Department of Economics
Discussion Paper Series

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Johannes Johnen, David Ronayne

Number 874
July, 2019

Department of Economics
Manor Road Building, Manor Road
Oxford, OX1 3UQ

ISSN 1471-0498

The Only Dance in Town: Unique Equilibrium in a Generalized Model of Price Competition*

Johannes Johnen[†] and David Ronayne[‡]

July 17, 2019

Abstract

We study a canonical model of simultaneous price competition between firms that sell a homogeneous good to consumers who are characterized by the number of prices they are exogenously aware of. This setting subsumes many used in the literature over the past several decades. Our result shows that there is a unique equilibrium if and only if there exist *some* consumers who are aware of *exactly two* prices. The equilibrium we derive is in symmetric mixed strategies. Furthermore, when there are no consumers aware of exactly two prices, we show there is an uncountable-infinity of asymmetric equilibria in addition to the symmetric equilibrium. Our result shows that the paradigm generically produces a unique equilibrium; that the commonly-sought symmetric equilibrium is robust; and that the asymmetric equilibria are knife-edge phenomena. *(JEL: D43, L11)*

Keywords: price competition; price dispersion; unique equilibrium

*We are grateful for helpful discussions with Simon Anderson, Mark Armstrong, Dan Bernhardt, David Myatt, Alexei Parakhonyak, Martin Peitz and Jidong Zhou.

[†]CORE, Université Catholique de Louvain. johannes.johnen@uclouvain.be.

[‡]Economics Dept. and Nuffield College, University of Oxford. david.ronayne@economics.ox.ac.uk.

Introduction

Prices for seemingly homogeneous goods are typically dispersed (see e.g., De los Santos, Hortaçsu, and Wildenbeest, 2012; Gorodnichenko, Sheremirov, and Talavera, 2018; Kaplan and Menzio, 2015; Lach and Moraga-González, 2017). The theoretical industrial organization literature offers an elegant rationalization of this phenomenon via games where firms simultaneously compete in prices for consumers who differ in the number of prices they compare.

We study the elementary and oft-studied such setting in which $n \geq 2$ firms each sell a homogeneous good at a common marginal cost, $c \geq 0$, and simultaneously set prices in a one-shot game. There is a mass of consumers who each demand one unit of the good for which they are willing to pay up to $r > c$. Consumers exogenously differ in the number of prices they know: $I_m \geq 0$ are aware of $m \in \{1, \ldots, n\}$ random prices, where we assume $I_1 > 0$ and $I_m > 0$ for at least some $m > 1$.[1] In this setting, it is well-known that equilibrium is in mixed strategies: price dispersion is generated by firms that "tango" (Baye, Kovenock, and De Vries, 1992).

If one restricts attention to symmetric pricing strategies, there is exactly one equilibrium. Researchers have almost exclusively relied upon this equilibrium in their analyses. However, a potentially-uncomfortable fact about these models is that they can produce very many equilibria. For example, under the assumption that $I_1, I_n > 0$ and $I_2, \ldots, I_{n-1} = 0$, Baye, Kovenock, and De Vries (1992) show there is an uncountable infinity of asymmetric equilibria in addition to the symmetric equilibrium.

We contribute by pin-pointing the source of this multiplicity and characterize when the symmetric equilibrium is in fact the only equilibrium. Specifically, we show that there is a unique equilibrium if and only if $I_2 > 0$. The result is stark: if $I_2 > 0$ the symmetric equilibrium is the unique equilibrium, but if $I_2 = 0$ we show there is a continuum of equilibria.

In the absence of consumers who make the minimal number of comparisons ($I_2 = 0$), we show there are infinitely-many asymmetric equilibria in which at least one firm charges consumers' reservation price, r, with positive probability. When these firms charge r, they sell only to their share of "captive" consumers (I_1/n) instead of competing for "contested" (non-captive) consumers by setting lower prices. In contrast, if $I_2 > 0$ each firm competes head-to-head with each other firm for some consumers. This gives firms an incentive to compete at every price (including r), which forces firms not to place mass points on any one price, dismantling these asymmetric equilibria to leave only the symmetric equilibrium.

We highlight two main implications of our result. More generally, the framework may become more attractive because equilibrium multiplicity is only a problem in the special case of

[1] It is well-known that if $I_1 = 0$ then at least two firms price at marginal cost and all firms earn zero profits, and if $I_1 > 0$ but $I_m = 0$ for all $m > 1$ then all firms set the monopoly price r.

$I_2 = 0$. Second, even if a researcher adopts $I_2 = 0$ and hence faces multiplicity, we provide a novel "stability" rationale for selecting the symmetric equilibrium: the symmetric equilibrium strategy when $I_2 = 0$ is equal to the unique equilibrium strategy as $I_2 \downarrow 0$.

The paper proceeds as follows: We next discuss related literature; then provide the model and derive our result. We finish by detailing the unique equilibrium strategy in several applied settings and discuss the implications of our result.

