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Optimal policies for resource-rich developing economies

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HARNESSING WINDFALL REVENUES:

Optimal policies for resource-rich developing economies[#]

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Abstract

A windfall of natural resource revenue (or foreign aid) faces government with choices of how to manage public debt, investment, and the distribution of funds for consumption, particularly if the windfall is both anticipated and temporary. Standard policy advice follows the permanent income hypothesis in suggesting a sustained in increase in consumption supported by interest on accumulated foreign assets (a Sovereign Wealth Fund) once resource revenues are exhausted. However, this strategy is not optimal for capital-scarce developing economies. Incremental consumption should be skewed towards present generations, relative to those in the far future. Savings should be directed to accumulation of domestic private and public capital rather than foreign assets. Optimal policy depends on instruments available to government. We study cases where the government can make lump-sum transfers to consumers; where such transfers are impossible so optimal policy involves cutting distortionary taxation in order to raise investment and wages; and where Ricardian consumers can borrow against future revenues so government only has indirect control of consumption.

Keywords: natural resource**,** windfall public revenues, risk premium on foreign debt, public infrastructure, private investment, credit constraints, optimal fiscal policy, debt management, Sovereign Wealth Fund, asset holding subsidy, developing economies.

JEL codes: E60, F34, F35, F43, H21, H63, O11, Q33

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Over the period 2000-05 exports of hydro-carbons and minerals accounted for more than 50% of goods exports in 36 countries. In 18 of these, revenues from natural resources contributed more than half of total fiscal revenue (IMF 2007). These earnings increased dramatically during the commodity boom of 2006-08 and remain at high levels. Sub-Saharan African exports of oil alone are nearly three times larger than its aid receipts, and new discoveries are presenting further countries (e.g. Ghana and Uganda) with the challenge of resource management. Challenges arise at all stages of resource management. The design of fiscal and contractual regimes to secure exploration and development; the capture of resource rents by government; the appropriate saving and spending of resource revenues; and the management of structural change associated with a foreign exchange windfall. Many countries have not handled these challenges effectively, as documented by the literature on the 'resource curse'.[1]

The objective of this paper is to provide a rigorous analysis of one key element of this decision chain. How should a temporary windfall of foreign exchange (from natural resource revenues or foreign aid) be managed by the recipient government? At the broadest level there are three choices. The first concerns the amount of the windfall that should be saved or, equivalently, the optimal time profile of consumption from the windfall. The second concerns the choice of whether to invest in the domestic economy or in foreign assets (by increasing foreign exchange reserves or creating a sovereign wealth fund, SWF). And the third concerns the balance between private or public domestic investment. Answers to these questions are, of course, country specific. What is appropriate for a high income and capital abundant country, such as Norway or Kuwait, is unlikely to be appropriate for a country with low income, widespread poverty and scarcity of capital, such as Ghana or Uganda. To analyse these choices we derive policy rules for a welfare-maximizing government that experiences a temporary windfall of foreign exchange. We build a family of models in increasingly complex economic environments and derive optimal time profiles for consumption, foreign debt/ assets, public investment, and tax and transfer policies. Our focus will be on showing how these policy rules depend on key features of developing countries, and how policy advice to these countries needs to be tailored accordingly.

We concentrate on three features of developing countries that interact importantly with a foreign exchange windfall. The first such feature is capital scarcity, implying a low capital-labour ratio, little public infrastructure, low wages and income, and a high domestic interest rate. We model this by assuming that capital-scarce countries have a low level of initial assets. They can add to domestic capital by borrowing on international capital markets, but may face an interest premium the size of which depends on the level of foreign debt. This premium implies that a foreign exchange windfall is particularly valuable to a developing country. Such countries are on a development path of

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[1] For a survey see van der Ploeg (2010c).

capital accumulation, debt reduction, and rising consumption and we examine how the windfall can be optimally used to alter this path.

The second feature concerns the set of instruments open to government. Developing country governments typically have a small tax base and consequently a high premium on public funds. Since a resource (or aid) windfall accrues largely to government, it relaxes this constraint. Our initial models will abstract from this by allowing lump-sum taxation, but we go on to remove this possibility so that distortionary taxation is the only source of fiscal revenue apart from the windfall. The optimal response to the windfall then involves particular emphasis on growing the non-resource economy by developing public infrastructure and reducing distortionary taxation.

The third feature concerns the behaviour of the private sector of the economy. In many countries households find it hard to borrow against future income so Ricardian debt neutrality is unlikely to hold. To capture this we initially suppose that households live entirely from current wage income and government transfers, having no access to capital markets. The presence of credit-constrained households means that there is a role for government to smooth consumption by varying taxes and transfers. In a final section we remove this assumption, and allow households access to capital markets. However, they may not internalise all the imperfections in the economy, so government policy has to address possible over-consumption from the resource boom.

Our approach contrasts with the existing literature which offers various prescriptions for use of a windfall, of which the best known is the permanent income hypothesis (PIH), familiar from the tax smoothing literature (Barro, 1979) or the optimal use of the current account (e.g. Sachs 1981). This and alternative recommendations are shown fig. 1, in which a flow of windfall revenue, (N, given for illustrative purposes by the step function) is discovered at date $t = T_0 = 0$ and flows at a constant rate until T_1, after which it ceases. Dashed lines give the increment in consumption, ΔC, under alternative prescriptions. The horizontal line is the PIH, giving a constant and permanent increase. This involves saving whilst revenue is flowing and building up a SWF large enough for interest on the fund to maintain the consumption increment in perpetuity. The PIH underlies much of the advice for the setting up of a SWF proffered by the International Monetary Fund (e.g. Davis et al., 2002; Barnett and Ossowski, 2003; Segura, 2006; Leigh and Olters, 2006; Olters, 2007; Basdevant, 2008).[2] A more conservative approach is the 'bird-in-hand' hypothesis (Bjerkholt, 2002; Barnett and Ossowski, 2003), under which all revenue is put in the SWF and incremental consumption is restricted to the interest earned on the fund. This extremely conservative strategy can be interpreted as being equivalent to the PIH, but with the windfall not valued until it has been banked.[3]

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[2] Much previous analysis of optimal resource depletion and associated asset accumulation (e.g. the Hartwick rule, van der Ploeg, 2010a) restricts attention to Rawlsian social welfare function and the issue of sustaining a constant level of consumption in the face of exhaustible resources by investing in foreign or other assets. An related discussion of investing resource revenue in foreign assets is contained in Collier and Gunning (2005).

[3] This is sometimes known as the Norwegian model, although it is only after the Tempo Committee in 1983 and the recommendations for setting up a financial hydrocarbon fund in combination with sophisticated financial
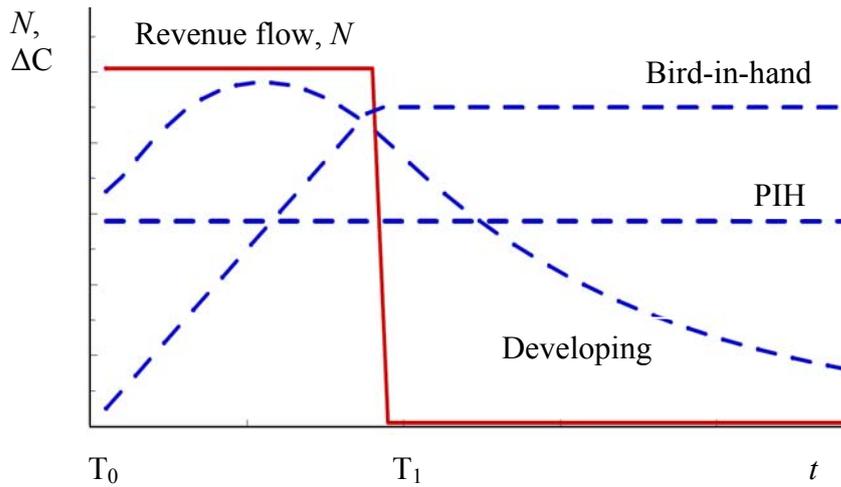
Fig. 1. *Alternative prescriptions; incremental consumption and revenue flow*
The solid line is the assumed flow of revenue, and dashed lines are the consequent increment to consumption under different policy rules.

Both the PIH and the bird-in-hand strategies have the effect of transferring much of the consumption increment to future generations. While this may be appropriate for high income countries, it ignores the features of developing economies in which there is capital scarcity, current incomes are low, there is scarcity of public funds, and there is a potential process of rapid growth and convergence. The curve labelled 'developing' is the optimal consumption increment for such an economy, as derived in a later section of this paper (fig. 6). It has three key features. First, consumption is skewed towards the present, because of the relative poverty of the present generation as compared to those in the far future. Second, capital scarcity requires that some of the revenue is saved, and this is invested in a combination of debt reduction and domestic capital stock. Third, the effect of this investment is to bring forward the development path of the economy. Once revenue flows have ceased consumption converges back to the level that it would have attained on the optimal growth path in the absence of the windfall, but is higher, at all dates, than it would have been without the windfall. Consumption is in any case on a rising path and optimal use of the windfall brings forwards the growth of consumption by accelerating development, rather than accumulating foreign assets to increase the consumption of far future generations.

Remaining sections of the paper set up a family of models in which we derive optimal policy rules including that illustrated in fig. 1. Section 1 sets up the benchmark for a country in which the home interest rate is pegged to the world interest rate and whose citizens are unable to smooth consumption but whose government can do it for them. This yields the PIH in which the windfall is converted to a constant-consumption increment by development and management of a SWF. We then

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instruments by the Steigum Committee in 1988 that the Norwegians implemented their bird-in-hand rule in 2001 (Harding and van der Ploeg, 2009). The rule says that all hydrocarbon revenues go into a Fund, and that

turn to economies which are capital scarce, having an interest rate greater than the world interest rate. Section 2 provides empirical evidence on the interest rates resource-rich developing economies have to pay on their debt, finding support for the hypothesis that highly indebted countries face a higher interest premium. Section 3 analyses the best way to harness a windfall in these circumstances. We initially assume that non-oil income is fixed, there is no physical capital, and the choice is simply between consumption and foreign debt/ asset management. As is suggested by fig.1, full consumption smoothing is no longer optimal. Instead, consumption is more skewed towards current (poorer) generations than is the case with the PIH benchmark. With small windfalls, the economy's consumption path is brought forward, but no SWF is built up. Only if windfalls are large relative to initial debt will it be optimal to use part of the windfall to build an SWF. We look also at cases where the windfall is anticipated ($T_0 > 0$) so there is a lag between discovery and revenue flow.

Section 4 develops a richer model of the non-resource economy and of policy options faced by government, adding private capital, public infrastructure and income taxation. The government now chooses not just how much to save, but also the mix of saving in domestic capital (infrastructure) versus financial debt/ assets. Infrastructure raises output directly, and also raises the return to private capital, so increasing the capital intensity and wage rate in the economy. We look both at cases where lump-sum transfers are possible, and where government has to use distortionary income tax to finance infrastructure. The windfall thus permits government to promote private investment through cutting the tax rate and investing more in public infrastructure. Section 5 further enriches the model to allow domestic consumers access to credit markets. This creates the possibility that, however prudent government may be, Ricardian consumers may over-expand consumption once the windfall is known. Government can respond by an asset holding subsidy to increase domestic saving and a capital holding tax to reduce domestic investment.

To focus on the main public finance issues at hand, we abstract from many important aspects of resource management. We take the size and timing of the windfall as exogenous. We use a single-sector model in which there are no problems in absorbing expenditure, either from appreciation of the real exchange rate and its impact on the traded sector (the Dutch disease, Corden and Neary, 1982; van Wijnbergen, 1984; Sachs and Warner, 1997) or from supply bottlenecks in non-traded goods sectors (van der Ploeg and Venables, 2010). We abstract from political economy concerns. And most critically, we work in an environment of certainty, so that resource revenue volatility and associated precautionary motives are ignored. The final section of the paper discusses the implications of relaxing these assumptions for our results as well as possible future research combining these issues. It also links our approach to the actual savings and spending decisions of countries that have experienced resource booms.

4% of the value of the Fund can be used to fund the general government deficit each year.

1. Benchmark: the permanent income hypothesis

We first consider, as a benchmark case, a small open economy that can borrow or lend unlimited amounts at the world interest rate. This economy has exogenous and constant non-resource output Y. Consumers have no assets and simply receive a lump-sum transfer or citizen dividend T from the government so their consumption is $C = Y + T$. The government is the only agent in the economy that has access to the international capital market, so foreign debt F corresponds to public debt. It chooses transfers T and public consumption G to maximise utility of its citizens,

$$U \equiv \int_0^\infty \left(\frac{C^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1-1/\sigma} \right) \exp(-\rho t)\mathrm{d}t, \text{where } \psi \geq 0 \text{ is the weight given to public consumption, } \sigma \text{ is}$$

the elasticity of intertemporal substitution and the rate of time preference is ρ. Maximisation is subject to the budget constraint $\dot{F} = r*F + G + C - Y - N$, with initial debt F_0 and exogenous world interest rate $r*$, assumed to equal the rate of time preference, $\rho = r*$. N stands for the flow of windfall revenue from the sale of resource or foreign aid, all of which accrues to government.

