Sticky prices and the transmission mechanism of monetary policy:
A minimal test of New Keynesian models

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Sticky prices and the transmission mechanism of monetary policy:
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Abstract
This paper proposes a minimal test of two basic empirical predictions that aggregate data should exhibit if sticky prices were the key transmission mechanism of monetary policy, as implied by the benchmark DSGE-New Keynesian models. First, large monetary policy shocks should yield proportionally larger initial responses of the price level and smaller real effects on output. Second, in a high trend inflation regime, prices should be more flexible, and thus the real effects of monetary policy shocks should be smaller and the response of the price level larger. Our analysis provides some statistically significant evidence in favor of a sticky price theory of the transmission mechanism of monetary policy shocks.

Keywords: Sticky prices, local projections, smooth transition function, time-dependent pricing, state-dependent pricing

JEL Codes: E30, E52, C22

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1 Introduction

In the last decade the DSGE-New Keynesian (NK) paradigm has become the new work- 
horse for the analysis of business cycle fluctuations and the effects of monetary and fiscal 
policy, both among academic researchers and policy institutions. Nominal rigidities lie 
at the very core of these models. The simplest version of this new paradigm (i.e., the 
so-called 3 equations model, see e.g., Woodford, 2003) assumes price stickiness to yield 
the New Keynesian Phillips Curve describing the supply side of the model. Indeed, 
the existence of real effects of monetary policy shocks (or demand shocks in general) 
rests on the assumption of sticky nominal prices and/or wages in these models. Whilst 
most recent models, introducing banking and financial frictions, can provide alternative 
mechanisms for the transmission of monetary policy shocks, price stickiness remains the 
major reason why monetary policy has effects on real variables in most versions of NK 
models.

This key assumption spurred large scale empirical investigations on the pricing be- 
behavior of individual firms (e.g., Bils and Klenow, 2004; Klenow and Kryvtsov, 2008; Na- 
kamura and Steinsson, 2008; Eichenbaum, Jaimovich, and Rebelo, 2011). This literature 
has emphatically shown that individual prices do change infrequently and interprets the 
documentation of this empirical fact as an external validation of the importance of sticky 
prices in the transmission of monetary policy shocks.

This paper proposes a minimal test of NK models on aggregate data, that is, a 
minimal test of the assumption that nominal rigidities are indeed at the root of the 
real effects of monetary policy. It does so by checking two basic empirical predictions 
that should appear in aggregate data if sticky prices were the main determinant of the 
transmission mechanism of monetary policy. First, large absolute value monetary policy 
shocks should lead more firms to adjust their prices, and hence yield a proportionally 
larger response of the aggregate price level and a smaller one of output. Second, the fre- 
quency of price changes should be an increasing function of underlying levels of inflation,
that is, prices should be more flexible in a high trend inflation regime than otherwise. Hence, the higher trend inflation, the smaller should be the real effects of monetary policy shocks and the larger the response of the aggregate price level. These two empirical predictions are quite intuitive and derive generally from a variety of state-dependent sticky price models in the literature (see Section 2), so they really get to the foundation of the NK paradigm. In this sense, our contribution is really a minimal, first pass test for NK theories: if sticky prices are pivotal, aggregate data should exhibit these two features.

This paper takes these two theoretical predictions to the data using local projections (Jordà, 2005). This estimation procedure will be combined with the smooth transition regression methodology of Granger and Teräsvirta (1993). Using US monetary policy shocks identified with the narrative method of Romer and Romer (2004), we investigate whether the response of a range of real and nominal variables support the sticky prices theory in the US between 1969 and 2007. The empirical methodology of this paper is most closely related to the work of Tenreyro and Thwaites (2016). Using local projections and a smooth transition function, they find that the effects of monetary policy are less powerful in recessions than in expansions.¹

By applying a similar approach to our research questions, our analysis provides some statistically significant evidence in favor of the sticky price theory of the propagation mechanism of monetary policy shocks. With regards to the first implication, large absolute value shocks have disproportionately larger effects on prices on impact, but are less persistent, matching the theoretical predictions. Testing the second implication, the impulse response functions in the high and low trend inflation regimes are significantly different and are in line with the theoretical prediction of higher price flexibility in the high trend inflation regime.

We conduct a thorough sensitivity analysis to establish the robustness of these results.

¹Cover (1992) and Ravn and Sola (2004) are examples of earlier studies on the non-linear effects of monetary policy shocks.
Furthermore, we conduct statistical tests to ensure that the non-linearity of the impulse response function is not due to a different feedback of monetary policy to inflation in different subsamples. Finally, we employ shocks recovered from a recursive VAR to test the robustness of the results with respect to the measure of monetary policy surprises.

Following Alvarez, Lippi, and Passadore (2016), our results also have a different interpretation. The majority of sticky price models can be categorized into two main types; state-dependent and time-dependent pricing models. The latter assume that the time between consecutive price changes of an individual firm is independent of economic conditions. In contrast, state-dependent models assume that the individual firm changes its price, subject to a fixed adjustment cost, whenever the economic state attains a critical level. Therefore, the frequency of price changes is endogenous to the model in this case, as well as both the number of firms that decide to change their prices following a monetary policy shock and the size of these changes (the so-called “selection effect”, see Golosov and Lucas, 2007). Hence, the two above predictions would hold in a state-dependent prices model, but not in a time-dependent one.\(^2\) However, there are likely to be both state-dependent and time-dependent price and wage contracts in the real economy, such that a relevant share of the former should make the two predictions above be present in aggregate data. Following this interpretation, our results are the first ones (to the best of our knowledge) that point towards a significant presence of state-dependent pricing in the US economy from an aggregate perspective, in accordance with what the empirical literature on individual firm price data suggests.

