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## **OxCarre Research Paper 66**

# **Bottlenecks in Ramping Up Public Investment**

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## BOTTLENECKS IN RAMPING UP PUBLIC INVESTMENT

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### Abstract

A windfall in a developing economy with capital scarcity and investment adjustment costs facing a temporary windfall should be used to give more consumption to poorer present generations and to speed up development by ramping up public investment and paying off debt taking due account of the increasing inefficiency as investment gets ramped up. The optimal strategy requires negative genuine saving; the permanent income requires zero genuine saving. The optimal real consumption increments are smaller once one allows for absorption constraints resulting from Dutch disease and sluggish adjustment of ‘home-grown’ public capital.

**Keywords:** optimal management of windfalls, economic development, capital scarcity, public capital, PIMI, investment adjustment costs, absorption constraints, genuine saving, Dutch disease

**JEL codes:** E60, F34, F35, F43, H21, H63, O11, Q33

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## 1. Introduction

Many developing economies enjoying a natural resource windfall face the challenge of how to use this once in a lifetime opportunity to speed up the process of economic development and structural transformation. In practice many resource rich economies show a dismal growth performance (Sachs and Warner, 1997), especially if they have poor institutions (Mehlum, et al., 2006; Bosschini, et al., 2007) or low degrees of financial development (van der Ploeg and Poelhekke, 2010). This dismal performance may be due to appreciation of the real exchange rate and decline of the growth enhancing manufacturing sectors, the notorious volatility of commodity prices, corruption, and sustaining bad policies such as import substitution or excessive borrowing or a too generous welfare state (e.g., van der Ploeg, 2011). In this paper we take a different tack and assess the optimal way of harnessing natural resource windfalls for economic growth and development in a variety of models of small open developing economies.

The orthodox policy view has been to save the windfall in a sovereign wealth fund and live off the interest on the fund afterwards (e.g., Barnett and Ossowski, 2003). This view gives rise to the *bird in hand* policy, which leads to no increase in consumption ahead of the windfall and a gradual buildup of consumption during the windfall. After the windfall, the sovereign wealth fund is gradually depleted which leads to a gradual withering away of the earlier increases in consumption. If future resource revenue can be used as collateral for borrowing, one obtains the *permanent income* policy which amounts to borrowing ahead of the windfall and paying off the debt and accumulating a sovereign wealth fund sufficient to sustain a permanent increase in consumption during the windfall. Although the bird in hand policy is often advocated on grounds of prudence, the volatility of consumption compared with the permanent income policy leads to large welfare losses. These two popular policies might be relevant for mature oil rich economies such as Norway or the Netherlands, but are unsuitable for developing economies.<sup>1</sup>

First, current generations of citizens in developing economies are poorer than future generations. Marginal utility of consumption is thus higher for current than for future generations. It is therefore optimal from this perspective to put more of the consumption increments upfront. The permanent income and bird in hand policies fail to do this and are thus not appropriate for developing economies.

Second, the permanent and bird in hand policies fail to take account of the capital scarcity that is prevalent in many developing economies. Such economies are typically not well integrated into international capital markets and borrowing for domestic investment often requires payment of a higher interest rate than the world interest rate whilst there are many public investment projects which could

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<sup>1</sup> Surveys of harnessing windfalls of foreign exchange in developing economies are offered in Collier et al. (2010) and van der Ploeg and Venables (2011c). A useful two-period analysis is presented in Venables (2010). Here we focus on the sources of bottlenecks that must be faced when ramping up public investment in developing economies.

generate a much higher rate of interest than the world interest rate. The interest premium increases with the degree of capital scarcity and decreases with the ability to pay, so we postulate that it increases with the ratio of outstanding debt to national income including the size of the natural resource windfall. In earlier work we have showed that is optimal to depart from the permanent income policy by hiking up consumption less strongly, using the remainder of the windfall to alleviate capital scarcity and gradually ramp up investment in the domestic economy, and thus speed up the process of economic development (van der Ploeg and Venables, 2011a).

Third, scaling up investment leads to absorption problems, so that investment is more costly in the early stages of economic development when investment rates are high. Recent studies suggest that of spending on public investment only 40 to 60 percent gets delivered and leads to effective accumulation of public sector capital (Dabla-Norris et al., 2011; Gupta et al., 2011). As the rate of public investment is ramped up, the efficiency of public investment deteriorates (cf., Berg, et al., 2011)<sup>2</sup>. We thus extend earlier work (van der Ploeg and Venables, 2011a) to allow for internal costs of adjustment of public investment and show that these also capture the increasing cost occurred when rapidly scaling up public investment. This extension captures that absorption problems frustrate rapid economic development. It also generates bigger returns on public investment and thus to a more realistic calibration of the model to developing economies. We calibrate our model to the low income countries and show the optimal way of harnessing a temporary windfall in a small economy with a relatively large sovereign debt.

Fourth, windfalls of foreign exchange lead to extra demand and pressure on the non-traded sectors to expand and are thus typically associated with increases in the price of non-tradables (appreciation of the real exchange rate) and reallocation of labor and capital from the traded to the non-traded sectors (Corden and Neary, 1982; Corden, 1984). The bird in hand policy and the permanent income policy fail to deliver an optimal response to such Dutch disease effects. It may be optimal to smooth the appreciation of the real exchange over time and thus have a small, long-lasting rather than a large, temporary decline of the traded sectors. It is thus optimal to put up with some Dutch disease (Torvik, 2001). We reconsider the optimal public investment and real exchange rate strategy in a fully specified general-equilibrium model of a two-sector Scandinavian economy with capital scarcity and a rising cost of public investment.

Related is that the optimal response should take account of the need of developing economies to ‘invest to invest’. Hence, developing economies need teachers to train teachers, nurses to train nurses, and roads to

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<sup>2</sup> Berg et al. (2011) provides a very interesting complimentary analysis of a fully specified, discrete-time DSGE model with a tradable, non-tradeable and resource sector where the cost of ramping up public investment also increases. The difference is, on the one hand, that the specification of these costs differs from our internal cost of adjustment approach which is derived from recent public investment measures of inefficiency (Dabla-Norris et al., 2011; Gupta, et al., 2011), and, on the other hand, the emphasis is on ad hoc saving, spending and investment rules whilst the emphasis in our continuous-time model is on deriving optimal responses to exogenous windfalls.

make roads. Home grown capital produced by the non-traded sector rather than imported from abroad is thus needed for successful economic development, but such capital takes time to deliver and leads to a different set of temporary absorption problems in the non-traded sector. This is why the optimal real consumption increments cannot be so large if a bigger share of consumption and investment goods has to be produced at home. In earlier work we show that in such a setting it is optimal to temporarily park some of the windfall in a sovereign wealth fund until the non-traded sectors are able to deliver the investment goods necessary for economic development (van der Ploeg and Venables, 2011b). We also extend this work to allow for the problem of scaling up investment.

The outline of the paper is as follows. Section 2 shows how the permanent income hypothesis can be adapted for macroeconomic windfall management (the benchmark). Section 3 discusses the inefficiency of public investment and the efficiency-adjusted measure of the capital stock. It allows for a framework of internal adjustment costs for public investment and shows that, as in section 2, investment decisions should be completely independent of foreign exchange windfalls. Section 4 presents some empirical evidence for interest spreads in developing economies, especially if their debt is high relative to national income. Section 5 contains the thrust of the paper. It shows that it is optimal to use part of the windfall to ramp up public investment and illustrates this with some simulations from a calibrated growth model with capital scarcity and increasing costs of ramping up public investment. Furthermore, section 5 discusses how bad the often recommended permanent income and bird in hand rules perform in a general equilibrium context with capital scarcity and absorption constraints. Section 6 analyzes the best way to harness resource windfalls in the presence of Dutch disease within the context of a two-sector developing economy with equal factor intensities. Absorption constraints will be more severe if capital cannot be imported but must be ‘home grown’. Section 7 concludes.

## **2. Benchmark: the permanent income hypothesis**

The benchmark is a partial equilibrium analysis, which decides how much of the resource windfall to save and how much to consume under the assumption that this does not affect prices, interest rates or factor intensities of the economy. We thus suppose that households receive exogenous production income  $Y$  and government transfers  $T$ . We also suppose that households have no access to the international capital market, so that their level of consumption is given by  $C = Y + T$ . All foreign assets  $A$  are supposed to be held by the government. These assets earn a return equal to the time-invariant world interest rate  $r$ . The economy’s budget constraint or equivalently the flow government budget constraint is thus given by:

$$(1) \quad \dot{A} = rA + N + Y - C = rA + N - T, \quad A(0) = A_0,$$

where  $N$  is the size of the exogenous resource windfall. The first part of (1) says that the current account must equal the increase in assets of the nation; the second part indicates that the government surplus equals the increase in government assets. The size of the sovereign wealth fund of this economy is thus indicated by  $A$ . Given that the economy is solvent and there are no Ponzi games, (1) implies that the present value of production income and the resource windfall plus the level of sovereign wealth should cover the present value of consumption. Alternatively, the present value of the resource windfall plus the level of sovereign wealth should cover the present value of government transfers. These two present value budget constraints can also be written as:

$$(2) \quad Y_p(t) + N_p(t) + rA(t) = r \int_t^{\infty} e^{-r(s-t)} C(s) ds, \quad N_p(t) + rA(t) = r \int_t^{\infty} e^{-r(s-t)} T(s) ds,$$

where the permanent values of production income and the resource windfall are given by, respectively,  $Y_p(t) \equiv r \int_t^{\infty} e^{-r(s-t)} Y(s) ds$  and  $N_p(t) \equiv r \int_t^{\infty} e^{-r(s-t)} N(s) ds$ . In situ resource wealth equals  $N_p/r$ , so the permanent component of the windfall  $N_p$  can be interpreted as the resource annuity that can be financed out of the windfall.

Private utility is given by:

$$(3) \quad \int_0^{\infty} \frac{C(t)^{1-1/\sigma} - 1}{1-1/\sigma} e^{-\rho t} dt \text{ if } \sigma \neq 1 \text{ or } \int_0^{\infty} \ln(C(t)) e^{-\rho t} dt \text{ if } \sigma = 1,$$

where  $\rho > 0$  is the rate of time preference,  $\sigma > 0$  is the coefficient of intertemporal substitution,  $1/\sigma$  is the coefficient of relative risk aversion, and  $1+1/\sigma$  is the coefficient of relative prudence.<sup>3</sup> The parameter  $\sigma$  is thus doing many jobs.<sup>4</sup> Relying on a positive third derivative of the utility function to capture prudence abstracts from prudence considerations in second-best economies.

The main decision of the government is whether to consume or accumulate sovereign wealth. It thus chooses the time paths of government transfers and saving to maximize private utility subject to the budget constraint (2). This yields the familiar Keynes-Ramsey rule or Euler equation which says that the

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<sup>3</sup> We abstract from population growth  $\pi$  and productivity growth  $\gamma$  but we can easily relax this by supposing that all quantity variables including the windfall are in intensive form and scaled by  $e^{(\gamma+\pi)t}$ . The interest rate  $r$  thus corresponds to the growth-corrected world interest rate, where the growth rate of the economy equals  $\gamma + \pi$ . Given the assumption that (3) corresponds to a utilitarian social welfare function, the parameter  $\rho$  corresponds to the rate of time preference minus the term  $\pi + (1-1/\sigma)\gamma$ .

