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Abstract

We employ a newly-developed partial cointegration system allowing for level shifts to examine whether economic fundamentals form the long-run determinants of the dollar-pound exchange rate in an era of structural change. The paper uncovers a class of local data generation mechanisms underlying long-run and short-run dynamic features of the exchange rate using a set of economic variables that explicitly reflect the central banks' monetary policy stances and the influence of a forward exchange market. The impact of the Brexit referendum is evaluated by examining forecasts when the dollar-pound exchange rate fell substantially around the vote.

Keywords: Exchange rates, Monetary policy, General-to-specific approach, Partial cointegrated vector autoregressive models, Structural breaks.

JEL classification codes: C22, C32, C52, F31.

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1 Introduction

Relationships between foreign exchange rates and economic fundamentals have been thoroughly investigated in the international economics and finance literature. However, it is still an open question whether economic fundamentals form the central element of the determinants of the long-run behaviour of floating exchange rates. This is puzzling from a macroeconomic perspective, discussed by Obstfeld and Rogoff (2000), as given the roles of foreign exchange rates in goods and asset markets, one would expect that exchange rates are closely related to prices and interest rates, thereby being also linked to monetary aggregates, outputs and inflation.

In fact, these macroeconomic variables constituted the heart of most of the theoretical and empirical exchange-rate models up to the 1970s; see Sarno and Taylor (2002, Ch.4) for a survey of fundamental-based exchange-rate models. Among them, both Dornbusch (1976) and Frankel (1979) are notable examples of formulations incorporating economic fundamentals. However, Meese and Rogoff (1983, 1988) changed the course of the literature, generating widespread doubt towards the existence of stable connections between exchange rates and economic fundamentals.

Equipped with advanced econometric methods and extended observation periods, substantial applied research subsequent to Meese and Rogoff (1983, 1988) found weak support for the view that macroeconomic variables are critical factors in exchange rate fluctuations; see Baxter (1994), Isaac and De Mel (2001), Cheung, Chinn, and Pascual (2005), and Sarno (2005), *inter alia.* This missing link has also sparked research interest in how to account for the deviations of observed exchange rates from their fundamental-based values, see Engel and West (2005), Sarno and Sojli (2009), Bacchetta and Van Wincoop (2013) and Balke, Ma, and Wohar (2013) for various explanations of this riddle from theoretical and empirical viewpoints.

In response to the unsettled state of the literature, this paper builds a fundamental-based empirical exchange rate model by (i) adopting a class of novel econometric techniques and (ii) selecting those variables that explicitly reflect monetary policy stances of central banks and influences of a forward market. The paper develops a cointegrated vector autoregressive (CVAR) system for processes integrated of order 1 (denoted $\mathsf{I}(1)$ hereafter), pioneered by Johansen (1988, 1995), which provides a basis for a feasible general-to-specific modelling scheme applicable to multivariate non-stationary time series data, see, e.g., Hendry (1995, 2000, 2018) and Hendry and Doornik (2014) for the general-to-specific methodology's development and current state. There have been various CVAR analyses conducted in the exchange rate literature, such as Johansen and Juselius (1992), Juselius and MacDonald (2004), Kurita (2007) and Juselius and Assenmacher (2017). Juselius and Stillwagon (2018) incorporate information on consensus forecasts of interest rates into their fundamental-based $\mathsf{I}(2)$ CVAR model so as to gain insight into various roles of expectations in the determination of the dollar-pound exchange rate.

The novel contributions of this paper are four-fold, all of which contribute to achieving a congruent, undominated, fundamental-based model of the dollar-pound exchange rate. First, a classical theory in international economics is adapted in such a manner that it directly reflects the monetary policy stances of the central banks in two countries (UK and US) and some other aspects of international financial markets. More specifically, both countries' monetary bases are incorporated into a theoretical model proposed by Frankel (1979), along with an expectation formation based on a foreign-exchange forward premium.

These theoretical reformulations then lead to the second distinguishing feature of this paper: the empirical investigation commences with a partial CVAR (PCVAR) system with level shifts, a member of a new class of econometric models introduced by Kurita and Nielsen (2018). Thanks to this new class of models, we can avoid an intractable large-scale econometric system subject to a number of problems such as a vanishing degrees of freedom. In this study, a PCVAR system including the dollar-pound rate is formulated conditional on the two countries' monetary bases and on the forward premium, by permitting a structural break caused by the global recession starting in September 2008. Guided by the preceding reformulated exchange-rate theory, the PCVAR system reveals the underlying long-run relationships between the two countries. It is further reduced to a trivariate vector equilibrium correction model (VECM) by exploiting revealed evidence for the existence of additional weakly exogenous variables.

Having developed the system for the dollar-pound exchange rate, we then focus on the exchange rate equation in the system and apply impulse-indicator saturation (IIS) to test for breaks and location shifts over the full sample period. The system models one location shift due to the financial crisis and subsequent great recession, whereas saturation techniques allow the data to identify where breaks may occur, including the very beginning and end of the sample, see Hendry, Johansen, and Santos (2008), Johansen and Nielsen (2009) and Castle, Doornik, Hendry, and Pretis (2015). This is particularly beneficial towards the end of sample when the Brexit referendum led to two large falls in the spot exchange rate in close succession.

Finally, we evaluate both the trivariate VECM and its single equation counterpart based on their forecast performance for different horizons over 2016-2018, which includes the Brexit referendum period. The forecasting models raise two questions that we address, including how to handle open models where there are unmodelled exogenous variables that need forecasting, see Hendry and Mizon (2012a, 2012b), and how to robustify equilibrium correction models to location shifts, see Hendry (2006) and Castle, Clements, and Hendry (2015). The forecasting models are compared to benchmark naïve devices, and the empirical results refute the results of Meese and Rogoff (1983a) who find that models based on economic fundamentals cannot beat the random walk in a forecasting context.

The rest of this paper is organised as follows. §2 re-visits a classical exchange-rate theory to obtain a set of candidate variables for long-run economic relationships underlying the UK and US. §3 outlines the econometric methods used in the empirical study, and §4 provides an overview of the data. The empirical analysis is given in §5, where a PCVAR system with a level shift is employed to reveal the long-run economic relationships, and the system is reduced using general-to-specific principles. The system-based analysis paves the way for a single-equation analysis in §6, where we focus on achieving a congruent model for the dollar-pound exchange rate using saturation methods. §7 conducts the forecasting exercise for the spot exchange rate and the change in the spot exchange rate, and finally, we conclude in §8.[1]

2 Resuscitating a classical exchange-rate theory

This section reformulates a theoretical model by Frankel (1979) in such a way that the model can be empirically relevant to modern financial and macroeconomic environments in which monetary policy and a forward exchange market play more critical roles than before. In view of its application to econometric analysis of the dollar-pound exchange rate data, the model presented here is composed of UK and US macroeconomic variables, the time series data of which are all realisations of processes that are wide-sense non-stationary, both integrated of order 1 (denoted as $\mathsf{I}(1)$), and subject to distributional shifts.

First, consider the uncovered interest parity (UIP) condition based on bond yields:

$$s^e_{t+1} - s_t = i_t - i^*_t + \rho_t, \tag{1}$$

where s_t is the log of the dollar-pound spot exchange rate, s^e_{t+1} is its one-period ahead expectation, i_t is the US bond yield and i^*_t is the UK counterpart (the superscript $*$ denotes a UK variable hereafter), and ρ_t represents a risk premium term. Introduce the following Frankel (1979)-type expectation formation in the foreign exchange market of the spot rate dynamics:

$$\Delta s^e_{t+1} = s^e_{t+1} - s_t = -\theta(s_t - \bar{s}_t) + \pi^e_{t+1} - \pi^{*,e}_{t+1} \qquad \text{for} \quad 0 < \theta < 1, \tag{2}$$

where $\bar{s}_t$ represents a fundamental-based value of the exchange rate and π^e_{t+1} denotes the market

[1]The software *OxMetrics* version 8 and *PcGive* (Doornik and Hendry, 2013a,b) was used in all the empirical analyses performed in this paper.

expectation of the US inflation rate. The level $\bar{s}_t$ was frequently specified using purchasing power parity (PPP) in the exchange rate literature (see MacDonald and Nagayasu, 1998, *inter alia*):

$$\bar{s}_t = p_t - p_t^*, \tag{3}$$

where p_t is the logs of the US price level. The specification (3) leads to the definition of the real exchange rate $q_t = s_t - p_t + p_t^*$, which, by noting $\pi_{t+1}^e = p_{t+1}^e - p_t$, enables us to restate (2) in terms of q_t as:

$$q_{t+1}^e = (1-\theta)\, q_t \qquad \text{for} \quad 0 < \theta < 1, \tag{4}$$

for $q_{t+1}^e = s_{t+1}^e - p_{t+1}^e - p_{t+1}^{*,e}$. Hence, if θ is small, (4) is consistent with vast econometric studies on near unit root properties of real exchange rates. As reviewed by Sarno and Taylor (2002, Ch.3), scalar unit root tests are liable to suffer power deficiency, with the result that we have to rely on long-span studies, such as a century-long data analysis, to estimate θ precisely in an AR regression. See Lothian and Taylor (1996) for a study employing two centuries of data for real exchange rates and Hendry (2001) who showed that very long-run PPP held for the UK versus the world; see also Johansen (2006) for inferential problems with modelling near unit root data as stationary in a small-sample context.

Furthermore, in a class of standard monetary models in the literature (see Sarno and Taylor, 2002, Ch.4, *inter alia*), each of the price levels in the PPP formulation (3) was replaced by a broad money stock measure (M2 and M3, for example), along with other factors such as real income. In this paper we explore a new approach, albeit in the spirit of monetary models, to modelling s_t such that it reflects the two countries' monetary policy stances explicitly:

$$\bar{s}_t = \ g\{\underset{+}{MB_t}, \underset{-}{MB_t^*}\} + \ \eta_t, \tag{5}$$

where $g\{\cdot\}$ denotes a monotonically increasing function of MB_t (the US monetary base) and a monotonically decreasing function of MB_t^*, and η_t denotes an intercept determining the level of $\bar{s}_t$ and it is changeable over time according to regime shifts in monetary policy. The formulation (5) can still be viewed as a PPP-based specification of $\bar{s}_t$ as its basis is given by (3), but it reflects the era of quantitative monetary easing and policy regime changes in response to economic shocks and crises. With the log of the US monetary base denoted by mb_t, we assume further the function $g\{\cdot\}$ is restricted in such a way that it is in a testable linear form:

$$g\{MB_t, MB_t^*\} = \delta mb_t - \xi mb_t^* \qquad \text{for} \quad \delta, \xi \geq 0, \tag{6}$$

in which the weak inequality is allowed with respect to the parameters δ and ξ.

Combining (1), (2), (5) and (6) then leads to:

$$s_t = \delta mb_t - \xi mb_t^* - \frac{1}{\theta}\left(i_t - i_t^* + \rho_t\right) + \frac{1}{\theta}\left(\pi_{t+1}^e - \pi_{t+1}^{*,e}\right) + \eta_t,$$

which indicates a close connection between the exchange rate and a class of macroeconomic variables. By assuming a mean-zero stationary risk premium, this equation allows us to conceive the following linear combination as a candidate of the underlying long-run economic relationships:

$$s_t - \delta mb_t + \xi mb_t^* + \frac{1}{\theta}\left(i_t - i_t^*\right) - \frac{1}{\theta}\left(\pi_{t+1}^e - \pi_{t+1}^{*,e}\right) - \eta_t \sim \mathsf{I}(0), \tag{7}$$

which means the linear combination is a mean-zero stationary or $\mathsf{I}(0)$ series. This combination, (7), lays a theoretical foundation for empirical cointegration analysis, in which π_{t+1}^e and $\pi_{t+1}^{*,e}$ need to be represented by some observable variables.