The conditions for optimal government policy are familiar.[4] The intratemporal efficiency condition requires that public and private consumption are in fixed proportion, $G = \psi^\sigma C$. The intertemporal efficiency condition states that consumption of government and its citizens are smoothed over time, so $\dot{G} = \dot{C} = 0$. The levels of C and G come from the budget constraint, incorporating the value of resource revenues. The present value of the windfall at the date of discovery, $t = 0$, is $V(0) \equiv \int_0^\infty N(t)e^{-r*t}dt$, the permanent income flow from this is $r*V(0)$, so the permanent level of consumption is $C + G = Y + r*(V(0) - F_0)$. The split between the levels of permanent private and public consumption depends on the weight given to public consumption in social welfare, ψ:

$$C = [Y + r*(V(0) - F_0)]/(1 + \psi^\sigma), \qquad G = [Y + r*(V(0) - F_0)]\psi^\sigma /(1 + \psi^\sigma). \tag{1}$$

If the flow of resource revenue is not constant through time, then permanent consumption levels are maintained by changing the level of debt/ assets held according to the flow budget constraint.[5] For an anticipated temporary windfall, there is borrowing (a current account deficit) before revenue flow, $t \in [0, T_0]$. During the period of revenue flow, $t \in [T_0, T_1]$, there is asset accumulation, paying off foreign

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[4] We provide an explicit solution of a more general version of this problem in the next section.
[5] This requires that the non-windfall deficit must equal the return on resource wealth, because $\dot{F} + N = r*V$ ensures that $V - F$ and thus C and G are held constant over time. Notice that $V \equiv V(t)$ is time varying and, with constant interest rate, $V(t) \equiv \int_s^\infty N(s)e^{-r*(s-t)}ds$, so $\dot{V} = N - r*V$.

debt and building a SWF by running a current-account surplus. The foreign assets that are built up at the end of the windfall, $-F(\text{T}_1) = V(0) - F_0$, generate just sufficient interest revenue to finance the permanent rise in total consumption, that is $r*V(0) = \Delta(C+G)$. This policy of borrowing, then saving and finally living on the return on the SWF thus transforms an anticipated temporary windfall revenue into a permanent increase in public and private consumption.[6]

2. Capital scarcity and the interest premium

The benchmark of using debt to smooth consumption may be applicable for countries able to borrow or lend unlimited amounts at a given world interest rate, yet many resource-rich developing economies are capital scarce and have high domestic interest rates. They are unable to remedy this by international borrowing as they are likely to face a high and increasing interest premium on such borrowing. Following Bardhan (1967), Obstfeld (1982) and Turnovsky (1997) and Chatterjee et al. (2003) we capture this with a supply schedule of foreign debt. We assume that for low values of foreign indebtedness the domestic interest rate equals the world interest rate, while for high levels of indebtedness it rises above the world rate. The domestic interest rate, r, is thus given by:

$$r = r* \text{ for } F \leq \overline{F} \text{ and } r = r*+\Pi(F) > r* \text{ for } F > \overline{F} \geq 0, \tag{2}$$

where $\Pi(F)$ is the interest rate premium and $\overline{F}$ the debt threshold below which the country is a price taker at the world rate of interest. For simplicity, we take this threshold to be zero, so $\Pi(0) = 0$ and $\Pi'(F) > 0, F > 0$. One can interpret $\Pi(F)$ as an international premium on foreign debt to capture the risk of default, but we do not model that.[7] Fig. 2 portrays the interest schedule.

Several remarks need to be made about our use of this schedule. First, our schedule differs from that employed elsewhere by going flat at $r = r*$ for $F < 0$. In the macroeconomics literature (e.g. Turnovsky, 1997, section 2.6) it is common to close small open economy models by specifying a supply schedule of foreign debt which slopes upwards for all F.[8] Although this is analytically convenient, it has the unattractive feature of implying a unique steady-state value of F at which the domestic interest rate equals the world rate and to which development converges. This level is

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[6] Clearly, net assets go from negative to positive only if the windfall is large enough relative to initial debt.
[7] An overview of country risk is offered by Eaton, Gersovitz and Stiglitz (1986); a micro-founded explanation of sovereign debt explaining negotiated partial default is given by Bulow and Rogoff (1989). For analysis of sudden stops of debt rollovers and recall of short-term debt in emerging economies, see Calvo,et al. (2004).
[8] Most small open economy models with incomplete asset markets have steady states that depend on initial conditions and furthermore have equilibrium dynamics with a random walk component. To ensure stationarity and a unique steady state one often postulates an upward-sloping supply schedule of foreign debt. Alternatives are to have an endogenous discount rate, convex portfolio adjustment costs or asset markets with a complete menu of state-contingent claims (Schmitt-Grohé and Uribe, 2003), but we do not explore these alternatives as a debt-elastic risk premium seems relevant for developing economies.

independent of windfall revenue, and therefore inconsistent with the permanent income hypothesis under which, as we saw in section 1, countries choose their steady-state value of F by, for example, building a SWF. It is to capture both the interest premium and the endogeneity of the steady-state value of F that we suppose that economies face a premium, $\Pi(F) > 0$, above some threshold level of indebtedness while below that level countries are price takers at r^*.
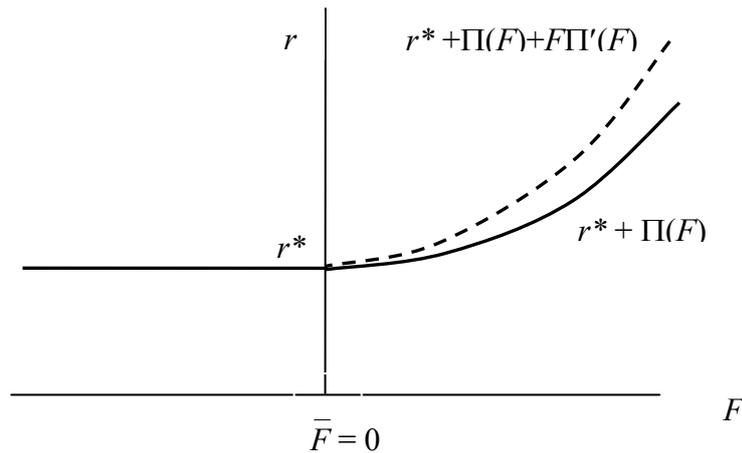
**Fig. 2.** *Debt and the cost of foreign borrowing*

Second, there is extensive empirical support for the existence of a relationship between foreign debt and the domestic interest rate. Fig. 3(a) shows a positive relationship between interest rate spreads and the ratio of public and publicly guaranteed (PPG) debt to GNI. Following Akitobi and Stratmann (2008), we estimate the interest rate spreads (not reporting country fixed effects and time dummies, reporting standard errors in parentheses) and find that interest spreads are significantly higher if the ratio of PPG debt to GNI is high, foreign reserves are low and the probability of default is high:

$$\text{Ln(spreads)} = 1.89^{**} \text{ PPG debt/GNI} - 4.14^* \text{ reserves/GDP} + 0.056 \text{ inflation}$$
$$(0.54) \qquad\qquad (1.72) \qquad\qquad (0.036)$$
$$- 0.0458^{**} \text{ output gap} + 0.296^{**} \text{ Ln(default)} + 0.0000866 \text{ regional spread}$$
$$(0.015) \qquad\qquad (0.096) \qquad\qquad (0.000124)$$

25 countries, 165 observations, within $R^2 = 0.732$, $^* p < 0.05$, $^{**} p < 0.01$.

Compared with Akitobi and Stratmann (2008), we find a stronger effect of debt on interest spreads; they find a coefficient of about unity, we find a coefficient of 1.9.[9] Figure 3(b) gives the conditional effect of PPG debt on interest rate spreads based on this regression.

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[9] For data sources and further discussion see appendix 1

|(a) Unconditional* | (b) Conditional** |
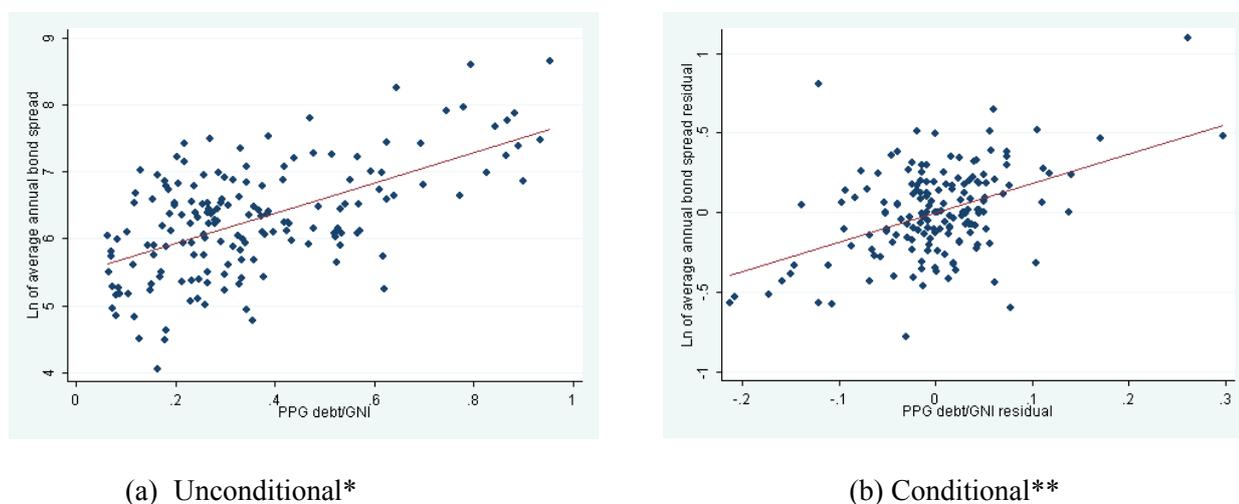
Fig. 3: *Interest rate spreads and public and publicly guaranteed foreign debt*

* The slope coefficient corresponding to the unconditional correlation for the pooled regression with N=165 and 25 countries is 2.270 with standard error 0.250, which is significantly different from zero at the 1% level, and the adjusted R^2 is 0.332.

** The slope coefficient is 1.89, as in the equation reported above.

It is possible that the relationship takes a different form for resource-rich countries, and may indeed by shifted by the presence of windfall revenues. Thus, international credit-worthiness may be directly improved by resource wealth, in addition to effects through the level of foreign debt. There are some instances where resource boom have led to improvements in countries' credit ratings, although our reading of selected credit rating country reports is that resources feature as a weakness (attributed to increased political risk, delayed fiscal reform, lack of diversification) as often as they do as a strength (e.g., witness recent rating improvements for Ghana and Russia).[10] It also possible that future resource revenues can be securitised as a way of gaining improved access to international capital markets. This has been used by some Latin American and former Soviet Union oil-rich countries, although the magnitude of credit obtained this way remains relatively small.[11]

To investigate the direct effects of resources on borrowing costs further we include resource exports as a fraction of GDP as an explanatory variable in our equation for interest spreads. The effects of PPG debt, reserves, the probability of default and the output gap on interest spreads remain significant. The effect of natural resource exports on interest rate spreads is not significant, although positive, indicating that resources *worsen* credit worthiness:

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[10] Drawn from http://www.standardandpoors.com/ratings/articles/en/us/
[11] See Ketkar and Ratha (2009) for discussion.

Ln(spreads) = 1.48* PPG debt/GNI − 3.94* reserves/GDP + 0.036 inflation
　　　　　(0.68)　　　　　　　　(1.74)　　　　　　　　(0.031)
　　　− 0.0443** output gap + 0.316** Ln(default) + 0.000138 regional spread
　　(0.013)　　　　　　　(0.104)　　　　　　　(0.000132)
　　　+ 3.132 resource exports/GDP
　　　(2.303)

25 countries, 165 observations, within R^2 = 0.736, * p < 0.05, ** p < 0.01.

To investigate robustness we tried various other specifications as well as corrections for endogeneity of PPG debt and resource exports. The qualitative nature of our results survives. If anything, our regression results suggest that resource exports have a positive effect on interest spreads, and therefore might, somewhat surprisingly, negatively affect credit worthiness. The mechanism might be through the impacts of resources on governance, political stability, and the risk of conflict (see Collier and Hoeffler, 2004; Fearon, 2005). We use the lack of a statistically significant impact of resources to justify working with the interest schedule of equation (2) in our context of resource-rich economies. However, in section 4.4 we discuss what happens to optimal policy if resource revenues also have a direct effect on the borrowing schedule.

3. Accelerating development in capital-scarce economies

We now derive optimal policy for an economy facing an interest premium, as described above. As in section 1, the government maximises utility of its citizens by choosing private and public consumption, C and G, subject to its budget constraint which now takes the form $\dot{F} = [r*+\Pi(F)]F + G + C - Y - N$, $F(0) = F_0$. The Hamiltonian for this problem is

$$\mathrm{H}(C,G,F,\lambda) \equiv \frac{C^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1-1/\sigma} + \lambda\left[(r*+\Pi(F))F + G + C - Y - N\right]$$

with costate λ for debt. Optimality conditions are:

$$\mathrm{H}_C = C^{-1/\sigma} + \lambda = 0, \quad \mathrm{H}_G = \psi G^{-1/\sigma} + \lambda = 0, \quad \rho\lambda - \dot{\lambda} = \mathrm{H}_F = \lambda\left[r*+\Pi(F) + F\Pi'(F)\right], \quad (3)$$

and the transversality condition $\lim_{t\to\infty}\left[\exp(-r*t)\lambda(t)F(t)\right] = 0$.