<sup>4</sup> Although it is possible to separate risk aversion from intertemporal substitution (Epstein and Zin, 1989) or to allow for a separate term to allow for prudence (van der Ploeg, 1993), this has seldom been done in macroeconomic policy applications.

growth in private consumption should respond to the difference between the interest rate and the rate of time preference, especially if intertemporal substitution is strong:

$$(4) \quad \dot{C} / C = \sigma(r - \rho).$$

Upon substitution of (4) into the present value budget constraint and supposing that  $r = \rho$ , we get the consumption function and the optimal level of government transfers handed out to citizens:

$$(5) \quad C = Y_p + N_p + rA, \quad T = N_p + rA - (Y - Y_p),$$

where the propensity to consume out of permanent production, the resource annuity and sovereign wealth income is unity. Upon substitution of (5) into the flow budget constraints (1), we get an expression for the optimal current account or the optimal government surplus:

$$(6) \quad \dot{A} = N - N_p + Y - Y_p.$$

Government transfers and thus private consumption respond to the permanent and not the actual value of resource windfall. The economy achieves this by borrowing ahead of a windfall, saving during the windfall, and financing the sustained increase in transfers and consumption after the windfall by the interest on the accumulated sovereign wealth fund. The government surplus and the current account thus respond to the temporary component of the windfall. The non-windfall primary deficit ( $N - \dot{A}$ ) is driven by the permanent component of the windfall. If the size of the windfall decays at the exponential rate  $\eta \geq 0$ ,

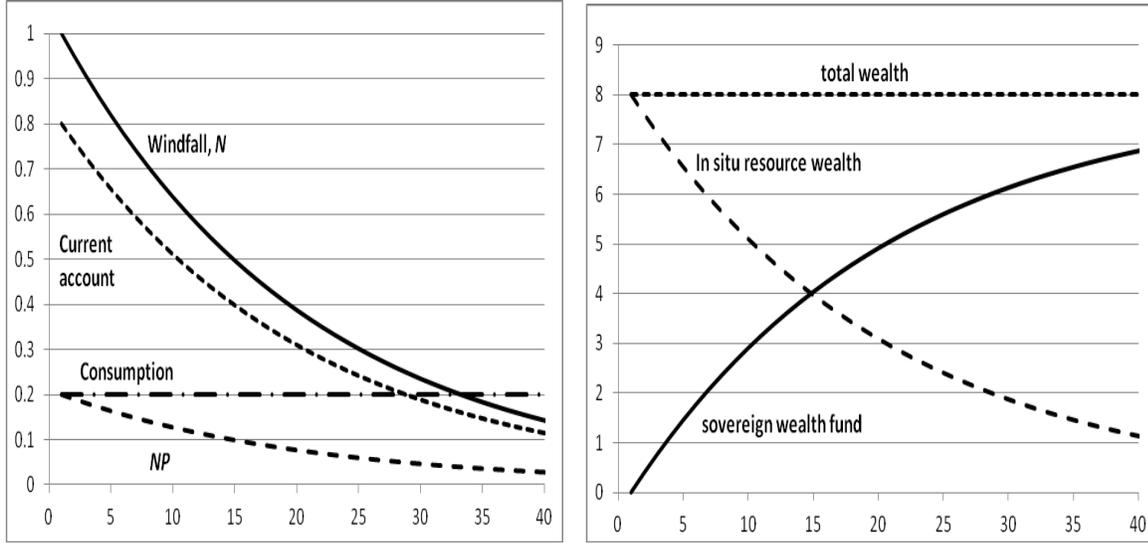
$$N_p = \left( \frac{r}{r + \eta} \right) N \leq N \text{ and } \dot{A} = \left( \frac{\eta}{r + \eta} \right) N + Y - Y_p. \text{ Hence, if the windfall is temporary and oil is expected}$$

to flow for another ten years ( $\eta = 0.1$ ) and thus (if  $r = 0.025$ ) one fifth of the windfall is the permanent component and can be consumed and four fifths is temporary and must be saved in sovereign wealth and shows up in temporary surpluses on the current account as illustrated in the time paths of fig. 1.

The first part of (5) and the right panel of fig. 1 reminds one of the celebrated Hartwick (1977) rule, since any depletion of in situ resource wealth must be made up by an accumulation of sovereign wealth (solid line) of equal magnitude. If the windfall is more or less permanent as in Iraq where oil may last for another 350 years ( $\eta \approx 0$ ) and all of the windfall is saved and there is no effect of the windfall on the current account.<sup>5</sup>

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<sup>5</sup> A derivation of the permanent income rule for the temporary oil windfall of Ghana, its comparison with the bird in hand rule and the constitutional rule, and its sensitivity to population growth, myopia, intertemporal substitution and finite lives is given in van der Ploeg et al. (2011).

**Fig. 1: Permanent income prescription for a temporary windfall**

From equations (5) and (6), a temporary fall in output due to a bad harvest or recession ( $Y < Y_p$ ) requires temporary borrowing from abroad to finance temporary higher government transfers in order to sustain the permanent increase in private consumption. Finally, the effects of a higher world interest rate ( $r > \rho$ ) depend on the elasticity of intertemporal substitution  $\sigma$ .<sup>6</sup>

### 3. Reinterpreting the PIMI: inefficiency and cost of ramping up public investment

It is well recognized that ramping up public investment in developing economies encounters many economic and political constraints. As public investment is often half or more of total investment in these economies, this is a serious constraint on potential growth. It takes a long time to recognize, implement and realize a public investment project, especially if it is a larger project. For example, a developing country may decide to make a large part of land suitable for modern agriculture which requires investments in large-scale irrigation, but this might involve many years of negotiation with local chiefs to get permission to use the land for this purpose. It may take years before the bureaucracy, local

<sup>6</sup> In general, we see that if  $r \neq \rho$ , equations (5) and (6) become

$$C = \left[ \frac{(1-\sigma)\rho + \sigma r}{r} \right] Y_p + N_p + rA, \quad T = \left[ \frac{(1-\sigma)\rho + \sigma r}{r} \right] N_p + rA - \left\{ Y - \left[ \frac{(1-\sigma)\rho + \sigma r}{r} \right] Y_p \right\} \text{ and}$$

$$\dot{A} = (1-\sigma)(r-\rho)A + N - \left[ \frac{(1-\sigma)\rho + \sigma r}{r} \right] N_p + Y - \left[ \frac{(1-\sigma)\rho + \sigma r}{r} \right] Y_p, \text{ where with } r > \rho \text{ the term in square brackets}$$

is less (greater) than one if  $\sigma$  is greater (less) than one. So, if  $\sigma > 1$ , the intertemporal substitution dominates the income effect and the propensity to consume out of permanent income is less than unity and from (4) private consumption and government transfers rise over time. If  $\sigma < 1$ , the income effect dominates. Hence, the propensity to consume out of resource wealth exceeds unity so the paths of private consumption and transfers fall over time.

government and national government agrees to undertake a particular project. There may also not be sufficient capacity to supply the necessary investment goods. For all these reasons, not all the amount of money that is spent on public investment will result in increases in the public capital stock. The cost of public investment is thus not the increment to the value of public capital (cf., Pritchett, 2000). Although much of sub-Saharan Africa has very high investment rates (often higher than in the Asian Tigers), many projects have not delivered the results for growth and welfare that were expected. But if the efficiency-adjusted measure of public capital is used, cross-country empirical evidence suggests that public capital is a significant determinant of economic growth (Gupta et al., 2011). The quality of public investment, measured by variables capturing the adequacy of project selection and implementation, are statistically significant in explaining growth, especially in low-income countries.

To better understand how this measure of the efficiency-adjusted capital stock is constructed, the public investment management index (the *PIMI*) by income group covering the period 2007-10 gives an average of 47 percent for 40 low income and of 57 percent for 31 middle income countries based on the data described in (Dabla-Norris et al., 2011). This implies that on average only about half of public investment effort translates into productive public capital. The *PIMI* captures all four stages of the investment process: (i) project appraisal; (ii) project selection; (iii) project implementation; and (iv) project evaluation. Since not all spending on public investment results in public capital, table 1 indicates that the gap with the traditional measure of public capital varies from about 7%-points of GDP in the 1960s to 43%-points of GDP in recent years. The *PIMI*-adjusted measure of public capital has on average for all countries declined from 57 to 36 percent of GDP. In low income countries the *PIMI*-adjusted measure of public capital is only 30 percent of GDP compared with 71 percent for the unadjusted measure.

**Table 1: Unadjusted and *PIMI*-adjusted public capital stocks**

	Unadjusted public capital stock (% GDP)				<i>PIMI</i> -adjusted public capital stock (% GDP)			
	1960-70	1970-90	1990-00	2000-09	1960-70	1970-90	1990-00	2000-09
Low income	64.9	73.4	84.0	71.0	57.9	40.0	38.4	30.1
Middle income	62.7	119.3	119.1	93.2	56.9	66.5	58.0	44.4
All countries	64.0	90.3	98.3	80.2	57.4	49.8	46.4	36.1

Source: Dabla-Norris et al. (2011)

Adjusted public capital has declined in low income countries due to the low efficiency of new investments whilst in middle income countries it has been offset by large investment efforts. This is why effective public capital has fallen much more in low than in middle income countries (by 27.8 instead of 12.5%-points of GDP). This contrasts sharply with the rise in the unadjusted measure of public capital.

### 3.1. Internal costs of adjusting public capital and the PIMI

To capture the flavor of recent research into the *PIMI* (Dabla-Norris et al., 2011), we assume the presence of internal costs of adjustment (e.g., Hayashi, 1982). We thus refer to  $J$  as the total costs of public investment and  $I$  as the actual increment in public capital. The difference between the two is the internal costs of adjustment, which are quadratic in  $I$ . We then define the *PIMI* as the ratio of the actual increment in public capital to the total costs of public investment. This gives the following expression for the *PIMI*:

$$(7) \quad J = I + 0.5\phi I^2 / S \Rightarrow PIMI = I / J = 1 / (1 + 0.5\phi I / S) < 1,$$

where  $\phi > 0$  indicates the public investment adjustment cost parameter. Our specification implies that the *PIMI* falls as the rate of public investment is ramped up, which captures that absorption and other constraints become more severe as the public investment rate rises. The accumulation of the efficiency-adjusted stock of public capital  $S$  is given by the following law of motion:

$$(8) \quad \dot{S} = I - \delta S, \quad S(0) = S_0,$$

where  $\delta > 0$  indicates the depreciation rate of the public capital stock.

We consider a small open economy which produces a traded good with private capital  $K$  and some fixed factors (labor, land, etc.) and that the productivity of production is boosted by the stock of public capital  $S$ . We assume the following (intensive form) production function:

$$(9) \quad Y = E' K^{\alpha'} S^{\beta'}, \quad 0 < \alpha' < 1, \quad \beta' > 0,$$

where  $E'$  is the efficiency of private production,  $K$  the private capital stock,  $\alpha'$  the share of private capital in value added, and  $\beta'$  the elasticity of private output with respect to public capital stock.

If the *PIMI*-adjusted public capital data are used to explain growth, it turns out that previous studies may have grossly underestimated the marginal productivity of public capital by not making corrections for the effectiveness of public capital. Cross-country dynamic system GMM estimates using *PIMI*-adjusted public capital yields  $\alpha' = 0.3$ ,  $\beta' = 0.15$  and a marginal product of public capital equal to  $\beta' Y/S = 0.69$  (Gupta et al., 2011).<sup>7</sup> Project implementation (competitive bidding, complaint mechanism for procurement, internal audit, etc.) turns out to be the big bottleneck in the overall investment process. We

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<sup>7</sup> The coefficient on skilled labor is estimated to be 0.336. We could combine skilled labor and private capital, but abstract from that. A recent meta-regression analysis from widely varying estimates suggests that the average output elasticity of public capital is significant and estimated at 0.15, but imposing constant returns to scale with respect to private capital and labor leads to larger estimates and they therefore use a benchmark estimate for  $\beta'$  of 0.17 (Bom and Ligthart, 2010). But as this study is based on available estimates from past research, it could not make use of an efficiency-adjusted measure of public capital to estimate the effect on growth.

base the calibration of section 5 on these results and thus set  $\alpha' = 0.3, \beta' = 0.15, \delta = 0.025$ . During the period 2000-09 low income countries had an investment ratio of  $J/Y = 4.2$  percent and a *PIMI*-adjusted public capital ratio of  $S/Y = 30.1$  percent whilst a ballpark estimate of the *PIMI* =  $I/J$  is 0.47. Using

$$\frac{I}{S} = \frac{J \times PIMI}{Y} / \left( \frac{S}{Y} \right) = 0.042 \times 0.47 / 0.301 = 0.0656, \text{ we use (7) to calibrate the adjustment}$$

$$\text{parameter } \phi = \frac{2(1 - PIMI)}{PIMI \times I / S} = \frac{2 \times 0.53}{0.47 \times 0.0656} = 34.4. \text{ Doubling the public investment rate from 6.56 to 13.12}$$

percent thus reduces the *PIMI* from 0.47 to 0.31, so that roughly two thirds rather than half of spending on public investment does not deliver. The long-run public investment rate ( $\delta$ ) is 2.5 percent is much lower and corresponds to a higher efficiency of public investment, i.e.,  $PIMI = 0.70$ .