\section{Theory and Testable Implications}

A large body of literature documents the importance of nominal price rigidities in monetary economics, using both time-dependent and state-dependent pricing models. Prom-

\(^2\)Alvarez, Lippi, and Passadore (2016) show that the impulse response after a monetary shock is size-independent in time-dependent models, whereas for state-dependent models shocks above a minimum size induce the economy to exhibit full price flexibility.
inent examples of the former include Fischer (1977), Taylor (1979, 1980), Calvo (1983) or Reis (2006). These models are typically characterized by the exogenous frequency or timing of price adjustments. In contrast, in state-dependent models, price adjustments of individual firms are triggered by changes in the economic environment. This feature can pose serious challenges to the idea of monetary non-neutrality as real effects can be either “small and transient” or entirely absent (Caplin and Spulber, 1987; Golosov and Lucas, 2007). Intuitively this can happen because of a strong “selection effect”. The firms that decide to change their prices by paying the menu costs after a monetary shock are the ones further off from their optimal price. Hence, these are not random firms and the sizes of the price changes are relatively large, such that the aggregate price level can mimic a flexible price environment. Nonetheless, a variety of later studies have subsequently investigated the validity of this result with extensions such as informational costs (Gorodnichenko, 2008; Bonomo, Carvalho, and Garcia, 2013; Alvarez, Lippi, and Paciello, 2010), multi-product firms (Alvarez and Lippi, 2014; Midrigan, 2011; Bhattacharai and Schoenle, 2014), or multiple sectors and intermediate inputs (Nakamura and Steinsson, 2010) and found that state-dependent models can also induce sizable real effects. The same result holds in a variety of state-dependent models that are carefully calibrated to match the main features of retail price microdata (Alvarez, Le Bihan, and Lippi, 2016; Eichenbaum et al., 2011; Costain and Nakov, 2011, 2018), and, more recently, of both the size and frequency of price and wage changes (Costain, Nakov, and Petit, 2018). Overall, the theoretical literature suggests that price stickiness is an important propagation mechanism. Crucially, these state-dependent models also offer two testable implications on aggregate data to verify the empirical validity of sticky price theories.

First Testable Implication: The impulse response functions of inflation and output to a monetary policy shock should depend on the size of the shock.
If there is a non-negligible fraction of state-dependent price-setters, the impulse response should be a non-linear function of the size of the shock, such that the larger the monetary policy shock, the larger should be the response in the aggregate price level at short horizons and the weaker the effect at large horizons (and vice versa for output). Alvarez and Lippi (2014) formally prove this result (see part (iii) of proposition 8 at p.109-110 and the right panel of Figure 3 therein): (i) for small monetary shocks, the impact effect on prices is second order compared to the shock size and, hence, the impact effect on output is on the order of the monetary shock, (ii) for large enough shocks, the economy exhibits full price flexibility. The intuition is clear and due to the endogenous frequency of price changes: if the shock is large enough, then many firms will decide to pay the menu costs and change their prices such that the reaction of the aggregate price level is increasing in the size of the monetary policy shock. The real effects of monetary policy shocks, instead, are hump-shaped with respect to the size of the shock. Intuitively, the larger the shock, \textit{ceteris paribus}, the larger the real effects, as in a time-dependent model. However, this effect faces an offsetting force due to proportionally larger reaction of the aggregate price level, because of the “selection effect”, as just explained. For a small shock, the first effect prevails, so that both the impact and the cumulative effect on output is increasing in the size of the shock. For large shocks the opposite occurs. It follows that big enough shocks should have lower real effects than smaller shocks.\footnote{Alvarez and Lippi (2014) show that the monetary shock that maximizes the cumulated effect on output (i.e., the area under the impulse response function) is about one-half of the standard deviation of price changes. Costain et al. (2018) show that a similar effect occurs in a model with state-dependent prices and wages (see Figure 10 therein).} Figure 1 displays the impulse response functions for two shocks of different size in the state-dependent pricing model in Costain and Nakov (2018). As explained, a larger shock will make more firms to adjust (see bottom-right panel), so that the strong reaction of the aggregate price level (see upper-right panel) counteracts the monetary policy stimulus. Hence, the response of consumption (which equals output in the model) is lower and less persistent than for the smaller shock.
Second Testable Implication: The impulse response functions of inflation and output to a monetary policy shock should depend on the average level of inflation. As outlined in Dotsey, King, and Wolman (1999) or Costain and Nakov (2011), average inflation has an effect on the frequency of price adjustments in state-dependent models. Inflation erodes a firm’s relative price so that firms are more likely to pay the fixed cost more often, and adjust prices more frequently. For example, in the model of Dotsey et al. (1999), firms adjust their prices at least once every 5 quarters with a trend inflation rate of 10%, while firms may not change their price for 13 quarters for a 2.5% inflation rate. Note that time-dependent models (as the widely used Calvo model) are criticized because they assume that the time between subsequent adjustments is a structural parameter independent from the trend inflation level (see, e.g., Bonomo and Carvalho, 2004; Levin and Yun, 2007). Indeed, the empirical analysis in Alvarez, Beraja, Gonzalez-Rozada, and Neumeyer (2018) provides solid evidence of how the frequency of price changes varies with inflation. Figure 5 and 6 therein show that the frequency of price changes do not react much for levels of annual inflation up to 5%, then it starts accelerating and finally it increases linearly for values of annual inflation above 14% with an elasticity of about two-thirds. This is in line with Sheshinski and Weiss (1977)’s menu cost model with no idiosyncratic shocks.4

Importantly, in a sticky price model, this evidence implies different impulse responses to a monetary policy shock in high trend inflation regimes compared to low trend inflation regimes, because in the former prices are more flexible. In particular, Alvarez, Le Bihan, and Lippi (2016) show that in a large class of models, the total cumulative output effect of a small unexpected monetary shock is inversely related to the average number of price changes per year. Figure 2 visualizes this testable implication, displaying the impulse response function to a monetary policy shock for different values

4Alvarez et al. (2018) show that the same holds for the dispersion of relative prices and the average size of price changes.
of trend inflation, again in the model in Costain and Nakov (2018).\footnote{The same holds in a model in which nominal rigidities in both wages and prices are state-dependent (see Figure 11 in Costain et al., 2018), and in a model that allows for temporary price changes, because firms can set a price plan, rather than a fixed price as in the standard menu cost model (see Figure 2 in Alvarez & Lippi, 2018).} This theoretical prediction provides the second testable implication: we should observe a quicker and less persistence reaction of prices in high inflation regimes.