Literature

Models of price competition with heterogeneously-informed consumers offered an early rationalization of price dispersion in homogeneous-goods markets (foundational studies include Rosenthal, 1980; Narasimhan, 1988; Shilony, 1977; Varian, 1980). Since then, the framework has been applied to, or featured in, a wide range of settings including: consumer search (both theoretical, e.g., Burdett and Judd, 1983; Janssen and Moraga-González, 2004; Stahl, 1989, and empirical, e.g., De los Santos, Hortaçsu, and Wildenbeest, 2012; De los Santos, 2018; Honka, 2014; Honka and Chintagunta, 2016; Pires, 2016); price discrimination (Armstrong and Vickers, 2018; Fabra and Reguant, 2018); product substitutability (Inderst, 2002); strategic clearinghouses such as comparison websites (Baye and Morgan, 2001; Moraga-González and Wildenbeest, 2012; Ronayne, 2019; Ronayne and Taylor, 2019; Shelegia and Wilson, 2017); competition with behavioral or boundedly-rational consumers (e.g., Carlin, 2009; Chioveanu and Zhou, 2013; Gu and Wenzel, 2014; Heidhues, Johnen, and Koszegi, 2018; Inderst and Obradovits, 2018; Johnen, 2018; Piccione and Spiegler, 2012; Spiegler, 2006, 2016); and switching-cost models (for a review, see Farrell and Klemperer, 2007).

The almost-ubiquitous assumption made is that consumers are aware of symmetrically- and randomly-drawn prices (without replacement), which is the setting we study. As such, each consumer's information can be characterized by the number of prices they are aware of. Perhaps the most well-known is the "Model of Sales" of Varian (1980, 1981), which assumes $I_1, I_n > 0$ and $I_{m \neq 1,n} = 0$. There, Baye, Kovenock, and De Vries (1992) show that there is an uncountable infinity of equilibria in addition to the symmetric equilibrium when $n > 2$. Most papers in the literature deal with this multiplicity by focusing on the symmetric equilibrium (for example, and in addition to the papers cited above: Armstrong, 2015; Armstrong, Vickers, and Zhou, 2009; Lach and Moraga-González, 2017; Moraga-González, Sándor, and Wildenbeest, 2017; Nermuth, Pasini, Pin, and Weidenholzer, 2013). Uniqueness has only been found in some special cases of our setting e.g., when $n = 2$ (Baye, Kovenock, and De Vries, 1992), or when consumer's awareness of any two prices is independent (Spiegler, 2006).[2] As we prove, these results follow because $I_2 > 0$, which we identify as the exact condition for uniqueness.

[2]Szech (2011) extends Spiegler's result to show uniqueness extends to the asymmetric independent case.

In the multiple equilibria identified by Baye, Kovenock, and De Vries (1992), firms earn the same profits, but charge very different price distributions. The price distributions themselves are of key interest in many models of consumer search. They drive consumers' incentive to become informed (see e.g., Armstrong, Vickers, and Zhou, 2009; Baye and Morgan, 2001; Burdett and Judd, 1983; Fershtman and Fishman, 1994; Moraga-González, Sándor, and Wildenbeest, 2017), and comparative statics of price distributions are the key interest in many related settings (e.g. Janssen and Moraga-González, 2004; Moraga-González, Sándor, and Wildenbeest, 2017; Nermuth, Pasini, Pin, and Weidenholzer, 2013).[3] Thus, the robustness of results in these aforementioned papers depends on the existence of asymmetric equilibria. Our finding that asymmetric equilibria are knife-edge phenomena implies that predictions derived with symmetric equilibria are the relevant ones.

Arguments in favor of the symmetric equilibrium have been made in some settings. In an extension of their main analysis, Baye, Kovenock, and De Vries (1992) allow those consumers willing to check exactly one firm to choose where they buy. They show that game has a unique solution where firms adopt symmetric pricing strategies. In our more general setting, we provide a distinct argument in favor of the symmetric equilibrium based on continuity, and without extending or otherwise changing the game's structure.

A few studies have made progress examining equilibria under limited asymmetric assumptions e.g., Baye, Kovenock, and De Vries (1992, Section V), Inderst (2002); Ireland (1993); McAfee (1994); Narasimhan (1988); Szech (2011). Allowing for totally general asymmtries in consumers' information sets is challenging and little is known about the nature of equilibria there; Armstrong and Vickers (2019) is an exception, offering a rich characterization when $n = 3$. In this paper, we analyze the standard (symmetric and random) configuration of consumers' information.

Model and Equilibrium

Model. There are $n \geq 2$ firms indexed $i = 1, \ldots, n$ that produce a homogeneous product to sell to consumers who wish to buy one unit of the good and have a common and finite willingness to pay, $r > 0$.[4] Firms face a constant marginal cost, $c \in [0, r)$, and simultaneously choose price, where the price of firm i is denoted p_i. Consumers differ by the number of prices they are exogenously aware of i.e., the size of their "information" or "consideration" sets.[5] Consumers

[3]Extending the framework by adding a preliminary stage in which firms set list prices (the upper bound of final retail prices), Myatt and Ronayne (2019) produce an equilibrium in pure strategies and compare the comparative statics to those of the mixed strategies of several of the single-stage models cited here.

[4]Under an alternative (unrealistic) assumption that demand is unbounded, mixed-strategy equilibria can exist, as shown by Kaplan and Wettstein (2000) in the case that $I_n > 0$ and $I_m = 0$ for $m < n$.

