Communication at the Zero Lower Bound: The Case for Forward Guidance

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Abstract

The zero lower bound (ZLB) acts as an informational curtain for adaptively learning agents as they cannot observe the path of the interest rate. In a canonical New Keynesian model with no policy change it is shown that this results in a disagreement between the Central Bank and the agents about the lift-off date from the ZLB. Consistent with data from the Swedish Riksbank, the agents expect an earlier lift-off than the Central Bank when the ZLB is binding.

The disagreement coupled with the learning of the agents results in explosive dynamics. Forward guidance is shown to restore stability at the ZLB by preventing spurious expectational drift. The paper calls for a necessary increase in transparency and communication by the Central Bank when constrained by the ZLB. Although such communication is welfare improving, the gains are modest and no forward guidance puzzle is present.

Keywords: Forward Guidance; Adaptive Learning; Central Bank Communication; Zero Lower Bound

JEL Classifications: E43; E52; E58; E61;
1 Introduction

The unprecedented length of the period when interest rates were limited by the zero lower bound after the Great Recession has spurred a large literature trying to understand the behaviour of the economy in such novel circumstances. This phenomenon was also accompanied by unconventional monetary policy instruments to which Central Banks resorted once the interest rates were no longer flexible.

This paper focuses on one of these instruments - namely, forward guidance, and strives to provide a structural justification for its use. The literature on forward guidance (FG) largely agrees that the main channel of influence of FG is the information conveyed for the future path of the policy rate. There are two main classifications of FG depending on the underlying reasons for its use. The seminal work of Krugman et al. (1998) and Eggertsson and Woodford (2003) showed that promises of lower interest rates for longer can largely mitigate the negative effects of a binding zero lower bound (ZLB) on interest rates. The stimulus comes through agents expecting low interest rates in the future (i.e. accommodative monetary policy) and higher inflation, hence cutting back less on present investment and consumption. Campbell et al. (2012) label this approach Odyssean Forward Guidance. Campbell et al. (2012) also acknowledge a more established form of FG, pursued by the Reserve Bank of New Zealand and the Riksbank in Sweden, for example. In essence, such CBs engage in regular forecasts of the path of their policy rate, hence it was dubbed Delphic Forward Guidance. This type of FG may be useful to the public if the CB has better information about the state of the shocks that hit the economy. Moreover, in Marinkov (2018) I propose another function of forward guidance as a communication strategy for policy change. There, the ZLB acts as an informational curtain for adaptive learners who fail to perceive a potential policy change as the policy rate is bound by the ZLB. Then, forward guidance is a useful tool in helping them learn the new policy regime through announcing future lift-off dates\(^1\).

\(^1\)Marinkov (2018) explores various communication and interpretation schemes for the FG signal. Wrong interpretation or small weights of the signal are shown to still be marginally better than no communication at all. The stimulative effects of a prolonged ZLB duration are modest and no forward guidance puzzle is present.
Here I built on this previous work but pursue a more fundamental reason for FG. Instead of considering a policy regime change, I show that the non-linearity introduced by the ZLB itself acts as a regime change for adaptive learners and this creates disagreement between their policy rate forecasts and the Central Bank’s forecasts, who knows the precise structure of the economy. Therefore, FG acts as a helping hand for learners to update their perceived law of motion of the economy under the ZLB regime. Such information revelation about the structure of the economy is akin to Delphic forward guidance. Although empirically supported by Campbell et al. (2017), these authors and others\(^2\) only incorporate Odyssean FG through anticipated monetary policy shocks in their models and do not study theoretically or numerically the effects and nature of Delphic FG. The model here allows for Delphic FG by showing a channel which could explain the observed policy rate forecast disagreement in the data between central banks and the private sector.

The main insight is that the zero lower bound calls for a necessary increase in transparency and communication by the Central Bank at the ZLB because it acts both as a regime change and an information curtain preventing agents from correctly adjusting their expectations about the path of the interest rate. First, forward guidance is shown to have a welfare-improving effect by helping the agents update their expectations even in the absence of interest rate observations. The benefit is modest and no forward guidance puzzle is present. Second, forward guidance helps prevent an expectational drift due to agents expecting an earlier lift-off from the ZLB. This is numerically shown to improve the stability of the system by keeping it tighter within the basin of convergence to the rational expectations equilibrium. This is a novel result which complements prior work on the stability implications of monetary policy in learning models (see Evans and Honkapohja (2003)). In the simple model this communication is achieved through forward guidance, yet in reality a combination of FG and asset purchases might be needed to achieve the necessary shift in expectations. For instance, Campbell et al. (2017) and Andrade et al. (2019) show that FG was successful at shifting short-term expectations but quantitative

\(^2\)see Eggertsson and Woodford (2003), Del Negro et al. (2012), Campbell et al. (2012), Ben Zeev et al. (2017) among others
2 Motivation

The Great Recession and the followed long spell of binding ZLB were unprecedented events that caught the public by surprise. Andrade et al. (2019) show that this led to very high levels of disagreement by historical standards among private forecasters. Additionally, agents often expected earlier lift-off than the Central Bank but this could be due to policy changes (Marinkov, 2018; Engen et al., 2015). To disentangle the disagreement between the CB and the private agents both their forecasts are needed. Among major central banks the Swedish Riksbank is one of the few who publish internal consensus interest rate forecasts along with private market forecasts. They began releasing their internal forecasts in the 2007 issue of their Monetary Policy Report.

Figure 1 plots the 1-year-ahead and 2-year-ahead repo rate forecasts for both the Riksbank (solid lines) and the public (dashed lines). As expected, they are not too disparate from one another, yet there are two important features of data. First, whenever interest rates are expected to be binding to some lower bound, the private forecasts are always supportive of an earlier lift-off than the Riksbank’s. Second, Sweden is a special case among developed economies because it dipped twice to the zero lower bound (ZLB), thus it provides more comparable data above and below the ZLB and allows for testing the theory that the ZLB causes disagreement between the CB and the agents.

