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OxCarre Research Paper 119

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Untapped Fossil Fuel and the Green Paradox:

A classroom calibration of the optimal carbon tax

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UNTAPPED FOSSIL FUEL AND THE GREEN PARADOX

A classroom calibration of the optimal carbon tax

Frederick van der Ploeg, University of Oxford*

Abstract

A classroom model of global warming, fossil fuel depletion and the optimal carbon tax is formulated and calibrated. It features iso-elastic fossil fuel demand, stock-dependent fossil fuel extraction costs, an exogenous interest rate and no decay of the atmospheric stock of carbon. The optimal carbon tax reduces emissions from burning fossil fuel, both in the short and medium run. Furthermore, it brings forward the date that renewables take over from fossil fuel and encourages the market to keep more fossil fuel locked up. A renewables subsidy induces faster fossil fuel extraction and thus accelerates global warming during the fossil fuel phase, but brings forward the carbon-free era, locks up more fossil fuel reserves and thus ultimately curbs cumulative carbon emissions and global warming. For relatively large subsidies social welfare is more likely to fall as the economic costs rises more than proportionally with the size of the subsidy. Our calibration suggests that such subsidies are not a good second-best climate policy.

Keywords: global warming, social cost of carbon, optimal carbon tax, renewables

JEL codes: D81, H20, Q31, Q38

revised August 2013

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* Department of Economics, University of Oxford. Also affiliated with the VU University Amsterdam. Support from the European Research Council is gratefully acknowledged. I have benefited from many helpful discussions with Armon Rezai and Cees Withagen and useful comments from Florian Habermacher, SamWills and Niko Jaakkola.

1. Introduction

Global warming is one of the biggest challenges facing our planet. The best way to deal with this is to price carbon via either a global carbon tax or a global market for tradable emission permits. The price of carbon should in a first-best world be equal to the social cost of carbon, which corresponds to the present value of all future marginal global warming damages. As a result of pricing carbon appropriately, carbon emissions are curbed by substituting away from fossil fuel to renewable energy and other production factors, more fossil fuel reserves are locked up in the crust of the earth and the carbon-free era is brought forward. Furthermore, pricing carbon will make it attractive to also mitigate global warming with carbon capture & sequestration and research & development into renewables.

It is crucial to get a good grasp of the mechanisms underlying the effectiveness of the global carbon tax and also to have a quantitative assessment of the optimal carbon tax. Many empirical integrated assessment models of climate change have been developed for this purpose and yield useful estimates of the social cost of carbon starting from 5 to 35 $ per ton of carbon in 2010 and rising to $16 to $50 per ton in 2050 (e.g., the DICE, PAGE and FUND models described in Nordhaus (2008), Hope (2006) and Tol (2002), respectively), but the Stern Review obtains much higher estimates of the social cost of carbon with a much lower discount rate (e.g., Stern (2007)). These models are often rather large and difficult to understand, so that it is not always clear what the underlying assumptions and the crucial parameters deriving the results are.

Our purpose is therefore to put forward a simple classroom model of climate change and to carefully discuss an illustrative and transparent calibration of this model. We then use this to derive the optimal carbon tax, the optimal amount of fossil fuel to leave untapped, and the optimal time for the commencement of the carbon-free era, and to compare these with the no-policy outcomes. We also use our model to discuss the adverse effects of the second-best policy of subsidizing renewables instead of the first-best policy of pricing carbon. Although this shortens the duration of the fossil fuel era and encourages the market to leave more fossil fuel unexploited compared with the no-policy scenario, it also leads to faster fossil fuel extraction rates and acceleration global warming which has been coined the Green Paradox by Sinn (2008). Large renewables subsidies have proportionally larger economic costs and are thus more likely to be counterproductive from a welfare point of view. The first-best optimal carbon tax leads to slower fossil fuel extraction rates than under “laissez faire” and does not suffer from Green Paradox effects. The optimal carbon tax also locks up more fossil fuel in the crust of the earth.