3 Empirical Methodology

We test these two predictions by analyzing the presence of non-linearities in the impulse response functions of inflation and output to a monetary policy shock for large and small shocks, and for high and low trend inflation. Our empirical methodology follows a growing body of literature employing local projections, as from Jordà (2005), and combining it with the smooth transition regression method of Granger and Teräsvirta (1993) (e.g. Auerbach and Gorodnichenko, 2012a, 2012b; Caggiano, Castelnuovo, and Groshenny, 2014; Caggiano, Castelnuovo, Colomb, and Nodari, 2015; Tenreyro and Thwaites, 2016; Ramey and Zubairy, 2014; Furceri, Loungani, and Zdienicka, 2016). The local projection framework allows to account for non-linearities and state-dependency in a very straightforward way. Adding squared and cubed terms of the structural shocks enables the analysis of sign- and size-dependencies. Conversely, smooth transition function interactions can account for state- or regime-dependency in a convenient way.

Inference on estimated coefficients needs to take three main characteristics of the estimation procedure into account: the possible correlation of residuals across dates $t$ (autocorrelation) and horizons $h$ (spatial correlation), the possibility of heteroskedasticity. We follow Ramey and Zubairy (2014) and Tenreyro and Thwaites (2016) and employ the Driscoll and Kraay (1998) method to obtain heteroskedasticity autocorrelation spatial correlation (HACSC) robust standard errors.

Moreover, we smooth local projections impulse responses such that the new, smoothed
coefficients are 9-period centered moving averages. This is true for all horizons, except for the starting and end regions where the moving averages are smaller in order to account for the reduced availability of lags or leads.\footnote{Consequently, the starting and end points are just the original coefficients. We provide the benchmark results with unsmoothed coefficients in Appendix A.}

**Data.** The monthly sample for our variables runs from 1969m3 up to 2007m12. Hence, the sample excludes the most recent financial crisis, mainly due to the fact that monetary policy may be have been very different during that time and that the zero-lower bound on nominal interest rates might have been binding.

The main shock variable used in this analysis is based on the narrative analysis of Romer and Romer (2004), extended by Coibion (2012). Romer and Romer (2004) identified monetary policy shocks by using a narrative approach, such that they infer the intended federal funds rate at every Federal Open Market Committee (FOMC) meeting from 1969 onwards. By regressing changes of this intended rate on Greenbook forecasts they then derive a measure of monetary policy surprises that is arguably exogenous to the Fed’s information set about the future state of the economy.

There are a number of other ways of identifying monetary policy surprises, for example High Frequency Identification (HFI) (Kuttner, 2001; Gürkaynak, Sack, and Swanson, 2005; Barakchian and Crowe, 2013; Gertler and Karadi, 2015) or identification restrictions in a VAR (Christiano, Eichenbaum, and Evans, 1999; Bernanke and Mihov, 1998; Kim and Roubini, 2000; Uhlig, 2005; Bernanke, Boivin, and Eliasz, 2005). We conduct robustness tests with respect to the latter and find largely similar conclusions. We do, however, not include HFI shocks due to two main reasons. Firstly, the available shock series are too short for our analysis. Even the backward extension up to 1979 by Gertler and Karadi (2015) omits a significant proportion of the Great Inflation period; a major source of variation in our smooth transition function. Secondly, Ramey (2016) argues that these shocks may not be robust to samples where anticipation effects are
important. Moreover, Ramey (2016, p. 109) shows that the impulse response function to HFI shocks can look very different depending on whether one uses a VAR or local projections; for example, a contractionary Gertler and Karadi (2015) shock in a local projection increases output and leaves the price level largely unchanged whereas it produces the literature standard effect in their proxy SVAR.

We analyze the response of the output, inflation, the price level and the nominal interest rate. More precisely, the series for output is the industrial production index, for the price is personal consumption expenditure (PCE) inflation, and for the nominal interest rate is the effective federal funds rate, all from the Federal Reserve Bank of St. Louis Database (FRED).

4 Results

4.1 Implication 1: Non-linear effects of monetary shocks

Non-linear local projections. In order to test the non-linearity of impulse responses we consider the following non-linear local projection:

\[ y_{t+h} = \alpha_h + \tau_h t + \beta_h \epsilon_t + \psi_h \epsilon_t^2 + \psi_h \epsilon_t^3 + \sum_{k=1}^{K} \eta_{h,k} x_{t,k} + \nu_{t+h} \]  

which is estimated for \( h = 0, 1, \ldots, H \). We set \( H = 48 \) which corresponds to an impulse response horizon of four years. \( y_{t+h} \) denotes the variable of interest, in our case either the industrial production index, PCE inflation or the federal funds rate. \( \epsilon_t \) are the narrative Romer and Romer (2004) shocks. \( x_{t,k} \) denotes the \( k \)th control variable and \( \nu_{t+h} \) the estimation error, possibly heteroskedastic and serially correlated. We include up to three months of lags of industrial production, PCE inflation and the federal funds rate.

\[ \text{The variables are all in natural log levels and then multiplied by 100 except for the federal funds rate which remains in percentage points. This transformation is standard in the literature and enables the interpretation of the strength of the coefficients as approximate percentage points.} \]
Moreover, we follow Ramey (2016) and include contemporaneous values of the industrial production index and PCE inflation. This is equivalent to assuming recursiveness between the three different variables of interest since inflation and industrial production can contemporaneously affect the federal funds rate but not vice versa.\footnote{As Ramey (2016) points out, relaxing this assumption would otherwise lead to a number of puzzles. A contractionary monetary policy shock would actually be expansionary for about a year and produce a very pronounced price puzzle.}

The inclusion of a squared and a cubed shock value accounts for non-linearities in the impulse response function. The coefficient with respect to squared shocks, $\vartheta_h$, captures possible asymmetries of the impulse response functions with respect to positive and negative shocks. In general, a $\vartheta_h$ with a sign equal to that of $\beta_h$ \\_amplifies\_ the linear coefficient of the impulse response with respect to positive shocks. On the contrary, a $\vartheta_h$ with an opposite sign to $\beta_h$ \\_counteracts\_ the linear coefficient of the impulse response with respect to positive shocks. Whenever $\vartheta_h = 0$, there is \textit{no asymmetry} of the impulse response function with respect to both negative and positive shocks.