### 3.2. Separation result: public investment decisions independent of windfall revenue

Since we wish to focus on the cost of ramping up public investment, we assume that private capital supplied by foreigners does not face any adjustment costs. The marginal product of private capital must thus equal the sum of the interest rate and depreciation rate  $\mu$ , i.e.,  $\alpha Y / K = r + \mu$ , so that (9) becomes:

$$(9') \quad K = \left( \frac{\alpha' E' S^{\beta'}}{r + \mu} \right)^{\frac{1}{1-\alpha'}} \Rightarrow Y = ES^{\beta}, \quad E \equiv E'^{\frac{1}{1-\alpha'}} \left( \frac{\alpha'}{r + \mu} \right)^{\frac{\alpha'}{1-\alpha'}}, \quad \beta \equiv \frac{\beta'}{1-\alpha'} > \beta'.$$

Households receive wage income,  $(1-\alpha')Y$ , whilst profits,  $\alpha'Y$ , are repatriated by the foreign owners of the private capital stock. Defining net foreign assets as sovereign wealth minus the stocks of foreign direct investment, i.e.,  $F \equiv A - K$ , we obtain the current account dynamics as:

$$(10) \quad \dot{A} = rA + N + (1-\alpha')Y - C - J \Rightarrow \dot{F} = rF + N + Y - C - J - (\dot{K} + \mu K).$$

The first part of (10) shows that savings of the nation and private investment are uncorrelated. Although this was not the case for many developed countries in the era where capital markets were not fully liberalized (Feldstein and Horioka, 1980), investment and saving decisions are nowadays much more independent in these economies. The second part of (10) shows that it is indeed optimal for the country to borrow from abroad to finance all private sector investment provided its return is high enough to cover the interest plus depreciation charges. If the economy is perfectly integrated into the international capital market, the optimal level of private investment is independent of the size of the foreign exchange windfall. However, many developing economies do not have good access to international capital markets and therefore the investment decisions depend much more on available savings. Before we consider this in section 4, let us see whether this separation result also holds for public investment.

The government chooses transfers and government investment to maximize private utility (3) subject to the government budget constraint

$$(11) \quad \dot{A} = rA + N - T - J, \quad A(0) = A_0,$$

the public investment dynamics (7) and (8), and the private sector budget constraint  $C = (1 - \alpha')ES^\beta + T$ , where private sector output has been substituted from (9'). The optimality conditions give rise to the familiar Keynes-Ramsey rule (4),

$$(12) \quad I / S = (q - 1) / \phi,$$

where  $q$  is the value of public capital in resource units, and the intertemporal efficiency condition<sup>8</sup>:

$$(13) \quad \dot{q} = (r + \delta)q - (1 - \alpha')\beta ES^{\beta-1} - 0.5(q - 1)^2 / \phi$$

(see appendix). Equation (12) indicates that the rate of public investment is high if the marginal value of public capital is high. Equation (13) implies that the boost to household wages plus the reduction in adjustment costs resulting from a marginal increase in public capital must equal the rental charge plus the depreciation charge minus the expected rate of change in the value of public capital.

The main point, however, is that equations (8), (12) and (13) can be solved for the time paths of  $I$ ,  $q$  and  $S$  and that these paths are completely *independent of the size of the resource windfall*  $N$ . Public and private investment depend only on supply factors; they are high if the world interest rate is low and private sector efficiency  $E'$  is high. To give a role for demand factors, we will next allow for capital scarcity.

#### 4. Capital scarcity and the interest spread on foreign debt

In developing economies the separation result discussed in section 3.2 does not necessarily apply. Policy makers will explain that it is asking too much to put all windfall revenue in sovereign wealth when US Treasury bills offer a much lower rate of interest than the interest that has to be paid on foreign debt and an even lower rate of return than the return that can be achieved on public investment projects. The key challenge for developing economies is to harness resource windfalls for growth and development; saving in a sovereign wealth fund does not seem the right strategy. To capture the problem of capital scarcity, we adopt a kinked cost of borrowing schedule where for a debt-GNI ratio larger than  $\bar{d} > 0$  the country pays an interest premium on its debt:

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<sup>8</sup> Note that average “ $q$ ” equals marginal “ $q$ ”, analogously to Tobin’s  $Q$  for private capital (cf., Hayashi, 1982).

$$(14) \quad r = r^* + \Pi(d), \quad \Pi' > 0, \quad d \equiv \frac{D}{Y+N} \geq \bar{d}, \quad r = r^*, \quad d < \bar{d},$$

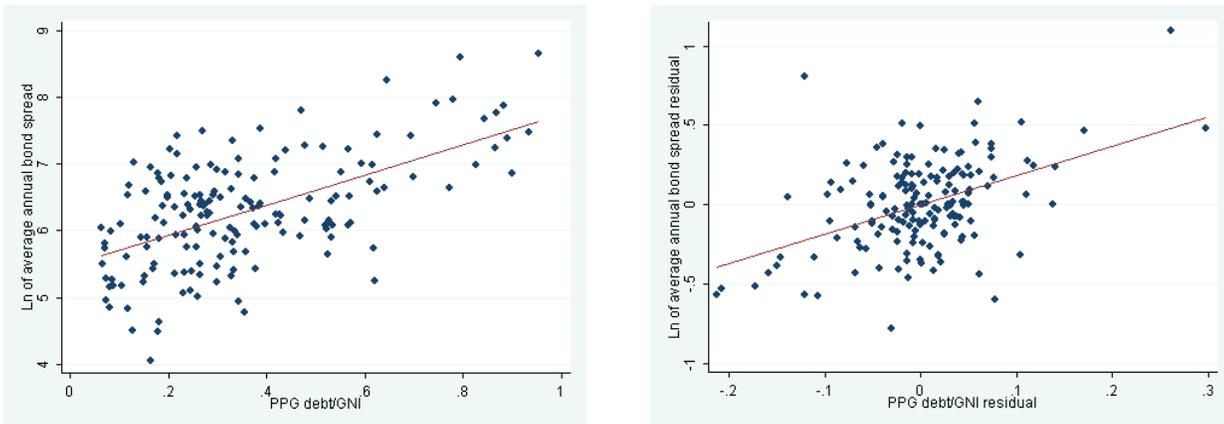
where  $r$  denotes the domestic interest rate,  $r^*$  the risk-free world rate of interest and  $D$  the level of sovereign debt. Although a large stock of public sector capital improves GNI and the ability to service sovereign debt, it cannot be collateralized and does not improve credit worthiness of the country (i.e.,  $\Pi(\cdot)$  does not depend on  $S$ ).

Fig. 2(a) suggests empirical support for the interest spread schedule (16) as there is a positive relationship between interest rate spreads and the ratio of public and publicly guaranteed (PPG) debt to GNI. Cross-country regressions suggest that interest spreads are significantly higher if the ratio of PPG debt to GNI is high, foreign reserves are low and the probability of default is high (van der Ploeg and Venables, 2011a; cf., Akitobi and Stratman, 2008) and the resulting conditional correlation between spreads and PPG debt is portrayed in fig. 2(b). The estimated semi-elasticity of the natural log of the spread with respect to the debt-GNI ratio is 1.9. In as far as resource income is part of GNI, it thus makes it easier for the country to service its debt and thus credit worthiness will be higher.

**Figure 2: Interest rate spreads and PPG debt**

(a) Unconditional\*

(b) Conditional\*\*



**Key:** \* Unconditional data for bond spread obtained from Akitobi and Stratman (2008) and for public and publicly guaranteed (PPG) debt/GNI data derived from the World Bank Development Indicators 2008. The slope coefficient corresponding to the unconditional correlation is 2.270 with standard error 0.250.

\*\* Conditional data are the errors from the estimated regression  $\ln(\text{spreads}) = 1.89 \text{ PPG debt/GNI} - 4.14 \text{ reserves/GDP} - 0.0458 \text{ output gap} + 0.296 \ln(\text{default})$ , where probability of default and reserves data are taken from Akitobi and Stratmann (2008). See van der Ploeg and Venables (2011a) for details of regression.

Of course, resource wealth may also directly improve international credit-worthiness. However, credit rating country reports suggests that natural resources feature as a weakness (attributed to increased political risk, delayed fiscal reform, lack of diversification) as often as they do as a strength (e.g., witness

recent rating improvements for Ghana and Russia). Future resource revenues might also be securitized as a way to gain improved access to international capital markets (as in some Latin American and former Soviet Union oil-rich countries), although the magnitude of credit obtained this way remains relatively small (Ketkar and Ratha, 2009). Cross-country empirical evidence suggests a very weakly significant positive effect of natural resource exports on interest rate spreads indicating, if anything, that resources *worsen* credit worthiness (van der Ploeg and Venables, 2011a). The mechanism might be through the impacts of resources on governance, political stability, and the risk of conflict. Given this insignificant effect we work with the interest schedule (16) and ignore the direct role of natural resources on credit worthiness.<sup>9</sup> Of course, substantial natural resource revenues do affect the ability to pay and improve credit worthiness in this way.

### 5. Invest in domestic economy rather than sovereign wealth: departure from separation result

So let us analyze how introducing the interest spread schedule (14) into the resource rich, small open economy with an increasing cost of ramping up public investment discussed in section 3 affects the optimal outcomes. In particular, we want to see how a temporary windfall of foreign exchange  $N$  affects the optimal public and private sector investment decisions, private consumption, borrowing and the current account when the country suffers from capital scarcity and cannot adjust its stock of public capital instantaneously. Two cases should be distinguished (cf., van der Ploeg and Venables, 2011a). The first is a windfall which is large enough to pay off sufficient debt so that the economy no longer has to pay an interest premium and can build up a sovereign wealth fund. This is relevant for rich countries with an integrated capital market and relatively large oil or other natural resource discoveries. The second is a windfall which is not large enough to fully alleviate the problem of capital scarcity. This case is relevant for resource rich, developing economies. We concentrate attention on the second case and thus extend our earlier work for the increasing cost of ramping up public investment and also relate the sovereign debt to the ability to pay of the nation, i.e.,  $Y + N$ , to reflect that poor countries have more problems borrowing on international markets than rich countries but less so if they enjoy a resource bonanza.

Private sector efficiency  $E \equiv E^{\frac{1}{1-\alpha'}} \left( \frac{\alpha'}{r^* + \mu} \right)^{\frac{\alpha'}{1-\alpha'}}$  depends on the world interest rate, since private sector

investment is done by foreign firms who do not suffer from capital scarcity. We set  $r^* = \rho$  as before. The

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<sup>9</sup> Hence, there is no empirical support for the alternative hypothesis  $\Pi = \Pi(d - N_p / [r(Y + N)])$ .

government thus chooses  $T$  and  $I$  to maximize (3) subject to (7), (8), (11),  $C = (1 - \alpha')ES^\beta + T$ , and (16??) taking  $E$  as given, where  $D = -A$ . The optimality conditions yield the following equations (see appendix):

$$(15a) \quad \dot{C} = \sigma C \left[ \Pi \left( \frac{D}{ES^\beta + N} \right) + \Pi' \left( \frac{D}{ES^\beta + N} \right) \frac{D}{ES^\beta + N} \right], \quad C(0) \text{ free},$$

$$(15b) \quad \dot{S} = \left[ \frac{1}{\phi}(q-1) - \delta \right] S, \quad S(0) = S_0,$$

$$(15c) \quad \dot{q} = \left[ r^* + \Pi \left( \frac{D}{ES^\beta + N} \right) + \Pi' \left( \frac{D}{ES^\beta + N} \right) \frac{D}{ES^\beta + N} + \delta \right] q - (1 - \alpha')\beta ES^{\beta-1} - \frac{1}{2\phi}(q-1)^2 - \beta ES^{\beta-1} \Pi' \left( \frac{D}{ES^\beta + N} \right) \left( \frac{D}{ES^\beta + N} \right)^2, \quad q(0) \text{ free},$$

$$(15d) \quad \dot{D} = \left[ r^* + \Pi \left( \frac{D}{ES^\beta + N} \right) \right] D + C + \frac{1}{2\phi}(q^2 - 1)S - ES^\beta - N, \quad D(0) = D_0$$

Equation (15a) is a modified version of the Keynes-Ramsey rule. The market does not internalize the interest spread externality and thus borrows too much from a social perspective. In contrast, the social planner modifies the interest rate (the world interest rate plus interest premium) to include the term  $\Pi'D/(Y+N)$  to correct for the interest spread externality. For an indebted economy it is thus optimal to have a rising path of consumption, since the economy consumes less upfront to pay of debt and lower the risk premium. Equation (15b) gives the same public sector capital stock dynamics as (8) and (12) above. Equation (15c) is same as the intertemporal efficiency condition for public investment (13) except that the interest premium on government debt,  $\Pi$ , plus the correction term to allow for the rising cost of public debt,  $\Pi'D/(Y+N)$ , have been added to the world interest rate and an extra term is included to allow for the reduction in the cost of borrowing resulting from a marginal increase in public capital, output and the ability to service the sovereign debt. Finally, equation (15d) gives the dynamics of government debt with the cost of public investment, transfers and output substituted.

