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Economic Growth and the Social Cost of Carbon:

Additive versus Multiplicative Damages

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ECONOMIC GROWTH AND THE SOCIAL COST OF CARBON:

ADDITIVE VERSUS MULTIPLICATIVE DAMAGES

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Abstract

In a calibrated integrated assessment model of Ramsey growth and climate change in the global economy we investigate the differential impact of additive and multiplicative global warming damages for both a socially optimal and business-as-usual scenario. Fossil fuel is available at a cost which rises as reserves diminish and a carbon-free backstop is supplied at decreasing cost. If damages are not proportional to aggregate production and the economy is along a development path, the optimal carbon tax is smaller. The economy switches later from fossil fuel to the carbon-free backstop and leaves less fossil fuel in situ. By adjusting climate policy in this way there is very little difference on the paths for global consumption, output and capital, and thus very little difference for social welfare despite the higher temperatures. For all specifications the optimal carbon tax is not a fixed proportion of world GDP but must follow a hump shape.

Keywords: climate change, multiplicative damages, additive damages, integrated assessment models, Ramsey growth model, fossil fuel, carbon-free backstop

JEL codes: H21, Q51, Q54

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1. Introduction

Integrated assessment models of climate change aim to integrate economics and climate science and to assess the impact the economy has on the climate and vice versa. This analysis is a crucial step in the design of optimal policies to fight the potentially negative effects of climate change on economic well-being. There are various ways in which the relationship between economics and climate can be and is addressed. Our main aim is to investigate the implications of different types of modeling damage and production technology in a simple but well-calibrated framework of aggregate economic growth. We are particularly interested in the different kinds of modeling of the potential damage inflicted upon the economy and the substitutability of energy in production.

Global warming stems from the negative externality associated with the emission of CO2 associated with the use of fossil fuels. This can be corrected by pricing carbon emissions at the social cost of carbon either via a carbon tax or a market for carbon emission permits. In the absence of distortions in raising public funds and other second-best issues, the social cost of carbon corresponds to the present value of all future global warming damages from burning an extra unit of fossil fuel.[1] It matters *how* global warming damages are specified. Higher global temperatures lead to respiratory and other diseases and thus induce lower levels of health, productivity and aggregate output. Global warming also destroys productivity of agriculture and reduces aggregate output in that way. Both of these channels justify a specification with damages proportional to output with the proportion increasing in temperature. This is customary in applied integrated economic assessment models and equivalent to the assumption of unit elasticity of substitution between output and damage. Nordhaus (2008) and Stern (2007) are prominent examples and this approach is also chosen in Tol (2002) and Golosov et al. (2014). However, Tol (2002) is noteworthy in explicitly considering effects on ecosystems, vector borne diseases and heat and cold stress rather than subsuming them under production losses. Higher global temperatures also lead to rising sea levels and destroy part of the capital stock in which case damages might be proportional to the aggregate capital stock rather than to aggregate output. Global warming also leads to destruction of natural habitats, e.g., the coral reefs, and to less biodiversity. In that case, one might suppose that global warming damages are specified in final goods units and increasing in temperature, not proportional to output or GDP as discussed in Stern (2013). We refer to this as *additive* global warming damages in contrast to the more usual case of *multiplicative* damages. The additive damage specification implies an infinite elasticity of substitution between global warming damages and aggregate output. It also implies that with a positive rate of economic growth

[1] Determining the social cost of carbon is less straightforward in a world with exhaustible fossil fuel, increasing efficiency of carbon-free alternatives, gradual and abrupt transitions from fossil fuel to renewables, fossil extraction cost, and endogenous growth and structural change. Here, one also needs to consider the interaction between carbon pricing and the market prices of fossil and renewable energy. These prices depend on expectations about future prices of fossil energy and the back-stop technology and the degree of learning by doing (e.g., Rezai and van der Ploeg, 2013).

and with additive rather than with multiplicative damages, marginal damages do not rise with world GDP as the economy grows.

Our first objective is to analyze the effects of these seemingly innocuous differences in specification for both the level and the time profile of the social cost of carbon and the optimal carbon tax. Of course, there are other ways of specifying global warming damages (e.g., Stern, 2013, pp. 846-850). For example, one might relax the assumption that the proportion of what is left from production after global warming damages and the level of total factor productivity (the state of technical progress) appear separable in the production function. More importantly, the rate of technical progress rather than the level of output may suffer from global warming. This should capture that the assumption of *exogenous* technical progress is simply not realistic given the scale of disruption to output that might result from global warming. In fact, global warming may also adversely affect production factors such as the stock of capital, infrastructure or land directly. We will focus at the difference between multiplicative and additive global warming damages and abstract from these other ways of specifying damages in a first approximation. We will also abstract from capturing global warming damages directly in the social welfare function or, more implicitly, via a ceiling as is often done in the theoretical literature on climate policy.[2] Global warming damages have been carefully split up into its multiplicative production and its additive utility components (e.g., Barrage, 2013). Our formulation of multiplicative production damages is equivalent to additive linear damages in utility. More generally, we wish to highlight the importance of how easy it is to substitute global warming damages for production output for the time profile and level of the optimal carbon tax.

Our second objective is to see how the optimal climate policy under multiplicative and additive specifications of global warming affects the amount of fossil fuel to be left in the crust of the earth and the timing at which the world switches to a carbon-free economy. We allow for scarcity of fossil fuel and stock-dependent extraction costs, so that costs of extraction rise as less reserves remain and reserves need not be fully depleted. This contrasts with many integrated assessment models in which fossil fuel reserves are typically abundant and extraction costs do not increase as fossil fuel reserves are depleted. Furthermore, in such models the elasticity of substitution between energy and a capital-

[2] First, a branch of the literature assumes catastrophic changes once the carbon concentration reaches or passes a deterministic or stochastic threshold because decay of atmospheric stops and climate change is drastic or irreversible (e.g., Tsur and Zemel (1996) and (1998)). A problem with this approach is that usually nothing goes wrong until the ceiling is reached, whereas one would expect damages to arise already for temperatures not too far from the ceiling (see also Dullieux et al. (2011) and Chakravorty et al. (2006)). Second, damages can appear as an externality in the social welfare function (e.g., van der Ploeg and Withagen (1991, 2012a, 2012b, 2013) and John and Pecchenino (1994)). Utility from consumption can be strongly separable from the damages from climate change or not. Bretschger and Smulders (2007) argue that in the latter case with Cobb-Douglas production balanced growth is feasible (assuming away exhaustibility of non-renewables). For the case of additive separability, Stokey (1998) shows that the growth process of the economy comes to an end, if more and more output is devoted to abatement. Weitzman (2009) considers additive and multiplicative damages in the social welfare function as well. He shows that the differences in optimal outcomes are considerable.

labor composite is typically set to zero[3] whilst we allow for substitution possibilities. Fossil fuel demand at any point of time in such models does not depend on expectations about the price of the future renewable backstops and consequently the transition times simply occur when the price of fossil fuel inclusive of the carbon tax reaches the price of the renewable. In contrast, we will determine the levels and the time profiles of the social cost of carbon and the market prices of fossil and renewable energy within the context of a fully calibrated integrated assessment model of climate change and Ramsey growth with exhaustible fossil fuel, transition to carbon-free renewable energy sources, stock-dependent extraction costs, and technical progress in the production of renewable energy.

The price of fossil fuel contains two forward-looking components: the scarcity rent of fossil fuel (the present discounted value of all future increases in extraction costs resulting from extracting an extra unit of fossil fuel) and the social cost of carbon (the present discounted value of all future marginal global warming damages). This complicates the calculation of the transition times, since expectations about future developments such as technological progress in using the renewable matter. We study not only the social cost of carbon and market prices of all energy sources but also the optimal transition times for abandoning fossil fuel altogether as well as the amount of untapped fossil fuel. We derive our results based on a calibrated and much richer version of the analytical growth and climate model put forward in van der Ploeg and Withagen (2014) and take into account recent empirical findings by Hassler et al. (2011) on the substitutability of energy in production.

Our third objective is to demonstrate that there is a hump-shaped relationship between the optimal carbon tax and world GDP. In contrast, Golosov et al. (2014) offer a tractable Ramsey growth model which generates the result that an optimal carbon tax which is proportional to GDP.[4] Their result depends on bold assumptions: logarithmic utility, Cobb-Douglas production, 100% depreciation of capital in each period, zero fossil fuel extraction costs, and multiplicative production damages captured by a negative exponential function. We find that their result is not robust in a general integrated assessment model of climate change and Ramsey growth with exactly the same carbon cycle, especially if the coefficient of intergenerational inequality aversion differs from unity.

