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Voracious Transformation of a Common Natural Resource into Productive Capital

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Frederick van der Ploeg[1]

Abstract

We analyze a power struggle about the control of natural resources where competing factions in society have a *private* stock of financial assets and a *common* stock of natural resources with inadequately defined private property rights. We solve a dynamic common-pool problem and obtain political economy variants of the Hotelling rule for resource depletion and the Hartwick saving rule necessary to sustain constant consumption in an economy with exhaustible natural resources. The rate of increase in the price of natural resources and resource depletion are faster than demanded by the Hotelling rule. As a result, the country substitutes away from resources to capital too rapidly so that it saves and invests more than a homogenous society. The power struggle boosts output, but depresses aggregate consumption and social welfare. Genuine saving is nevertheless zero in a fractionalized society, since the too rapid depletion of natural resources is exactly in line with the too rapid accumulation of physical capital. World Bank measures of genuine saving are likely to be over-estimated. This exacerbates the puzzle of why many resource-rich countries experience negative genuine saving rates.

Running head: Voracious resource transformation

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I. Introduction

For societies with competing groups and lack of effective property rights, Lane and Tornell (1996) and Tornell and Lane (1999) have demonstrated that a higher raw return on a common assets may increase the extent of rent seeking and thus reduce the rate of economic growth and make a country worse off. This has been coined the *voracity effect*. It has been shown in a context where rival factions accumulate both a private asset and a common asset, where the common asset has a higher rate of return than the private asset. The voracity effect arises from a dynamic common-pool problem whereby each group tries to grab more of the common asset before the the other groups do so. The main objective of this paper is explore these ideas within the context of the canonical model of transforming exhaustible natural resources into productive capital in order to sustain a constant level of consumption developed by Hartwick (1977) and based on Solow (1974).We are thus interested in how fractionalization of society affects saving and investment in a closed economy whereby each group accumulates private assets and depletes a common exhaustible natural resource. Another way of stating our objective is to investigate how the well known Hotelling (1931) rule for optimal resource depletion and the Hartwick (1977) rule for reinvesting natural resource rents[2] should be modified in countries with competing rival factions and lack of effective property rights.

We thus develop a political economy explanation of why resource-rich countries deplete their natural resources relatively faster and why such countries end up with lower levels of consumption than homogenous societies. Each one of the rival groups tries to grab a share of natural resource revenues before other groups can do so. The problem is that property rights for natural resources do not exist or are badly defined. We show that the power struggle makes competing groups more impatient and thus the country depletes natural resources faster than suggested by the Hotelling rule. As a result, a fractionalized resource-rich country substitutes away from natural resources to capital in production at a too rapid rate so that it saves and invests more than a homogenous society. The power struggle thus boosts output. Nevertheless, fractionalization depresses aggregate consumption and social welfare (the voracity effect). Genuine saving is nevertheless zero in a fractionalized society, since the too rapid depletion of natural resources is exactly in line with the too rapid accumulation of physical capital.

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[2] The Hartwick rule is first put forward by Hartwick (1977) for the closed economy. Dixit, Hammond and Hoel (1980) and Dasgupta and Mitra (1983) discuss it from the point of view of max-min egalitarianism.

We establish that the political distortions in the Hotelling and Hartwick rules are bigger if the country is more fractionalized. We derive our results for a closed economy with natural resources as a factor of production and competing factions. We show that the non-cooperative saving rate is greater than the production share of natural resources, but less than the production share of capital. The interest rate and the output-capital ratio gradually fall to zero. A fractionalized country sustains a sub-optimally low level of consumption. We show that even in a fractionalized society genuine saving is zero if the proper accounting price of natural resources is used following Arrow, Dasgupta and Mäler (2003). This suggests that World Bank estimates of genuine saving based on the *market* price of natural resources may be an over-estimate.

The outline of the paper is as follows. Section II sets up a model of depletion of a common natural resource and private accumulation. Section III gives the optimality conditions for a feedback Nash equilibrium outcome and section IV shows how the maxi-min outcome for this game permits an outcome with constant levels of consumption and output. Section V shows how the homogenous case without competing factions results in the familiar Hotelling and Hartwick rules where all resource rents are reinvested. Section VI shows that with competing factions natural resources are depleted too fast, savings and output are too high, and consumption is too low. Section VII establishes that genuine saving is also zero in society with competing factions in society and suggests that World Bank estimates of genuine saving are an over-estimate. Section VIII qualifies the results and section IX confronts them with empirical evidence. Section X concludes.

II. Competing Factions, Resource Depletion and Capital Accumulation

We set up a model of a closed economy with a common-pool exhaustible natural resource. There are various groups in society who invest in private capital and there is no population growth. Each group also depletes the stock of common resources and uses the resource together with private capital (and possibly labour and other factor inputs in fixed supply) to produce output according to a Cobb-Douglas production function. To focus on the interactions between private asset accumulation and depletion of a common resource, we abstract from trade between the various groups in society. We also abstract from natural resource exports, imports of produced goods, and investment in foreign assets.

There are thus N rival groups who struggle for power over the control of natural resources. Each group i grabs a share σ_i of natural resource reserves S. Depletion of the common stock of natural reserves is thus given by

(1) $$\dot{S} = -\sum_{j=1}^{N} R_j, \quad S(0) = S_0 \quad \text{or} \quad \int_0^\infty \sum_{j=1}^{N} R_j(s)\,\mathrm{d}s = S_0,$$

where R_j denotes the depletion rate of group j in society. The natural resource stock is a *common* stock, since it is depleted by all groups in society. This captures the idea that property rights on natural resources do not exist or are badly defined.

Each group i also accumulates private assets K_i. Since we abstract from adjustment costs, taxes, etc., the relative price of financial assets is unity and the value of private assets exactly equals the capital stock. The capital stock of each group can be viewed as physical capital or human capital. Each group i employs capital, natural resources R_i and labor L_i to produce output Y_i. The production function for each group Y_i = F(K_i, L_i, R_i) satisfies the Inada conditions and displays constant returns to scale. We assume that natural resources are *necessary* for production, so F(K_i, L_i, 0) = 0. We also assume that natural resources are *inessential* for production to avoid that feasible consumption vanishes as natural resources run out. If there are sufficient substitution possibilities between resources and capital or labor, it is possible to generate positive levels of output by switching from resource-intensive to capital-intensive modes of production. With a CES production function, natural resources are neither necessary nor essential if the elasticity of substitution between factors of production exceeds unity. If the elasticity of substitution is less than unity, capital accumulation cannot compensate for the unavoidable decline in the use of natural resources and output and consumption must inevitably decline to zero. The economy is then doomed, so that natural resources are essential for production. We therefore assume that each group has a Cobb-Douglas production with a unit elasticity of factor substitution and a share of capital in value added greater than that of natural resources, i.e., $Y_i = K_i^\alpha R_i^\beta L_i^{1-\alpha-\beta}, \alpha > \beta > 0, \alpha + \beta < 1$. Natural resources are thus necessary, but not essential for production.[3] We assume that there is no depreciation of capital. We assume that total labor supply in the country is one and that each group is the same. If consumption by group i is denoted by C_i, the evolution of private wealth of group i is given by

(2) $$\dot{K}_i = Y_i - C_i, \quad \text{where} \quad Y_i = K_i^\alpha R_i^\beta L_i^{1-\alpha-\beta} \text{ and } L_i = 1/N,$$

[3] If $\alpha < \beta$, capital does not add enough to production to compensate for the declining use of natural resources and sustain a positive level of private consumption. Resources are then essential for production.

