$

$$\pi_s^R = \frac{y^*(\mu-1)(1-d)\lambda_n}{\gamma\phi} \quad (28)$$

$$(\rho^R - \gamma^2\phi)(\lambda_n^2 + \rho^R) - \rho^R\lambda_n^2d = 0 \quad (29)$$

Given (27)-(29) the annuity value of the minimised total expected loss under the rule is given by (30) which involves the long run average deviation of output (if  $\mu > 1$ ) and inflation (if  $\pi_s^R > 0$ ) from target and the loss from shocks in the last term. To avoid less informative transitional terms, (30) assumes that inflation at time -1 is equal to its long run value in the regime. This does not affect the results below (except when noted).

$$(1-d)E_{-1}[L_0] = [y^*(\mu-1)]^2 + (\pi_s^R)^2\phi + \frac{\sigma^2\rho^R}{\gamma^2(1+\rho^R)} \quad (30)$$

Under discretion the policy rule and outcomes are given by (31)-(34) which are analogous to (27)-(30). In this case  $\rho$  is the unique positive root to the cubic (33). The analytic expression for this is much too cumbersome to use but comparative static results may still be derived.

$$\pi_t = \frac{\rho}{(\lambda_n + \rho)}\pi_s + \frac{\lambda_n}{\lambda_n + \rho}\pi_{t-1} + \frac{1}{1+\rho}\theta_t \quad (31)$$

$$\pi_s = \frac{y^*(\mu-1)[\lambda_n(1-d) + \rho(1-d\lambda_n)]}{\gamma\phi(\lambda_n + \rho)} \quad (32)$$

$$(\rho - \gamma^2\phi)(\lambda_n + \rho)^2 - d\lambda_n^2\rho(1+\rho) = 0 \quad (33)$$

$$(1-d)E_{-1}[L_0] = [y^*(\mu-1)]^2 + \phi(\pi_s)^2 + \frac{\sigma^2}{\gamma^2}\left(\frac{\rho}{1+\rho}\right) \quad (34)$$

It may be noted that the coefficient on  $\pi_{t-1}$  in (31) corresponds to 'observed' inflation persistence

in the sense that it is equal to the simple correlation coefficient between values of inflation in successive time periods. While  $\lambda_n$  in (24) shows the structural effect of capital endogeneity on inflation persistence, this coefficient in (31) is of interest because it demonstrates how optimal policy under discretion responds to  $\lambda_n$  and thus the impact of capital flexibility. In addition, from (24), the output gap will have the same serial correlation properties as inflation, the common correlation coefficient being shown in Figure 4 for different values of  $V$  (with additional parameter assumptions of  $\phi=1, d=0.96$ ). Taking Figures 1 and 4 together a more flexible capital stock raises  $\lambda_n$  which in turn raises the degree of serial correlation in both inflation and output.

#### 4. Results

We state the results of the paper in propositions 1-8 below of which 1-2 concern the dynamics of inflation and output, 3-6 are general policy results corresponding to greater inflation persistence with flexible capital and 7-8 relate to the possibility in this model of  $\lambda_n > 1$ . These results are expressed in terms of greater flexibility of the capital stock (higher  $V$ ) which encompasses the case of a fixed vs. endogenous capital stock ( $V=0$  vs.  $V>0$ ). For the results concerning policy we make use of the derivatives in (35), which come from differentiating (29) and (33). The sign of  $d\rho^R/d\lambda_n$ , from which  $d\rho^R/dV > 0$  also, is straightforward, That for  $d\rho/d\lambda_n$  is more problematic given the cubic structure to (33) but we may show that  $d\rho/d\lambda_n$  and hence  $d\rho/dV$  are positive if  $\lambda_n < 2$ . This is a sufficient (but not necessary) condition and covers the whole empirically plausible range of  $\lambda_n$  so we assume  $\lambda_n < 2$  and hence  $d\rho/dV > 0$ .

$$\frac{d\rho^R}{d\lambda_n} = \frac{2d\lambda_n(\rho^R)^2}{(\lambda_n^2 + \rho^R)^2 - d\lambda_n^4} > 0 \quad ; \quad \frac{d\rho}{d\lambda_n} = \frac{2d\rho^2\lambda_n(1+\rho)}{(\lambda_n + \rho)^3 - d_b\lambda_n^2(\lambda_n + 2\rho\lambda_n - \rho)} \quad (35)$$

*Proposition 1. Greater flexibility of the capital stock increases the degree of inflation persistence in the reduced form Phillips curve if the proportion of backward looking wage setters is strictly positive ( $d\lambda_r/dV > 0$  if  $\lambda > 0$ ). This follows directly from (24).*