As before, intratemporal efficiency requires that G and C are in fixed proportion $G = \psi^{\sigma}C$. This can be used to write the current-account dynamics as

$$\dot{F} = [r*+\Pi(F)]F + (1+\psi^{\sigma})C - Y - N, \quad F(0) = F_0. \tag{4}$$

The intertemporal efficiency condition (the first-order condition for the optimal consumption path) can be derived from equations (3) as,

$$\dot{C} = \sigma C\left[r^* + \Pi(F) + F\Pi'(F) - \rho\right],\tag{5}$$

which is the usual Ramsey rule. In contrast to the PIH, perfect consumption smoothing is no longer optimal, since (even with $r^* = \rho$) the marginal cost of borrowing is not equal to the pure time preference rate. The marginal cost of foreign borrowing now includes the premium $\Pi(F)$ and the cost of any change in the premium, $F\Pi'(F)$.[12] At this higher rate a country with $F > 0$ has an incentive to postpone consumption and save, placing consumption on a rising path.

Fig. 4 portrays the phase-plane diagram corresponding to (4)-(5). Looking first at the lower part of the figure, $\left[\dot{F} = 0\right]_{N=0}$ is the locus of stationary values of F in the absence of windfall revenues. It has negative gradient, with F increasing above (consumption is high) and decreasing below. The differential equation for consumption (5) has $\dot{C} = 0$ wherever $\Pi(F) + F\Pi'(F) = 0$, which is for all values of $F \leq 0$, and $\dot{C} > 0$ for $F > 0$. Countries with foreign debt $F > 0$ face high domestic interest rates and have rising consumption, while countries with $F \leq 0$ have constant consumption. Combining the F and C stationaries gives a set of stationary points, the line S–S. For an economy which finds itself with $F \leq 0$ this line segment is unstable, so C must jump to S–S; this is precisely the permanent income hypothesis. For an economy with $F > 0$ there is a unique saddlepath, illustrated by the dashed line (see appendix 1).

Our focus is on a developing country, which is initially indebted and which starts out at point E_0 on fig. 4.[13] In the absence of a windfall the economy simply moves along the saddlepath with relatively high saving and growing consumption. As it pays off its foreign debt the domestic interest rate falls so that the propensity to save and the growth of consumption decline. In the long run the economy has paid off its foreign debt ($F = 0$), the domestic interest rate has fallen to the world interest rate, and private and public consumption have risen to their steady-state values.

3.1. A small temporary windfall

We suppose that the flow of windfall revenue, N, takes a constant value $N > 0$ during an exogenously given interval $t \in [T_0, T_1]$, and is zero outside this interval, $t < T_0$ and $t > T_1$. As is clear from equation (4), this shifts the F stationary upwards, giving the upper locus, $\left[\dot{F} = 0\right]_{N>0}$, on fig. 4. This sets the dynamic during the period when $N > 0$ and the associated set of stationary points is S′-S′.

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[12] Government controls consumption directly. If consumption was chosen by decentralised households they would not internalise $F\Pi'(F)$. We look at this case in section 5.
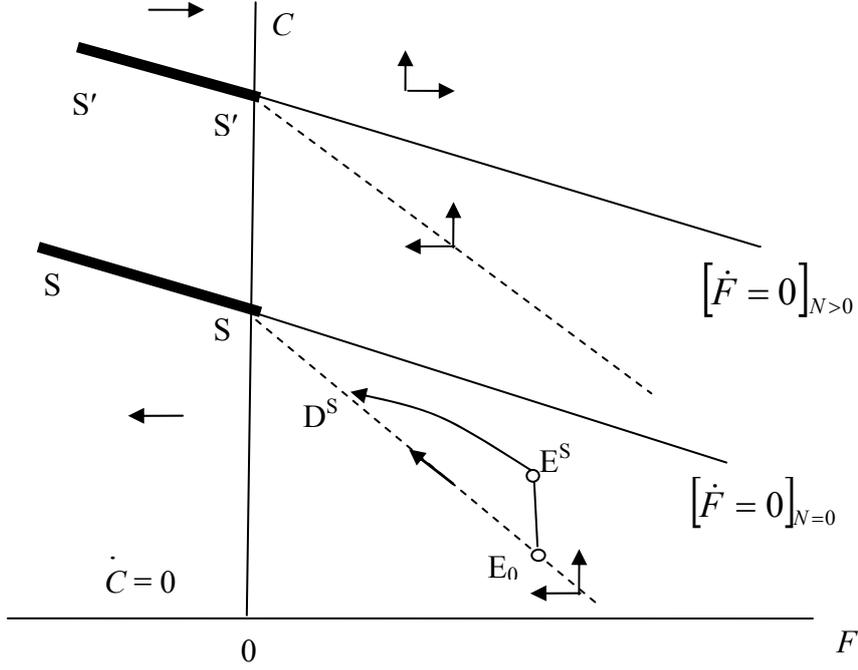
Fig. 4. *Consumption and debt responses to a small temporary windfall*

Our first case is a windfall that is known to be temporary, and for which revenue flows from date of announcement (e.g. there is no lag between discovery and extraction) so $N > 0$ at date $t = T_0 = 0$. Furthermore, the windfall is 'small' in a sense which we will make precise later on. The dynamics are illustrated on fig. 4. Prior to the windfall the economy is on the saddlepath, at point E_0. During the period of revenue flow dynamics are subject to $[\dot{F} = 0]_{N>0}$, but must return to the original saddlepath at date $t = T_1$. This involves an upwards jump in consumption at $t = 0$, followed by movement along the line $E^S D^S$. The size of the jump (distance $E_0 E^S$) is determined so that the path regains the initial saddlepath when the windfall ceases at $t = T_1$.

We establish two sets of results concerning this path. First, comparing the situation with the windfall to that without, proposition 1 of appendix 2 uses the linearised model to show that with a windfall of (infinitesimally small) size $N > 0$ lasting from time 0 to time T_1, the initial jump in total consumption equals

$$\Delta C(0) + \Delta G(0) = [1 - \exp(-\lambda_u T_1)] N > 0, \tag{6}$$

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[13] In section 4 we endogenise initial indebtedness by having an economy with low initial wealth choose to taken on foreign debt to finance public capital.

where $\lambda_u = \dfrac{1}{2}r* + \dfrac{1}{2}\sqrt{r*^2 + 8\sigma\Pi'Y} > r* > 0$ and Δ indicates deviations from the benchmark

trajectories. Equation (6) indicates that the initial jumps in private and public consumption are proportionate to the magnitude of the temporary windfall and are larger if the windfall is more prolonged (higher T_1). A steeper interest schedule (higher Π'), a higher elasticity of intertemporal substitution (higher σ) and a higher level of non-windfall endowment income (higher Y) imply a bigger value of λ_u and thus a larger initial increase in consumption. If the interest schedule is flat, we have $\Pi'=0$ and thus $\lambda_u = r*$, which corresponds to the outcome under the PIH. A windfall thus leads to a larger upwards jump in consumption in a capital-scarce economy (with $\Pi' > 0$)[14]. Proposition 2 of appendix 2 establishes that during the windfall the country runs a surplus and reduces debt, and after the windfall it runs a deficit. This proposition also proves that the magnitude of the reduction in debt at the end of the windfall increases with the size (N) and duration of the windfall (T_1), that is

$$\Delta F(T_1) = -\left[\frac{1 - \exp\left((\lambda_s - \lambda_u)T_1\right)}{\lambda_u - \lambda_s} \right] N < 0, \tag{7}$$

where $\lambda_s = \dfrac{1}{2}r* - \dfrac{1}{2}\sqrt{r*^2 + 8\sigma\Pi'Y} < 0,$ and that the incremental change in consumption at the end of the windfall is less than the initial change in consumption:

$$0 < \Delta C(T_1) + \Delta G(T_1) = -\lambda_u \Delta F(T_1) < \Delta C(0) + \Delta G(0). \tag{8}$$

Comparing the situation with the windfall to that without, we can establish that consumption is higher at all dates and converges asymptotically to its previous path; debt is lower at all dates, and converges asymptotically to its previous path: the point D^S (and all subsequent points on the saddlepath) are attained sooner than they otherwise would have been. These results establish that optimal use of the windfall does not involve raising consumption in perpetuity, but instead using it to bring forward the development path of the economy.

The second set of results compares the path to the prescriptions of the PIH, i.e. increasing consumption at all dates by the annuity value of the windfall.[15] Proposition 1 of appendix 2 indicates that the initial jump in total consumption is bigger than suggested by the PIH,

$$\Delta C(0) + \Delta G(0) = \left[1 - \exp(-\lambda_u T_1)\right]N > \left[1 - \exp(-r* T_1)\right]N \quad \text{as} \quad \lambda_u > r*, \tag{9}$$

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[14] The consumption increment does not have the hump of fig.1 (and fig. 6) which arises when capital stocks, output and wages are endogenised, instead falling steadily. As in fig.1 it starts larger than under the PIH and falls below it after date T_1.

[15] This now calculated using the time varying interest rate r.

especially if the interest schedule is steep, intertemporal substitution is easy and non-windfall income is substantial (as then λ_u is high). Proposition 3 of appendix 2 establishes that the incremental increase in consumption at the end of the windfall remains larger than that which would prevail under the permanent income hypothesis,

$$\Delta C(0) + \Delta G(0) > \Delta C(T_1) + \Delta G(T_1) = \left(\frac{\lambda_u}{\lambda_u - \lambda_s}\right)\left[1 - \exp\left((\lambda_s - \lambda_u)T_1\right)\right] N$$
$$> \left[1 - \exp(-r * T_1)\right] N.$$

(10)

In summary, following the discovery (and revenue flow) consumption jumps up and the consumption increment then declines through time. During the period of revenue flow ($t < T_1$) consumption is higher than it would be under the PIH, but eventually falls below this level as the consumption increment goes asymptotically to zero. Debt is correspondingly higher (assets lower) than under the PIH for all $t > 0$. This is because the economy is initially poor and on a rising consumption path. It is optimal to skew consumption (relative to the PIH) towards the present poor generation rather than transfer too much to future relatively rich generations, and there is no rationale for building a SWF.

To draw out results clearly, we have in this sub-section assumed that revenue flows from the date of discovery, $T_0 = 0$. It is straightforward to extend this to the case where there is a lag between discovery and revenue flow, $T_0 > 0$. Comparing the situation with the windfall to that without, proposition 1 of appendix 2 proves that the initial jump in total consumption equals

$\Delta C(0) + \Delta G(0) = \exp(-\lambda_u T_0)\left[1 - \exp\left(-\lambda_u (T_1 - T_0)\right)\right] N > 0$. The initial jump in consumption is thus proportionate to the windfall and larger if the windfall is more prolonged and starts earlier. As before, a steeper interest schedule (higher Π') implies a larger initial increase in consumption. In terms of the phase diagram, there are three stages. At date of discovery consumption jumps up from E_0 and during $t \in [0, T_0]$ the system remains under the influence of $\left[\dot{F} = 0\right]_{N=0}$ so both C and F increase, i.e. debt is being incurred to finance increasing consumption. From $t = T_0$ onwards the dynamics of F are controlled by $\left[\dot{F} = 0\right]_{N=0}$, and a path similar to $E^S D^S$ is followed, with consumption rising and debt being paid down. Proposition 3 of appendix 2 establishes that the longer the anticipation period T_0, the bigger the reduction in debt at the end of the windfall. The height of the initial jump is determined by the condition that the trajectory hits the original saddlepath when the revenue flow ceases, at date $t = T_1$.

3.2. Large windfalls: encompassing the permanent income hypothesis

In the analysis of fig. 4 optimal policy returns the economy to the original saddlepath (point D^S) at date T_1. The larger and longer is the period of revenue flow, the larger is the initial jump in

consumption and the closer is D^S to the stationary. For a large enough windfall the path becomes that illustrated in fig. 5. The initial jump in consumption is to point E^L. As with the small-windfall case, during the period of revenue flow, $t \in [T_0, T_1]$, consumption is higher and debt is being repaid, but now it is optimal to repay all debt (reach $F = 0$) before the revenue flow ceases. Consumption increases until this point is reached and the marginal cost of capital reaches the rate of time preference. Once consumption has reached its permanent value an SWF is established and foreign assets are built up to the level sufficient to sustain this consumption in perpetuity. At date T_1 when the revenue flow ceases (point D^L), the economy becomes stationary. The size of the jump to E^L is determined by the requirement that the economy reaches the original stationary, S–S, at this date. The size of the long-run SWF is now endogenously determined. For example, a lower initial debt or a larger and more protracted windfall will increase the size of the terminal SWF, as will a starting point with lower initial debt.
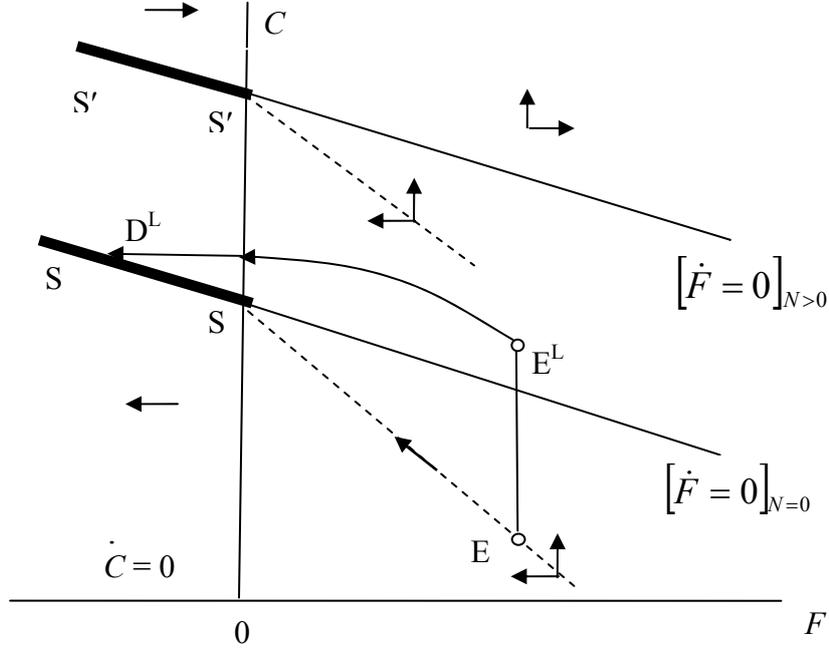
Fig. 5. *Consumption, debt reduction and foreign saving: responses to a large temporary windfall*

The boundary between the 'large' and 'small' windfall is when points D^S and D^L coincide at foreign debt level $F(T_1) = 0$. For the linearised solution derived in appendix 2, we have a 'small' windfall and thus no building up of a SWF and no long-run increase in consumption if

$$F_0 + \Delta F(T_1) = F_0 - \left[\frac{1 - \exp\left((\lambda_s - \lambda_u)T_1\right)}{\lambda_u - \lambda_s} \right] N > 0,$$ that is a windfall is 'small' if the initial level of

debt is high, the windfall is small in magnitude and does not last very long. However, if this

inequality is reversed the economy will pay off all debt during the period of revenue flow, following which a SWF will be built up and a permanent increase in consumption will be optimal.