Suppose that π_{t+1}^e and $\pi_{t+1}^{*,e}$ are approximated as π_t and π_t^*, respectively, as a result of a random-walk expectation formation; we are then justified in suggesting:

$$s_t - \delta mb_t + \xi mb_t^* + \frac{1}{\theta}\left\{(i_t - \pi_t) - (i_t^* - \pi_t^*)\right\} - \eta_t \sim \mathsf{I}(0), \tag{8}$$

as a candidate long-run combination incorporating an observable real interest differential. Alternatively, $\pi_{t+1}^e - \pi_{t+1}^{*,e}$ may be approximated as a linear function of the forward premium fp_t defined as $fp_t = f_{t+1}^t - s_t$, where f_{t+1}^t is the dollar-pound forward exchange rate applicable at $t+1$ (but contracted at t). The justification of this approximation stems from the combination of covered interest rate parity (CIP) and a forward guidance strategy in inflation targeting policy, which utilises short-term interbank interest rates as policy tools to influence inflation expectations. The CIP tells us $fp_t = i_t^s - i_t^{s,*} + \sigma_t$, where i_t^s is the US interbank rate and σ_t denotes transaction costs, which could be marginal in the current liberalised global financial market. The conjunction of CIP and monetary policy with forward guidance thus leads to

$$\pi_{t+1}^e - \pi_{t+1}^{*,e} = \psi\left(i_t^s - i_t^{s,*}\right) \approx \psi fp_t \qquad \text{for} \quad \psi > 0. \tag{9}$$

Using (9), we then arrive at the other observable long-run relationship:

$$s_t - \delta mb_t + \xi mb_t^* + \frac{1}{\theta}\left(i_t - i_t^*\right) - \frac{\psi}{\theta} fp_t - \eta_t \sim \mathsf{I}(0). \tag{10}$$

We can employ $i_t^s - i_t^{s,*}$ instead of fp_t to derive (10), but the use of fp_t is justified in that it may have the advantage of capturing the underlying complex interdependency between the spot and forward foreign exchange markets.

The set of conceivable long-run combinations, (8) and (10), allows us to formulate an empirical system for X_t defined as:

$$X_t = (s_t, i_t, i_t^*, \pi_t, \pi_t^*, mb_t, mb_t^*, fp_t)', \tag{11}$$

which gives a basis for the starting point of a general unrestricted model, thereby allowing us to proceed to a nested specific model centering on the dollar-pound exchange rate. The next section discusses the econometric methodology before undertaking the empirical analysis in §5.

3 Econometric methods for feasible general-to-specific modelling

This section reviews a class of econometric methods to be utilised in the rest of this study. §3.1 explains a PCVAR model with level shifts, which provides a basis for our system-based modelling strategy, while §3.2 reviews IIS, which allows us to pursue a parsimonious single-equation representation of the data under study.

3.1 A PCVAR system with structural breaks

A new class of CVAR models introduced by Kurita and Nielsen (2018) consists of PCVAR systems allowing for structural breaks in deterministic components, such as a linear trend or intercept. The new models have been derived by combining the results of Harbo, Johansen, Nielsen, and Rahbek (1998) with those of Johansen, Mosconi, and Nielsen (2000). This new class of models enables us to avoid an intractable large-dimensional econometric system, which suffers from vanishing degrees of freedom. The PCVAR model with level shifts is reviewed below.

Let us consider a p-variate $\mathsf{I}(1)$ vector sequence X_t for $t = 1, ..., T$, which is further decomposed as $X_t = (Y_t', Z_t')'$, in which the dimensions of Y_t and Z_t are m and $p - m$ respectively under $m < p$. A system for X_t is a stochastic autoregressive system driven by the innovation sequence ε_t, which is assumed to be a martingale difference sequence with time-varying conditional variances and the average of the variances is assumed to converge to a positive-definite matrix $\Omega \in \mathbf{R}^{p\times p}$; see Kurita and Nielsen (2018) for further details. Corresponding to the decomposition $X_t = (Y_t', Z_t')'$, the innovation sequence and variance matrix are also broken down as follows, along with the introduction of two combinational expressions:

$$\varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{z,t} \end{pmatrix}, \; \Omega = \begin{pmatrix} \Omega_{yy} & \Omega_{yz} \\ \Omega_{zy} & \Omega_{zz} \end{pmatrix}, \; \varepsilon_{y.z,t} = \varepsilon_{y,t} - \omega\varepsilon_{z,t} \text{ for } \omega = \Omega_{yz}\Omega_{zz}^{-1}.$$

The PCVAR system proposed by Kurita and Nielsen (2018) is based on the assumption that Z_t is weakly exogenous (see Engle, Hendry, and Richard, 1983) with respect to parameters of interest, cointegrating vectors $(\beta', \gamma) \in \mathbf{R}^{r\times(p+q)}$ and adjustment vectors $\alpha_y \in \mathbf{R}^{m\times r}$. In order to permit deterministic shifts in the level of X_t, we pre-determine the number of sub-sample periods, q, corresponding to the length of each sub-sample period: $0 = T_0 < T_1 < \cdots < T_q = T$. It is also assumed that the underlying VAR system for X_t satisfies all conditions stated in Theorem 2.1 in Johansen, Mosconi, and Nielsen (2000), so that X_t has an $\mathsf{I}(1)$ moving-average representation presented in that theorem.

Given a period j under $1 \leq j \leq q$ and a time point t under $T_{j-1} + k < t \leq T_j$, we are in a position to present the following PCVAR(k) system for Y_t given Z_t, allowing for the presence of $q - 1$ level shifts and k lagged dynamics:

$$\Delta Y_t = \omega\Delta Z_t + \alpha_y (\beta', \gamma) \begin{pmatrix} X_{t-1} \\ E_t \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_{y.z,i}\Delta X_{t-i} + \sum_{i=1}^{k}\sum_{j=2}^{q} \Psi_{j,i} D_{j,t-i} + \Phi_{y.z} d_t + \varepsilon_{y.z,t} \quad \text{for} \quad t = k+1, ..., T, \tag{12}$$

where both $\alpha_y \in \mathbf{R}^{m\times r}$ and $\beta \in \mathbf{R}^{p\times r}$ are of full column rank r, while the other parameters are subject to: $\gamma \in \mathbf{R}^{r\times q}$, $\Phi_{y.z} \in \mathbf{R}^{m\times s}$, $\Gamma_{y.z,i} \in \mathbf{R}^{m\times p}$ for $i = 1, \ldots, k-1$, and $\Psi_{j,i} \in \mathbf{R}^{m}$ for $i = 1, \ldots, k$ and $j = 2, \ldots, q$. With regard to various other variables appearing in (12), $D_{j,t}$ is defined as:

$$D_{j,t} = \begin{cases} 1 & \text{for } t = T_{j-1}, \\ 0 & \text{otherwise,} \end{cases} \qquad \text{for } j = 1, ..., q \quad \text{and} \quad t = 1, ..., T,$$

which results in $D_{j,t-i} = 1$ if $t = T_{j-1} + i$, allowing us to construct a conditional likelihood function for each sub-sample period, and this variable is also employed to define:

$$E_{j,t} = \sum_{i=1}^{T_j - T_{j-1}} D_{j,t-i} = \begin{cases} 1 & \text{for } T_{j-1} < t \leq T_j, \\ 0 & \text{otherwise,} \end{cases} \qquad \text{and} \quad E_t = (E_{1,t}, \ldots, E_{q,t})',$$

so that E_t enables us to include level shifts in the system with their corresponding parameters

γ, or more specifically, $\gamma = (\gamma_1, \ldots, \gamma_q)$. Finally, d_t represents an unrestricted s-variate vector of dummy variables and second-order difference variables, which are included in the system for the purpose of capturing various irregularities in the process. See Doornik, Hendry, and Nielsen (1998) and Kurita and Nielsen (2009) for further details.

In this system, the underlying long-run economic relationships are interpreted as being represented by the combinations $\beta' X_{t-1} + \gamma E_t$, which are of testable linear form in view of the reformulated theory in §2. The combinations also allow for the possibility of level shifts in the process. In order to reveal the underlying long-run linkages, we follow a multi-step modelling strategy: we begin by estimating a well-formulated general unrestricted partial VAR (PVAR) model assuming known location shifts and then proceed to test the cointegrating rank r by using a quasi-likelihood testing procedure in Kurita and Nielsen (2018); after determining r, we seek those structures of the quasi-maximum likelihood estimates $(\hat{\beta}', \hat{\gamma})$ which are consistent with the theoretical formulations given in §2. In addition, the adjustment vectors α_y represent how each variable in the system reacts to disequilibrium errors expressed by $\beta' X_{t-1} + \gamma E_t$; see equation (12). Hence, checking the structure of $\hat{\alpha}_y$ is also important in terms of detailed economic interpretations of $(\hat{\beta}', \hat{\gamma})$.

The PCVAR system (12) is a tractable model for exploring long-run dynamics subject to regime shifts. The partial model focusing on Y_t given information on Z_t, allows us to avoid a large-dimensional system which poses difficulties in correctly choosing the cointegrating rank in finite samples. Furthermore, the partial system can be reduced to a smaller system by testing for evidence indicating the presence of additional weakly exogenous variables in the set of Y_t; let $Y_t = (Y_{1t}', Y_{2t}')'$ and let $\alpha_y = (\alpha_{y1}', \alpha_{y2}')'$ accordingly, and if $\alpha_{y2} = 0$, then Y_{2t} is judged to be weakly exogenous for α_{y1} and $\left(\beta', \gamma\right)$ in the same sense as the assumption about Z_t. This small system then acts as a bridge connecting the system analysis with a single-equation analysis based on IIS, which is reviewed in the next sub-section.

3.2 Impulse indicator saturation

The reduction from the system to single equation analysis requires that there is no loss of information by focusing on the conditional model for one variable, marginalizing with respect to the other variables. A conditional-marginal factorization can be applied to any joint density, but the parameter space from the joint model must be the cross product of the individual parameter spaces, ensuring the parameter spaces of the marginal models are not linked to the parameter spaces of the conditional model. If these conditions are satisfied we can proceed to a single equation analysis for the spot exchange rate. Although endogeneity is no longer modelled, the single equation approach allows for a more general analysis of structural breaks and outliers by applying IIS.[2]

We take the PCVAR as given from the multivariate modelling approach outlined above and check for robustness of the model specification to outliers and location shifts. IIS creates an indicator for every observation, taking the value 1 for that observation and 0 for all other observations, resulting in T impulse indicators for T observations. A subset of these indicators are included in the model for the spot exchange rate and the model selection algorithm *Autometrics* (Doornik, 2009) undertakes a tree search to select the relevant indicators at a significance level α, checking for evidence of mis-specification or a lack of encompassing en route. The algorithm can search over the indicators whilst forcing the economic variables in the model to be retained. The algorithm then includes a different set of impulse indicators and retains significant impulses after search. Many different blocks will be tried as the number of regressors (economic and indicator variables) $N > T$. Finally, the union of the retained impulse indicators will be included and the selected impulse indicators from this final stage will form the final model. Hendry and Doornik (2014) provide details of the algorithm and the underlying theory justifying IIS, where the average false null retention of impulse indicators is controlled at αT and there is a very small efficiency loss despite testing for T breaks at any point in sample. Although all impulse indicators are orthogonal, the algorithm does not exploit this, which allows generalisations to other functional form saturation such as steps to detect location shifts (SIS).

4 An overview of the data

Figure 1 records the monthly log dollar-pound spot exchange rate (s_t), along with its time series properties and distribution.[3] The estimation period for the subsequent analysis is October 2003 to April 2018, denoted as 2003.10 - 2018.4. The spot exchange rate is non-stationary, with a stochastic trend and location shifts, highlighted by panel b which records a time-varying mean obtained using SIS at 0.1% with a fixed intercept. There is a large location shift in 2008.9 corresponding to the financial crisis and subsequent great recession, with a bimodal distribution for the data, but there are additional location shifts, notably a decline in s_t over 2015-2017 which also need to be modelled. It is important to jointly model the shifts and stochastic non-stationarity in the data for a congruent model.