where labor supply is exogenous and equal to $1/N$. We abstract from extraction costs for natural resources. Rather than assuming an open-loop Nash equilibrium outcome where each rival group i when deciding on its depletion level R_i supposes that the depletion levels of the other factions $R_j,\ j \neq i,$ remain constant[4], we are interested in a feedback Nash equilibrium outcome in which account is taken of the possibility that each of the rival factions will deplete more if the common stock of remaining natural reserves is higher and the private stock of capital lower. Suppose therefore that the depletion of the common resource by each group is conjectured to be proportional to the remaining stock of resources divided by the accumulated wealth of the group:

(3) $$R_j = \sigma_j S / K_j, \quad j \neq i,$$

where σ_j will be referred to as the *depletion coefficient*.[5] Each rival group i thus assumes that the depletion levels of the other groups are given by (3) and takes the coefficients $\sigma_j,\ j \neq i,$ as constant when determining its optimal R_i. If ρ indicates the pure rate of time preference employed by each group, each group i chooses C_i and R_i to maximize its utility

(4) $$U_i = \int_0^{\infty} \mathrm{u}(C_i) \exp(-\rho t) \mathrm{dt}, \quad \mathrm{u}' > 0, \mathrm{u}'' \leq 0,$$

subject to the evolution of the common stock (1), the evolution of the private stock (2) and the conjectures (3) where the depletion coefficients of the other groups in society, σ_j , $j \neq i$, are supposed to be constant.

III. Optimality Conditions for the Feedback Nash Equilibrium Outcome

We will derive for this differential game a feedback Nash equilibrium solution. We will establish that the conjectures (3) will in fact be consistent with this equilibrium solution. The resulting

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[4] It is straightforward to demonstrate that the open-loop Nash equilibrium outcome yields the efficient solution which would prevail in a homogenous society without rival factions.

[5] It is possible to derive the results for the case $R_j = \sigma_j^* S$, $j \neq i$. In fact, it is easy to establish that this yields exactly the same outcome as we derive in sections III-IV when the transformation $\sigma_i^* = \sigma_i / K_i$, $\forall i$ is used. The reason is that the private capital stocks do not affect the common-stock externality.

solution will be summarized in Proposition 1 and characterized in Proposition 2. The Hamiltonian for group i is defined by

(5) $$H_i \equiv \mathrm{u}(C_i) + \lambda_i \left(K_i^{\alpha} R_i^{\beta} \left(\frac{1}{N} \right)^{1-\alpha-\beta} - C_i \right) - \mu_i \left(R_i + S \sum_{j \neq i} \frac{\sigma_j}{K_j} \right),$$

where λ_i and μ_i denote the marginal utility for group i of an extra unit of private capital and the common stock of natural resources, respectively. Application of Pontryagin's maximum principle yields the following first-order conditions for each of the groups:

(6) $$\frac{\partial H_i}{\partial C_i} = \mathrm{u}'(C_i) - \lambda_i = 0, \quad \frac{\partial H_i}{\partial R_i} = \beta \frac{Y_i}{R_i} \lambda_i - \mu_i = 0, \quad \rho\lambda_i - \dot{\lambda}_i = \frac{\partial H_i}{\partial K_i} = \alpha \frac{Y_i}{K_i} \lambda_i \equiv r_i \lambda_i$$
$$\text{and } \rho\mu_i - \dot{\mu}_i = \frac{\partial H_i}{\partial S} = -\left(\sum_{j \neq i} \frac{\sigma_j}{K_j} \right) \mu_i, \quad i = 1, \ldots, N.$$

The following transversality conditions should also be satisfied:

(7) $$\lim_{\mathrm{t}\to\infty} \left[\exp(-\rho \mathrm{t}) \lambda_i(\mathrm{t}) K_i(\mathrm{t}) \right] = 0 \ \text{ and } \ \lim_{\mathrm{t}\to\infty} \left[\exp(-\rho \mathrm{t}) \mu_i(\mathrm{t}) S(\mathrm{t}) \right] = 0, \quad i = 1, \ldots, N.$$

Equation (6) implies that the marginal product of natural resources should equal the price of natural resources, $p_i \equiv \mu_i / \lambda_i$, and that the price of natural resources should rise at the rate of interest r_i plus an extra term proportional to the sum of the depletion rates of all the rival groups. Furthermore, the marginal product of capital should equal the rate of return on capital for each group. Since in symmetric equilibrium the interest rates and natural resource prices are the same for each group, we can drop group subscripts (i.e., $r = r_i$ and $p = p_i$, i=1,…,N) and write these efficiency conditions as

(8) $$\frac{\dot{p}}{p} = r + (N-1)\frac{\sigma}{K} \quad \text{where} \quad p = \beta \frac{Y_i}{R_i}, r = \alpha \frac{Y_i}{K_i}, i = 1, \ldots, N, K \equiv \sum_{i=1}^{N} K_i \ \text{ and } \ \sigma \equiv \sum_{i=1}^{N} \sigma_i .$$

Equation (8) is the political variant of the Hotelling rule. If there is no fractionalization of society, N = 1 and (8) reduces to the familiar Hotelling rule which states that the expected rate of increase in natural resources should exactly equal the market rate of interest. This follows from the following arbitrage condition. On the margin society should be indifferent between keeping natural resources under the ground and receiving a capital gain $\dot{p}/p$, and digging the resources up, selling them and receiving a rate of return *r*. Rival groups in society, however, drive a wedge in the Hotelling rule. Effectively, each group depletes the stock of natural resources faster as it expects other groups to deplete this stock if it postpones depletion. As a result, the political Hotelling rule implies a bigger rate of increase in the price of natural resources than is socially optimal. This distortion appears to be smaller if the groups have accumulated a lot of non-resource wealth, but in the feedback Nash equilibrium solution with constant levels of consumption and output (derived in section IV) the rate of interest also falls as the capital stock rises over time. Equation (8) thus indicates that the rate of change of natural resource prices is inversely related to the capital stock. It also exceeds the rate of interest in a fractionalized society.

The first-order conditions (6) also imply the familiar Keynes-Ramsey rule for growth in private consumption:

(9) $$\frac{\dot{C}_i}{C_i} = \theta_i \left(r_i - \rho \right), \text{ where } \theta_i \equiv - u'(C_i)/C_i u''(C_i) \geq 0.$$

IV. Sustaining Constant Levels of Private Consumption in the Feedback Nash Equilibrium Outcome

Following the literature on the Hartwick rule, we are interested in maxi-min egalitarian outcomes with zero elasticities of intertemporal substitution (i.e., $\theta_i = 0$). We are thus looking for dynamic general equilibrium paths with constant levels of private consumption, $C_i(t) = C/N > 0$, $\forall t \geq 0$ with C > 0 a constant. To obtain a feedback Nash equilibrium solution with constant levels of consumption and output, we suppose a constant savings propensity *s* and thus hypothesize the following feasible program:

(10) $$K_i(t) = (s\, Y\, t + K_0)/N > 0, \ \forall t \geq 0,$$

where K_0/N is the initial private stock of assets of each group and the output level $Y > 0$ is a positive constant. We will now verify that such a program (10) indeed satisfies the optimality conditions of the non-cooperative Nash equilibrium (6)-(7). Since investment is constant in such a program, output of each faction Y_i(t) = (sY + C)/N and aggregate output $Y = sY + C$ are constant as well. Aggregating and making use of (2) and (3) yields aggregate use of natural resources:

$$R(\mathrm{t}) \equiv \sum_{j=1}^{N} R_j(\mathrm{t}) = \frac{N\sigma S(t)}{K(t)} = Y^{1/\beta}\left(sY\mathrm{t} + K_0\right)^{-\alpha/\beta}. \tag{11}$$

Substituting R(t) from (11) into the depletion equation (1) and integrating, we obtain the trajectory for the remaining stock of natural resources

$$S(\mathrm{t}) = \left(\frac{K_0}{K_0 + sYt}\right)^{(\alpha-\beta)/\beta} S_0 \to 0 \quad \text{as} \quad \mathrm{t} \to \infty. \tag{12}$$

The stock of natural resources is thus asymptotically fully depleted. We see that the transformation of the exhaustible natural resource stock into a reproducible stock of capital manages to keep the level of production and thus of consumption constant. Let us now see whether this hypothesized feasible program with an appropriate choice of Y, s and σ indeed corresponds to a feedback Nash equilibrium with constant levels of consumption and output.