Our results demonstrate that, if damages are not proportional to aggregate production output and the economy is along a development path, the social cost of carbon and the optimal carbon tax are smaller than with multiplicative damages, as damages can more easily be compensated for by higher output and damages do not increase with a growing economy. As a result, the economy switches later from fossil fuel to the carbon-free backstop and leaves less oil in situ. If intergenerational inequality aversion is weaker (i.e., the elasticity of intertemporal substitution is larger), we show that the optimal

[3] For example, the seminal study of Nordhaus (2008) assumes energy demand is exogenously decreasing.
[4] This formula is used already by others too (e.g., Hassler and Krusell, 2012; Gerlagh and Liski, 2012). Copeland and Taylor (1994) prose a similar framework.

carbon tax is still smaller with additive damages, but that the effect is less substantial. We find that with an elasticity of intertemporal substitution of 0.5 the social cost of carbon for additive damages from global warming is about half that for multiplicative damages from global warming.

In contrast, Weitzman (2009) finds for the same elasticity of intertemporal substitution, 0.5, that the optimal willingness to forsake current consumption to avoid future global warming is 7 times larger with additive damages and a growth rate of 2% per annum. This effect disappears in a stagnant economy. Our results are not comparable as Weitzman (2009) has a partial equilibrium model whilst we derive our results within a fully specified general equilibrium model. This has the advantage that we can allow for growth and development from an initial capital stock that is below the steady state and for the exhaustibility of fossil fuel whilst Weitzman (2009) assumes an exogenous growth path. Furthermore, he deals with damages in utility and we focus on damages in production. However, Weitzman's insights reverse once we look at *production* damages instead of *utility* damages in a fully-fledged integrated climate assessment model along a growth trajectory.

The outline of this paper is as follows. Section 2 sets out our general equilibrium model of climate change and Ramsey economic growth with additive and multiplicative global warming damages, exogenous population growth and labor productivity growth. Production combines energy with a capital-labor composite. Energy and the composite are imperfect substitutes in production. The two sources of energy, however, carbon-free energy and exhaustible fossil fuel, are perfect substitutes. The cost of carbon-free energy is exogenous and benefits from exogenous technical progress. The extraction cost of fossil fuel increases as fewer reserves are left in the crust of the earth. Our carbon cycle allows for permanent and transient components of atmospheric carbon and abstracts from positive feedback loops. We provide intuition for the different effects of additive and multiplicative damage and derive closed-form solutions for the social cost of carbon under simplifying assumptions. We show that the implications of different specifications of damage hinge on the growth prospects of the economy. In a growing economy environmental policy is less ambitious with multiplicative than with additive damages. Section 3 uses a calibrated version of the model of section 2 and presents the simulations paying particular attention to the level and time profile of the optimal carbon tax as well as to how this tax affects the moments in time that the economy switches from fossil fuel to the carbon-free renewable and shows how the tax and transition times depend on whether global warming damages are additive or multiplicative and on how elastic energy use is. We present details for the price dynamics of fossil and renewable energy and also investigate the sensitivity of our core results with respect to the elasticity of substitution in production, the discount rate and the elasticity of intertemporal substitution. Section 4 concludes.

2. An integrated assessment model of Ramsey growth and energy transitions

Using a simple Ramsey growth model we derive the social cost of carbon for two different ways of modeling damages, additive and multiplicative. The social welfare function is utilitarian, with instantaneous per capita utility U depending on per capita consumption C_t / L_t, where C_t is aggregate consumption and L_t is the population size, possibly non-constant over time, but exogenous. With ρ the constant rate of time preference social welfare is given by

$$(1) \qquad \sum_{t=0}^{\infty} (1+\rho)^{-t} L_t U_t (C_t / L_t).$$

The elasticity of intertemporal substitution (*EIS*) at time t is defined as $\eta_t \equiv -\frac{U'(C/L)}{(C_t / L_t) U''(C_t / L_t)} > 0$.

The ethics of climate policy depend on how much weight is given to welfare of future generations (and thus on how small ρ is) and on how small intergenerational inequality aversion is or how easy it is to substitute current for future consumption per head (how low $1/\eta_t$ is). Ceteris paribus, climate policy is most ambitious if society has a low rate of time preference and little inequality aversion (low ρ, high η_t).

Optimal climate policy takes place under a number of constraints in the form of a set of difference equations governing the global economy. First, output at time *t*, $Z(K_t, L_t, F_t + R_t)$, is produced using three inputs, manmade capital K_t, labor, L_t, and energy. We model two types of energy: fossil fuels like oil, natural gas and coal, F_t., and renewables, R_t, such as solar and wind energy. The aggregate general production function $Z(K_t, L_t, F_t + R_t)$ allows for imperfect factor substitution. Renewable energy is infinitely elastically supplied at potentially exogenously decreasing cost, b_t. Fossil fuel extraction cost at time *t* is $G(S_t)F_t$, with S_t the existing stock of fossil fuel reserves at the start of period *t*. Extraction becomes more costly as the less accessible fields have to be explored, $G' < 0$. We also allow for technical progress in aggregate and renewable energy production and for an exogenous profile for the time path of population growth. Mean temperature or the concentration of atmospheric carbon creates a convex combination of (ξ) multiplicative and ($1-\xi$) additive climate damages (with $0 < \xi < 1$). This specification allows us to contrast multiplicative global warming damages in production with additive damages in production.

What is left of production after covering the cost of resource use and climate damage is allocated to consumption C_t, investments in manmade capital $K_{t+1} - K_t$, depreciation δK_t with a constant rate of depreciation δ:

$$(2) \qquad K_{t+1} = (1-\delta)K_t + \left[1-\xi D(T_t)\right] Z(K_t, L_t, F_t + R_t) + (1-\xi) D(T_t) Z_0 - G(S_t) F_t - b_t R_t - C_t,$$

where damages, $D(T_t)$, increase with temperature and the initial stock of capital K_0 is given.

With $\xi = 1$, there are purely multiplicative damages and with $\xi = 0$ purely additive damages which are proportional to $Z_0 = Z(K_0, L_0, F_0 + R_0)$. The development of the finite fossil fuel stock follows from:

(3) $$S_{t+1} = S_t - F_{1t}, \quad \sum_{t=0}^{\infty} F_t \le S_0,$$

where initial reserves S_0 are given. We follow Golosov et al. (2014) in modeling a three-stock carbon cycle with carbon as fossil fuel reserves in the crust of the earth S, and a permanent component E_1 and a transient component E_2 of the stock of carbon in the atmosphere.[5] Equations (4), and (5) show the dynamics of the permanent and transient component of the stock of atmospheric carbon with φ_L the fraction of emissions that stays up permanently in the atmosphere, φ the speed at which the temporary component of the atmospheric stock of carbon decays, and φ_0 a coefficient to calibrate how much of carbon is returned to the surface of the oceans and earth within a decade.

(4) $$E_{1t} = E_{1t-1} + \varphi_L F_t,$$

(5) $$E_{2t} = \varphi E_{2t-1} + \varphi_0 (1 - \varphi_L) F_t,$$

Ignoring the lags between stocks of atmospheric carbon and global warming discussed by Gerlagh and Liski (2012), we define global mean temperature, T_t, as the deviation from the pre-industrial temperature in degrees Celcius. The equilibrium climate sensitivity, ω, defines the rise in global mean temperature following a doubling of the total stock of carbon in the atmosphere, $E_t = E_{1t} + E_{2t}$. The usual formulation for radiative forcing capturing this relationship is:

(6) $$T_t = \omega \ln\left(\frac{E_t}{596.4}\right) / \ln(2),$$

where 596.4 GtC is the IPCC figure for the pre-industrial stock of atmospheric carbon. Using (6) we can thus write damages just as well as a function of the total stock of carbon in the atmosphere, $D(E_t)$.

[5] We focus on the effects of fossil fuel use on global warming in a detailed calibrated model of growth and climate change, but following Golosov et al. (2014) and based on Archer (2005) and Archer et al. (2009) we adopt a tractable model of the carbon cycle which is linear and allows for decay of only part of the stock of atmospheric carbon. This model of the carbon cycle abstracts from a delay between the carbon concentration and global warming and the dynamics of multiple carbon reservoirs (e.g., Gerlagh and Liski, 2012). Abstracting from such a lag biases the estimate of the social cost of carbon and the carbon tax upwards. A more realistic model of the carbon cycle should also model the dynamics of the stocks of carbon in the upper and lower parts of the ocean and the time-varying coefficients originally put forward in the path-breaking paper of Bolin and Erikkson (1958).We also capture catastrophic losses at high levels of atmospheric carbon but abstract from positive feedback effects and the uncertain climate catastrophes that can occur in climate and growth models once temperature exceeds certain thresholds (e.g., Lemoine and Traeger, 2013; van der Ploeg and de Zeeuw, 2013).