Figure 2 records other market-based variables (bond and goods markets), with the US (do-

[2]Saturation techniques can also be applied to the system but the impulse indicators are detected based on tests for joint restrictions across all equations in the system.

[3]See Appendix A for further details including data sources.

mestic) bond yield and inflation rate in solid red and the UK (foreign) bond yield and inflation rate in dashed blue. The nominal interest rates exhibit similar declines over the last decade, although the differences in annual inflation over the period are more stark.

Figure 3 records a class of policy-oriented variables, with the first panel plotting the US (domestic) and UK (foreign) monetary base, scaled to match means to view on the same figure (with original units in billions of US dollars and millions of pounds sterling before transformations). The periods of expansion of the monetary base due to active policy are evident, but are not aligned in the two countries. The US quantitative easing (QE) policy started around the end of 2008, while the QE policy in the UK commenced in March 2009. The second panel records the forward premium, which is also subject to two countries's monetary policies under the CIP condition.

All the data series we use are 'wide-sense' non-stationary, in that they exhibit stochastic non-stationarity in the form of unit roots and are treated as $\mathsf{I}(1)$, and they are subject to distributional shifts, with abrupt changes in mean and variance. Hence, the econometric methodology employed must handle both forms of non-stationarity to obtain a congruent model, which we now explore.

We have given here an overview of all the variables in a system for (11), that is $X_t = (s_t, i_t, i_t^*, \pi_t, \pi_t^*, mb_t, mb_t^*, fp_t)'$. Referring to the PCVAR model with structural breaks in (12), we are justified, on several grounds, in classifying X_t into endogenous or modelled variables Y_t and conditioning variables Z_t which are assumed to be weakly exogenous for $(\alpha_y', \beta', \gamma)$ in the following manner:

$$Y_t = (s_t, i_t, i_t^*, \pi_t, \pi_t^*)' \quad \text{and} \quad Z_t = (mb_t, mb_t^*, fp_t)'. \tag{13}$$

The first ground for this classification is that all the variables in Z_t in (13) are of policy-driven nature, so that they exhibit too erratic behaviour to be modelled in the linear VAR framework; see Figure 3. This type of argument is found in Section 4 of Harbo, Johansen, Nielsen, and Rahbek (1998), and it provides a strong basis for a modelling strategy based on a partial model, instead of a large-dimensional full system. The second argument is that, given our research interest, we can regard Y_t in (13) as a class of variables of interest, which contains s_t and several other market-based variables, while modelling Z_t in (13) is considered to be a secondary interest. Lastly, we may be encouraged by the finding that a short-term interest rate, a representative policy-oriented variable, was judged to be approximately weakly exogenous in an empirical UK money demand model in Ericsson, Hendry, and Mizon (1998). Our US-UK empirical system may also be of some parallel structure such that (13) could be justifiable. The above classification of variables in view of the PCVAR system (12) sets the stage for the general-to-specific econometric modelling of the dollar-pound exchange rate.

5 A partial-system analysis with a shift in the level

A system-based analysis is conducted in this section. §5.1 determines cointegrating rank in an empirical PVAR system after checking its residual diagnostic test statistics. §5.2 then specifies cointegrating vectors consistent with the reformulated theoretical model presented in §2 and seeks evidence for the validity of further model reduction. §5.3 then arrives at a trivariate equilibrium correction model reduced from the original PVAR system.

5.1 Testing for cointegrating rank

First, we introduce an unrestricted PVAR system for Y_t given Z_t, which is defined in (13), so that, with $p = 8$ and $m = 5$, the overall vector of variables is $X_t = (Y_t', Z_t')'$. See the Appendix for further details of the data. As noted in the previous section, the effective sample period for estimation is 2003.10 - 2018.4. The PCVAR system (12) in §3.1 is nested in this 5-dimentional unrestricted PVAR system as a result of the cointegrating-rank restriction, which is determined by a sequence of partial log-likelihood ratio (PLR) tests examined by Kurita and Nielsen (2018). As described in §3.1, Z_t is assumed to be weakly exogenous for the underlying parameters of interest, and, given the wide-sense non-stationarity of the system subject to distributional shifts we can also test for super exogeneity which combines weak exogeneity with the invariance of conditional parameters to interventions changing marginal parameters, see Hendry and Santos (2010).

Before testing for cointegrating rank in the system, we need to determine the number of breaks q, the lag length k and a class of unrestricted variables d_t; see equation (12). The data overview indicates a structural break around the end of 2008, which corresponds to the global economic recession triggered by the US financial crisis starting in 2008.9. Thus, selecting $q = 2$ with a break point 2008.9 is justifiable based on both the observation of the data and the historical record. This selection results in a classification of two regimes for the whole sample period, with the first regime's relative length $T_1/T = 0.352$, for $T_1 = 63$ and $T = 179$, when the lag length is 4 ($k = 4$). This lag order is chosen on the basis of F test statistics for the lag-order selection in preliminary regression analysis.

The impact of the Brexit referendum (23 June 2016) was also noticeable in 2016.7 in the data overview, but its magnitude was much smaller than the global recession starting in 2008.9, reflecting the effect of shocks on the dominant economy versus a shock on the UK alone which did not feed back to the US economy. Thus, as the initial model setting, we stick to the specification of $q = 2$ with the break point 2008.9, and adopt a strategy of employing unrestricted dummy variables for possible outliers due to the Brexit referendum. We then check diagnostic tests for the estimated PVAR system, in order to verify the specification adopted here is judged to be a valid one as a statistical representation of the data.

In preliminary regression analysis we find outliers in 2016.7 and 2016.10 in the residuals of the s_t equation, which are attributable to the Brexit referendum and its aftermath. These outliers are thus handled by an unrestricted dummy variable being 1 in both 2016.7 and 2016.10 while 0 otherwise. There is also an outlier in 2006.9 in the residuals of the π_t equation, caused by a decrease in petrol prices in the US. This outlier is modelled by an unrestricted dummy being 1 in 2006.9 and 0 otherwise. In addition, there is some evidence indicating seasonality-related autocorrelations in the residuals of the equation for π_t. Hence, an unrestricted 12-lagged second-order differenced series, $\Delta^2\pi_{t-12}$, is added to the PVAR(4) system, as proposed by Kurita and Nielsen (2009); this adjustment addresses the autocorrelation without affecting the underlying asymptotic theory in cointegration analysis.

Single-equation tests	s_t	i_t	i_t^*	π_t	π_t^*
Autocorr.[$F_{AR}(7,124)$]	1.50[0.17]	1.29[0.26]	1.33[0.24]	1.00[0.43]	1.22[0.30]
ARCH [$F_{ARCH}(7,161)$]	0.43[0.89]	0.99[0.44]	1.10[0.37]	0.34[0.94]	0.36[0.93]
Hetero.[$F_{HET}(74,95)$]	0.90[0.69]	1.21[0.19]	1.01[0.48]	0.64[0.98]	1.24[0.17]
Normality [$\chi^2_{ND}(2)$]	1.44[0.49]	2.49[0.29]	4.19[0.12]	4.71[0.10]	2.27[0.32]

Vector tests		
	Autocorr.[$F_{AR}(175,461)$]	1.08[0.25]
	Hetero.[$F_{HET}(370,459)$]	1.16[0.07]
	Normality [$\chi^2_{ND}(10)$]	11.43[0.33]

Note. Figures in square brackets are p-values.

Table 1: Diagnostic tests for the PVAR(4) system with a level shift

Table 1 displays a battery of residual diagnostic test outcomes for the PVAR(4) model. They are rounded to two decimal places, and most of them are presented in the form $F_j(\cdot, \cdot)$, which indicates an approximate F test against the alternative hypothesis j. A class of the alternative hypotheses is given as follows: 7th-order serial correlation (F_{AR}: see Godfrey, 1978; Nielsen, 2006), 7th-order autoregressive conditional heteroscedasticity (F_{ARCH}: see Engle, 1982), heteroscedasticity (F_{HET}: see White, 1980), and a chi-squared test for normality (χ^2_{ND}: see Doornik and Hansen, 2008). There is no evidence indicating residual diagnostic problems, so we interpret the PVAR system as a well-formulated general model; the lack of mis-specification in the table also indicates the validity of the level shift specification characterised by $q = 2$ and the break point 2008.9. We can thus proceed to the analysis of cointegrating rank in this PVAR system based on the results of Kurita and Nielsen (2018).

	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$	$r \leq 4$
PLR	141.02[0.00]**	94.17[0.04]*	58.33[0.15]	25.93[0.61]	6.03[0.93]

Note. Figures in square brackets are p-values according to Kurita and Nielsen (2018). $**$ and $*$ significance at the 1% and 5% level, respectively.

Table 2: Testing for cointegrating rank in the PVAR(4) system with a level shift

A sequence of PLR test statistics is presented in Table 2, in which the p-values of the statistics are calculated by using a response-surface table reported in Kurita and Nielsen (2018). We conclude that $r = 2$ is selected at the 5% significance level, which is also supportive of the theoretical exchange-rate model proposed; there is a possibility that either of the estimated cointegrating combinations may correspond to one of the conceivable long-run relationships derived in §2. In order to pursue this possibility, we next explore the underlying structure of the two cointegrating vectors.

5.2 Theory-consistent long-run economic relationships

Having selected $r = 2$, we can examine various restrictions on $(\hat{\beta}', \hat{\gamma})$, along with those on $\hat{\alpha}$. What we intend to illuminate here is whether or not we can reveal empirical cointegrating relationships consistent with the theoretical long-run formulations, such as (10), developed in §2. This approach towards theory-consistent long-run linkages was also adopted by Kurita (2007); see also Almaas and Kurita (2018). It will also be useful, in terms of further model reduction, if some of the variables in Y_t are judged to be weakly exogenous for the cointegrating parameters, as described in §3.1.

	H1	H2	H3
PLR	$2.3[0.129]_{(df=1)}$	$2.618[0.270]_{(df=2)}$	$7.352[0.118]_{(df=4)}$

Note. Figures in square brackets are p-values according to $\chi^2(df)$, in which df denotes degree of freedom.

Table 3: Preliminary tests for a class of sub-hypotheses

We first check the validity of sub-hypotheses for the theory-consistent long-run relationships. We have a class of three sub-hypotheses represented as a set of restrictions on $\hat{\beta}$ under $r = 2$, which are specified as follows:

H1 : Exclusion of π_t and π_t^* from the first cointegrating relation

H2 : H1 $\cap$ The interest rate spread $i_t - i_t^*$ in the first cointegrating relation

H3 : H2 $\cap$ Exclusion of s_t, i_t and i_t^* from the second cointegrating relation

The test results are recorded in Table 3, which indicates all the hypotheses are not rejected at the 5% level. The non-rejection of H1, the theoretical formulation excluding the observed inflation rates, equation (10), captures one of the underlying long-run relationships. Next, H2 supports the preceding inference, also suggesting an important role played by the differential between i_t and i_t^* in the candidate relationship. Lastly, H3 conveys useful structural information on the other long-run relationship concerning π_t and π_t^*.