3.3. Resource wealth and the cost of capital

Our analysis to this point assumes that the windfall brings down interest rates only in so far as it is associated with lower foreign debt. However, as discussed in section 3, it is possible that the windfall has a direct effect on creditworthiness, changing the borrowing rate at given F. While the evidence reported in section 3 found no statistically significant support for this, the effect might hold for particular countries. The model can easily be extended to incorporate this possibility by letting the interest premium be a function of both debt and the present value of the resource windfall, so $r = r * + \Pi(H)$, where $H \equiv F - \omega V$ and ω is a constant parameter which is positive if the resource discovery improves creditworthiness, and negative if it worsens it. The present value of resources remaining at each date, $V(t) \equiv \int_t^\infty N(z) \exp\left[-\int_t^z r(v)dv\right]dz$, is changing through time according to $\dot{V} = rV - N$. The change in H is therefore $\dot{H} = \dot{F} - \omega\dot{V} = \dot{F} - \omega(rV - N)$. The economy's flow budget constraint, equation (4), is $\dot{F} = \left[r * + \Pi(H)\right]F + (1+\psi^\sigma)C - Y - N$ so equations (4) and (5) are replaced by:

$$\dot{H} = \left[r * + \Pi(H)\right]H + (1+\psi^\sigma)C - Y - (1-\omega)N, \tag{11}$$

$$\dot{C} = \sigma C\left[\Pi(H) + H\Pi'(H)\right]. \tag{12}$$

This pair of differential equations is analogous to that analysed above (equations 4 and 5) and can be studied with similar phase-plane diagrams, but with H replacing F on the horizontal axis. Analysis of the effect of the windfall is altered in two ways. First, whereas $F(0)$ has given initial value F_0, credit worthiness, $-H$, jumps at the date of discovery, with sign and magnitude depending on ω. Second, the upwards shift of the $\dot{H} = 0$ stationary is smaller than the shift in the $\dot{F} = 0$ stationary if $0 < \omega < 1$, does not shift at all if $\omega = 1$, and is larger if $\omega < 0$. Looking at the case with $\omega = 1$ (which can be thought of as full and immediate securitization of future flows) the economy remains on an unchanged (original) saddlepath at all dates. The effect of the windfall is to abruptly reduce $H(0)$, this causing an upwards jump in C to stay on this saddlepath. Consumption is higher at all dates than it is without the windfall, and converges to the non-windfall level as the economy converges to the steady-state. While H is lower along this path than it is without the windfall, debt F is higher as the economy borrows against the present value of resource revenues. Compared to the PIH, the increment to consumption is once again skewed towards near generations and (for a small windfall) no SWF is constructed.

4. Domestic investment in public and private capital

We established in the previous section that optimal policy for a developing economy with an interest rate greater than the rate of time preference involves using the windfall to accelerate the growth of consumption towards its long-run value, rather than increasing that value through investment in a SWF. Whereas the PIH suggests that a SWF should be built up in response to a temporary windfall, this is only true if the windfall is so large that it moves the economy out of the regime in which it faces a premium on its foreign debt. However, the analysis of the previous section allowed government to invest only in foreign assets, either by debt reduction or construction of an SWF. We now turn to the next question. If there are domestic assets – private and public capital stock – as well as foreign, how should optimal policy combine current consumption, debt reduction, public investment, and incentives to private investment?

To answer these questions we make non-resource output endogenous by including private capital and public infrastructure. Non-resource domestic income is given by a production function with constant returns to scale with respect to private capital and labour, expressed as $Y = f(K, S)$ where the labour force is normalised at unity, K denotes the private capital stock and S is the stock of public infrastructure. We assume that $f(K, S)$ exhibits decreasing returns in K and S together, to rule out ever-increasing growth. Infrastructure can be thought of as consisting of seaports, airports, roads and railroads, but also education, health or any public investment that boosts the productivity of private production.

We retain until section 5 the assumption that there are no private domestic asset holders. Public infrastructure is owned by government, while private capital is rented from foreign owners who face the world interest rate, r^*. They are subject to host country income taxation at a proportional rate τ. Profit maximisation requires that the after-tax marginal product of capital, net of depreciation δ_K, equals the world interest rate, so that

$$(1 - \tau)f_K(K, S) = r^* + \delta_K. \tag{13}$$

The equilibrium private capital stock follows from (13), and can be written as $K = K(S, \tau)$, this giving wage $W(S, \tau) \equiv (1 - \tau)\big[f(K(S, \tau), S) - K(S, \tau)f_K(K(S, \tau), S)\big]$ and output $Y(S, \tau) \equiv f(K(S, \tau), S)$.[16] Since there are no private domestic asset holders, private consumption is wage income, $W(S, \tau)$, plus transfers from government, T. Capital stock, wages and income are all increasing S and decreasing in τ. Differentiating (13) and $W(S, \tau)$, the effect of a tax change on the wage rate, W_τ, satisfies $W_\tau = -Y$

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[16] Note that we use roman letters throughout to indicate functions and italics to indicate variables.

(see appendix 3): given the fixed supply price of capital, the first order effect of a reduction in income tax is to transfer income to workers.

In this structure the only debt is that of government, D. It is held entirely by foreigners, and the interest rate becomes $r = r^* + \Pi(D)$.[17] The dynamics of government debt come from the government budget constraint in the familiar way,

$$\dot{D} = [r^* + \Pi(D)]D + G + I_S + T - \tau Y - N,$$ (14)

where I_S is spending on infrastructure investment and the final terms are lump-sum transfers to consumers, income taxation and resource revenues. The stock of infrastructure evolves according to $\dot{S} = I_S - \delta_S S$ where δ_S is the depreciation rate. Analysis is simplified by working with net government assets defined as the stock of public infrastructure minus government debt, $B \equiv S - D$.[18] The budget constraint is then

$$\dot{B} = [r^* + \Pi(S - B)](B - S) + N + \tau Y - G - T - \delta_S S, \quad B(0) = B_0$$ (15)

with the initial value of net government assets given by B_0. The no-Ponzi game condition must be satisfied, $\lim_{t \to \infty} B(t) \exp\left(-\int_0^t r(v)dv\right) = 0$, so that initial net government assets plus the present value of the stream of future income taxes and resource revenue must cover the present value of the stream of future spending on public consumption, government transfers and infrastructure services.

The government's problem is now to choose the public capital stock S, public consumption G, together with the rate of income taxation τ and transfers to households T, where households' consumption is $C = W + T$. Its objective is social welfare,

$U \equiv \int_0^\infty \left(\frac{C^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1 - 1/\sigma} \right) \exp(-\rho t)dt$, and the constraints are the budget constraint (15), together

with initial conditions, the no-Ponzi condition and equilibrium levels of private capital (and hence wages and income) as captured by $K(S, \tau)$, $W(S, \tau)$ and $Y(S, \tau)$. The Hamiltonian is

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$$H(\tau, T, G, S, \mu) \equiv \left(\frac{\left[W(S,\tau) + T \right]^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1 - 1/\sigma} \right)$$
$$+ \mu \left[\{ r^* + \Pi(S-B) \}(B-S) + N + \tau Y(S,\tau) - G - T - \delta_S S \right] \tag{16}$$

with co-state for net government assets μ. This yields the optimality conditions

$$H_\tau = C^{-1/\sigma} W_\tau + \mu(Y + \tau Y_\tau) = 0, \tag{17}$$

$$H_T = C^{-1/\sigma} - \mu = 0, \tag{18}$$

$$H_G = \psi G^{-1/\sigma} - \mu = 0, \tag{19}$$

$$H_S = C^{-1/\sigma} W_S + \mu \left[\tau Y_S - \{ r^* + \Pi(S-B) + (S-B)\Pi'(S-B) + \delta_S \} \right], \tag{20}$$

$$r^* \mu - \dot{\mu} = H_B = \mu \left[r^* + \Pi(S-B) + (S-B)\Pi'(S-B) \right], \tag{21}$$

and the transversality condition

$$\lim_{t \to \infty} \left[\exp(-r^* t)\mu(t)B(t) \right] = 0. \tag{22}$$

We analyse temporary windfalls in this system in two stages, first looking at the case in which lump-sum transfers – the instrument T – are possible, and then in section 4.2 removing this instrument.

4.1. Policy with lump-sum taxes/transfers

To focus on the role of public infrastructure investments, we first suppose that government can make lump-sum transfers to consumers. It is then optimal to set the income tax rate τ at zero (from (17) and (18) together with $W_\tau = - Y$). Infrastructure is set optimally to satisfy (20), so

$$W_S(S,0) = r^* + \Pi(S-B) + (S-B)\Pi'(S-B) + \delta_S. \tag{23}$$

The marginal value of infrastructure is simply its effect on national income which, absent income taxation and given foreign ownership of private capital, is its effect on the wage. Its marginal cost is the full marginal cost of public borrowing including the marginal cost of the interest premium, so the optimal level of infrastructure is lower in a capital-scarce economy.

The optimal path of consumption is as in section 3; using (18) in (21),

$$\dot{C} = \sigma C \left[\Pi(S-B) + (S-B)\Pi'(S-B) \right] \text{ for } S - B > 0 \text{ and } \dot{C} = 0 \text{ otherwise.} \tag{24}$$

This private consumption path is supported by transfers $T = C - W(S, 0)$. The optimal level of public consumption satisfies $G = \psi^\sigma C$ from (18) and (19). Debt is $D = S - B$ and the dynamics of net government assets B are, from equation (15)

$$\dot{B} = \left[r* + \Pi(S-B) \right](B-S) + N + W(S,0) - C(1+\psi^\sigma) - \delta_S S, \quad B(0) = B_0. \tag{25}$$

The dynamic system (24) and (25) in B and C is qualitatively similar to the system (4) and (5) in F and C illustrated in figs. 4 and 5, although S is now endogenous through equation (23), and $K =$ K(S,0), $Y =$ Y(S,0) and $W =$ W(S,0) are now changing along the optimal path. It is straightforward to extend the analytical results of appendix 2, but more revealing to illustrate results by simulation of an example presented in the panels of figs. 6 and 7. Time is on the horizontal axis, and scaling of the vertical axes is achieved by normalising the long-run stationary value of income at unity. Production is Cobb-Douglas, $Y = AK^\alpha L^{1-\alpha} S^\gamma$, where A scales long-run output to unity, and parameters are set to α = 0.4, γ = 0.25, $\rho = r* = 0.05$, $\sigma = 0.75$, $\psi = 0$ and $\delta_K = \delta_S = 0.05$. In figs 6 and 7 $\Pi = \rho(F_+)^2$. Simulations are done with a reverse multiple shooting algorithm with a horizon of 130 and using the computer package GAUSS. We set the time dimension such that the horizontal axis can be (loosely) interpreted as years.

The solid lines in figs. 6 and 7 give the path of an economy which starts out with national wealth, B_0, set at 0, and which experiences no shocks. Optimal policy involves this economy taking on positive initial foreign debt in order to finance public infrastructure. Along the development path there is smooth convergence to long-run equilibrium with accumulation of assets, decumulation of foreign debt, falling interest rates, and rising income and consumption.

The effect of a temporary 'small' windfall is given by the dashed lines. In fig. 6 the windfall is unanticipated, with revenue equal to 10% of long-run stationary non-resource income flowing from period $T_0 = 0$ to $T_1 = 18$. The figure illustrates that there is an immediate jump in consumption, financed by an increase in transfer payments (bottom centre).[19] The increase in consumption is less than the revenue flow so savings increase, this being divided between debt reduction and infrastructure investment and putting the stocks of debt D and public infrastructure S on steeper paths. In the first year the division of resource revenues is 68% to higher consumption, 11% to debt reduction, and 21% to infrastructure investment. Private capital formation also increases as does growth of income. Growth of income and wages enables the optimal consumption profile to be attained – beyond a certain point – with declining public transfers.