To quantify the disagreement between the agents and the Riksbank Table 1 computes the difference between the forecasts of the Bank and those of the market. The measure is set up such that a positive disagreement means that the Riksbank expects higher future repo rate than the market. The data is split in two regimes...
2 Motivation

Figure 1: Forecasts of Swedish repo rate

Source: Riksbank’s Monetary Policy Report 2007-2018

- Low and High, where Low is classified as expected 1-year-ahead repo rate to be smaller than 0.25, and High - to be larger than 0.25. The table shows the classification according to future expected repo rates by both Riksbank and the market. Further robustness classifications on horizons and cut-offs are performed in Table A.1 in appendix A.1.

Table 1 shows that regardless of the classification private agents expect an earlier lift-off than the CB (negative and significant average disagreement) when the economy is a Low regime of near zero interest rates. Moreover, the High regime of normal times exhibits no systematic forecast bias for either party. As a case in point, Figure 2 shows that during the first ZLB spell in 2009 agents expected a higher interest rate path than the Bank, but already a year later when the interest rate left the ZLB expectations aligned perfectly.

It is worth noting that disagreement between the Riksbank and the market continued throughout the ZLB spell. This is an unexpected fact because of Riksbank’s open and explicit interest rate forecasts which one would expect are one of the most
2 Motivation

Figure 2: Forecasts of Swedish repo rate 2009-2010

Source: Riksbank’s Monetary Policy Report 2009-2010
Table 1: 1-year-ahead disagreement on Swedish repo rate

<table>
<thead>
<tr>
<th></th>
<th>count</th>
<th>mean</th>
<th>se(mean)</th>
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<td>Private agents’</td>
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<tr>
<td>Low</td>
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<td>-.1135</td>
<td>.0085</td>
<td>-.18</td>
<td>-.05</td>
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<tr>
<td>High</td>
<td>24</td>
<td>.0107</td>
<td>.0755</td>
<td>-.79</td>
<td>.91</td>
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<td>Riksbank’s</td>
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<tr>
<td>Low</td>
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<td>-.1391</td>
<td>.0192</td>
<td>-.37</td>
<td>-.05</td>
</tr>
<tr>
<td>High</td>
<td>21</td>
<td>.0616</td>
<td>.0804</td>
<td>-.79</td>
<td>.91</td>
</tr>
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Note: ‘High’ and ‘Low’ states refer to 1-year-ahead expected repo rate above or below 0.25, respectively. The first block defines ‘High’ and ‘Low’ based on private agent’s expectations and the second - on Riksbank’s forecast.


transparent and informative means of CB communication. Perhaps, the market did not put a high enough weight on their routine announcements while the unprecedented forward guidance by the Federal Reserve and the Bank of England among others had a notable effect on market expectations as shown by Engen et al. (2015), Andrade et al. (2019) and Campbell et al. (2017). See Marinkov (2018) on the implications of imprecise or unconvincing forward guidance in a model with learning agents.

Finally, a similar study of disagreement is not possible for the USA because the Federal Reserve does not publish its internal consensus forecasts. Yet, Figure 3 shows the average expectations of professional forecasters in the US. It is seen that the period of explicit date- and state-contingent forward guidance (2011-2013) saw market expectations converging closely to what ended up being the actual rate. Yet, before that period and even after it around 2015 market expectations were higher than what the T-bill rate ended up being. This is indirect evidence of a similar pattern as observed in Sweden above.

Abstracting from disagreement, Campbell et al. (2012) find that future monetary policy tightening lowers unemployment expectations and increases inflation expectations in the US, contrary to the predictions of New Keynesian models. They interpret this finding as evidence for successfully communicated Delphic forward guidance by the FOMC. Campbell et al. (2017) study empirically the hypothesis that FOMC’s meeting announcements carried Delphic forward guidance. They classify
private information of the FED by the difference between the Greenbook forecasts on inflation, GDP growth and unemployment rate and Bluechip survey of private forecasters’ expectations. They find that the four-quarter ahead futures contract rate is statistically positively correlated with policy makers’ forecast of future GDP growth being higher than the market expects (and lower for unemployment). This is evidence that the committee’s private information about the future of the economy was transmitted through the FOMC’s announcements - supporting a Delphic forward guidance interpretation.

Figure 3: Private Forecasts of US T-bill rate

3 Model

To explain why disagreement between the Central Bank and the private agents arises at the zero lower bound I build a simple New Keynesian model featuring adaptive learning as an expectation formation framework for the agents. Marinkov (2018) outlines in detail the difference between rational and learning agents and how each model interacts with the non-linearity of an occasionally binding ZLB. Notable differences in the current paper are the reduced form knowledge of the Taylor rule
by the agents and the presence of the lagged interest rate as a state variable. As will
later become clear the first assumption eliminates the simultaneity in determining
the output gap, inflation and the policy rate, while the second makes the learners’
forecasts of output gap and inflation more responsive to the ZLB - a necessity pointed
out in Marinkov (2018).