To capture the dynamics of a temporary resource windfall, we add the following state equation to (15):

$$(16) \quad \dot{N} = -\eta N, \quad N(0) = N_0.$$

The economy (15)-(16) defines a five-dimensional system of ordinary differential equations in the predetermined state variables  $D$ ,  $S$  and  $N$  and in the non-predetermined variables  $C$  and  $q$ . Hence,  $C(0)$

and  $q(0)$  adjust instantaneously to ensure that the economy is on its three-dimensional stable manifold. The system (15)-(16) can be solved with a multiple shooting algorithm.<sup>10</sup>

So how is a windfall of oil income likely to affect the economy on impact? First, a windfall constitutes an increase in oil wealth and thus an immediate boost to aggregate demand, so that consumption immediately jumps up on impact (higher  $C(0)$ ). This follows from the present-value budget constraint of the economy. Second, the windfall implies that more funds will be spent on public capital and thus that the social value of capital jumps upwards on impact and that the efficiency of public investment falls (higher  $q(0)$ , lower  $PIMI(0)$ ). Although the stock of public capital is unaffected by the windfall on impact, its value jumps up (higher  $q(0)S(0)$ ). Debt is also unaffected by the windfall on impact. Afterwards, the dynamic evolution is according to equations (15)-(16) and is discussed in detail in section 5.2.

### 5.1. Calibration of the model and steady state

The parameters which we have used to calibrate the model (15)-(16) are given in Table 2. The chosen elasticity of intertemporal substitution implies an elasticity of intergenerational risk aversion of 2. As before, the interest and discount rate have been set to the same value so that in the absence of capital scarcity there is perfect smoothing of consumption. The production shares are derived from the cross-country dynamic GMM estimates using *PIMI*-adjusted public capital data (Gupta et al., 2011) and imply a production share of public capital of  $\beta = 0.21$  (see section 3.1). The implied expected life of public capital is forty years. Total factor productivity has been normalized to unity. The adjustment cost parameter for public investment has been matched to public investment being 4.2 percent of GDP during 2000-09 and a *PIMI* of 047 for the low-income countries and so that it (see section 3.1). The rate of decline of the windfall is 10 percent per year (as in fig. 1 of section 2).

**Table 2: Calibration parameters**

Elasticity of intertemporal substitution, $\sigma = 0.5$	Interest and discount rate, $r^* = \rho = 0.025$
Production share of private capital, $\alpha' = 0.3$	Production share of public capital, $\beta' = 0.15$
Depreciation rate of physical capital, $\delta = 0.025$	Total factor productivity, $E = 1$
Adjustment cost parameter for public capital, $\phi = 34.4$	Windfall, $N_0 = 0.72$ , $\eta = 0.1$
Initial public capital and debt, $S_0 = 0.22$ , $D_0 = 0.72$	$\Pi(d) = 10^{-4} \exp(6.294) [\exp 1.9d - 1]$

<sup>10</sup> We use a Runge-Kutta algorithm to solve (15)-(16) from time zero to some horizon  $T$  with initial conditions  $K(0) = K_0$ ,  $D(0) = D_0$ , and guesses for  $C(0)$  and  $q(0)$ . A Newton-Raphson method is then used to adjust  $C(0)$  and  $q(0)$  until  $C(T) = C^*$  and  $q(T) = q^*$  (as well as  $D(T) = D^*$  and  $S(T) = S^*$ ) are satisfied.  $T$  is then increased until it no longer has an effect on the  $C(0)$  and  $q(0)$  that are needed to make the economy jump to its stable manifold. Alternatively, a spectral decomposition algorithm (e.g., Buiter, 1984) is used to solve the linearized model.

We have set the initial size of the windfall and initial debt equal to the initial value of non-resource GDP,  $Y(0) = 0.72$ . The initial stock of public capital has been set in line with the empirical evidence to 0.22 (see section 3.1). Hence, our calibration implies we start away from steady state. Finally, the last expression in table 2 for the interest spread schedule is inspired by the empirical discussion in section 4, where 6.294 is the mean log of the spread. It implies that a 10%-point increase in the debt-GNI ratio pushes up the interest differential by 6.9%-points if the economy starts out with a debt-GNI ratio of 100 percent (or 1.3%-points if it starts off with zero foreign debt).

In steady state there is no debt and the ratio of public investment to public capital equals the rate of

depreciation,  $\delta$ , hence in steady state  $D^* = 0$ ,  $q^* = 1 + \phi\delta$ ,  $S^* = \left( \frac{\beta^* E}{(r^* + \delta)(1 + \phi\delta) - 0.5\phi\delta^2} \right)^{\frac{1}{1-\beta}}$  and

$C^* = ES^{*\beta} - \delta(1 + 0.5\phi\delta)S^*$ . The long-run stock of public capital decreases with

$(r^* + \delta)(1 + \phi\delta) - 0.5\phi\delta^2$ , which exceeds the rental plus depreciation charge, 0.05, especially if costs of adjusting public capital  $\phi$  are high. So a high value of  $\phi$  corresponds to an absorption constraint in that it requires higher marginal returns on public capital. Investment is thus relatively inefficient in the early stages of economic development when public investment rates have to be high. In the steady state we have a *PIMI* of  $I/J = 1/(1 + 0.5\phi\delta) = 0.70$ , but in the early stages of development relevant for low income countries only 47 percent of investment outlays is delivered as current public investment rates ( $I/S$ ) are much higher (see section 3.1).

### 5.2. Optimal development paths without a windfall

The dashed lines in fig. 3 indicate the optimal trajectories in the absence of a windfall. Since the economy starts out of steady state, it moves along a development path with the stock of public capital, wages, output and private consumption gradually rising towards their steady-state values. The high initial debt implies a high initial interest rate and social cost of borrowing. This induces households to save a lot in the beginning and thus consumption rises steeply in the initial phases of the development path. Over time the economy becomes richer and gradually pays of its foreign debt, which lowers the interest premium on foreign debt and thus reduces both the private and the social cost of borrowing. This reduces the incentive to save and thus the growth in consumption flattens off in the later phases of the development path.

To understand the paths of the social value of public capital, the *PIMI* and public capital, we note that the economy starts off with an under-supply of public capital. This is why the initial marginal value of public capital and thus the initial rate of public investment is quite high, which drives the fast rates of economic growth in the early phases of the development path. In fact, during the early part of the windfall when

revenue is at its highest, the rate of public investment continues to rise. This ramping up of public investment deteriorates the efficiency of public investment (lower *PIMI*), which is only reversed when the rate of public investment starts to decline and economic growth flattens out again. An interesting feature of the no-windfall simulations is thus that the shadow value of public capital, the public investment rate and the *PIMI* overshoot.

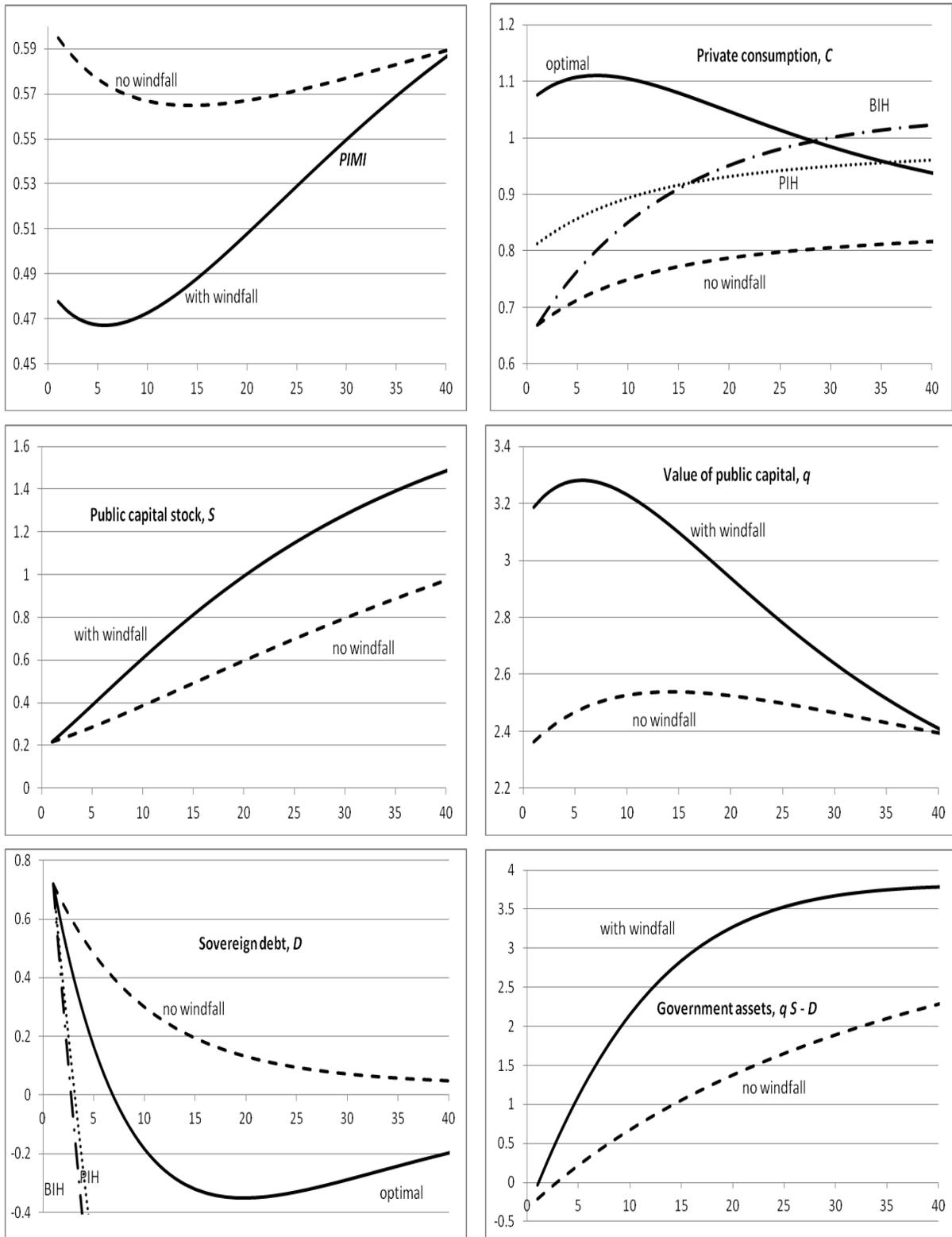
### 5.3. Optimal strategies for harnessing a temporary resource windfall

The solid lines in fig. 3 present the optimal paths for the situation where it becomes apparent that from time zero there will be a temporary windfall. The windfall corresponds to boost to wealth (i.e., the present value of expected windfall income) and thus a boost to aggregate demand. The social value of public capital immediately jumps up, thus making it attractive to invest in public capital. The mechanism by which this works is that part of the windfall is gradually used to pay off debt, so the expected future social costs of borrowing fall and thus the social value of public investment rises. It is apparent from comparison with the dashed lines in fig. 3 that the windfall speeds up the process of economic development, since the public capital stock (and thus inflows of private capital and non-windfall income as well) indeed rise much more rapidly towards their unchanged steady-state values of 2.15 (1.18). The temporary boom in public investment, over and above the normal no-windfall boom discussed in section 5.2, is triggered by a temporary spike in the value of public sector capital, which results from the anticipation of the windfall-induced boom in demand. This also fuels the temporary boom in consumption which speeds up the process of development considerably. We make two comments on the boom in public investment.

