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EQUILIBRIUM SELECTION AND PUBLIC GOOD PROVISION

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Abstract. Collective action problems arise in a variety of situations. The economic theory of public good provision raises a number of important questions. Who contributes to the public good, and who free rides? How might a social planner exploit the interdependence of decision-making to encourage contributions? Under what conditions will such actions result in public good provision? Using a simple game theoretic framework and recent results from the study of equilibrium selection, this paper attempts to answer some of these questions. Under reasonable assumptions of asymmetry and less than complete information, the more efficient agent will contribute. Contributions can be elicited by "integrating" the production process when agents are sufficiently *optimistic* about the success of the project. When this is not the case, the social planner may be better off "separating" the project so that individual contributions are independent of each other.

1. Collective Action Problems

1.1. **Public Good Provision.** A variety of community projects involve collective action.[1] For example, local environmental projects benefit everyone, yet only those who give up their time to contribute to such projects bear any of the costs. Alternatively, consider the provision of open-source software. All members of the community

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[1]The classic analysis of collective action problems appears in Olson (1965). The literature which followed is vast and continues to grow in both applied and theoretical directions, see for example, Bardhan, Ghatak and Karaivanov (2002) and Marx and Matthews (2000) respectively.

have access to the finished product, but only those who write the software spend time and energy on production.[2] Of course, the individuals involved will also benefit from the completion of the project, but not necessarily to an extent which outweighs their private cost. The social benefit may exceed the total cost, and yet due to the lack of incentives to privately contribute, a project might fail. In standard economic parlance this is a problem of public good provision.[3]

The under-provision of a public good stems from the positive externality which any level of production induces. Since no agent can be prevented from consuming the good (public goods are non-excludable) the social benefit of provision far outweighs an individual's private benefit. A common response to this under-provision problem is for the social planner themselves to provide the public good. This is the case for some of the most important public goods — the classic example being national defence. However, in many interesting cases, such governmental intervention is neither practical nor desirable. In these instances, private provision of the public good is associated with two distinct problems.

Firstly, even if private benefit outweighs cost, there is a problem of *free riding.* It is always in the interests of an individual to free ride on the efforts of others. Since public goods are non-excludable, as long as the good is being provided by some group of agents, each individual would rather be one of the agents who do not contribute to production. This leads to a coordination problem. Secondly, when private benefits are less than private costs, no individual will be willing to contribute to the project unilaterally and hence no provision will take place at all.[4] The situation resembles a Prisoners' Dilemma where it is socially optimal for all agents to contribute toward the project but none are willing to do so.[5]

1.2. **Eliciting Contributions.** There are three separate questions that arise from the previous discussion. In the case of free riding, who free rides and who provides?

[2]Raymond (1998) studies the way in which the operating system Linux has developed via the "part time hacking by several thousand developers scattered all over the planet". This is a prime example of a community project involving collective action and has already received the attention of economists — see for example, Johnson (2002).

[3]The now standard economic approach to the problem of public good provision is presented in the central contribution of Bergstrom, Blume and Varian (1986) and a useful diagrammatic exposition can be found in Ley (1996).

[4]Both these problems have attracted a great deal of interest, and not only from theorists. For a summary of the experimental approach to these problems, see Ledyard (1995).

[5]The use of game theoretic concepts has become standard in the public goods literature, text books commonly employ such language to introduce the problem, for example Cornes and Sandler (1996).

This question can only be answered meaningfully if there is some difference between the agents. In this paper one of the agents is assumed more efficient than the other (in the sense of having lower costs of contribution). It is shown in Result 1 that the more efficient player will contribute whilst the less efficient free rides. This is the socially optimal outcome.

The second question applies when agents have private values that are exceeded by their private costs. In this case they will not contribute to the production of a public good even though the social benefit (due to the positive externality effect) might easily dominate total cost. The question here is straightforward: How can contributions to the community project be elicited from the agents? In particular, can a social planner exploit the interdependence of decision making in such situations to extract optimal provision? Result 2 demonstrates that a possible solution to this problem is "integration" of the public good production technology.

A collective action problem is integrated if values are not realised unless all agents contribute to provision. If private benefits can be obtained by unilateral contribution to the project alone, the technology is said to be "separated". Consider the example of open source software provision. Suppose two agents are involved in the production of a piece of software, one is writing the source code whilst the other writes a graphical interface. If the two contributions are tied to each other so that neither is of any use without the other, the project is integrated. If, on the other hand, the two contributions are made so that they stand alone and are of independent value, the project is separated.[6] The immediate and final question is then: When does integration as opposed to separation provide higher levels of public good provision?[7] Result 3 shows that the benefits of integration depend critically upon the level of "optimism" the agents have concerning the cost of the project.

1.3. **Games and Coordination.** This paper uses the approach of the global games literature.[8] For simplicity, attention is restricted to 2×2 games. Two players simultaneously choose whether or not to contribute to a given community project or not.