3.1 Rational Expectations

The model environment is the canonical New Keynesian model with a representa-
tive consumer and monopolistically competitive firms subject to Calvo pricing.
As extensively discussed in Woodford (2003), under rational expectations (RE) the
linearised aggregate economy can be summarized by the following two equations:

\[ x_t = \mathbb{E}_t x_{t+1} - \frac{1}{\sigma} (i_t - \mathbb{E}_t \pi_{t+1} - r_t) \]  \hspace{1cm} (1)

\[ \pi_t = \kappa x_t + \beta \mathbb{E}_t \pi_{t+1} + u_t \]  \hspace{1cm} (2)

with shock processes

\[ r_t = \rho_r r_{t-1} + \varepsilon^r_t, \quad \varepsilon^r_t \sim N(0, \sigma^2_r) \]  \hspace{1cm} (3)

\[ u_t = \rho_u u_{t-1} + \varepsilon^u_t, \quad \varepsilon^u_t \sim N(0, \sigma^2_u) \]  \hspace{1cm} (4)

where \( x_t \) is the current output gap, defined as the difference between output and its
natural rate in an economy with fully flexible prices; \( \pi_t \) denotes the inflation rate; \( i_t \)
the nominal interest rate; \( \beta \) is the discount factor; \( \sigma \) is the elasticity of inter-temporal
substitution of consumption; and \( \kappa \) is a convolution of structural parameters. All
endogenous variables are expressed as log-deviations from their steady state values.
Thus, in steady state \( x = \pi = i = 0 \). Finally, \( r_t \) and \( u_t \) stand for exogenous natural
rate and cost-push shocks, respectively, and follow \( AR(1) \) processes.

The model is closed with a Taylor rule subject to the zero lower bound (ZLB):

\[ i_t = \max \{ i^*, \rho_i i_{t-1} + (1 - \rho_i)(\chi_x \pi_t + \chi_x x_t) \} \]  \hspace{1cm} (5)
where the reaction parameters satisfy $\chi_x > 1$ and $\chi_x > 0$, and the interest rate smoothing $\rho_t \in (0, 1)$. The constant $i^* = 1 - 1/\beta < 0$ represents the effective lower bound on interest rates since, otherwise, as explained in Eggertsson and Woodford (2003) agents would choose to hold all their assets in cash. I will refer to it as the ZLB to be consistent with the arguments in the Introduction and with real world analogies.\(^3\)

### 3.2 Expectations formation

The specification of expectations employed is adaptive learning (ADL). In particular, agents do not know the true structure of the economy and make forecasts as econometricians using simple regression models\(^4\). Namely, they make forecasts according to the aggregate policy functions from the minimum state-variable RE solution to the model:

\[
\begin{bmatrix}
x \\
p \\
i_t
\end{bmatrix} =
\begin{bmatrix}
u_t \\
r_t \\
i_{t-1}
\end{bmatrix} = \Gamma Z_t
\]

where due to the smoothing in the Taylor rule, the lagged interest rate becomes a state variable\(^5\).

Adaptive learning is a linear updating procedure, yet the ZLB creates a non-linearity in the expectations for the path of the interest rate, because agents must understand it cannot be realised below $i^*$. To get around this issue I model the agents as forming expectations about the shadow interest rate and then applying the ZLB to their expectations. Hence, importantly, they use realised rather than shadow prices when forming expectations of $x_t$ and $\pi_t$. The shadow rate is needed during a period of binding ZLB such that agents could form lift-off expectations consistent with the known policy prior to the ZLB. If they were to use $i_{t-1} = i^*$

\(^3\)Appendix A.2 provides estimates for the policy coefficients in the monetary policy rule.

\(^4\)Following the 'consistency principle' of Evans and Honkapohja (2001)

\(^5\)Note that (6) represents the solutions of the model under RE without a ZLB. If the ZLB is respected, when binding the solution of the model will be piece-wise linear featuring a sequence of different policy transformations $\Gamma^i$ for every period $i$ when the ZLB is binding.
as a basis for expectations for \( t = 0, 1, \ldots \), they would have an upward bias in their projected paths for the interest rate because the ZLB \( i^* \) is higher than the shadow rate at \( t = -1 \). Hence, as described below, I assume that above the ZLB agents rely solely on realised prices. When the ZLB binds, on the other hand, due to a lack of exact observable data on the policy rate, they rely on their shadow rate projections for the full path of realisations of \( i_t \). Thus, even though the use of a shadow rate complicates the notation, this dichotomy is necessary for more realistic and sophisticated expectations. In this sense the imperfect knowledge of the agents here is conservative.

The agents have similar forecasting models to (6), as shown below. But each period as additional data becomes available, they update the coefficients to their perceived transition matrix \( \Phi_t \) following a recursive constant gain algorithm. They are assumed to observe the disturbances \( r_t \) and \( u_t \) and to know their autoregressive coefficients.\(^6\)

**Adaptive Learning**

Denote by \( S_t \equiv \begin{bmatrix} u_t & r_t & s_{t-1} \end{bmatrix}' \) the state variables vector where \( s_t \) is the shadow interest rate. Note that above the ZLB the Taylor rule (5) implies that the actual and shadow interest rates coincide - that is \( i_t = s_t \) if \( s_t > i^* \), while \( s_t \) could go below the ZLB and then \( i_t = i^* \). This distinction is vital for the correct formulation of expectations of the agents because output gap and inflation are determined by actual prices (i.e. by the actual interest rate \( i_t \)). As mentioned above, on the other hand, the trajectory for the interest rate is determined by the shadow rate since otherwise the policy smoothing in the Taylor rule would create artificial upward drift in the interest rate due to the ZLB being an inefficiently high rate last period - \( i_{t-1} = i^* \) but \( s_{t-1} < i^* \).