First, ramping up public investment worsens absorption constraints and thus increases the inefficiency of public investment as may be witnessed from the substantial and persistent falls in the *PIMI*. On impact the boost to public investment under the windfall depresses the *PIMI* from 59.5 to 47.8 percent. Second, net government assets (i.e., the value of public capital minus sovereign debt,  $qS - D$ ), are *not* predetermined even though the initial debt and the initial stock of public capital ( $D_0$  and  $S_0$ ) are predetermined, since the shadow value of public capital jumps up on impact from 2.36 to 3.19 and therefore the initial value of public capital jumps up from 0.51 to 0.69. Hence, the initial value of net government assets jumps up from -0.21 to -0.03.

The boom in public investment is associated with bigger government surpluses. This results over time in more rapid and substantial drops in sovereign debt which rapidly turn into sovereign wealth. As a result, interest spreads and the costs of borrowing are brought down which gives the government more scope to hand out transfers to its citizens and to ramp up public capital despite the lower efficiency of public investment (witness the falls in the *PIMI*). As is the case of the no-windfall trajectories, the shadow value

**Figure 3: Harnessing windfall with capital scarcity and rising cost of ramping up public investment**



**Key:** Dashed = no windfall; solid = optimal; dotted = PIH rule; dashed-dotted = BIH rule

of public capital, the public investment rate and the *PIMI* overshoot. In contrast to the no-windfall trajectories, in the windfall simulations private consumption overshoots, so that in the initial phases consumption is kept low and rises gradually in order to make room for a rapid rise in public investment. This is in sharp contrast to the separation result derived in section 3.2.

#### *5.4. Comparison with the permanent income rule*

The permanent income (PIH) rule discussed in section 3.2 is, in contrast to the optimal policies discussed in section 5.2, a partial-equilibrium rule in the sense that no account is taken of the effects of the windfall on wages, the social cost of borrowing, the social value of public capital, public investment and output. The effects of such a rule are indicated with the dotted lines in fig. 3. As we have seen in section 2, the increment in private consumption equals the permanent value of the windfall at the time the windfall starts (the annuity value), i.e., 0.144. Over time the annuity return on in situ resource wealth falls while the increase in interest income on the balance of the sovereign wealth fund rises by an equivalent amount until it has reached 0.144. The sum of the annuity return on the remaining in situ resource wealth and the return on the sovereign wealth fund thus always equals exactly 0.144. Consumption thus immediately jumps up by 0.144 and stays by this much higher for every moment of time thereafter. The huge increase in sovereign wealth does not lead to a boost to economic development, since public capital, private capital inflows and non-resource production are unaffected. The increase in government transfers yields a sustained increase in consumption of 0.144. Interestingly, during the initial phases the optimal strategy puts much less in sovereign wealth or paying off government debt and thus leads to a much bigger increase in private consumption than the PIH rule. After about 35 years the consumption increment under the optimal strategy falls below that under the PIH rule. The optimal strategy thus permits much more consumption upfront and leads to a boost in public capital rather than putting much needed revenue in a sovereign wealth fund. The reason is clear: the economy is in the initial phases of development where consumption is low and thus the marginal utility of consumption is high. As the PIH rule fails to take account of this feature of developing economies, it is far from optimal and does not sufficiently serve the interests of current generations of citizens.

#### *5.5. Comparison with the bird in hand rule*

The dashed-dotted lines in fig. 3 show what happens to consumption and sovereign wealth if the bird in hand (BIH) rule is used. Just like the PIH rule and in contrast to the optimal policies, the BIH rule does not take account of the general-equilibrium interactions as it does not take internalize the effects on the social cost of borrowing, the social value of public capital, public investment, public capital, output and saving. All windfall revenue is accumulated into a sovereign wealth fund and no transfers and boosts to private consumption are allowed until sovereign wealth is accumulated. The withdrawal rate is 4 percent

from the balance of the fund (as in Norway), so saving under the bird in hand rule is  $\dot{A} = rA + N - 0.04A$  where  $r < 0.04$ . We see that consumption does not jump on impact at all and rises only gradually.

Eventually it does reach a higher value of private consumption than under the PIH rule before falling back to the no-windfall path in the very long run. The BIH rule performs even worse than the PIH rule. It does not offer any prospect for improved economic development and gives bigger consumption increments than the PIH rule only after ten years and higher consumption increments than the optimal rule only after 23 years before dropping off to zero in the very long run.

Since the PIH and BIH rules fail to deliver sufficient consumption to the present, relatively poor generations and do not allocate a part of the windfall to public investment, they fare worse from a social welfare perspective than the optimal policies. Indeed, the gain in the present value of the utility of the path of present and future consumption (normalized by the initial marginal value of wealth to convert from utility to resource units) resulting from the windfall is 1.24 under the optimal policies, which exceeds the gain under the PIH rule (0.88) and the BIH rule (0.72). Note that the BIH rule performs worse than the PIH rule. However, although advanced oil-rich economies satisfy the conditions of the separation theorem (see section 3.2) and should thus finance investment by borrowing on world capital markets, for capital-scarce economies using a BIH rule it makes sense to allocate some of the withdrawals of the fund for domestic investment purposes as well as consumption. This adjustment to the BIH rule may make it more attractive from a welfare point of view.

### 5.6. *Genuine saving should be negative, not zero or positive*

Many practitioners adhere to the Hartwick rule which states that any depletion of subsoil natural resource assets must be exactly offset by accumulation of other assets such as sovereign wealth and public, private or human capital, leaving total subsoil and sovereign wealth unchanged (Hartwick, 1977). In other words, genuine saving must be zero. The PIH satisfies the Hartwick rule. To see this, note that for the PIH rule

(see section 2) the optimal accumulation of sovereign wealth  $\dot{A} = N - N_p = \eta N / (r + \eta) > 0$  exactly equals the decline in natural resource wealth  $\dot{N}_p / r = -\eta N / (r + \eta) < 0$ , so that genuine saving  $\dot{A} + \dot{N}_p / r$

is indeed zero. The time path of genuine saving under the PIH rule corresponds to the dotted line (the

horizontal axis) in fig. 4. Accumulation of sovereign wealth under the BIH rule is  $\dot{A} = N - (0.04 - r)A$ , so

that the genuine saving increments are given by  $\dot{A} + \frac{\dot{N}_p}{r} = \frac{r}{r + \eta} N - (0.04 - r)A$ . The time paths of these

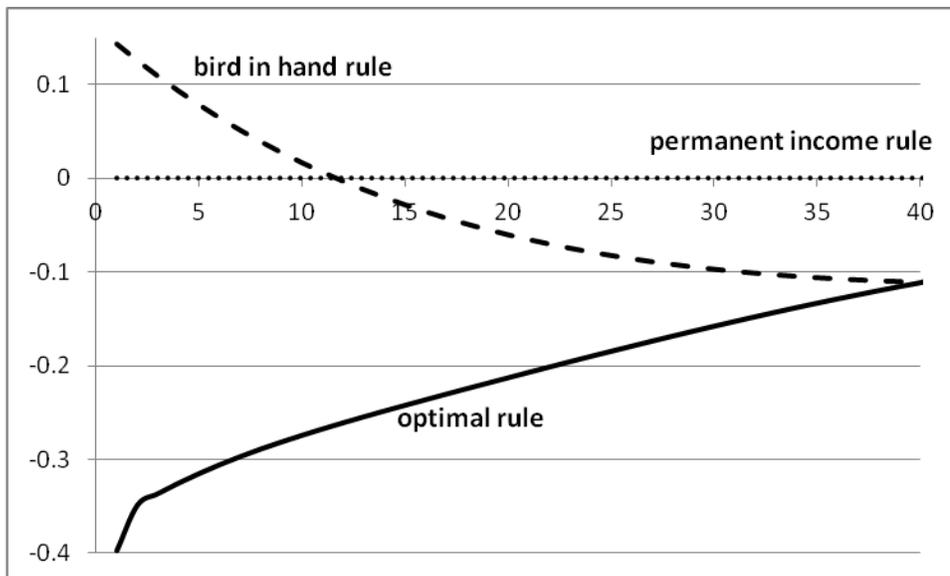
genuine savings increments for the BIH rule are portrayed as the dashed line in fig. 4. They start off at the

start of the windfall being positive with genuine saving equal to  $\dot{A} + \frac{\dot{N}_p}{r} = \frac{r}{r + \eta} N_0 = 0.144 > 0$ . Whilst

sovereign assets are being built up and the consumption increments still trail behind, the genuine savings increments remain positive which reflects the conservative nature of the BIH rule. After a while, the genuine saving increments turn negative. In the very long run (not shown in fig. 4), the accumulated fund is run down completely and the windfall has ceased so that consumption returns to its original level. The long-run genuine savings increment is thus zero under the BIH rule.

In contrast, the solid line in fig. 4 indicates that developing economies with capital scarcity and increasing costs of ramping up public investment require *negative* genuine saving increments to speed up the process of growth and development so that the positive increment in net assets (public capital minus sovereign debt) at each point of time is less than the negative increment in subsoil wealth.<sup>11</sup>

**Figure 4: Genuine saving increments under optimal, BIH and PIH rules**



There may be other reasons than capital scarcity and increasing costs of public investment for negative genuine saving. One of these is anticipation of better times which may happen if natural resource exporters anticipate either future reductions in the costs of extracting natural resources or future increases in the world price of natural resources (van der Ploeg, 2010). It is then better for those countries to borrow on the international capital market and deplete their natural resources later when they can extract them more cheaply and fetch a higher price. It can be shown that genuine saving must equal minus the sum of expected extraction cost reductions and expected capital losses on subsoil natural resource wealth.

<sup>11</sup> The level of genuine savings under the optimal rule without the windfall is positive, since the country is accumulating assets along its development path. With the windfall, the level of genuine savings turns negative as the country uses it to pay off its debt more rapidly.

## 6. Investing to invest: Dutch disease with intra-sectoral factor mobility

In developing economies ‘investing to invest’ is a crucial feature of the development process and leads to a different type of absorption constraints than the ones resulting from the rising cost of ramping up public investment discussed in sections 3-5. The problem is that public capital has to be ‘home grown’, i.e., produced by the non-traded sector, but the capacity of the non-traded sector can only be increased if it has more public capital. Put differently, teachers are needed to educate more teachers, nurses are needed to train more nurses, roads are needed to produce more roads, etc. No absorption problems arise if public capital can be imported from abroad or if airports, roads, etc. are delivered by foreigners (as the Chinese have often done in Africa and Brazil). However, if public capital must be home grown and/or private demand for consumption falls on non-traded products, the economy faces the challenge of Dutch disease. It arises if the extra demand induced by a windfall of foreign exchange causes an appreciation of the real exchange rate or an increase in the relative price of non-tradables, which triggers factors of production to move from the traded sector to the non-traded sector (Cordon and Neary; 1982; Corden, 1984). The fall in production of tradables and the increase in demand for tradables are met by higher import of tradables, financed by the windfall of foreign exchange; the rise in production of non-tradables is needed to meet the increase in demand for non-tradables.