Given the results in Table 3, we next identify a set of interpretable cointegrating combinations in a theory-consistent manner. We have normalised the two cointegrating vectors for s_t and π_t^*, respectively, and checked various restrictions which agree with the theoretical model in §2, along with those restrictions which indicate weak exogeneity of some elements of Y_t. As a result, we have reached the structure of $\hat{\alpha}$ and $(\hat{\beta}', \hat{\gamma})$ recorded in Table 4:

$$\widehat{\alpha}' \quad \begin{bmatrix} s_t & i_t & i_t^* & \pi_t & \pi_t^* \\ \underset{(0.032)}{-0.126} & \underset{(0.0018)}{-0.005} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} \\ \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(0.022)}{-0.157} \end{bmatrix}$$

$$(\widehat{\beta}', \widehat{\gamma}) \quad \begin{bmatrix} s_t & i_t & i_t^* & \pi_t & \pi_t^* & mb_t & mb_t^* & fp_t & E_{1,t} & E_{2,t} \\ \underset{(-)}{1} & \underset{(2.437)}{16.456} & \underset{(-)}{-16.456} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(0.0015)}{-0.056} & \underset{(-)}{0} & \underset{(0.081)}{-0.224} & \underset{(0.02)}{-0.257} & \underset{(-)}{0} \\ \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{-1} & \underset{(-)}{1} & \underset{(0.012)}{0.044} & \underset{(0.009)}{-0.014} & \underset{(-)}{0} & \underset{(0.042)}{-0.132} & \underset{(0.048)}{-0.184} \end{bmatrix}$$

Note. The PLR test statistic for the joint restrictions is 19.45[0.194], where the figure in square brackets denotes a p-value according to $\chi^2(15)$.

Table 4: Restricted adjustment and cointegrating vectors

The rounded estimates and standard errors, along with the corresponding PLR test statistic, are provided in the table. The hypothesis of the overall restrictions is not rejected at the 5% level, hence allowing us to infer that the set of restrictions represents the structure of the underlying data generation mechanism. The second panel in Table 4 shows that the first cointegrating relationship leads to an equilibrium correction term, $ecm_{1,t}$, defined as:

$$ecm_{1,t} = s_t - 0.055 mb_t + 16.456 \left(i_t - i_t^*\right) - 0.224 fp_t - 0.257 E_{1,t},$$

for $E_{1,t} = 1$ for $0 < t \leq T_1$ and 0 otherwise with the break point 2008.9. The relationship $ecm_{1,t}$ matches (10), so that we can argue that all the coefficients in $ecm_{1,t}$ are interpretable from the theory developed in §2. The finding of $ecm_{1,t}$ allows us to argue that Frankel (1979)'s theoretical model is empirically relevant, as a result of incorporating the US monetary base and the forward premium into the PCVAR system with a regime shift around the end of 2008. Note that the US monetary base is highly significant judging from its relatively small standard error, while that of the UK is insignificant and thus removed from the cointegrating combination. This asymmetry indicates that US monetary policy has played a more dominant role in the determination of the dollar-pound exchange rate. Since i_t and i_t^* in $ecm_{1,t}$ are both annual rates, it is necessary to interpret their coefficients in terms of monthly rates $16.456\,(i_t - i_t^*) \approx 197.47\,(i_t - i_t^*)\,/12$, resulting in $\hat{\theta} = (197.47)^{-1} \approx 0.0051$. This estimate is seen as evidence for a near unit value of $1 - \theta$ in equation (4), consistent with numerous econometric studies on persistent deviations from PPP-based values in small samples, as reviewed by Sarno and Taylor (2002, Ch.3).

It should also be noted that the level parameter is significant only in the first sub-sample period, suggesting a structural change represented by a shift down in the cointegrating relationship. This level shift is interpreted as a structural shift in the policy-oriented value of the exchange rate $\bar{s}_t$, as defined in equation (6); this could reflect the impact of quantitative easing adopted by the US monetary authority, a policy regime change in response to the global financial crisis. In addition, as shown in the first panel in Table 4, the adjustment coefficient for s_t (the first entry in the first row vector in $\hat{\alpha}'$) is -0.127 and is also judged to be highly significant. This finding implies that the dollar-pound exchange rate reacts to $ecm_{1,t-1}$ in such a manner that it steadily adjusts disequilibrium errors from its long-run value. Overall, the revealed structure here is seen as strong evidence supporting the fundamental-based view of the long-run dollar-pound exchange rate.

Similarly, the second cointegrating combination in Table 4 leads to:

$$ecm_{2,t} = \pi_t^* - \pi_t - 0.014 mb_t^* + 0.044 mb_t - 0.132 E_{1,t} - 0.184 E_{2,t},$$

in which $E_{1,t} = 1$ for $0 < t \leq T_1$, $E_{2,t} = 1$ for $T_1 < t \leq T$ and both 0 otherwise with the break point 2008.9. This combination is interpreted as a long-run representation connecting the two countries' monetary base measures with the rates of inflation. This combination indicates that, for example, monetary expansion in the US given UK monetary policy tends to increase inflation in the US relative to that in the UK, although we have to bear in mind only a marginal role played by the UK monetary base in the relationship. Since one of the primary objectives of monetary policy is to control inflation, this empirical evidence is informative in terms of macroeconomic policy coordination between the two countries. In addition, the adjustment coefficients in the table show that there is a sole significant feedback mechanism for the UK inflation rate π_t^*, so that it is justifiable to view $ecm_{2,t-1}$ as critical for the stability of UK inflation. Judging from the marginal significance of mb_t^* in $ecm_{2,t}$ reported in Table 4, one can infer that the level shift captured by $E_{1,t}$ and $E_{2,t}$ corresponds to a step shift observed in mb_t (see Figure 3), which reflects the US quantitative monetary easing implemented as a policy measure for the financial crisis in 2008.

	No level shift in $ecm_{1,t-1}$	No level shift in $ecm_{2,t-1}$
PLR	30.885[0.009]**	34.565[0.003]**

Note. Figures in square brackets are p-values according to $\chi^2(15)$.
$**$ denotes significance at the 1% level.

Table 5: Testing for no level shift

The level shifts found in the two cointegrating linkages are noteworthy, implying the importance of employing a PCVAR model allowing for structural breaks in the data. In order to strengthen the validity of this model specification, we will check how significant the level shift is in each of the cointegrating combinations. For each of them, the coefficients for $E_{1,t}$ and $E_{2,t}$ are restricted to be of the same value, with the zero restriction imposed on the first vector (see Table 4) being lifted, *i.e.* no level shift is imposed on each cointegrating relationship so that we can check the PLR test statistic for this restriction. Table 5 records the test statistics and their p-values, which indicate that the null hypothesis of no level shift is rejected at the 5% level for both of the relationships.

Given the results in Table 5, Figure 4 then displays a class of cointegrating combinations under different level-shift specifications. Figure 4 (a) and (b) record plots of $ecm_{1,t}$ and $ecm_{2,t}$, which are based on Table 4 and are mean-reverting stationary series except for some noticeable fluctuations around the global recession in 2008 - 2009, whereas Figure 4 (c) and (d) are the no level shift versions of $ecm_{1,t}$ and $ecm_{2,t}$ (denoted as $\widetilde{ecm}_{1,t}$ and $\widetilde{ecm}_{2,t}$ in the figure), which are based on Table 5; recall that this restriction was rejected for both of them. The series displayed in panels (c) and (d) suffer influences of the uncaptured level shift around 2008 - 2009. Both $\widetilde{ecm}_{1,t}$ and $\widetilde{ecm}_{2,t}$ appear to be non-stationary indicating a significant problem with the restriction of no level shift. The overall evidence supports the level shift specification in Table 4, enabling the derivation of theory-consistent cointegrating linkages.

We also test whether some of the variables in Y_t are judged to be weakly exogenous with respect to the parameters of interest. Following §3.1, let $Y_t = (Y_{1t}', Y_{2t}')'$ and $\alpha_y = (\alpha_{y1}', \alpha_{y2}')'$, and if $\alpha_{y2} = 0$, then Y_{2t} is long-run weakly exogenous for $(\alpha_{y1}', \beta', \gamma)$. As shown in the first panel in Table 4, the adjustment coefficients for i_t^* and π_t are zero, implying that these two variables are weakly exogenous for the parameters of interest β. Thus, we classify $Y_{1t} = (s_t, i_t, \pi_t^*)'$ and $Y_{2t} = (i_t^*, \pi_t)'$ in the rest of this paper.

Lastly, we employ a set of *ex post* tests suggested in Section 4 of Harbo, Johansen, Nielsen, and Rahbek (1998), *inter alia*, to check if each of $Z_t = (mb_t, mb_t^*, fp_t)$ can be viewed as weakly exogenous for the parameters of interest fixed at the estimated values. First, we set up an $\mathsf{I}(0)$ marginal VAR system for the three variables $\Delta Z_t = (\Delta mb_t, \Delta mb_t^*, \Delta fp_t)'$, which contains the two restricted cointegrating relationships ($ecm_{1,t-1}$ and $ecm_{2,t-1}$) as lagged regressors. Second, the marginal VAR model is estimated to check if the cointegrating relationships can be excluded from each equation in the model. The estimated marginal model is nothing but a preliminary data-representation, inevitably suffering from a number of diagnostic problems due to policy-driven irregularities in the data (see Figure 3), so any likelihood-based tests for this model should be seen as quasi-type tests. The observed test statistics for exclusion restrictions on the cointegrating combinations are: 4.3[0.117] for the Δmb_t equation, 20.544[0.000]** for the Δmb_t^* equation and 2.015[0.365] for the Δfp_t equation, where the figures in square brackets are p-values according to $\chi^2(2)$. We may have to doubt the weak-exogeneity property of mb_t^*, due to the quasi likelihood-based evidence for some feedback effects from the cointegrating combinations on Δmb_t^*.

$$\widehat{\alpha}' \quad \begin{array}{cccccc} s_t & i_t & i_t^* & \pi_t & \pi_t^* & mb_t^* \end{array}$$
$$\begin{bmatrix} \underset{(0.031)}{-0.142} & \underset{(0.0016)}{-0.0045} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(0.055)}{-0.204} \\ \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(0.028)}{-0.136} & \underset{(0.407)}{0.953} \end{bmatrix}$$

$$(\widehat{\beta}', \widehat{\gamma}) \quad \begin{array}{cccccccccc} s_t & i_t & i_t^* & \pi_t & \pi_t^* & mb_t & mb_t^* & fp_t & E_{1,t} & E_{2,t} \end{array}$$
$$\begin{bmatrix} \underset{(-)}{1} & \underset{(2.102)}{18.076} & \underset{(-)}{-18.076} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(0.0013)}{-0.056} & \underset{(-)}{0} & \underset{(0.07)}{-0.251} & \underset{(0.018)}{-0.261} & \underset{(-)}{0} \\ \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{0} & \underset{(-)}{-1} & \underset{(-)}{1} & \underset{(0.012)}{0.054} & \underset{(0.008)}{-0.023} & \underset{(-)}{0} & \underset{(0.042)}{-0.1} & \underset{(0.047)}{-0.151} \end{bmatrix}$$

Note. The quasi PLR test statistic for the joint restrictions is 18.323[0.246], where the figure in square brackets denotes a p-value according to $\chi^2(15)$.

Table 6: Restricted adjustment and cointegrating vectors when treating mb_t^* as non-weakly exogenous variable

Given the evidence concerning the Δmb_t^* equation, we have re-estimated the above restricted PCVAR model recorded in Table 4 but treated mb_t^* as an additional endogenous or modelled variable, so the model is now a 6-dimentional system. Table 6 reports a set of updated estimates and test statistic, which are almost equal to those in Table 4 in terms of the non-rejection of the joint hypotheses as well as coefficients' significance, values and signs. The updated PCVAR system is subject to the same restrictions as the PCVAR system studied in Table 4. The results recorded in Tables 4 and 6 are thus in support of the theory-consistent long-run structure revealed from the data, regardless of a difference in the two model specifications.

5.3 An $\mathsf{I}(0)$ vector equilibrium correction system

This sub-section plays the role of connecting the above PCVAR analysis with an IIS-based single equation analysis performed in the next section. Thus, the results of an $\mathsf{I}(0)$ trivariate system are briefly presented here; they provide an impetus for a reduced single-equation model for the dynamics of the dollar-pound exchange rate.