*Proposition 2. Greater flexibility of the capital stock decreases the optimal speed at which inflation returns to target after a shock and increases the serial correlation of inflation and output in reduced form under the optimal rule and discretion. This follows from differentiating the coefficient on  $\pi_{t-1}$  in (27) and (31) with respect to  $\lambda_n$  using (35) subject to a sufficient but not necessary condition for the discretion case of  $\lambda_n \leq 1$ . Numerical analysis shows that this result holds with  $\lambda_n$  substantially above unity. These outcomes were shown in Figures 2-3.*

*Proposition 3. Greater flexibility of the capital stock matters for optimal monetary policy (under both rules and discretion) when inflation is persistent ( $\lambda > 0$ ) unless the policy maker's loss function places zero weight on output deviations ( $\phi$  tends to  $\infty$ ). The policy maker's optimisation of (26) is subject to the constraint (24) which depends on  $\lambda_n$  which in turn depends on  $V$  from (25) unless  $\lambda=0$  (or  $\phi$  tends to infinity).*

*Proposition 4. Greater flexibility of the capital stock increases the total loss from optimal monetary policy under a rule. The total loss under the optimal rule is given by (30) which is increasing in  $\lambda_n$  and thus  $V$ . The first term is a constant, the second is positive and increasing in  $\lambda_n$  from (28) unless  $\mu=1$ ,  $d=1$  or  $\phi$  tends to infinity in which case it is constant at zero irrespective of  $\lambda_n$  and  $V$ , and the third term is also increasing in  $V$  because  $d\rho^R/dV > 0$ .*

*Proposition 5. Greater flexibility of the capital stock increases the loss arising from shocks away*

from the steady state under optimal discretionary policy. This follows from the third term in (34) being increasing in  $\rho$  and thus  $V$  given  $d\rho/dV > 0$  as noted above. This result readily extends to the total loss under discretion, through the second term in (34), if either  $\mu=1$  (when the first two terms in (34) disappear) or  $d=1$  for any  $\mu$  in which case the absolute value of the steady state inflation rate in (32) and thus the second term of (34) is increasing in  $\rho$ . We do not consider the case of  $\mu > 1$  and  $d < 1$  further since it requires an explicit analysis of the initial inflation rate at the start of this regime.

*Proposition 6. Greater flexibility of the capital stock increases the optimal contemporaneous inflation-stabilising response to shocks under both the optimal rule and discretion.* From the last terms of (27) and (31), optimal policy immediately offsets a larger amount of the effect of a shock on inflation the higher is  $V$  given that  $d\rho^R/dV > 0$  and  $d\rho/dV > 0$ .

*Proposition 7. There is a state-contingent inflation bias during the return to steady state if  $\lambda_n < 1$  but a state-contingent disinflation bias if  $\lambda_n > 1$ .* This result draws on the analysis of Clark et. al. (1999), though the capital stock is fixed in that paper, and was demonstrated in Figures 2-3. It concerns the speed of return of inflation to its steady state value after a shock under discretion relative to the optimal rule which is independent of the policy makers output target. From the coefficients on  $\pi_{t-1}$  in the rule and discretion regimes shown by (27) and (31), it is clear that there will be a disinflation bias if condition (36) is satisfied.

$$\frac{\lambda_n}{\lambda_n + \rho} < \frac{\lambda_n^2}{\lambda_n^2 + \rho^R} \Leftrightarrow \rho^R < \lambda_n \rho \quad (36)$$

The cubic equation (33) in  $\rho$  makes direct verification or otherwise of this condition difficult.

Clark, Goodhart and Huang (1999) show that it is not satisfied if  $\lambda_n < 1$ , so there is state contingent inflation bias, leaving open what occurs if  $\lambda_n > 1$ . We do, however, know that  $\rho = \rho^R$  if  $\lambda_n = 1$  and hence we differentiate both sides of the right hand version of (36) and use (35) evaluated at  $\lambda_n = 1$  and find that the derivatives of  $\rho$  and  $\rho^R$  are equal (and  $\rho$  is strictly positive) and hence  $\lambda_n \rho$  increases more quickly than  $\rho^R$ . Hence the condition is satisfied for small increases in  $\lambda_n$  above one and thus a disinflation bias is present in this case. This result may also be reinforced by an indirect argument. If  $\lambda_n > 1$ , condition (36) is satisfied if  $\rho > \rho^R$ . From (30) the loss from shocks under a rule is proportional to  $\rho^R / (1 + \rho^R)$  while under discretion, from (34), it is proportional to  $\rho / (1 + \rho)$ . These expressions are increasing in  $\rho^R$  and  $\rho$  respectively (and equal if  $\lambda_n = 1$ ) so if the loss from shocks under discretion is higher than under the rule, it must be the case that  $\rho > \rho^R$ . Given that the rule based policy maker can choose the discretionary outcome the loss from shocks under a rule must be no worse than that under discretion.