To capture the consequences of Dutch disease and the cost of ramping up public investment, we extend the model discussed in section 5 in the following manner. We define GNP as  $Y \equiv Y_T + pY_N$ , where output of tradables and of non-tradables are denoted by, respectively,  $Y_T$  and  $Y_N$ , and the relative price of non-tradables (the real exchange rate) by  $p$ . Capital used in the traded and non-traded sectors,  $K$ , is supplied as foreign direct investment and is driven by the user cost of private capital  $r^* + \mu$ . The price of tradables has been normalized to unity. The GNP function  $Y = Y(p, r^*, L, S)$  gives maximum GNP for a competitive economy given the price of non-tradables  $p$ , the world interest rate  $r^*$ , exogenous labor supply  $L$  and the stock of public capital  $S$  (cf., Neary, 1988). The partial derivatives of the GNP function give, respectively, the supply of non-tradables, the supply of capital  $K$ , the wage  $w$ , and the marginal product of public capital. The GNP function is constructed under the assumption that the markets for labor and capital clear. Factors of production are thus mobile across sectors, so factor returns are equalized across sectors.

To highlight the absorption problem resulting from Dutch disease, we suppose that not all of the stock of public capital is imported from abroad and a (substantial) part of it has to be produced at home in the non-traded sector. We capture this with the following unit-expenditure function  $e(p)$ ,  $e'(p) > 0$ ,  $e''(p) < 0$  with  $e(p)J = J_T + pJ_N$ , where public sector investment spending on tradables and non-tradables are

$J_T = (1 - \xi)e(p)J$  and  $J_N = e'(p)J$ , where  $0 < \xi \equiv pe'(p)/e(p) < 1$ , denotes the share of non-traded goods in the basket of public investment goods. Similarly we have spending on consumption goods  $e(p)C = C_T + pC_N$ , where private demand for tradables and non-tradables are  $C_T = (1 - \xi)e(p)C$  and  $C_N = e'(p)C$ , respectively.<sup>12</sup>

Total consumption and investment demand for non-tradables,  $e'(p)(C+J)$ , should equal supply of non-tradables,  $Y_p$ . This market clearing condition for non-tradables can be solved for the price of non-tradables, which increases with demand for private consumption and public investment, decreases with the supply of private and public capital, and decreases with the supply of labor in the economy:

$$(17) \quad Y_p(p, r^*, L, S) = e'(p)(C + J) \Rightarrow dp = \frac{e'(dC + dJ) - Kdr^* - wdL}{Y_{pp} - e''(C + J)}.$$

The real exchange has to respond much more if supply of non-tradables and private consumption and public investment demand for non-tradables are more inelastic.

We will illustrate our model with the functional specification  $e(p) = p^\xi$ , where  $0 < \xi \equiv pe'(p)/e(p) < 1$  is now the constant share of non-traded goods in private consumption and in public investment and the elasticity of demand is easily seen to be  $0 < 1 - \xi < 1$ . Further, we suppose  $Y_T = L_T$  and  $Y_N = EL_N^{1-\alpha} S^\beta$ .

Hence, tradables are only produced with labor and non-tradables are produced with labor and some fixed factors (e.g., land) and benefits from public capital. Also, we abstract from private capital inflows. It

follows that the wage is  $w = 1$  and demands for labor are  $L_N = (1 - \alpha)pS^\beta E^{\frac{1}{\alpha}}$  and

$L_T = L - (1 - \alpha)pS^\beta E^{\frac{1}{\alpha}}$ , so labor shifts from the traded to the non-traded sector if the price of non-tradables is high and if the stock of public capital (and relative TFP in the non-traded sector  $E$ ) is high. Setting  $L = 1$ , it can be easily shown that the GNP function can be written as:

$$(18) \quad Y(p, S) = 1 + \alpha(1 - \alpha)^{\frac{1-\alpha}{\alpha}} pS^\beta E^{\frac{1}{\alpha}}.$$

If the government maximizes utilitarian welfare as before, we obtain (see appendix):

$$(15a') \quad \dot{C} = \sigma C \left[ \Pi \left( \frac{D}{Y(p, S) + N} \right) + \Pi' \left( \frac{D}{Y(p, S) + N} \right) \frac{D}{Y(p, S) + N} - \xi \frac{\dot{p}}{p} \right], \quad C(0) \text{ free,}$$

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<sup>12</sup> For simplicity, we suppose the same unit-expenditure functions for private consumption and public investment.

$$(15c') \quad \dot{q} = \left[ r^* + \Pi \left( \frac{D}{Y(p, S) + N} \right) + \Pi' \left( \frac{D}{Y(p, S) + N} \right) \frac{D}{Y(p, S) + N} + \delta - \xi \frac{\dot{p}}{p} \right] q - \frac{1}{2\phi} (q-1)^2 - \frac{Y_S(p, S)}{e(p)} \left[ 1 + \Pi' \left( \frac{D}{Y(p, S) + N} \right) \left( \frac{D}{Y(p, S) + N} \right)^2 \right], \quad q(0) \text{ free},$$

$$(15d') \quad \dot{D} = \left[ r^* + \Pi \left( \frac{D}{Y(p, S) + N} \right) \right] D + e(p)C - Y(p, S) + e(p) \frac{1}{2\phi} (q^2 - 1)S - N, \quad D(0) = D_0.$$

The interest rate in the Keynes-Ramsey rule (15a') now has the real consumption interest rate, i.e., the interest rate minus the rate of the change in the consumer price index (i.e.,  $\dot{e}/e = \xi \dot{p}/p$ ), instead of the interest rate as in (15a). Similarly, the real interest rate in the arbitrage equation for public investment (15c') is adjusted for the rate of change in the price index of public investment ( $\xi \dot{p}/p$ ). The government budget constraint (15d') now includes the cost of private consumption  $e(p)C$  (when calculating transfers) and the cost of private investment. Equations (15b) and (16) are unaffected. The real exchange rate  $p$  and its time derivative  $\dot{p}$  follow from:

$$(17') \quad dp = \left( \frac{e(p)}{Y_N} \right) \left( \frac{\alpha \xi}{1 - \alpha \xi} \right) \left[ dC + \frac{qS}{\phi} dq + \frac{J}{I} dS \right] - \left( \frac{p}{S} \right) \left( \frac{\beta}{1 - \alpha \xi} \right) dS.$$

From (15a') we see that steady-state debt is zero,  $D^* = 0$ , and from (15b) we obtain  $q^* = 1 + \phi\delta$ . Using (15c'), (15d') and (17), we can solve for the steady-state values ( $S^*, p^*, C^*$ ) from:

$$(19a) \quad S^* = \left[ \frac{\beta(1-\alpha) \frac{1-\alpha}{\alpha} p^{*\frac{1-\alpha\xi}{\alpha}}}{(r^* + \delta)(1 + \phi\delta) - 0.5\phi\delta^2} \right]^{\frac{\alpha}{\alpha-\beta}},$$

$$(19b) \quad e(p^*) \left[ C^* + \frac{1}{2\phi} [(1 + \phi\delta)^2 - 1] S^* \right] = Y(p^*, S^*) \Rightarrow dC^* = r^* q^* dS^*,$$

$$(19c) \quad Y_p(p^*, S^*) = e'(p^*) \left[ C^* + \delta(1 + 0.5\phi\delta^2) S^* \right] \Rightarrow dp^* = \left( \frac{e'(p^*) [r^* q^* + \delta(1 + 0.5\phi\delta^2)] - Y_{pS^*}}{Y_{pp} - e'' [C^* + \delta(1 + 0.5\phi\delta^2) S^*]} \right) dS^*.$$

Given that the production elasticity of public capital is less than that of the fixed factors in the non-traded sector (i.e.,  $\alpha > \beta$ ), (19a) indicates that the long-run supply of capital is high if the world interest rate is low and the price of non-tradables is high,  $dS^* = S^* [(1 - \alpha\xi) / (\alpha - \beta)] dp^* / p^*$ . Total differentiation of (19b) and substitution of (19c) gives  $dC^* = r^* q^* dS^*$ , so steady-state consumption rises with the long-run

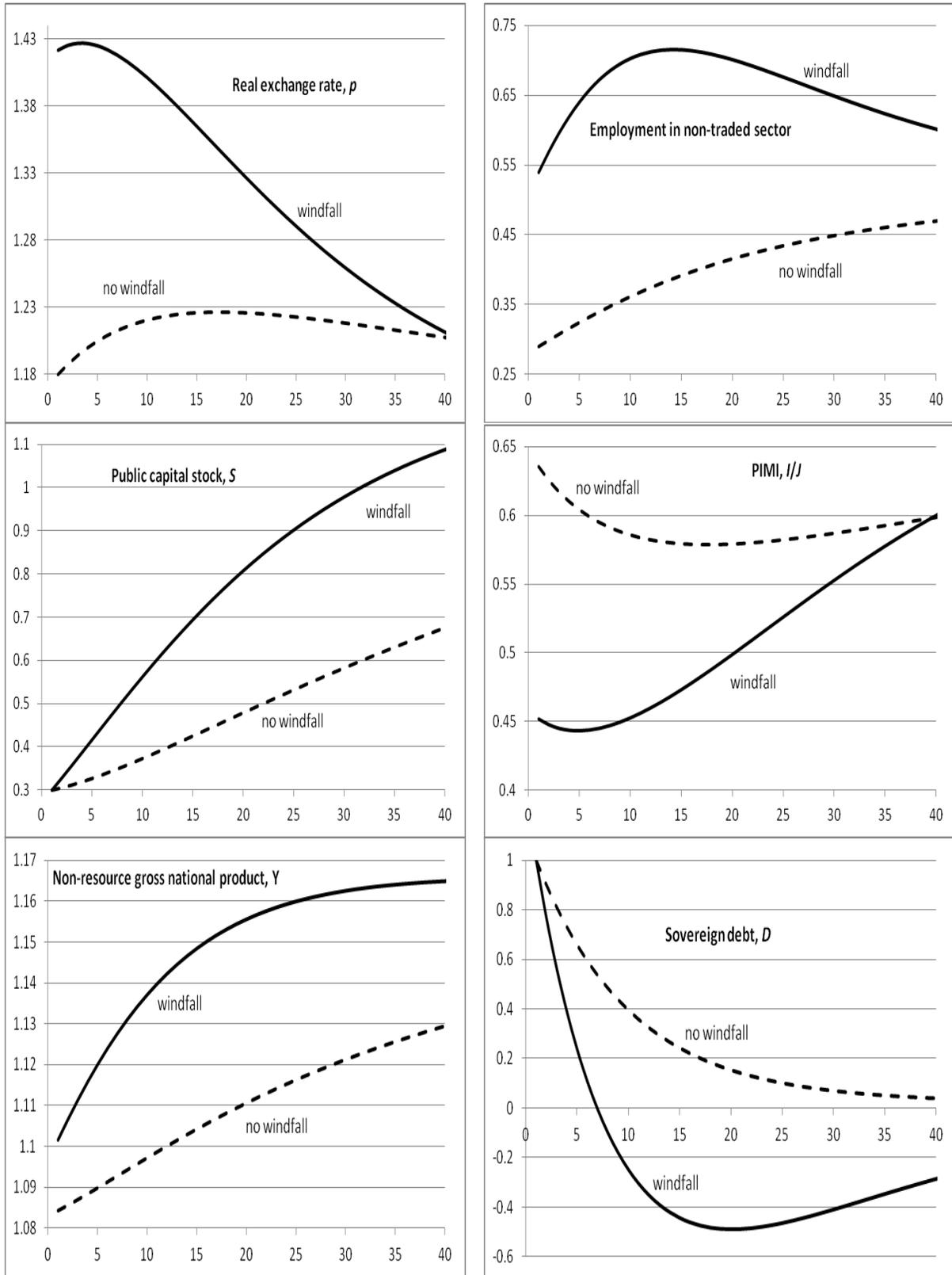
stock of public capital. Equation (19c) indicates that the price of non-tradables must fall to clear the market for non-tradables given the boost resulting to consumption and investment demand resulting from a higher stock of public capital, especially if supply of and demand for non-tradables are not very elastic. However, if public capital boosts supply of non-tradables sufficiently, this effect may be reversed.