We abstract from a lag between temperature and atmospheric carbon stock, but Rezai and van der Ploeg (2013) discuss how the analysis is modified with such a lag. [6]

The Lagrangian for the social planner's problem (maximize (1) subject to (2)-(6)) of our model of Ramsey growth and climate change reads as follows:

$$L \equiv \sum_{t=0}^{\infty}(1+\rho)^{-t}\left[L_t U_t(C_t/L_t) - \mu_t(S_{t+1}-S_t+F_t) + \nu_{1,t}(E_{1,t+1}-E_{1,t}-\varphi_L F_t) + \nu_{2,t}\left\{E_{2,t+1}-(1-\varphi)E_{2,t}-\varphi_0(1-\varphi_L)F_t\right\}\right]$$
$$-\sum_{t=0}^{\infty}(1+\rho)^{-t}\lambda_t\left[K_{t+1}-(1-\delta)K_t - D(E_t)\{\xi Z(K_t,L_t,F_t+R_t)+(1-\xi)Z_0\} + G(S_t)F_t + b_t R_t + C_t\right],$$

where μ_t denotes the shadow value of in-situ fossil fuel, ν_{1t} and ν_{2t} the shadow disvalue of the permanent and transient stocks of atmospheric carbon, and λ_t the shadow value of manmade capital. Necessary conditions for a social optimum are:

(7a) $U'(C_t/L_t) = \lambda_t,$

(7b) $[1-\xi D(E_t)]Z_{F_t+R_t}(K_t,L_t,F_t+R_t) \leq G(S_t) + \left[\mu_t + \varphi_L \nu_{1t} + \varphi_0(1-\varphi_L)\nu_{2t}\right]/\lambda_t, \quad F_t \geq 0, \quad \text{c.s.},$

(7c) $[1-\xi D(E_t)]Z_{F_t+R_t}(K_t,L_t,F_t+R_t) \leq b_t, \quad R_t \geq 0, \quad \text{c.s.},$

(7d) $\left\{1-\delta+[1-\xi D(E_{t+1})]Z_{K_{t+1}}(K_{t+1},L_{t+1},F_{t+1}+R_{t+1})\right\}\lambda_{t+1} = (1+\rho)\lambda_t,$

(7e) $\mu_{t+1} = (1+\rho)\mu_t + G'(S_{t+1})F_{t+1}\lambda_{t+1},$

(7f) $\nu_{1t+1} = (1+\rho)\nu_{1t} - D'(E_{t+1})\{\xi Z_{t+1}(K_{t+1},L_{t+1},F_{t+1}+R_{t+1})+(1-\xi)Z_0\}\lambda_{t+1},$

(7g) $(1-\varphi)\nu_{2t+1} = (1+\rho)\nu_{2t} - D'(E_{t+1})\{\xi Z_{t+1}(K_{t+1},L_{t+1},F_{t+1}+R_{t+1})+(1-\xi)Z_0\}\lambda_{t+1}.$

Equations (7a) and (7d) give the Euler equation for the growth in consumption per capita as an increasing function of the return on capital and decreasing function of the rate of time preference:

(8) $U'(C_t/L_t) = \left(\frac{1+r_{t+1}}{1+\rho}\right)U'(C_{t+1}/L_{t+1}), \quad r_{t+1} \equiv [1-\xi D_{t+1}(E_{t+1})]Z_{K_{t+1}}(K_{t+1},L_{t+1},F_{t+1}+R_{t+1})-\delta,$

For a constant elasticity of intertemporal substitution (*EIS*), we have $\frac{C_{t+1}/L_{t+1}}{C_t/L_t} = \left(\frac{1+r_{t+1}}{1+\rho}\right)^{\eta}$ so that the effect of the return on capital (r_{t+1}) on per-capita consumption growth is stronger if the *EIS* is high or intergenerational inequality aversion is weak ($1/\eta$ is low).

Equation (7b) implies that, if fossil fuel is used, its marginal product should, again, equal its marginal extraction cost (which now equals $G(S_t)$) plus its scarcity rent (defined as $s_t \equiv \mu_t / \lambda_t$) plus the social

[6] The implications of a lag on optimal climate policy within our IAM model are discussed in section 3.3 below.

cost of carbon ($\tau_t \equiv [\varphi_L \nu_{1t} + \varphi_0(1-\varphi_L)\nu_{2t}]/\lambda_t$). The scarcity rent and the social cost of carbon are defined in units of final goods (not utility units). If the marginal product of fossil fuel is below the total marginal cost (extraction cost *plus* scarcity rent *plus* social cost of carbon), it is not used. Equation (7c) states that, if the renewable is used, its marginal product must equal its marginal cost, b_t. We get:

(9a) $$[1-\xi D(E_t)]Z_{F_t+R_t}(K_t,L_t,F_t+R_t) \le G(S_t)+s_t+\tau_t, \quad F_t \ge 0, \quad \text{c.s.},$$

(9b) $$[1-\xi D(E_t)]Z_{F_t+R_t}(K_t,L_t,F_t+R_t) \le b_t, \quad R_t \ge 0, \quad \text{c.s.}$$

The dynamics of the scarcity rent follows from (7e) and (7d) and yields the Hotelling rule:

(10) $$s_{t+1} = (1+r_{t+1})s_t + G'(S_{t+1})F_{t+1} \quad \text{or} \quad s_t = -\sum_{\varsigma=0}^{\infty}\left[G'(S_{t+1+\varsigma})F_{t+1+\varsigma}\Delta_{t+\varsigma}\right],$$

where the compound discount factors are $\Delta_{t+\varsigma} \equiv \prod_{\varsigma'=0}^{\varsigma}(1+r_{t+1+\varsigma'})^{-1}, \varsigma \ge 0$. Hence, the scarcity rent of keeping an extra unit of fossil fuel unexploited must equal the present discounted value of all future reductions in fossil fuel extraction costs.

Finally, using (7f), (7g) and (7d), the social cost of carbon (the *SCC*), i.e., the present discounted value of all future marginal global warming damages from burning an additional unit of fossil fuel, equals:

(11) $$\tau_t = -\sum_{\varsigma=0}^{\infty}\left[\left\{\varphi_L + \varphi_0(1-\varphi_L)(1-\varphi)^{\varsigma}\right\}\Delta_{t+\varsigma}D'(E_{t+1+\varsigma})\left\{\xi Z_{t+1+\varsigma}(K_{t+1+\varsigma},L_{t+1+\varsigma},F_{t+1+\varsigma}+R_{t+1+\varsigma})+(1-\xi)Z_0\right\}\right].$$

It takes into account that one unit of carbon released from burning fossil fuel affects the economy in two ways: the first part remains in the atmosphere for ever and the second part gradually decays over time at a rate corresponding to roughly 1/300 per year.

2.1 Additive global warming damages

The expression for the social cost of carbon (11) already allows us to make a first comparison between additive and multiplicative damages on two levels:

First, suppose we propose two feasible paths to the economy, which at each instant of time are identical with regard to investments, renewable inputs and fossil fuel use. They yield the same capital stock, resource stock and CO2 concentration. The difference between the resulting consumption paths are given by $C_t^A - C_t^M = D(E_t)[Z(K_t,L_t,F_t+R_t)-Z_0]$, where superscripts refer to additive and multiplicative climate damages. Hence, additive damages allow higher consumption levels (and the social cost of carbon is lower) if production is above Z_0, i.e. if the rate of economic growth is positive.

Second, consider the conditions from profit maximization: $[1-D(E_t^M)]Z_{F_t}^M(K_t,L_t,F_t+R_t)$ $= G(S_t^M)+s_t^M+\tau_t^M$ and $Z_{F_t}^A(K_t,L_t,F_t+R_t) = G(S_t^A)+s_t^A+\tau_t^A$. The equations imply that the marginal product of fossil fuel equals the direct extraction cost plus the social cost of carbon plus the scarcity rent. The first equation holds for multiplicative damages and the latter holds for additive damages. If two economies would follow identical paths then the extraction costs are identical, the marginal potential products of fossil fuels are identical, as well as the scarcity rents. It then follows that this would require higher carbon taxes in the economy with multiplicative damages than in the economy with additive damages. Hence, the *SCC* is higher with multiplicative damages too.

