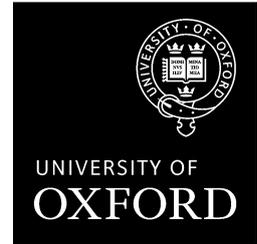


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# **OxCarre Research Paper 110**

## **Cumulative Carbon Emissions and the Green Paradox**

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# CUMULATIVE CARBON EMISSIONS AND THE GREEN PARADOX

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## Abstract

The Green Paradox states that a gradually more ambitious climate policy such as a renewables subsidy or an anticipated carbon tax induces fossil fuel owners to extract more rapidly and accelerate global warming. However, if extraction becomes more costly as reserves are depleted, such policies also shorten the fossil fuel era, induce more fossil fuel to be left in the earth and thus curb cumulative carbon emissions. This is relevant as global warming depends primarily on cumulative emissions. There is no Green Paradox for a specific carbon tax that rises at less than the market rate of interest. Since this is the case for the growth of the optimal carbon tax, the Green Paradox is a temporary second-best phenomenon. There is also a Green Paradox if there is a chance of a breakthrough in renewables technology occurring at some random future date. However, there will also be less investment in opening up fossil fuel deposits and thus cumulative carbon emission will be curbed.

**Keywords:** global warming, Green Paradox, carbon tax, renewables subsidy, second best, technological breakthrough

**JEL codes:** D81, H20, Q31, Q38

*Revised May 2013*

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\* Department of Economics, University of Oxford and the VU University Amsterdam. Support from the European Research Council and the BP funded Oxford Centre for the Analysis of Resource Rich Economies is gratefully acknowledged. I have benefited much from very helpful comments by Florian Habermacher and Cees Withagen.

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## 1. Introduction

The idea that well-intended climate policy may have undesirable unintended consequences has gained traction in recent years.<sup>1</sup> By levying a steeply rising carbon tax or subsidizing renewables, fossil fuel producers are encouraged to extract and sell their fuel more quickly, thereby exacerbating carbon emissions and global warming. This counterintuitive result has been coined the Green Paradox (Sinn, 2008, 2012). However, if fossil fuel extraction becomes more costly as fewer reserves are left, the total amount of fossil fuel extracted from the earth is endogenous and not all reserves are necessarily fully exhausted. Over time, fossil fuel will then become less attractive relative to the carbon-free backstop. A sharply rising schedule for the carbon tax or a renewables subsidy thus makes it more attractive to keep more fossil fuel reserves unexploited in the crust of the earth. This offsets and can overturn the adverse effect on global warming, thus rendering the Green Paradox a ‘red herring’. The Green Paradox is conceptually not wrong, but it is questionable how relevant it is for the big issue of global warming which is mainly driven by cumulative carbon emissions (e.g., Allen et al., 2009ab).

Our objective is to formally illustrate the Green Paradox and how it is offset or reversed in a variety of economic models. We first establish in section 2 the Green Paradox in the simplest setting: a partial-equilibrium, two-period model of a market for fossil fuel with exhaustible reserves and no potential substitutes. We show that announcing a future carbon tax causes today’s fossil fuel prices to increase relative to those in the future, thus encouraging fossil fuel producers to produce more crude oil, natural gas and coal today and thus less tomorrow. The acceleration of carbon emissions brings forward global warming, which is the essence of the Green Paradox. Since fossil supply reserves are completely elastic and fossil fuel demand disappears completely when the carbon-free backstop kicks in, we extend our model in section 3 to a continuous-time, infinite-horizon model of a market with extraction costs rising as less reserves are left and with a carbon-free perfect substitute for fossil fuel which is currently too pricy to be used but may become competitive at some point in the future. This extension allows for both an internal margin (how fast to deplete a *given* amount of fossil fuel) and an external margin (how much of the total stock of fossil fuel to exploit). We establish that in such a market a renewables subsidy, a sufficiently steeply rising carbon tax or an anticipated carbon tax always leads to short-run Green Paradox effects (provided that the tax does not make fossil fuel extraction unprofitable), but also to a higher stock

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<sup>1</sup> The idea that rising carbon taxes may harm global warming is found in Long and Sinn (1985) and Sinclair (1992) and Strand (2007). Recent papers on the Green Paradox are Sinn (2008, 2012), Gerlagh (2011), Hoel (2008, 2010, 2011a, 2012), Smulders et al. (2010), van der Ploeg (2011), Edenhofer and Kalkuhl (2011), Grafton et al. (2012), van der Ploeg and Withagen (2012a,b,c), Long (2012), van der Werf and Di Maria (2012) and Habermacher and Kirchgässner (2012).

of fossil fuel left in situ and thus to lower cumulative carbon emissions given that we abstract from natural decay of atmospheric carbon. The latter effect typically dominates and is what ultimately matters for global warming (Allen et al., 2009a,b; Solomon et al., 2009).

We then derive the optimal carbon tax in section 4 and show that this grows at a lower rate than the market rate of interest and compared with “laissez faire” thus never leads to a Green Paradox. We also show that the second-best policy of a renewables subsidy which ensures that the socially optimal level of fossil fuel is left in situ gives rise to temporary Green Paradox effects.

Subsequently, we reexamine the Green Paradox in a context without stock-dependent extraction costs but with initial outlays on exploration investment (Gaudet and Lasserre, 1988). This also gives two margins: how quickly to extract fossil fuel and how much fossil fuel in total to extract from the earth. We show that the prospect of some breakthrough in the bringing to the market of a carbon-free substitute at some *uncertain* future moment induces a given stock of fossil fuel to be exhausted more quickly which is another manifestation of the Green Paradox. As a result, carbon is more quickly emitted into the atmosphere and global warming is exacerbated. These effects are less strong if the carbon-free backstop is a worse substitute for oil.<sup>2</sup> However, we also show that the prospect of cost-effective renewables becoming available at some random moment in the future curbs exploration investment and thus the total stock of available oil reserves. This curbs cumulative carbon emissions and global warming. Subsidizing green R&D to speed up the introduction of breakthrough renewables thus leads to more rapid fossil fuel extraction before the breakthrough, but more fossil fuel is left in situ as exploration investment is reduced. The resulting drop in cumulative carbon emissions offsets and is likely to reverse the Green Paradox.

Finally, section 6 concludes and sums up our reservations against using the Green Paradox as a rationale to give up on levying an ambitious carbon tax or subsidies for a renewables subsidy or green R&D.

## 2. Two-period illustration of the Green Paradox

Here we illustrate the Green Paradox in a two-period framework. Let  $S$  be the given total stock of fossil fuel reserves and  $F_t$  the amount of fossil fuel extracted in period  $t$ . Suppose there are no extraction costs, so that it pays to exhaust all fossil fuel reserves and thus  $S = F_1 + F_2$ . Fossil fuel demand in each period is isoelastic with price elasticity equal to the constant  $\varepsilon > 0$ . Consumers face an *ad valorem* carbon tax equal to  $\tau_t$  and the demand schedule for fossil fuel at time  $t$  is  $F_t = \psi [p_t(1 + \tau_t)]^{-\varepsilon}$ ,  $\psi > 0$ , where  $p_t$  is the price

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<sup>2</sup> This is related to Long (2012) who finds that technical change that marginally increases the degree of substitutability of bio-fuels for fossil fuels causes fossil fuel producers to expect lower future demand and thus react by increasing current extraction and accelerating global warming (the Green Paradox).

fetched by fossil fuel producers. The inverse fossil fuel demand schedule is  $p_t = (\psi / F_t)^{1/\varepsilon} / (1 + \tau_t)$ .

Extraction rates are chosen to maximize the present discounted value of profits subject to the constraint that fossil fuel reserves are fully exhausted:

$$(1) \quad \text{Max}_{F_1, F_2} p_1 F_1 + \frac{p_2 F_2}{1+r} = p_1 F_1 + \frac{p_2 (S - F_1)}{1+r},$$

where  $r > 0$  denotes the exogenous interest rate. We suppose that fossil fuel producers operate under perfect competition and take the industry-wide fossil fuel price as given. Optimality then requires that the Hotelling rule holds: the return on keeping a unit of fossil fuel in the ground must equal the return of extracting a unit, selling it and getting a rate of return on the sale proceeds:

$$(2) \quad p_2 - p_1 = r p_1.$$

Using (2) and the condition that all fossil fuel is fully exhausted, we solve for extraction rates:

$$(3) \quad F_1 = \left( \frac{1}{1 + (1+r)^{-\varepsilon} [(1+\tau_2)/(1+\tau_1)]^{-\varepsilon}} \right) S, \quad F_2 = \left( \frac{1}{1 + (1+r)^{\varepsilon} [(1+\tau_2)/(1+\tau_1)]^{\varepsilon}} \right) S.$$

If the market expects a rising carbon tax,  $\tau_2 > \tau_1$ , (3) implies that more fossil fuel will be extracted today and less oil tomorrow than in the “laissez-faire” outcome,  $F_1 > (1+r)^{-\varepsilon} F_2$ . This market reaction to an anticipated future carbon tax implies that there are relatively more carbon emissions today than tomorrow. The bringing forward of global warming has become known as the Green Paradox (Sinn, 2008, 2012).