[6]This distinction is reminiscent of the concepts "Cathedral" and "Bazaar" introduced in Raymond (1998). The cathedral corresponds (loosely) to integration and the bazaar to separation. Johnson (2002) uses the term "modularity" in place of separation.

[7]The production technology of a public good has long been known to be critical. For example, Varian (1994) compares provision when contributions are made sequentially as opposed to simultaneously.

[8]This literature began with the seminal contribution of Carlsson and van Damme (1993). An excellent overview of the key developments and a snapshot of the current state of the literature is provided by Morris and Shin (2002a).

The approach involves adding noisy components to the payoffs in the game. Players receive signals concerning these payoffs, both publicly and privately. Public signals are commonly observed by both agents whereas private signals are only received by the individual in question, although the distribution from which they are drawn is common knowledge. With this information players calculate their posterior on the variable of interest and equilibria are found in Bayesian strategies.[9]

Typically in such games, agents will adopt a cut-off rule, where if the posterior (or equivalently the signal) is above a critical level the agent chooses to play a particular strategy.[10] As the private signal becomes increasingly precise, agents select their action based on the public information alone which results in coordination on one or other of the pure strategy Nash equilibria in the underlying game. In this way equilibrium selection takes place — risk-dominance plays the crucial role and generates answers to the first two questions raised above.[11] Even without allowing private information to be extremely precise, however, something can be said about the equilibrium cut-off values themselves. It is this analysis that yields the third result mentioned above.

In the next section, the simple games used in the current analysis are introduced and discussed. Following that, the criterion of risk-dominance is brought to bear to select between multiple equilibria in symmetric and asymmetric games respectively (Sections 3 and 4). Finally the role of optimism within the integration/separation paradigm is discussed in Section 5. All mathematical proofs are in the appendix.

2. Some Basic Game Representations

2.1. **The Free-Rider Problem.** The simplest possible way in which to model the free-rider problem is illustrated in Figure 1. There is a single public good with some value to each agent, v. There are two potential contributors who must choose between contributing to the project (at a cost of c_i to agent i) or choosing not to — in which case they incur no cost. Represent these two strategies as C and D respectively. If

[9]The results presented here (and proven formally in Appendix A) can be readily applied to the case of many agents. This generates additional technicality and some new and important insights but is not crucial for the key ideas raised here.

[10]Applications, such as those found in Morris and Shin (1998) and (2002b), verify this claim.

[11]The risk-dominance criterion for selecting between multiple equilibria is due to Harsanyi and Selten (1988). The global game literature is not alone in its support of risk-dominance. The evolutionary stochastic adjustment dynamics literature, typified by Kandori, Mailath and Rob (1993) and Young (1993) also provides independent theoretical justification for the selection of risk-dominant equilibria.

neither contribute, no payoff is received. Initially, suppose if either agent contributes there is no way to prevent the other from extracting a value v from the provision of the good and that if both contribute no additional value is generated for either agent. The payoffs are as depicted in Figure 1(i). This will be referred to as a "separated" technology — the idea being that it is possible to obtain provision from the separate efforts of either individual.

	C	D
C	$v - c_2$ $v - c_1$	v $v - c_1$
D	$v - c_2$ v	0 0

(i) Separated

	C	D
C	$v - c_2/2$ $v - c_1/2$	0 $-c_1/2$
D	$-c_2/2$ 0	0 0

(ii) Integrated

FIGURE 1. The Free Rider Problem

One interesting case is when $v > c_i$ for $i = 1, 2$. In this instance, both $\{C, D\}$ and $\{D, C\}$ are equilibria. The selection problem that arises — who will contribute? — will be examined in Sections 3 and 4. If the higher of the costs exceeds the value then D will be dominant for that player and hence there will be a unique Nash equilibrium in which the lower cost agent contributes. In other words, there is no free-rider problem at all. However, if *both* of the costs exceed the value of the good there is a unique Nash equilibrium in which neither agent contributes. As long as the total value of the good ($2v$) exceeds the smaller of the costs, it is socially optimal for the good to be provided, but it will not be. The problem here is not one of selection, but one of provision.

A plausible solution is that of "integration". If technology is integrated provision *requires* the participation of both agents. The costs are assumed (for simplicity) to halve for both agents — the total cost of the project is $(c_1 + c_2)/2$.[12] If either agent chooses not to contribute to the project, it fails. This is the situation described in Figure 1(ii). The value of provision remains v to each agent and this value is only realised if both agents play C. When $2v > (c_1 + c_2)/2$ it is socially optimal to provide the good. It is an equilibrium for both agents to play C if $2v > c_i$ for $i = 1, 2$. However, it remains an equilibrium for both agents to play D — there is another selection problem. The solution to this problem is discussed in Sections 3 and 4.

[12]The results would change in an obvious way if the cost of the project were to be split in some other way. The case of most interest might be if only the lower of the two costs were to be paid when both agents contribute.