Defining $Z \equiv K/Y$ and using the production function in differentiated form, we obtain

$$\dot{Z} = \frac{K}{Y}\left(\frac{\dot{K}}{K} - \frac{\dot{Y}}{Y}\right) = \frac{K}{Y}\left[\frac{(1-\alpha-\beta)\frac{\dot{K}}{K} + \beta\left(\frac{\dot{Y}}{Y} - \frac{\dot{R}}{R}\right)}{1-\beta}\right]. \tag{13}$$

With the aid of the modified Hotelling rule (8) and using the static efficiency condition $p = \beta Y/R$, we obtain from (13) the following expression for the aggregate saving propensity:

$$s \equiv \frac{\dot{K}}{Y} = \frac{(1-\alpha-\beta)s + \beta\left[\alpha + (N-1)\sigma / Y\right]}{1-\beta} = \beta + \frac{\beta\sigma}{\alpha Y}(N-1) > \beta \text{ if } N > 1. \tag{14}$$

We also have from (11) that the following relationship must hold at time zero:

(15) $$N\sigma S_0 = Y^{1/\beta} K_0^{-(\alpha-\beta)/\beta}.$$

The natural resource must asymptotically be fully depleted. Hence, using (11), (15) and $K(\mathrm{t})=K_0+sY\mathrm{t}$, we obtain from

$$S_0 = \frac{Y^{1/\beta} K_0^{-(\alpha-\beta)/\beta}}{N\sigma} = \int_0^\infty R(\mathrm{t})\mathrm{dt} = \int_0^\infty Y^{1/\beta} K_0^{-\alpha/\beta}\left(1+\frac{sY\mathrm{t}}{K_0}\right)^{-\alpha/\beta} \mathrm{dt}$$

the following relationship between the natural resource depletion intensity and aggregate saving:

(16) $$N\ \sigma\beta = (\alpha-\beta)sY.$$

Expressions (14)-(16) can be solved for Y, s and σ, so that the resulting solution indeed satisfies the first-order conditions (6) corresponding to the feedback Nash equilibrium solution. Upon substitution of (16) into (14), we obtain the following expressions for the aggregate savings propensity and the natural resource depletion coefficient:

(17) $$s = \left[\frac{\alpha}{\alpha-(\alpha-\beta)(N-1)/N}\right]\beta > \beta \text{ and}$$

(18) $$\frac{\sigma}{Y} = \left[\frac{\alpha}{\alpha+\beta(N-1)}\right](\alpha-\beta) < \alpha-\beta \text{ for } N>1.$$

We note from (17) and (18) that $\partial s/\partial N > 0$ and $\partial(\sigma/Y)/\partial N < 0$. Upon substitution of (17) and (18) into (15), we obtain the solution for aggregate output:

(19) $$Y = \left[\frac{\alpha(\alpha-\beta)S_0 K_0^{\frac{\alpha-\beta}{\beta}}}{\alpha-(\alpha-\beta)(N-1)/N}\right]^{\frac{\beta}{1-\beta}} \equiv \mathrm{Y}\left(\overset{+}{K_0}, \overset{+}{S_0}, \overset{+}{N}\right).$$

National saving thus follows from (17) and (19):

(20) $$sY = \left[\frac{\alpha(\alpha-\beta)^{\beta} S_0{}^{\beta} K_0{}^{\alpha-\beta}}{\alpha-(\alpha-\beta)(N-1)/N} \right]^{\frac{1}{1-\beta}} \beta \equiv \Upsilon\left(\overset{+}{K_0}, \overset{+}{S_0}, \overset{+}{N} \right).$$

Aggregate consumption is subsequently obtained from (19) and (20):

(21) $$C = (1-s)Y = \left(\alpha(\alpha-\beta) S_0 K_0{}^{\frac{\alpha-\beta}{\beta}} \right)^{\frac{\beta}{1-\beta}} \left(\frac{\alpha(1-\beta)-(\alpha-\beta)(N-1)/N}{\left[\alpha-(\alpha-\beta)(N-1)/N\right]^{1/(1-\beta)}} \right)$$
$$\equiv \mathrm{C}\left(\overset{+}{K_0}, \overset{+}{S_0}, \overset{-}{N} \right),$$

where

(22) $$\frac{\partial C}{\partial N} = -\left[\frac{(\alpha-\beta)^2 \beta s^2 C}{\alpha^2 (1-\beta)(1-s)} \right] \left(\frac{N-1}{N^3} \right) < 0 \text{ for } N > 1.$$

Since the inequality $0 \le \frac{N-1}{N} < 1 < \frac{\alpha(1-\beta)}{\alpha-\beta}$ holds, we have a meaningful solution with positive levels of aggregate consumption, output and saving/investment provided that capital is more important in production than natural resources (i.e., $\alpha > \beta$). If $\alpha < \beta$, output cannot be sustained at a constant level with a finite stock of natural resources even if all of output is saved. Consequently, if $\alpha < \beta$, private consumption eventually vanishes.[6] We thus assume $\alpha > \beta$. In that case, the levels of aggregate consumption and output that can be sustained are obviously larger if the initial stock of private assets and common stock of natural reserves are higher.

Finally, using $R(0) = Y^{1/\beta} K_0{}^{-\alpha/\beta}$, $p = \beta Y / R$ and the expression for aggregate output (19), the modified Hotelling rule (8) and the initial natural resource price can be written as

(23) $$\frac{\dot{p}(\mathrm{t})}{p(\mathrm{t})} = \left(\frac{\alpha}{\beta} \right) \left(\frac{sY}{K_0 + sY\mathrm{t}} \right) \text{ with } p(0) = \beta \left[\frac{\alpha+\beta(N-1)}{\alpha(\alpha-\beta)} \right] \left(\frac{K_0}{S_0} \right).$$

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[6] Natural resources are also essential if physical capital depreciates in a radioactive manner, but not if depreciation is linear or proportional to output.

The initial natural resource price is thus low if the initial stock of natural resource reserves is high and the initial capital stock is low. Over time, natural resource prices increase. This induces continuous factor substitution, so that gradually the capital stock grows and the use of natural resources declines. Furthermore, we see from (23) that both the initial natural resource price and its rate of increase are higher in a more fractionalized society. Over time, the rate of increase in natural resource prices decreases more in a less homogenous society. Initial resource depletion is higher in a less homogenous society.