Equations (3) imply that a positive, constant *ad valorem* carbon tax does not affect the optimal path of fossil fuel depletion and carbon emissions. In practice, a carbon tax is typically a *specific* tax, so demand is given by  $F_t = \psi (p_t + \tau_t)^{-\varepsilon}$ . The Hotelling rule (2) is still valid, so extraction rates are solved from:

$$(3') \quad \left( \frac{\psi}{S - F_1} \right)^{\frac{1}{\varepsilon}} = (1+r) \left( \frac{\psi}{F_1} \right)^{\frac{1}{\varepsilon}} + [\tau_2 - (1+r)\tau_1], \quad F_2 = S - F_1.$$

We thus establish that, only if the specific carbon tax rises at a rate faster than the market rate of interest, will there be a Green Paradox in the sense that  $F_1$  is lower and  $F_2$  correspondingly bigger (cf., Sinn, 2008). An anticipated specific carbon tax ( $\tau_2 > 0$ ,  $\tau_1 = 0$ ) always leads to a Green Paradox.

### 3. Does the Green Paradox really hold in the market economy?

To gain more insight into the Green Paradox, one must examine both the external margin (how much of fossil fuel reserves to leave untouched) and the internal margin (how much fossil fuel to extract from the earth at a given instant of time). To analyze these issues, we switch from a two-period, discrete-time model with no substitutes for fossil fuel to an infinite-horizon, continuous-time model where an initial phase of fossil fuel production is at some endogenously determined future instant of time ended and replaced by a final phase where a carbon-free substitute has priced fossil fuel out of the market altogether.

#### 3.1. Stock-dependent extraction costs and a carbon-free substitute for fossil fuel

We suppose that fossil fuel firms face stock-dependent extraction costs,  $G(S)F$ , where  $G' < 0$  and  $\lim_{S \rightarrow 0} G(S) = \infty$ . Hence, the cost of fossil fuel extraction rises without bound as reserves diminish and less accessible fields, wells or mines have to be exploited so that it is not profitable to fully exhaust fossil fuel reserves. Fossil fuel firms face a time path for a *specific* carbon tax  $\tau$ . We suppose that renewables, supplied at fixed unit cost  $b > 0$ , are a perfect substitute for fossil fuel. Fossil fuel is used until it is priced out of the market by renewables at which point the carbon-free era starts.

Fossil fuel firms operate under perfect competition to maximize the discounted value of future profits,

$$(4) \quad \text{Max}_F \int_0^{\infty} [pF - G(S)F] e^{-rt} dt,$$

subject to the constraint that cumulative fossil fuel extraction cannot exceed initial reserves,

$$(5) \quad \dot{S} = -F, \quad F \geq 0, \quad S(0) = S_0, \quad \int_0^{\infty} F(t) dt \leq S_0.$$

The Maximum Principle gives the following first-order conditions:

$$(6a) \quad \left. \begin{array}{l} p \leq G(S) + \lambda \\ F \geq 0 \end{array} \right\} \text{c.s.},$$

$$(6b) \quad \dot{\lambda} = r\lambda + G'(S)F, \quad \lim_{t \rightarrow \infty} e^{-rt} \lambda(t) F(t) = 0,$$

where  $\lambda$  denotes the value of a marginal unit of reserves (the 'scarcity rent'). Equation (6a) states that, if fossil fuel is extracted and sold, the marginal revenue must equal the extraction cost plus the scarcity rent. If the marginal revenue falls short of that, it is not profitable to extract fossil fuel. The price of fossil fuel inclusive of the carbon tax rises until time  $t = T > 0$  when it reaches the cost of renewables after which fossil fuel is uncompetitive. Given that the final scarcity rent must be zero,  $\lambda(T) = 0$ , we have:

$$(7) \quad p(T) + \tau(T) = G(S(T)) + \tau(T) = b \quad \Rightarrow \quad S(T) = G^{-1}(b - \tau(T)).$$

Hence, the market leaves more fossil fuel in the crust of the earth if the cost of renewables  $b$  is lowered or the carbon tax is raised. Using (5) and combining (6a) and (6b) for the fossil fuel era, we obtain:

$$(8a) \quad \dot{S} = -F, \quad S(0) = S_0, \quad S(T) = G^{-1}(b - \tau(T)),$$

$$(8b) \quad \dot{p} = r[p - G(S)], \quad p(T) = b - \tau(T),$$

$$(8c) \quad \frac{\dot{F}}{F} = -\varepsilon r \left[ 1 - \left( \frac{F}{\psi} \right)^{\frac{1}{\varepsilon}} \{G(S) + \tau - \dot{\tau} / r\} \right], \quad F(T) = \psi b^{-\varepsilon},$$

where  $F = \psi(p + \tau)^{-\varepsilon}$ . Equation (8b) states that the capital gains on holding a marginal unit of fossil fuel in situ must equal the return on extracting a marginal unit. The final rate of fossil fuel extraction is affected by the cost of renewables  $b$ , but not by the specific carbon tax  $\tau$ . The system (8) can be solved for the time trajectories of reserves  $S$ , depletion rates  $F$  and fossil fuel prices  $p$  as well as for the final stock of fossil fuel to be left in situ and the time it takes to transition to the carbon-free era  $T$ . We suppose that the carbon tax is not so high that it makes fossil fuel extraction unprofitable altogether.

The phase diagram for the system (8a) and (8c) is given in fig. 1. The horizontal axis from the origin to  $S = S_0$  gives the locus of zero fossil fuel extraction and time-invariant stocks of reserves. Above this locus fossil fuel extraction rates are positive and stocks of reserves decline. The locus corresponding to those combinations of  $F$  and  $S$  for which the fossil fuel extraction rate is constant slopes upwards (the red line in fig. 1), because a higher stock of reserves implies lower extraction costs per unit of fossil fuel and thus warrants a lower price and a higher fossil fuel extraction rate for  $\dot{F}$  to be zero. Below this locus the fossil fuel extraction rate is lower and thus prices are higher, so that the fossil fuel extraction rate is falling with time. Above this locus the fossil fuel extraction rate is rising.

### 3.2. Renewables subsidy and the Green Paradox<sup>3</sup>

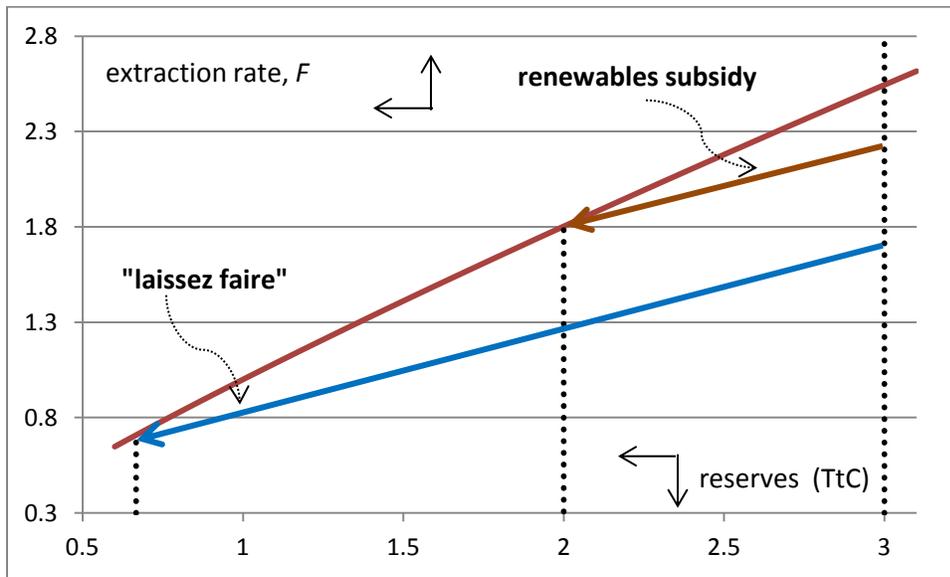
Many governments find it politically unattractive to increase carbon taxes. However, subsidizing renewables is politically less costly. We note from (8) that using a subsidy to lower the cost of renewables from  $b$  to  $b'$  raises the stock of fossil fuel to be left unexploited in the earth,  $G^{-1}(b') > G^{-1}(b)$ . It corresponds to higher fossil fuel use at time  $T$ . Before the subsidy the economy was to travel along the saddlepath indicated by the blue line in south-westerly direction. Upon news of the subsidy the extraction rate jumps up instantaneously and then travels along the new saddlepath indicated by the brown line in

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<sup>3</sup> Early studies that showed that increased current emissions can result from the emergence of a clean substitute for fossil fuel are Strand (2007) and Hoel (2008). See van der Ploeg and Withagen (2012c) and van der Meijden (2012) for similar result in a Ramsey growth model and an R&D growth model with imperfect substitution between fossil fuel and the backstop, respectively.

south-westerly direction to the new steady state. Over time extraction rates fall, energy prices rise and the stock of atmospheric fossil fuel rises. Since the renewables subsidy leads to higher extraction rates and more fossil fuel to be left in situ, the duration of the fossil fuel phase must be shortened and renewables are phased in earlier ( $T' < T$ ). The higher fossil fuel extraction rates and carbon emissions during the fossil fuel phase are the essence of the Green Paradox, but the fossil fuel phase will be shorter and the total volume of carbon emitted into the atmosphere in the atmosphere will be less. There may be some temporary acceleration of global warming, but ultimately the stock of atmospheric carbon and global warming will inevitably be less.<sup>4</sup>

**Fig. 1: Renewables subsidy gives rise to Green Paradox**



Although the qualitative shape of fig. 1 holds generally, we have drawn it using the following.