2.2 The social cost of carbon under additive global warming damages

A tractable model of the optimal carbon tax has been put forward by Golosov et al. (2014) based on a decadal Ramsey growth model. Relying on logarithmic utility, Cobb-Douglas production function for capital, labor and energy, 100% depreciation each period (and thus has a coarse calibration grid), exponential damages, and labor-only energy production costs, they show that the *social cost of carbon* (*SCC*) is proportional to GDP and independent of technology. Following the exposition of Rezai and van der Ploeg (2014), the optimal carbon tax of Golosov et al. (2014) can be generalized to different specifications of damage. Under these assumptions, the general carbon tax of the Ramsey model, equation (11), simplifies to

$$(11') \qquad \tau_t = \left[\left(\frac{1+\rho}{\rho - n} \right) \varphi_L + \left(\frac{1+\rho}{1+\rho-\varphi(1+n)} \right) \varphi_0 (1-\varphi_L) \right] D'(E_t) \left[\xi Z(K_t, L_t, F_t + R_t) + (1-\xi) Z_0 \right].$$

Comparing equations for different specifications of damage (i.e. different values of ξ), we see that

$$[\tau_t]^{additive} < [\tau_t]^{multiplicative} \Leftrightarrow Z_0 < Z(K_t, L_t, F_t + R_t).$$

The social cost of carbon is, thus, higher under multiplicative damages if the rate of economic growth (in potential output) is positive (i.e., $Z(K_t, L_t, F_t + R_t) > Z_0, t > 0$). The reason is that positive economic growth implies that marginal damages are growing under multiplicative but not under additive global warming damages and consequently the optimal social cost of carbon as a fraction of GDP is higher under multiplicative than under additive damages. Since the elasticity of GDP with respect to the carbon tax is likely to be less than one, we conclude that the social cost of carbon will be higher under additive than under multiplicative damages.

2.3 Policy scenarios

The missing market for carbon permits is the only externality in our model and the social optimum can be implemented in the market economy with a specific carbon tax τ_t which is set to the social cost of carbon (11). Under “laissez-faire” the climate externality remains uncorrected, i.e. $\tau_t = 0$.

In principle, three regimes can occur in our fully specified Ramsey model: a regime with only fossil fuel use, a regime with only use of renewable energy and a regime with simultaneous use of fossil fuel and renewable energy. In our numerical IAM it is optimal to start with an initial phase with only fossil fuel use since initially renewable energy is not competitive. This holds for additive as well as multiplicative damages. After some time renewable energy is phased in and an intermediate phase with simultaneous use of fossil fuel and renewable energy commences. After some more time fossil fuel is phased out and the final carbon-free era starts. Since fossil fuel extraction costs become infinitely large as reserves are exhausted, fossil fuel reserves will not be fully exhausted and thus some fossil fuel will be left untapped in the crust of the earth at the end of the intermediate phase. From that moment on the in-situ stock of fossil fuel will remain unchanged, but the carbon in the atmosphere will gradually decay leaving ultimately only the permanent component of the carbon stock. [7]

During the initial phase fossil fuel demand follows from (7b), holding with equality. Setting its marginal product, $[1-\xi D(E_t)]Z_{F_t}(K_t, L_t, F_t + R_t)$ to the sum of extraction cost, scarcity rent and carbon tax, $G(S_t) + s_t + \tau_t$. We have strict inequality in the second part of (7c). During the intermediate phase fossil fuel and renewable demand follow from $(1-\xi D(E_t))Z_{F_t+R_t}(K_t, L_t, F_t + R_t) = G(S_t) + s_t + \tau_t = b_t$. Since fossil fuel and renewable energy are perfect substitutes, simultaneous use in a competitive economy beyond one period of time, requires a renewable subsidy or a carbon tax.[8] During the final phase we have $(1-\xi D(E_t))Z_{R_t}(K_t, L_t, R_t) = b_t$, which gives renewable use as an increasing function of capital and a decreasing function of global mean temperature or the concentration of carbon in the atmosphere.

The time profile of the carbon tax is crucial in determining whether it is optimal to return to simultaneous use after a carbon-free era. The carbon tax suggested by Golosov et al. (2014) to be rephrased in (11′) sets it proportional to output. We find that this rule is a poor approximation to the optimal carbon tax in our model of economic growth and climate change with exogenous growth in technical progress and population. We find that it is optimal for the carbon tax to be hump-shaped, since it must fall in the carbon-free era as the temporary component of atmospheric carbon dissipates. If the market price of fossil energy falls below the market price of renewables in this transition, (partial) re-switching to fossil fuel is optimal.

One of our key objectives is to study the optimal timing of transitions from introducing renewable energy alongside fossil fuel and from phasing out fossil fuel altogether because in most of the prevailing integrated assessment models these transitions are exogenous. We are interested in how the timing of these transitions is affected by different assumptions on the climate-economy and energy-

[7] For our calibration it is never optimal to use oil again, in spite of the decrease in the damages.

[8] Technological progress lowers the market price of renewable energy. Since both energy sources are perfect substitutes, simultaneous use would imply that it is optimal to sell energy at a lower price in the future rather than meeting full demand at a higher price today. This cannot be the case under positive discounting.

output relationships; for example, by how much do optimal carbon taxes bring forward the carbon-free era when damages are additive or multiplicative damage and elasticity of substitution between energy and capital is high or low. The stock of fossil fuel to leave untapped in the earth at the end of the intermediate phase follows from the condition that the economy is indifferent between fossil fuel and the renewable and that the scarcity rent has vanished:

(12) $G(S_t)+\tau_t < b_t,\ 0 \le t < t_{CF}$, and $G(S_t)+\tau_t \ge b_t,\ S_t = S_{t_{CF}},\ \forall t \ge t_{CF}$.

where t_{CF} is the time at which the economy for the first time relies on using only the renewable (carbon free). The amount of fossil fuel to be left in situ increases in the renewable subsidy and the carbon tax.

3. Policy simulation and optimization

In our numerical simulations time runs from 2010 till 2600 and is measured in decades, t=1,2,.., 60, so period 1 corresponds to 2010-2020, period 2 to 2020-2030, etc. The final time period is t=60 or 2600-2610, but we highlight the transitional dynamics in the earlier parts of the simulation. The functional form and calibration of the carbon cycle, temperature module and global warming damages are discussed in more detail in the appendix. The functional forms and benchmark parameter values for the economic part of our IAM of growth and climate change are also discussed in the appendix. On the whole our benchmark parameter values assume relatively low damages, low fossil fuel extraction cost and a high cost for renewable energy. This biases our model toward fossil fuel use.

We report full results for three simulations with multiplicative damages $(\xi = 1)$ and another three simulations with additive damages $(\xi = 0)$. We consider three scenarios for each type of damages. First, the first best optimum. Second, the "laissez faire" outcome (not taking damages into account and setting the carbon tax to zero). Third, a scenario based on a carbon tax set according to a proportional rule, as proposed by Golosov et al. (2014), (11′). The reported simulations use an elasticity of substitution between energy and the capital-labor aggregate equal to $\vartheta = 0.5$.