Then, just like in the rational expectations solution in (6) the learning agents use the state variables vector and a transition matrix to forecast the endogenous variables vector \( \begin{bmatrix} x_t & \pi_t & s_t \end{bmatrix}' \). Unlike RE, however, they do not know the correct

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\(^6\)Eusepi and Preston (2010) show that this assumption can be dispensed with and instead agents would estimate those coefficients. For simplicity, it is maintained.
transition matrix $\Gamma$ from (6) and instead use their perceived 3-by-3 transition matrix $\Phi_{t-1}$ from the end of period $t-1$. Remember that the RE state variables vector with actual prices is $Z_t \equiv \begin{bmatrix} u_t & r_t & i_{t-1} \end{bmatrix}'$. Given the discussion above I assume agents use $Z_t$ to form expectations of the output gap and inflation, but they use $S_t$ to forecast the interest rate as follows:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} Z_t$$

$$\hat{E}_t s_t = \phi_{3,t-1} S_t$$

$$\hat{E}_t i_{t+j} = \max \{ i^*, \hat{E}_t s_{t+j} \}, \quad j \geq 0$$

(7)

where $\hat{E}$ is the expectations operator for the learners and $\phi_{n,t}$ is the $n^{th}$ row of their perceived 3-by-3 transition matrix $\Phi_t$. Agents update this perceived law of motion (LOM) by a recursive constant gain algorithm using the discrepancies between their expectations of endogenous variables $\hat{E}Y_t$ and the actual realizations $Y_t$. They weigh this discrepancy by the historical variance-covariance matrix $R_{t-1}$ of the endogenous variables and use the weighted forecast discrepancy for error correction. Each error correction term is given a constant gain weight $\tau$ against their prior beliefs from $t-1$. Finally, they update the VCV matrix $R_t$ in a similar fashion.

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}_t = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}_{t-1} + \tau R_{t-1}^{-1} Z_t \left( \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} - \hat{E}_t \begin{bmatrix} x_t \\ \pi_t \end{bmatrix} \right)'$$

$$\phi_{3,t} = \phi_{3,t-1} + \tau R_{t-1}^{-1} S_t (i_t - \hat{E}_t i_t)$$

$$R_t = R_{t-1} + \tau (Z_t Z_t' - R_{t-1})$$

(8)

### 3.3 Bounded Rationality and the Actual Law of Motion

Replacing RE with ADL means that the structural equations of the economy (1)-(2) need to be modified accordingly. For a related class of models Preston (2005) and

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Note that here I assume constant gain learning instead of the decreasing gain learning used in Evans and Honkapohja (2001). The reason is that the former is more useful for tracking regime changes, while the latter is useful for studying asymptotic convergence properties of learning models to their RE counterparts. Given the current emphasis on the ZLB, tracking is a necessary feature of the model.
Eusepi and Preston (2018) argue that under ADL aggregate expectations \( \hat{E}_t \) are an average of the expectations of heterogeneous households and firms who know only their own objectives, constraints and beliefs and cannot compute aggregate probability laws, i.e. cannot obtain model-consistent expectations like RE. Thus, agents act rationally when it comes to their own objective functions but unlike rational agents fail to anticipate the aggregate laws of motion and resort to econometric learning as in section 3.2. A representative agent occurs when a symmetric equilibrium is assumed in which although everyone’s problem is identical, no individual is aware of that and as a result the representative agent cannot compute aggregate probability laws. This breaks the law of iterated expectations (LIE) for the operator \( \hat{E} \), and hence the recursion from which the aggregate demand (1) and Phillips curve (2) equations are derived. These two equations under ADL and \( \hat{E} \) then depend on a long horizon expectations reading:

\[
\begin{align*}
x_t &= \hat{E}_t \sum_{T=t}^{\infty} \beta^{T-t} \left[ (1 - \beta)x_{T+1} - \frac{1}{\sigma} (i_T - \pi_{T+1} - r_T) \right] \\
\pi_t &= \hat{E}_t \sum_{T=t}^{\infty} \left( \alpha \beta \right)^{T-t} \left[ \kappa (x_T + u_T) + (1 - \alpha) \beta \pi_{T+1} \right]
\end{align*}
\]

where \( \hat{E}_t \) again stands for the expectations of the adaptive learners and \( \alpha \) is the Calvo probability of not being able to reset prices. I will refer to these two equations as the actual law of motion (ALM) of the economy.

Yet, Honkapohja et al. (2012) point out that assuming a continuum of symmetrical agents as is the case in the used NK model, one could still apply the LIE and resort to one period ahead Euler equation learning. I keep the infinite horizon learning for two reasons.

First, it allows for incorporation of FG as information about future policy rates into the law of motion for the aggregate variables, contrary to the Euler equation learning approach. Second, because agents do not know the structure of the economy, they cannot foresee how the ZLB will change the actual law of motion of the system. Take equation (1) and apply \( \hat{E} \) instead of the RE operation like the Euler equation learning approach advocated by Honkapohja et al. (2012) would prescribe. Now, if
the ZLB is expected to be binding for a few periods ahead, its effect should come through expectations of the output gap ($\hat{E}_t x_{t+1}$) and inflation ($\hat{E}_t \pi_{t+1}$). But these are only gradually updated (as described in section 3.2), implying that although agents respect the ZLB in their forecasts for the interest rate, they are completely oblivious of its future effects on inflation and the output gap when only the Euler equation (1) is used. In contrast, suppose that agents expect $t = T^{ZLB}$ as the last period of binding ZLB. Then in the long horizon approach (9) they could set $\hat{E}_t i_{t+s} = i^*$ for all $t + s \leq T^{ZLB}$. Then, the expected duration of the ZLB has an effect on the realisations of the output gap both through the current and future binding periods, which in turn is reflected on future inflation as in (10). Thus, the economy driven by the learners features minimal deviations from the rational expectations economy which are reflected only in the recursively updated $\hat{E}_t x_{t+s+j}$ and $\hat{E}_t \pi_{t+s+j}$ for $s \geq 1$, $j \geq 0$.

These arguments are related to the discussion of the use of long horizon expectations for anticipated structural changes in Evans et al. (2008).