The coefficient with respect to cubed shocks, $\psi_h$, captures possible non-linearities with respect to the size of the shock. In general, a $\psi_h$ with a sign equal to the one of $\beta_h$ \textit{amplifies} the latter coefficient after larger shocks. On the contrary, a $\psi_h$ with a sign equal to the one of $\beta_h$ \textit{counteracts} the latter coefficient after larger shocks, possibly even tilting the overall effect from one sign to another for large enough shocks. Whenever $\psi_h = 0$, then, the impulse response function is linear with respect to the shock size. Therefore, $\psi_h$ is the main coefficient of interest to test our first theoretical prediction. Let $y_t$ signify prices. If firms exhibit state-dependent pricing we should see a negative $\psi_h$ at small horizons as we expect $\beta_h$ to be close to zero or negative. This means that firms change prices quicker and so prices are disproportionately more negative at small horizons. Furthermore, we would then expect to see a positive $\psi_h$ at larger horizons, weakening the price response, as more firms have already changed prices earlier. Consequently a combination of a negative $\psi_h$ at small horizons, as more firms change prices right away,
and a positive $\psi_h$ at larger horizons, as persistence is lower due to earlier price changes, would speak in favor of state-dependent pricing and of sticky prices as a valid aggregate propagation mechanism of monetary policy shocks.

**Estimates.** The resulting linear coefficient and the various non-linear terms are reported in the panels of Figure 3. Panel (a) corresponds to the results with respect to the price factors; PCE inflation and the corresponding price level (the cumulative inflation response). Panel (b) depicts the results with respect to industrial production and the federal funds rate. The first column combines all three coefficients of the non-linear impulse response, $\beta_h$ (green, solid line), $\vartheta_h$ (red, dashed line) and $\psi_h$ (blue, dashed-pointed line) into one graph. The linear terms in the projection deliver a familiar picture in Panel (a). After a positive (contractionary) monetary policy shock the linear coefficients yield a muted response of inflation for about two years and a significant decline thereafter. The first column also displays the non-linear effects induced by the squared and the cubed terms, which are also reproduced together with the 90\% confidence intervals in column two and three, respectively. The fourth column shows the t-statistics of the three coefficients and the 10\% significance interval ($-1.65, 1.65$), shaded in gray.

The coefficient for the squared shock in the inflation response is first marginally negatively significant and then positive at larger horizons. The price level coefficient is mostly negative, albeit largely statistically insignificant. Hence, there is some evidence that prices reacts more (negatively) at short horizon for positive (contractionary) monetary policy shocks, reducing somewhat the linear price puzzle, than for negative (expansionary) ones, where the initial price puzzle is actually amplified by the non-linear term.

Thirdly, and most importantly for the scope of our analysis, the cubic shock coefficient for the price response is supporting our theoretical predictions. There is a disproportionately stronger effect of a large monetary policy shock on prices and inflation, as
the coefficients are statistically significantly negative after short horizons of about one to two years after the monetary policy shock. Consequently a sufficiently large shock counteracts the small linear coefficient and switches the sign of the overall impulse response of prices or inflation. This removes the price puzzle and induces a more price flexible response at these horizons.

Moreover, inflation and the price index respond disproportionately weaker after larger shocks at long horizons. This again seems to point towards state-dependency as, for large shocks, the price level reacts stronger at short, and thus weaker at long horizons.

Panel (b) also reports literature standard results for the linear coefficient in the industrial production and federal funds rate local projection. The output coefficient starts to fall (after an initial small positive response), reaching its trough about three years after the shock, and then recovers. The federal funds rate coefficient remains positive after the shock for about three years before it becomes negative.

Further, the coefficient for the squared shock for output is negatively significant at the beginning of the horizon. The negative sign suggests that there is a smaller output puzzle with respect to positive (contractionary) monetary policy shocks compared to negative (expansionary) shocks, again strengthening the results of panel (a) with respect to the asymmetric effects of monetary policy shocks. This result is consistent with the findings of Cover (1992) that positive money-supply (expansionary) shocks have a weaker effect on output than negative money-supply (contractionary) shocks. However, the coefficient with respect to the federal funds rate is positively significant at the beginning of the horizon, suggesting that monetary policy reacts stronger after positive shocks. Yet, this effect is marginal and dies away quite quickly such that, overall, the results suggest asymmetric responses to monetary policy shocks, in accordance with the previous literature on this issue, as the ones in Tenreyro and Thwaites (2016).

Looking at the cubic coefficient in panel (b), we also see that the output response is in accordance with the predictions of state-dependent pricing models. If large monetary
policy shocks do indeed predict a higher degree of price flexibility, as the evidence suggests, then the natural conclusion is that larger monetary policy shocks should also have weaker real effects. The limiting case is the flexible price allocation where monetary policy shocks have no effect on real variables at all and the classical dichotomy holds. Our analysis suggests that this conclusion is true since output exhibits a weaker response for large shocks as the cubed shock coefficient is significant, has the opposite sign of the linear coefficient, and thus counteracts the latter for both short and long horizons.

Crucially, it seems that these results with respect to the cubic shock coefficient are not due to a stronger response of monetary policy since the associated coefficient is negative for most of the short to medium horizons, suggesting a proportionally weaker response of monetary policy to a larger shock.

In sum, there is some evidence of asymmetry of monetary policy shocks, with stronger output effects (and weaker price effects) for contractionary shocks. Most importantly, prices exhibit a non-linear, size-dependent impulse response function, reacting stronger at short and weaker at long horizons for large shocks. Coherently, the output response seems to be weaker, and monetary policy feedback does not seem to drive the above results. Hence, we interpret this finding as evidence in favor of sticky prices as the central propagation mechanism of monetary policy, as larger shocks induce more firms to change prices early and thus reduce the real effects of a monetary shock.