**Calibration I:**  $E_0 = 0.85$  TtC,  $S_0 = 3$  TtC,  $E = 2.35 - 0.5 S$ ,  $G(S) = 1/S$ ,  $b = 1.5$ ,  $b' = 0.5$ ,  $\varepsilon = 0.85$ ,  $\psi = 1$ .

Large-scale climate models indicate that the change in global peak temperatures depends principally on cumulative past carbon emissions (Allen et al., 2009a,b; Solomon et al., 2009). Half of emitted carbon returns to the surface of the earth and the oceans very quickly and the other half stays in the atmosphere forever or for hundreds of years. The initial stock of atmospheric carbon is roughly 0.85 trillion tons of carbon (or 400 ppm CO<sub>2</sub>) and current fossil fuel reserves are 3 trillion tons of carbon. Cumulative carbon emissions are thus  $E(T) = 0.85 + 0.5[S_0 - S(T)] = 2.35 - 0.5 S(T)$  trillion tons of carbon.

<sup>4</sup> With zero extraction costs,  $F(t) = \psi b^{-\varepsilon} \exp(-\varepsilon r(t - T))$ ,  $T = \ln(1 + \varepsilon r b^\varepsilon S_0 / \psi)$  and  $S(T) = 0$ , so that cheaper renewables boost emissions and shorten the fossil fuel phase but do not affect cumulative emissions.

Ensemble simulations of simple climate-carbon-cycle models constrained by observations and projects from more comprehensive models to simulate the temperature response to a broad range of carbon dioxide carbon pathways suggest that limiting cumulative emissions to 1 TtC leads to a mostly likely peak global mean warming of 2° C above pre-industrial temperature with a 5-95% confidence interval of 1.3° to 3.9° C (Allen et al., 2009b). These results also suggest that the relationship between cumulative emissions and peak warming is remarkably insensitive to the timing of emissions or the peak emission rate. Hence, policy targets based on limiting cumulative emissions of carbon dioxide are likely to be more robust to scientific uncertainty than emission-rate or concentration targets. Leaving more fossil fuel reserves unexploited is thus more important than how fast a given amount of carbon reserves is extracted and released into the atmosphere.

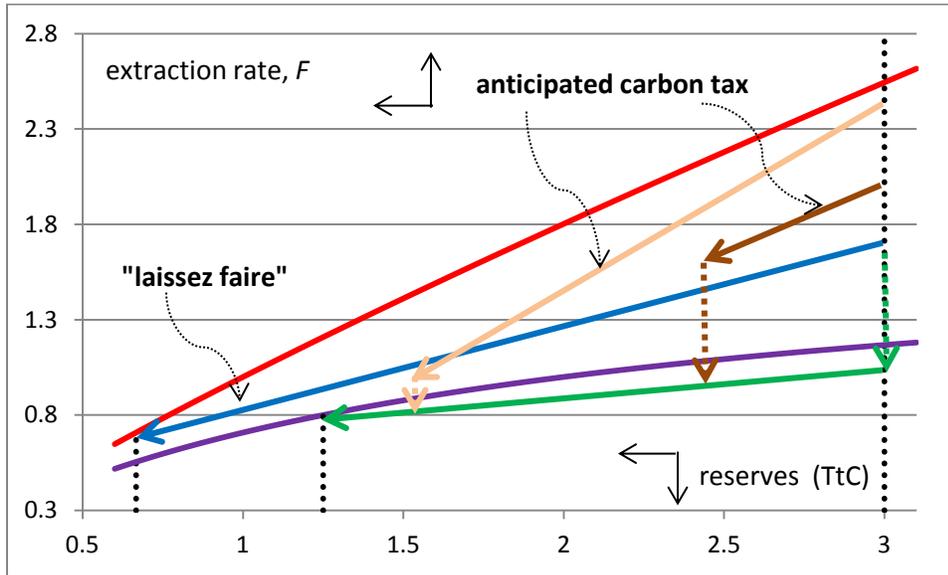
Under “laissez faire” 0.67 TtC of reserves are left unused and cumulative carbon emissions are 2.0 TtC,<sup>5</sup> which will lead to substantial and unacceptable global warming of 4.5° C. This follows from our temperature module,  $T = 2 + 3.683 \ln(E)$ , where  $T$  is peak global mean warming in °C above pre-industrial temperature and  $E$  is measured in TtC. This module has been calibrated so that  $T = 0°$  C if  $E$  equals its pre-industrial value of 0.581 TtC and has  $T = 2°$  C if  $E = 1$  TtC and abstracts from the lag between the carbon stock and temperature. This temperature module implies a climate sensitivity (the increase in temperature resulting from a doubling of the global carbon stock) of  $\beta \ln(2) = 2.55°$  C. The definition of the equilibrium climate sensitivity is the change in the global mean surface temperature resulting from the sustained doubling of the atmospheric (equivalent) CO<sub>2</sub> concentration, which seems close enough. The IPCC Fourth Assessment Report (2007) estimates the climate sensitivity as likely to be in the range of 2 to 4.5° C with a best estimate of about 3° C, and is very unlikely to be less than 1.5° C. (Values of the climate sensitivity substantially higher than 4° C cannot be excluded, but agreement of models with observations is not as good for those values.) Our climate sensitivity of 2.55° C is thus within the ballpark range and, if anything, a bit on the low side.

### 3.3. Anticipated carbon tax and the Green Paradox

We saw in section 2 that a specific carbon tax that grows at the market rate of interest does not affect the path of fossil fuel extraction or cumulative emissions. The full burden is then borne by fossil fuel producers as the price they get drops by the full extent of the carbon tax. This result no longer holds with stock-dependent extraction costs. Although (8c) is unaffected, (8a) indicates that the final stock of fossil fuel to be kept in situ is curbed. Hence, the extraction path must start from a lower rate.

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<sup>5</sup> This comes from current extraction costs being a third of the current market price of fossil fuel rising to a 1.4 times the current market price under “laissez faire” and the renewables cost being 1.5 times the current market price.

**Fig. 2: Anticipated carbon taxes cause Green Paradox**

An *unanticipated* permanent increase in the carbon tax of 0.7 times the current market price, i.e., 330 US\$/tC,<sup>6</sup> leads to an immediate downward jump in fossil fuel use and corresponding increase in the consumer price of fossil fuel; afterwards fossil fuel use and reserves diminish until the new steady state is reached (see the green lines and compare with the blue line for “laissez faire” in fig. 2). There is no Green Paradox. The unanticipated permanent carbon tax has the desired effect reducing emissions throughout and unambiguously reducing global warming.

More interesting are the effects of an *anticipated* sustained increase of 330 US\$/tC in the carbon tax. It incentivizes the market to conserve fossil fuel and leave more in the earth,  $S(T)' = 1.25 > S(T) = 0.7$  (see (7)). The upward-sloping  $\dot{F} = 0$  locus in the right panel shifts down from the red line to the purple line as result of the increase in the carbon tax, but the  $\dot{S} = 0$  locus being the left part of the horizontal axis is unaffected. The saddlepath shifts down from the blue line to the solid green line. On impact of the news that the carbon tax will rise in the future the extraction rate jumps up instantaneously and then falls but continues to be higher (at least initially) during the announcement period than in the “laissez faire” (compare the brown with the blue lines). This is the Green Paradox effect. During the announcement period fossil fuel prices increase. At the end of the announcement period the carbon tax jumps up. The price fetched for fossil fuel by the owners of the deposits is unaffected, but the consumer price of fossil fuel jumps up and the extraction rate jumps down by a discrete amount. Afterwards, fossil fuel is depleted

<sup>6</sup> In 2011 total fossil fuel use was 9.3 GtC, world GDP 70 trillion US\$ and the GDP share of total fossil fuel 6.2%, so we use a market price of fossil fuel of  $62 \times 70/9.3 \cong 470$  US\$/tC.

at decreasing rates. The economy moves along the new saddlepath in south-westerly direction (indicated by the green line) towards the new steady state with higher  $S(T)$ , higher  $F(T)$  and lower  $E(T)$ .