The outline of this paper is as follows. Section 2 discusses and calibrates a simple classroom model of how burning fossil fuel leads to more atmospheric carbon and thus to more global warming. Section 3 discusses various specifications that have been used to capture damages for global warming and puts forward a simple specification which captures that at higher levels of mean global temperature damages increase more than proportionally with temperature. Section 4 puts forward a partial equilibrium model of fossil fuel depletion, renewables and climate change and uses it to discuss and highlight how increases in the carbon tax rate and the renewables subsidy can result in Green Paradox effects. Section 5 discusses social welfare and shows that large renewables subsidies can not only damage green welfare, but can also damage overall welfare and thus be counterproductive. Section 6 derives the social optimum and the optimal carbon tax. Section 7 concludes.

2. Burning fossil fuel, atmospheric carbon and global warming

Burning fossil fuel leads to carbon emissions. About half of these return quickly to the oceans and the surface of the earth and the other half remains forever up in the atmosphere (e.g., Allen et al., 2009ab). We abstract from atmospheric decay of greenhouse gases, which is less unreasonable for CO_2 than for methane. We also abstract from positive feedback effects, which may occur at higher temperatures. Hence, the stock of carbon in the atmosphere E (measured in trillions ton of carbon or TtC) is simply:

(1) $$E = E_0 + 0.5\,(S_0 - S) = 1.71 - 0.5\,S,$$

where S is the stock of fossil fuel (also measured in TtC) and the subscript 0 indicates current levels. We set the current stock of atmospheric carbon to E_0 = 0.85 TtC, which corresponds to 2.13 $TtCO_2$ (using the conversion factor 1kg carbon = 3.664 kg CO_2) or to 400 parts per million of CO_2 (in May 2013). Cumulative use of fossil fuel from now onwards is given by $S_0 - S$. Proven reserves refer to reserves that are profitable to exploit current price and cost conditions. Initial reserves (S_0) are much larger and to a certain extent arbitrary, since it is difficult to know how much reserves will be profitable when fossil fuel prices have reached a much higher level. Typically, more less accessible fields will become worthwhile as prices rise. Hence, specification of the dependence of extraction costs on cumulative fossil fuel use is more important than the initial value of S_0. We will calibrate the initial stock of fossil fuel reserves so that it produces the current price of fossil fuel (see section 4 for this calibration), which gives S_0 = 1.72 TtC. This is more than twice proven and listed reserves[1] but if the market price is right it will be economically feasible to extract more fossil fuel from the crust of the earth.

The temperature response to a broad range of carbon dioxide carbon pathways can be found from ensemble simulations of simple climate-carbon-cycle models which are constrained by observations and projects from more comprehensive models (Allen et al., 2009b). These simulations suggest that limiting

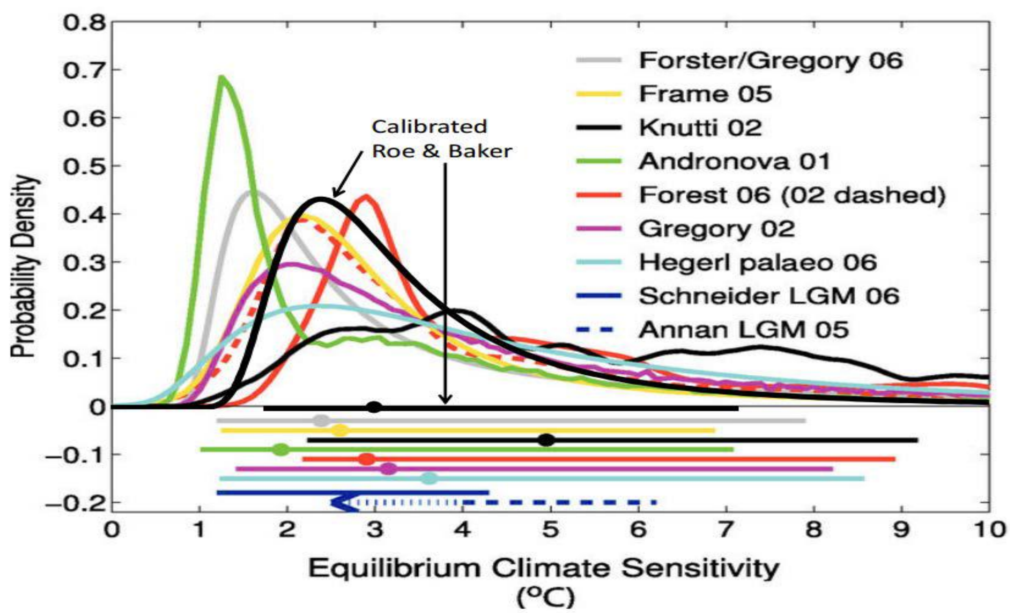
[1] The International Energy Agency suggests in its World Energy Outlook that *proven* international reserves contain 2,860 $GtCO_2$ = 781 GtC, but with higher prices potential reserves can be much higher.