Proposition 1: *The feedback Nash equilibrium sustains a constant level of consumption, ensures that the price of natural resources satisfies the modified Hotelling rule (8), is characterized by (11), (12) and (17)-(23), and satisfies the transversality conditions (7).*

Proof: By construction our solution satisfies the first-order conditions (6). To check that the transversality conditions (7) are satisfied, we note that along our derived solution trajectory

$$\frac{\dot{K}_i}{K_i}+\left(\frac{\dot{\lambda}_i}{\lambda_i}-\rho\right)=\frac{sY}{NK_i}-\frac{\alpha Y}{NK_i}=\frac{(s-\alpha)Y}{K}\to 0 \text{ and } \frac{\dot{S}}{S}+\left(\frac{\dot{\mu}_i}{\mu_i}-\rho\right)=-\frac{\sigma}{K}\to 0 \text{ as } t\to\infty$$

as σ and Y are constant and the aggregate capital stock K increases linearly at the rate sY towards infinity. It follows that (11), (12) and (17)-(23) corresponds to a feedback Nash equilibrium. □

Armed with the explicit solutions (17)-(23) and (11)-(12), we can characterize the non-cooperative equilibrium outcome more precisely. Before we do that, it is useful to discuss the benchmark Hotelling and Hartwick rules that prevail in a homogenous society with N=1. These would also be the outcomes that prevail under a social planner (see Solow (1974)).

V. Case: Homogenous Society

Consider a homogenous society without any rival factions. In that case, N = 1 and (17)-(21) yield

$$\text{(24)}\quad s=\beta,\quad \sigma/Y=\alpha-\beta,\quad Y=\left[(\alpha-\beta)S_0K_0^{\frac{\alpha-\beta}{\beta}}\right]^{\frac{\beta}{1-\beta}} \text{ and } C=(1-\beta)\left((\alpha-\beta)S_0K_0^{\frac{\alpha-\beta}{\beta}}\right)^{\frac{\beta}{1-\beta}}.$$

The saving rate of a homogenous society s thus equals the share of natural resources in value added β. In other words, the value of depleted natural resources is fully saved and invested (i.e., $pR = \beta Y = sY$). This is the celebrated Hartwick rule. The genuine saving rate of the economy without rival factions is given by

$$(25) \qquad s_G(\mathrm{t}) \equiv \frac{\dot{K}(\mathrm{t}) + p(\mathrm{t})\dot{S}(\mathrm{t})}{Y(\mathrm{t})} = \frac{\beta Y(\mathrm{t}) - p(\mathrm{t})R(\mathrm{t})}{Y(\mathrm{t})} = 0.$$

The Hartwick rule thus requires that the depletion of natural wealth is exactly compensated by accumulation of physical capital, hence genuine saving must be zero. By transforming exhaustible natural resources into productive capital, it is possible to sustain constant levels of private consumption, output and investment. Investment in capital is positive and compensates exactly for the loss in natural wealth. The value of natural resources extracted at each point of time pR does not change over time, since the depletion level of resources falls at exactly the same rate as the price of resources appreciates. This rate is, of course, the market interest rate in a homogenous society, which declines over time and vanishes asymptotically ($\dot{p}/p = -\dot{R}/R = r$). The fraction appropriated from the stock of natural resources (i.e., $R/S = \sigma/K$) also vanishes asymptotically.

VI. Under-Consumption and Too Rapid Depletion in a Fractionalized Society

Table 1 summarizes our comparative statics results for a fractionalized society based on the solution (17)-(23). A fractionalized society saves more than the natural resource rents, so that the saving rate exceeds β. As the number of rival factions increases, the saving rate rises gradually from β towards α. The constant level of output is readily seen to be higher in more fractionalized societies. Nevertheless, consumption is less when there are rival factions as (22) indicates that $\partial C/\partial N < 0$ if $N > 1$. The inefficient allocation in this economy arises from the lack of effective property rights for natural resources. Each group thus consumes less than they would do in the absence of the voracity effect. Effectively, fractionalization boosts saving and investment more than output so that each group ends up consuming less. Rapacious rent seeking thus hurts consumption by the members of each group and harms social welfare.

Total initial wealth consists of financial capital, human wealth (i.e., the net present value of the return on the fixed factor) and natural resource wealth. Natural resource wealth is the value of selling all reserves at once (i.e., $p(0)S_0$), which must equal the present value of present and future resource rents ($\int_0^\infty p(\mathrm{t})R(\mathrm{t})\exp[-\int_0^{\mathrm{t}} r(\mathrm{v})\mathrm{dv}]\mathrm{dt} = \beta K_0/(\alpha - s)$). Human wealth is proportional to natural resource wealth and equals $(1-\alpha-\beta)K_0/(\alpha-s)$. Total initial wealth can thus be written as

$$(26) \qquad K_0 + \left(\frac{1-\alpha-\beta}{\alpha-s}\right)K_0 + \left(\frac{\beta}{\alpha-s}\right)K_0 = \left(\frac{1-s}{\alpha-s}\right)K_0 = \left[\frac{\alpha-\beta+\beta(1-\alpha)N}{\beta(\alpha-\beta)}\right]K_0.$$

Interestingly; initial wealth is independent of the initial stock of natural resource reserves. Initial wealth corresponds to the net present value of the stream of present and future consumption. Total initial wealth increases in the number of rival factions *N*. Still, we know from (22) that consumption decreases in the number of rival factions. The reason is that, even though the interest rate is initially higher, it falls more rapidly in a fractionalized society and eventually becomes less than in a homogenous society. Consequently, the present value of the lower level of the stream of constant consumption levels is higher which matches the higher value of initial wealth in a fractionalized society. Finally, despite natural resource reserves being depleted all the time, natural resource wealth, human wealth, financial wealth and thus total wealth increase throughout as the capital stock rises and the interest rate falls as time proceeds.

Figure 1 shows some illustrative simulations for a homogenous and two fractionalized societies.[7] We immediately see that in a more fractionalized society, consumption is lower and output higher. With two groups, output is 5% higher and the saving propensity increases from 0.1 to 0.16. With five rival factions, output is 11% higher and the savings propensity is 0.25. Consequently, capital rises much faster in the fractionalized societies. Resource use is initially higher and then falls more rapidly in fractionalized societies. Fractionalization depresses sustainable consumption. Figure 2 shows that, even though the interest rate tapers off more rapidly, the price of natural resource rises to a much higher level in fractionalized societies. This is a direct result of the intertemporal distortion in the Hotelling rule.

We summarize the above comparative statics results in the following proposition.

[7] We have chosen α=0.4, β=0.1, K_0=1 and S_0=10/3, so that Y=1 in case N=1.

Proposition 2: *In the absence of well-defined property rights for natural resources and rival groups in society, the rate of resource depletion and appreciation of the price of natural resources are faster than suggested by the Hotelling rule. Consequently, the substitution of natural resources for capital occurs too rapidly and thus the savings rate of the country is too high. Output is too high, but consumption too low. These political distortions are bigger if the country is more fractionalized.*

VII. Genuine Saving is Zero Despite Competing Factions[8]

The economy with competing factions has an imperfect mechanism for resource allocation and thus yields an inefficient allocation with too rapid extraction and too low levels of consumption from a social point of view. It is interesting to apply the theoretical framework for national accounting in economies with imperfect allocation mechanisms developed by Dasgupta and Mäler (2000), Dasgupta (2001b) and Arrow, Dasgupta and Mäler (2003) to our economy. They show that the sign of the genuine saving indicator in a model with two capital goods (not like the present model) depends on the accounting price of the natural resource in terms of capital. This accounting price equals the relative effect of a marginal increase in the initial stock of natural resources on the social objective function divided by the relative effect of a marginal increase in the initial capital stock on the social objective function.

In our model all groups in society have a maxi-min objective function. Since we know that the intertemporal preferences of all groups are aligned, the social objective function will be maxi-min as well. Equation (21) gives an expression for sustainable consumption C(K_0,S_0,N), which gives an indication of social welfare. Since only the relative price matters, the numeraire for the social welfare indicator does not matter. The appropriately corrected accounting price of natural resources, $p_G(0)$, to be used in the genuine saving indicator is thus given by

$$p_G(0) \equiv \frac{\partial \mathrm{C}(K_0, S_0, N) / \partial S_0}{\partial \mathrm{C}(K_0, S_0, N) / \partial K_0} = \left(\frac{\beta}{\alpha - \beta} \right) \frac{K_0}{S_0}. \tag{27}$$

We define the genuine saving indicator by $s_G(0) \equiv [\dot{K}(0) + p_G(0)\dot{S}(0)] / Y(0)$ and prove that it is zero even when there are rival factions.