2.2. **The Positive Externality Problem.** Another case of interest is that of "positive externalities". The difference here is that when an agent contributes (again at a cost c_i) to the project, a value of v, common to both agents, is generated. Again, there is no way to prevent either agent from obtaining this value — the good is non-excludable. However, if both agents contribute a total value of $2v$ is generated. This situation is described in Figure 2(i). If $v > c_i$ for either player there is a dominant strategy (for agent i) to contribute. There is therefore no problem of provision if this is true for both agents. The interesting case is therefore when at least one $c_i > v$. Agent i will now have a dominant strategy D. As long as $2v > (c_1 + c_2)/2$, however, it is socially optimal for both agents to contribute. How might this be achieved?

	C	D
C	$2v - c_1$, $2v - c_2$	$v - c_1$, v
D	v, $v - c_2$	0, 0

(i) Separated

	C	D
C	$2v - c_1$, $2v - c_2$	$-c_1$, 0
D	0, $-c_2$	0, 0

(ii) Integrated

Figure 2. The Positive Externality Problem

Again, a plausible solution is that of "integration". The game in Figure 2(ii) illustrates this case. Here the agents no longer obtain a value from individual contributions — it takes both agents combined to generate the good. By making the project integrated in this fashion, the game becomes a simple coordination game (as long as $2v > c_i$ for $i = 1, 2$) with two pure Nash equilibria at $\{C, C\}$ and $\{D, D\}$. This game is identical to the one in Figure 1(ii) with all payoffs multiplied by a factor of two. It seems that in some instances integration might offer a solution by turning the socially optimal joint contribution outcome into a Nash equilibrium. However, an equilibrium selection problem results. The next section resolves this problem by the use of global game techniques and shows that the initial promise of integration does not altogether hold up to a closer analysis.

3. Equilibrium Selection

3.1. **Risk-Dominance.** The methods of the global game literature can be applied to problems such as this in order to select between multiple equilibria. In fact, as shown for the example of Figures 1(ii) and 2(ii) in the appendix, Section A.5, such techniques result in the selection of the risk-dominant equilibrium in 2×2 games such as the ones under consideration here. This is a consequence of a much more general result due to Carlsson and van Damme (1993).

In a 2×2 coordination game with two pure strategy Nash equilibria, an equilibrium is risk-dominant when the product of the differences between the payoff received in equilibrium and the payoff an agent would receive were they to unilaterally deviate is larger than the same quantity calculated for the other equilibrium. Suppose, for the duration of this section, that the game is symmetric, so that $c_1 = c_2 = c$. The risk-dominant equilibria can be found for each of the games presented in Section 2.

3.2. **The Free Rider Problem.** Consider a symmetric version of the games in Figure 1. Firstly, consider game (i). If $v > c$ there are two equilibria where one player contributes. Neither equilibrium is risk-dominant as the product of the equilibrium payoff minus the deviation payoff is $c(v-c)$ for both equilibria. Of course, symmetry is a strong (and unreasonable) assumption. Asymmetry alone can help to solve the selection problem here and will be dealt with in Section 4. If $v < c$ there is no selection problem as the game has a dominant strategy equilibrium $\{D, D\}$.

Now consider game (ii). As long as $v > c/2$ then $\{C, C\}$ is an equilibrium. This equilibrium will be risk-dominant if:

$$\left(v - \frac{c}{2}\right)^2 > \left(\frac{c}{2}\right)^2 \quad \Leftrightarrow \quad v > c$$

Recall that $v > c$ is precisely the condition for there to be no provision problem in any case: In fact, either agent would be willing to contribute, and the problem was one of *which* agent ought to contribute. When integration is desirable it is ineffective. The "wrong" equilibrium will be selected. This rather negative result is alleviated when weakening two of the critical assumptions. Firstly, asymmetry in Section 4 allows selection in the first game and under strong conditions equilibrium $\{C, C\}$ becomes risk-dominant in the second. Secondly, when weaker claims are made about the information available to players in the model (particularly concerning their costs) integration can have a positive role in the provision of public goods (see Section 5).

3.3. **The Positive Externalities Problem.** Consider symmetric versions of the games in Figure 2. Problems arise in the separated case when $v < c$ — the Nash equilibrium involves neither player contributing to the good, even though, as long as $2v > c$ it is socially optimal to do so. Integration seems to offer a possible solution to this problem since, in the case of interest ($2v > c$), there is an equilibrium where both players contribute. This equilibrium is risk-dominant whenever:

$$(v-c)^2 > c^2 \quad \Leftrightarrow \quad v > c$$

Again, in precisely the instance when integration might be of service, it is ineffective. The equilibrium where both players contribute is risk-dominated by the equilibrium where neither player contributes. Therefore it is not selected in a global game environment. However, as before, when the strong assumptions on symmetry and on the information available to players are replaced with more general formulations, integration does offer a solution to the public good provision problem. Sections 4 and 5 present precise conditions for which this is true.