All the variables are transformed into the $\mathsf{I}(0)$ series by using $ecm_{1,t-1}$, $ecm_{2,t-1}$ and first-order differences. Starting with the estimation of a general trivariate model for ΔY_{1t} conditional on ΔY_{2t} and ΔZ_t, we have tested for the reduction of the model and finally arrived at the following parsimonious vector equilibrium correction (VEC) system:

$$
\begin{aligned}
\Delta s_t \;=\;& \underset{(0.061)}{0.179}\ \Delta s_{t-1} + \underset{(0.061)}{0.163}\ \Delta s_{t-3} - \underset{(0.449)}{1.308}\ \Delta \pi^*_{t-2} - \underset{(0.026)}{0.114}\ ecm_{1,t-1} + \underset{(0.785)}{1.883}\ \Delta i^*_t \\
&+ \underset{(0.273)}{0.722}\ \Delta \pi_t - \underset{(0.023)}{0.114}\ (\Delta fp_t - \Delta mb^*_t + \Delta mb_t) - \underset{(0.012)}{0.075}\ d_{t,Brexit}, \\
\Delta i_t \;=\;& -\underset{(0.003)}{0.009}\ \Delta s_{t-1} + \underset{(0.015)}{0.03}\ \Delta \pi_{t-1} + \underset{(0.003)}{0.007}\ \Delta s_{t-3} - \underset{(0.015)}{0.052}\ \Delta \pi_{t-3} \\
&- \underset{(0.0014)}{0.005}\ ecm_{1,t-1} + \underset{(0.042)}{0.931}\ \Delta i^*_t + \underset{(0.0014)}{0.0068}\ \left(\Delta mb^*_{t-1} - \Delta mb^*_{t-3}\right) \\
&+ \underset{(0.0024)}{0.0085}\ \Delta fp_t + \underset{(0.0025)}{0.0057}\ \Delta fp_{t-3} - \underset{(0.001)}{0.0026}\ d_{t,2008.9} - \underset{(0.0011)}{0.006}\ d_{t,2008.12}, \\
\Delta \pi^*_t \;=\;& \underset{(0.042)}{0.132}\ \Delta \pi_{t-1} - \underset{(0.024)}{0.076}\ ecm_{2,t-1} + \underset{(0.043)}{0.266}\ \Delta \pi_t - \underset{(0.006)}{0.015}\ \Delta mb_t \\
&+ \underset{(0.0022)}{0.0059}\ d_{t,2008.9} + \underset{(0.0028)}{0.0065}\ d_{t,2008.11},
\end{aligned}
\tag{14}
$$

Vector tests : $F_{AR}(63{,}430) = 1.13[0.24]$, $F_{HET}(222{,}791) = 1.10[0.19]$, $\chi^2_{ND}(6) = 4.15[0.66]$,

where standard errors are given in parentheses. A number of contemporaneous regressors play significant roles in accounting for the dynamics of the three endogenous variables. If we focus on $\Delta Y_{2t} = (\Delta i^*_t, \Delta \pi_t)'$, we find Δi^*_t is significant in the equations for Δs_t and Δi_t while $\Delta \pi_t$ is in the equations for Δs_t and $\Delta \pi^*_t$; their coefficients are all interpretable in a manner consistent with standard reasoning based on economic theory. Various influences from $\Delta Z_t = (\Delta mb_t, \Delta mb^*_t, \Delta fp_t)'$ are also noted; in particular, they play important roles in the Δs_t and Δi_t equations. Recall the CIP condition justifies $fp_t \approx i^s_t - i^{s,*}_t$. We can, therefore, interpret

Δfp_t as a difference between the dynamics of the two countries' short-term policy interest rates, implying that a relative increase in the US rate to the UK rate results in an appreciation in the US dollar against the UK pound as well as a rise in the US long-term interest rate. Positive and negative coefficients for Δmb_t^* and Δmb_t in the Δs_t equation indicate the presence of an overshooting phenomenon in the foreign exchange market. A monetary expansion in the UK, for example, brings about a rapid overshooting depreciation in the pound sterling, so that it then moves back afterwards to cancel out the overshooting part. This cancelling-out effect is captured by Δmb_t and Δmb_t^* in the above system. It should be noted that various dummy variables are significant in this conditional system, indicating the failure of an invariance condition required for super exogeneity of ΔY_{2t} and ΔZ_t with respect to the parameters of interest; see Engle and Hendry (1993) for a procedure for checking super exogeneity using a class of dummy variables.

Figure 5 (a), (c) and (e) compare the actual values of $\Delta Y_{1t} = (\Delta s_t, \Delta i_t, \Delta \pi_t^*)'$ with their fitted values derived from the VEC system, while Figure 5 (b), (d) and (f) present a set of scaled residuals of the equations in this system. As consistent with the vector diagnostic tests reported above, there is no sign of significant mis-specification problems in the figure. Overall, the VEC system is judged to be a data-congruent representation subject to economic interpretations.

The equation for Δs_t in the system, equation (14), contains lagged regressors along with a class of contemporaneous explanatory variables. This structure indicates the possibility that the dynamics of the dollar-pound exchange rate are predictable to some extent by means of lagged information. We have checked that the lagged variables are all highly significant in equation (14) even after we removed the contemporaneous variables (that is, Δi_t^*, $\Delta \pi_t$ and $\Delta fp_t - \Delta mb_t^* + \Delta mb_t$) from it. The finding of forecastability is remarkable but considered to be in line with some of the existing studies such as MacDonald and Marsh (1997), and it is encouraging for a comparative forecasting analysis pursued in §7. We will focus on the refinement of equation (14) in §6 to assess the impact of the Brexit referendum on the dollar-pound exchange rate dynamics in a single-equation framework. Since neither i_t nor π_t^* was judged to be weakly exogenous for the parameters of interest, focusing on the single-equation model for the exchange rate dynamics may give rise to some loss of information for statistical inference, but this is of second order importance when forecasting in §7.

6 A single-equation analysis of the spot exchange rate

We apply IIS to the Δs_t equation to check for unmodelled outliers. The retained variables from (14) are not selected over, but the Brexit dummy is omitted to see if saturation picks it up. The significance level for selection of impulse indicators is $\alpha = 0.0057\,(\approx 1/T)$. The resulting model picks up 6 impulse indicators. The two Brexit dummies for 2016.7 and 2016.10 are selected and are highly significant ($\mathsf{t}_{(2016.7)} = -4.94$ and $\mathsf{t}_{(2016.10)} = -4.56$) and could be combined as in the multivariate model. We also detect four further outliers, namely 2004.3, 2004.12, 2006.5 and 2016.1. They are recorded in figure 6, with the $\pm 2\sigma$ error bars around the impulse indicators. The additional indicators do not change the model significantly. The equation standard error is reduced from $\widehat{\sigma} = 1.67\%$ to $\widehat{\sigma}_{IIS} = 1.57\%$ and the short run effect of i_t^* weakens to become marginally insignificant at the 5% level. Interestingly, there are no indicators picked up over the financial crisis period, so the level shift included in the equilibrium correction mechanism is sufficient to model this period.

Clearly the Brexit dummies play an important role. We next look at forecasts for the period to see what impact the Brexit vote had on the determinants of the spot exchange rate and how our forecasts would perform.

7 Forecasting the spot exchange rate

Forecasting exchange rates is notoriously difficult. Ever since Meese and Rogoff (1983a, 1988), attempts to outperform a random walk in out-of-sample forecasting have struggled at short horizons, although the relative performance of structural models tends to improve at longer horizons. Forecasting theory is well developed for models that are assumed to match the DGP and the data are assumed stationary (often after differencing) but our models are at best good approximations to the unknown DGP and there are distributional shifts over the forecast horizon, see Clements and Hendry (1999). We use the VECM model derived above to undertake an out-of-sample forecasting exercise over 2016.1-2018.4. As highlighted in §6, the Brexit referendum led to a sudden fall in the value of sterling relative to the dollar from over 1.45$/*£* in 2016.5 to less than 1.24$/*£* in 2016.10. This large location shift will question the ability of models with embedded equilibria to forecast over periods of structural change. Clements and Hendry (2011) and Castle, Clements, and Hendry (2015), *inter alia*, discuss the main culprit of forecast failure, namely unanticipated location shifts. As such, we also consider a number of robust devices based on the VECM to forecast both the spot exchange rate and the change in the spot exchange rate at horizons of 1-month, 6-months and 12-months ahead. Section §7.1 outlines the models considered and §7.2 presents the results.

7.1 Forecasting Models

Six forecasting models are considered, including multivariate and single equation equilibrium correction models, a robust variant, and some naïve benchmarks. Forecasts are produced from the models for $\widehat{\Delta s}_{T+h+j|T+j}$, where T is the last in-sample observation ($T = 2015.12$), h defines the forecast horizon ($h = 1, 6, 12$) and j defines the forecast period ($j = 2016.1 - 2018.4$). Median level forecasts for the spot exchange rate are obtained using $\widehat{S}_{T+h+j|T+j} = \exp\left(\sum_{i=1}^{h} \widehat{\Delta s}_{T+h+j|T+j} + s_{T+j}\right)$ for $h > j$, with dynamic level forecasts for the first h forecasts.[4]

The forecasting models belong to the class of open models, where there are a number of exogenous regressors which must be forecast in order to produce *ex ante* spot exchange rate forecasts. Hendry and Mizon (2012b) develop forecast error taxonomies for open models and demonstrate that forecasts may not be improved by forecasting regressors offline relative to excluding them from the forecast model. The eight regressors in the system are $X_t = (s_t, i_t, i_t^*, \pi_t, \pi_t^*, mb_t, mb_t^*, fp_t)'$. To produce forecasts of the regressors offline we estimate a VAR(1) in ΔX_t with IIS at $\alpha = 0.001$ over 2003.10-2015.12, where the lagged regressors are forced so selection is applied to the impulse indicators only. IIS is applied jointly to all rows in the ΔX vector, so a significant impulse indicator in one equation of the VAR will entail the impulse indicator being retained in all other equations of the VAR, even if insignificant. We denote the offline forecasts by $\tilde{\cdot}$; $\widetilde{\Delta X}_{T+h+j|T+j}$, other than s_t, are stored to be utilized in the forecasting models outlined below, where forecasts for $h = 6, 12$, are iterated using the Wiener-Kolmogorov prediction formula, see Hamilton (1994, Ch.4), conditioning on the known information set h periods prior. Table 8 in Appendix B reports forecast statistics for the offline variables. Contrary to forecast theory for stationary regressors, forecast accuracy for many of the variables improves at longer horizons. The exogenous monetary bases and forward premium variables are particularly difficult to forecast, which is unsurprising from such a simple model given the out-of-sample behaviour of the variables shown in figure 3.

The first forecasting model is [1] the VECM derived in §5.3, but estimated over 2003.10-2015.12 to exclude the forecast period. The imposed restrictions are now marginal but the cointegrating relations are very similar to their full sample counterparts, see figure 7. The same trivariate specification is imposed, so model reduction is not applied again from the 5 variable system.[5] The equation for Δs_t has $\widehat{\sigma} = 1.69\%$ which compares to $\widehat{\sigma} = 1.67\%$ for the full sample.

[4] We compute median level forecasts rather than mean level forecasts which require an additional variance correction. Assuming $s_{T+h|T} \sim \mathsf{N}\left[\widehat{s}_{T+h|T}, \widehat{\sigma}^2_{s,T+h|T}\right]$, then $\mathsf{E}\left(S_{T+h|T}\right) = \exp\left(\widehat{s}_{T+h|T} + \frac{1}{2}\widehat{\sigma}^2_{s,T+h|T}\right)$, see Doornik and Hendry (2013a, ch.18).