[8] This section has benefited enormously from the very helpful suggestions of one of the referees.

Proposition 3: *Genuine saving is also zero in a fractionalized society.*

Proof: We can use expressions (1) and (19) to write the expression for genuine saving as $s_G(0) = [s\Upsilon(K_0, S_0, N) - p_G(0)R(0)]/Y(0).$ It follows from expressions (17) and (18) that $\sigma/Y = (\alpha-\beta)s/\beta$ and thus (11) gives $R(0) = N\sigma\left(\frac{S_0}{K_0}\right) = \left(\frac{\alpha-\beta}{\beta}\right)s\left(\frac{S_0}{K_0}\right)\Upsilon(K_0, S_0, N).$ It follows from substituting this expression and (27) into the definition of genuine saving that

$$s_G(0) = \left[1 - \left(\frac{\beta}{\alpha-\beta}\right)\frac{K_0}{S_0}\left(\frac{\alpha-\beta}{\beta}\right)\left(\frac{S_0}{K_0}\right)\right]s = 0. \qquad \square$$

It is interesting to note that the *accounting* price $p_G(0)$ as function of the relative stock of physical capital to natural resources for a *fractionalized* society is exactly the same as the *market* price of natural resource in a *homogenous* society – see expression (23) for $p(0)$ with $N = 1$. This reflects that the trajectory of physical capital and natural resource in (K,S)-space are exactly the same in the homogenous and fractionalized societies. This is why genuine saving is zero and not negative and why development in this economy with competing factions and a common natural resource is sustainable. The problem from a social perspective is that movement along this trajectory is too fast in a fractionalized society, thus leading to an inefficiently low constant level of consumption.

The World Bank (2006) calculates, however, its empirical estimate of the genuine saving indicator with the *actual* market price, but as Arrow, Dasgupta and Mäler (2003) point out relying on market observables to infer social welfare can be misleading in imperfect economies. In our model the World Bank approach corresponds to using the market price $p(0)$ with $N > 1$ instead of the accounting price $p_G(0)$ (i.e., $p(0)$ with $N = 1$). Expression (23) indicates that the correct accounting price $p_G(0)$ that should be used for the calculation of genuine saving is lower than the market price, especially if there are many competing factions.[9] Hence, the World Bank indicator of genuine saving would in our framework show up as *positive* for a fractionalized society:

(25′) $$s_G^{WB} = \frac{sY - pR}{Y} = s - \beta = \beta s\left(\frac{\alpha-\beta}{\alpha}\right)\left(\frac{N-1}{N}\right) > 0 \text{ if } N > 1.$$

[9] With $\alpha = 0.4$, $\beta = 0.1$ (0.3) and $N = 5$, the accounting price should be a half (quarter) of the market price.

Since the properly calculated measure of genuine saving should be zero, the World Bank estimate is thus likely to over-estimate genuine saving for countries with many rival factions.

VIII. Comment

A strong government could correct for the political distortions by levying a tax on natural resource use by each group and rebating the revenues in a lump-sum fashion. This would correct for the intertemporal distortion in the Hotelling rule and return the economy back to the Hartwick rule. Such a policy slows down the rate of natural resource depletion and returns private consumption back to its socially optimal level. However, a resource-rich country with strife about natural resources is unlikely to have a strong government.

In the closed economy model of capital accumulation and natural resource depletion capital grows ad infinitum while the rate of interest and the depletion rate decline to zero. If positive total factor productivity growth is introduced, there may be a steady state with a positive interest rate and a positive depletion rate as discussed in Dasgupta and Heal (1974). It can be shown that the qualitative insights of the dynamic common-pool problem and the political variants of the Hotelling and Hartwick rules are not affected.

Like most discussions of the Hartwick rule, we have adopted a max-min egalitarian perspective and used a zero elasticity of intertemporal substitution. If groups adopt a positive elasticity of intertemporal substitution ($\theta_i > 0$), levels of private consumption will not be constant. If consumption is initially held for some time below its max-min level, capital is accumulated at a sufficiently fast pace to ensure that later generations enjoy ever-increasing levels of consumption. While resource use declines to zero, unlimited growth in consumption and output is feasible. The Keynes-Ramsey rule (9) implies that, as long as the rate of time preference is strictly positive, the capital stock must ultimately go to zero as well to ensure that growth in private consumption is non-negative. It is thus optimal to let consumption, output and capital vanish in the long run even though it is feasible to avoid such a doomsday scenario. Future generations are thus doomed, but from a utilitarian perspective that does not matter as the benefit to early generations exceeds the loss to later generations. Rival groups in a fractionalized society bring forward consumption even more, which is also happens if groups are impatient and find intertemporal substitution easy (large ρ or θ). Obviously, it is hard on ethical grounds to defend the socially optimal outcome for a homogenous society. This is why the max-min egalitarian outcome seems preferable.

In general, in a market economy without externalities constant genuine saving corresponds to constant instantaneous utility and thus constant consumption (Dixit et al. 1980).

More generally, in the model without rival factions discussed in section V, Hamilton and Withagen (2007) demonstrate that prescribing genuine saving as a constant positive fraction of output yields a path with unbounded consumption and higher wealth than the standard Hartwick rule of zero genuine saving and constant consumption. It is interesting to explore how this result would hold up within the context of a fractionalized society.

IX. Empirical Evidence

Our theory explains why fractionalized societies have more rapid extraction of natural resources, more rapid accumulation of productive capital, and lower levels of consumption and welfare than homogenous societies. Some of the obvious inefficiencies resulting from squabbling about natural resources are thus captured. Countries rich in natural resources have indeed poor growth performance even after controlling for the quality of institutions, openness, the investment rate and initial income per capita (e.g., Sachs and Warner, 2000). Our results suggest that one reason for this may be that resource-rich countries are also often highly fractionalized. The rule of thumb based on the Hartwick rule is to save the rents from extracting and selling natural resources and invest them in physical capital, infrastructure or education.

Hamilton and Hartwick (2005), Hamilton, Ruta and Tajibaeva (2005) and the World Bank (2006) present estimates of *genuine* saving. Genuine saving is defined as public and private saving at home and abroad, net of depreciation, *plus* current spending on education to capture changes in intangible human capital *minus* depletion of natural exhaustible and renewable resources *minus* damage of stock pollutants (CO_2 and particulate matter). Dasgupta and Mäler (2000) show within the context of a social planner that genuine saving thus defined corresponds to the increase in wealth of the nation and that realization of the constant maxi-min level of consumption demands *zero* genuine saving.[10] [11] Any depletion of natural resources or damage

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[10] In fact, Dasgupta (2001a) shows that wealth per capita is the correct measure of social welfare if the population growth rate is constant, per capita consumption is independent of population size, production has constant returns to scale, *and* current saving is the present value of future changes in consumption.

[11] The Hartwick rule is related to Hicksian real income. Asheim and Weitzman (2001) and Sefton and Weale (2006) show that the rule ensures no change in the present discounted value of current and future utility and requires use of the Divisia index of real consumption prices. Capital gains represent the capitalization of the future changes in factor prices and thus constitute a transfer

done by stock pollutants must thus be compensated for by increases in non-human and/or human capital. However, our analysis suggests that, if societies are fractionalized as many resource-rich countries are, not the actual market price of natural resources but the market price which would prevail in a homogenous society should be used in the calculation of genuine saving. World Bank figures of genuine saving may thus be an over-estimate of the true indicator of genuine saving.