*Proposition 8. There is steady state inflation bias if  $\lambda_n < 1$  and  $\mu > 1$  and steady state disinflation bias if  $\lambda_n > 1$  and  $\mu > 1$ . There is no bias in the steady state inflation rate if  $\mu = 1$  for any  $\lambda_n$ .* From (32) and (28) we may derive (37), the sign of which indicates the presence of inflation bias (positive) or disinflation bias (negative). If  $\mu = 1$  neither bias is present.

$$\pi_s - \pi_s^R = \frac{y^*(\mu - 1)(1 - \lambda_n)[\rho + \lambda_n(1 - d)]}{\gamma\phi(\lambda_n + \rho)} \quad (37)$$

## 5. Conclusion

This paper has presented a microfounded model of monetary policy with an endogenous capital stock when there is a source of inflation persistence in the form of a backward looking fraction

of wage setters. The model captures linkages between expectations of the policy stance and levels of the capital stock, and the capital stock and firms' price setting behaviour which affects monetary policy outcomes. In particular it has analysed i) the effect of an endogenous capital stock on the dynamics of aggregate supply and inflation, ii) the impact of that endogeneity on optimal policy, and iii) the interaction between an endogenous capital stock and time inconsistency issues.

It was shown that, for a given (non-zero) degree of backward lookingness in wage setting, capital stock endogeneity makes inflation more persistent in reduced form (with numerical analysis showing that this linkage is quantitatively significant), thus helping to explain the empirical strength of this phenomenon. When this structural persistence effect is combined with optimal discretionary policy it was shown that capital endogeneity increases the degree of serial correlation in both inflation and output. The model also shows that expectations of output as well as inflation may be important in the Phillips curve with persistence.

Under both discretion and a credible rule, capital endogeneity gives rise to a more vigorous optimal contemporaneous response to shocks, but a slower optimal speed of return to steady state thereafter. It can also give rise to a *disinflation* bias in discretionary policy both in steady state if the output target is above the natural rate and in the return to steady state for any output target. We do not place undue weight on the disinflation bias results since they require parameter values which generate a coefficient on lagged inflation in the Phillips curve above unity but their possibility is of interest nevertheless. More generally, capital endogeneity was shown to be important for optimal policy and our understanding of the microfoundations of how that coefficient may take a value significantly above zero, perhaps close to unity, with much less

backward looking or indexing behaviour elsewhere in the economy than would otherwise be necessary.

Given the scarcity of research on the monetary policy implications of an endogenous capital stock the paper has aimed to present core results in a comparatively simple model. It is clear that the analysis could usefully be extended in a number of directions of which we highlight two. The first is that the capital stock was assumed to depend solely on expected profitability whereas in practice firms' investment decisions may be affected by the volatility of the return to capital. Volatility depends in part on policy choices and this effect could be brought in together with richer modelling of capital setting in general. The second is that the paper assumed wage and price setting decisions of the "Barro-Gordon-Rogoff" type leaving open the effect of capital endogeneity with New Keynesian staggered wage or price setting such that a term in expected future inflation appears in the Phillips curve. The major implication of that term (see Clarida et al., 1999) is that optimal policy (under a rule) after an inflation increasing shock, for example, commits to more negative inflation rates and output gaps in the future than would occur under discretion. With forward looking investment behaviour the capital stock will be lower than otherwise in response to those output gaps, thus at the margin reducing the desirability of that commitment path.

Figure 1:  $\lambda_n$  as a function of  $\lambda$ .