Table 1: Policy Scenarios for the setting of the global carbon tax

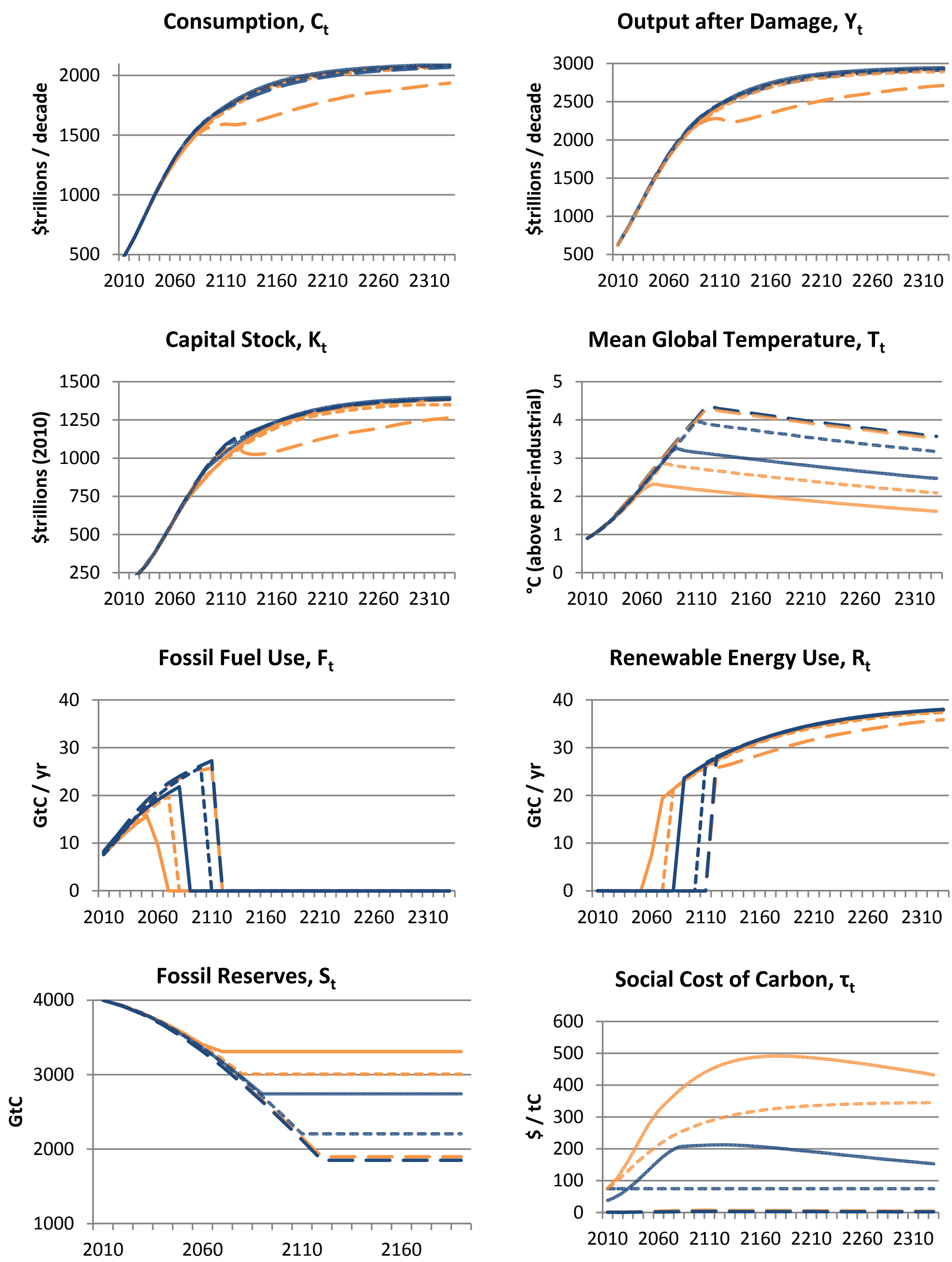
	First best τ_t	*Zero* *("Laissez faire")*	*Proportional to GDP* *(11')*
multiplicative damages $(\xi = 1)$	———	— — —	- - - - -
additive damages $(\xi = 0)$	———	— — —	- - - - -

Table 1 illustrates the six simulations and the coding that is used to distinguish them in the simulation figures. We also analyze the sensitivity of the social cost of carbon with respect to the elasticities of intertemporal and factor substitution and the social rate of discount.

3.1. Climate policy is more ambitious under multiplicative than under additive damage

We start with the *first-best* outcomes and first compare additive and multiplicative damages. These correspond to, respectively, the solid light and dark lines in figure 1. The first, second and third panels show aggregate consumption, total net output and the aggregate capital stock. Over the entire period of time under consideration output net of damage is monotonically increasing. Moreover, net output is almost the same in both situations. The same holds for the capital stock and consumption. Surprisingly, this implies that the total welfare is hardly affected by whether the function capturing global warming damages is additive or multiplicative. Still, there are essential differences in how this is achieved under these two types of specification for damages. These differences concern mostly the use of fossil fuel and the timing of the transition to renewable energy. The economy's endowments and technological change allow the economy to grow. So, if the economy with multiplicative global warming damages would use the same rate of fossil fuel, damages in terms of loss of production would be much higher over time. Therefore, the economy with multiplicative damages uses less fossil fuel, leaves more fossil fuel unexploited and makes the transition to renewables at an earlier stage. Despite the effects on consumption, capital and output being small, the differences in timing of energy use and how much fossil fuel is stranded are considerable – see also table 2. With multiplicative damages 700 GtC are burnt, and the transition to renewables takes place as soon as 2050. For the additive case much less fossil fuel is left in situ, i.e. 1250 GtC are burnt, and renewables are phased in much later, i.e., in 2090. This difference in fossil fuel consumption leads to higher temperature trajectories under additive damages than under multiplicative damages, but despite the higher damages the effects on consumption, output and capital and thus on welfare is not large. This leads us to conclude that climate change, if addressed through optimal policy, can be avoided at relatively low costs. Depending on the nature of climate damages, the costs of inaction are potentially large.

Under multiplicative damages temperature slightly overshoots the 2°C warming limit, peaking at 2.3°C, whereas with additive damages temperature peaks at 3.2°C above pre-industrial levels. The transition to the carbon-free era is helped by the imposition of the carbon tax. In both cases the optimal path for the social cost of carbon and global carbon tax is inverted U-shaped, which results from the fact that CO2 emissions first rise and then come to an end. The location of the two curves is, of course, different. In the multiplicative case the social cost of carbon starts at a level of 75 $/tC and reaches a maximum of 490 $/tC in the year 2180. With additive damages the carbon tax starts with half that under multiplicative damages, 37.5 $/tC, and the maximum is reached at 210 $/tC in the year 2120.



Key: Social optima: multiplicative damages (———); additive damages (———)
"Laissez faire": multiplicative damages (— —); additive damages (— —)
Proportional carbon tax: multiplicative damages (- - - -); additive damages (- - - -)

Figure 1: Simulations with CES production technology $(\vartheta = 0.5)$

Weitzman (2010) finds that with additive global warming damages (in instantaneous welfare) the willingness to sacrifice current consumption to avoid future global warming is seven times higher than with multiplicative damages. In contrast, we find that additive damages lead to half the social cost of carbon at each point of time compared to multiplicative damages. However, this stark difference should serve as a reminder that the additive utility damages used in Weitzman and the additive production damages used in our model are not comparable. Weitzman's number applies to the willingness to pay to avoid any change in temperature along an exogenous consumption trajectory, whereas the social cost of carbon of our simulations reports the willingness to pay to avoid a marginal increase in atmospheric carbon along the optimal path. Our numerical results are in line with the analytical findings in section 2: the social cost of carbon is higher under multiplicative damages provided that the rate of economic growth (in potential output) is positive. Within a fully specified integrated climate assessment model additive damages lead to a less ambitious climate policy. The nature of climate damage is irrelevant for laissez-faire fossil fuel use and the climate dynamics it implies.

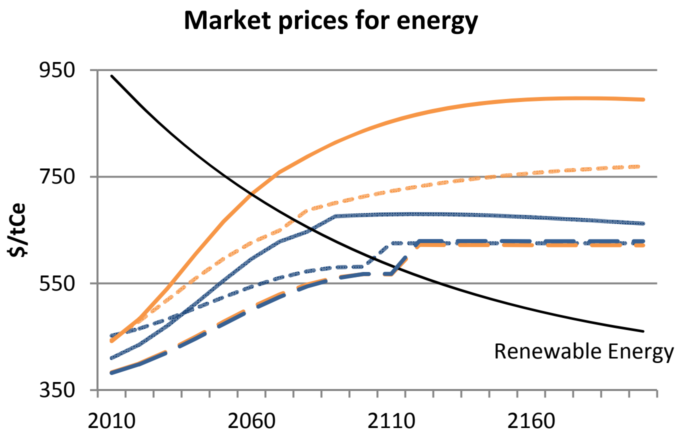
Table 2: Transition times and carbon budget

		Fossil fuel Only	*Simultaneous use*	*Renewable Only*	*Carbon used*	*max. T*
multip.	*First best*	2010-2050	2060	2070 –	690 GtC	2.3 °C
	"Laissez faire"	2010-2110	x	2120 –	2100 GtC	4.3 °C
	Proportional tax	2010-2070	x	2080 –	990 GtC	2.8 °C
additiv.	*First best*	2010-2050	x	2090 –	1250 GtC	3.2 °C
	"Laissez faire"	2010-2110	x	2120 –	2150 GtC	4.3 °C
	Proportional tax	2010-2100	x	2110 –	1800 GtC	3.9 °C

3.2. Time paths for the market price of fossil fuel and the renewable in the various scenarios

The imposition of a carbon tax increases the too low (relative to the first-best) market price of fossil energy. The prices of all energy sources under the different scenarios are depicted in figure 2. The energy price in the first-best outcome is the shadow price of fossil fuel, which consists of the marginal extraction cost, the Hotelling rent (the present discounted sum of all extraction cost savings due to a higher fossil fuel stock) and the social cost of carbon.

The shadow price of fossil fuel increases initially because all three components of the social cost increase. Since the social cost of carbon is higher under multiplicative damages, the carbon tax is higher initially and rises faster than under additive damages. This leads to a higher market price of fossil energy and induces lower fossil fuel consumption and higher in situ stocks as discussed above. Once the market price of fossil energy exceeds the market cost of renewable energy, renewable energy takes over. From then on the marginal extraction cost of fossil fuel is constant and the Hotelling rent is zero. The reason is that some fossil fuel is left in the crust of the earth, but no extraction takes place.