The scatter diagram and estimated regression line in Figure 3 indicate that countries with a large percentage of mineral and energy rents of GNI typically have *negative* genuine saving rates as calculated by the World Bank.[12] True figures of genuine saving are likely to be even more negative. Many countries become poorer each year despite have abundant natural resources. They squander their natural resources at the expense of future generations without investing sufficiently in other forms of intangible or productive wealth. This may help to explain why oil-rich Venezuela has negative economic growth while Botswana, Ghana and China with positive genuine saving rates enjoy substantial growth. Highly resource-dependent Nigeria and Angola have genuine saving rates of minus 30 percent, thus clearly impoverishing future generations. The oil/gas states of Azerbaijan, Kazakhstan, Uzbekistan, Turkmenistan and the Russian Federation also have negative genuine saving rates. Oil-rich states such as Nigeria or Venezuela, oil/gas-rich Trinidad and Tobago and copper-rich Zambia would have enjoyed an increase in productive capital by a factor four or five if the Hartwick rule would have been followed during the past three decades. Venezuela, Trinidad and Tobago and Gabon could have been as wealthy as South Korea if they would have reinvested their resource rents. All these countries (except Trinidad and Tobago) have suffered declines in per capita income from 1970 to 2000.

To get an idea of whether World Bank indicators of genuine saving are negatively affected by the degree of fractionalization, we offer the scatter plots and drawn regression lines in Figures 4 and 5. They give a weak indication that countries with a share of mineral rents greater than 5 percent have more negative genuine saving rates if they have a high degree of ethnic fractionalization and suffer from internal conflict. Corruption is also associated with negative genuine saving rates in resource-rich countries.

The negative genuine saving rates calculated by the World Bank for resource-rich country are cause for concern, especially as true genuine saving is likely to be even more negative

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from one factor to another. In the closed economy net gains are zero and should not be included in real income.

[12] The following resources are included: bauxite, copper, iron ore, lead, zinc, phosphates, silver, gold, oil, natural gas, brown coal, hard coal, tin, and nickel.

for countries with rivalry among groups for the resource. This suggests that the resource is not only extracted too fast in many countries due to the common-pool problem, but also that each group's reinvestment is smaller than what is needed to sustain the rate of consumption. In fact, for many of the poorest resource-rich economies the situation is even worse as they must cope with high population growth rates. Such countries therefore need *positive* rather than *zero* genuine saving rates to maintain constant consumption per head, since genuine saving may be positive while wealth per capita declines (e.g., World Bank 2006, Table 5.2). Such countries are on a treadmill and need to save more than their exhaustible resource rents, but rarely manage that.

There are two reasons why many resource-rich countries experience negative genuine saving. The first reason is economic. Countries want to save less than their natural resource rents and postpone extraction if they anticipate the world price of resources to rise over time as discussed in Asheim (1986, 1996) and Vincent, Panayotou and Hartwick (1997). However, Hamilton and Bolt (2004) show that the adjustments to allow for future changes in resource prices are relatively small if historical price trends are extrapolated. Resource-rich countries may also expect the cost of natural resource extraction to fall[13] or governments spending to fall in the future. In those cases, it is also socially optimal to have negative genuine saving rates.

The second reason for negative genuine saving is political. Countries with a lot of fighting about natural resources are more likely to suffer from corruption and erosion of the quality of the legal system, which discourages saving and investment in productive capital as in Hodler (1996). Also, countries plagued by infighting about natural resources suffer from shortsighted politicians who are too much concerned about present rather than future generations.

X. Conclusion

A key question is what happens to saving and investment in a country with a badly functioning legal system, an exhaustible common natural resource and rival factions depleting it. If there were no rival factions or property rights to natural resources could be credibly allocated, the country

[13] The US historical experience suggests that under the right circumstances anticipated falls in the cost of extraction and thus the downward effect on the nation's saving may be substantial. The US supremacy as mineral producer was driven by big falls in exploration costs from the mid-nineteenth to mid-twentieth century, collective learning, leading education in mining/engineering/metallurgy, increasing returns, private initiative and an accommodating legal environment; see Habbakuk (1962) and David and Wright (1997).

would transform its exhaustible resource into productive capital by reinvesting all resource rents (the Hartwick rule). This way the country is able to sustain a constant level of consumption and output. The rate of appreciation of the price of natural resources would equal the interest rate (the Hotelling rule), which gradually decreases over time. Although resources are being run all the time, natural resource wealth increases throughout. Matters are very different in a heterogeneous society. Now the country can still sustain a constant level of consumption and output, but these levels are lower than in a homogenous society. The reason is that the common-pool externalities imply that the rate of appreciation of the price of natural resources is higher and thus the substitution of resources for productive capital occurs at a faster rate. As a consequence, a fractionalized country has a higher savings propensity than a homogenous society and extracts natural resources more rapidly. Fewer resources are thus available for consumption (the voracity effect), especially in countries with a large degree of fractionalization and poor legal systems. People really are worse off.

Our theory predicts *zero* genuine saving rates even in fractionalized societies. Genuine saving indicators for many resource-rich countries as calculated by the World Bank are actually *negative*, and the true figures are likely to be even more negative as one should really use the true *accounting* price (i.e., the market price that would prevail in a homogenous society) rather than the *market* price in the calculation of genuine saving. This is a real cause for concern, even more so for countries with high population growth rates which should perhaps be saving more than their resource rents. One reason for this may be anticipation of higher price of natural resources or lower cost of extraction in the future. A more likely reason for negative genuine saving rates is probably the erosion of the legal system due to infighting about natural resources and the resulting depressing effects on saving and investment.

More work is needed on how the Hotelling rule and the Hartwick rule should be modified for practical policy formulation. Natural resource revenues may be siphoned off by the political elite and their cronies and thus not reach the people. Less natural resource rents will thus be saved. Furthermore, natural resource bonanzas may induce exuberant public spending based on the incorrect assumption that windfall natural resource revenues are permanent. This leads to unsustainable spending levels with painful adjustments when resource revenues run out. It is important to study how advice on optimal rates of resource depletion, government spending, saving and investment survives when politicians seek office and grab resource rents for themselves or to pay off political opponents and get away with it due to poor institutions, bad legal systems and poor checks and balances in the political system. Rapacious rent seeking implies that many resource-rich, fractionalized countries with poor legal systems squander their natural resource rents and suffer disastrous economic and social outcomes.