**Disagreement between the CB and the learning agents**

Suppose the economy exists for a long enough period with no extreme shocks that bring it to the ZLB. Then, following the forecast and updating procedures from section 3.2 the learning agents converged to the RE solution of the model in (6). This implies that at some period $t - s$ the perceived transition matrix has converged to the actual one - $\Phi_{t-s} = \Gamma$. Therefore, the agents have fully learned the model with no binding ZLB. The period of the Great Moderation is a useful analogy for this scenario.

Now, suppose the economy is hit by a demand shock $\varepsilon_r$ at period $t$ which brings the interest rate to the ZLB for at least 2 periods. Since the agents respect the ZLB in their expectations they know that today the interest rate will be at the ZLB - $\hat{E}_t i_t = i^*$. Hence, from (8) the error correction term for the interest rate’s law of motion is zero and no updating occurs - $\phi_{3,t} = \phi_{3,t-1}$. On the other hand, their perceived LOMs for output gap and inflation ($\phi_{1,t-1}$ and $\phi_{2,t-1}$) are the first and second rows of the transition matrix for a world with no binding ZLB ($\Phi_{t-1} =$
A model prescribed by $\Gamma$ is characterised by a fully flexible Taylor rule which accommodates demand shocks. This, however, is no longer true with a binding ZLB which locks the interest rate at an inefficiently high level $i^*$. Therefore the agents’ forecasts for time $t$ will be based on $t - 1$ beliefs of the Great Moderation and will be too optimistic. At the end of period $t$ they will observe the realisations and update their expectations as in (8). Overall, during the expected period of the ZLB the agents will not update their perceived Taylor rule for the shadow rate but will update their beliefs for the laws of motion of output gap and inflation.

I assume the Central Bank knows the ALM of the model (9) and (10) and observes agents’ expectations $\hat{E}Y_t$ for all endogenous variables. Upon observing agents’ expectations the CB plugs them into the ALM equations (9) and (10) and obtains model-consistent forecasts. Given its projections for output gap and inflation it uses the Taylor rule (5) and forms projections for the shadow rate. Because the CB’s shadow rate forecasts are based on constantly updated $\hat{E}_t$ expectations through the ALM, it is better able to anticipate the trajectory of the interest rate than the agents, who due to their fulfilled expectations of a binding ZLB in the immediate future fail to adjust the law of motion for the interest rate ($\phi_{3,t+s} = \phi_{3,t-1}$ if $\hat{E}_{t+s-1}i_{t+s} = i^*$) and expect an earlier lift-off. Thus, they gradually update their output gap and inflation expectations, but the binding ZLB prevents them from understanding how the new regime changes the dynamics of the Taylor rule even in the absence of an explicit policy change. The only source of change in the system is the ZLB which affects the propagation of the state variables $Z_t$ to the endogenous variables $Y_t$.

**Proposition 1.** Suppose the economy is brought to the zero lower bound after a period of convergence to a rational expectations model with no binding ZLB. Then, the mechanics described above result in a disagreement between the agents and the Central Bank about the future path of the interest rate even in the absence of any policy change. Namely, the agents expect an earlier lift-off from the ZLB than the Central Bank.

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8Considering the vast amounts of information collected and processed by central banks as well as their sophisticated forecast models this does seem like a realistic assumption.
3.4 Forward guidance

Henceforth, I assume that in order to correct the disagreement about the future path of interest rates the CB uses forward guidance (FG) by truthfully revealing its expected lift-off date during every period of a binding ZLB. Next I describe how forecasts are made and the considered experiments of the use of FG. The next section presents simulations for each experiment and discusses their implications and effectiveness.

3.4.1 Forecasting

Every period the agents form long-run expectations \( \hat{\mathbb{E}}_t \{ x_j, \pi_j, i_j, s_j \} \) as outlined in section 3.2. This allows them to estimate the last period of binding ZLB defined as:

\[
T^{ag} \text{ such that } \begin{cases} 
\hat{\mathbb{E}}_t i_{T^{ag}} = i^* \\
\hat{\mathbb{E}}_t i_{T^{ag}+1} > i^* 
\end{cases}
\] (11)

The Central Bank is assumed to have rational model-consistent expectations, but no choice variable and to truthfully reveal its expectations, thus abstracting from strategic behaviour. It observes agents’ expectations \( (\hat{\mathbb{E}}_t Y_{t+j}, \, j > 0 \text{ and } T^{ag}) \) and uses them to form expectations according to the structural equations of the model (9)-(10). Then it sets its instrument \( i_t \) according to the Taylor rule (5) and in a similar fashion to (11) obtains its expectation of the last period of binding ZLB - \( T^{cb} \). As per Proposition 1, we would have \( T^{cb} > T^{ag} \), because agents’ expectations adjust to reflect the new regime\(^9\) brought by the ZLB only gradually through observations. This disagreement about the path of the interest rate is the rationale for FG.

3.4.2 Experiments

At period \( t = 1 \) the economy is in its RE equilibrium above the ZLB. Then a large persistent natural rate shock \( (\varepsilon^\tau_2 \text{ - see Table B.1}) \), pushes it to the ZLB. Both the agents and the CB anticipate a lift-off date according to the described procedures

\(^9\text{as manifested through the transition matrix } \Phi_t.\)
above. Whenever forecasts disagree, there is scope for forward guidance. Three cases of such CB communication are considered. In all cases where communication occurs, the CB is assumed to release its beliefs truthfully, abstracting from strategic behaviour.

1. **Baseline no FG** - the agents expect a lift-off at $T^{ag}$ and are surprised by the continuing ZLB. They gradually update their beliefs by comparing $s_t$ and $i_t$.