[5]Some evidence indicates that such "fixed-sample" or "simultaneous" search often describes consumer behavior well. De los Santos, Hortaçsu, and Wildenbeest (2012) and Honka and Chintagunta (2016) examine data from markets for books and auto insurance, respectively. Both studies fail to find a relationship between the prices

buy from the firm in their consideration set with the lowest price. Where there is a tie in the lowest price, any interior tie-breaking rule may be assumed. The mass of consumers informed of $m \in \{1, \dots, n\}$ prices is denoted $I_m \geq 0$, where $I_1 > 0$ and $I_m > 0$ for some $m > 1$. For each type of consumer, consideration sets are symmetrically and randomly distributed across firms. This means, for example, that I_2 comprises of the same share, equal to $I_2/\binom{n}{2}$, of consumers with each consideration set $\{1, 2\}, \{1, 3\}, \{1, 4\}, \dots$, etc. We refer to the I_1 consumers as captive and all others as contested. Before deriving our result we illustrate it for the case of $n = 3$.

Example. Consider a triopoly in which no consumer sees exactly two prices ($I_2 = 0$), but some consumers observe one ($I_1 > 0$) and others all three ($I_3 > 0$).

Here, Baye, Kovenock, and De Vries (1992) show that the following equilibria exist. Two firms, say 1 and 2, randomize continuously over the interval $\left[\underline{p}, r\right]$.[6] The remaining firm, 3, randomizes continuously over some interval $\left[\underline{p}, x\right) \cup r$ with $x \in \left(\underline{p}, r\right]$, placing a mass point at r whenever $x < r$.[7] This is an equilibrium for all $x \in \left[\underline{p}, r\right]$, hence there is an uncountable infinity of equilibria.[8] In the special case of $x = r$, equilibrium strategies are symmetric. The asymmetric equilibria require firm 3 to have a mass point on r. In all equilibria, each firm's profit is determined by what they can earn from charging the monopoly price to their captive consumers, their minmax payoff, $(r - c)I_1/3$.

In each equilibrium, firms 1 and 2 trade-off exploiting captive consumers and competing for the I_3 contested consumers. But the asymmetric equilibria "sideline" firm 3: firms 1 and 2 compete head-to-head for I_3 by mixing over a common interval such that firm 3 has nothing to gain from joining the competition, and can therefore afford to focus more on exploiting its captive consumers by placing a mass point on r.

In contrast, when $I_2 > 0$ each firm competes head-to-head with each of its rivals for some consumers. This incentivizes each firm to compete for contested consumers: no firm can sit on the sideline in equilibrium so the asymmetric equilibria no longer exist. To see this more precisely, take our example, but add some consumers who compare two prices so that $I_2 > 0$, and suppose asymmetric strategies are played in which firm 3 places a mass point at r. Now when charging r, firm 3 loses all contested consumers and earns $(r-c)I_1/3$. In contrast, because $I_2 > 0$ and firm 3 has a mass point on r, firms 1 and 2 each sell to $I_2/3$ contested consumers with positive probability, even at prices arbitrarily close to r. Thus, firms 1 and 2 earn strictly more than $(r - c)I_1/3$. But then firm 3 can increase its profit by competing for the $2 \cdot I_2/3$ contested consumers who see firm 3 and exactly one other firm by charging prices below r (it has a strict

consumers have seen and their decision to search on, consistent with the assumption of simultaneous search.

[6] The price $\underline{p}$ will be shown to be the lowest undominated price, defined in (2).

[7] We omit the equilibrium strategies here to avoid clutter; they are detailed in the proof of Lemma 16.

[8] More generally for $n \geq 2$ (and $I_1, I_n > 0$ and $I_2, \dots, I_{n-1} = 0$), at least two firms continuously mix over $\left[\underline{p}, r\right)$ in any equilibrium, while all other firms may have mass points on r.

incentive to shift the probability mass from r to lower prices).

Our result generalizes this intuition for any $n \geq 2$. If $I_2 > 0$, each firm competes head-to-head with each rival for at least some contested consumers. This rules out the possibility of sidelined firms in equilibrium and implies that the symmetric equilibrium is unique.

Analysis. Firms are guaranteed a profit of at least $\pi_i = (r - c)I_1/n > 0$ by setting a price of r which sells to their I_1/n captive consumers regardless of other prices. Profit is zero for $p_i > r$, and so such prices are strictly dominated by r. The highest profit a price p_i below r can generate is found when the firm sells with certainty to all consumers who are aware of its price:

$$(p_i - c) \sum_{m=1}^{n} I_m \frac{\binom{n-1}{m-1}}{\binom{n}{m}} = (p_i - c) \sum_{m=1}^{n} I_m m/n, \tag{1}$$

hence the lowest undominated price, $\underline{p}$, is

$$(p_i - c) \sum_{m=1}^{n} I_m m/n = (r - c)I_1/n \iff \underline{p} = \frac{rI_1 + c \sum_{m=2}^{n} I_m m}{\sum_{m=1}^{n} I_m m} > c, \tag{2}$$

so only prices in $\left[\underline{p}, r\right]$ are charged in any equilibrium.[9]

Proposition 1. *There is a unique equilibrium if and only if $I_2 > 0$. The equilibrium is symmetric: firms continuously mix over the support $\left[\underline{p}, r\right]$ via a common CDF that solves* (12). *When $I_2 = 0$, there are uncountably-many equilibria.*

We prove Proposition 1 via two lemmas. Lemma 15 shows there is a unique equilibrium when $I_2 > 0$, and Lemma 16 shows there are uncountably-many equilibria when $I_2 = 0$. We construct our proof of Lemma 15 through a sequence of intermediate Lemmas.