Key: Social optima: multiplicative damages (——); additive damages (——)
"Laissez faire": multiplicative damages (— —); additive damages (— —)
Proportional carbon tax: multiplicative damages (- - -); additive damages (- - -)

Figure 2: The market prices of energy

However, the social cost of carbon continues to rise for some time, because decay of atmospheric carbon is limited and consumption is increasing, yielding smaller marginal utility of consumption and thus a higher social cost of carbon, expressed in the numeraire (see equation (11)). However, after some point of time decay of atmospheric carbon dominates the decrease in marginal utility of consumption and the social cost of carbon starts to fall.[9] Under "laissez faire" the *SCC* is not imposed on the market price of fuel which consequently is lower. We are not finding any Green Paradox effects in our simulations (cf. van der Ploeg and Withagen, 2012a). Figure 2 also illustrates the sub-optimality of the proportionality rule in (11′). With additive damages the proportional carbon tax increases the market price of fossil energy beyond its optimal level initially but fails rise rapidly enough to curb carbon emissions sufficiently. Figure 3 presents a decomposition of the market price of fossil energy for selected scenarios.

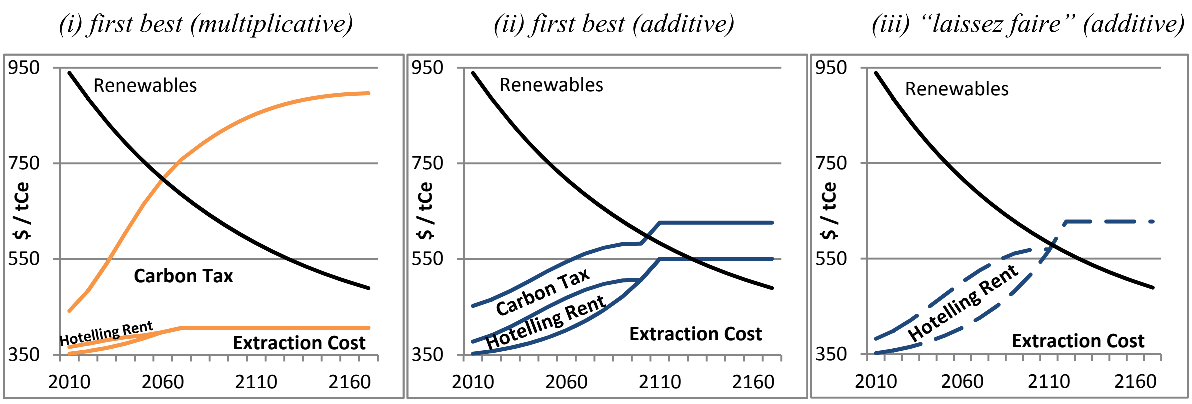


Figure 3: Decomposition of the market price of fossil relative to renewable energy

[9] In theory the falling social cost of carbon can decrease the market price of fossil energy sufficiently to make fossil fuel competitive again. As figure 2 indicates, the permanent stock of atmospheric carbon and extraction costs are too high and the time horizon too short to make re-switching optimal.

Under multiplicative damages the *SCC* is large relative to the other cost components of the market price of fossil energy, namely the scarcity rent and extraction cost. The carbon tax rises rapidly and induces a transition to renewable energy mid of the current century. Under additive damages the *SCC* is significantly lower, allowing for more extraction of fossil fuel. Higher extraction increases the value of the in situ resource and the scarcity rent.

The dynamics of the "laissez faire" market price of fossil energy are essentially identical under additive and multiplicative damages as pointed out in section 2 and can be seen in figure 2. As the carbon tax is set to zero in both cases, almost the same amount of fossil fuel is used for approximately the same period of time, till 2120.[10] Again, higher cumulative extraction increases the Hotelling rent. At the end of the fossil era slightly more fossil fuel is left in situ in the economy with multiplicative damages than in the one with additive damages (see figure 2). But, as is to be expected, damages to production are much higher in the multiplicative case, and therefore consumption will be lower. This becomes particularly manifest toward the end of this century. An interesting feature of the simulations is that with multiplicative damages capital is decreasing for several decades immediately after the economy stops using fossil fuel. We also see that, in spite of higher input of fossil fuel, net output decreases over a short period of time, preceding the transition to renewable energy. This indicates that in the fossil fuel phase capital is over-accumulated, which is then corrected in the phase where renewable energy is used. We also observe that much more fossil fuel is used in the absence of a carbon tax and that the transition to renewable energy takes place much later.

3.3. No policy leads to overinvestment and too little use of the renewable energy

Although not our primary focus, it is interesting to see how the market outcome differs from the first-best outcome. Output, consumption and capital accumulation take place at very similar levels for the first-best and "laissez-faire" outcomes under additive damages. The reason for this is mainly that in a growing economy net output is not much affected by temperature changes if affordable mitigation is available (as in the first-best scenarios) or damages are low (as in the additive BAU scenario). Under multiplicative damages of global warming the impacts of the climate externality are large enough to drastically change accumulation paths.

Table 3: Welfare gains and losses in % of initial GDP (relative to multiplicative first best)

	multiplicative	*additive*
First best	0%	5%
"Laissez faire"	- 17%	2%

[10] In the "laissez faire" scenario the social cost of carbon does not vanish completely, which is due to the numerical implementation of the program where each individual agent is aware of the fact that she is responsible for less than 1% of total emissions and thus for some damage (see appendix for details on "laissez faire").

Table 3 summarizes total welfare relative to multiplicative first-best. “Laissez-faire” yields a welfare loss of 17% of initial GDP relative to first-best under multiplicative damage. If damages are additive, welfare falls by mere 3% due to a zero tax on carbon.

Sinn (2007) and Stern (2010) point out that “laissez-faire” leads to an inefficient allocation of resources, because economic decision-makers do not recognize the deleterious effects of greenhouse gas (GHG) emissions. Private and social cost calculations diverge; agents overvalue the returns to conventional capital stock and undervalue the investments in green energy sources. Imperfect price signals (λ, μ, ν) induce excess fossil fuel extraction and capital accumulation, leading to high climate damages over the time horizon. The inefficiency of “laissez faire” manifests itself in low consumption to allow accumulation in early periods of the program leading to low consumption in the future due to high climate damages.[11]

Comparing welfare across different specification in table 3, scenarios under additive global warming damages yield higher welfare than multiplicative damages even if no carbon tax is imposed. The reason is, again, that positive economic growth implies that marginal damages are growing under multiplicative but not under additive global warming damages and rising production costs of fossil energy drive the transition to renewable energy rather than mounting environmental damage. The higher social cost of carbon under multiplicative damages brings forward the carbon-free era but also increases the cost of the energy transition (market prices of energy increase by at most 10%) and lowers consumption (by at most 3%). This implies that welfare decreases by 5% of today’s GDP under multiplicative damages under first-best relative to the outcome under additive damages.

We conclude from table 3 that the climate problem is potentially large if not addressed by optimal policy. Under “laissez faire” the nature of global warming damages matters greatly. In the social optimum the problem of climate change can be managed at relatively modest cost. Interestingly, whether damages are additive or multiplicative leads to small differences in welfare under the optimal carbon tax but to large differences in welfare under “laissez faire”.

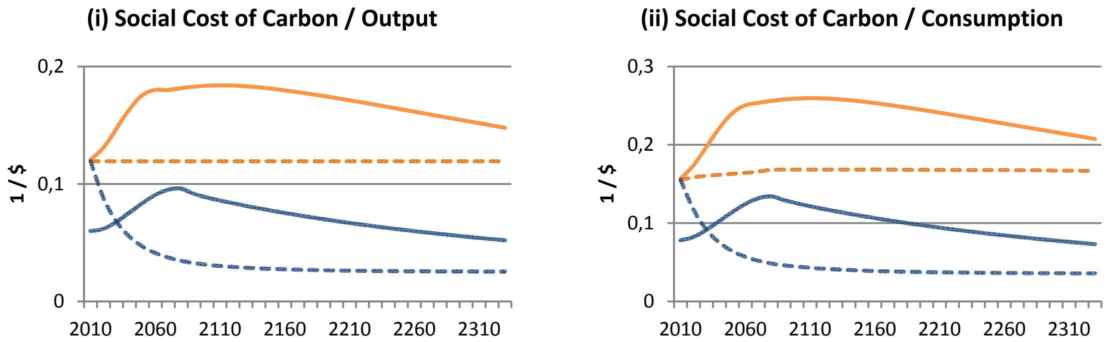
3.4. The optimal carbon tax is not proportional to aggregate consumption or world GDP

To examine whether the linear formula for the optimal carbon tax is really proportional to global GDP as suggested by Golosov et al. (2014) and demonstrated in equation (11’), we examine whether it holds up in a more general integrated assessment model of Ramsey growth and climate change. Figure 4 therefore plots the ratio of the optimal carbon tax to both world GDP and aggregate consumption; the short-dashed lines in Figure 1 provide further details. For sake of comparison we also use equation

[11] Rezai et al. (2012) discuss this mechanism in more detail and demonstrate the relevance of this inefficiency for the debate on the (opportunity) cost of climate change in a simpler model of Leontief production technology and unlimited stocks of oil.