**Non-linear IRF vs. linear IRF.** In order to further assess and clarify the importance of the non-linear effect, we compute the two different impulse response functions, a linear and a non-linear one, for a large structural innovation in the interest rate ($\delta = 200$ b.p.). The linear impulse response function is simply the linear coefficient from the benchmark projection (1), multiplied by the shock. The non-linear impulse response function includes both the linear coefficient, the quadratic and the cubic term in (1)
such that:

\[
\theta_{h}^{NL} = \beta_{h}(200\text{ b.p.}) + \vartheta_{h}(200\text{ b.p.})^2 + \psi_{h}(200\text{ b.p.})^3
\]

\[
\theta_{h}^{L} = \beta_{h}(200\text{ b.p.})
\]

Figure 4 reports the corresponding results. The first column of panels (a) and (b) shows both the non-linear impulse response function and the linear impulse response function for all four headline variables after a 200 b.p. shock. The second column gives the bootstrapped, one standard deviation confidence intervals in order to give a second, non-parametric measure for coefficient confidence. The final column reports the t-statistics of the two coefficients, corresponding to the confidence intervals in column one.\(^9\) First, one can see that the non-linear impulse response functions for prices and inflation quickly decline, and become statistically significantly different from both zero and the linear response after about 10 and 20 months, respectively. Both responses then recover relatively quickly in the following periods. Contrary to that, the linear impulse responses stay around zero for about two years before eventually declining.

Second, this is also corroborated by the response of output after such a shock. The output response of the non-linear impulse response function is not statistically significant from zero, whereas it is for the linear function. Consequently, this is also in accordance with the sticky prices hypothesis since state-dependent price setters change prices, not output after such a large shock.

Finally, the response of the federal funds rate is also different. Whilst the linear impulse response stays positive for much longer, the persistence is quite low when it comes to the non-linear impulse response.

This visualization illustrates our previous empirical results on the significant size-dependent effects of monetary policy shocks. Large shocks - around 200 b.p. or more - induce firms to change prices early and thus reduce the real effects of such a monetary shock, in accordance with our first implication.

\(^9\)We estimated confidence intervals and t-statistics using the delta method. Additionally, following Coibion (2012), we bootstrap the coefficients to provide an alternative confidence interval.
4.2 Implication 2: Increased flexibility with high trend inflation

Smooth transition local projections. We use smooth transition local projections to test whether the impulse responses after a monetary policy shock are different in high and low inflation regimes. Auerbach and Gorodnichenko (2012b) and Tenreyro and Thwaites (2016) popularize this method, and we follow their approach to a large extent. Using this technique, the impulse response of variable of interest $y_t$ at horizon $h$ in state $s = HI, LO$ to a unitary structural shock $\epsilon_t$ is the estimated coefficient $\beta_s^h$ in:

$$y_{t+h} = \tau_t + F(z_t)(\alpha_{HI}^h + \beta_{HI}^h \epsilon_t + \sum_{k=1}^{K} \eta_{HI}^{h,k} x_{t,k}) + (1 - F(z_t))(\alpha_{LO}^h + \beta_{LO}^h \epsilon_t + \sum_{k=1}^{K} \eta_{LO}^{h,k} x_{t,k}) + u_{t+h}$$

(3)

for $h = 0, 1, ..., H$. Again, we include up to three month lags of industrial production, PCE inflation and the effective federal funds rate and contemporaneous values of industrial production and PCE inflation. $F(z_t)$ is a smooth transition function which indicates the state of the economy (Granger and Teräsvirta, 1993). We use a logistic function with the following form:

$$F(z_t) = \frac{\exp \left( \gamma \frac{(z_t-c)}{\sigma_z} \right)}{1 + \exp \left( \gamma \frac{(z_t-c)}{\sigma_z} \right)} \in [0, 1]$$

(4)

If sticky prices are an important propagation mechanism we would expect $\beta_{HI}^h$ to be statistically significantly more negative than $\beta_{LO}^h$, especially for short horizons. Prices should be more flexible and so react both quicker and stronger to monetary policy shocks in a high inflation regime.

The state variable. In the main specification of our smooth transition function local projection, i.e., equation (3), we choose smoothed personal consumption expenditure.

$^{10}$HI stands for high inflation, LO for low inflation.
(PCE) inflation as the state variable $z_t$. The smoothing of the variable is done by taking a 23 month centered moving average (MA) to capture trend inflation appropriately. We set $\gamma = 5$ as this gives an intermediate degree of regime switching intensity. This is relatively standard in the literature and also fits our inflation data well. Finally, $c$ corresponds to the 75th percentile of the historical trend inflation distribution. This is equivalent to assuming that about 70% of the time trend inflation is classified as negligible (i.e. $F(z_t) \in [0, 0.1]$) and 30% of the time there is some trend inflation (i.e. $F(z_t) \in (0.1, 1]$). Figure 5 illustrates this chosen calibration of the smooth transition function. The blue line, measured on the left vertical axis measures PCE inflation on an annual basis. The red and green line show the smooth transition function based on our MA- and a HP-filtered measure ($\lambda = 14400$) of PCE inflation, measured on the right vertical axis.\textsuperscript{11} The period of the Great Inflation from around 1974 to 1983 is characterized by two pronounced spikes of inflation of up to 11%. The smooth transition function reaches 1 around these two peaks and stays above 0.4 for the entire period of the Great Inflation, classifying the latter period mostly as a high inflation regime. We take this to be a reasonable approximation for periods of high and low trend inflation in the United States.\textsuperscript{12}

**Estimates.** The panels of Figure 6 displays the results for this specification. The first column shows the point estimates for the impulse response without any state-dependency (green, solid line), the impulse response in the high inflation regime (red, dashed line) and the impulse response in the low inflation regime (blue, dash-dot line). These responses are then also depicted individually with 90\% confidence intervals in column one, two and three, respectively. Column four shows the t-statistic that tests the null of equality of the high and low inflation regime coefficients (purple, solid line).

\textsuperscript{11}The moving average is our benchmark smoothing procedure. However, the results from the local projections are very similar with HP-Filter smoothing procedure, see Appendix A.

\textsuperscript{12}This calibration is also in accordance with Alvarez et al. (2018) who show that the frequency of price changes starts increasing significantly from annual inflation rates of 5\%. Our smooth transition function indicates a value of approximately 0.5 with such an annual inflation rate.
A positive value means that the high inflation response is larger whereas a negative value of the t-statistic indicates the opposite. Panel (a) again correspond to the PCE inflation and the corresponding price level whereas panel (b) depicts the results for industrial production and the federal funds rate.