When the resource revenue ceases (at $t = 18$), the rates of growth of capital and income slow. The economy resumes its previous growth path but, crucially, it reaches income and consumption levels some 20 years sooner than it would have without the windfall. Thus, revenues have been used to bring forwards the development of the economy, not to transfer consumption to distant future generations. This is brought out in the bottom right panel of fig. 6, which is the source of fig. 1. The

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[19] Notice that overall lump-sum transfers to households are negative due to the need to finance public infrastructure.

revenue flow is the solid line and incremental consumption is the smooth dashed curve. Other lines are incremental consumption if the economy had followed the PIH (flat line) or bird-in-hand hypothesis (piece-wise linear). Since the current generation is relatively poor, optimal policy skews consumption towards the present. Computing the change in the social objective along these alternatives, the permanent-income path yields 85% of the gains delivered by the optimal policy, while the bird-in-hand path delivers just 72%.
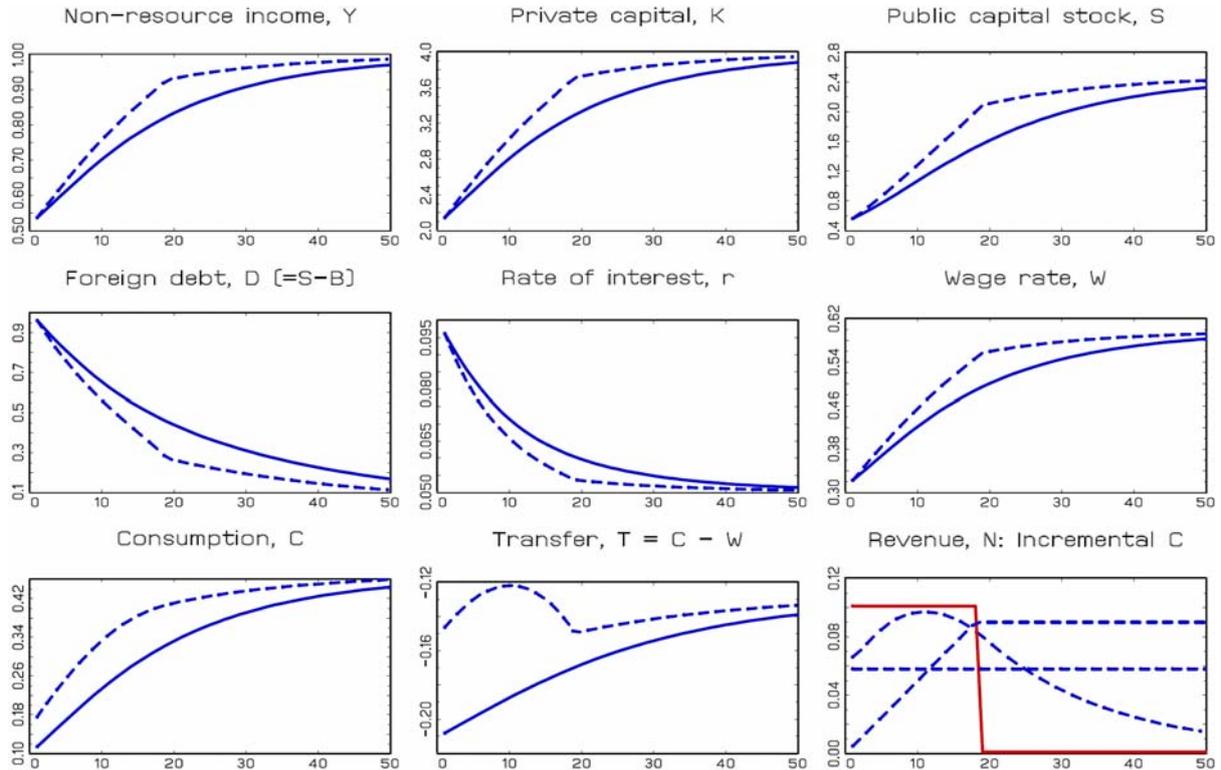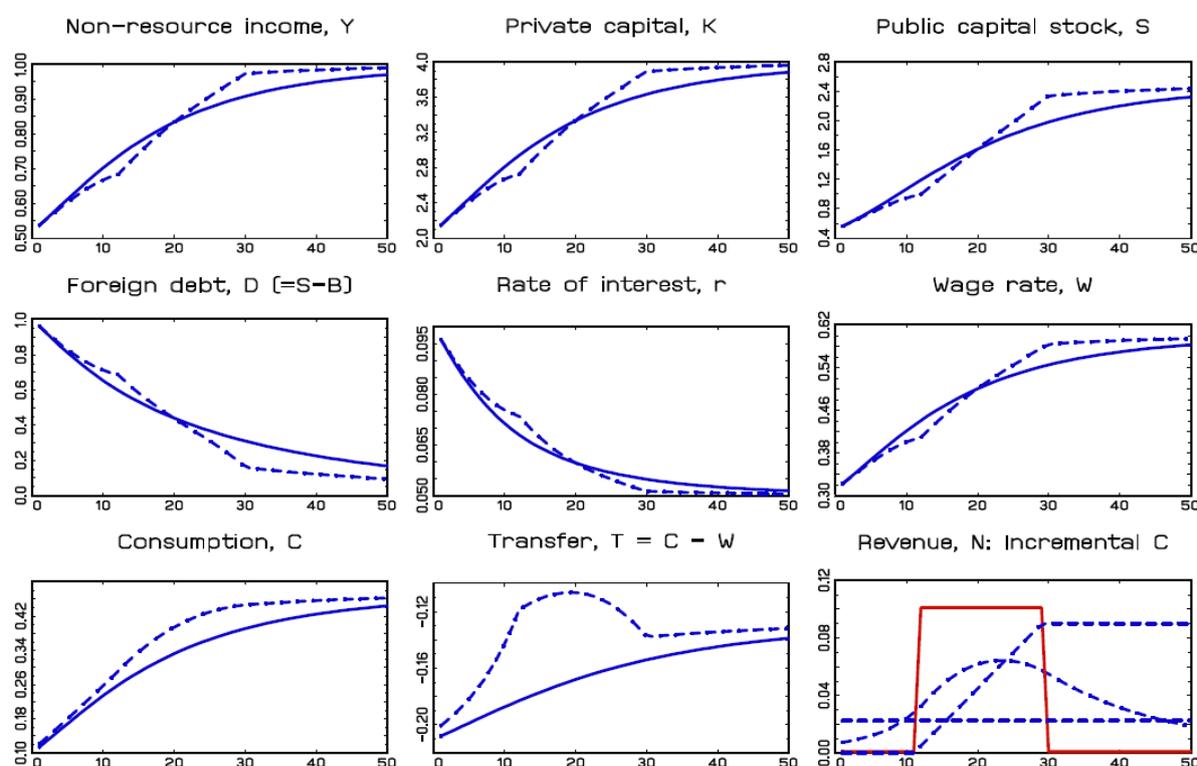

Fig. 6: *Optimal development with and without the windfall. Lump-sum transfers possible*

Solid lines give times paths of variables in the absence of the windfall and dashed lines with a windfall. In the bottom right panel the solid line is windfall revenue, *N*, and dashed lines are incremental consumption.

Fig. 7 illustrates the same economy and the same size windfall, but now shifted 12 periods into the future, so there is an interval in which policy is set in anticipation of revenues. There is a small upward jump in consumption *C* at the date of announcement, followed by a further increase. This incremental consumption in the interval [0, T_0] is less than it would under the PIH (bottom right) and is financed by foreign borrowing and lower infrastructure investment (see the paths of *D* and *S*). These translate into higher *r* and lower private capital *K* and non-resource income, *Y*. Once revenue flow commences (*t* = 12) there is a sharp increase in the rate of debt reduction and public infrastructure investment, this putting capital stocks and income on a steeply rising path which

overtakes the non-windfall trajectory. At the date when the windfall revenue ceases ($t = 30$, at the kink) domestic public and private capital stocks are something over 10% higher than they otherwise would have been, and foreign debt at half the level. Once again, the economy reverts to its previous path, but earlier than would otherwise have been the case.

Fig. 7: *Optimal development with and without the windfall. Lump-sum transfers possible*

Solid lines give times paths of variables in the absence of the windfall and dashed lines with a windfall. In the bottom right panel the solid line is windfall revenue, *N*, which is now anticipated from date 0 but only flows from date 12.

4.2. Second best: resource revenues and public funds

The possibility of lump-sum transfers makes it easy for government to control the level of private consumption, but is rather implausible. One of the key features of developing economies is limited tax raising capacity, and an important aspect of a windfall is that it relaxes this constraint. We therefore now move to a case in which lump-sum taxes and transfers are ruled out, so consumption can only be controlled indirectly, via the wage rate. Policy must therefore be directed to using resources to raise the wage rate, and two instruments affect this. More public infrastructure raises the marginal product of labour and the wage rate directly, and also by attracting private investment. Lower distortionary taxation attracts private investment, thereby raising wages. Of course, these instruments are linked by the budget constraint. Resource revenues relax this constraint, and the ensuing second-best optimal policy response is outlined below.

The optimal policy is found from the first-order conditions above, but with $T = 0$, so instead of first-order condition (18) we have simply $C = W(S, \tau)$. It is helpful to define the marginal cost of public funds as the shadow price of public funds relative to the marginal utility of private consumption $\phi \equiv \mu / C^{-1/\sigma}$. In the preceding subsection $\phi = 1$, but the fact that the government now has to raise funds by distortionary taxes implies $\phi > 1$. The relationship between the optimal income tax and the marginal cost of public funds is given by equation (17) (using $W_\tau = - Y$) as

$$\tau = \left(\frac{1-\phi}{\phi}\right)\left(\frac{Y}{Y_\tau}\right) \quad \text{or} \quad \phi = \frac{1}{1+\tau Y_\tau / Y} . \tag{26}$$

Since $Y_\tau < 0$ a positive tax rate is associated with marginal cost of funds greater than unity. A higher cost of funds depresses the public consumption relative to private,

$$G = \left(\frac{\psi}{\phi}\right)^\sigma W(S,\tau) . \tag{27}$$

The first-order condition for infrastructure becomes (from (20) with $\phi \equiv \mu / C^{-1/\sigma}$),

$$W_S(S,\tau) = \phi\left[r^* + \Pi(S - B) + (S - B)\Pi'(S - B) + \delta_S - \tau Y_S\right]. \tag{28}$$

Thus, ϕ greater than unity raises the cost of capital which tends to reduce the optimal level of public infrastructure, although this may be offset as an increase in infrastructure raises income and tax revenue, $\tau Y_S > 0$.

We once again illustrate the optimal development paths, with and without resource revenue, by numerical example. Fig. 8 describes the same economy as fig. 6, but with this restricted set of instruments. Government funding of infrastructure requires distortionary taxation ($\tau > 0$) and hence a shadow premium on public funds ($\phi > 1$). Along the development path without resources there is steady payback of debt, increasing capital stock and rising income and consumption. This is accompanied by a declining cost of public funds and rate of income tax as accumulation of infrastructure reduces the flow of infrastructure investment (relative to the size of the economy) to be financed. The presence of the distortion means that income and consumption are lower at all dates than they are when lump-sum transfers can be used.

Resource revenue now has additional value as it accrues as government revenue and so causes an immediate decrease in the cost of public funds. There is an abrupt reduction in the tax rate which leads to a jump in the private capital stock and hence in income. Wages therefore rise, giving the substantial jump in consumption that is illustrated, followed by a period of continuing rapid growth. A higher value of the private capital stock also raises the marginal product of the public capital stock, which therefore increases more rapidly following the windfall. It is the need to fund this public

infrastructure that creates the initial increase in debt and in the rate of interest illustrated on fig. 8. This can be seen clearly in the Cobb-Douglas case in which the cost of public funds and optimal stock of infrastructure, (26) and (28), simplify to

$$\phi = \frac{1}{1 - \left(\dfrac{\alpha}{1-\alpha}\right)\left(\dfrac{\tau}{1-\tau}\right)} \quad \text{and} \quad \frac{S}{Y} = \frac{\gamma}{r^* + \Pi(D) + D\Pi'(D) + \delta_S} \tag{29}$$