An anticipated future rise in the specific carbon tax thus leads to short-run Green Paradox effects, but also leads to more fossil fuel left in the earth and less cumulative carbon emissions.<sup>7</sup> A longer announcement period implies bigger initial Green Paradox effects (compare light brown with dark brown lines).

#### 4. Welfare aspects of the Green Paradox<sup>8</sup>

The arguments for a Green Paradox have been examined for a market economy in which the government announces a gradually more progressive climate policy. Here we use the model of section 3 to derive the optimal carbon tax which maximizes social welfare and show that the *first-best* optimal carbon tax is not subject to the Green Paradox. We also analyze the *second-best* case where the carbon tax is infeasible and one has to resort to a renewables subsidy.

##### 4.1. The optimal carbon tax and amount of fossil fuel to be left in situ

We specify a concave utility function  $U(F + R)$  to show how utility depends on energy use  $F + R$  and a convex function  $D(E)$  describing global warming damages resulting from cumulative carbon emissions  $E$  (see the discussion in section 3.2). The social planner thus maximizes social welfare,

$$(9) \quad U = \int_0^{\infty} [U(F + R) - G(S)F - bR - D(E_0 + 0.5(S_0 - S))] e^{-rt} dt, \quad U(F + R) = \frac{(F + R)^{1-1/\varepsilon} - 1}{\psi(1 - 1/\varepsilon)},$$

subject to the dynamics of fossil fuel depletion (5), where  $r$  denotes the discount rate. The social optimum is characterized by an initial phase where only fossil fuel is used and a final phase where only renewables are used. Simultaneous use of the two energy sources is thus not optimal in this setting.<sup>9</sup> At the time of the switch to the carbon-free era  $T$ , the scarcity rent is zero so that the social cost of the final unit of fossil fuel consists of the extraction cost,  $G(S(T))$ , plus the social cost of carbon,  $0.5 D'(E(T))/r$  (i.e., the present value of marginal global warming damages). This cost must at the time of the switch be equal to the cost of renewables for society to be indifferent between the two sources of energy:

$$(10) \quad G(S(T)) + 0.5D'(E_0 + 0.5(S_0 - S(T))) / r = b.$$

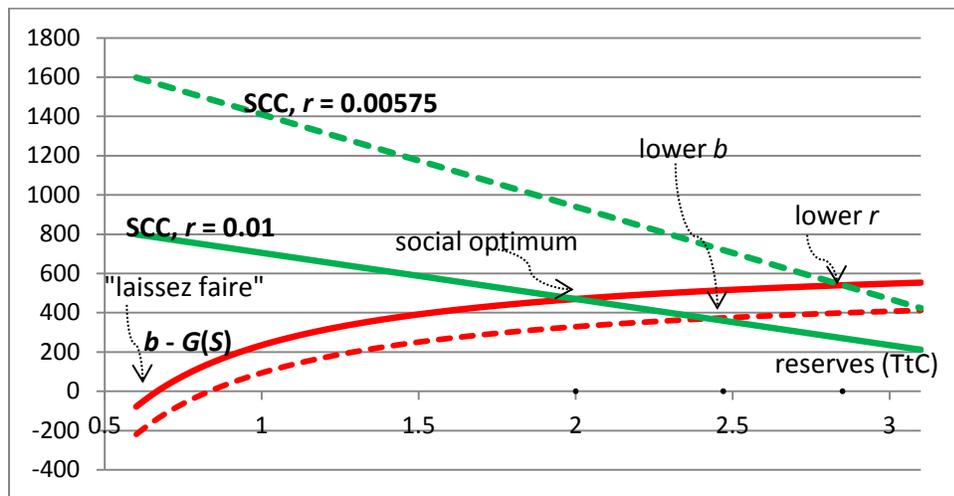
<sup>7</sup> Habermacher and Kirchgässner (2012) show in a similar model that even for very high discount rates that green welfare (the present value of cumulative emissions) improves despite initial Green Paradox effects for anticipated increases in the carbon tax.

<sup>8</sup> More details and formal derivations of results are in van der Ploeg and Withagen (2012a). The Green Paradox is analyzed in general equilibrium setting with growth and capital accumulation by van der Ploeg and Withagen (2012c).

<sup>9</sup> With a convex cost function, simultaneous use can be optimal (van der Ploeg and Withagen, 2012a).

Fig. 3 can be used to solve (10) for the stock of fossil fuel to be left in the earth. The upward-sloping locus portrayed by the solid red line gives the cost advantage fossil fuel has in the market over the carbon-free backstop,  $b - G(S(T))$ . When reserves are depleted to a lower level, extraction costs rise and the competitive advantage of fossil fuel in the market diminishes. If reserves are small enough, the cost advantage of fossil fuel disappears. The downward-sloping locus portrayed by the green line in fig. 3 gives the social cost of carbon at the time of the transition to the fossil fuel era,  $0.5D'(E + S_0 - S(T))/r$ . As reserves are depleted and more carbon is emitted into the atmosphere, the marginal damages of global warming and thus the social cost of carbon increase. The intersection of the two loci gives the optimal amount of fossil fuel to be left in situ.

**Figure 3: Socially optimal amount of fossil fuel to be left in situ**  
(SCC and  $b - G(S)$  evaluated at time of switch to the carbon-free era, measured in US \$/tC)



**Calibration II:**  $D(E) = 0.005 (E - 0.35)^2$ ,  $r = 1\%$ ,  $T = 2 + 3.683 \ln(E)$ .

Although the qualitative shape of fig. 3 is general, we have drawn it using our stylized calibration with quadratic damages. The social optimum then leaves 2 TtC fossil fuel in situ whilst under “laissez faire” much left, 0.67 TtC, is left in situ.<sup>10</sup> This corresponds to a stock of atmospheric carbon of 1.35 GtC and a global warming of a little more than 3<sup>0</sup> C.

The Stern Review argues that it is ethical to use a very low discount rate, because there is no reason to weigh welfare of future, unborn generations less than that of current generations (Stern, 2007). This implies that the green locus describing the social cost of carbon in fig. 3 shifts up so that it is optimal to leave more fossil fuel in situ and thus emit less carbon. Also, more fossil fuel is left in situ if renewables

<sup>10</sup> Equation (10) gives  $G(S(T)) + [2 - 0.5 S(T)] = 1.5$ , so  $S(T) = 2$  TtC .

are cheaper (lower  $b$ ) and fossil fuel extraction is dearer; both of these shocks correspond to a downward shift of the red locus describing the competitive advantage of fossil fuel.

Since the energy price in the carbon-free area is  $b$ , renewables demand follows from  $U'(R(t)) = p$ :

$$(11) \quad R(t) = \psi b^{-\varepsilon}, \quad \forall t \geq T.$$

The time paths for the social price of fossil fuel  $q$  and the social value of fossil fuel  $\lambda$  follow from:

$$(12a) \quad q(t) \equiv [\psi / F(t)]^{1/\varepsilon} = G(S(t)) + \lambda(t), \quad 0 \leq t \leq T, \quad q(t) = b, \quad \forall t \geq T,$$

$$(12b) \quad \begin{aligned} \dot{\lambda}(t) &= \rho \lambda(t) + G'(S(t))F(t) - 0.5D'(E_0 + 0.5(S_0 - S(t))), \quad 0 \leq t \leq T, \\ \lambda(T) &= b - G(S(T)), \quad \forall t \geq T. \end{aligned}$$

The *social* price of fossil fuel has to equal the marginal utility of fossil fuel and is denoted by  $q$  to distinguish it from the *market* price of fossil fuel  $p$ . Equation (12a) thus sets the social price to the extraction cost plus the scarcity rent of fossil fuel. Equation (12b) states that the Hotelling rise in the scarcity rent of fossil fuel is mitigated by the marginal increases in the extraction cost of fossil fuel and global warming damages. Combining equations (12a) and (12b) we get:

$$(13) \quad \dot{q}(t) = \rho [q(t) - G(S(t))] - 0.5D'(E_0 + 0.5(S_0 - S(t))), \quad 0 \leq t \leq T, \quad q(T) = b.$$

The social optimum thus demands that the marginal return on keeping a marginal unit of fossil fuel in the ground ( $\dot{q}$ ) must equal the social rent of extracting a marginal unit *minus* the marginal global warming damage of that unit. Comparing (13) with (8b) and making use of  $q = p + \tau$  we see that the social optimum can be sustained in a decentralized market economy if there is a specific carbon tax  $\tau$  which is set to the present discounted value of marginal global warming damages:

$$(14) \quad \tau(t) = 0.5 \int_t^{\infty} D'(E_0 + 0.5(S_0 - S(s))) e^{-r(s-t)} ds, \quad 0 \leq t < T, \quad \tau(t) = \frac{D'(E_0 + 0.5(S_0 - S(T)))}{2r}, \quad \forall t \geq T.$$

where we have used  $r = \rho$ . The optimal specific carbon tax rises until the time of the switch after which it is constant as there are no longer emissions in the carbon-free era. The rise in the optimal carbon tax becomes less steep over time during the fossil fuel phase, because rising extraction costs induce the market to conserve more energy and thus less internalization of the climate externality is needed. With the same calibration as that of fig. 3 the optimal carbon tax is  $0.5D'(E)/r$  and rises at a decreasing rate to  $\tau(T) = 470$  US\$/tC. It ensures that 2 TtC rather than 0.7 TtC is left in situ. By limiting the stock of atmospheric carbon to 1.35 TtC rather than 1.7 TtC (achieved by leaving 2 GtC rather than 0.7 TtC in situ), global peak temperatures are 1.5 degrees Celcius less in the social optimum than under “laissez faire”.