4. Asymmetry and Selection

Asymmetry assists most directly with the problem of free-riding. Again utilising the techniques of the global game literature, risk-dominance (at least in these 2×2 games) resolves the equilibrium selection problem. Looking initially at the first game of Figure 1, when agents have different costs and there are two equilibria, $\{C, D\}$ risk-dominates $\{D, C\}$ if and only if:

$$c_2 (v - c_1) > c_1 (v - c_2) \quad \Leftrightarrow \quad c_2 > c_1$$

That is, if agent i is more efficient, the equilibrium involving that agent i contributing to the project is risk-dominant and hence selected. Combined with the fact that if $v > c_i$ for exactly one of the agents, there is only one equilibrium where the more efficient agent contributes, this yields the following result:

Result 1. *When public good provision involves a free-riding problem, but at least one agent obtains a value greater than their private cost, the most efficient agent will bear the cost of provision.*

Now consider the possibility of integration. Here, as long as $v > c_i/2$ for both agents, there is an equilibrium where both contribute. This will be risk-dominant if:

$$\left(v - \frac{c_1}{2}\right)\left(v - \frac{c_2}{2}\right) > \frac{c_1}{2}\frac{c_2}{2} \quad \Leftrightarrow \quad v > \frac{c_1 + c_2}{2}$$

This simply restates the earlier observation that when integration is desirable, it is ineffective. Integration is only desirable if both $v < c_1$ and $v < c_2$ but the social optimum involves a positive level of contribution. This rules out the above condition for risk-dominance — $\{C, C\}$ might be an equilibrium under integration but it is always risk-dominated by $\{D, D\}$. Nothing is gained by integration. It is in this case that the informational assumption needs to be relaxed (see Section 5).

In the case of Figure 2(i) it is possible that only one agent contributes when it is socially optimal for both to. Here, relaxing the symmetry result can be of value. Integration yields the second game. The risk-dominance condition for $\{C, C\}$ is:

$$(2v - c_1)(2v - c_2) > c_1 c_2 \quad \Leftrightarrow \quad v > \frac{c_1 + c_2}{2}$$

Given that one agent found it optimal to contribute in the separated case, it is quite possible that equilibrium $\{C, C\}$ is risk-dominant and therefore selected. Hence integration can be effective when it is desirable. The following result states this fact:

Result 2. *When there is a positive externality involved in the provision of a good, and at least one agent has a private value which exceeds their private cost, then integration can result in the socially optimal level of provision.*

Of course, this is only a possibility. Social optimality of dual provision corresponds to $v > (c_1 + c_2)/4$, whereas actual dual provision only takes place if $v > (c_1 + c_2)/2$. Hence there is a range of values v for which the good is not provided to the socially optimal level. Integration then results in no provision at all, whereas separation would at least result in partial provision if $v > c_i$ for some i.

In the next section, the assumption of complete information is relaxed. The results from this analysis reinforce the idea that integration *can* generate higher levels of public good provision, but will not always do so.

5. Optimism and Integration

So far, agents have been perfectly informed of the values and costs associated with contributing to the project. A much more likely scenario, however, is one where agents do not know the exact cost of the project before they carry it out. In Appendix A, a model is presented in which both agents receive a common public signal of the cost of the project and in addition a private signal that they alone observe.[13] The analysis considers an asymmetric game of the form found in Figures 1(ii) and 2(ii), a special

[13] As these private signals become more and more precise, only public (or common) information remains. It is precisely in this case that the game becomes one of complete information. This is the method by which an equilibrium is selected in Sections 3 and 4.

case of which is the symmetric game discussed in Section 3. The results, although tighter for symmetric games, produce similar qualitative statements.

When agents do not receive full information concerning the payoffs in the game, they must use the information available to them to form their strategies. Each agent has to choose whether or not to contribute to the public good project. Given an opponent's strategy, the cost of contribution determines the relative expected payoff of the two strategies available. The higher the cost the more unattractive contribution becomes. Given the public and private signal received, the agent calculates their posterior. If this posterior cost is sufficiently low they choose to contribute and if it is too high they choose not to. When both agents play such strategies, interest focuses upon the cut-off value of cost such that the other agent is exactly indifferent between their two strategies. Neither agent has any incentive a priori to change their strategy and hence this characterises an equilibrium. The analysis implicitly solves for such cut-off values and examines their properties.

In a symmetric game the equilibrium cut-off values (unsurprisingly) are the same for both players (see Proposition 2). In the asymmetric case, the agent with the cost advantage (the more efficient) has a higher cut-off value, and hence contributes with higher probability (see Proposition 1). However, the interesting results concern the way in which cut-off values change as public information changes. As public information worsens, the cut-off values fall. The higher the public cost signal, the lower the cut-off value and hence the lower the probability with which agents contribute to the public good. This key result is discussed in Proposition 3 for the symmetric case and Proposition 4 for the asymmetric case.