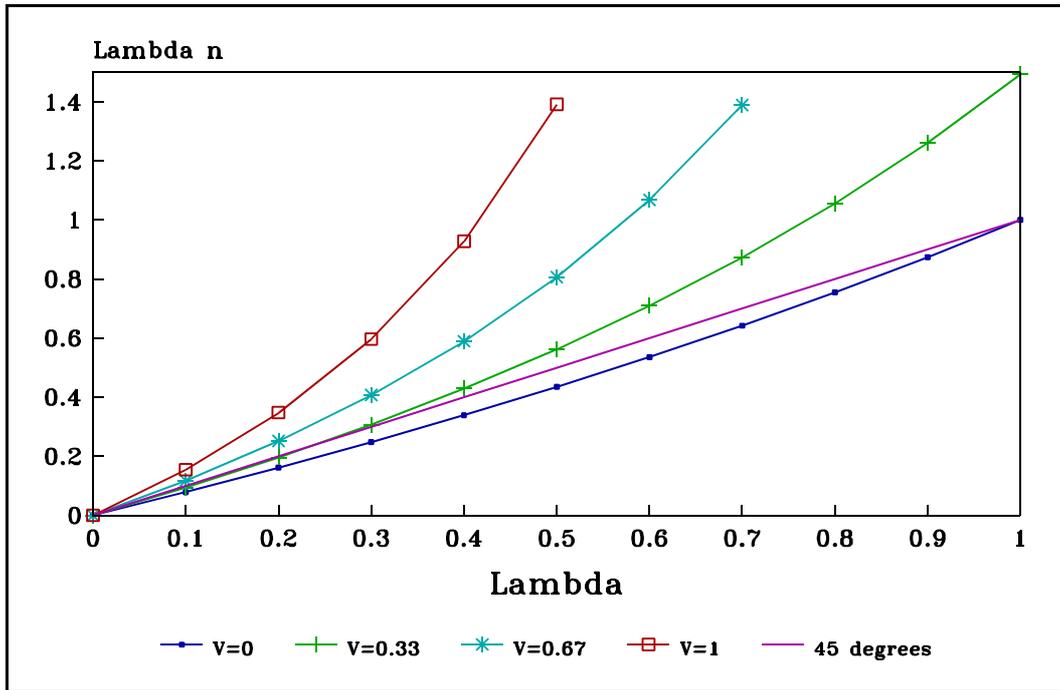


Figure 2: Policy Outcomes With  $\lambda_n < 1$

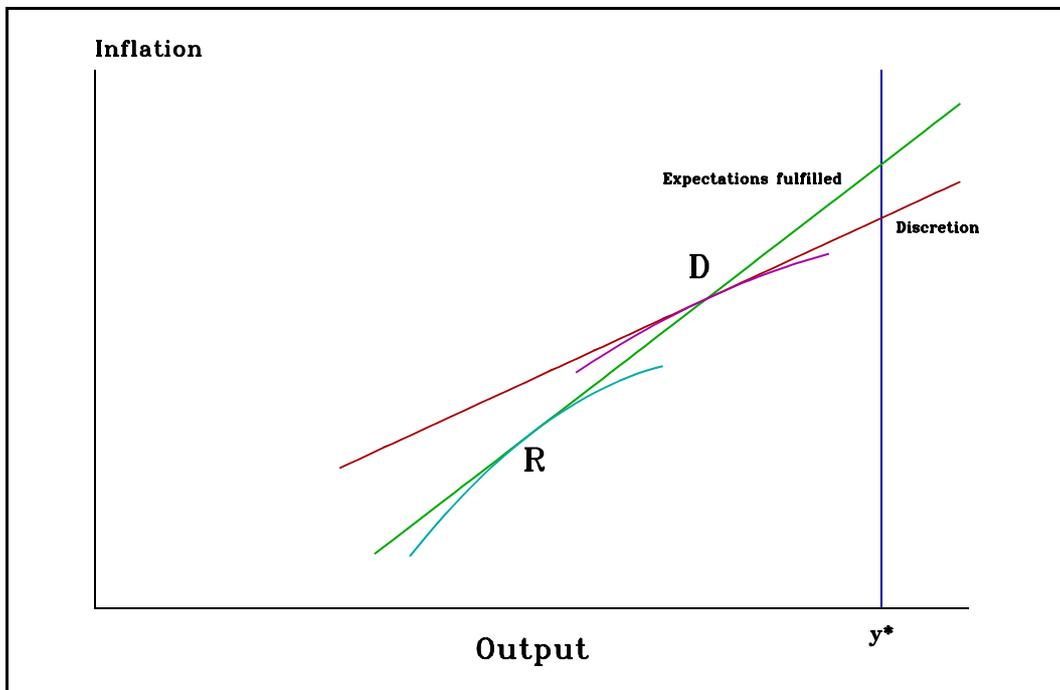


Figure 3: Policy Outcomes With  $\lambda_n > 1$

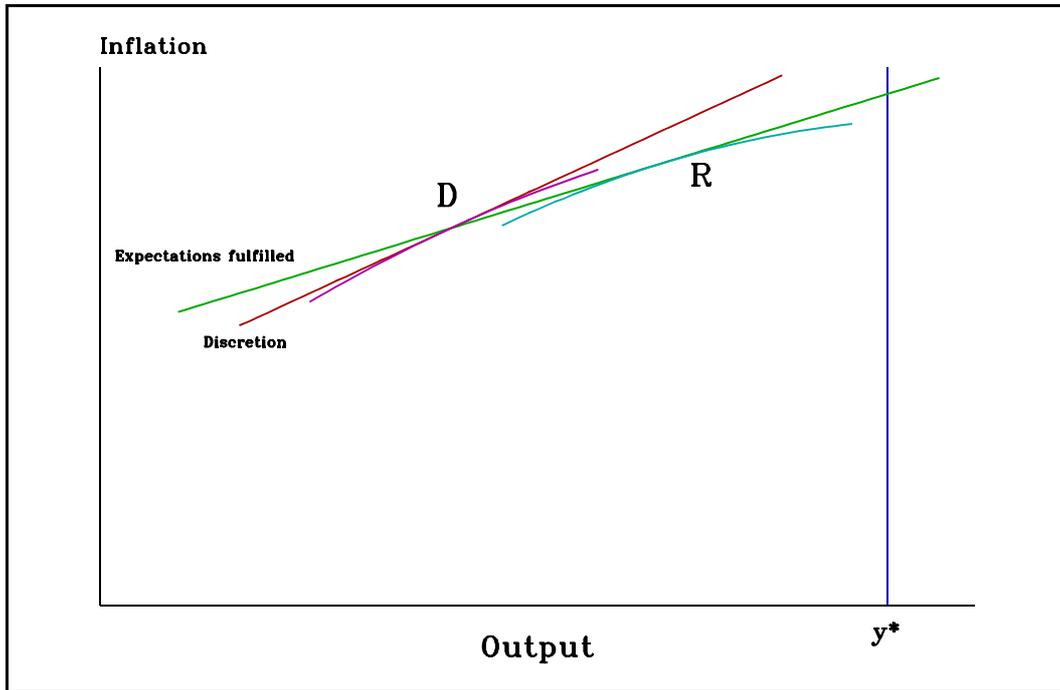
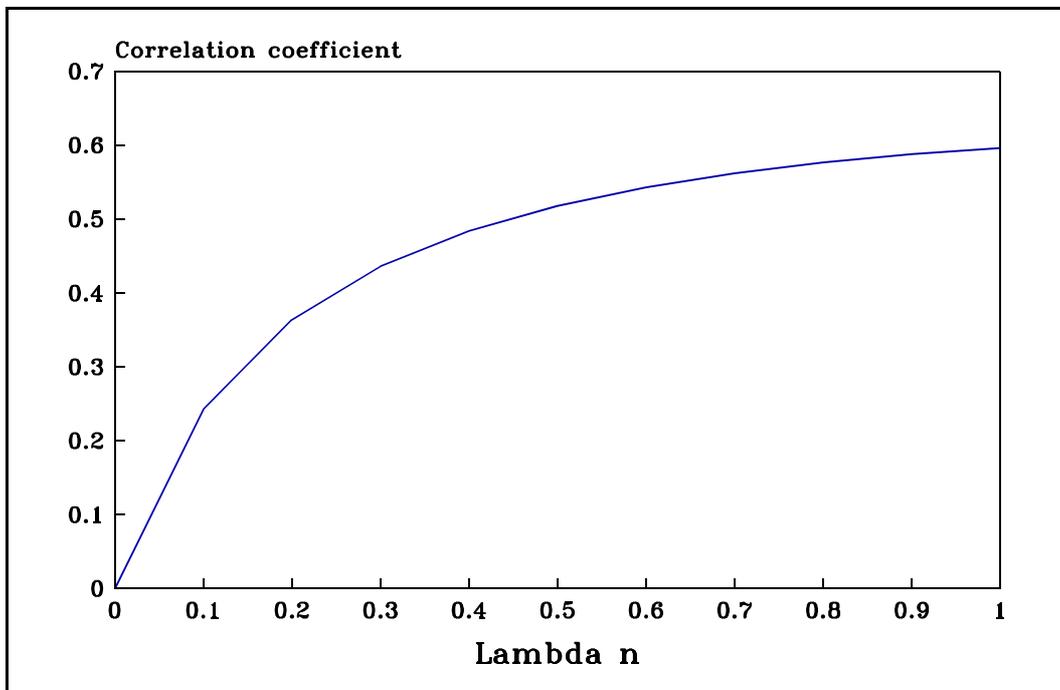


Figure 4: Simple (common) correlation coefficient between successive values of inflation and output ( $\pi_i, \pi_{i-1}$ ) & ( $y_i, y_{i-1}$ )



## Appendix

### Derivation of the cost of variable capital

We derive the cost of variable capital,  $c_{Kv}$  and in doing so state further assumptions concerning capital stock setting. First we assume that "variable" capital goods are an amalgam of other goods (so for simplicity there is no separate capital goods producing sector) and hence their price,  $P_{Kt}$ , is a constant multiple of  $P_t$ , the general price level so  $P_K/P$  is constant. This means that there are no non-linear adjustment costs from capacity constraints in capital goods supply. We also assume that any installation costs are linear in investment and are incorporated in  $P_K$ . Against this background we assume that firms order and pay for capital goods at  $t-1$  when their price is  $P_{K(t-1)}$ . We want to derive the nominal cost of capital evaluated at time  $t$  and thus the effective purchase price of a capital good at time  $t$  is  $P_{K(t-1)}(1+i_{t-1})$  where  $i_{t-1}$  is the nominal interest rate between  $t-1$  and  $t$  and is present because firms must pay for capital goods when they are ordered<sup>19</sup>. At time  $t$ , when the firm will order new capital for time  $t+1$ , the value of a unit of capital ordered at  $t-1$  and used during  $t$  is  $(1-\delta)P_{Kt}$  where  $\delta$  is the rate of depreciation. The cost of a unit of capital in period  $t$  is thus  $P_{K(t-1)}(1+i_{t-1})-(1-\delta)P_{Kt}$ . Making use of  $(1+i) = (1+r)(1+E[\Delta P/P])$ , where  $r$  is the real interest rate, and the assumed constancy of  $P_K/P$  this simplifies to (17) in the text.