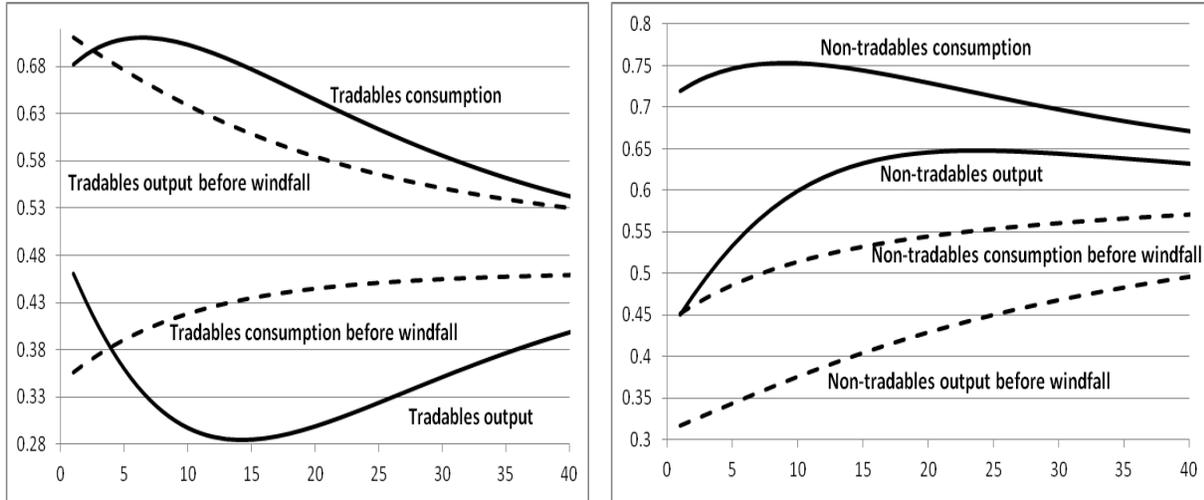
Upon substitution of  $p$  and  $\dot{p}$  from (20') into the system (15a'), (15b), (15c'), (15d') and (16), we can solve and obtain a system in the state equations  $(C, S, q, D)$ . This system can be solved in the same way as the system (15) and (16) in section 5. Note that if private consumption and public investment demand are entirely imported ( $\xi = 0$ ), the real exchange rate adjusts immediately and there are no Dutch disease effects to take account of (cf., the simulations discussed in section 5). However, the benchmark simulations presented in fig. 5 set the consumption share of non-tradables equal to  $\xi = 0.8$ , the production elasticities in the non-traded sector to  $\alpha = 0.3$  and  $\beta = 0.15$ , efficiency to  $E = 1$ , and the other parameters to the values in section 5. We start off the system with  $S(0) = 0.3$  and  $D(0) = N(0) = 1$ .

The simulations reported in fig. 5 show that the price of non-tradables jumps up on impact of the news of the windfall and over time, as labor shifts from the traded to the non-traded sector, the capacity of the non-traded sector expands. The stock of public capital expands and, as the demand for non-tradables is met, the initial appreciations of the real exchange rate are undone. The absorption constraints resulting from Dutch disease are more severe if a greater part of consumption and public investment has to be produced at home. It then takes much longer for the economy to move along its development path. As in section 5 the efficiency of public investment is reduced considerably as public investment is ramped up, which aggravates the absorption constraints resulting from Dutch disease, and the windfall is also used to bring down sovereign debt more quickly. Interestingly, this optimal harnessing strategy yields a sustained increase in private consumption and non-resource gross national product  $Y$ . The last two panels of fig. 5 show that the windfall increases private consumption of both non-tradables and tradables, but output of tradables falls considerably in order to make room for a boost to production of tradables. The windfall of foreign exchange finances the resulting current account deficits as well as the more rapid reduction in sovereign debt.

The solid, long-dashes and short-dashes lines in the left panel of fig. 6 show the time paths of incremental real consumption if, respectively, zero, 20 and 40 percent of consumption and investment goods are imported. These simulations indicate that the incremental change in real consumption is higher if a greater part of consumption and investment goods is imported than produced at home as then absorption constraints are less severe. Interestingly, despite the absorption constraints, the optimal policy ensures that the increments in consumption are more upfront if a greater proportion of consumption and investment

**Figure 5: Harnessing windfall with Dutch disease**

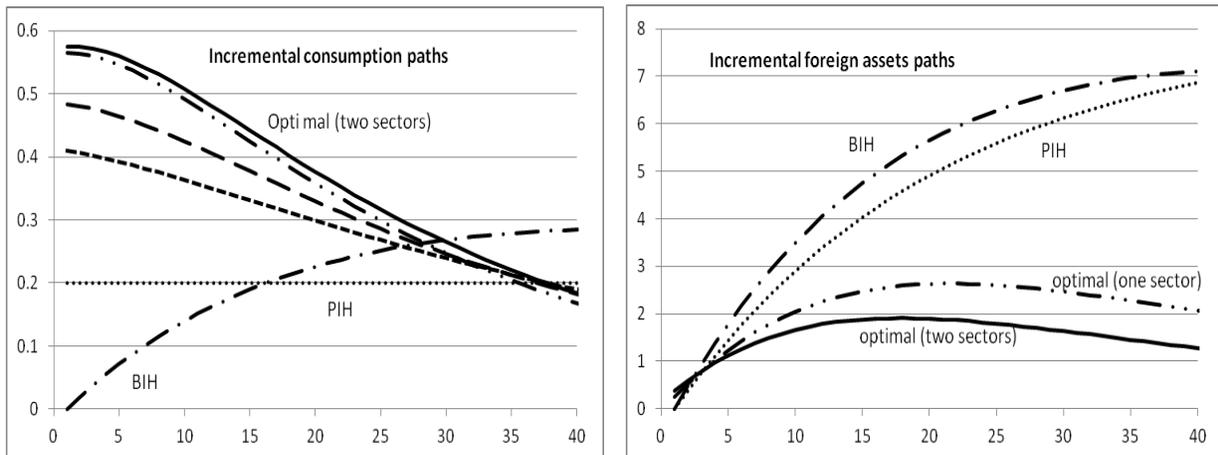




**Key:** Solid lines = windfall trajectories; dashed lines: no-windfall trajectories

goods have to be home grown. The dotted and long-dashes-dots lines give the real consumption increments under the PIH and the BIH rule, respectively. The optimal real consumption increments in the Dutch disease model are thus concentrated much more upfront than the ones under the PIH and BIH rules. The reason is that the optimal increments take account of the high marginal utilities of real consumption in the early phases of the development paths where consumption is still low. The right panel of fig. 6 shows the corresponding time paths of the increments in net foreign assets under the optimal policies and under the PIH and BIH rules. The BIH rule is a more conservative than the PIH rule and thus leads to a bigger accumulation of sovereign wealth. The optimal strategies give priority to having extra real consumption upfront in the beginning of the development phase when the marginal utility of real consumption is high and thus lead to less accumulation of sovereign wealth.

**Figure 6: Incremental real consumption and foreign assets paths**



**Key:** solid = optimal (two sectors); long dashes = optimal (two sectors) with  $\xi = 0.8$ ; short dashes = optimal (two sectors) with  $\xi = 1$ ; long dashes, double dots = optimal (one sector)

Fig. 6 also includes a comparison of the two-sector model of section 6 with the one-sector model of section 5. These two models are not very comparable, since in the model of section 5 tradables are produced using labor according to decreasing returns to scale and benefiting from infrastructure whilst the tradables production technology of section 6 is linear in labor only. Still, we see that the qualitative feature of the optimal real consumption increments under the models of sections 5 and 6 are very similar (and the same is true for sovereign debt and the *PIM*). Both models realize that current generations are poor and thus put more real consumption upfront than the PIH or BIH rules. However, the more realistic two-sector model of this section indicate the optimal responses of the real exchange rate (i.e., the sharp temporary appreciation) and the reallocations from the non-traded to the traded sector that are required to achieve these real consumption increments. The key message we obtain from section 6 is, however, that absorption constraints (arising when a bigger proportion of consumption and investment goods have to be produced at home) limit the size of the feasible real consumption increments.

The above analysis can be modified in two directions. First, if the traded sector is intensive in public capital rather than the non-traded sector, the real exchange rate does not respond and adjustment including the reallocation of production factors from the non-traded to the traded sector is instantaneous. But in practice public capital cannot easily be unbolted and shifted between sectors. The non-traded sector can then only expand if the stock of public capital in the traded sector is gradually winded down via wear and tear. This also leads to temporary appreciations of the real exchange rate and a gradual reallocation of workers from the traded to the non-traded sector, but the root cause of it is wholly different (van der Ploeg and Venables, 2011b). Second, if learning by doing is the engine of the growth in the traded sector, the temporary decline of the traded sector resulting from Dutch disease has a permanent negative effect on total factor productivity of the traded sector (e.g., van Wijnbergen, 1984; Torvik, 2001). The essence of Dutch disease in developing economies is, however, better captured by absorption constraints, investing to invest and capital scarcity than by learning by doing.

## 7. Conclusion

Many countries experiencing a temporary windfall of foreign exchange have been advised to put the revenue into a sovereign wealth fund to either allow a sustained increase in consumption as under the permanent income rule or a gradual increase in consumption followed by a gradual fall in consumption as under the more conservative bird in hand rule. These rules may be relevant for an advanced mature economy as Norway, but is wholly inappropriate for developing economies for a number of reasons. First, current generations in developing economies are worse off than future generations. It is therefore optimal

to deliver a lot of the real consumption increments upfront rather than in the future. The permanent income and bird in hand rules fail to do that. Second, developing economies are often not very well integrated into world capital markets and face capital scarcity. They also have to cope with an increasing cost of ramping up public investment. For such developing economies it does not make sense use their windfall to buy US T bills if they could pay down costly sovereign debt or invest in public projects with a much higher rate of return. The optimal strategy to harness a windfall is thus to gradually ramp up public investment, tolerate a temporary fall in the efficiency of public investment, and gradually boost public capital, inflows of private capital and non-resource output. As a result of this strategy wages and consumption rise in the initial phases much more than under the permanent income and a fortiori the bird in hand rules and sovereign debt is brought down to bring down the cost of borrowing and alleviate capital scarcity. In contrast to the permanent income which implies zero genuine saving, the optimal policies for harnessing a foreign exchange windfall require negative genuine saving so that the positive increment in net assets (public capital minus sovereign debt) at each point of time is less than the negative increment in subsoil wealth.

In practice, absorption constraints are amplified as private demand and especially public sector demand have a strong bias for non-traded products. This results in appreciation of the real exchange rate and Dutch disease. Nevertheless, this optimal harnessing strategy yields a sustained increase in real consumption and non-resource gross national product but the real consumption increments will be smaller if a bigger part of consumption and investment has to be produced at home. It is then more difficult for the economy to use the windfall to speed up the process of economic development. The bigger the component of public capital that has to be home grown, the bigger the absorption constraints. This is the challenge of ‘investing to invest’.

The kind of 'public capital' that matters most for raising the growth potential is non-rival public capital. In practice, important examples of non-rival public capital that drive economic growth are public infrastructures not subject to congestion, and individual knowledge generated by better public education. In developing countries congestion in infrastructures is typically not an issue in the short or medium run. Hence, one may suggest investing primarily in public infrastructure, because it takes more time to get returns from stimulating “ideas and human knowledge”. However, one must be careful to avoid the trap of “white elephants”, so it is best to aim for growth-enhancing public infrastructure projects rather than partisan, illiquid projects which are often used for patronage. Moreover, the empirical evidence suggests that public infrastructure and ideas and human knowledge are complements, not substitutes. Investing in a mix thus seems desirable in view of these two types of public capital being non-rival and mutual complementary. In future work we will investigate how our policy for harnessing windfalls of foreign

exchange can be used to boost growth in an endogenous growth context. We will then also allow for the implications of poverty traps and the associated policy prescriptions of enacting "big push" policies and redirecting the direction of technical change (cf., Acemoglu et al., 2012). The modern theory of directed technical change suggests that a big-push policy, directed at public infrastructure and knowledge creation, and financed by the resource windfall, may induce a sudden transition from the phase of stagnating TFP to a phase of sustained TFP growth.