cumulative emissions to 1 TtC leads to a mostly likely peak global mean warming of 2^o C above pre-industrial (1900) temperature, with a 5-95% confidence interval of 1.3^o to 3.9^o C. These simulations also suggest that the relationship between cumulative emissions and peak warming is remarkably insensitive to the timing of emissions or the peak emission rate. Hence, policy targets based on limiting cumulative emissions of carbon dioxide are likely to be more robust to scientific uncertainty than carbon emission rate or concentration targets. We use these findings to calibrate the following temperature module:

(2) $T = \alpha + \beta \ln(E) = 2 + 3.683 \ln(E)$,

where T denotes peak global mean warming in degrees Celsius above pre-industrial temperature. Since 1 TtC of carbon leads to a warming of 2^o Celsius, we have $\alpha = 2$. The pre-industrial concentration of carbon is 0.581TtC and corresponds to 0^o C, hence we have $0 = 2 + \beta \ln(0.581)$ which gives $\beta = 3.683$. This corresponds to the current temperature of 1.4^o Celsius above pre-industrial global mean temperature. Although there is a lag between global mean temperature and the carbon stock of about 70 years, we ignore it. This will bias our estimate of the social cost of carbon and optimal carbon tax upwards as can be seen from comparing Liski and Gerlagh (2012) with Golosov et al. (2013).

Figure 1: Estimates of equilibrium climate sensitivity



Source: IPPC (2007)

The equilibrium climate sensitivity (*ECS*) is defined as the global mean temperature increase which results from a doubling of the atmospheric carbon stock. The simple temperature module (2) implies an *ECS* of 2.55, since doubling the atmospheric carbon stock gives global warming of $\beta \ln(2) = 2.55^o$ Celsius. The IPCC Fourth Assessment Report (2007) estimates a range 2 to 4.5^o Celsius for the *ECS* with a best estimate of about 3^o Celsius, and the *ECS* is very unlikely to be less than 1.5^o Celsius. Values of the climate sensitivity substantially higher than 4^0 Celsius cannot be excluded. The probability density functions for the *ECS* shown in fig. 1 confirm that our climate sensitivity of 2.55^o Celsius is within the ballpark range. If anything, it is a bit on the low side, but it is in line with recent estimates which suggest that the *ECS* based on the energy budget of the most recent decade is 2^o Celsius with a 5-95% confidence interval of 1.2-3.9^o Celsius (Otto, et al., 2013).[2]

The carbon budget and the risk of unburned fossil fuel

Given E_0 = 0.85 TtC, we can burn a total of $S_0 - S$ = 0.3TtC of fossil fuel and keep E below 1 TtC and limit global warming to 2^o Celsius. If we are less prudent and aim to limit temperature to 3^o Celsius, we must limit the stock of atmospheric carbon to exp(1/β) = 1.3 TtC and can afford to burn three times as much, 0.9 TtC. BP (2012) states that in 2011the global economy emitted 34 billion tons of CO2 or GtCO2 or 9.3 GtC, so limiting global mean temperature to 2^o Celsius means that we can emit for another 300/9.28 = 32 years at this level before moving fully to a carbon-free economy.[3] If we allow temperature to rise by 3^o Celsius, then we can emit like this for almost another century before having to abandon fossil fuel altogether. Of course, if fossil fuel use declines as prices rise over time, fossil fuel reserves can last much longer. But if the *ECS* is higher, we have to abandon fossil fuel more quickly.