Firstly, the linear terms in panel (a) show the familiar picture described earlier. Inflation and prices decline eventually. Secondly, the responses of the inflation and price level in a high and low inflation regime are significantly different at the early horizons. The price level in a high inflation regime declines right away after a monetary policy shock, suggesting that firms are willing to change prices more frequently on average. On the other hand, the impulse response function in a low inflation regime does not exhibit such a pattern. Here a statistically insignificant price puzzle is observable for about two years, mirrored by the response of inflation. This suggests that in this regime firms are not as willing to change price as frequently as the price level stays persistently at around zero for a longer period. We interpret this as evidence in favour of sticky prices models as key propagation mechanism of monetary policy shocks, because they predict a faster reaction to a monetary disturbance in a high trend inflation regime. This is exactly what the impulse response functions show.

Looking at panel (b), the linear projections show that output exhibits a hump shaped dynamic and the interest rate increases after a monetary policy shock and stays positive for about two years. Secondly, as our second theoretical prediction predicts, the point estimate suggests that output reacts stronger in a low inflation regime compared to a high inflation regime (although this difference is not statistically significant).

Finally, note that monetary policy initially reacts differently to a shock in the two regimes. When there is high trend inflation, it seems a shock raises the interest rate by more for one month but the rate stays positive for a shorter period. This pattern may explain the initially quicker reaction of the price level in the high inflation regime and the reaction of prices at the end of the horizon which is somewhat weaker under high...
inflation. But prices in the low regime react considerably more sluggish even though the interest rate is positive for a longer period of time. Hence, it is difficult to impute that the differing monetary policy behavior is driving the different response of prices between the high and low inflation regimes.

To conclude, we find evidence in favor of staggered prices as a key propagation mechanism of monetary policy also regrading our second prediction. Theory predicts a more price flexible reaction in a high trend inflation regime. Our results show that inflation and the price level decline right away and do not exhibit a price puzzle in such a regime. Moreover, the price level is more persistent in a low inflation regime, despite the interest rate staying positive for a longer amount of time.

5 Robustness

5.1 Non-linear Local Projection

5.1.1 Stability

What if monetary policy exhibits a different dynamic reaction after a large shock compared to a small shock or in a high inflation regime compared to a low inflation regime? It is crucial for our analysis to be relatively confident that the non-linear and state-dependent dynamic behavior of the impulse response functions of output and inflation is not driven by the non-linear and state-dependent monetary policy feedback. As discussed in sections 4, several of our results seem to contradict this possibility by showing an opposite contemporaneous correlation between inflation/output and monetary policy to what standard theory would predict. For example, as explained in more detail in section 4.1, monetary policy seems to react weaker after a large shock whereas prices react by more at the beginning of the horizon. Subsequently, when monetary policy reverses to a more restrictive reaction at the end of the horizon, the price level decline seems to
be weaker. The same applies to a large extent for the test of the second implication in section 4.2 as the funds rate stays positive for longer in the regime where prices seem to be more sticky. These patterns seem to indicate that the dynamics of inflation and output are not defined by the reaction of monetary authorities only.

In what follows, we conduct further robustness tests to show that monetary policy feedback only plays a limited role with respect to large and small shocks. We appeal to the idea of stability of the estimated crucial parameter in the local projection regression. More specifically, we check if the output and inflation coefficients with respect to the cubed shock do not exhibit structural breaks when monetary policy conduct may have changed. If these estimates stay stable over a variety of monetary policy regimes throughout time then we can reject the thesis of monetary feedback driving the result of size-dependent effects.

The two main coefficient stability tests used in this analysis are recursive estimates of the local projection coefficients and the Hansen (1992) test for structural breaks in individual coefficients of a regression.

**Recursive estimates.** We recursively reestimate equation (1) adding three months for every new estimation, after a starting window for the first sub-sample of 150 months. Figure 7 plots the results for the coefficient of the cubed shock with respect to prices. The x axis shows the horizon of the impulse response, the y axis describes the ending period of that estimate and the z axis measures the coefficient size. We focus on the price level coefficients with respect to the cubic shock as these show the potential non-linearity of impulse responses with respect to the shock size most clearly. Visual inspection clarifies that the recursive coefficient sequences are relatively stable. Qualitatively, the negative and then positive dynamics of this coefficient are certainly constant over the recursive estimations. Furthermore, the value is also quite similar across the different samples, apart from some fluctuations for the long horizons at the beginning of the

\[ \text{The other relevant results are summarized in the Appendix A.} \]
Table 1: Estimated test statistics for the Hansen (1992) test for parameter constancy.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Hansen (1992) test statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$h = 1$</td>
</tr>
<tr>
<td>Linear ($\beta_h$)</td>
<td>0.18</td>
</tr>
<tr>
<td>Quadratic ($\vartheta_h$)</td>
<td>0.04</td>
</tr>
<tr>
<td>Cubic ($\psi_h$)</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Individual critical values are $c_{0.01} = 0.75$, $c_{0.05} = 0.47$ and $c_{0.10} = 0.35$.

sample until the mid '80s.

**Hansen (1992) test.** We apply the Hansen (1992) stability test to the coefficients in the local projection (1) for PCE inflation. This test has locally optimal power and needs no *a priori* assumption concerning the breakpoint. Furthermore, this test is robust to heteroskedasticity, a potential concern in this analysis. For a test on an individual coefficient, we can reject the null of parameter constancy at the 5% significance level, if the relevant test-statistic is larger than the asymptotic critical value of 0.47.\(^{14}\) Table 1 shows the results for the coefficients at a 1, 10, 20, 30 and 40 month horizon. These results clearly underpin the graphical analysis just presented in the previous paragraph. We cannot reject the null of individual parameter constancy for any of the three individual impulse response coefficients at any of the horizons. This suggests that, even though the dynamic feedback of monetary policy may have changed throughout time, the shape, asymmetry and non-linearity of the price impulse response has stayed relatively constant throughout the sample.