### Derived constants and steady state values

We derive  $y^*$ , the steady state level of log output. For the time being this is done in terms of  $k^*$ , the steady state capital stock. The steady state is defined as the equilibrium of the system if there are no shocks, or equivalently the position to which it will return after a shock and inflation expectations are fulfilled. We may use (12) which shows the level of employment if wages could

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<sup>19</sup>It would make no difference if firms paid 'on delivery' at time  $t$  with the purchase price adjusted for the time cost of capital in the same way.

be negotiated ex post. Hence we derive  $Y^*$  as the level of output given by (1) when employment is given by (12) with  $Y$  in (12) equal to  $Y^*$ . This gives  $Y^*$  in log form and in terms of  $k^*$  by (A1). Once  $K$  is endogenised (see Section 1),  $k^*$  may be derived to give (A2) and thus  $y^*$  with  $k^*$  substituted out by (A3).

$$y^* = \frac{(1-\alpha)(1+\beta)k^* + \alpha\beta \text{Ln}[\alpha(1-\frac{1}{\epsilon})]}{1+\beta(1-\alpha)} \quad (\text{A1})$$

$$k^* = \frac{c_3\epsilon[1-\alpha(1-\frac{1}{\epsilon})][[1+\beta(1-\alpha)][\text{Ln}[V(1-\alpha)(1-\frac{1}{\epsilon})]-\text{Lnc}_1] + \alpha\beta \text{Ln}[\alpha(1-\frac{1}{\epsilon})]}{1+\beta(1-\alpha) - c_3(1-\alpha)[1+\beta-\alpha(\epsilon-1)]} \quad (\text{A2})$$

$$y^* = \frac{c_3[\epsilon - \alpha(\epsilon-1)](1-\alpha)(1+\beta)[\text{Ln}[V(1-\alpha)(1-\frac{1}{\epsilon})]-\text{Lnc}_1] + \alpha\beta[1+c_3(\epsilon-1)(1-\alpha)]\text{Ln}[\alpha(1-\frac{1}{\epsilon})]}{1+\beta(1-\alpha) - c_3(1-\alpha)[1+\beta-\alpha(\epsilon-1)]} \quad (\text{A3})$$

$$c_2 = \frac{V[1-\alpha(1-\frac{1}{\epsilon})][\text{Ln}[V(1-\alpha)(1-\frac{1}{\epsilon})]-\text{Lnc}_1] + \frac{\alpha(1-\frac{1}{\epsilon})\text{Ln}[\alpha(1-\frac{1}{\epsilon})]}{1-\alpha(1-\frac{1}{\epsilon})}}{1-\alpha(1-\frac{1}{\epsilon}) - V(1-\alpha)(1-\frac{1}{\epsilon})} \quad (\text{A4})$$

$$c_3 = \frac{1}{1 + \epsilon(\frac{1-V}{V})[1-\alpha(1-\frac{1}{\epsilon})]} \quad (\text{A5})$$

### Derivation of optimal policy with commitment

From the linear-quadratic structure of the model, the optimal policy rule will be of the form  $\pi_t = a + b\pi_{t-1} + c\theta_t$  where  $a$ ,  $b$  and  $c$  are pre-committed at values to be determined below. It is convenient to define the expected value of  $\pi_t$  by  $E_{t-1}[\pi_t] = \bar{\pi}_t = a + b\pi_{t-1}$  and use the notation  $\pi_t = \bar{\pi}_t + c\theta_t$ . Given  $E[\theta_t] = 0$  and the linearity of the policy rule in  $\theta$ , the optimal choice of  $c$  may be made independently from the optimal choice of the coefficients  $a$  and  $b$  that determine  $\bar{\pi}_t$  for given  $\pi_{t-1}$  so we may think of the policy maker optimising separately with respect to both  $\bar{\pi}_t$  and the response to shocks. As a first step towards deriving the optimal rule we may substitute  $\pi_t = \bar{\pi}_t + c\theta_t$  into (24) to yield (A6). We assume that the choice of rule is made in period  $t = -1$  and hence analyse the effect of that choice from period 0 onwards.

$$y_0 - \mu y^* = -y^*(\mu - 1) + \frac{\lambda_n}{\gamma}(\bar{\pi}_0 - \pi_{-1}) - \theta_0 \frac{(1-c)}{\gamma} \quad (\text{A6})$$

$$L_0 = \phi \pi_0^2 + (y_0 - \mu y^*)^2 + d[\eta_0^R + \eta_1^R \pi_0 + \eta_2^R \pi_0^2] \quad (\text{A7})$$