(11′) to plot a similar simple formula for the optimal carbon tax when global warming damages are additive.

We immediately observe that the optimal carbon taxes (solid lines) are not well described by a constant proportion of world GDP or aggregate consumption. The general pattern is that during the initial phases of fossil fuel use the social cost of carbon rises as a proportion of world GDP as more carbon emissions push up marginal damages of global warming whilst during the carbon-free phases the social cost of carbon falls as a proportion of world GDP as a significant part of the stock of carbon in the atmosphere is gradually returned to the surface of the oceans and the earth.



Key: Social optima: multiplicative damages (———); additive damages (———)
Proportional carbon tax: multiplicative damages (– – –); additive damages (– – –)

Figure 4: The social cost of carbon as ratio of aggregate world GDP and consumption

Further sensitivity analysis shows that setting the carbon tax to a constant proportion of world GDP is a poor approximation to the optimal carbon tax under multiplicative or additive climate damages, regardless of whether the elasticity of factor substitution is zero (Leontief) or one (Cobb-Douglas), the elasticity of intertemporal substitution is high or low, and the pure rate of time preference is high or low. These simple formulas for the optimal carbon tax are, in fact, non-optimal and induce more fossil fuel to be burnt (about 50% more relative to the first best, see also table 2) and more severe climate change. They are an especially poor description of the optimal carbon tax under additive global warming damages, since the carbon tax then falls during the fossil-fuel era as a result of economic growth whilst the optimal carbon tax should be increasing during that period (see figure 4 for details). In general, the social cost of carbon under multiplicative damage is about 2 to 2½ times its value under additive damages. Weitzman (2009) uses a partial equilibrium model and finds that the willingness to pay, in terms of giving up present consumption, for reducing future temperature is 7 times higher in the additive case compared to the multiplicative case along a suboptimal trajectory. In our fully-fledged integrated climate assessment model we consistently find the opposite result.

4. Conclusions

How global warming damages are modeled and calibrated matters for the social cost of carbon and climate policy. We find that the climate policy is less ambitious, energy use higher, the stock of fossil fuel left in situ lower, global mean temperature higher and the optimal carbon tax lower with additive damages provided that the rate of economic growth (in potential output) is positive. This ranking is independent of how tough society finds it to substitute present for future consumption, the social rate of discount, and the elasticity of substitution of energy in production. Interestingly, the time paths for global consumption, capital and GDP are not much affected by whether damages are additive or multiplicative despite the temperature path being higher under additive damages.

Our integrated assessment model also indicates that a higher elasticity of intertemporal substitution and a lower social rate of discount lead to a higher optimal carbon tax and a quicker phasing in of renewables and more fossil fuel left in the crust of the earth, less so under additive than multiplicative global warming damages. A higher elasticity of intertemporal substitution corresponds to a lower coefficient of intergenerational inequality aversion. Since society is more concerned with fighting global warming than with avoiding big differences in consumption of different generations, the carbon tax is borne much more by earlier generations than by later generations, both in the additive and in the multiplicative case.

Stern (2013) criticizes the current generation of IAMs for focusing on too limited a set of functional and parametric relationships. Our analysis is only a first step in broadening the scope of the type of climate damages that might be considered, admittedly, leaving ample room for further improvement. Our analysis, however, also leads us to conclude that climate change, if addressed through optimal policy, can be avoided at relatively low costs. Depending on the nature of climate damages, the costs of inaction are potentially large even if many additional reasons for concern Stern highlights are not taken into account. Our analysis suggests that further empirical work is needed on whether climate damages are multiplicative or additive and, more generally, on how substitutable they are for economic production.

Golosov et al. (2014) employ unrealistically low damages at higher temperatures and need to make some very bold assumptions to ensure that the optimal carbon tax is a fixed proportion of world GDP. Our model of Ramsey growth and climate change has more realistic global warming damages, capital-intensive extraction costs, CES instead of Cobb-Douglas utility and production functions, and realistic time profiles for the evolution of population and technology. We find that the optimal carbon tax is then a hump-shaped function with the carbon tax falling in the carbon-free era as temporary component of the atmospheric stock of carbon fades away. Our findings of a hump-shaped carbon tax and a lower carbon tax are robust to changes in key parameters of the model.

Finally, future developments in the productive capacity of the economy are important determinants of the optimal carbon tax. Future prices of clean and dirty sources of energy and their necessity in the general production process heavily influence relative prices and the allocation of resources today. We have examined the effects of variations in the substitutability between energy and conventional capital through a CES production function with a fixed rate of technological progress. Recent contributions by Acemoglu et al. (2012) and Mattauch et al. (2012) highlight the importance of learning and lock-in effects by making the rate of technical progress as endogenous. It is possible to use the empirical estimates of the determinants of growth rates in total factor and energy productivities given in Hassler et al. (2011) in our model of economic growth and climate change. This will allow much more substitution possibilities between energy and the capital-labor aggregate in the long run than in the short run. The logic of directed technical change suggests that it is more important to have substantial R&D subsidies for green technology to kick-start green innovation and fight global warming.

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Appendix: Functional forms, calibration and computational implementation

Preferences

In the simulations we will use an iso-elastic utility function $U(C/L)=\frac{(C/L)^{1-1/\eta}-1}{1-1/\eta}$, where the elasticity of intertemporal substitution is $EIS=-\frac{U'}{U''C}=-\frac{(C/L)^{-1/\eta}}{(-1/\eta)(C/L)^{-1/\eta}}=\eta$. We set the elasticity of intertemporal substitution to η = ½ and thus intergenerational inequality aversion to 2. The pure rate of time preference ρ is set to 10% per decade which corresponds to 0.96% per year.

Cost of energy

We employ an extraction technology of the form $G(S)=\gamma_1(S_0/S)^{\gamma_2}$, where γ_1 and γ_2 are positive constants. This specification implies that reserves will not be fully be extracted; some fossil fuel remains untapped in the crust of the earth. Extraction costs are calibrated to give an initial share of energy in GDP between 5%-7% depending on the policy scenario. This translates to fossil production costs of \$350/tC (\$35/barrel of oil), where we take one barrel of oil to be equivalent to 1/10 ton of carbon. This gives approximately $G(S_0)=\gamma_1=0.75$. The IEA (2008) long-term cost curve for oil extraction gives a doubling to quadrupling of the extraction cost of oil if another 1000 GtC are extracted. Since we are considering all carbon-based energy sources (not only oil) which are more abundant and cheaper to extract, we assume a more doubling but less than quadrupling of production costs if a total 3000 GtC is extracted. With $S_0=4000$ GtC,[12] this gives $\gamma_2=0.75$.[13] In general, we assume very low extraction costs and a high initial stock of reserves.

The unit cost of renewable energy is calibrated to the percentage of GDP necessary to generate all energy demand from renewables. Under a Leontief technology, with $\vartheta\to 0$, energy demand is σZ_t with Z_t potential, pre-damage output and σ the carbon intensity of output. The cost of generating all energy carbon free is $\sigma Z_t b_t/Z_t=\sigma b_t$. Nordhaus (2008) assumes that it costs 5.6% of GDP to achieve this. We take double this number $\sigma b_1=0.12$ (i.e. we assume 12%) or, with $\sigma=0.62$ as derived below, $b_1=2$. In the future this cost falls to current prices of fossil energy (with energy amounting to

[12] Stocks of carbon-based energy sources are notoriously hard to estimate. IPCC (2007) assumes in its A2-scenario that 7000 $GtCO_2$ (with 3.66 tCO2 per tC this equals 1912 GtC) will be burnt with a rising trend this century alone. We roughly double this number to get our estimate of 4000 GtC for initial fossil fuel reserves. Nordhaus (2008) assumes an upper limit for carbon-based fuel of 6000 GtC in the DICE-07.