Denote firm i's price distribution by G_i, and $\underline{s}_i$ and $\bar{s}_i$ as the minimum and maximum of the support of firm i's prices. Lemmas 1 to 5 do not require $I_2 > 0$. They say that at least two firms have r as the maximum of their support in equilibrium.

Lemma 1. *If some firm i has a mass point at $\bar{\bar{s}} \equiv \max_j\{\bar{s}_j\}$, i sells only to its I_1/n captive consumers when it sets $p_i = \bar{\bar{s}}$.*

Proof. Suppose instead there was a positive probability that i sells to some contested consumers when setting $p_i = \bar{\bar{s}}$. Then some other firm, $j \neq i$, also has a mass point at $\bar{\bar{s}}$, implying $\lim_{p \uparrow \bar{\bar{s}}} \pi_i(p) > \pi_i(\bar{\bar{s}})$, a contradiction. □

Lemma 2. $\exists\, i\colon\ \bar{s}_i = r$.

[9]When stating and discussing our results, we follow common convention and ignore the possibility that firms choose suboptimal prices with probability zero.

Proof. Denote i: $\bar{s}_i = \max_j\{\bar{s}_j\}$ and suppose $\bar{s}_i < r$. Suppose some firm, j, has a mass point at $\bar{s}_i$. By Lemma 1, $\pi_j(\bar{s}_i) = (\bar{s}_i - c)I_1/n < (r - c)I_1/n = \pi_j(r)$. If no firm has a mass point at $\bar{s}_i$, $\lim_{p\uparrow\bar{s}_i} \pi_i(p) < \pi_i(r)$. □

Lemma 3. $\exists\, i, j\colon i \neq j \;\&\; \bar{s}_i = \bar{s}_j = r$.

Proof. From Lemma 2 we know one firm, say i, has $\bar{s}_i = r$. Denote $j \neq i$ as a firm with the second-highest support-maximum and suppose $\bar{s}_j < r$. Note that as a result, firm i places no mass on prices in $(\bar{s}_i, r)$. Suppose some firm, k, has a mass point at $\bar{s}_j$. By the same argument as in Lemma 1, firm k only ever sells to two types of consumers when it sets $p_k = \bar{s}_j$: its captive consumers, and contested consumers who only see the price of i and k. But then $\pi_k(\bar{s}_j) < \lim_{p\uparrow r} \pi_k(p)$. If no firm has a mass point at $\bar{s}_j$, $\lim_{p\uparrow\bar{s}_j} \pi_j(p) < \lim_{p\uparrow r} \pi_j(p)$. □

Lemma 4. $\pi_i = \pi_j \;\forall\, i, j$.

Proof. Suppose $\pi_i < \pi_j$ for some i and j. Then $\lim_{p\uparrow\bar{s}_j} \pi_i(p) = \pi_j > \pi_i$. □

Lemma 5. *No firm places a mass point at any* $p \in \left[\underline{p}, r\right)$.

Proof. Suppose that firm i has a mass point at $p_i \in \left[\underline{p}, r\right)$. There exists some interval $(p_i, p_i + \epsilon)$ in which no other firm puts probability mass (suppose some firms did, and let $j \neq i$ be a firm with $p_j > p_i$ in its support such that no other firm $k \neq i, j$ has (p_i, p_j) in its support: because $I_m > 0$ for some $m > 1$, there are consumers informed of i's and j's price, hence $\lim_{p\uparrow p_i} \pi_j(p) > \pi_j(p_i + \delta)$ for $\delta \in (0, p_j - p_i)$). But then $\pi_i(p_i + \delta) > \pi_i(p_i)$ for $\delta \in (0, \epsilon)$. □

Lemma 6. *If* $I_2 > 0$*, at most one firm places a mass point at* r*.*

Proof. Suppose i and j place mass points at r. Because $I_2 > 0$, there are consumers who are informed of i's and j's price and no other price, so $\pi_i(r) < \lim_{p\uparrow r} \pi_i(p)$. □

Lemma 7. *If* $I_2 > 0$*, no firm has a mass point at* r*.*

Proof. By Lemma 6, at most one firm has a mass point at r. Suppose exactly one firm, i, has a mass point at r. By Lemma 1, $\pi_i = (r - c)I_1/n$. By Lemma 3 there is some $j \neq i$ with $\bar{s}_j = r$. Because $I_2 > 0$ there are consumers who are informed of the prices from firm i and j and no other price, hence $\lim_{p\uparrow r} \pi_j(p) > \pi_i$, contradicting Lemma 4. □

Lemma 8. *If* $I_2 > 0$, $\pi_i = (r - c)I_1/n \;\forall\, i$.