We define the value function under commitment by  $V^R(\pi_t) = \eta_0^R + \eta_1^R \pi_t + \eta_2^R \pi_t^2$  which is the expected value of the loss function from period  $t+1$  onwards, given adherence to the rule, discounted to period  $t+1$ , as a function of  $\pi_t$  (the value of the state variable inherited at the start of period  $t+1$ ). Using this value function and the short run Phillips curve under commitment from (A6), the loss function (26) may be expressed in terms of  $\pi_0$  by (A7), in which the last term represents the effect of the period 0 choice on the loss in future periods. Expanding (A7) using (A6) and taking its expected value gives (A8) where  $\rho^R = \gamma^2(\phi + d\eta_2^R)$ . Since the rule coefficients  $a$ ,  $b$  and  $c$  must be chosen in advance (at time  $-1$ ) they are optimally set to minimise the expected loss from period 0 onwards shown by (A8).

$$\begin{aligned} \gamma^2 E_{-1}[L_0] = & \bar{\pi}_0^2(\rho^R + \lambda_n^2) + \sigma^2[c^2\rho^R + (1-c)^2] \\ & + \bar{\pi}_0[\gamma^2 d\eta_1^R - 2\lambda_n^2\pi_{-1} - 2\lambda_n\gamma y^*(\mu-1)] \\ & + \gamma^2 d\eta_0^R + [\gamma y^*(\mu-1)]^2 + \lambda_n^2\pi_{-1}^2 + 2\lambda_n\gamma y^*(\mu-1)\pi_{-1} \end{aligned} \quad (A8)$$

From (A8) the first order conditions for  $\bar{\pi}_0$  and  $c$  are given by (A9) and (A10) which in turn yield (A11).

$$\gamma^2 \frac{\partial E[L_0]}{\partial \bar{\pi}_0} = 2(\rho^R + \lambda_n^2)\bar{\pi}_0 + \gamma^2 d\eta_1^R - 2\lambda_n^2\pi_{-1} - 2\lambda_n\gamma y^*(\mu-1) = 0 \quad (A9)$$

$$\gamma^2 \frac{\partial E[L_0]}{\partial c} = \sigma^2[2c\rho^R - 2(1-c)] = 0 \quad (A10)$$

$$\pi_0 = \frac{2\lambda_n\gamma y^*(\mu-1) - \gamma^2 d\eta_1^R}{2(\lambda_n^2 + \rho^R)} + \frac{\lambda_n^2}{\lambda_n^2 + \rho^R}\pi_{-1} + \frac{1}{1 + \rho^R}\theta_0 \quad (A11)$$

To solve for the  $\eta$  coefficients in the value function we set up the recursion equation (A12) using (A6).

$$\begin{aligned} \gamma^2(\eta_0^R + \eta_1^R\pi_{-1} + \eta_2^R\pi_{-1}^2) = & E_{-1}[[-\gamma y^*(\mu-1) + \lambda_n(\bar{\pi}_0 - \pi_{-1}) - (1-c)\theta_0]^2] \\ & + \gamma^2\phi E_{-1}[\pi_0^2] + E_{-1}[\gamma^2 d(\eta_0^R + \eta_1^R\pi_0 + \eta_2^R\pi_0^2)] \end{aligned} \quad (A12)$$

Substituting (A11) into (A12), with  $E[\theta_0]=0$ , and simplifying gives (A13).

$$\begin{aligned} \gamma^2(\eta_0^R + \eta_1^R\pi_{-1} + \eta_2^R\pi_{-1}^2) = & \gamma^2 d\eta_0^R + [\gamma y^*(\mu-1)]^2 - a^2(\lambda_n^2 + \rho^R) + \frac{\sigma^2\rho^R}{1 + \rho^R} \\ & + \pi_{-1}\left(\frac{2\lambda_n\gamma y^*(\mu-1)\rho^R + \gamma^2 d\eta_1^R\lambda_n^2}{\lambda_n^2 + \rho^R}\right) + \pi_{-1}^2\left(\frac{\rho^R\lambda_n^2}{\lambda_n^2 + \rho^R}\right) \end{aligned} \quad (A13)$$

Equating the coefficients on either side of (A13) in  $\pi_{-1}$  and its square, and the terms that do not involve  $\pi_{-1}$ , gives us Riccati equations for  $\eta_1^R$ ,  $\eta_2^R$ , and  $\eta_0^R$  respectively. These yield the quadratic in  $\eta_2^R$  given by (29) together with (A14) and (A15).