2. **FG as the length of the ZLB spell** - the CB releases $T^{cb}$ and if different from $T^{ag}$, the agents adopt it outright in their expectations. This is reflected in the aggregate demand equation (9). Note that in this case the law of motion for the interest rate is not updated, so even at lift-off date ($T^{cb}$) there might be some disagreement between the CB and the agents.

3. **FG interpreted by adjusting** 

$$\hat{E}_t s_t = \phi_{3,t-1} \begin{bmatrix} u_t \\ r_t \\ s_{t-1} \end{bmatrix}$$

$$\phi'_{3,t-1} = \phi_{3,t-1} + \lambda R_{t-1}^{-1}S_{T^{cb}} \left( i^* - \hat{E}_t i_{T^{cb}} \right)$$  

Equation (12) shows that now when the CB announces $T^{cb}$ the agents try to adjust their perceived LOM for the shadow rate such that as of today their expectations for date $T^{cb}$ are for $\hat{E}_t i_t = i^*$.

• here $\lambda = \tau$ gives weight to FG announcements as 1 quarter worth of data. Variation in $\lambda$ can proxy how credible or well understood FG is.

### 4 Experiments

This section presents the conducted experiments and results. Throughout, I use the parameter and shock values from Table B.1 in appendix B.2 and the estimated monetary policy rule from Table A.2b in appendix A.2. Impulse responses are calculated as point-wise median from 5000 simulations of random iid $\varepsilon_r$ and $\varepsilon_u$ shocks. This is done in order to provide enough variability for the learners to update
their perceived transition matrix. The zero lower bound is respected throughout and the only commonality between simulations is the negative natural rate shock at period \( t = 2 \).

### 4.1 No forward guidance

Figure 4 below shows the impulse response functions (IRFs) of \( x_t \), \( \pi_t \) and \( i_t \) (both expected and realised) to an -8% natural rate shock in period 2, which results in a prolonged period of binding ZLB. The bold black line shows the actual end-of-period realisations of the endogenous variables, while the dashed red line is the beginning-of-period expectations of the agents. Both expectations and realisations are, as expected, below the schedules which would have occurred were there not ZLB constraint. Moreover, as explained in previous sections, agents’ expectations of future output gap and inflation only change with observations even if they understand what the ZLB means for the path of the interest rate. Thus, initially they expect a faster recovery, yet since the ZLB changes the economy’s response to shocks, the actual output gap and inflation turn out to be lower. The constant gain learning results in a quick updating of beliefs and convergence of the dotted and solid lines.

![Figure 4: no FG - IRFs](image)

Note that without FG the agents’ projected shadow rate will be identical to
the hypothetical one if no lower bound constraint existed (green thin solid line). Figure 5 below zooms in on the end of the ZLB spell to highlight the disagreement about the lift off date between the agents and the CB. Even in this parsimonious model disagreement does occur and it is around 150 basis points at period 9 when the agents expect lift-off next period. A richer model featuring more persistence (e.g. habit formation or price indexation) as used by central banks today is likely to produce even larger disagreements. Finally, Figure 6 plots the expected duration of the ZLB of both agents and the CB when asked at every period. Disagreement persists with agents consistently expecting a 3-4 quarters shorter ZLB duration than the more informed CB.

Figure 5: no FG - interest rate paths

\[\text{Figure 5: no FG - interest rate paths}\]

4.2 "Period" forward guidance

Now suppose whenever disagreement occurs at the beginning of a period (as in Figure 6), the CB announces \(T^{cb}\) and the agents outright adopt it without changing their perception of the law of motion for the interest rate. Naturally, now the expected durations of the ZLB coincide throughout (Figure B.1 in appendix B.3). This situation is akin to the framework of forward guidance as anticipated shocks by Del Negro et al. (2012). Agents understand the length of the ZLB spell will be
different but do not update their perceived LOM of the interest rate. Notably the agents’ perceived LOM during the ZLB is misspecified but since no updating has occurred, it is in fact the correct one upon exit from the ZLB.

4.3 Update from forward guidance

Such smooth transfer of information as above is not very likely in practice. In this scenario the CB again announces $T^{cb}$ but instead of directly adopting it, the agents use their usual learning procedure aiming to adjust their expectations for the interest rate at time $T^{cb}$ ($\hat{E}_t i_{T^{cb}}$) to equal $i^*$. Note that this communication scheme resembles the conditional FG that CBs have implemented in practice.

There are two differences with this learning step compared to their usual updating. First, it regards further than 1 period ahead forecasts. Second, the learning gain ($\lambda$) here can be varied to emulate the credibility of the message released by the CB. Here it is assumed that $\lambda = \tau = 0.02$, or the agents view CB’s FG announcements as just another data point. See Marinkov (2018) for comparisons of the effects of different $\lambda$’s. The learning for FG announcements (12) is restated below.

$$\phi_3' = \phi_{3,t-1} + \lambda R^{-1}_{t-1} S_{T^{cb}} (i^* - \hat{E}_t i_{T^{cb}})$$
A benefit of the "learning FG" scenario is that it could be beneficial in cases of earlier or delayed lift-offs than announced due to future shocks. Agents could better anticipate those if they have updated their perceived LOM for the interest rate. A potential downside compared to the "period" FG above is that this communication causes an expectational change in the perceived law of motion of the agents which might threaten the stability of the system.

Figure 7 shows the corresponding anticipated ZLB durations. Given that the agents solve a linear problem in order to match the announced lift-off date (12), it is no surprise that their perceived duration of the ZLB coincides with the CB’s announcement. Notice that in period 4 the common perceived duration drops below the value of the no communication case in Figure 6. This happens because of the feedback of the updated long-run agents’ expectations from (12) into the ALM (9) and (10).