5.1.2 Excluding the NBR targeting period

Coibion (2012) and others have suggested that the exclusion of the NBR targeting period October 1979 and September 1982 can account for a difference in results between the

\(^{14}\)The null hypothesis is that each coefficient in (1) is constant and the respective distribution is non-standard and depends on the number of parameters tested for stability (see Appendix B).
Romer and Romer (2004) and VAR approach. Critically, the largest absolute shock values lie in this period and may thus play a significant role for the conclusion on our first implication. In order to account for this suggestion we exclude this part of the shock sample and modify the non-linear local projection by interacting the linear, squared and cubed shock variables with a time-dummy that takes a value of 1 for the sample between October 1979 and September 1982 and 0 otherwise. Figure 9 depicts the results for this specification. The results change somewhat in terms of significance and, especially, in terms of magnitude. The reduced shock sample does not include the large shocks of the NBR targeting period, and it features low sample variation with values mostly below 1. Hence, the effects of large shocks are more difficult to identify, inducing wide confidence intervals. However, the impulse response functions are qualitatively similar to the benchmark results.

5.1.3 Alternative specification: absolute value interaction

An alternative local projection specification to investigate non-linearities in the impulse response function with respect to the size of the shock uses the absolute value of the shock as an interaction variable:

$$y_{t+h} = \alpha_h + \tau_h t + \beta_h \epsilon_t + \zeta_h (\epsilon_t \cdot |\epsilon_t|) + \sum_{k=1}^{K} \eta_{h,k} x_{t,k} + \nu_{t+h}. \tag{5}$$

In this specification $\zeta_h$ captures any non-linear effects of shocks on the response variable $y_{t+h}$ that are due to the absolute value of the shock. Thus, similar to $\psi_h$ in equation (1), it indicates an amplification of the linear coefficient after larger shocks if they have same sign term and counteracts it in the case of opposite signs.

The results of this set of local projections are summarized in Figure 8. The first row displays shows the responses of industrial production, the second row of the PCE price level and the final row of the federal funds rate.\textsuperscript{15} The first column shows both the

\textsuperscript{15} We omit the response of PCE inflation for the robustness checks in order to economize on the
linear (green, solid line) and interaction point estimates (blue, dashed-dotted line), the second row shows only the interaction effect with 90% confidence bands and the final column depicts the relevant t-statistics. A similar pattern compared to the main specification of subsection 4.1 emerges. The literature standard conclusions with respect to the linear coefficients still hold to a large extent, albeit less statistically significant. More importantly, this respecification provides a consistent picture with respect to the non-linear interaction terms. Larger monetary policy shocks seem to prompt a weaker output puzzle at the beginning of the period and a weaker peak effect around 30 months after the shock. The price impulse response exhibits a weaker price puzzle at the beginning of the period and thus a negative overall effect for sufficiently large shocks. Furthermore, there seems to be less persistence as the interaction effect becomes significantly positive after around four years whilst the linear effect is negative. Consequently, a weaker effect on output and a more price flexible price response are in line with the main specification of subsection 4.1 and confirm our first theoretical predictions once again.

5.2 Smooth transition function

We now turn to the robustness of the results regarding the response during high and low inflation regimes with respect to the parameters of the smooth transition function. Firstly, a change of the regime switching parameter from $\gamma$ to $\gamma = 1$ or $\gamma = 10$ does not have any significant impact on any of the impulse responses (see Figure 10 and 11). The only main difference relates to the case when $\gamma = 1$ and the fact that output in the low inflation case is significantly different from the high inflation response at the end of the horizon.

Secondly, lowering $c$ to the 70th percentile of trend inflation reinforces the results of the main specification even further. There is a significantly faster decline of prices for about 25 months (see Figure 12), albeit at a lower level of significance. The results are
also robust to an increase in $c$ to the 80th percentile. We see a significantly stronger decline of prices under the high trend inflation regime for early horizons, similar to the main specification (see Figure 13). In conclusion, the main results with regards to the second hypothesis are robust to changes in the parameters of the smooth transition function.

5.3 Alternative price measure: CPI

This robustness test uses the Consumer Price Index (CPI) instead of PCE as the inflation measure. Figure 14 and 15 provide the results for the two main local projections using the CPI measure. It is clear that the results are quite robust to this change in the inflation measure.

5.4 Controlling for financial frictions

Financial frictions (Bernanke, Gertler, and Gilchrist, 1999; Kiyotaki and Moore, 1997) is another prominent propagation mechanism of monetary policy shocks in the literature. Variations in financial frictions over time could then affect our estimate, making them spurious. We control for financial frictions by including the contemporaneous value and three lags of the highly informative corporate bond credit spread, introduced by Gilchrist and Zakrajsek (2012), as a proxy for financial frictions (hereafter GZ-spread). This series is available from January 1973, so we are estimating our local projection on a truncated sample.

The results, reported in Figures 16 and 17 are largely unchanged. With respect to our first implication, we still can discern a weaker effect of large shocks on output and a more pronounced, yet less persistence response of prices. However, the latter effect is now insignificant. The graph for the second hypothesis shows that prices behave as in the baseline specification, with a statistically significantly different response in the high inflation regime compared to the low inflation regime. Altogether, we conclude that
our evidence in favor of a sticky price theory holds up to controlling for the presence of financial frictions.

5.5 Non-linear Romer and Romer (2004) regression

The shocks used in the main local projections are residuals of the estimated reaction function of the central bank. More specifically, Romer and Romer (2004) assume a concrete form of reaction function by regressing the change in the intended federal funds rate ($\Delta FFR_t$) on a measure of forecast variables primarily obtained from the Greenbook. This method assumes that the reaction function is linear. Following Tenreyro and Thwaites (2016), we reestimate the Romer and Romer (2004) regression using the smooth transition function of the main specification, as:

$$\Delta FFR_t = F(z_t)(X_t\beta^H) + (1 - F(z_t))(X_t\beta^L) + \epsilon_{t}^{NL}. \quad (6)$$