$$\eta_1^R = \frac{2y^*(\mu-1)\lambda_n \rho^R}{\gamma[\lambda_n^2(1-d)+\rho^R]} \quad (\text{A14})$$

$$\eta_0^R = \frac{1}{1-d} \left[ [y^*(\mu-1)]^2 \left[ 1 - \frac{\lambda_n^2(1-d)^2(\lambda_n^2+\rho^R)}{[\rho^R+\lambda_n^2(1-d)]^2} \right] + \frac{\sigma^2 \rho^R}{\gamma^2(1+\rho^R)} \right] \quad (\text{A15})$$

Substituting (A14) into (A11) allows us to derive (27) and the long run average inflation rate under the rule,  $\pi_s^R$  given by a/(1-b) shown by (28). This is greater than its target level of zero because, unlike the static case, the backwards looking element in (24) gives rise to a (small) tradeoff between inflation and output, even under a rule, and the quadratic structure of preferences means that it is optimal to exploit it to some extent. From the definition of the value function the expected loss from  $t=0$  onwards is given by (A16) which gives (30) once the  $\eta$  values are substituted from the expressions above together with the simplifying assumption that  $\pi_{-1}$  equals the subsequent steady state value given by (28).

$$(1-d)E_{-1}[L_0] = (1-d)[\eta_0^R + \eta_1^R \pi_{-1} + \eta_2^R \pi_{-1}^2] \quad (\text{A16})$$

### Derivation of optimal policy under discretion

It is convenient to define  $\pi_t^c = \lambda_n \pi_{t-1} + (1-\lambda_n)E_{t-1}[\pi_t]$ , the first two terms in (24) which, in the absence of commitment and unlike the case of a credible rule, the policy maker must treat as given. Once the (constrained) optimal choice of  $\pi_t$  has been made, its expected value solves for  $\pi_t^c$  given  $\pi_{t-1}$  and thus  $\pi_t$  in full. Using this notation (24) may be expressed by (A17).

$$y_t - \mu y^* = -y^*(\mu-1) + \frac{1}{\gamma}(\pi_t - \pi_t^c - \theta_t) \quad (\text{A17})$$

We make use of the value function  $V(\pi_t) = \eta_0 + \eta_1 \pi_t + \eta_2 \pi_t^2$  which is defined analogously to that under the rule. Using this and (A17) we may express the loss function of the policy maker from

period 0 onwards by (A18) in which the term  $dV(\pi_0)$  captures the effect of the period 0 decision on the loss from period 1 onwards.

$$L_0 = \phi \pi_0^2 + [-y^*(\mu-1) + \frac{1}{\gamma}(\pi_0 - \pi_0^c - \theta_0)]^2 + dV(\pi_0) \quad (A18)$$

The first order condition for the minimisation of (A18) implies (A19) in which  $\rho = \gamma^2(\phi + d\eta_2)$ .

$$\pi_0 = \frac{2\gamma y^*(\mu-1) + 2(\pi_0^c + \theta_0) - \gamma^2 d\eta_1}{2(1+\rho)} \quad (A19)$$

Taking expectations of (A19), noting that  $E_{-1}[\theta_0] = 0$ , allows us to solve for  $\pi_0^c$  in (A20) which may be substituted back into (A19) to give (A21).

$$\pi_0^c = \frac{2\lambda_n(1+\rho)\pi_{-1} + 2(1-\lambda_n)\gamma y^*(\mu-1) - (1-\lambda_n)\gamma^2 d\eta_1}{2(\lambda_n + \rho)} \quad (A20)$$

$$\pi_0 = \frac{2\gamma y^*(\mu-1) - \gamma^2 d\eta_1}{2(\lambda_n + \rho)} + \frac{\lambda_n}{\lambda_n + \rho} \pi_{-1} + \frac{1}{1+\rho} \theta_0 \quad (A21)$$

Rearranging (A18), making use of (A20), into terms in  $\pi_0$  and its square allows us to express the recursion equation for the policy maker's choice problem by (A22).

$$\eta_0 + \eta_1 \pi_{-1} + \eta_2 \pi_{-1}^2 = [y^*(\mu-1)]^2 + d\eta_0 + \left[\frac{\gamma}{2} d\eta_1\right]^2 + E_{-1}\left[\pi_0^2 \frac{\rho}{\gamma^2} (1+\rho) + \pi_0 d\eta_1 (1+\rho)\right] \quad (A22)$$

Substituting (A21) into (A22) and equating coefficients on both sides on  $\pi_{-1}$  and its square gives us two Riccati equations from which  $\eta_2$  and  $\eta_1$  may be determined. The first yields (33) and the second gives (A23).

$$\eta_1 = \frac{\eta_2 2\rho y^*(\mu-1)}{\gamma \lambda_n \phi} \quad (\text{A23})$$

Substituting (A23) into (A21) allows us to express the optimal choice of  $\pi_t$  in the absence of commitment by (31) where  $\pi_s$  is the long run average inflation rate given by (32). The expected loss under discretion in (34) follows in analogous fashion from that under the rule above.

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