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Figure 1: Fractionalization, macroeconomic outcomes and resource depletion

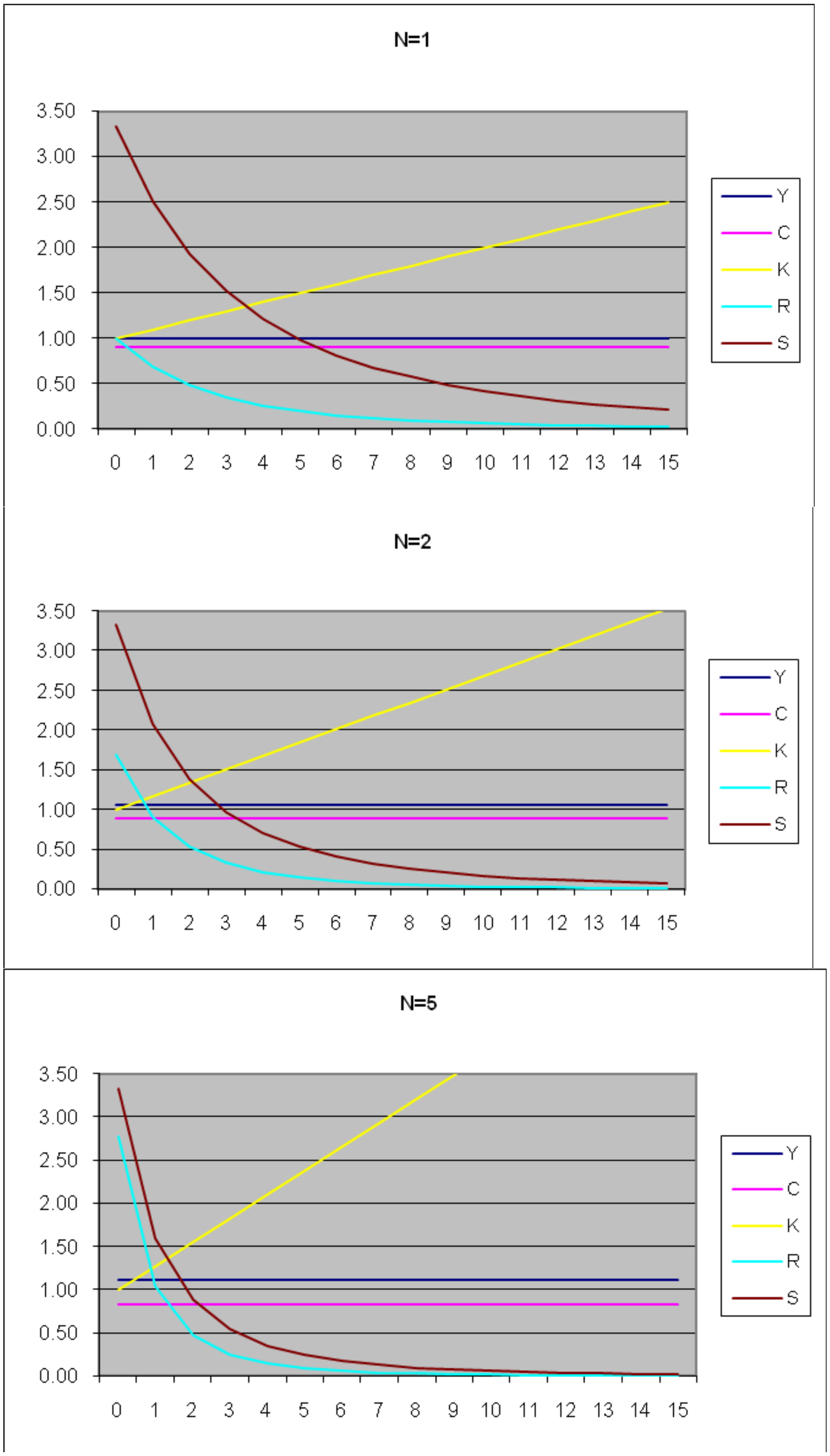


Figure 2: Impact of fractionalization on the interest rate and price of resources

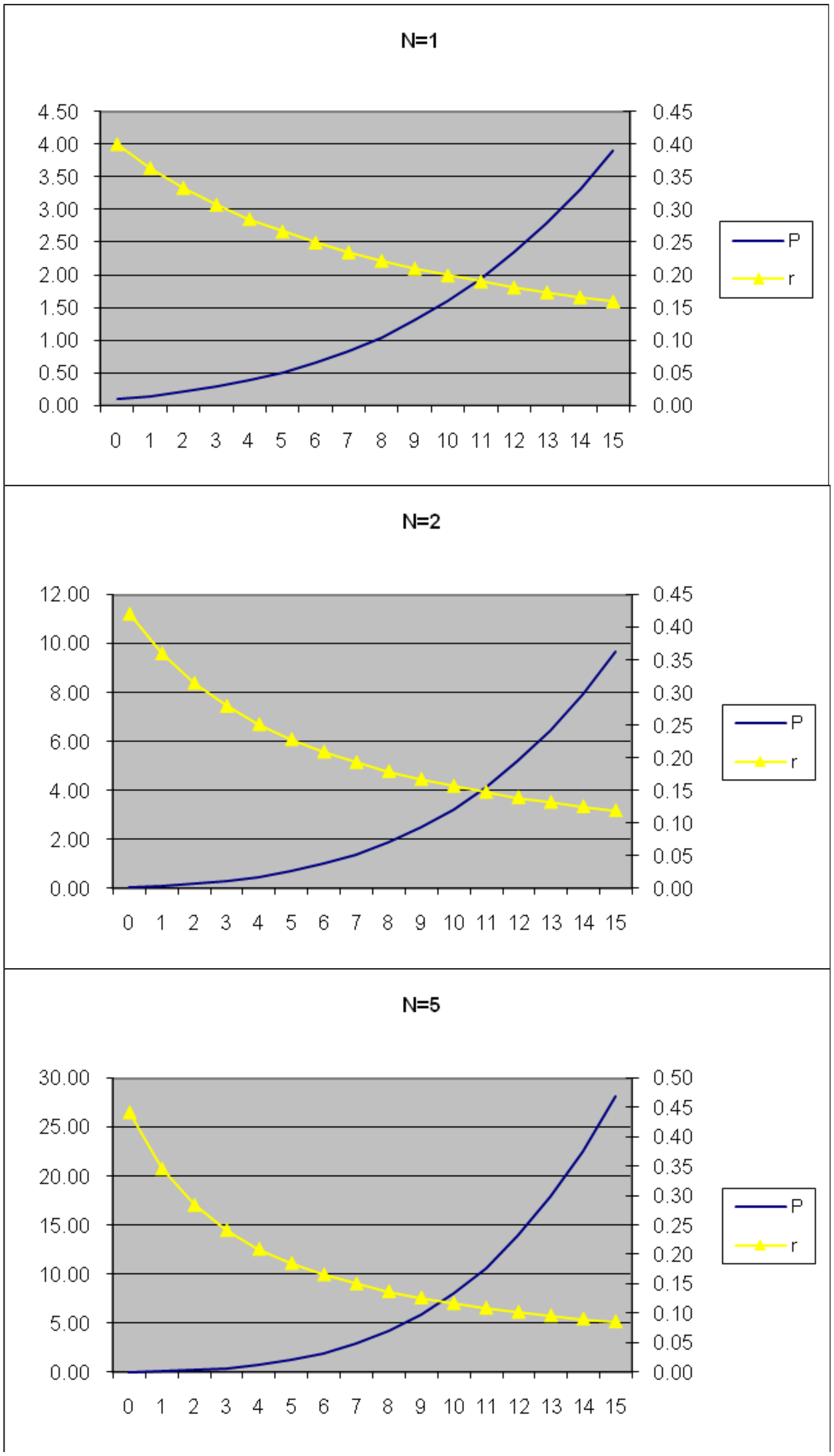
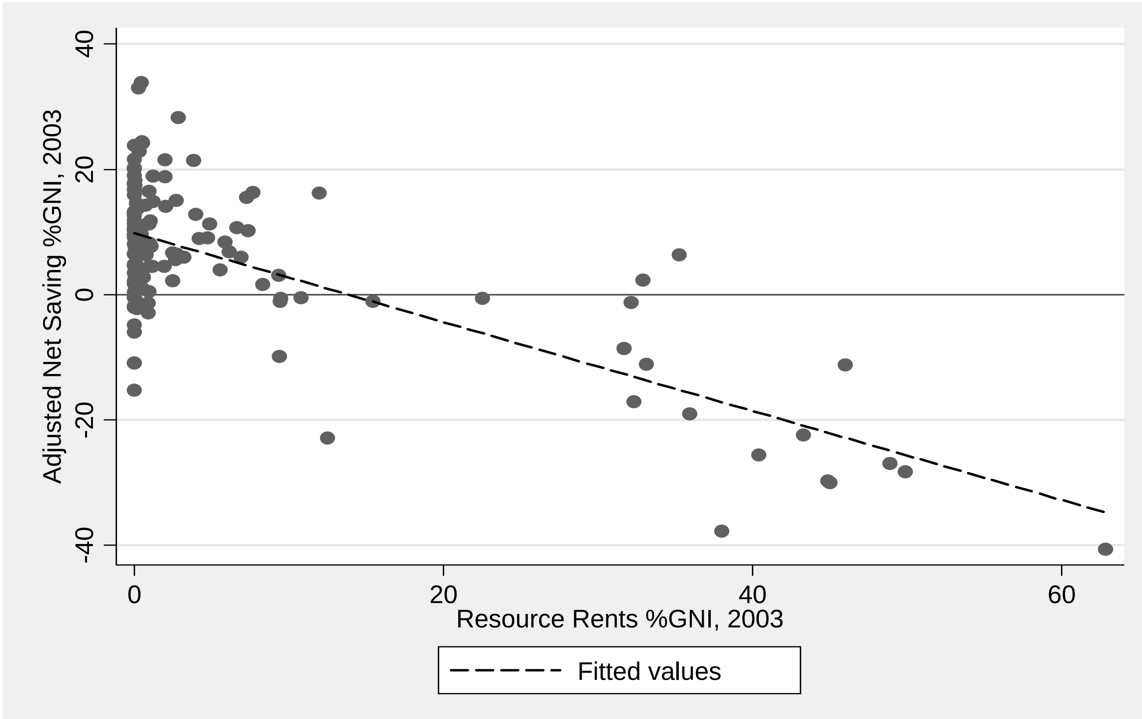
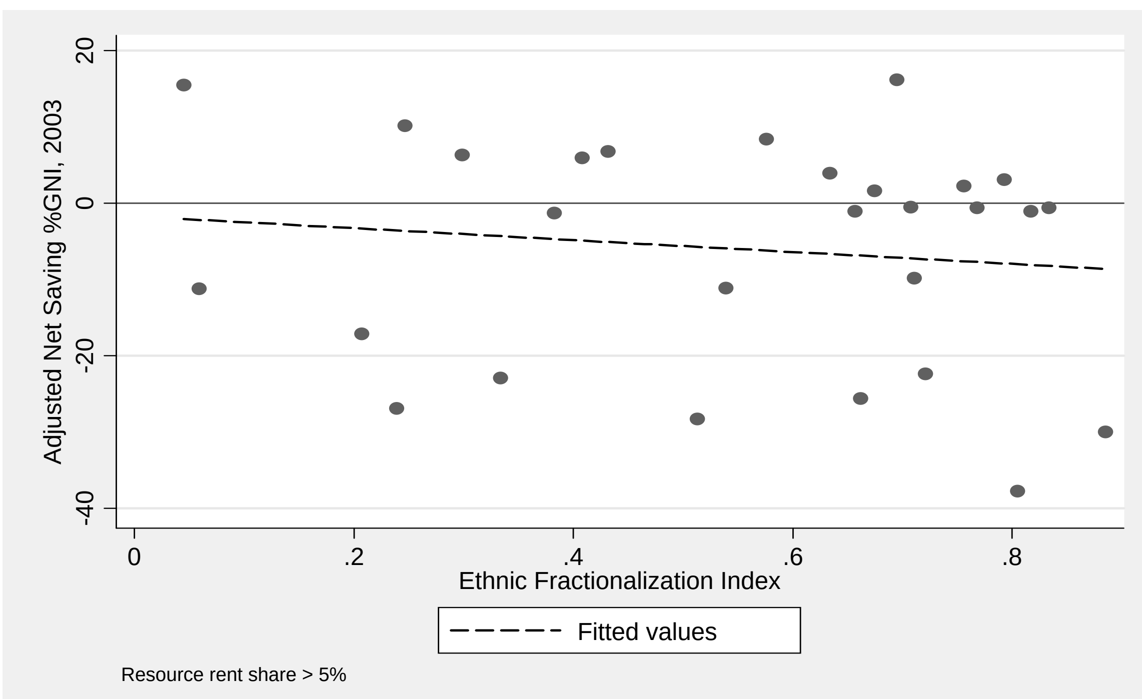