In doing so, the identified shocks now account for the possibility that monetary authorities reacted differently to forecasts in a high and low trend inflation regime. The new series of non-linear shocks ($\epsilon_t^{NL}$) has a correlation of 0.92 with the linear shocks. This suggests that, whilst the new reaction function is picking up some non-linearities, the original shocks are still a relatively good instrument. Figures 18 and 19 shows that the results are very similar to the original ones. The non-linear local projection coefficients exhibit the same dynamics although the cubed shock coefficient in the output projection is now mostly insignificant. The smooth transition function local projection point estimates are also very similar to the original ones. But, importantly, note that the larger effect on the federal funds rate at small horizons in the high inflation regime has disappeared. Despite that, we still see a quicker decrease of prices in that regime reinforcing our original results.
5.6 VAR estimates

This subsection investigates the sensitivity of the results with respect to the shock measure. More specifically, we use a classical three equation recursive VAR including industrial production, PCE inflation and the federal funds rate to recover the structural VAR monetary policy shocks and then use these in our local projections. Figure 20 and 21 are very similar to our benchmark, with some minor quantitative differences, notably the state-dependent responses in the price level which are largely insignificantly different in this case at long horizons. However, the figures show once again that our results are extremely robust to different specifications.

6 Conclusion

The assumption of sticky prices lies at the very center of the current workhorse model for the analysis of business cycle fluctuations and, particularly, monetary policy effects. As long as a significant fraction of state-dependent prices exists in the economy, then a sticky price theory of the transmission mechanism of monetary policy shock yields two testable implications; the impulse response function of the aggregate price level should be more flexible both after a large shock and during high trend inflation regimes. Employing the methodology of local projections, we tested these predictions on aggregate US data. We found some evidence in favor of the New Keynesian paradigm. With regards to the response to large shocks, the coefficient of the cubed shock projections matched our theoretical prior, both in terms of output and inflation. The empirical investigation during large trend inflation regimes also showed that prices reacted significantly quicker to a monetary policy shock in times of high trend inflation.

Hence, ‘Prices are sticky after all’, just as recent literature has shown (Kehoe and Midrigan, 2015) - but less so when shocks are large or inflation is high. These results are in line with what would be expected if state-dependent pricing played a significant
role in the US economy. This supports the theoretical implication that the frequency of changing prices is, at least to some extent, endogenous to the economic environment, as in Alvarez, Lippi, and Passadore (2016). So, although the Calvo (1983) model may work quite well for “normal” times, when considering situations where high-trend inflation is present or large shocks are likely, a state-dependent supply side that accounts for these phenomena seems more appropriate (as e.g., Alvarez and Lippi, 2014; Alvarez, Le Bihan, and Lippi, 2016; Costain and Nakov, 2011, 2018; Costain et al., 2018).

Nevertheless, whilst our analysis highlights the importance of state-dependency and non-linearities at the aggregate level, it should not be read by any means as a rejection of the entire class of time-dependent models. Our results seem to suggest that elements of both time- and state-dependent pricing play an essential role in inflation dynamics, as we neither do see a fully price flexible response after large shocks or during high inflation, nor is the impulse response devoid of any non-linearities. The formation of prices remains a complex and incomplete field of research where both theories deserve their place. In more general terms, this study contributes to the already extensive recent research in the literature that highlights the importance and the role of non-linearities.

There is a variety of future research directions. We believe that the most natural one would be a similar analysis at a more granular level, taking network effects into account. Studying the response of prices to monetary policy shocks at a more disaggregated level could allow to identify whether only a few sectors are responsible for this aggregate behavior, and how the network structure of the economy influences the non-linearities in the propagation of monetary policy shocks to aggregate variables.
Bibliography


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Figure 5: PCE inflation (left axis) and related smooth transition functions (right axis)
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(a) PCE inflation and price level
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Appendix

A Additional Figures

Figures A.1 and A.2 report the unsmoothed results of the benchmark non-linear and smooth transition local projections, respectively.

Figure A.3 reports the results of the smooth transition local projection with HP-filtered ($\lambda = 14400$) PCE inflation as a state variable.

Figures A.5 and A.4 provide the recursive cubic estimates with respect to the industrial production and federal funds rate.
Figure A.1: Non-linear local projection: Unsmoothed results
Figure A.2: Smooth transition local projection: Unsmoothed results
Figure A.3: Smooth transition local projection: HP filtered state variable
Figure A.4: Recursive federal funds local projection: Cubed shock coefficient
Figure A.5: Recursive industrial production local projection: Cubed shock coefficient
B  Hansen (1992) test procedure

Here we use the following specification for non-linear effects of shocks on prices:

\[ y_{t+h} = \alpha_h + \beta_h \epsilon_t + \vartheta_h \epsilon_t^2 + \psi_h \epsilon_t^3 + \sum_{k=1}^{K} \eta_{h,k} x_{t,k} + v_{t+h} \equiv b_h' x_t + v_{t+h} \]  

(7)

and assume \( E(v_{t+h}|x_t) = 0 \) and \( E(v_{t+h}^2) = \sigma_{t,h} \) and \( \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \sigma_{t,h}^2 = \sigma_h^2 \). Furthermore, the variables are assumed not to contain any deterministic or stochastic trends. Accordingly, the original specification is modified by excluding the deterministic time trend and taking first differences of the output control variables to ensure the fulfillment of this assumption. Estimating the above by least squares yields the following system of first-order conditions:

\[ 0 = \sum_{t=1}^{T} f_{k,t,h} \quad k = 1, \ldots, K + 1 \]

(8)

with

\[
f_{k,t,h} = \begin{cases} x_{k,t} \hat{v}_{t+h} & \text{for } k = 1, \ldots, K \\ (\hat{v}_{t+h}^2 - \hat{\sigma}_h) & \text{for } k = K + 1 \end{cases} \]

(9)

where \( K \) is the number of coefficients to be estimated in the local projection. The Hansen (1992) individual test statistic is then based on the cumulative of these first order conditions. Defining \( S_{k,t,h} = \sum_{j=1}^{t} f_{k,j,h} \), the final individual test statistic is:

\[ L_{k,h} = \frac{1}{TV_k} \sum_{t=1}^{T} S_{k,t,h}^2 \]

(10)

where \( V_k = \sum_{t=1}^{T} f_{k,t,h}^2 \). The null hypothesis is that \( b_{k,h} \) is constant and the respective distribution is non-standard and depends on the number of parameters tested for stability.