Proof. From Lemma 3, $\exists\, i, j\colon i \neq j$ and $\bar{s}_i = \bar{s}_j = r$. By Lemma 7, no firm has a mass point at r, hence $\pi_i = \pi_j = (r - c)I_1/n$. By Lemma 4, all firms make this profit. □

Lemma 9. *If* $I_2 > 0$, $\exists\, i, j\colon \underline{s}_i = \underline{s}_j = \underline{p}$.

Proof. Index firms such that $\underline{s}_1 \leq \underline{s}_2 \leq \cdots \leq \underline{s}_n$ and suppose $\underline{s}_1 < \underline{s}_2$. Firm 1 strictly increases profit by shifting the mass it places on prices in $[\underline{s}_1, \underline{s}_2)$, to prices slightly below $\underline{s}_2$, hence

$\underline{s}_2 = \underline{s}_1$. Next, suppose $\underline{s}_1 = \underline{s}_2 > \underline{p}$. By Lemma 5 no firm places a mass point on $\underline{s}_1$. But then $\lim_{p\uparrow\underline{s}_1} \pi_i(p) > (r-c)I_1/n$ for any i, contradicting Lemma 8. □

We now introduce some additional notation to characterize i's expected profits. First, the expected share of consumers that i sells to among consumers who see m prices by setting a price $p_i < r$, D_i^m. The set of all vectors of length $m-1$ of distinct firms that do not include i is:

$$S_{m-1}(i) \equiv \{a | a = (j_1, ..., j_{m-1}) \not\ni i, \text{ and } j_k \neq j_l \,\forall j_k, j_l \in a\}, \tag{3}$$

which allows us to write

$$D_i^m \equiv \begin{cases} n^{-1} & m = 1, \\ \binom{n}{m}^{-1} \sum\limits_{a \in S_{m-1}(i)} \prod\limits_{k \in a} (1 - G_k) & m = 2, \dots, n. \end{cases} \tag{4}$$

Term D_i^1 is the share of consumers who see one price and buy from i. More generally, let a be a consideration set of a consumer of length m that includes firm i and $m-1$ other firms. Consumers with this consideration set buy from i if $p_i \leq p_k$ for all $k \in a$.[10] The probability of this is $\prod_{k\in a}(1 - G_k)$. Summing over all sets of firms $a \in S_{m-1}(i)$ that are in a consideration set with i, and dividing by the number of consumers who see m prices, $\binom{n}{m}$, gives the expected share of consumers that firm i attracts among consumers who see m prices.

Similarly, we define the set vectors of length $m-1$ of non-equal firms other that i and $j \neq i$:

$$S_{m-1}(ij) \equiv \{a | a = (j_1, ..., j_{m-1}) \not\ni i, j \colon i \neq j, \text{ and } j_k \neq j_l, \forall j_k, j_l \in a\}, \tag{5}$$

which allows us to write,

$$D_{ij}^m \equiv \begin{cases} n^{-1} & m = 1 \\ \binom{n}{m}^{-1} \sum\limits_{a \in S_{m-1}(ij)} \prod\limits_{k \in a} (1 - G_k) & m = 2, \dots, n-1 \\ 0 & m = n. \end{cases} \tag{6}$$

Relating D_i^m and D_{ij}^m, we find

$$D_i^m = D_{ij}^m + \frac{\binom{n}{m-1}}{\binom{n}{m}} (1 - G_j) D_{ij}^{m-1}, \qquad m = 2, \dots, n \text{ and } i \neq j. \tag{7}$$

This decomposition will be useful for the proceeding results. Note that $D_i^n = \prod_{j=1; j\neq i}^n (1 - G_j)$, $D_{ij}^{n-1} = \frac{1}{n} \prod_{k=1;k\neq i,j}^{n-1} (1 - G_k)$, and $D_i^n = (1 - G_j) n D_{ij}^{n-1}$. The total profits at a price p from consumers

[10] By Lemma 5, ties are zero-probability events so we do not need to specify a tie-breaking rule.

who see m prices is $K^m(p) \equiv (p-c)I_m$ for $m = 1, \ldots, n$, so that the expected profit of firm i is

$$B_i(p_i) \equiv \sum_{m=1}^{n} K^m(p_i) D_i^m(p_i), \qquad i = 1, \ldots, n. \tag{8}$$

$K^m(p_i)$ is the total profit from consumers who see m prices when they pay p_i. $D_i^m(p_i)$ is the expected share of consumers of firm i when charging p_i among consumers who see m prices. Thus, $K^m(p_i)D_i^m(p_i)$ is the expected profit of firm i from consumers who see m prices, and $B_i(p_i)$ is the expected profit of firm i given that each rival j plays G_j.