Figure 3: Genuine saving and exhaustible resource share



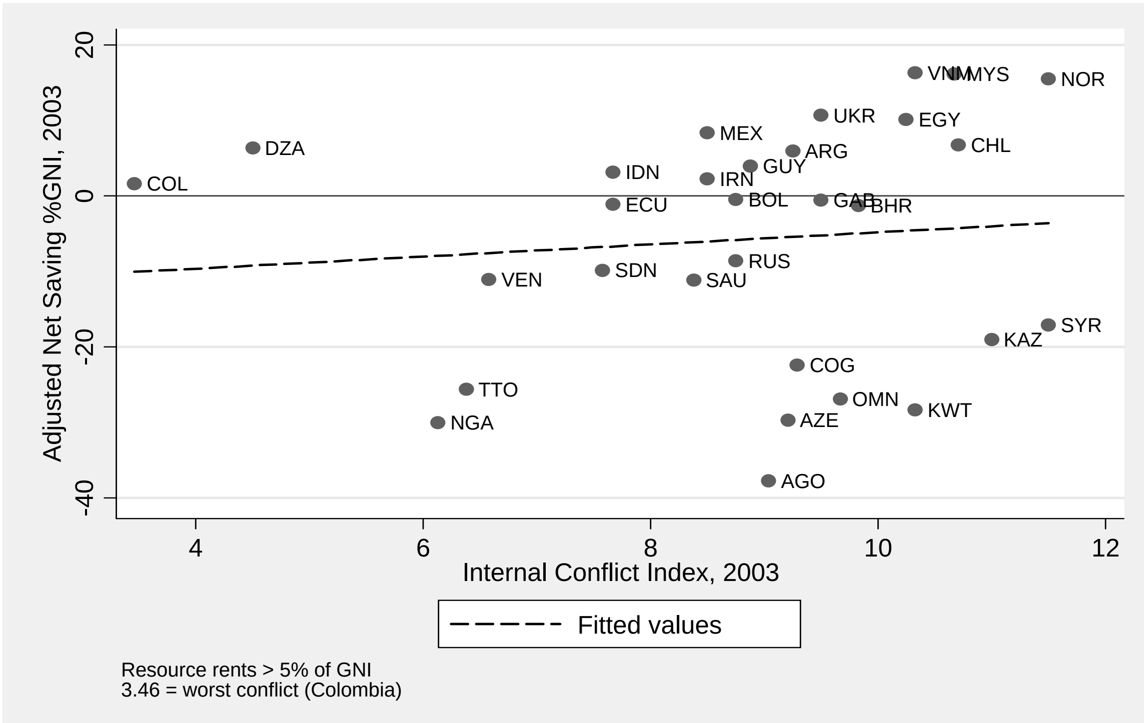
Source: World Bank (2006)

Figure 4: Genuine saving and ethnic fractionalization for resource-rich countries



Source: International Country Risk Guide and World Bank (2006)

Figure 5: Genuine saving and internal conflict for resource-rich countries



Source: International Country Risk Guide and World Bank (2006)

Table 1: Comparative Statics Results

	Y	*sY*	*s*	*C*	*R(0)*	*p(0)*	s_G	s_G^{WB}
K_0	+	+	0	+	–	+	0	0
S_0	+	+	0	+	+	–	0	0
N	+	+	+	–	+	+	0	+