With a lower rate of discount of 0.575 percent per annum, it is optimal to leave even more fossil fuel locked up in the crust of the earth, 2.7 TtC, thus limiting the stock of carbon in the atmosphere to 1 TtC and limiting global warming to a much more comfortable 2° C. This requires a carbon tax which rises to 530 US\$/tC at the time of the switch to the carbon-free era. The results so far are summarized in table 1.

**Table 1: Global warming and the social cost of carbon at the time of switch to the carbon-free era**

	Fossil fuel in ground, $S$	Fossil fuel in atmosphere, $E$	Global mean temperature, $T$	Social cost of carbon
Pre-industrial	3.54 TtC	0.58 TtC	0° C	0.23
Now	3 TtC	0.85 TtC	1.4° C	0.5
“Laissez faire”	0.67 TtC	2.01 TtC	4.6° C	1.67
Social optimum I	2 TtC	1.35 TtC	3.1° C	1
Social optimum II	2.7 TtC	1 TtC	2° C	1.13

**Note:**  $E = E_0 + 0.5(S_0 - S) = 2.35 - 0.5 S$ ;  $G(S) = 1/S$ ;  $b = 1.5$ ;  $r = 0.01$ ;  $SCC = D'(E)/r = 0.01 (E - 0.35)/r$ ; climate sensitivity = 2.55;  $T = 2 + 3.683 \ln(E)$ ; SCC and  $b$  as fraction of market price, 470 US\$/tC.

Initial reserves are to a certain extent arbitrary, since they are largely unknown and depend in any case on market prices and extraction costs for reserves to be not only proven but also economically worthwhile to extract. Specification of extraction costs is therefore more important than the specification of  $S_0$ . We therefore followed Allen et al. (2009a) and took  $S_0 = 3$  TtC. It is much more than proven and listed reserves but if the market price is right it will be economically feasible to extract much more fossil fuel from the crust of the earth. Given  $E_0 = 0.85$ , we can burn a total of  $S_0 - S = 0.3$ TtC of fossil fuel and limit  $E$  to 1 and thus limit the global mean temperature increase to 2° C. If we want to limit the temperature increase to 3° C, we must limit the stock of atmospheric carbon to  $\exp(1/\beta) = 1.3$  TtC and can then burn three times as much, i.e., 0.9 TtC. BP (2012) states that in 2011 the global economy emitted 34 GtCO<sub>2</sub> or 9.28 GtC, so limiting global mean temperature to 2° C means that we can emit for another  $300/9.28 = 32$  years like that before moving fully to a carbon-free economy. If we allow temperature to rise by 3° C, then we can emit like this for almost another century before having to abandon fossil fuel altogether. Of course, if the climate sensitivity is higher, then we have to abandon fossil fuel much more quickly.

An interesting article in The Economist of 4 May 2013 discusses joint research by Carbon Tracker and the Grantham Institute (<http://www.carbontracker.org/wastedcapital>) why governments are either not serious about climate change or energy firms are overvalued if the risk of unburnable fossil fuel is inadequately factored in market prices. Their analysis takes a global carbon budget of  $1000 \text{ GtCO}_2 = 273 \text{ GtC}$  that can maximally be emitted between now and 2050 for global mean temperatures to be restricted

to 2° C (which corresponds to our 0.3 TtC derived above), but existing fossil fuel reserves contain much more than that. The International Energy Agency suggests in its World Energy Outlook that *proven* international reserves contain 2,860 GtCO<sub>2</sub> = 781 GtC, which is almost three times the carbon budget. Proven reserves of listed energy firms contain 762 GtCO<sub>2</sub> = 208 GtC, which can prudently be burned before 2050 without jeopardizing the 2° C-target. But listed reserves are 1,541 GtCO<sub>2</sub> = 421 GtC, which cannot all be burnt without unacceptable global warming. Hence, either energy firms are overvalued or climate policy is incredible. Oil and gas companies are either betting on a failing climate policy or on a geo-engineering or on a carbon sequestration fix.

#### 4.2. Carbon sequestration and lower cost of renewables

If carbon capture and storage is allowed for, it can be shown that the marginal cost of sequestration must rise at the rate of discount. If it is optimal to have, say, 0.3 TtC sequestered at the end of the fossil fuel era, (10) becomes  $1/S(T) + 2 - 0.5S(T) - 0.3 = 1.5$ , so that  $S(T) = 1.6$  TtC,  $E(T) = 1.25$  and  $\tau(T) = 420$  US\$/tC. Hence, in an optimal mix of sequestration and carbon taxes, cumulative carbon emissions are lower and global warming is less severe despite less fossil fuel left in situ and the carbon tax being lower.

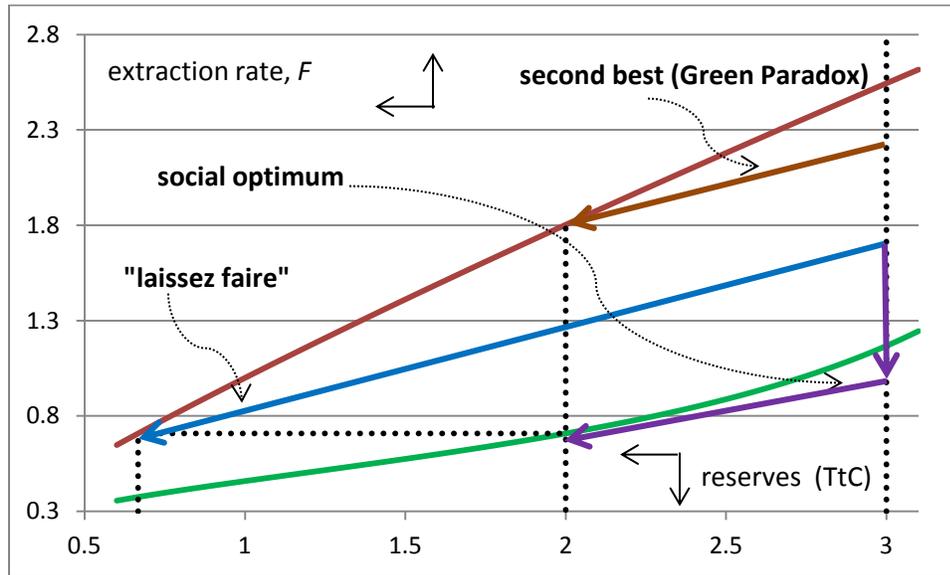
If there is technological progress in renewables (see also section 5), it can be shown that this always boosts green welfare if there is *partial* exhaustion but curbs green welfare if there are no extraction costs and there is *full* exhaustion of reserves (see proposition 4 in van der Ploeg and Withagen, (2012a)).

#### 4.3. Comparing the first-best optimum with the laissez-faire” outcome

Rewriting equation (13) in terms of fossil fuel use, we have:

$$(8c') \quad \frac{\dot{F}}{F} = -\varepsilon r \left\{ 1 - \left( \frac{F}{\psi} \right)^{1/\varepsilon} \left[ G(S) + \frac{D'(E_0 + 0.5(S_0 - S))}{2r} \right] \right\}, \quad F(T) = \psi b^{-\varepsilon}.$$

Fig. 4 gives the calibrated phase diagram corresponding to equations (8c') and (5). The red locus corresponds to the one in fig. 1 and gives the combinations of  $F$  and  $S$  for which  $F$  is stationary in the “laissez-faire” economy. The green locus corresponds to those  $F$  and  $S$  for which  $F$  is stationary in the social optimum (from (8c')). The cost of renewables  $b$  pins down the same  $F(T)$  for both loci. The blue saddlepath indicates the “laissez-faire” outcome. If at time zero the social optimum is implemented, fossil fuel use jumps down discretely and then gradually falls along the purple saddlepath. Extraction rates are smaller and more fossil fuel is left in situ in the social optimum outcome than under “laissez faire” so the Green Paradox is clearly a second-best phenomenon. Since the optimal specific carbon tax grows at less than the market rate of interest, i.e.,  $\dot{\tau} = r\tau - 0.5D'(E) < r\tau$ , there simply cannot be a Green Paradox (see equation (8c)).