Lemma 10. *If $I_2 > 0$, $B_i(p_i)$ is constant and equal to $(r-c)I_1/n$ at the points of increase of G_i on $\left[\underline{p}, r\right)$ for all i.*

Proof. If p_i is a point of increase of G_i, then firm i must earn the equilibrium profit at p_i, which by Lemma 8 is $(r-c)I_1/n$. □

Lemma 11. *If p is a point of increase of G_i and G_j, then $G_i = G_j$ at p almost surely. $B_i(p)$ is continuous at every point of increase of i.*

Proof. First, we can use (7) to rewrite

$$B_i(p) = \tfrac{1}{n}K^1(p) + \sum_{m=2}^{n-1} D_{ij}^m(p)K^m(p) + (1 - G_j(p)) \sum_{m=2}^{n} \frac{\binom{n}{m-1}}{\binom{n}{m}} D_{ij}^{m-1}(p)K^m(p). \tag{9}$$

Because p is a point of increase of G_i and G_j, Lemma 4 implies that $B_i(p) = B_j(p)$. Using $D_{ij}^m = D_{ji}^m$ and $D_{ij}^{m-1} = D_{ji}^{m-1}$, it follows that $B_i(p) = B_j(p)$:

$$\begin{aligned} &\Rightarrow (1 - G_j(p)) \sum_{m=2}^{n} \frac{\binom{n}{m-1}}{\binom{n}{m}} D_{ij}^{m-1}(p)K^m(p) = (1 - G_i(p)) \sum_{m=2}^{n} \frac{\binom{n}{m-1}}{\binom{n}{m}} D_{ij}^{m-1}(p)K^m(p) \\ &\Rightarrow G_i(p) = G_j(p). \end{aligned} \tag{10}$$

Note that because $D_{ij}^1 = n^{-1}$, the summations are strictly positive, which guarantees that the last implication is true.

To show that $B_i(p)$ is continuous at every point of increase p of i, we use that all firms j who also have a point of increase at p set $G_j(p) = G_i(p)$. For all other firms k, $G_k(p)$ is constant around p. Additionally, observe that $K^m(p)$ is continuous for all m and p, and $(r-c)I_1/n$ is continuous. Thus, $B_i(p) = (r-c)I_1/n$ pins down a continuous function $G_i(p)$ around p. We conclude that $B_i(p)$ is continuous at p. □

Lemma 12. *If $I_2 > 0$, for every i and every point of increase p of G_i in $\left[\underline{p}, r\right)$, there is at least one G_j with $j \neq i$ such that G_j increases at p.*

Proof. Suppose instead there is no such firm $j \neq i$. Because p is a point of increase of G_i, $dB_i(p) = 0$ by Lemma 10. By Lemma 11, we can differentiate $B_i(p)$ with respect to p, leading to

$$\sum_{m=1}^{n} D_i^m(p) dK^m = 0. \tag{11}$$

Note that because no firm $j \neq i$ has a point of increase at p, $dD_i^m = 0$ for all $m > 1$; hence dD_i^m terms are absent from (11). Because $D_i^m(p)dK^m \geq 0$ for all m and $D_i^1 dK^1 = \frac{1}{n}dK^1 > 0$, the left-hand-side of (11) is strictly positive, but the right-hand-side is zero, a contradiction. We conclude that $dD_i^m(p) < 0$ for some m, implying that at least one G_j has to increase at p. □

Lemma 13. *If $I_2 > 0$ and G_i is strictly increasing on some open interval (x, y), $\underline{p} < x < y < r$, then G_i is strictly increasing on the whole interval $\left[\underline{p}, y\right)$.*

Proof. Suppose instead that G_i is, without loss of generality, constant on (z, x) for some $z \in \left[\underline{p}, x\right)$. By Lemma 5, there are no mass points at z or x, hence $G_i(z) = G_i(x)$. At least two firms, k and l, must charge prices with positive probability in $(x - \epsilon, x)$ for some $\epsilon > 0$. (If no firm charges a price in $(x - \epsilon, x)$, take $\bar{\epsilon}$ to be the supremum of the set of $\epsilon > 0$ such that this is true. Firms charging prices in some neighborhood around $x - \bar{\epsilon}$ strictly increase profits by moving probability mass into $(x - \bar{\epsilon}, x)$. Thus, at least one firm charges prices with positive probability in $(x - \epsilon, x)$ $\forall$ $\epsilon > 0$. By Lemma 12, this extends to two firms.)

By Lemma 10, for every $p \in (x - \epsilon, x)$, $B_k(p) = B_l(p) = (r - c)I_1/n$. And because there are no mass points on $\left[\underline{p}, r\right]$ (Lemmas 5 and 7), $B_i(x) = B_k(x) = B_l(x) = (r - c)I_1/n$. Because $x < r$ and i charges prices in (x, y) with positive probability, Lemma 11 implies that

$$G_k(x) = G_l(x) = G_i(x) < 1.$$

But with $B_i(x) = B_l(x) = B_l(p)$ for all $p \in (x - \epsilon, x)$, it must be that $B_i(p) \leq B_l(p)$ for all $p \in (x - \epsilon, x)$, because these p are not in i's support. But then by the same arguments used in Lemma 11, $B_i(p) \leq B_l(p)$ implies $G_i(p) \leq G_l(p)$. But this contradicts $G_i(x) = G_l(x)$, because $G_l(p)$ is increasing on $(x - \epsilon, x)$, but $G_i(p)$ is constant. □

Lemma 14. *If $I_2 > 0$, all firms mix continuously via some symmetric $G(p)$ over the support $\left[\underline{p}, r\right]$, in any equilibrium.*

Proof. We first show that $\bar{s}_i = r$ $\forall i$. Suppose instead there exists a firm i with $\bar{s}_i < r$. Then by Lemmas 5 and 13, this firm charges prices on $\left[\underline{p}, \bar{s}_i\right)$. By Lemma 11, all firms that charge prices $p \in \left[\underline{p}, \bar{s}_i\right)$ play symmetric CDFs $G(\cdot)$ for these prices. Because some firms have r in their support (Lemma 3) and charge prices in $\left[\underline{p}, r\right)$ (Lemmas 7 and 13), we have $G(\bar{s}_i) < 1$, a contradiction. We conclude that $\bar{s}_i = r$ $\forall i$.