Suppose that the public information indicated that costs were exactly equal to v (and, for simplicity assume symmetry). In the "separated" case of Figure 2(i), public information alone leaves the agents entirely indifferent between their two strategies. Their decision to contribute or not will be based solely upon their private information. In the integrated case of Figure 2(ii) however, they *are* concerned with with whether or not their opponent will contribute. Indeed, if their opponent contributes with a probability of at least one half, they too find it optimal to contribute.

Now suppose that the public information indicated that costs were lower that v — an "optimistic" piece of news. In the separated case, little changes. Agents still base their actions upon their own private signals. Due to the separated nature of the technology, neither agent cares whether or not the other is contributing, and hence bases their decision entirely upon their own information — they will contribute if and only if their information indicates that their cost is below v. Under integrated technology, however, the action of the other player is of paramount importance. Now that the public information indicates that costs are low, an agent will believe it relatively more likely that their opponent has received a good signal and hence are themselves more likely to contribute. This reinforcement of good news causes their equilibrium cut-off value to fall below v. Therefore, the probability that the $\{C, C\}$ equilibrium is played increases by more than it does in the separated case. Integration yields a higher contribution level (probabilistically) when public information is optimistic.

The reverse argument holds when the public information indicates that the cost level is higher than v (a "pessimistic" piece of news). In this case, the fact that an opponent is more likely to have bad information leads to an unwillingness to risk contributing in the integrated case. Hence the probability that the $\{C, C\}$ equilibrium is played falls substantially. In the separated case, on the other hand, each agent continues to base their contribution decision on their own information alone — it only matters whether their costs lie above or below v.[14] This intuition is formalised in the appendix and summarised in the below result:[15]

Result 3. *When information is imperfect, if the public signal indicates that private costs lie below (above) private valuations, integration yields higher (lower) levels of public good provision than separation.*

A social planner that is able to integrate or separate public good provision technology in this manner may therefore have good reason to do so.

Appendix A. The Global Game Framework

A.1. **The Game.** This appendix presents the mathematical model underlying the previous discussions. Consider the following asymmetric two player (integrated) "positive externality" public good contribution game:[16]

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[14] Of course, according to Raymond (1998), the success of the Linux project was due to the "bazaar" (or separated) nature of the production technology. Was there then substantial pessimism over the potential benefits of the project at the outset? The opening paragraph of Raymond (1998) begins "... Who would have thought [...] that a world-class operating system could coalesce as if by magic..." Perhaps there was at least some pessimism in the early stages!

[15] A similar result and intuition holds with asymmetry, although the mathematics is less appealing.

[16] Notice that the asymmetric general costs c_1 and c_2 have been replaced by $c - \lambda$ and c respectively. Hence player 1 has a cost advantage throughout, and this is known. The uncertainty revolves around the actual value of c. This makes the analysis easier to follow, without affecting the main results.

	C	D
C	$2v-c$ $2v-c+\gamma$	0 $-c+\gamma$
D	$-c$ 0	0 0

The interesting case is when $2v > c > \gamma$, which is assumed throughout. Notice that $\{C, C\}$ is risk-dominant if and only if $v \geq c - \gamma/2$. Suppose that c is unknown, but that each player receives a public signal $\overline{c}$ and a private signal c_i such that:

$$\overline{c} = c + \varepsilon \quad \text{and} \quad c_i = c + \varepsilon_i \quad \text{where} \quad \varepsilon \sim \mathrm{N}(0, 1/\alpha) \quad \text{and} \quad \varepsilon_i \sim \mathrm{N}(0, 1/\beta).$$

A.2. **The Players' Beliefs.** Calculating player 2's posterior $\overline{c}_2$ yields:

$$\overline{c}_2 = \frac{\alpha\overline{c} + \beta c_2}{\alpha+\beta} = \frac{\alpha\overline{c}}{\alpha+\beta} + \frac{\beta c}{\alpha+\beta} + \frac{\beta\varepsilon_2}{\alpha+\beta} \tag{1}$$

Suppose player 2 will play C whenever $\overline{c}_2 \leq c_2^*$. Now, from player 1's perspective, $c \sim \mathrm{N}(\overline{c}_1, 1/(\alpha+\beta))$. From equation (1), $(\alpha+\beta)\overline{c}_2 - \alpha\overline{c} = \beta c + \varepsilon_2$ and hence:

$$(\alpha+\beta)\overline{c}_2 - \alpha\overline{c} \sim \mathrm{N}\left(\beta\overline{c}_1, \frac{\beta^2}{\alpha+\beta} + \beta\right)$$

Finally, from player 1's perspective:

$$\overline{c}_2 \sim \mathrm{N}\left(\frac{\beta\overline{c}_1 + \alpha\overline{c}}{\alpha+\beta}, \frac{\beta^2}{(\alpha+\beta)^3} + \frac{\beta}{(\alpha+\beta)^2}\right)$$