**Fig. 4: Social optimum, second-best and “laissez-faire” outcomes**

#### 4.4. Second-best policy: renewables subsidy

The second-best policy of setting the renewables subsidy to 60% (bringing down  $b$  from 1.5 to 0.5) portrayed by the brown lines in fig. 4 curbs cumulative emissions to exactly what they should be in the social optimum. Since the brown saddlepath lies unambiguously above both the blue and the green saddlepaths, we immediately see that this particular second-best policy suffers from the curse of the Green Paradox. Fossil fuel use is thus not only bigger than in the social optimum but also than under “laissez-faire”. Given that cumulative carbon emissions matter ultimately for global warming and not the speed of carbon emissions (Allen et al., 2009a,b; Solomon, et al., 2009), this second-best policy may be quite good. A lower than 60% renewables subsidy attenuates the Green Paradox effect, but leads to deeper depletion of fossil fuel reserves and thus to more cumulative carbon emissions. If the discount rate was halved for precautionary reasons,  $S(T) = 2.8$  TtC (see fig. 3) so  $E(T) = 0.6$  TtC. As a result, the optimal cost of carbon rises to US 564 US \$/tC. Less fossil fuel is used and thus global warming is less severe with a lower discount rate.

## 5. Breakthrough renewables, cost of exploration investment and the Green Paradox<sup>11</sup>

What happens if the market anticipates a breakthrough in renewables technology at significantly lower costs at some random future moment? To highlight the resulting inefficiencies, we suppose isoelastic

<sup>11</sup> More details and formal derivation of the results in this section are in van der Ploeg (2012). The results are akin to the idea that greenhouse gases increase in the interim period between policy announcement and the uncertain time of implementation, which holds even without fossil fuel scarcity (Smulders et al., 2012).

demand and zero variable extraction costs so that monopolistic resource extraction is efficient (Stiglitz, 1976). The idea of a discrete change in demand resulting from a breakthrough technology at some unknown date in the future goes back to Dasgupta and Heal (1974) and Dasgupta and Stiglitz (1981). Here we explore how this affects the merits of certain second-best climate policies.

The economy uses fossil fuel  $F$  and/or renewables,  $R$ . In contrast to sections 2, 3 and 4, these two types of energy are *imperfect* substitutes. Fossil fuel needs investment outlays  $I$  which lead to *proven* initial reserves  $S_0$ . The breakthrough occurs at time  $T > 0$ . Before the breakthrough ( $t < T$ ), renewables are infinitely elastically supplied at cost  $\tilde{b}(t) = b$ . After the breakthrough ( $t \geq T$ ), they come at cost  $\tilde{b}(t) = b - \Delta$  where  $0 < \Delta \leq b$ . The owner of fossil fuel deposits chooses exploration investment and extraction rates to maximize the present value of profits,

$$(15) \quad \text{Max}_{F,I} \text{E} \left[ \int_0^{\infty} p(t)F(t)e^{-rt} dt \right] - qI$$

subject to the depletion equations (5), the exploration investment schedule,

$$(16) \quad S_0 = \Theta(I), \quad \Theta' > 0, \Theta'' < 0,$$

the fossil fuel demand schedule,

$$(17) \quad F(t) = \psi p(t)^{-\varepsilon} b^{\sigma}, 0 \leq t < T, \quad F(t) = \psi p(t)^{-\varepsilon} (b - \Delta)^{\sigma}, \forall t \geq T,$$

the probability that the breakthrough technology occurs in the interval ending at time  $t$ ,

$$(18) \quad \Pr(T \leq t) = 1 - \exp(-ht), \forall t \geq 0, \quad h \geq 0,$$

where  $q$  and  $I$  denote the price and volume of exploration investment. The price of exploration investment and the market rate of interest are determined on world markets and constant over time. Concavity of  $\Theta(\cdot)$  ensures decreasing returns to exploration investment. The own price elasticity of fossil fuel demand ( $\varepsilon$ ) exceeds unity, so marginal revenue is positive.<sup>12</sup> With the demand function (17), marginal revenue is always finite and fossil fuel reserves are asymptotically fully exhausted. Fossil fuel and renewables are gross substitutes, so the constant cross price elasticity of fossil fuel demand ( $\sigma$ ) is positive. Inverse demand is  $p = (\psi \tilde{b}^{\sigma} / F)^{1/\varepsilon} \equiv p(F, \tilde{b})$ . The probability that the breakthrough has not taken place before time  $t$  is  $\Pr(T > t) = \exp(-ht)$ . There is a constant hazard rate  $h$  with expected time of the breakthrough equal to  $\text{E}[T] = 1/h$ .

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<sup>12</sup> Aggregate demand is relatively inelastic, but the relevant elasticity for an individual firm is much higher as it cannot easily manipulate the price without losing market share.

We use dynamic programming first to solve the problem from unknown time  $T$  onwards when the cheap carbon-free substitute is on the market, then to solve for the fossil fuel extraction rate before the substitute has arrived on the market, and finally to solve for the optimal exploration investment. We denote the fossil fuel extraction problems *after* and *before* the breakthrough with superscripts  $A$  and  $B$ , respectively.

### 5.1. After and before the breakthrough

Marginal fossil fuel revenue,  $(1 - 1/\varepsilon)p^A$ , must equal the scarcity rent,  $\lambda$ , which according to the Hotelling rule must rise at the market interest rate,  $r$ . The Hotelling rules are  $\dot{p}^A / p^A = r > 0$  and  $\dot{F}^A / F^A = -\varepsilon r < 0$ . We solve for the outcomes *after* the breakthrough:

$$(19) \quad \begin{aligned} F^A(t) &= \varepsilon r S(t) = \varepsilon r e^{-\varepsilon r(t-T)} S(T), & S^A(t) &= e^{-\varepsilon r(t-T)} S(T) \leq S(T) < S_0, \\ p^A(t) &= e^{r(t-T)} \left[ \frac{(b-\Delta)^\sigma \psi}{\varepsilon r S(T)} \right]^{1/\varepsilon}, & \forall t > T. \end{aligned}$$

Equations (19) imply that a cheaper backstop ( $\Delta > 0$ ) pushes down the fossil fuel price, especially if the backstop is a good substitute (high  $\sigma$ ), but does not affect the path of depletion rates. The present value of profits after the breakthrough is:

$$(20) \quad V^A(S(t), b - \Delta) = \left[ \frac{(b-\Delta)^\sigma \psi}{\varepsilon r} \right]^{1/\varepsilon} S(T)^{1-1/\varepsilon}, \quad \forall t \geq T.$$

A future breakthrough ( $\Delta > 0$ ) cuts the cost of the substitute and thus curbs the future price of fossil fuel. As a result, the present value of fossil fuel profits is lower. Profits after the breakthrough are high if remaining reserves at the time of the breakthrough are high.

The Hamilton-Jacobi-Bellman equation for the optimization problem *before* the breakthrough is:

$$(21) \quad \text{Max}_{F^B} \left[ p(F^B, b) F^B - V_S^B(S, b, \Delta, h) F^B \right] - h \left[ V^B(S, b, \Delta, h) - V^A(S, b - \Delta) \right] = r V^B(S, b, \Delta, h),$$

where  $V^B(S, b, \Delta, h)$  denotes the value function (excluding the cost of the initial outlay on exploration investment) before the breakthrough. The maximization in (21) requires marginal revenue,

$(1 - 1/\varepsilon)p^B(t)$ , to be set to the marginal value of in situ reserves,  $V_S^B(S(t), b, \Delta, h)$ . Using the fossil fuel demand curve (17), we obtain:

$$(22) \quad F^B(t) = \psi b^\sigma \left( \frac{V_S^B(S(t), b, \Delta, h)}{1 - 1/\varepsilon} \right)^{-\varepsilon}, \quad 0 \leq t < T.$$

Using (22) into (21), we can verify that the value function  $V^B(S, b, \Delta, h) = KS^{1-1/\varepsilon}$ , where  $K$  satisfies

$$(23) \quad \frac{1}{\varepsilon} \psi b^\sigma K^{1-\varepsilon} + h \left[ \frac{(b-\Delta)^\sigma \psi}{\varepsilon r} \right]^{1/\varepsilon} = (r+h)K,$$

solves (21). We thus get  $p^B(t) = KS(t)^{-1/\varepsilon}$ ,  $F^B(t) = LS(t)$ ,  $0 \leq t < T$ , where  $L \equiv \psi K^{-\varepsilon} b^\sigma$ , and thus:

$$(24) \quad p^B(t) = e^{Lt/\varepsilon} K S_0^{-1/\varepsilon}, \quad F^B(t) = L e^{-Lt} S_0, \quad S^B(t) = e^{-Lt} S_0, \quad 0 \leq t < T.$$

### 5.2. Breakthrough uncertainty causes aggressive fossil fuel depletion rates

We now characterize the solution to (23). A zero hazard rate,  $h = 0$ , gives  $K = (\psi b^\sigma / \varepsilon r)^{1/\varepsilon}$  and thus

$$F^A(t) = F^B(t) = \varepsilon r S(t), \quad \forall t \geq 0. \quad h \rightarrow \infty \text{ gives } K \rightarrow [\psi(b-\Delta)^\sigma / \varepsilon r]^{1/\varepsilon} \text{ and } L \rightarrow \varepsilon r \left( \frac{b}{b-\Delta} \right)^\sigma > \varepsilon r. \text{ The}$$