Because $\bar{s}_i = r$ $\forall i$ and no firm has a mass point at r (Lemma 7), Lemma 13 implies that all firms charge prices on every open subinterval of $\left[\underline{p}, r\right]$ with strictly positive probability. Lemma 11 then implies that all firms play a symmetric price distribution over the whole support. $\square$

Lemma 15. *If $I_2 > 0$, there is a unique equilibrium which is symmetric. The equilibrium CDF, G, solves*

$$(p-c)\sum_{m=1}^{n}\frac{m}{n}I_m(1-G(p))^{m-1} = (r-c)\frac{I_1}{n}, \quad \forall p \in \left[\underline{p}, r\right]. \tag{12}$$

Proof. By Lemma 14 all firms play symmetric price distributions, G over the support $\left[\underline{p}, r\right]$, in any equilibrium. By Lemma 10, G must solve $B(p) = (r-c)I_1/n$ (12) for all $p \in \left[\underline{p}, r\right]$. Equation (12) is satisfied at $G\left(\underline{p}\right) = 0$ and $G(r) = 1$. Because $(p-c)$ is strictly increasing and continuous in p, the equation pins down a unique strictly increasing and continuous $G(p)$ for all $p \in \left[\underline{p}, r\right]$. To complete the specification, let $G(p) = 0$ for $p < \underline{p}$, and $G(p) = 1$ for $p \geq r$. $\square$

Lemma 16. *If $I_2 = 0$, infinitely many equilibria exist.*

Proof. We show that there are uncountably-many asymmetric equilibria in which one firm has a mass point at r when $I_2 = 0$. Specifically, the following strategies constitute Nash Equilibria, parameterized by $x \in \left(\underline{p}, r\right)$. Firms $i = 1, \ldots, n-1$ have the support $\left[\underline{p}, r\right]$, while one firm, n, has the support $\left[\underline{p}, x\right] \cup \{r\}$. Over $\left[\underline{p}, x\right)$, all firms play the symmetric mixed strategy $G(\cdot)$ that solves (12). Firm n places its remaining probability mass $1 - G(x)$ at r. Over $[x, r)$, firms $i = 1, \ldots, n-1$ play the symmetric mixed strategy $\hat{G}(\cdot)$. Adjusting (9) appropriately, we see that $\hat{G}(\cdot)$ solves

$$\frac{1}{n}K^1(p)+\sum_{m=2}^{n-1}\frac{\binom{n-2}{m-1}}{\binom{n}{m}}(1-\hat{G}(p))^{m-1}K^m(p)+(1-G(x))\sum_{m=2}^{n}\frac{\binom{n-2}{m-2}}{\binom{n}{m}}(1-\hat{G}(p))^{m-2}K^m(p) = (r-c)\frac{I_1}{n}. \tag{13}$$

As $p \uparrow r$, the left-hand-side converges to $K^1(p)/n = (p-c)I_1/n$. Because $I_2 = 0$, firms that charge prices arbitrarily close to r cannot attract any contested consumers from firm n when n charges r. Thus, even arbitrarily close to r, firms $i \neq n$ still only earn $(p-c)I_1/n$ (in contrast to the proof of Lemma 7 when $I_2 > 0$).

These equilibria exist for $I_2 = 0$ and any $I_3, I_4, \ldots, I_m \geq 0$. If $I_3 = 0$ in addition to $I_2 = 0$, there are equilibria where two firms have a mass point at r, and so on. $\square$

Lemmas 15 and 16 together imply that Proposition 1 holds.

Proposition 1 generalizes the intuition from the motivating triopoly example. In each equilibrium at least two firms have no mass point at r. These firms compete for contested consumers. If $I_2 = 0$, however, the remaining firms might be sidelined. Sidelined firms do not benefit from joining the competition for contested consumers and place a mass point at r, focusing on exploiting their captive consumers.

If $I_2 > 0$, however, no firm is sidelined in equilibrium. Each firm competes head-to-head with each rival for at least some consumers, giving each firm an incentive to compete for contested consumers. This is why no firm can have a mass point on r and only the symmetric equilibrium survives.

Discussion

Proposition 1 tells us there is a unique equilibrium when $I_2 > 0$. The unique equilibrium CDF (the solution to the polynomial (12)) does not generally have an analytic solution except in some cases. We now report the unique equilibrium for some such settings that feature $I_2 > 0$. To do so, we normalize (without loss of generality) $\sum_{m=1}^{n} I_m = 1$, and express (12) more parsimoniously as a probability generating function associated with the number of rivals faced by each firm:[11]

$$\phi(x) = \sum_{m=1}^{n} a_m x^{m-1}, \tag{14}$$

where $a_m \equiv I_m m/n$ and the equilibrium CDF is the solution to

$$\frac{\phi(1 - G(p))}{\phi(0)} = \frac{r - c}{p - c}, \quad \text{where} \quad \underline{p} \equiv \frac{r\phi(0) + c(\phi(1) - \phi(0))}{\phi(1)}. \tag{15}$$

Example 1. Suppose consumers only check one or two prices i.e., $I_1, I_2 > 0$ and $I_{m>2} = 0$. Then $\phi(x) = a_1 + a_2 x$. This setting is particularly relevant. First, it is the model of Varian (1980) (i.e., $I_1, I_n > 0$ and $I_1 + I_n = 1$) with $n = 2$. Here, the unique equilibrium is that reported by Baye, Kovenock, and De Vries in their full characterization of equilibria in Varian's model. Our Proposition 1 shows uniqueness is obtained not because $n = 2$ per se, but because $I_2 > 0$.