[5] While using the same model specification could imply that information in the forecast period has been used to

Forecasts for 6- and 12-months ahead iterate the endogenous variables Δs and $\Delta \pi^*$ forward in the system, but offline $h-$step ahead forecasts are used for the conditioning variables. Forecasts of the cointegrating relations use offline forecasts weighted by the in-sample estimates of $\widehat{\beta}$, ignoring $\widehat{\gamma} E_{T+j}$ as this is zero in the forecast period.[6] The longer horizon forecasts smooth out the cointegrating relations as the component forecasts converge to in-sample unconditional means for longer horizons. For the first h periods the forecasts coincide with dynamic forecasts, but differ thereafter.

The second forecasting device is [2] the differenced VECM (DVECM). Castle, Clements, and Hendry (2015) propose a class of robust forecasting devices in which the VECM can adapt to changes in the equilibrium mean with a varying degree of smoothness, based on Hendry (2006). The robust device obtained by differencing (12) is given by:

$$\Delta Y_{T+j+h|T+j} = \frac{1}{r}\sum_{i=1}^{r} \Delta Y_{T+j-i} + \widehat{\alpha}_y \left[\left(\widehat{\beta}', \widehat{\gamma}\right) \begin{pmatrix} X_{t+j-1} \\ E_{t+j} \end{pmatrix} - \frac{1}{m}\sum_{i=1}^{m} \left(\widehat{\beta}', \widehat{\gamma}\right) \begin{pmatrix} X_{t-j-1-i} \\ E_{t+j-i} \end{pmatrix} \right], \tag{15}$$

for $h = 1, 6, 12$, where Y_t denotes the vector of five endogenous variables in the system. When $r = m = 1$, the differenced device is that given in Hendry (2006), but differencing (12) results in second differenced short run dynamics, $\Delta^2 Z_{T+j}$ and $\Delta^2 X_{T+j-i}$ which are $\mathsf{I}(-1)$ and therefore add noise to the forecasts. As these are mean zero on average we omit the Δ^2 terms, so exogenous regressors are excluded from the forecasting model.

The in-sample estimates of $\widehat{\alpha}, \widehat{\beta}, \widehat{\gamma}$ are used, so the economic content of the VECM is retained in the forecasting device, but the embedded in sample mean is removed. $\frac{1}{r}\sum_{i=1}^{r} \Delta Y_{T+j-i}$ estimates the system growth rate and $\frac{1}{m}\sum_{i=1}^{m} \left(\widehat{\beta}', \widehat{\gamma}\right) \begin{pmatrix} X_{t-j-1-i} \\ E_{t+j-i} \end{pmatrix}$ is an estimate of the equilibrium mean. Shifts in both the growth rate or equilibrium mean (in addition to those modelled via E_t) are likely to cause forecast failure, so replacing their in-sample estimates with adaptive counterparts results in a forecasting device which is more robust to location shifts than the VECM. When $r = m = 1$, estimates of both the growth rate and equilibrium mean are highly adaptive using just the last observation, but using more observations as r and m increase will result in smoother forecasts that are less inclined to overshoot at the end of a break, so would be better designed when there are no location shifts.

We can treat each equation in the DVECM separately as there are no feedbacks given that

determine the forecasting model, commencing from the PCVAR and applying model reduction over the in-sample period only results in an almost identical model specification.

[6] For ecm_2, $E_{2,t} = 1$ in the forecast period but would enter as $\Delta E_{2,T+h+j|T=j}$ which is 0 as the shift occurs in-sample.

we discard contemporaneous short-run double differenced dynamics and therefore the regressors in each equation are pre-determined. Three alternative DVECMs are considered, including [2a] $r = m = 1$, [2b] $r = m = 6$ and [2c] $r = m = 12$, where longer averages introduce smoothing, but will reduce adaptability to shifts. This robust method uses the last available observation(s) to estimate the equilibrium mean and growth rate of the system (where α_1 is the coefficient on $ecm_{1,t-1}$):

$$\widehat{\Delta s}_{T+h+j|T+j} \quad = \quad \frac{1}{r}\sum_{i=0}^{r-1} \Delta s_{T+j-i} + \alpha_1 \left(\widetilde{ecm}_{1,T+h+j-1} - \frac{1}{m}\sum_{k=1}^{m} ecm_{1,T+j-k} \right). \tag{16}$$

The third forecasting device [3] considers a single equation ECM as the VECM equation for the spot exchange rate does not rely on contemporaneous feedback from the other two endogenous variables. Following §6 we fix the retained regressors for the Δs_t equation of the parsimonious VECM and apply IIS at $\alpha = 0.0068$ for $T = 147$ observations. Some retained impulses are then combined following joint restriction tests. We also find that the change in UK inflation ($\Delta\pi^*$) enters at t rather than $t-2$. $\widehat{\sigma} = 1.40\%$ and the model passes in sample diagnostics. Three alternative specifications are considered depending on how we treat the exogenous regressors in the open model. Offline variables can be [3a] forecast, [3b] included as direct projections, or [3c] ignored, where [3c] does include the lagged ecm_1 term estimated over the in-sample period. The online Appendix reports all the forecasting models.

Three benchmarks are computed for comparison including [4] a VAR(1) including all 8 variables defined in X with one lag and [5] an AR(1) for Δs_t. There is no recursive or rolling updating and the $h > 1$-step ahead forecasts are computed by iterating forward by $h-$steps. Finally, we compute the Random Walk [6].

7.2 Forecasting Results

Table 7 reports summary statistics including the Root Mean Square Forecast Error (RMSFE) and the log determinant of the general matrix of the forecast-error second moment (GFESM), see Clements and Hendry (1993) and Hendry and Martinez (2017). Define the forecast errors as $\widehat{v}_{T+h+j|T+j} = \Delta s_{T+h+j} - \widehat{\Delta s}_{T+h+j|T+j}$ or $\widehat{v}_{T+h+j|T+j} = S_{T+h+j} - \widehat{S}_{T+h+j|T+j}$, where $\widehat{\cdot}$ denotes the forecast from one of the models 1-6 listed above. Stack the forecast errors over j such that $\underset{(1\times J)}{\widehat{\mathbf{v}}_h} = \left(\widehat{v}_{T+1|T}, \ldots, \widehat{v}_{T+h+1|T+1}, \ldots, \widehat{v}_{T+J|T+J-h}\right)$, where the first h forecast errors are dynamic, and $h-$step ahead from thereon, and then stack the different horizon forecast errors to obtain $\underset{(3\times J)}{\widehat{\mathbf{W}}} = (\widehat{\mathbf{v}}_1, \widehat{\mathbf{v}}_6, \widehat{\mathbf{v}}_{12})'$. The forecast statistics are given by $\mathsf{RMSFE}_h = \sqrt{\left(\widehat{\mathbf{v}}_h \widehat{\mathbf{v}}_h'\right)}$, and $\mathsf{GFESM} = \log\left|\frac{1}{J}\left(\widehat{\mathbf{W}}\widehat{\mathbf{W}}'\right)\right|$.

	Δs_t				S_t			
	RMSFE ($\times 100$)			GFESM	RMSFE ($\times 100$)			GFESM
	1m	6m	12m	$\forall h$	1m	6m	12m	$\forall h$
1. VECM	2.54	2.53	2.53	-29.82	3.40	11.36	19.72	-15.69
2a. Robust (1)	3.11	3.04	<u>3.19</u>	-25.55	4.05	9.60	**11.64**	-15.92
2b. Robust (6)	**2.41**	2.92	3.12	-24.44	**3.20**	13.99	22.35	<u>-14.41</u>
2c. Robust (12)	2.50	3.02	2.97	-26.29	3.33	<u>15.35</u>	21.00	-14.71
3a. ECM	2.79	2.64	2.65	-29.78	3.73	12.96	<u>23.59</u>	-15.36
3b. ECM ($t-1$)	3.06	*2.40*	**2.38**	-25.88	4.10	10.52	13.52	*-17.48*
3c. ECM (exc. t)	3.03	2.70	2.45	-25.54	4.07	13.51	14.80	**-17.83**
4. VAR	2.50	**2.38**	*2.38*	*-31.98*	3.33	**8.91**	*13.14*	-16.88
5. AR	*2.49*	2.41	2.41	**-37.84**	*3.31*	*9.26*	14.16	-16.73
6. RW	<u>3.27</u>	<u>3.16</u>	3.06	<u>-21.44</u>	<u>4.24</u>	9.39	17.48	-14.67

Table 7: RMSFE $\times 100$ and GFESM for forecast period 2016.1–2018.4. Bold indicates smallest RMSFE/GFESM and italic indicates second smallest RMSFE/GFESM, with underline indicating largest RMSFE/GFESM. Note that the 1−step ahead forecasts in levels and differences are not identical because the levels are given by the spot exchange rate (S_t) and not the log spot exchange rate (s_t).

GFESM has advantages over RMSFE because its rankings for forecasts can differ, both depending on whether the levels or differences are evaluated, and over the horizon. GFESM is invariant to the difference transform, so should give the same rankings on both the levels and differences. However, in our exercise there are two factors that affect this result. First, we evaluate the spot exchange rate rather than the log of the spot exchange rate, so the mapping between levels and differences is subject to a monotonic transformation and would, therefore, not be identical. Second, the first h forecasts are dynamic, switching to $h-$step thereafter, so the mis-specification for the forecast models varies over the forecast horizon, making the different $h-$step forecasts not comparable. This does not question the validity of the forecast accuracy statistic though. Furthermore, Christoffersen and Diebold (1998) highlight that standard forecast accuracy measures (the trace MSE, of which the reported RMSFE is the univariate equivalent) fail to value the maintenance of cointegrating relations at longer horizons. Whether imposing cointegrating relations helps long term forecasts is difficult to tell when using RMSFE as the forecast accuracy criterion, so this is a second reason for using the more general GFESM.

Figure 8 records the forecasts for the various models at the 1-step ahead horizon, with the online Appendix recording the equivalent figures for 6-step ahead and 12-step ahead forecasts. No model is able to predict the unanticipated shocks to the spot exchange rate in 2016.7 and 2016.10 due to Brexit. Indeed, forecasting Δs_t or S_t is extremely difficult, as even for 1-month ahead the smallest RMSFE is 2.41, which compares to the VECM equation standard error over the full sample of $\widehat{\sigma} = 1.67\%$.

Model rankings vary depending on both forecast horizon and dependent variable transformation. Following Meese and Rogoff (1983b), the random walk has dominated as the benchmark forecasting device, with models evaluated on their ability to produce superior forecasts to the random walk. In our out-of-sample analysis, the random walk forecast performs worst across most horizons, particularly in differences. The large Brexit related falls in sterling are temporary in differences, so the random walk misses the shift, but then predicts further falls which do not materialize. Even at short horizons such a naïve device is not recommended.

The VECM, our congruent, theory consistent and data admissible model, does not produce the best forecasts on either criterion, but is not ranked amongst the worst either. As Castle and Hendry (2011) discuss, the model should not be judged by the accuracy of its forecasts. Good models can forecast poorly, and bad models can forecast well. However, a variant of the VECM designed to be robust to shifts with 6 months of smoothing (model 2b) dominates at short horizons, both in levels and differences, suggesting that there is value in imposing the long-run cointegrating relations but that the deterministic terms need to adapt to shifts. The robust VECM is superior to the random walk forecast, which it nests, which signifies the importance of the economic content of the model captured in the long-run cointegrating relations. However, the robust devices are not designed for long range forecasts where the available information set does not allow for rapid updating. The GFESM for levels finds the robust device with 6 months of smoothing to be the worst ranked method, and yet at just the 1 month horizon it is preferred.

At the medium horizon of 6 months the VAR dominates, and performs well at the long horizon as well. The VAR and AR models converge to their unconditional means, which is approximately 0 in differences. The uninformative forecasts mitigate risks, but the levels forecasts systematically overpredict during sterling's decline, and then underforecast when sterling begins to recover.