[13] Since $G(1000)/G(4000)=(4000/1000)^{\gamma_2}=4^{\gamma_2}$ and $4^{0.75}=2.8$.

about 5% of GDP), that is b_t approaches 0.8). We assume that exogenous technical progress lowers the unit cost at a falling rate starting at a reduction of 1% per year. Specifically, $b_t = 0.8 + 1.2e^{-0.1t}$. This calibration is done for a Leontief technology. We assume that for a more general technology the same parameter values can be applied. Our calibration assumes that renewable energy is initially very expensive and falls to current levels only in the very long run. This, together with the assumption about fossil energy, biases the model against rapid de-carbonization.

Initial capital stock and depreciation rate

The initial capital stock is set to 200 (US$ trillion), which is taken from Rezai et al. (2012). We set δ to be 0.5 per decade, which corresponds to a yearly depreciation rate of 6.7%.

Population growth and labor-augmenting technical progress

Population in 2010 (L_1) is 6.5 billion people. Following Nordhaus (2008) and UN projections population growth is given by $L_t = 8.6 - 2.1e^{-0.35t}$. Population growth starts at 1% per year and falls below 1% percent per decade within six decades and flattens out at 8.6 billion people. Without loss of generality the efficiency of labor $A_t^L = 3 - 2e^{-0.2t}$ starts out with $A_1^L = 1$ and an initial Harrod-neutral rate of technical progress of 2% per year. The efficiency of labor stabilizes at 3 times its current level.

Global production and global warming damages

Output before damages is $Z_t = \left[(1-\beta)\left(AK_t^{\alpha}(A_t^L L_t)^{1-\alpha} \right)^{1-1/\vartheta} + \beta\left(\frac{F_t + R_t}{\sigma}\right)^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}$, $\vartheta \geq 0, 0 < \alpha < 1$ and $0 < \beta < 1$. This is a constant-returns-to scale CES production function in energy and a capital-labor composite with ϑ the elasticity of substitution, β the share the parameter for energy, and σ the carbon intensity of output. The capital-labor composite is defined by a constant-returns-to-scale Cobb-Douglas function with α the share of capital, A total factor productivity and A_t^L the efficiency of labor. The two types of energy are perfect substitutes in production. Damages are calibrated so that they give the same level of global warming damages for the initial levels of output and mean temperature. It is convenient to rewrite production before damages as

$$Z_t = Z_0 \left[(1-\beta)\left(\frac{AK_t^{\alpha}(A_t^L L_t)^{1-\alpha}}{Z_0} \right)^{1-1/\vartheta} + \beta\left(\frac{F_t + R_t}{\sigma Z_0} \right)^{1-1/\vartheta} \right]^{\frac{1}{1-1/\vartheta}}.$$

We set the share of capital to $\alpha = 0.35$, the energy share parameter to $\beta = 0.05$, and the elasticity of factor substitution to $\vartheta = 0.5$. World GDP in 2010 is 63 $trillion. The energy intensity of output σ is calibrated to current energy use. In the Leontief case energy demand (only fossil fuel initially) is

$F_0 = \sigma D_0 Z_0$. With carbon input equal to 8.36GtC in 2010, we obtain $\sigma = (8.36/2.13)/63 = 0.062$. Finally, given $A_1^L = 1$ we can back out A = 34.67.

Climate Dynamics and Damage

Following Golosov et al. (2014), E_t^P denotes the stock of carbon (GtC) that stays thousands of years in the atmosphere and E_t^T the stock of atmospheric carbon (GtC) that decays at rate $\varphi = 0.0228$, This carbon cycle supposes that 20% of carbon emissions stay up 'forever' in the atmosphere and the remainder has a mean life of about 300 years, so $\varphi_L = 0.2$. The parameter $\varphi_0 = 0.393$ is calibrated so that about half of the carbon impulse is removed after 30 years. We set current atmospheric carbon at $E_0^P = 103$ GtC and $E_0^T = 699$ GtC. It is commonly assumed that an increase of atmospheric carbon to double its pre-industrial level, leads to a temperature increase of 3°C, so $\omega = 3$.

Nordhaus (2008) supposes that with global warming of 2.5^{o} C damages are 1.7% of world GDP and uses this for purposes of his DICE-07 model to calibrate the following function for the fraction of output what is left after damages from global warming: $D(T_t) = \dfrac{1}{1+0.00284T_t^2} = \dfrac{1}{1+(T_t/18.8)^2}$.

Weitzman (2010) argues that global warming damages rise more rapidly at higher levels of mean global temperature than suggested by Nordhaus (2008). With output damages equal to 50% of world GDP at 6^{o} C and 99% at 12.5^{o} C, Ackerman and Stanton (2012) calibrate what is left of output after global warming damages as: $D(T_t) = \dfrac{1}{1+(T_t/20.2)^2 + (T_t/6.08)^{6.76}}$. The extra term in the denominator captures potentially catastrophic losses at high temperatures.

Computational implementation

The transversality condition for the model is $\lim_{t\to\infty} e^{-\rho t}(\lambda_t K_t + \mu_t S_t + \eta_{1t} E_t^P + \eta_{2t} E_t^T) = 0$. In our simulations we solve the model for finite time and use the turnpike property to approximate the infinite-horizon problem. All equilibrium paths approach the steady state quickly such that the turnpike property renders terminal conditions essentially unimportant. We allow for continuation stocks to reduce the impact of the terminal condition on the transitions paths in the early periods of the program. We use the computer program GAMS and its optimization solver CONOPT3 to solve the model numerically. The social planner optimum in which the externality is taken into account fit the program structure readily. To solve for the "laissez faire" equilibrium paths, we adopt the iterative approach discussed in detail in Rezai (2011). Briefly, to approximate the externality scenario, the aggregate economy is fragmented into N dynasties. Each dynasty has *1/N*th of the initial endowments and chooses consumption, investment and energy use in order to maximize the discounted total utility of per capita consumption. The dynasties understand the contribution of their own emissions to the climate change, but take carbon emissions of others as given. The climate dynamics are affected by the decisions of all dynasties. This constitutes the market failure.

It might seem easier to simply assume that there is one dynasty that ignores the externality but this would not be a rational expectation equilibrium. The externality problem is not an optimization but an equilibrium problem. The CONOPT3 solver of GAMS is powerful in solving maximization problems and it is more efficient to adopt an iterative routine in which a planner of a fragmented economy solves an optimization problem representatively than to attempt solving the equilibrium conditions directly. Given our specifications, the computation of the equilibrium problem takes less than one minute. To introduce this approximate externality, we make the following adjustments to the initial stocks $K(0) = K_0 / N, S(0) = S_0 / N$ and $L(0) = L_0 / N$. All production and cost functions are homogeneous of degree 1 and therefore invariant to *N*. The introduction of the pollution externality only requires a modification of the transition equation of atmospheric carbon to include emissions regarded as exogenous by each dynasty:

$$E_{t+1}^P = E_t^P + \varphi_L(F_t + Exg_t) \quad \text{and}$$

$$E_{t+1}^T = (1-\varphi)E_t^T + \varphi(1-\varphi_L)(F_t + Exg_t) \ .$$

In the “laissez faire” scenario dynasties essentially play a dynamic non-cooperative game, which leads to a Nash equilibrium in which each agent forecasts the paths of emissions correctly and all agents take the same decisions as all dynasties are identical. Equilibrium requires $Exg_t = (N-1)F_t$. Under “laissez faire” the planner only adjusts her controls to take into account the effects of her own decisions (i.e. 1/*N*th of the climate externality). If $N = 1$ the externality is internalized and we obtain the social optimum. As $N \to \infty$, we obtain the “laissez faire” outcome characterized in section 2.

Following Rezai (2011), the numerical routine starts by assuming a time path of emissions exogenous to the dynasty's optimization, Exg_t , at an informed guess. GAMS solves for the representative dynasty's welfare-maximizing investment, consumption, and energy use choices conditional on this level of exogenous emissions. $(N-1)$ times the dynasty's emission trajectory implied by these choices, F_t, defines the time profile of exogenous emissions in the next iteration. The same applies for the knowledge trajectory. The routine is repeated and Exg_t are updated until the difference in the time profiles between iterations meets a pre-defined stopping criterion. In the reported results iterations stop if the deviation $|(N-1)F_t/Exg_t - 1|$ in each time period is at most 0.001%.

We set $N = 400$ to account for the fact that in the present world economy, the externality in the market of GHG emissions is already internalized to a very small extent through the imposition of carbon taxes or tradable emission permits and non-market *regulation* (e.g. through the Kyoto Protocol or the establishment of the European Union Emission Trading Scheme). In our “laissez faire” simulations, the dynastic planner takes into account less than 0.25% of global emissions.