where α and γ are elasticities of output with respect to K and S, respectively. The jump in income implies an instantaneous increase in the *stock* of public infrastructure unless mitigated by an increase in r, raising the denominator of the right hand side. The optimal policy is therefore a substantial increase in infrastructure investment financed partly by an increase in debt, D, which in turn raises r and reduces the optimal S/Y ratio.
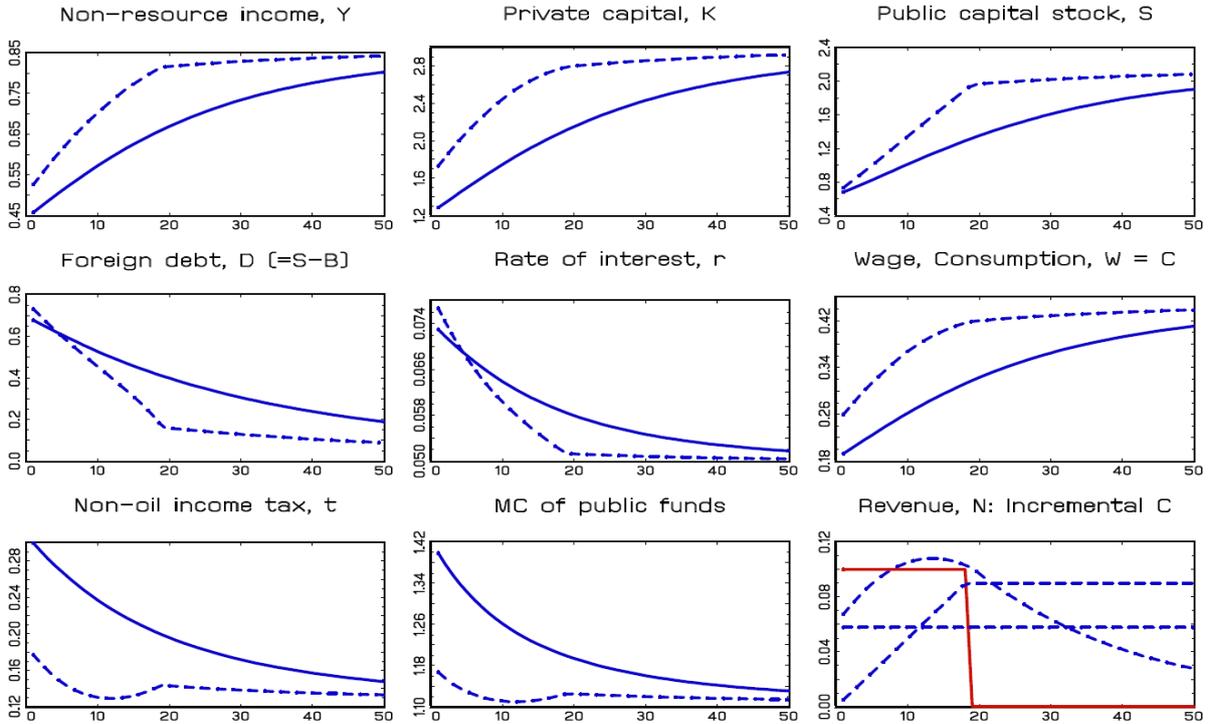
Fig. 8. *Optimal development with and without the windfall. Lump-sum transfers not possible*

Solid lines give times paths of variables in the absence of the windfall and dashed lines with a windfall. In the bottom right panel the solid line is windfall revenue, *N*, and dashed lines are incremental consumption.

At the date the windfall revenue ceases ($t = 20$, at the kink) there is a small increase in the tax rate, but domestic capital stocks (public and private) are significantly higher than they otherwise would have been, non-resource income is some 20% higher, and wages and consumption some 25% higher. Comparing the bottom right panels of figs. 6 and 8, it is noteworthy that the consumption

increment associated with the windfall is larger at all dates in the present case, precisely because the windfall reduces the impact of the public finance constraint and associated distortions.

In summary, this case captures the important point that windfall revenue relaxes the government's public finance constraint, reducing the marginal cost of public funds and permitting a reduction in other distortions in the economy. In particular, the government is able to increase private sector investment both by cutting distortionary taxes and increasing public capital. The consequent growth of non-resource income creates the large increase in wages and consumption. In this case the PIH would yield only 75% of the social welfare gain of the optimal second-best policy, and the bird-in-hand rule only 66%. The simulations indicate also that the welfare losses from the permanent income and bird-in-hand rules are larger than the social welfare cost of distortionary taxes.[20]

5. Public and private savings: the Ricardian curse

Up to this point consumers have been assumed to be credit constrained, so their consumption is determined by current income and transfers. We now remove this assumption and allow for forward-looking private agents who own assets and adjust savings and consumption decisions in response to current and future resource revenues. This is important, because it raises the possibility that Ricardian consumers may offset – or even negate – the effect of government policy. They fully anticipate their future shares in resource revenues and adjust consumption accordingly, but not necessarily socially optimally, a 'Ricardian curse'. Thus, even if government is seeking to save a substantial share of resource revenues, policy may be undermined by a private consumption boom fuelled by private borrowing, as has happened in some countries.[21] How should government react to this? To address this question we make two main changes to our model. First, households may own private assets. And second, the foreign debt of the economy is no longer just that of government, but also includes that of households and firms. In order to focus our analysis, we shall ignore income taxation and public infrastructure (so $\tau = 0$ and $S = 0$), but add two tax instruments that directly influence saving and investment, an asset holding subsidy τ_A and capital holding tax τ_K.

Aggregate household wealth is denoted by A and can be held in either domestic equity or government bonds which we assume to be perfect substitutes. Thus, the physical assets in the economy, K, are owned by foreigners (foreign debt F), government (net assets B), and households (wealth A), so $K = F + B + A$. The interest premium is a function of foreign debt, so $r = r^* + \Pi(F) =$

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[20] With σ < 1 welfare levels are negative and reported below:

	$N = 0$	$N > 0$, opt	$N > 0$, PIH	$N > 0$, bird-in-hand
First best	-87.045	-81.012	-81.903	-82.704
Distortionary tax	-88.724	-81.454	-83.298	-83.932

$r* + \Pi(K - E)$ where it is convenient to work with the combined assets of households and government, $E \equiv A + B$. Private households have access to domestic capital markets and smooth consumption C. They are subject to two government instruments, an asset holding subsidy at rate τ_A and a lump-sum transfer of T_A. Their budget constraint is $\dot{A} = (r + \tau_A)A + W + T_A - C$, and privately optimal growth in consumption is proportional to the gap between the return on assets (interest rate r plus asset holding subsidy τ_A) and time preference rate ρ, so

$$\dot{C}/C = \sigma\big[\{r* + \Pi(K - E)\} + \tau_A - \rho\big].$$ (30)

The government borrows and issues debt $D (= -B)$ at rate of interest r. Its budget constraint is $\dot{B} = rB + N - G - T_A - \tau_A A + \tau_K K,\ B(0) = B_0$. Ricardian equivalence implies that the intertemporal profile of government transfers T_A does not affect private consumption, so we may as well use the consolidated private and public budget constraint[22]

$$\dot{E} = \{r* + \Pi(K - E)\}(E - K) + f(K) + N - G - C - \delta_K K,\ E(0) = A_0 + B_0.$$ (31)

Turning to the production side of the economy, firms now face the domestic interest rate and their borrowing contributes to foreign debt, so profit maximisation implies that the marginal product of capital equals the user cost of capital,

$$f_K(K) = r* + \Pi(K - E) + \tau_K + \delta_K.$$ (32)

Social welfare is as before, and we maximise with respect to the asset holding subsidy τ_A, the capital holding tax τ_K, and public consumption G. Notice that lump-sum transfers T_A are not an effective government instrument because Ricardian consumers know the combined budget constraint (31) and hence the implicit value of these payments. The formal statement of the government's problem is that it maximises welfare given differential equations (30) and (31) with co-states λ and μ and constraint (32) with shadow price ν. The Hamiltonian and first-order conditions are:

$$H(\tau_A, \tau_K, G, K, E, C, \lambda, \mu) \equiv \frac{C^{1-1/\sigma} + \psi G^{1-1/\sigma}}{1 - 1/\sigma} + \lambda\sigma\big[r* + \Pi(K - E) + \tau_A - \rho\big]C$$
$$+ \mu\big[\{r* + \Pi(K - E)\}(E - K) + f(K) + N - G - C - \delta_K K\big] + \nu\big[f_K(K) - \{r* + \Pi(K - E)\} - \tau_K - \delta_K\big]$$

$$H_{\tau_A} = \lambda\sigma C = 0,$$ (33)

[21] Notably Kazakhstan where public saving has been offset by private borrowing (Esanov and Kuralbeyeva, 2009).

[22] Although noting that the asset holding tax still affects private consumption.

$$H_{\tau_K} = -v = 0, \tag{34}$$

$$H_G = \psi G^{-1/\sigma} - \mu = 0, \tag{35}$$

$$H_K = \lambda\sigma C\Pi' + \mu\left[f_K - \delta_K - \left\{r^* + \Pi + (K-E)\Pi'\right\}\right] + v\left[f_K - \Pi'\right] = 0, \tag{36}$$

$$\rho\mu - \dot{\mu} = H_E = -\lambda\sigma C\Pi' + \mu\left[r^* + \Pi + (K-E)\Pi'\right] + v\Pi', \tag{37}$$

$$\rho\lambda - \dot{\lambda} = H_C = C^{-1/\sigma} + \lambda\sigma\left[r + \tau_A - \rho\right] - \mu, \tag{38}$$

$$\lim_{t\to\infty}\left[\exp(-\rho t)\lambda(t)C(t)\right] = 0 \text{ and } \lim_{t\to\infty}\left[\exp(-\rho t)\mu(t)E(t)\right] = 0. \tag{39}$$

We first look at the case in which government can use τ_A and τ_K. First-order conditions (33) and (34) then imply $\lambda = 0$ and $v = 0$ and hence from (38) and (35), $\mu = C^{-1/\sigma} = \psi G^{-1/\sigma}$. Consumption paths follow from (37) : $-\dot{\mu}/\mu = \dot{C}/\sigma C = r^* + \Pi + (K-E)\Pi' - \rho$. From (36), capital allocation satisfies $f_K = r^* + \Pi + (K-E)\Pi' + \delta_K$. Comparison of these equations with the private behavioural equations (30) and (32) indicates that optimal tax/ subsidy rates are $\tau_A = \tau_K = (K-E)\Pi' \geq 0$. The economy as a whole is not a price taker on the international capital market (as long as it has debt, $K - E > 0$), but private agents fail to internalise this. This distortion is corrected by an asset holding subsidy on households and a capital holding tax on firms. The distortion is essentially a terms of trade effect, and the asset holding subsidy and capital tax could be combined into a border tax increasing the domestic rate of interest to equal the full marginal cost of capital.

The interesting question arises when government does not have this degree of control over private agents, but is constrained to set $\tau_A = \tau_K = 0$, so conditions (33) and (34) do not apply. We illustrate simulation results for this case in fig. 9, presenting all results as deviations of consumption from the situation where there is no windfall and no asset/ capital holding subsidies or taxes, $\{\tau_A = \tau_K = 0, N = 0\}$.[23] The first-best outcome optimises all instruments (given no resource windfall) and is given by the curve $\{\tau_A = \tau_K = \tau^*; N = 0\}$. This first-best policy gives lower initial consumption, which rises steeply overtaking that when $\{\tau_A = \tau_K = 0, N = 0\}$ after around 12 periods. The intuition is that domestic saving is initially too low (as households and firms do not internalise the beneficial terms of trade effect of lower foreign debt); setting τ_A and τ_K optimally raises saving giving the consumption increment indicated.

The other two curves on fig. 9 illustrate the effect of the windfall. Without the appropriate asset-holding subsidies and capital-holding taxes the second-best consumption increment is illustrated by the solid curve $\{\tau_A = \tau_K = 0; N > 0\}$, this having the familiar shape of an upwards jump in

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[23] Parameters in this simulation are as before except that there is no public infrastructure ($\gamma = 0$) and the share of private capital has been increased correspondingly to $\alpha = 0.66$. Initial wealth of the economy is set at 33% of its terminal wealth (previously zero) to accommodate the fact that private capital is now domestically owned not rented from abroad. This level of wealth gives initial non-resource income equal to 60% of its long-run value, consistent with fig. 6 (top left panel).

consumption followed eventually by convergence back to the original level. The remaining curve, $\{\tau_A = \tau_K = \tau^*;\ N > 0\}$, gives the effect of the windfall *and* introduction of first-best optimal policy at date 0. The key point to note is that the jump in consumption is very much smaller, so a much higher proportion of the windfall is saved as a consequence of internalising the interest spread externality. The difference between these lines tracks closely the effect of the policy change alone; this is because, while the windfall and policy change both have large effects, the *interaction* between the two is second order.
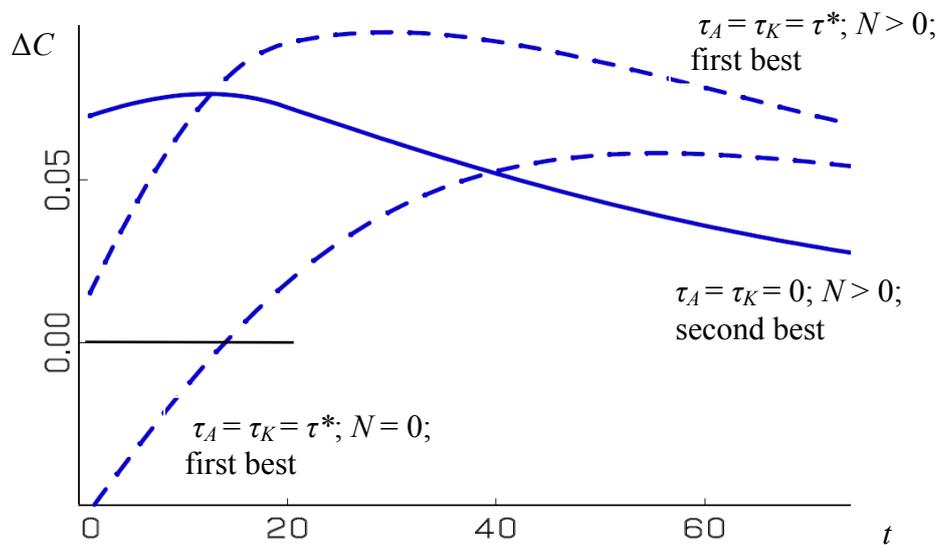
Fig 9: *The effects of policy and the windfall on the consumption*

Curves give deviations in the level of consumption from the case in which $\tau_A = \tau_K = 0$, $N = 0$.