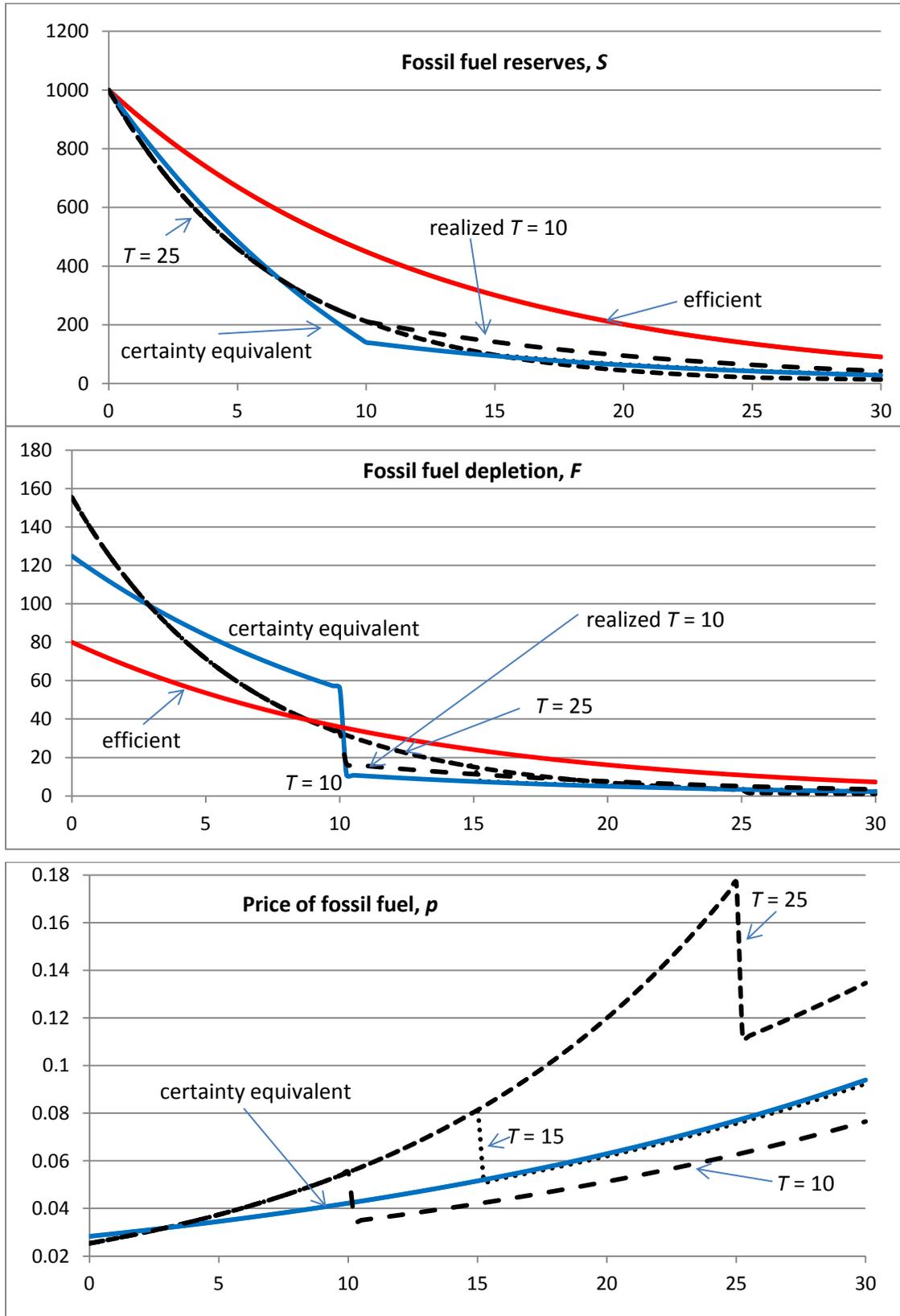
extraction rate is bigger if the breakthrough is imminent than if it never occurs. The possibility of fossil fuel being made obsolete depresses expected profits to go. It also leads to more aggressive depletion of reserves. Total differentiation of (23) shows that  $K$  decreases and  $L$  increases in  $h$ . This reflects that a higher probability of a renewables breakthrough curbs expected profits of fossil fuel firms, lifts up the depletion rate path, and depresses the fossil fuel price path before the switch. Hence, we have

$$\varepsilon r < L < \varepsilon(r+h) \text{ for any } 0 < h < \infty. \text{ Further, if } h > 0, K \text{ increases in } b - \Delta \text{ for all } 0 < \Delta < 1.$$

Anticipation of a future breakthrough in renewables thus boosts the initial depletion rate and depresses the initial fossil fuel price, especially if the chance of a breakthrough occurring and the expected cost reduction are high. As long as the breakthrough has not occurred, the depletion rate falls at the rate  $L > \varepsilon r$  and the fossil fuel price rises too rapid. Once the breakthrough arrives on the market, the depletion rate jumps down and the fossil fuel price jumps down by a discrete amount if the breakthrough yields a big enough cost reduction and renewables are a good enough substitute, else the fossil fuel price jumps up by a discrete amount. From then on depletion rates and fossil fuel prices follow Hotelling paths, but starting out from an inefficiently low level of fossil fuel reserves.

Fig. 5 offers various simulations with  $\varepsilon = 2$ ,  $\sigma = 1$ ,  $\psi = 1$ ,  $r = 0.04$ ,  $b = 100$ ,  $\Delta = 80$ , and  $S_0 = 1000$ . The hazard rate is set to  $h = 0.1$ , so the expected time for the breakthrough is 10 years. Hence,  $K = 0.802$  and the speed of fossil fuel depletion before the breakthrough,  $L = 0.155$ , is nearly twice as high as the speed afterwards,  $\varepsilon r = 0.08$ . Simulations with realized times of the breakthrough occurring at times 10, 15 and 25 (long dashes, dots and short dashes, respectively) confirm that the Green Paradox effect of too rapid extraction rates occur for as long as it takes for the breakthrough to occur. Although not shown, the initial fossil fuel price if there is never a breakthrough is higher (0.0354) than if there is an immediate

Figure 5: Threat of a renewables breakthrough induces Green Paradox



breakthrough in renewables (0.0158). From then on fossil fuel prices follow a Hotelling path in each of these two cases. The paths for depletion rates and reserves do not depend on whether there is never or an immediate breakthrough. The certainty-equivalent in blue path starts off with a fossil fuel price in between (0.0283) and then also follows a Hotelling path. Extraction rates are affected by the certainty of a breakthrough at some future date: until the breakthrough, reserves are depleted at a rapid rate and at a lower rate after the breakthrough.

Not knowing the date of the breakthrough also speeds up initially the rate of extraction and lowers fossil fuel prices before the breakthrough compared with the blue certainty-equivalent (and a fortiori the red efficient) path. But after some time this Green Paradox effect disappears when the extraction rate is lower and fossil fuel prices higher than in the certainty-equivalent outcome. At the moment the breakthrough comes to market, both the rate of depletion and the fossil fuel price jump down and thereafter continue along their Hotelling paths, albeit from an inefficient base.<sup>13</sup> The Green Paradox effects thus arise from the unknown date at which the breakthrough will occur; if the date of the breakthrough is certain, there is no Green Paradox effect in this setting with isoelastic demand and zero extraction costs. If the threat of a breakthrough and the size of the breakthrough become more substantial, fossil fuel producers start extracting more rapidly before renewables come to market and fossil fuel prices fall.

#### 5.4. Inefficiency in exploration investment

The final stage of the dynamic programming problem is to solve for the optimal  $I$ . We use (16) and  $V^B(\Theta(I)) = K\Theta(I)^{1-1/\varepsilon}$  to set the marginal return on exploration investment to its cost:

$$(25) \quad (1 - 1/\varepsilon)K\Theta(I)^{-1/\varepsilon} \Theta'(I) = q - \theta,$$

where  $\theta$  denotes an investment exploration subsidy. Total differentiation of (25) shows that optimal exploration investment declines with its cost  $q$ , the breakthrough hazard  $h$  and the size of the reduction in the cost of renewables after the breakthrough  $\Delta$ , and increases with the exploration subsidy  $\theta$ . In the efficient outcome with never a breakthrough or a breakthrough from the outset, one has

$(1 - 1/\varepsilon)(\varepsilon r)^{-1/\varepsilon} \Theta(I)^{-1/\varepsilon} \Theta'(I) = q$ . The optimal exploration investment subsidy increases with the breakthrough hazard and the potential cost advantage of breakthrough renewables:

$$(26) \quad \theta = \left[ (\varepsilon r)^{-1/\varepsilon} - K \right] (1 - 1/\varepsilon) \Theta(I)^{-1/\varepsilon} \Theta'(I) \equiv \theta(b - \Delta, h) > 0, \quad \theta_{b-\Delta} < 0, \theta_h > 0.$$

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<sup>13</sup> With a smaller cost reduction or renewables being a worse substitute, the fossil fuel price would jump up by a discrete moment at the time of the breakthrough but not if  $(b / (b - \Delta))^\sigma > (r + h) / h = 3.5$  holds.