The single equation ECMs (3a-c) perform similarly at short horizons, all missing the fall in the exchange rate and therefore producing systematic forecast failure after the shift until the spot exchange rate recovers. Differences between the specifications are more pronounced at longer horizons, where it pays to either produce direct forecasts for the exogenous regressors by lagging those variables by h periods, or by excluding them rather than forecasting offline. This is in keeping with the results from Hendry and Mizon (2012b) who show that many more mistakes can be made when forecasting exogenous regressors offline in addition to the variable of interest. The single equation ECMs without forecasting regressors offline dominate the levels forecasts across all horizons based on GFESM.

In some cases, forecast accuracy (of Δs_t) can improve with forecast horizon (e.g. the ECM excluding contemporaneous regressors). Such a result refutes standard forecasting theory that forecasts worsen as the forecast horizon increases, demonstrating both the wide-sense non-stationarity of the data and model mis-specification.

The results highlight the distinction between empirical modelling to test theories or understand economic phenomena and forecasting. The VECM model struggles to forecast over the second half of 2016, when the Brexit shocks occured. The model could not predict the sudden shifts in the exchange rate. However, from mid 2017 onwards, the forecasts from the VECM for Δs_t begin to perform well again at all horizons, despite parameter estimates not being updated as the forecast origin moves forward. The effect is stark in levels, as systematic misforecasting occurs when there are shifts that are not addressed in the forecasting model, but the forecasts are back on track after the structural change. The implications of the forecasting exercise are that congruent models should not be discarded if they forecast poorly, but it indicates that structural breaks have occured over the forecast period and alternative forecast devices that can adapt more rapidly should be used temporarily until the breaks have passed and the model dominates once more.

8 Conclusions

The paper develops a fundamental-based model of the dollar-pound exchange rate in which the monetary stances of the central banks are incorporated into a sticky-price monetary model, with the foreign exchange forward premium capturing expectations. The model hinges on the explicit modelling of a structural break for the financial crisis and subsequent global recession, triggered in September 2008. Modelling the structural break is essential to finding stable, mean-reverting long-run equilibrium relationships that capture the cointegrating relations in the model. The partial cointegrated VAR with level shifts is a new class of econometric models and reveals theory-based relationships that would otherwise have been obscured by the non-stationary data subject to distributional shifts. The resulting parsimonious vector equilibrium system is a data-congruent representation with theoretically justified economic interpretations.

The financial crisis and global recession is not the only structural break that needs modelling. The Brexit referendum took place near the end of the sample period, and had a large effect on the dollar-pound exchange rate, detected using IIS. IIS also checks the robustness of the VECM specification, demonstrating that the modelled level shift in the cointegrating relations was sufficient to capture the structural break with no additional short-run shocks to the system.

A forecasting exercise is undertaken to assess how well the model performs over the Brexit referendum period. No forecast device was able to predict the shifts, but at short horizons a robust form of the VECM forecast better than other models, and was superior to a random walk forecast which dominates in the literature. This result highlights the importance of adapting to shifts in the forecast period, while incorporating economic fundamentals in the forecasting model. At longer horizons, a single equation variant of the VECM performed well, but it is preferable to either exclude exogenous regressors or include direct forecasts of those regressors rather than forecast the exogenous regressors offline. Our forecasting results highlight that a model should not be judged on its forecast performance as this could be influenced by unanticipated events occuring in the forecast period.

In this paper we do not claim that we could forecast the exchange rate movements in advance of the Brexit referendum. However, our results indicate that there is a need to rapidly update to unanticipated shifts when they occur. We show that (i) forecast accuracy need not decline with forecast horizon; (ii) that level shifts (which are outliers in differences) need to be modelled immediately *ex post* to avoid systematic forecast failure; (iii) that in open models it may be preferable to use mis-specified models to avoid forecasting exogenous regressors offline if these regressors are also subject to structural breaks; and (iv) there is a difference between forecasting the level of the exchange rate and changes in the exchange rate, where it is hard to beat the unconditional mean of the exchange rate at longer horizons.

Our research highlights the importance of taking the data seriously when modelling the exchange rate. Non-stationarity, both in the form of unit roots and distributional shifts, are handled with care to develop a congruent, theory-consistent and data-admissible empirical model of the dollar-pound exchange rate. The importance of both forms of non-stationarity are also emphasized when forecasting the exchange rate. Anticipating the distributional shifts would resolve the apparent disconnect between modelling and forecasting, but that is left for future research.

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Appendix A: Data

(Data definitions, sources and notes)

Data definitions (figures in angle brackets represent source numbers):

s_t	The log of the spot dollar-pound exchange rate; Series code: XUMAUSS in <1>.
i_t	US 10-year treasury constant maturity rate in decimal form; Series code: GS10 in <3>.
i_t^*	UK 10-year government securities yield in decimal form; IUMAMNPY in <1>.
p_t	The log of the US consumer price index for all commodities; Series code: CPIAUCNS in <4>.
p_t^*	The log of the UK consumer price index for all commodities; Series code: D7BT in <2>.
π_t	12th-order difference of p_t, *i.e.* $\Delta^{12}p_t$
π_t^*	12th-order difference of p_t^*, *i.e.* $\Delta^{12}p_t^*$.
m_t	The log of the US monetary base (seasonally adjusted); Series code: AMBSL in <5>.
m_t^*	The log of the UK monetary base (constructed series; see the notes below); <1>.
fp_t	Forward premium (discount) rate, 1 month; Series code: XUMADF1 in <1>.

Sources (all the sources below were accessed on 7th and 9th Jul. 2018):

<1> Bank of England (BoE) Statistical Interactive Database (http://www.bankofengland.co.uk/boeapps/iadb/).

<2> Office for National Statistics (https://www.ons.gov.uk/economy/inflationandpriceindices/timeseries/d7bt/mm23#).

<3> Board of Governors of the Federal Reserve System (US), 10-Year Treasury Constant Maturity Rate, retrieved from FRED, Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/GS10).

<4> U.S. Bureau of Labor Statistics, Consumer Price Index for All Urban Consumers: All Items, retrieved from FRED, Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/CPIAUCNS).

<5> Federal Reserve Bank of St. Louis, St. Louis Adjusted Monetary Base, retrieved from FRED, Federal Reserve Bank of St. Louis (https://fred.stlouisfed.org/series/AMBSL).

Notes:

i_t, i_t^* The original series were divided by 100 to obtain interest rates in decimal form.

m_t^* Since UK data equivalent to those for the US monetary base appear to be unavailable, we constructed data for the UK monetary base by adding data for notes and coins in circulation (Series code: LPMB8H4; seasonally adjusted) to those for bank reserves (Series codes: RPWAEFI and LPMBL22). That is, we constructed the following series, depending on the availability of the data:

$$M_t^* = (\text{LPMB8H4} + \text{RPWAEFI for 2003.6 - 2006.4})$$
$$\text{and } (\text{LPMB8H4} + \text{LPMBL22 from 2006.5 onwards}),$$

so that $\log M_t^* = m_t^*$. Note that the data for RPWAEFI are available on a weekly basis, so we averaged the weekly data to obtain the corresponding monthly data.

Appendix B: Forecast results for offline variables

	Mean Error			RMSFE			$\widehat{\sigma}$
	1-step	6-step	12-step	1-step	6-step	12-step	
Δi_t	0.026	0.036	0.037	0.159	0.158	0.158	0.172
Δi_t^*	0.014	0.015	0.014	0.154	0.153	0.153	0.147
$\Delta \pi_t$	0.102	0.089	0.085	0.290	0.269	0.267	0.341
$\Delta \pi_t^*$	0.102	0.104	0.100	0.254	0.231	0.225	0.243
Δmb_t	-0.218	-0.508	-0.527	2.488	2.223	2.232	1.935
Δmb_t^*	0.250	0.680	0.733	2.001	1.738	1.751	2.057
Δfp_t	-0.497	-0.276	-0.221	6.311	4.550	4.534	1.618

Table 8: Mean Error (ME) and Root Mean Square Forecast Error (RMSFE) $\times 100$ over 2016.1-2018.4 from a VAR(1) with IIS at $\alpha = 0.001$, with in-sample (2003.10-2015.12) equation standard error ($\widehat{\sigma}$) for comparison.

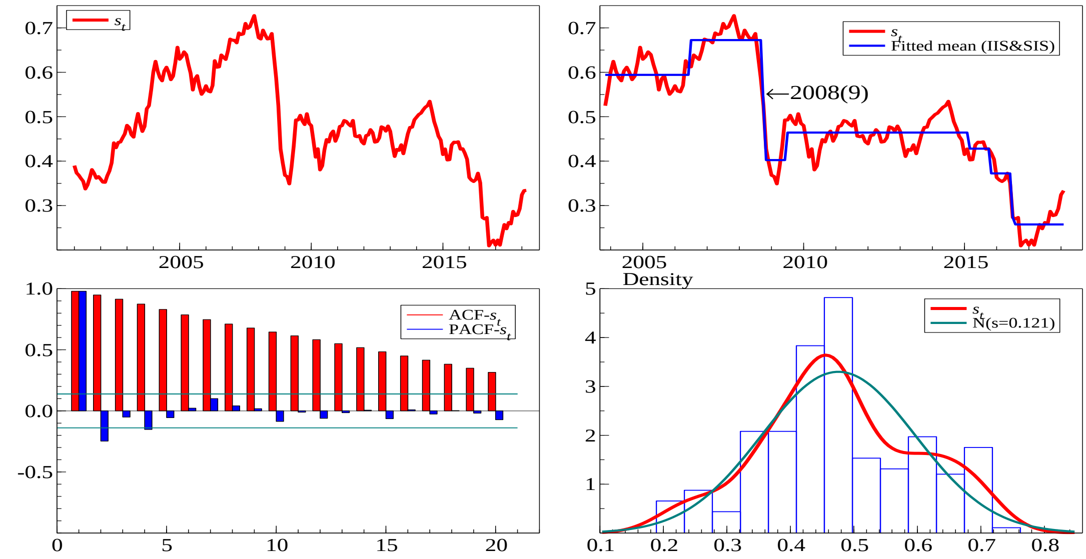

Figure 1: The log dollar-pound spot exchange rate (s_t), with a fitted mean using SIS (panel b), The autocorrelation and partial autocorrelation function for s_t and the histogram for s_t, recorded against a normal distribution.