### 4.4 Welfare comparisons

Figure 8 plots the cumulative welfare loss associated with the cases for forward guidance described above. It is computed through a standard central bank welfare loss function: \( L_t = L_{t-1} + \beta^{t-1} (\pi_t^2 + 0.5x_t^2) \). Naturally, period forward guidance
4 Experiments

4.5 Beliefs’ drift and Stability

This section discusses the underlying updating of beliefs in the three experiments. Figure 9 shows the drifts in the elements of the transition matrix $\Phi_t$ mapping states into expectations of endogenous variables. Although in the long-run these converge back to their equilibrium values under RE\textsuperscript{10}, they exhibit a prolonged drift away that lasts much longer after the ZLB is no longer binding. Regarding the welfare of the economy in the presence of future volatility this may be important in richer models or if shocks had larger variances such that future binding ZLB periods were more likely. Note that it is also consistent with the findings in section 2 where

\textsuperscript{10}Due to constant gain learning instead of decreasing gain learning they converge to a distribution centered around their RE values (Evans and Honkapohja, 2001)

Figure 8: Welfare losses

has the best welfare outcome since it results in full agreement and in contrast to the learning forward guidance it does not create any expectational drift from the announcements. Thus, after lift-off agents still hold their pre-crisis beliefs about the law of motion of the interest rate, which are in fact the correct ones for the case of above the ZLB. Although this is welfare improving, the gains are marginal and no forward guidance puzzle is present.
private expectations in Sweden remained higher than the CB’s even in the end of the second ZLB spell.

Importantly, the drift is especially dangerous in the case of no communication. As already established, the agents expect an earlier lift-off than the CB. After their last anticipated period of ZLB - $T_{ag}$, they expect an interest rate above the ZLB but observe it still remains at the ZLB. This causes them to increase their perceived persistence of the policy rate through their learning algorithm (8). All figures above are median outcomes from Monte Carlo simulations. Nonetheless, during these simulations I find that over 10,000 draws and 500 periods over half of the draws end up in instability due to perceived unit root in the law of motion of the interest rate. Figure 10 plots the impulse responses of an identical economy as in the no communication case but it allows for moderate future shocks after the initial period. The familiar disagreement about the lift-off date and the severity of the crisis are still present. This time, however a sequence of very small negative demand shocks in period 12 push the economy beyond the bounds of stability. As established above, after their expected lift-off date (period 9) the agents observe a still binding
ZLB which causes them to increase their perceived persistence of the interest rate. Iterated in their medium-run expectations in the ALM (9), this creates a boom in the economy around period 12. The Central Bank increases the interest rate in order to tame the boom, but this creases more disagreement in the interest rate forecasts with the agents. Given the small negative shocks at period 12 and the increasing policy rate, the agents again are lead to belief that the interest rate depends more on its past value rather than shocks. This again affects the medium to long-run expectations of the agents who now (around period 18) expect very high interest rates in the future, thus causing the economy to experience a recession. The CB, following its Taylor rule, quickly lowers interest rates, thus creating yet another big disagreement between with the agents. This causes even higher perceived persistence of the interest rate until around period 30 it surpasses 1 (unit root) and renders the economy explosive. The trajectory of the policy rate disagreement and the continual drift towards a unit root of the perceived persistence of the interest rate are depicted in Figure 11.
The learning literature has long established that the stability of the economy is greatly improved by a CB which reacts not to actual data as assumed here, but to the expectations of the agents (see Evans and Honkapohja (2003)). This is a remarkable analytical result in environments with no regime changes such as a zero lower bound. Here, I numerically make the case that FG can greatly improve the stability of a system with occasionally binding ZLB even when the CB reacts to contemporaneous data. The reason for this is that the communication provided by the CB provides a workaround for the unobservable shadow rate to the agents who adjust their expectations. This helps minimize the initial expectational drift caused by the ZLB period and keeps the economy tighter within the basin of convergence, which greatly improves its stability.

Forward guidance in both of its iterations considered above has a stabilizing effect on the economy by keeping expectational drift at bay, thus preserving stability. Figure 11 shows on the first row the disagreement between the agents and the CB for the interest rate nowcast and on the second the AR(1) persistence in the perceived law of motion of the interest rate in the same Monte Carlo draw as in Figure 10. Although the two FG schemes exhibit some disagreement after the lift-off date, it is very contained and does not cause big drifts in the perceived persistence of the interest rate. The case of no communication, however, shows that disagreement keeps growing even after the lift-off and this is fuelled by an upward drifting perceived interest rate persistence. Once the perceived interest rate reaches 1, the system becomes explosive due to the long-horizon expectations in (9).

Thus, the last result of the paper is the forward guidance can be used at the ZLB to restore stability to the system. This is so because if no communication is issued, the learners will wrongly think the prolonged ZLB reflects higher persistence in the interest rate. Their updating quickly leads them to believe there is a unit root in the interest rate’s law of motion since it does not react to shocks (the shadow rate does, but it is unobserved). When this happens, the economy becomes unstable. Forward guidance prevents this spurious drift in expectations and preserves the stability of

\footnote{Figure B.2 in the Appendix shows how an economy with the same sequence of shocks as in Figure 10 but with period FG preserves its stability has suffers less volatility.}
The economy.

5 Discussion

This paper shows that the zero lower bound calls for a necessary increase in transparency and communication by the Central Bank because the ZLB kink distorts private agents’ expectations of the trajectory of the policy rate. The private agents’ and Central Bank’s expectations diverge because the Bank is better able to understand the effects of the new ZLB regime on the aggregate law of motion of the economy. In particular, a binding ZLB causes private agents to expect and earlier lift-off than the CB does. In the simple model communication is achieved through forward guidance, yet in reality a combination of FG and asset purchases might be needed to achieve the necessary shift in expectations. The discrepancy is not negligible, but neither is it huge, so no forward guidance puzzle is present. Importantly, forward guidance can be used as a stabilizing tool to ensure stability at the ZLB by preventing spurious expectational drift.
Avenues for future work include allowing for Central Bank learning and considering optimal policy. A different expectation formation in the form of rational inattention also has the potential to explain disagreement and the effectiveness of forward guidance.\footnote{Note that this is similar to varying the weight on CB announcements $\lambda$ studied in Marinkov (2018)}.