The inefficiencies induced by the uncertain timing of a breakthrough in renewables are thus exacerbated by a drop in exploration investment, especially if the threat of a better carbon-free substitute and the potential cost reduction are higher. These inefficiencies can be eliminated by subsidizing exploration investment at the rate (26).<sup>14</sup> Since oil and other fossil fuel producers are typically in different jurisdictions to their customers, such a subsidy is unlikely to be implemented.

### *5.5. Subsidizing green R&D and the Green Paradox*

A green R&D subsidy  $\nu$  brings forward the introduction of carbon-free substitutes for fossil fuel,  $h = H(\nu)$ ,  $H' > 0$ ,  $H'' < 0$ . The subsidy thus increases the hazard rate  $h$  and cuts the expected time of the breakthrough,  $1/h$ . But this makes fossil fuel more likely to become obsolete and thus depresses exploration investment  $I$  and curbs the total of recoverable reserves,  $S_0$ , and thus cumulative carbon emissions. We also know that a higher hazard rate slows down the speed of fossil fuel extraction and thus the speed at which carbon is emitted into the atmosphere (see also fig. 5). Subsidizing green R&D to speed up the development of carbon-free substitutes thus leads to a Green Paradox in the short run. However, as the total amount of carbon that is emitted into the atmosphere is curbed, the Green Paradox is likely to be reversed in the long run.

In sum, subsidizing green R&D speeds up fossil fuel extraction and exacerbates global warming (the Green Paradox) but also discourages exploration of oil and gas fields and coal mines and thus limits cumulative carbon emissions which matters for global warming.

## **6. Concluding remarks**

The Green Paradox has been coined by Sinn (2008, 2012) to describe the idea that a climate policy that becomes more ambitious during the decades ahead is an announced expropriation for the owners of fossil fuel resources, thus inducing them to accelerate resource extraction and global warming. It works as follows. A gradual tightening of climate policy over the coming decades exerts a stronger downward pressure on future prices than on current ones, thereby curbing the expected rate of capital appreciation of fossil fuel reserves. The owners of these fossil fuel deposits will try to avert this by increasing extraction rates and converting the sales revenue into investments in the capital markets, which offer higher yields. Hence, a climate policy which becomes more aggressive over time acts as an announced expropriation

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<sup>14</sup> The exploration investment is thus subject to the holdup problem (cf., Rogerson, 1992; Holmström and Roberts, 1998). One solution is vertical integration; for example, nationalization of oil companies. There may also be contractual solutions. There is another potential holdup problem as well related to the lack of commitment to the time path of future carbon taxes. This depresses long-run investments in greenhouse mitigation and postpones extraction; the net effect depends crucially on the supply side of fossil fuel markets, especially on how rapid extraction rates increase with cumulative extraction (Hoel, 2011a).

that provokes owners to react by accelerating the rate of extraction of their fossil fuel stocks and thereby exacerbating global warming. Sinn (2008) deserves credit for intensifying efforts to better understand the channels by which one can fight global warming. Furthermore, the problem is even more severe in an international context when carbon leakage and the Green Paradox might reinforce each other.<sup>15</sup> Countries which do not join in efforts to curb fossil fuel demand thus benefit in two ways: by burning the lower priced carbon set free by the carbon-reducing countries and by burning the additional carbon extracted as a result from the announced and expected price cuts following from the gradual increase in the carbon tax.

It follows that an effective climate policy must focus on the often neglected supply side of the carbon market in addition to the demand side. It is thus crucial that not all the fossil fuel that is in the crust of the earth is burned. This can be achieved with the establishment of a seamless global emissions trading system that effectively puts a cap on worldwide fossil fuel consumption, thus achieving the desired reduction in carbon extraction rates. Alternatively, Sinn (2008) has suggested a withholding tax on the capital gains on the financial investments of oil sheikhs and other fossil fuel resource owners (see Jaakkola (2012) for an analysis).

One can only mitigate the accumulation of atmospheric carbon by either extracting less carbon from the ground and thus leaving more in situ or by injecting carbon back into the ground by sequestration or by planting more forests. Roughly one trillion tons of carbon must thus be either left unused or be sequestered (Allen, 2009a,b). Both the sequestration effort and the increased stock left unused can be realized via a carefully designed rising carbon tax: mandatory sequestration is thus not necessary. Proponents of the Green Paradox lament that climate policy in many countries is not focused at leaving more fossil fuel in the ground, but at promoting alternative carbon-free energy substitutes and more efficient use of fossil fuel energy. These advocates ignore that a renewables subsidy or a demand-side policy for choking off the demand for fossil fuel such as a specific carbon tax also operate on the supply side by quickening the transition to the carbon-free economy, inducing more fossil fuel to be left in situ and curbing cumulative carbon emissions. It is too simple to replace a demand-side cartel with a supply-side cartel. The best way to convince the oil sheikhs and other fossil fuel producers to not extract all of their reserves is a carefully designed carbon tax or second-best renewables subsidy; a withholding tax on capital gains or buying up all fossil fuel reserves will be unrealistic and ineffective.<sup>16</sup>

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<sup>15</sup> Eichner and Pehtig (2011) analyze the interrelationships between carbon leakage and the Green Paradox in a two-period, three-country general equilibrium model without capital mobility and Hoel (2011b) analyzes these interactions in partial equilibrium model.

<sup>16</sup> If one can trade fossil fuel deposits before climate and trade policies are set, all problems including the Green Paradox can be overcome by appeal to the Coase (1960) theorem: fossil fuel importers simply buy up the fossil fuel deposits and deplete them conservatively taking full account of the social cost of carbon (Harstad, 2012a). Since this solution is only efficient if transaction costs are zero, property rights for polluting or not polluting the atmosphere

Although the Green Paradox proposition is in principle correct, the question is how relevant it is. Sinn (2008) argues that the Green Paradox requires that fossil fuel is scarce in the sense that there is a Hotelling scarcity rent, so that the price of fossil fuel is always higher than the unit extraction and exploration costs combined. For the coming decades this condition is likely to be satisfied as carbon-free backstop technologies will at best offer a perfect substitute for electricity, but not for fossil fuels. The prices of coal and crude oil are currently many times higher than the corresponding exploration and extraction costs combined. But what matters is whether fossil fuel is going to be priced out of the market by some carbon-free substitute at some point in the future. That is why we have explored why and how the Green Paradox will disappear if either renewables are phased in at some endogenous moment in the future once fossil fuel prices have increased sufficiently or a breakthrough in renewables technology occurs at some random future moment. In both cases, lowering the cost of renewables or having a rising ramp for the carbon tax brings forward the date that renewables take over from fossil fuel. They also reduce cumulative emissions and mitigate global warming by leaving more fossil fuel unused.

The anticipation of a carbon-free substitute for fossil fuel coming to market at an uncertain future moment in time induces higher extraction rates and pushes down the oil price which exacerbates global warming (the Green Paradox). An uncertain introduction date for the carbon-free substitute also depresses exploration investment and thus more fossil fuel is left in situ. Hence, the total amount of carbon emitted into the atmosphere is reduced, albeit that what is emitted is emitted more rapidly. This is likely to overturn the adverse effect of the Green Paradox on global warming, especially as cumulative carbon emissions are what really affect peak temperatures. The consequences of cheaper renewables are thus not as bleak as suggested by proponents of the Green Paradox. Furthermore, an optimal carbon tax rises at less than the market rate of interest and thus leads to lower extraction rates and lower cumulative carbon emissions than in the “laissez-faire” outcome. The Green Paradox is thus at most a temporary, second-best phenomenon and the priority should be to design appropriate carbon taxes, renewables subsidies and green R&D incentives to curb cumulative emissions by encouraging more fossil fuel to be kept in situ and quickening the transition from a fossil-fuel to a carbon-free economy.

Finally, the design of climate policy should realize that in an imperfect world where the backstop when oil and gas reserves run out is in many parts of the world cheap, abundant and dirty coal rather than carbon-free renewables. In that case, it is optimal to conserve oil and gas reserves by extracting more slowly than the market would in order to replace or postpone the use of coal which does much more damage to global warming than oil or gas (e.g., van der Ploeg and Withagen, 2012c; Michielsen, 2011).

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are well defined and there are no effects on non-negotiating countries, one may doubt its practical relevance except perhaps for buying up forests with the REDD funds. However, even under these strong conditions the coalition of carbon-reducing countries has an incentive to postpone the purchase of deficits (Harstad, 2012b).

This also goes against the Green Paradox. Such design should not rely only on a credible rising profile for the carbon tax, but also on substantial green R&D subsidies to kick-start green innovation and redirect technical change in the direction of green growth (e.g., Acemoglu et al., 2012).

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