Player 1 can now calculate the probability that player 2 will contribute to the project:

$$\mathrm{Prob}(\overline{c}_2 \leq c_2^* | \overline{c}_1) = \Phi\left[\frac{c_2^* - (\beta\overline{c}_1 + \alpha\overline{c})/(\alpha+\beta)}{(1/(\alpha+\beta) + 1/\beta)^{1/2}\beta/(\alpha+\beta)}\right]$$

Now suppose player 1 will choose to contribute if and only if $\overline{c}_1 \leq c_1^*$. Evaluating the above when player 1 is exactly indifferent between the two strategies obtains:

$$\mathrm{Prob}(\overline{c}_2 \leq c_2^* | \overline{c}_1 = c_1^*) = \Phi\left[\frac{\frac{\alpha}{\beta}c_2^* + c_2^* - \frac{\alpha}{\beta}\overline{c} - c_1^*}{\sqrt{\frac{1}{\alpha+\beta} + \frac{1}{\beta}}}\right]$$

The following two equations then solve to give the cut-off values c_1^* and c_2^*.

$$2v\Phi\left[\frac{\frac{\alpha}{\beta}c_2^* + c_2^* - \frac{\alpha}{\beta}\overline{c} - c_1^*}{\sqrt{\frac{1}{\alpha+\beta} + \frac{1}{\beta}}}\right] = c_1^* - \gamma \tag{2}$$

$$2v\Phi\left[\frac{\frac{\alpha}{\beta}c_1^* + c_1^* - \frac{\alpha}{\beta}\overline{c} - c_2^*}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\right] = c_2^* \tag{3}$$

Proposition 1. *The cut-off values are such that:* $\gamma > c_1^* - c_2^* > 0$.

Proof. Suppose $c_1^* \leq c_2^*$. Then from equations (2) and (3):

$$2v\Phi\left[\frac{\frac{\alpha}{\beta}c_2^* + c_2^* - \frac{\alpha}{\beta}\overline{c} - c_1^*}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\right] + \gamma \leq 2v\Phi\left[\frac{\frac{\alpha}{\beta}c_1^* + c_1^* - \frac{\alpha}{\beta}\overline{c} - c_2^*}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\right]$$

Since $\gamma > 0$ it follows that $c_1^* > c_2^*$ — a contradiction. Subtracting the left hand side of equation (3) from the left hand side of equation (2) yields a negative number. The right hand side is $c_1^* - c_2^* - \gamma$. So $\gamma > c_1^* - c_2^* > 0$ as required. □

A.3. **The Symmetric Case.** Consider $\gamma = 0$. The following result is immediate.

Proposition 2. *If* $\gamma = 0$ *then* $c_1^* = c_2^* = c^*$ *where:*

$$c^* = 2v\Phi\left[\frac{\frac{\alpha}{\beta}(c^* - \overline{c})}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\right] \tag{4}$$

Proof. Again subtracting equation (3) from equation (2) yields $c_1^* - c_2^*$. Suppose $c_1^* > c_2^*$. The left hand side is negative, a contradiction. Likewise for $c_1^* < c_2^*$ — which implies $c_1^* = c_2^*$. Substitute into either equation to yield the value above. □

Notice that if the public information is just equal to the individual value, $\overline{c} = v$, then a solution is $c^* = v$. A sufficient condition for equation (4) to have a unique solution is that the right hand side has a slope less than 1 at c^*. So:

Condition 1. *There is a unique solution to equation* (4) *if:*

$$\frac{2\beta^2 + \alpha\beta}{\alpha^2(\alpha+\beta)} > \frac{2v^2}{\pi}$$

Proof. Differentiate the right hand side of equation (4) to yield:

$$2v\phi\left[\frac{\frac{\alpha}{\beta}(c^* - \overline{c})}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\right]\frac{\frac{\alpha}{\beta}}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}$$

This must be less than one. Notice that $\phi(\cdot)$ is maximised at $1/\sqrt{2\pi}$. After some manipulation the above condition is obtained. □

This condition will be satisfied for small α or large β — which are the cases of interest and is assumed to hold in the remainder. Hence the following key proposition in the symmetric case is available:

Proposition 3. *If Condition 1 is met,* $\bar{c} > v \Rightarrow c^* < v$ *and* $\bar{c} < v \Rightarrow c^* > v$.

Proof. Notice that $\Phi(\cdot)$ is an increasing function. A fall in $\bar{c}$ results in a rise in c^*. Combining that with the fact that if $\bar{c} = v$ then $c^* = v$ yields the result. □

This is precisely the optimism result discussed in Section 5. When public information is "bad" ($\bar{c} > v$), agents adopt a lower cut-off value $c^* < v$ and hence are *less* likely to contribute in the integrated case than they would be in the separated case.[17] A social planner would be better selecting a separated public good technology. The reverse is true when public information is "good" — in an optimistic world integration of public good technology is advantageous.

A.4. **The Asymmetric Case.** The following lemma is a first step toward a similar optimism argument to that given in the symmetric case.