Second, our analysis treated consumers' information as exogenous. In their canonical consumer search model, Burdett and Judd (1983) show ex-ante symmetric consumers endogenously choose to search either once or twice (i.e., $I_1, I_2 > 0$ and $I_{m>2} = 0$) and derive a symmetric equilibrium.[12] Our Proposition 1 shows this is in fact the unique equilibrium in such a setting.

Example 2. Extending the first example, suppose that consumers are still only aware of few prices, but this time either one, two, or three (i.e., $I_1, I_2, I_3 > 0$ and $I_{m>3} = 0$). Then $\phi(x) = a_1 + a_2 x + a_3 x^2$, for which only one of the two solutions to (15) is valid, and by our result is the unique equilibrium. More generally, the solution to the setting with $I_1, \ldots, I_k > 0$ and $I_{m>k} = 0$ quickly becomes intractable as k grows. However, there are approaches that simplify (14) when $I_m > 0$ for all m, as the next example shows.

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[11] We are grateful to Mark Armstrong for this suggestion and adopt the notation of Armstrong and Vickers (2019).

[12] Burdett and Judd (1983) assume a continuum of firms, but the equilibrium strategy they derive can be obtained for any finite $n \geq 2$.

Example 3. Consider a setting in which each consumer is independently aware of each firm's price with probability $\alpha \in (0, 1)$. Then $I_m = \binom{n}{m}\alpha^m(1 - \alpha)^{n-m} > 0$ for $m = 1, \ldots, n$. Application of the binomial theorem to (14) yields $\phi(x) = \alpha(1 - \alpha(1 - x))^{n-1}$. This is the symmetric version of the models of awareness and advertising by Ireland (1993) and McAfee (1994), shown to be unique by Spiegler (2006). Our more general result shows that uniqueness follows in this special setting because $I_2 > 0$, which is implied by $\alpha \in (0, 1)$.

Textbook Bertrand. Our results also help to select equilibria in perhaps the simplest version of Bertrand competition where all consumers compare prices ($I_1 = 0$). With more than two firms, it is well-known that price competition produces infinitely-many equilibria (two firms charge marginal cost while others charge any positive prices). Our results offer an intuitive reason to focus on the symmetric (price-equals-marginal-cost) equilibrium: When $I_1 > 0$ and $I_2 > 0$ Proposition 1 shows the only equilibrium is symmetric; and as $I_1 \downarrow 0$ ceteris paribus, profits go to zero and the equilibrium pricing distribution converges to the degenerate distribution with all its probability mass on marginal cost.

Other Implications. More generally, our result may make this classic price competition framework more attractive to researchers. For those adopting it, we argue that our result makes the symmetric equilibrium the practically- and theoretically-relevant one.

Firstly, the assumption that some consumers check two prices ($I_2 > 0$) appears quite reasonable. It seems intuitive that a wide range of search-cost distributions would produce some consumers who compare two prices. As a case in point, Burdett and Judd (1983) find that even with homogeneous (and linear) search costs, a positive mass of consumers gather two price quotations in equilibrium.[13] Empirical evidence also broadly supports this. The Consumer Financial Protection Bureau (2015) show that around 90% of home-buyers consider either one or two mortgage lenders/brokers, a finding consistent with the estimate of Woodward and Hall (2012). Similar results have also been found in markets for simpler products. For example, De los Santos, Hortaçsu, and Wildenbeest (2012) find that around 70% of consumers visited either one or two online book stores before purchasing, while De los Santos (2018) finds the average number to be 1.27. Studying auto-insurance in the US, Honka (2014) finds that around 10% and 35% of consumers obtain one and two quotes respectively.

From a theoretical perspective, the framework as a whole may be more attractive to researchers because we now know that multiple equilibria only arise in the special case of $I_2 = 0$. Furthermore, even in that special case, the symmetric equilibrium is robust in the sense that the strategy is right-continuous at $I_2 = 0$: as $I_2 \downarrow 0$, the equilibrium strategy is equal to the symmetric equilibrium strategy when $I_2 = 0$. Conversely, suppose one predicts an asymmetric

[13]Such consumer behavior is also produced in the "high-intensity search" equilibrium of Janssen and Moraga-González (2004).

equilibrium when $I_2 = 0$ and consider a perturbation or shock to the number of consumers such that $I_2 = \epsilon > 0$ ceteris paribus. Following the small change, the unique and symmetric equilibrium strategies of each firm following the shock jumps, and can be qualitatively very different from firms' supposed asymmetric strategies before the change. This offers a stability justification for the restriction to symmetric equilibria made in prior work.

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