Summarising this section, we see that our central result on the shape of the consumption increment associated with a windfall remains intact when it is not possible to directly internalise the interest spread externality with a border tax or a combination of asset holding subsidy and capital holding tax. The government then loses direct control of the time profile of consumption, because Ricardian consumers understand the impact of the windfall on the government budget constraint. It is not meaningful to talk about government prudence, since consumers borrow against the expectation of receiving the windfall revenue in the future. However, when they do so they anticipate the future path of the interest rate and this gives a consumption path tilted towards the present (qualitatively similar to previous cases and in contrast to the PIH.

The private sector consumption and investment paths are not socially optimal, because the domestic interest rate is not the full marginal cost of foreign debt. The private sector runs up more foreign debt than is socially optimal and consumption – with and without the windfall – is higher than

is optimal. This effect applies to consumption as a whole and not solely (or even disproportionately) to the consumption increment from the windfall. However, the best policy is for government to take the opportunity of the windfall to introduce taxes to correct the distortion (the asset holding subsidy and investment tax or border tax). In the example of the simulation, the ensuing path (curve $\{ \tau_A = \tau_K = \tau^*; N > 0 \}$) gives high saving out the windfall and rapid debt reduction and capital accumulation.

6. Concluding remarks

Many countries face the challenge of resource revenue management, and the historical record has often been poor. The 'resource curse' has many aspects, one of which has been the failure to save adequate amounts of resource revenue. In response to this many countries have set up short- or medium- run stabilisation funds to smooth commodity price fluctuations. Some countries have also set up long-run inter-generational savings funds (SWFs) with the objective of building up a stock of foreign assets from which interest income can be used to finance a 'permanent' increase in consumption. Examples of these funds are Norway's Pension Fund, Kuwait's Reserve Fund for Future Generations and Gabon's Fund for Future Generations.[24]

The use and scale of a SWF should be based on decisions about how much to save, and where to invest; in the domestic economy or in the acquisition of foreign assets. Answers to these questions are, of course, country specific. What is appropriate for a high income and capital abundant country, such as Norway or Kuwait, is unlikely to be appropriate for a country with low income, scarcity of capital, and limited tax capacity, such as Gabon. This paper provides a rigorous framework which encompasses these cases and, in particular, shows how developing countries should deviate from the frequently recommended prescriptions of the permanent income hypothesis. We have established a number of results.

The first result is that a developing economy which is capital scarce and on a rising consumption path should skew incremental consumption from resource rents more towards the present generation than should a high income country which is following the permanent income hypothesis. Despite the high rate of interest implied by capital scarcity, it is optimal for a developing country to have a lower savings rate from resource revenue than would a high-income country. If, however, there is a substantial time lag before revenues flow, then the initial jump in consumption should be smaller than suggested by the permanent income hypothesis.

The second result is that investment should take the form of a combination of investment in the domestic economy and foreign debt reduction which may bring down the interest rate in the economy. Spending that is complementary to private investment is particularly valuable, and we have

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[24] See Ossowski et al (2008) and http://www.sovereignwealthfundsnews.com for details.

shown how it is optimal to use resource revenues both to raise spending on public infrastructure capital and to reduce distortionary taxes. The gains from spending in the domestic economy are particularly large in economies with highly distorting tax systems, precisely because the windfall reduces the impact of the public finance constraint and associated distortions. These results on saving and investment together imply that resource revenue should be used to accelerate growth of the non-resource economy. Instead of transferring rent to future generations (through building an SWF) revenue should be used to bring forward the path of consumption that is in any case rising.

A consequence of these results is that for capital-scarce countries it is not likely to be attractive to build up a long-run SWF. Only if the extent of capital scarcity is limited and the windfall is big and protracted enough does it become desirable to build such a fund. Indeed, Table 1 indicates that resource-rich Chile, Thailand, Malaysia and Tunisia with relatively low interest spreads have instituted a SWF. The oil-rich Gulf economies are not capital-scarce developing economies either and are also setting up SWFs.[25] In contrast, the resource-rich countries with the highest spreads, Ukraine, Ecuador, Argentina and Côte d'Ivoire, do not have a SWF. The Russian Federation only instituted its SWF, the National Welfare Fund, in 2008. Capital-scarce countries thus seem less likely to set up a SWF. We have focused most of our attention on those countries.

Table 1: Interest spreads for resource-rich developing economies

Lowest five spreads		Highest five spreads	
Chile (1999-2003):	173.06 (21.1)	Ukraine (2000-2003):	1125.67 (13.9)
Thailand (1997-2003):	190.68 (11.7)	Ecuador (1995-2003):	1563.45 (20.8)
Malaysia (1996-2003):	220.04 (19.1)	Russian Federation (1998-2003):	1566.46 (21.3)
Tunisia (2002-2003):	248.17 (6.0)	Argentina (1994-2003):	1778.50 (7.8)
South Africa (1995-2003): 269.54 (8.2)		Côte d'Ivoire (1998-2003):	1847.93 (31.3)

Basis points. Average resource exports as a percentage of GDP in brackets.
Source: Sovereign Wealth Institute

The final set of results concerns the interaction between public saving and private saving. Ricardian consumers can undo government prudence. Private access to capital markets means that consumption can be separated from the date at which transfers are made, but private smoothing need not be socially optimal. Market failures (such as that created by imperfect access to international capital markets) may need to be offset by a time-varying asset holding subsidy or capital holding tax. However, if it is not feasible to internalise the interest spread externality, the second best optimal policy is broadly similar to that when consumers cannot borrow/save on the capital market.

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[25] Fast-growing countries with large export surpluses such as China are reinvesting their accumulated earnings of exports of traded goods in a SWF as well, this raising different a number of further issues not addressed here.

Of course, the analysis we have presented abstracts from many important issues, and we briefly discuss the possible implications of these issues for our results. First, we have worked with a single sector economy that has no absorption or structural change (Dutch disease) problems (e.g., Cordon and Neary, 1982). There are short-run and long-run angles to this. In the short run, it is likely that a resource-fuelled increase in domestic spending on non-tradeables will encounter bottlenecks of various sorts arising both from the productive capacity of the economy and the administrative capacity of government. Van der Ploeg and Venables (2010) look at the case where shortages of non–tradeable capital (e.g. human capital) give rise to such such bottlenecks. This creates a case for parking some revenue in foreign assets, although only on a temporary basis until absorption issues have been overcome and domestic supply curves have flattened out. The long-run aspect relates to the impact of resources on the real exchange rate and hence the non-resource tradable sector. This becomes a policy issue if there are market failures, such as learning economies in the tradable sector (e.g., van Wijnbergen 1984). Placing revenues in a SWF may appear to mitigate exchange rate appreciation and address this issue, but two points need to be made. One is that it is not first-best policy; the problem is failure to internalise an externality, and micro-policy should be used to solve it. The other is that the growth of the non-traded sector, real exchange appreciation, and associated contraction of tradeables is avoided only if the SWF provides a commitment to *never* allow extra spending on non-tradeables – a situation which is neither credible nor efficient.

The second issue is to do with political economy. Governments of resource-rich countries come under pressure to spend (through both formal and illicit channels) and the record of many countries is one of saving too little from resource revenues.[26] The time horizon of governments is too short to handle many aspects of resource management – revenue management, and also the establishment of credible fiscal regimes or efficient depletion paths. Numerous policy initiatives are directed at this problem, for example the Extractive Industries Transparency Initiative and Natural Resource Charter.[27] The approach of this paper is to establish what a benevolent and far-sighted government *should* do, but it is interesting to note that this involves putting relatively higher weight on the current (poor) generation and more attention on spending in the domestic economy than does a standard PIH recommendation. Delivering benefits this way is, perhaps, more politically sustainable than placing revenues in a SWF. The relatively liquid nature of a SWF makes it easy for politicians to raid for short-term electoral gains or populist reasons, this reducing its efficiency relative to physical investment in the domestic economy.[28]

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[26] Ossowski et al. (2008) estimated that the average change in government expenditure per unit change in resource revenue was 93% during the period 1974-81, although dropped to 50% during 2000-05.

[27] http://eiti.org/ and http://www.naturalresourcecharter.org/ respectively.

[28] There are numerous political economy approaches. For example, there may be voracious depletion of resources with competing factions (Tornell and Lane, 1999; van der Ploeg, 2010c), over-spending on partisan public goods (Alesina and Tabellini, 1990) and under-investment in a SWF as the latter is easily raided by political rivals (Collier et al., 2010). Resource-rich economies may also get addicted to high public spending and find it difficult to curb spending once resource revenues dry up (e.g., Leigh and Olters, 2006; Olters, 2007).

Third, we have abstracted from risk arising from geological, environmental, and above all price uncertainty. How does this alter revenue management? There are several arguments. First, diversification suggests reducing dependence on a volatile revenue stream. This might involve both relatively rapid depletion of the resource itself, and investment in assets (either domestic or foreign) whose returns are not correlated with the price of the resource. Second, prudence (i.e. a negative third derivative of utility) would suggest a somewhat higher level of saving, although has no direct bearing on the form that this saving should take (van der Ploeg, 2010b). Third, and most importantly, volatility creates a case for smoothing the impact of resources on the domestic economy through the development of a short- or medium-run stabilisation fund into which payments are made (withdrawn) as the resource price is high (low). There are important remaining research questions concerning the size and operating rules of such funds, the answer depending on the stochastic process governing resource price movements, the marginal costs and benefits of fluctuations in domestic activity, and the possibility of alternative hedging strategies.[29],[30] However, it is important to draw a clear distinction between the different objectives, operating rules, and asset structures of a short- or medium-run stabilization fund from those of a long-term SWF, a distinction which is made by some, but not all, countries operating such funds.[31] Our arguments have focussed on the latter, and sought to clarify policies for long-term consumption, saving, and investment decisions.

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[29] See Gelb and Grassman (2008) for discussion of these issues.

[30] An alternative is to hedge resource price risk, a strategy that has been successfully used by Mexico but has proved politically infeasible elsewhere. See http://cachef.ft.com/cms/s/0/f177cdcc-9bdf-11de-b214-00144feabdc0.html

[31] Ossowski et al (2008) provide listings and descriptions of funds currently operating.

Appendix 1: Data (section 2)

Apart from the public and publicly guaranteed debt and GNI variables which we obtained from World Bank Development Indicators (April 2008), we use the same years, countries, and explanatory variables as Akitobi and Stratmann (2008). When we use their definition of PPG debt and GNI, we replicate their regression results but get different standard errors as we report standard errors that are robust to serial correlation as well as heteroskedasticity (Arellano, 1987; Stock and Watson, 2008). *Country/ years*: Argentina (1994-2003), Brazil (1994-2003), Bulgaria (1994-2003), Chile (1999-2003), Colombia (1997-2003), Cote d'Ivoire (1998-2003), Croatia (1996-2003), Ecuador (1995-2003), Egypt, Arab Rep. (2001-2003), El Salvador (2002-2003), Malaysia (1996-2003, Mexico (1994-2003), Morocco (1998-2003), Pakistan (2003), Panama (1996-2003), Peru (1997-2003), Philippines (1998-2003), Russian Federation (1998-2003), South Africa (1995-2003), Thailand (1997-2003), Tunisia (2002-2003), Turkey (1996-2003), Ukraine (2000-2003), Uruguay (2001-2003), Venezuela, (1994-2003).

Appendix 2: Saddlepath dynamics (section 3)

We establish three propositions, which are used in the discussion of section 3.2.

Proposition 1: With a temporary 'small' windfall of size $N > 0$ from time $T_0 \geq 0$ to $T_1 > T_0$, we have

$$\Delta C(0) + \Delta G(0) = \exp(-\lambda_u T_0)\left[1 - \exp(-\lambda_u T)\right] N > 0, \tag{40}$$

where $\lambda_u = \frac{1}{2} r* + \frac{1}{2}\sqrt{r*^2 + 8\sigma\Pi'Y} > r* > 0$, $T \equiv T_1 - T_0 > 0$, and Δ indicates deviations from the benchmark trajectories.