Lemma 1. *The cut-off values for both agents are decreasing in public information:*

$$\frac{dc_1^*}{d\bar{c}} < 0 \quad \text{and} \quad \frac{dc_2^*}{d\bar{c}} < 0$$

Proof. Begin by totally differentiating equations (2) and (3). This gives:

$$\frac{2v\phi_1}{\sqrt{\frac{1}{\alpha+\beta} + \frac{1}{\beta}}} \left\{ \frac{\alpha}{\beta}\frac{dc_2^*}{d\bar{c}} + \frac{dc_2^*}{d\bar{c}} - \frac{dc_1^*}{d\bar{c}} - \frac{\alpha}{\beta} \right\} = \frac{dc_1^*}{d\bar{c}} \tag{5}$$

$$\frac{2v\phi_2}{\sqrt{\frac{1}{\alpha+\beta} + \frac{1}{\beta}}} \left\{ \frac{\alpha}{\beta}\frac{dc_1^*}{d\bar{c}} + \frac{dc_1^*}{d\bar{c}} - \frac{dc_2^*}{d\bar{c}} - \frac{\alpha}{\beta} \right\} = \frac{dc_2^*}{d\bar{c}} \tag{6}$$

$$\text{where} \quad \phi_i = \phi\left[\frac{\frac{\alpha}{\beta}c_j^* + c_j^* - \frac{\alpha}{\beta}\bar{c} - c_i^*}{\sqrt{\frac{1}{\alpha+\beta} + \frac{1}{\beta}}}\right] > 0 \quad \text{for} \quad i \neq j \in \{1,2\}$$

Label $D_i = dc_i^*/d\bar{c}$. If $D_i > 0$ then $(1 + \alpha/\beta)D_j - D_i - \alpha/\beta > 0$ and hence $D_j > 0$ for all i and j. Hence without loss of generality, suppose $D_1 \geq D_2$ and $D_2 > 0$. This implies that $(1 + \alpha/\beta)D_2 - D_1 - \alpha/\beta > 0$ and hence $(1 + \alpha/\beta)D_1 - D_1 - \alpha/\beta > 0$.

[17] In the separated case agents simply contribute when their information suggests cost is less than value — their cut-off level is always v.

So $D_1 > 1$. By equation (5) and $D_1 \geq D_2$:

$$\frac{2v\phi_1}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}}\left\{\frac{\alpha}{\beta}D_1 + D_1 - D_1 - \frac{\alpha}{\beta}\right\} \geq D_1$$

Which implies (since $D_1 > 1$) that:

$$\frac{\alpha 2v\phi_1/\beta}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}} \geq \frac{D_1}{D_1-1} > D_1$$

Recall that $\phi(\cdot)$ is maximised at $1/\sqrt{2\pi}$ and hence the above equation implies:

$$\frac{\alpha 2v/\beta\sqrt{2\pi}}{\sqrt{\frac{1}{\alpha+\beta}+\frac{1}{\beta}}} > D_1 > 1$$

This is ruled out by Condition 1 yielding a contradiction. So $D_1 < 0$ and $D_2 < 0$. □

The cut-off values are strictly decreasing in public information, guaranteeing uniqueness, and providing the first part of the "optimism" result of Section 5 in the asymmetric case. The central result is then:

Proposition 4. *Assume Condition 1 holds. Then* (i) $\bar{c} \leq v - \gamma\beta/\alpha \Rightarrow c_1^* > v + \gamma$ *and* $c_2^* > v$, *and* (ii) $\bar{c} \geq v + \gamma + \gamma\beta/\alpha \Rightarrow c_1^* < v + \gamma$ *and* $c_2^* < v$.

Proof. Consider $c_2^* = v$. By equation (2), $\alpha(c_1^* - \bar{c})/\beta + c_1^* - c_2^* = 0$. Hence $c_1^* = (\beta v + \alpha\bar{c})/(\alpha+\beta)$. Now from Proposition 1, $v + \gamma > c_1^* > v$. Substituting in the value for c_1^* yields:

$$v + \gamma + \frac{\beta}{\alpha}\gamma > \bar{c} > v$$

By Lemma 1, if $\bar{c} \geq v + \gamma + \gamma\beta/\alpha$ then $c_2^* < v$ and therefore via another use of Proposition 1, $c_1^* < c + \gamma$. This gives part (ii). For part (i) consider $c_1^* = v + \gamma$ and use equation (3) in a similar way. □

Again this reveals a very similar story to the symmetric case of Proposition 3. If public information is sufficiently "good" ($\bar{c} \leq v - \gamma\beta/\alpha$), the cut-off values are respectively above $v + \gamma$ and v. In the game with separated contributions, the optimal cut-off values for the two agents are exactly $v+\gamma$ and v respectively. Hence, integration will increase the likelihood of contribution. If public information is sufficiently "bad", the reverse is true. The difference with asymmetry is that, unlike in the symmetric case, there is now a range of ambiguity. The mid-point of this range is where public information is exactly equal to the critical risk-dominant value ($v + \gamma/2$). Even for very precise public information ($\alpha \to \infty$) the range is bounded away from this point, on $[v, v+\gamma]$. It remains to generate the standard selection results.