## References

- Acemoglu, D., P. Aghion, L. Bursztyjn and D. Hemous (2012). The environment and directed technical change, *American Economic Review*, to appear.
- Akitobi, B. and T. Stratmann (2008). Fiscal policy and financial markets, *Economic Journal*, 118, 533, 1971-1985.
- Barnett, S. and R. Ossowski. (2003). Operational aspects of fiscal policy in oil-producing countries, in J. Davis, R. Ossowski and A. Fedelino (eds.), *Fiscal Policy Formulation and Implementation in Oil-Producing Countries*, International Monetary Fund, Washington, D.C.
- Berg, A., R. Portillo, S.-C. S. Yang and L.-F. Zanna (2011). Government investment in resource abundant low-income countries, mimeo., International Monetary Fund, Washington, D.C.
- Bom, P. and Ligthart J.E. (2010). What have we learned from three decades of research on the productivity of public capital, Working Paper No. 2206, CESifo, Munich.
- Boschini, A.D., J. Petterson and J. Roine (2007). Resource curse or not: a question of appropriability, *Scandinavian Journal of Economics*, 109, 593-616.
- Buiter, W.H. (1984). Saddlepoint problems in continuous time rational expectations models: a general method and some macroeconomic examples, *Econometrica*, 52, 3, 665-680.
- Collier, P., F. van der Ploeg, M. Spence and A.J. Venables (2010). Managing resource revenues in developing economies, *IMF Staff Papers*, 57, 1, 84-118.
- Cordon, W.M. (1984). Booming sector and Dutch disease economics: survey and consolidation, *Oxford Economic Papers*, 36, 359-380.
- Cordon, W.M. and J.P. Neary (1982). Booming sector and de-industrialisation in a small open economy, *Economic Journal*, 92, 825-848.
- Dabla-Norris, E., J. Brumby, A. Kyobe, Z. Mills and C. Papageorgiou (2011). Investing in public investment: an index of public investment efficiency, Working Paper 11/37, International Monetary Fund, Washington, D.C.
- Epstein, L.G. and S.E. Zin (1989). Substitution, risk aversion, and the temporal behavior of consumption growth and asset returns: I. A theoretical framework, *Econometrica*, 57, 4, 937-969.
- Feldstein, M. and C. Horioka (1980). Domestic saving and international capital flows, *Economic Journal*, 90, 358, 314-329.

- Gupta, S., A. Kangur, C. Papageorgiou and A. Wane (2011). Efficiency-adjusted public capital and growth, Working Paper 11/217, International Monetary Fund, Washington, D.C.
- Hartwick, J.M. (1977). Intergenerational equity and the investment of rents from exhaustible resources, *American Economic Review*, 67, 972-974.
- Hayashi, F. (1982), Tobin's marginal Q and average Q: a neoclassical interpretation, *Econometrica*, 50, 1, 213-224.
- Ketkar, S. and D Ratha (2009) New paths to funding, *Finance and Development*, 46.2
- Mehlum, H., K. Moene and R. Torvik (2006). Institutions and the resource curse, *Economic Journal*, 160, 1-20.
- Neary, J.P. (1988). Determinants of the equilibrium real exchange rate, *American Economic Review*, 78, 1, 210-215.
- Ploeg, F. van der (1993). A closed-form solution for a model of precautionary saving, *Review of Economic Studies*, 60, 2, 385-396.
- Ploeg, F. van der (2010). Why do many resource-rich countries have negative genuine saving? Anticipation of better times or rapacious rent seeking, *Resource and Energy Economics*, 32, 28-44.
- Ploeg, F. van der (2011). Natural resources: curse or blessing?, *Journal of Economic Literature*, 49, 2, 366-420.
- Ploeg, F. van der and S. Poelhekke (2010). Volatility and the natural resource curse, *Oxford Economic Papers*, 61, 4, 727-760.
- Ploeg, F. van der and A.J. Venables (2011a). Harnessing windfall revenues: optimal policies for resource-rich developing economies, *Economic Journal*, 121, 1-31.
- Ploeg, F. van der and A.J. Venables (2011b). Absorbing a windfall of foreign exchange: Dutch disease dynamics, Research Paper 52, OxCarre, Department of Economics, University of Oxford.
- Ploeg, F. van der and A.J. Venables (2011c). Natural resource wealth: how not to squander it, *Annual Reviews of Economics*, to appear.
- Ploeg, F. van der, R. Stefanski and S. Wills (2011). Harnessing oil revenues in Ghana, Policy Paper 12, OxCarre, University of Oxford and International Growth Centre, LSE.
- Pritchett, L. (2000). The tyranny of concepts: CUDIE (Cumulated, Depreciated, Investment Effort) is not capital, *Journal of Economic Growth*, 5, 361-384.
- Sachs, J.D. and A.M. Warner (1997). Natural resource abundance and economic growth, in G. Meier and J. Rauch (eds.), *Leading Issues in Economic Development*, Oxford University Press, Oxford.
- Torvik, R. (2001). Learning by doing and the Dutch disease, *European Economic Review*, 45, 285-306.
- Venables, A.J. (2010). Resource rents: when to spend and how to save, *International Tax and Public Finance*, 17, 340-356.
- Wijnbergen, S.J. van (1984). The Dutch disease: a disease after all?, *Economic Journal*, 94, 373, 41-55.

## Appendix

### Section 3.2

The optimality conditions for the problem of maximizing (3) subject to (7), (8), (11) and

$C = (1 - \alpha')ES^\beta + T$  follow from the Hamiltonian function

$$(A1) \quad H \equiv \frac{\left[ (1 - \alpha')ES^\beta + T \right]^{1-1/\sigma} - 1}{1-1/\sigma} + \lambda_A \left[ rA + N - T - I - 0.5\phi I^2 / S \right] + \lambda_S \left[ I - \delta S \right],$$

where  $\lambda_A$  is the shadow price of  $A$  and  $\lambda_S$  of  $S$ . They are given by:

$$(A2a) \quad \partial H / \partial T = C^{-1/\sigma} - \lambda_A = 0,$$

$$(A2b) \quad \partial H / \partial I = -\lambda_A \left[ 1 + \phi I / S \right] + \lambda_S = 0,$$

$$(A2c) \quad \rho \lambda_A - \dot{\lambda}_A = \partial H / \partial A = r \lambda_A,$$

$$(A2d) \quad \rho \lambda_S - \dot{\lambda}_S = \partial H / \partial S = (1 - \alpha')\beta ES^\beta C^{-1/\sigma} + 0.5\phi(I/S)^2 \lambda_A - \delta \lambda_S,$$

Equations (A2a) and (A2c) can be combined to yield (4). Defining  $q \equiv \lambda_S / \lambda_A$ , we obtain (12) from

(A2b). Equation (A2a), (A2c), (A2d) and (12) can be combined to give (13).

### Section 3.2

The government maximizes (3) subject to (7), (8), (11),  $C = (1 - \alpha')ES^\beta + T$ , and (14), where  $D = -A$ .

The Hamiltonian function becomes

$$(A1') \quad H \equiv \frac{\left[ (1 - \alpha')ES^\beta + T \right]^{1-1/\sigma} - 1}{1-1/\sigma} + \lambda_A \left[ \left( r^* + \Pi \left( \frac{-A}{ES^\beta + N} \right) \right) A + N - T - I - 0.5\phi I^2 / S \right] + \lambda_S \left[ I - \delta S \right],$$

and yields the optimality conditions (A2a), (A2b),

$$(A2c') \quad \rho \lambda_A - \dot{\lambda}_A = \partial H / \partial A = \left[ r^* - \Pi' \left( \frac{-A}{Y + N} \right) \frac{A}{Y + N} \right] \lambda_A,$$

$$(A2d') \quad \rho \lambda_S - \dot{\lambda}_S = \partial H / \partial S = (1 - \alpha')\beta ES^\beta C^{-1/\sigma} + \left[ 0.5\phi(I/S)^2 + \frac{\beta Y}{S} \Pi' \left( \frac{-A}{Y + N} \right) \left( \frac{A}{Y + N} \right)^2 \right] \lambda_A - \delta \lambda_S.$$

### Section 6

To obtain the first-order conditions in section 6, we define the Hamiltonian function:

$$(A3) \quad H \equiv U(C) + \lambda_S (I - \delta S) + \lambda_D \left[ \left\{ r^* + \Pi \left( \frac{D}{Y(p, S) + N} \right) \right\} D + e(p) C + I(1 + 0.5\phi I / S) - Y(p, S) - N \right] \\ + \lambda_N \left[ Y_p(p, S) - e'(p) C + I(1 + 0.5\phi I / S) \right],$$

where  $\lambda_S, \lambda_D, \lambda_N$  denote the shadow value of  $S$ , minus the shadow cost of  $D$ , and the Lagrange multiplier corresponding to the condition for equilibrium in the market for non-tradables. This yields:

$$(A4a) \quad \partial H / \partial C = U'(C) + e(p)\lambda_D - e'(p)\lambda_N = 0,$$

$$(A4b) \quad \partial H / \partial J = \lambda_S + e(p)\lambda_D - e'(p)\lambda_N (1 + \phi I / S) = 0,$$

$$(A4c) \quad \partial H / \partial p = \left[ e'(p)(C + J) - Y_p(p, S) \right] \lambda_D + \left[ Y_{pp} - e''(p)(C + J) \right] \lambda_N = 0,$$

$$(A4d) \quad \rho \lambda_S - \dot{\lambda}_S = \partial H / \partial S = -\delta \lambda_S - Y_S(p, S) \Pi'(d) d^2 \lambda_D - 0.5e(p)\phi(I / S)^2 \lambda_D \\ - Y_S(p, S) \lambda_D + Y_{pS}(p, S) \lambda_N + 0.5e'(p)\phi(I / S)^2 \lambda_N,$$

$$(A4e) \quad \rho \lambda_D - \dot{\lambda}_D = \partial H / \partial D = \left[ r^* + \Pi(d) + \Pi'(d)d \right] \lambda_D.$$

Combining (A4a) and (A4b) we get  $I = (q - 1)S / \phi$  and (17b) where  $q \equiv \lambda_S / U'(C)$ . Putting (A4c) into

$$(A4a) \text{ we get } U'(C) = - \left\{ e(p) + \left[ \frac{e'(p)^2(C + J) - e'(p)Y_p(p, S)}{Y_{pp}(p, S) - e''(p)(C + J)} \right] \right\} \lambda_D = -e(p)\lambda_D, \text{ where use has been}$$

made of  $e'(p)(C + J) = Y_p(p, S)$ . Using this and  $r^* = \rho$  in (A4e) gives (15a'). Using (A4a) in (A4d)

gives  $(\rho + \delta)\lambda_S - \dot{\lambda}_S = -Y_S \Pi'(d) d^2 \lambda_D - Y_S(p, S) \lambda_D + Y_{pS}(p, S) \lambda_N + 0.5U'(C)\phi(I / S)^2$  or

$$\dot{\lambda}_S / \lambda_S = \rho + \delta - Y_{pS}(p, S) \frac{\lambda_N}{\lambda_S} - Y_S \left[ 1 + \Pi'(d) d^2 \right] \frac{1}{e(p)q} - \frac{\phi(I / S)^2}{2q}.$$
 From (15a') we get

$$\frac{\lambda_N}{\lambda_S} = \frac{U'(C) + e(p)\lambda_D}{e'(p)\lambda_S} = 0, \text{ so we have } \frac{\dot{q}}{q} = r^* + \delta - Y_S \left[ 1 + \Pi'(d) d^2 \right] \frac{1}{e(p)q} - \frac{\phi(I / S)^2}{2q}$$

$+ \Pi(d) + \Pi'(d)d - \xi \frac{\dot{p}}{p}$  from  $q \equiv \lambda_S / U'(C)$ . and thus (15c'). Equation (15d') follows from substituting

$e(p)C - Y(p, S)$  for government transfers. To obtain (17'), we make use of the GNP function (18)

including  $Y_{pp}(p, S) = \frac{(1 - \alpha)Y_N}{\alpha p} > 0$  and  $Y_{pS}(p, S) = \frac{\beta Y_N}{\alpha S} > 0$  to totally differentiate (17).