## Appendices

### A Data

#### A.1 Robustness policy rate forecasts disagreement - Swedish Riksbank

Table A.1 performs robustness checks on disagreement between the Riksbank and private agents. As in Table 1 in Low regimes agents expect on average higher interest rates than the CB (disagreement is negative and significant). In High states there is no significant disagreement between the CB and the agents at 3-months and 1-year forecast horizons. Interestingly, there is some evidence that at 2-year forecast horizons the Riksbank expect higher interest rates than the agents (disagreement is positive and significant). This might be due to better long-run forecasting abilities of the Central Bank or it might reflect a private agents’ perception of more past-dependent policy compared to what the CB claims.

#### A.2 Estimating the Taylor rule

Abstracting from the ZLB, the theory in (5) implies that the following relationship holds for the nominal interest rate:

$$i_t - \bar{r} = \rho_i (i_{t-1} - \bar{r}) + (1 - \rho_i) (\chi_\pi (\pi_t - \bar{\pi}) + \chi_x x_t)$$

or equivalently

$$i_t = \bar{r} (1 - \rho_i) + \rho_i i_{t-1} + (1 - \rho_i) (\chi_\pi (\pi_t - \bar{\pi}) + \chi_x x_t)$$

(13)
Table A.1: Disagreement on Swedish repo rate

Based on private agents’ expected 1-year-ahead repo rate

\[ E_t i_{t+1y} \leq 0.25 \]

Based on Riksbank’s expected 1-year-ahead repo rate

\[ E_t i_{t+1y} \leq 0.75 \]

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<td>-0.0426</td>
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where \( \bar{r} \) stands for the natural rate of interest and \( \bar{\pi} \) - for the inflation target of the central bank, both of which are netted out in the theoretical model in (5) since all variables in the model are in deviations from steady state. Thus, I estimate the following empirical interest rate model from which I then back out the implied the Taylor parameters in (13) and, respectively, (5):

\[
i_t = a_0 + a_i i_{t-1} + a_\pi \pi_t + a_x x_t + \varepsilon_t
\]  

(14)

The data used is from the Federal Reserve Economic Data and includes the official output gap, the Personal Consumption Expenditures (PCE) Excluding Food and Energy (chain-type price index) and the FED funds rate (FFR). The frequency is quarterly and the sample is chosen for the period January 1st 1987 - October 1st 2007 in order to coincide with the end of the Volcker administration at the FED and the onset of the financial crisis. The same sample period has also been chosen by Taylor (1993) and Kahn (2012).

Table A.2 shows the estimates for the regression in (14) as well as the implied Taylor coefficients in (13) and (5).
Table A.2: Monetary policy rules

(a) empirical interest rate models

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<th>smoothing</th>
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<td>ffr_{-1}</td>
<td></td>
<td>0.85^{***}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.02)</td>
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<tr>
<td>inflation</td>
<td>1.54^{***}</td>
<td>0.31^{***}</td>
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<td></td>
<td>(0.13)</td>
<td>(0.06)</td>
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<td>output gap</td>
<td>0.74^{***}</td>
<td>0.22^{***}</td>
</tr>
<tr>
<td></td>
<td>(0.09)</td>
<td>(0.03)</td>
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<tr>
<td>Constant</td>
<td>2.79^{***}</td>
<td>0.43^{***}</td>
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<tr>
<td></td>
<td>(0.15)</td>
<td>(0.07)</td>
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<tr>
<td>Observations</td>
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<tr>
<td>Adjusted $R^2$</td>
<td>0.568</td>
<td>0.970</td>
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(b) implied Taylor coefficients

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<td>$\chi_x$</td>
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<tr>
<td>$\bar{r}$</td>
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Note that the a monetary rule with persistence implies stronger reaction to deviations from target for inflation and output gap. This is because a higher persistence means that inflation or output gap shocks have a longer lasting effect on the interest rate and hence the CB would choose to react more strongly to such shocks as they happen.

Lastly, Figure A.1 plots the actual FED funds rate versus the implied monetary rules in Table A.2. Since the sample for the estimates in Table A.2 ends in October 1st 2007, all values this date show the out of sample fit of the two empirical models. Notably, both visually and evidenced by the higher $R^2$ in Table A.2a the model with smoothing fits the data much better not only during the sample period including the Great Moderation but also out of sample suggesting a long period of binding ZLB.

B Model

B.1 Model derivations

Follow Eusepi and Preston (2018) ...
Figure A.1: Estimated US Taylor rules and the FFR

B.2 Calibration

Table B.1: Calibration

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<td>$\beta$</td>
<td>0.99</td>
<td>implying 4.1% annual rate of return</td>
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<td>$\kappa$</td>
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<td>Woodford(2003)</td>
</tr>
<tr>
<td>$\sigma$</td>
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<td>implying IES of $\frac{1}{3}$</td>
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<td>$\rho_u$</td>
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<tr>
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<td>consistent with staff estimates</td>
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<tr>
<td>$\sigma_r, \sigma_u$</td>
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<td>only for welfare loss calculations</td>
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<tr>
<td>$\varepsilon_{2}^r$</td>
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<td>a &quot;Great Recession&quot; shock</td>
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<tr>
<td>$\tau$</td>
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B.3 Figures

Figure B.1: "Period FG" - Anticipated duration of ZLB

Figure B.2: period FG - single simulation IRFs
Bibliography