Proof: Linearizing equations (4) and (5) around a steady state with zero N, that is $F_\infty = \bar{F} = 0$, $\Pi_\infty = 0$ and $C_\infty = Y/(1+\psi^\sigma)$, yields:[32]

$$\dot{x} = Ax - \begin{pmatrix} \Delta N \\ 0 \end{pmatrix} \text{ with } x \equiv \begin{pmatrix} \Delta F \\ \Delta C \end{pmatrix}, \; A \equiv \begin{pmatrix} r* & 1+\psi^\sigma \\ \Sigma & 0 \end{pmatrix} \text{ and } \Sigma \equiv 2\sigma\Pi'C_\infty > 0.$$

The eigenvalues of the matrix A are:

$$\lambda_s = \frac{1}{2} r* - \frac{1}{2}\sqrt{r*^2 + 4\Sigma(1+\psi^\sigma)} < 0 \text{ and } \lambda_u = \frac{1}{2} r* + \frac{1}{2}\sqrt{r*^2 + 4\Sigma(1+\psi^\sigma)} > r* > 0.$$

Since one of the eigenvalues is positive and the other one is negative, the system displays saddlepath dynamics. We solve this system with spectral decomposition of the matrix A (cf., Buiter, 1984):

$$A \equiv \begin{pmatrix} r* & 1+\psi^\sigma \\ \Sigma & 0 \end{pmatrix} = N^{-1} \begin{pmatrix} \lambda_s & 0 \\ 0 & \lambda_u \end{pmatrix} N \text{ with } N \equiv \begin{pmatrix} N_{ss} & N_{su} \\ N_{us} & N_{uu} \end{pmatrix},$$

where the columns of the matrix N stack the eigenvectors of the matrix A. The eigenvectors are calculated from the equations:

$$N A = \begin{pmatrix} r* N_{ss} + \Sigma N_{su} & (1+\psi^\sigma)N_{ss} \\ r* N_{us} + \Sigma N_{uu} & (1+\psi^\sigma)N_{us} \end{pmatrix} = \begin{pmatrix} \lambda_s N_{ss} & \lambda_s N_{su} \\ \lambda_u N_{us} & \lambda_u N_{uu} \end{pmatrix} = \begin{pmatrix} \lambda_s & 0 \\ 0 & \lambda_u \end{pmatrix} N.$$

Normalizing such that $N_{uu} = 1$, we obtain $N_{us} = \lambda_u/(1+\psi^\sigma) = \Sigma/(\lambda_u - r*) > 0$ from equating the two elements in the bottom row of the above matrix equation. Note that the top row gives $N_{ss} = -\Sigma/(1+\psi^\sigma) < 0$. Defining the vector $z \equiv N x$, we obtain

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[32] We assume $\Pi'>0$ at the steady state. That is not strictly necessary as long as $\Pi'>0$ at all points on the adjustment path towards the steady state.

$z_u(t) = \int_0^\infty \exp(-\lambda_u t) N_{us} \Delta N(t) \mathrm{d}t$ provided we assume that $\lim_{t\to\infty} \exp(-\lambda_u t) z_u(t) = 0$ holds. Restricting the solution to the stable manifold, we obtain $\Delta C(t) = -N_{uu}^{-1} N_{us} \Delta F(t) + N_{uu}^{-1} z_u(t)$ or

$$\Delta C(t) = -N_{us}\Delta F(t) + z_u(t) = \left(\frac{\lambda_u}{1+\psi^\sigma}\right)\left[\int_t^\infty \exp(-\lambda_u(t'-t))\Delta N(t')\mathrm{d}t' - \delta F(t)\right].$$

With the step function $\Delta N(t) = N$, $T_0 < t \le T_1$ and zero at all other instants of time, this equation becomes:

$$\Delta C(t) = \left(\frac{\lambda_u}{1+\psi^\sigma}\right)\left[\exp\left(-\lambda_u(T_0 - t)\right)\left(\frac{1-\exp(-\lambda_u T)}{\lambda_u}\right)N - \Delta F(t)\right], \quad 0 < t \le T_0$$

$$\Delta C(t) = \left(\frac{\lambda_u}{1+\psi^\sigma}\right)\left[\left(\frac{1-\exp\left(-\lambda_u(T_1 - t)\right)}{\lambda_u}\right)N - \Delta F(t)\right], \quad T_0 < t < T_1, \qquad (41)$$

$$\Delta C(t) = -\left(\frac{\lambda_u}{1+\psi^\sigma}\right)\Delta F(t), \quad t \ge T_1.$$

Since $\Delta F(0)=0$, it follows that $\Delta C(0) = \exp(-\lambda_u T_0)\left[\dfrac{1-\exp(-\lambda_u T)}{1+\psi^\sigma}\right]N > 0$. The initial jump in total consumption thus equals (40). Q.E.D.

Proposition 2: With a temporary 'small' windfall of size N starting at time 0 and finishing at $t = T_1$, we have $\Delta F(t) < 0$ for all $t > 0$, $\Delta\dot{F}(t) < 0$ for $0 < t < T_1$ and $\Delta\dot{F}(t) > 0$ for $t > T_1$. We also have (7) and (8). Comparing the outcome in the capital scarce economy with that under the PIH, we have (10).

Proof: Setting $T_0 = 0$ and substituting the last two expressions for $\Delta C(t)$ given in (41) into $\Delta\dot{F}(t) = r*\Delta F(t) + (1+\psi^\sigma)\Delta C(t) - \Delta N(t)$, and making use of $r* - \lambda_u = \lambda_s$ yields:

$$\Delta\dot{F}(t) = -\exp\left(-\lambda_u(T_1 - t)\right)N + \lambda_s\Delta F(t), \quad 0 < t < T_1,$$
$$\Delta\dot{F}(t) = \lambda_s\Delta F(t), \quad t \ge T_1.$$

Solving these differential equations with the initial condition $\Delta F(0) = 0$, we obtain:

$$\Delta F(t) = -\left[\frac{\exp(\lambda_u t) - \exp(\lambda_s t)}{\lambda_u - \lambda_s}\right]\exp(-\lambda_u T_1)N < 0, \quad 0 < t \le T_1,$$
$$\Delta F(t) = \exp\left(\lambda_s(t - T_1)\right)\Delta F(T_1) \to 0 \text{ as } t\to\infty, \quad t > T_1, \qquad (42)$$

where $\Delta F(T_1)$ is as given in (7). It follows that $\Delta F(t) < 0$ for all $t > 0$. Differentiation of (42) yields:

$$\Delta\dot{F}(t) = -\left[\frac{\lambda_u\exp(\lambda_u t) - \lambda_s\exp(\lambda_s t)}{\lambda_u - \lambda_s}\right]\exp(-\lambda_u T_1)N < 0, \quad 0 < t \le T_1,$$
$$\Delta\dot{F}(t) = \lambda_s\exp\left(\lambda_s(t - T_1)\right)\Delta F(T_1) > 0, \quad t > T_1, \qquad (43)$$

Hence, $\Delta\dot{F}(t) < 0, 0 < t < T_1$ and $\Delta\dot{F}(t) > 0, t > T_1$. Using (41), $\Delta C(T_1) + \Delta G(T_1) = -\lambda_u\Delta F(T_1)$ and thus using (7):

$$\Delta C(T_1) + \Delta G(T_1) = \left(\frac{\lambda_u}{\lambda_u - \lambda_s}\right)\left[1 - \exp\left((\lambda_s - \lambda_u)T_1\right)\right] N > 0. \tag{44}$$

Hence, expression (8) follows. To establish the inequality sign in (10), we note that (44) becomes equal to the outcome under the PIH, $\left[1 - \exp(-r * T_1)\right]N$, if $\Sigma^* \equiv 4\Sigma(1 + \psi^\sigma) = 0$. If we differentiate and use $\lambda_u + \lambda_s = r*$, we find that:

$$\frac{\partial\left[\Delta C(T_1) + \Delta G(T_1)\right]}{\partial \Sigma^*} = \left[\frac{\exp\left((\lambda_s - \lambda_u)T_1\right)N}{4(\lambda_u - \lambda_s)^3}\right]\left\{-r^*\left[1 - \exp\left((\lambda_s - \lambda_u)T_1\right)\right] + 2T_1\lambda_u(\lambda_u - \lambda_s)\right\}.$$

To sign the expression in the curly brackets, we note that it equals zero and its derivative with respect to T_1 is positive, $(2\lambda_u - r*)(\lambda_u - \lambda_s) > 0$, if $T_1 = 0$. For positive T_1, this derivative is even more positive. Hence, the term in curly brackets is positive for all $T_1 > 0$ and therefore consumption at $t = T_1$ is more than under the PIH as indicated in expression (10). Q.E.D.

Proposition 3: With an anticipated 'small' windfall of size $N > 0$ starting at $t = T_0$ and finishing at $t = T_1$, we have

$$\Delta \dot{F}(t) > 0 \text{ for } 0 < t < T_0, \quad \Delta \dot{F}(t) < 0 \text{ for } T_0 < t < T_1 \text{ and } \Delta \dot{F}(t) < 0 \text{ for } t > T_1.$$

A bigger T_0 implies a bigger reduction in foreign debt at the end of the windfall.

Proof: Upon substitution of (41) into the differential equation for $\Delta F(t)$, we obtain:

$$\Delta \dot{F}(t) = \exp\left(-\lambda_u(T_0 - t)\right)\left[1 - \exp(-\lambda_u T)\right]N + \lambda_s \Delta F(t), \quad 0 < t \leq T_0,$$

$$\Delta \dot{F}(t) = \left[1 - \exp\left(-\lambda_u(T_1 - t)\right)\right]N + \lambda_s \Delta F(t), \quad T_0 < t < T_1,$$

$$\Delta \dot{F}(t) = \lambda_s \Delta F(t), \quad t \geq T_1.$$

Forward integration of this differential equation yields:

$$\Delta F(t) = \exp(\lambda_s t)\left[1 - \exp(-r * t)\right]\exp(-\lambda_u T)\left[1 - \exp(-\lambda_u T)\right]N / r* > 0, \quad 0 < t \leq T_0,$$

$$\Delta F(t) = \left[\frac{\exp(\lambda_s t) - 1}{\lambda_s}\right]\exp(-\lambda_u T_0)\left[1 - \exp(-\lambda_u T)\right]N + \left[\frac{1 - \exp\left(\lambda_s(t - T_0)\right)}{\lambda_s}\right]N, \; T_0 < t \leq T_1,$$

$$\Delta F(t) = \exp\left(\lambda_s(t - T_1)\right)\Delta F(T_1) \rightarrow 0 \text{ as } t \rightarrow \infty, \quad t > T_1.$$

Ahead of the windfall ($t \leq T_0$), the country borrows to make possible an increase in consumption ($\Delta F(t) > 0$). During the windfall ($T_0 < t \leq T_1$), the country starts to pay back its debt and eventually build up assets. At the end of an anticipated windfall, we have:

$$-\Delta F(T_1) = -\left(\frac{N}{\lambda_s}\right)\left\{1 - \exp\left(\lambda_s T_1\right) - \left[1 - \exp(\lambda_s T_1)\right]\exp(-\lambda_u T_0)\left[1 - \exp(-\lambda_u T)\right]\right\} > 0.$$

Note that

$$\frac{\partial - \Delta F(T_1)}{\partial T_0} = -\left(\frac{N}{\lambda_s}\right)\left\{-\lambda_s \exp\left(\lambda_s T_1\right) + \right.$$

$$\left[\lambda_u \exp(-\lambda_u T_0) - (\lambda_u - \lambda_s)\exp\left(-(\lambda_u - \lambda_s)T_0 + \lambda_s T\right)\right]\left[1 - \exp(-\lambda_u T)\right]\right\}$$

and thus $\frac{\partial - \Delta F(T_1)}{\partial T_0} = -\left(\frac{N}{\lambda_s}\right)\left\{-\lambda_s \exp\left(\lambda_s T\right) + \left[\lambda_u - (\lambda_u - \lambda_s)\exp(\lambda_s T)\right]\left[1 - \exp(-\lambda_u T)\right]\right\} > 0$

at $T_0 = 0$. Hence, the bigger T_0, the more debt is reduced at the end of the windfall. After the windfall ($t > T_1$), the second term in the expression for $\Delta F(t)$ gradually vanishes as $t \to \infty$. Q.E.D.

Appendix 3: Comparative statics of production

Section 4: The production function is $Y = f(K, S)$ (with labour force normalised at unity and constant return to private capital and labour). Profit maximisation implies $(1-\tau) f_K(K, S) = r^* + \delta_K$ this implicitly defining $K(S, \tau)$. Totally differentiating, $f_{KK} \, dK = \dfrac{f_K \, d\tau}{1 - \tau} - f_{KS} \, dS$. The wage is

$W(S, \tau) = (1 - \tau)\big[f(K(S, \tau), S) - K(S, \tau) f_K(K(S, \tau), S)\big]$ so that

$dW = W_\tau d\tau + W_S dS = -f(K(S, \tau), S) d\tau + (1 - \tau) f_S(K(S, \tau), S) dS$ (so $W_\tau = -Y$)

Income is $Y(S, \tau) = f(K(S, \tau), S)$. Totally differentiating gives

$$dY = Y_\tau d\tau + Y_S dS = \frac{(f_K)^2}{f_{KK}}\left(\frac{d\tau}{1 - \tau}\right) + \left[f_S - \frac{f_K f_{SK}}{f_{KK}}\right] dS$$

For the Cobb-Douglas case $Y = L^{1-\alpha} K^\alpha S^\gamma$ (with $L = 1$), we have

$$Y_\tau = \frac{(f_K)^2}{f_{KK}}\left(\frac{1}{1 - \tau}\right) = \left(\frac{\alpha}{\alpha\text{-}1}\right)\left(\frac{1}{1 - \tau}\right), \quad Y_S = \left[f_S - \frac{f_K f_{SK}}{f_{KK}}\right] = \left(\frac{\gamma}{1 - \alpha}\right)\left(\frac{Y}{S}\right)$$

and $W_S = (1 - \alpha)(1 - \tau) Y_S$.

Section 5: Profit maximisation implies $f_K(K) - \delta_K = r = r^* + \Pi(K - E)$, this implicitly defining $K(E)$. Hence $r(E) = f_K(K(E)) - \delta_K$ and $W(E) = f(K(E)) - K(E) f_K(K(E))$. Comparative statics are; $K_E = \Pi' / (\Pi' - f_{KK})$, $r_E = \Pi' f_{KK} / (\Pi' - f_{KK})$, $W_E = -\Pi' K f_{KK} / (\Pi' - f_{KK})$, so $W_E + K r_E = 0$.

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