A.5. **Equilibrium Selection.** Fix $\alpha > 0$ and consider $\beta \to \infty$. This represents increasingly precise private information and hence an increasingly good approximation to the complete information game. In the complete information game, there are multiple equilibria. In the incomplete information game, uniqueness results (when Condition 1 is satisfied) and note that $\beta \to \infty$ will guarantee this eventually.

Proposition 5. *Equilibrium $\{C, C\}$ is played if and only if $c \leq v + \gamma/2$ as $\beta \to \infty$.*

Proof. As $\beta \to \infty$ suppose $c_1^* - c_2^* \nrightarrow 0$. Then, from equations (2) and (3), $2v\Phi(\infty) = c_2^*$ and $2v\Phi(-\infty) = c_1^* - \gamma$ so that $c_2^* = 2v$ and $c_1^* = \gamma$. But $2v > \gamma$ by assumption. The contradiction implies that $c_1^* - c_2^* \to 0$ as $\beta \to \infty$. Moreover, (informally) writing equations (2) and (3) as:

$$\begin{aligned} 2v\Phi\left[0 + \frac{c_1^* - c_2^*}{\to 0}\right] &= \quad c_2^* \\ 2v\Phi\left[0 - \frac{c_1^* - c_2^*}{\to 0}\right] &= c_1^* - \gamma \end{aligned}$$

Note that $\Phi(-x) = 1 - \Phi(x)$ and hence:

$$2v = c_1^* + c_2^* - \gamma \quad \text{as} \quad \beta \to \infty$$

Since (in the limit) $c_1^* = c_2^* = c^*$ then $c^* = v + \gamma/2$. Hence both agents will play strategy C if their posterior $\bar{c}_i$ is less than this value. But the posterior becomes their private information precisely as $\beta \to \infty$: see equation (1). Moreover $c_i \to c$ as $\beta \to \infty$, which yields the required result. □

The risk-dominant equilibrium is selected as discussed in Section 3. Similar arguments can be used to obtain all of the risk-dominance selection results referred to throughout the current paper.

References

Bardhan, P., M. Ghatak, and A. Karaivanov (2002): "Inequality, Market Imperfections, and the Voluntary Provision of Collective Goods," University of California at Berkeley, mimeo.

Bergstrom, T. C., L. Blume, and H. R. Varian (1986): "On the Private Provision of Public Goods," *Journal of Public Economics*, 29(1), 25–49.

Carlsson, H., and E. van Damme (1993): "Global Games and Equilibrium Selection," *Econometrica*, 61(5), 989–1018.

Cornes, R., and T. Sandler (1996): *The Theory of Externalities, Public Goods and Club Goods, 2nd Edition*. Cambridge University Press, London.

Harsanyi, J. C., and R. Selten (1988): *A General Theory of Equilibrium Selection in Games*. MIT Press, Cambridge, MA.

Johnson, J. P. (2002): "Open Source Software: Private Provision of a Public Good," *Journal of Economics and Management Strategy,* forthcoming.

Kandori, M., G. J. Mailath, and R. Rob (1993): "Learning, Mutation and Long Run Equilibria in Games," *Econometrica*, 61(1), 29–56.

Ledyard, J. O. (1995): "Public Goods: A Survey of Experimental Research," in *The Handbook of Experimental Economics*, ed. by J. H. Kagel, and A. E. Roth, chap. 2, pp. 111–194. Princeton University Press, Princeton, NJ.

Ley, E. (1996): "On the Private Provision of Public Goods: A Diagrammatic Exposition," *Investigaciones Económicas*, 20(1), 105–123.

Marx, L. M., and S. A. Matthews (2000): "Dynamic Voluntary Contributions to a Public Project," *Review of Economic Studies*, 67(2), 327–358.

Morris, S., and H. S. Shin (1998): "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review*, 88(3), 587–597.

——— (2002a): "Global Games: Theory and Application," in *Advances in Economics and Econometrics: Theory and Applications*, ed. by M. Dewatripont, L. P. Hansen, and S. J. Turnovsky. Cambridge University Press, London, forthcoming.

——— (2002b): "Heterogeneity and Uniqueness in Interaction Games," Cowles Foundation, Yale University, mimeo.

Olson, M. (1965): *The Logic of Collective Action: Public Goods and the Theory of Groups*. Harvard University Press, Cambridge, MA.

Raymond, E. S. (1998): "The Cathedral and the Bazaar," *First Monday*, 3(3), 1–20, available at `http://www.firstmonday.dk/issues/issue3_3/raymond/`.

Varian, H. R. (1994): "Sequential Contributions to Public Goods," *Journal of Public Economics*, 53(2), 165–186.

Young, H. P. (1993): "The Evolution of Conventions," *Econometrica*, 61(1), 57–84.