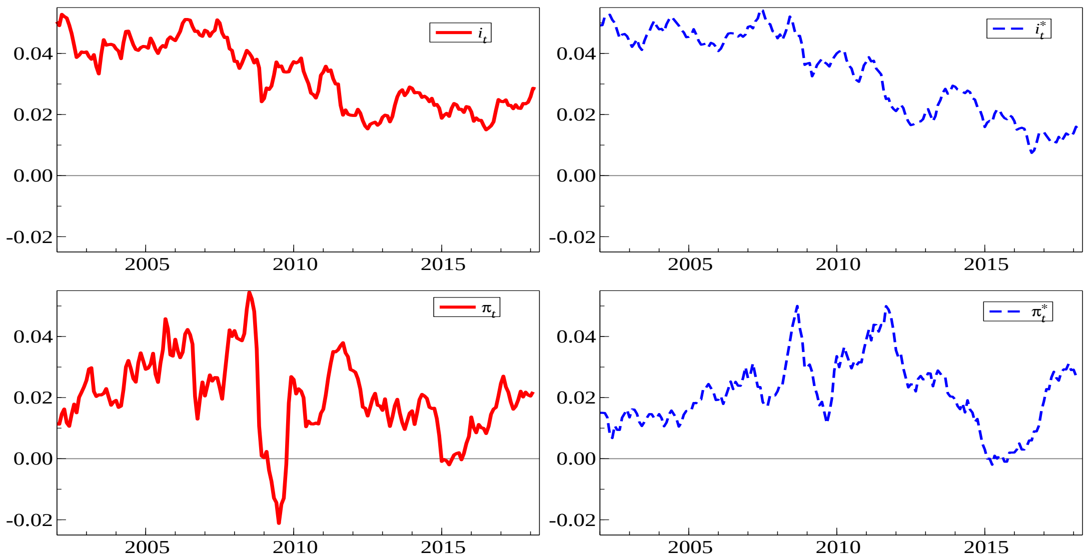

Figure 2: The US 10-year bond yield (i_t), the UK 10-year bond yield (i^*_t), the US annual CPI inflation rate (π_t) and the UK annual CPI inflation rate (π^*_t)

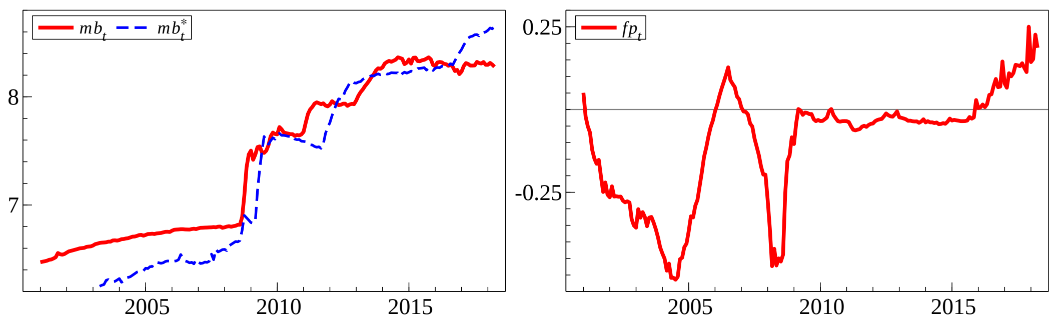

Figure 3: The US and UK monetary base in logs (mb_t, mb^*_t) and the forward premium (fp_t)

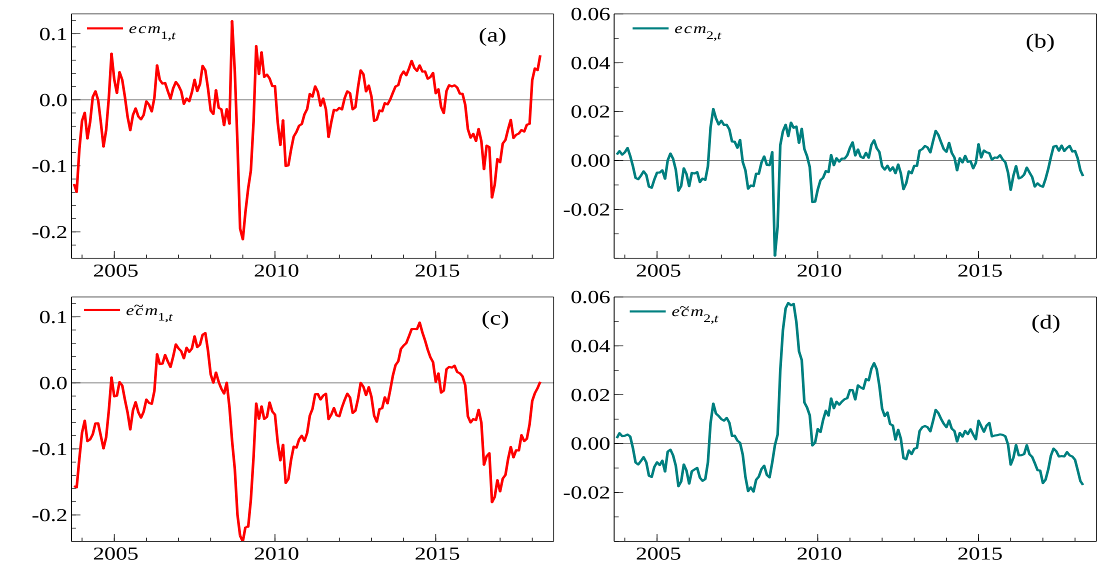

Figure 4: Plots of cointegrating combinations with level shift (a) and (b), and without level shift (c) and (d).

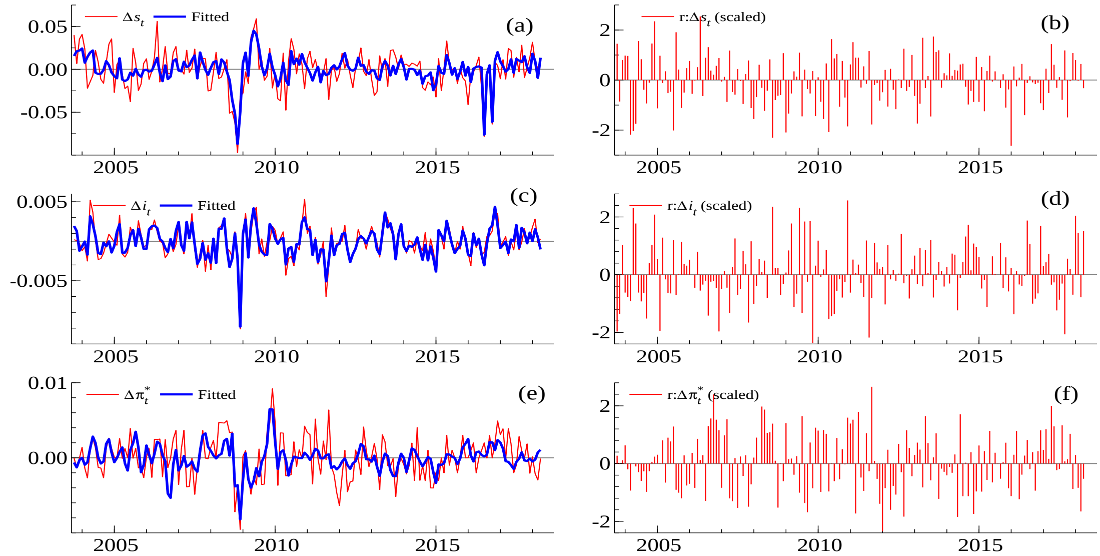

Figure 5: Actual values, fitted values and scaled residuals

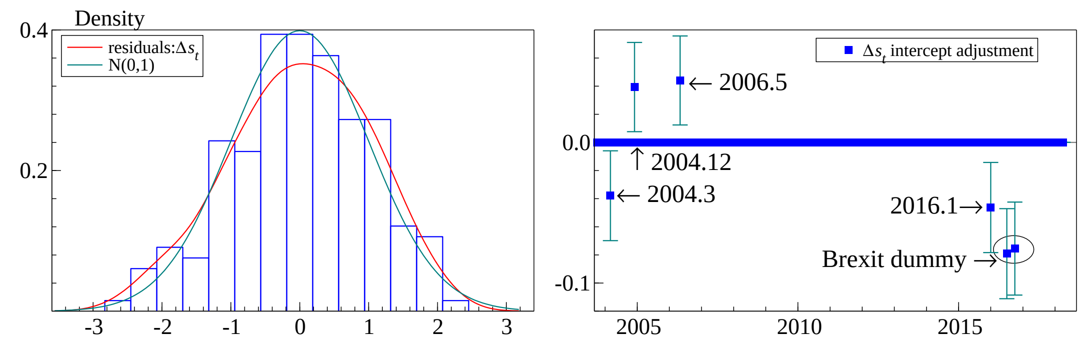

Figure 6: Single equation model of Δs_t with IIS residual density and intercept adjustment with error bars plotting $\pm 2\sigma$ for retained impulse indicators.

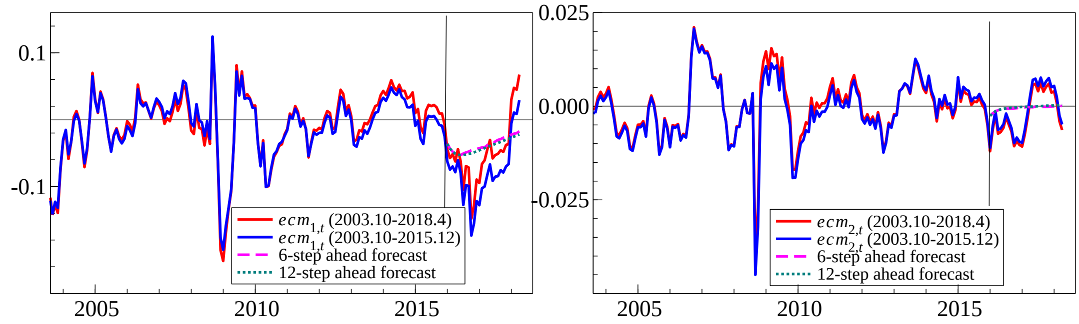

Figure 7: Cointegrating relations estimated over the full-sample (2003.10-2018.4) and in-sample (2010.3-2015.12) periods, with 6-step and 12-step ahead forecasts of ecm_1 and ecm_2. The cointegrating relations for the in-sample period use parameters estimated over 2010.3-2015.12, but data over the out-of-sample period to extend the cointegrating relations in the figure to 2018.4.

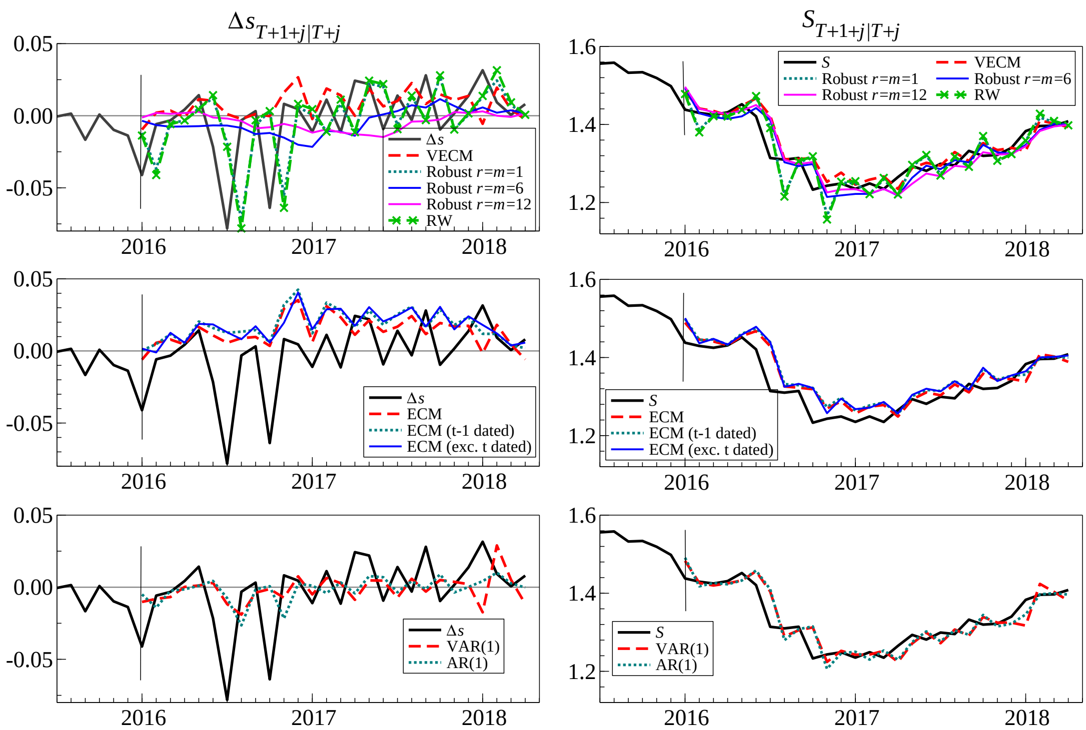

Figure 8: 1−step ahead forecasts for $\Delta s_{T+1+j|T+j}$ (left hand panels) and $S_{T+1+j|T+j}$ (right hand panels). Top panels record the VECM and robust VECM forecasts in comparison to the RW, middle panels record the single equation ECM forecasts, and bottom panels record the benchmark forecasts.