3. Production damages from global warming at low and higher temperatures

Nordhaus (2008) assumes for his DICE model that production damages from global warming are 1.7% of GDP at 2.5^0 Celsius and uses it for a particular chosen functional form to calibrate the ratio of global warming damages to world GDP before damages:

$$D^{\text{Nordhaus}}(T) = 1 - \frac{1}{1+0.00284T^2} = 1 - \frac{1}{1+(T/18.8)^2}.$$

[2] The transient climate response (*TCR*), defined as the temperature increase at the point of doubling the CO2 concentration following a linear ramp of increasing greenhouse gas forcing, is also sometimes used. Estimates based on the most recent decade suggest a *TCR* of 1.3^o Celsius with a 5-95% confidence interval of 0.9-2.0^o Celsius (see Otto et al., (2013)), which is about 0.3^o Celsius lower than the estimates based on the 1990s.

[3] The Economist of 4 May 2013 discusses joint research by Carbon Tracker and the Grantham Institute (http://www.carbontracker.org/wastedcapital) which supposes a global carbon budget of 1000 GtCO2 = 273 GtC that can maximally be emitted between now and 2050 for global mean temperatures to be restricted to 2^o Celsius (cf. our 0.3 TtC derived above). Listed reserves of energy firms are 421 GtC, which cannot be burnt without jeopardizing the 2^o Celsius-target. Hence, governments are either not serious about climate change or energy firms are overvalued as the risk of fossil fuel that never is allowed to be burnt is inadequately factored in market prices. Alternatively, oil and gas companies are betting on a failing climate policy or on a geo-engineering or CCS fix.

At pre-industrial temperature ($T = 0^o$ Celsius), damages are zero. Hanemann (2008) supposes production damages from global warming are 4.2% of GDP at 2.5^o Celsius, which yields damage ratio:

$$D^{\text{Hanemann}}(T) = 1 - \frac{1}{1 + (T/12.0)^2}.$$

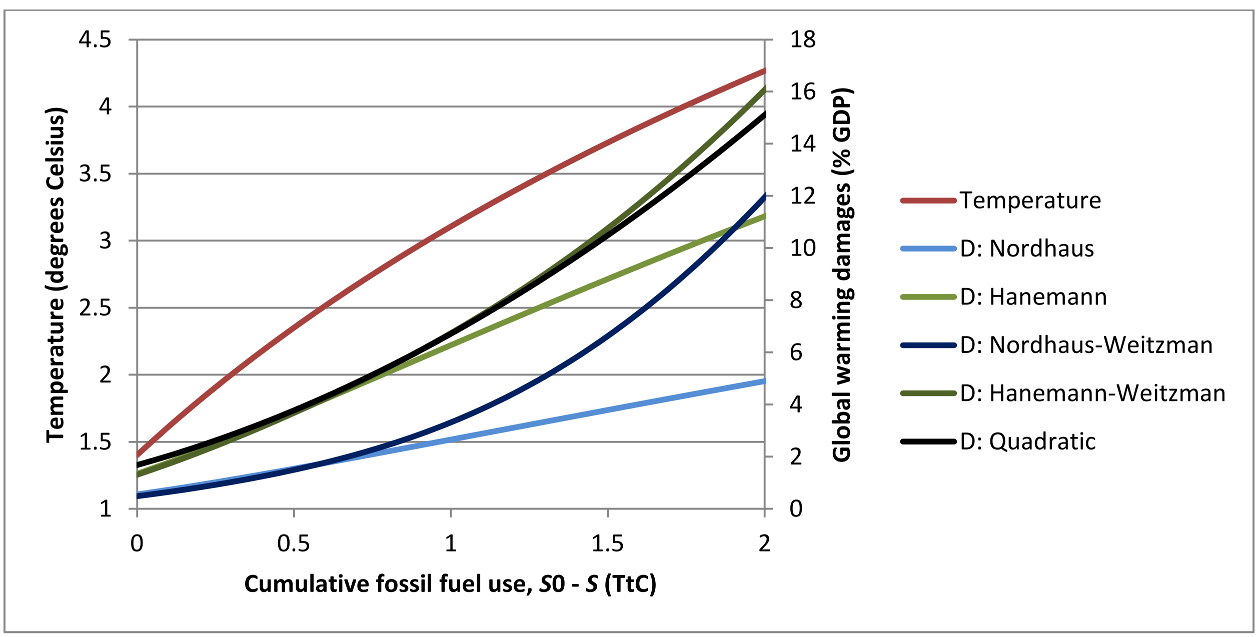
These two damage ratios imply that half of world GDP is lost at, respectively, 19^o and 12^o Celsius. This seems rather too small, because human mankind let alone economic production will cease at these temperatures. To get higher and more realistic damages at higher temperatures, Weitzman (2012) suggests that output damages might be 50% of world GDP at 6^o C and 99% at 12^o C. Combining the low-temperature damages of Nordhaus (2008) and Hanemann (2008) with the high-temperature damages of Weitzman (2012), Ackerman and Stanton (2012) arrive at the following net GDP ratios:

$$D^{\text{Nordhaus-Weitzman}}(T) = 1 - \frac{1}{1 + (T/20.2)^2 + (T/6.08)^{6.76}} \text{ and}$$

$$D^{\text{Hanemann-Weitzman}}(T) = 1 - \frac{1}{1 + (T/12.2)^2 + (T/6.24)^{7.02}}.$$

Fig. 2 plots on the left-hand side the concave relationship between global mean temperature (T) and cumulative fossil fuel use ($S_0 - S$) and on the right-hand side the percentage of global warming damages ($D \times 100\%$) against cumulative fossil fuel use for the various models of global warming damages.

Figure 2: Temperature and global warming damages



The Nordhaus damages are in the relevant range very well described by the linear reduced-form function $D \times 100\% = 0.49 + 2.19\ (S_0 - S)$ (with $R^2 = 0.9997$) and the Hanemann damages are well described by $1.41 + 4.83\ (S_0 - \mathrm{S})$ (with $R^2 = 0.9989$), but damages are clearly convex for the Nordhaus-Weitzman and Hanemann-Weitzman specifications. This reflects Weitzman's hypothesis that at higher temperatures marginal increases in global warming lead to proportionally bigger damages. Since we are interested in an upper estimate of the cost of global warming, we use a reduced-form quadratic specification for global warming damages which approximates the Hanemann-Weitzman specifications in a range of cumulative fossil use of up to 2 TtC (or an atmospheric carbon stock E of less than 1.85 TtC):

$$(3) \qquad D(E) = 0.0001(E - 0.35)^2\,\overline{p}_0 / r \quad \text{or} \quad D(S) = 0.000025(1 + S_0 - S)^2\,\overline{p}_0 / r, \quad \overline{p}_0 = 470 \text{ US\$/tC.}$$

where $\overline{p}_0$ denotes the current market price of fossil fuel which corresponds to about 9 US$ per million BTU or 470 US$ per ton of carbon. This quadratic specification does not fit too badly at both low and higher levels of global warming.

Social cost of carbon

The social cost of carbon (*SCC*) is defined in the usual manner as the present value of all future marginal global warming damages resulting from burning an extra unit of fossil fuel:

$$(4) \qquad SCC \equiv \int_0^\infty 0.5 e^{-r^* t} D'\big(E(t)\big) Y_0 e^{gt} dt = \int_0^\infty 35 \times e^{-rt} D'\big(E_0 + 0.5(S_0 - S(t)\big) dt,$$

where r^* denotes the interest rate, g denotes the trend rate of growth in real world GDP and Y_0 is the initial level of world GDP (which is set to its 2012 value of 70 trillion US$). We use the interest rate rather than the rate of time preference, since we are discounting marginal damages in units of lost production. For our calculations we use an exogenous and constant growth-corrected real interest rate of $r \equiv r^* - g = 1\%$.

For the Nordhaus and Hanemann specifications we can use the linear approximations of fig. 2 with constant marginal damages to get simple expressions for the social cost of carbon which are constant and independent of cumulative fossil fuel use:

$$SCC^{\text{Nordhaus}} \cong \frac{2.19 \times 70}{100 \times 0.03} = 153 \text{ US\$/tC} = 42 \text{ US\$/tCO2} \quad \text{and}$$

$$SCC^{\text{Hanemann}} \cong \frac{4.83 \times 70}{100 \times 0.03} = 338 \text{ US\$/tC} = 93 \text{ US\$/tCO2}.$$

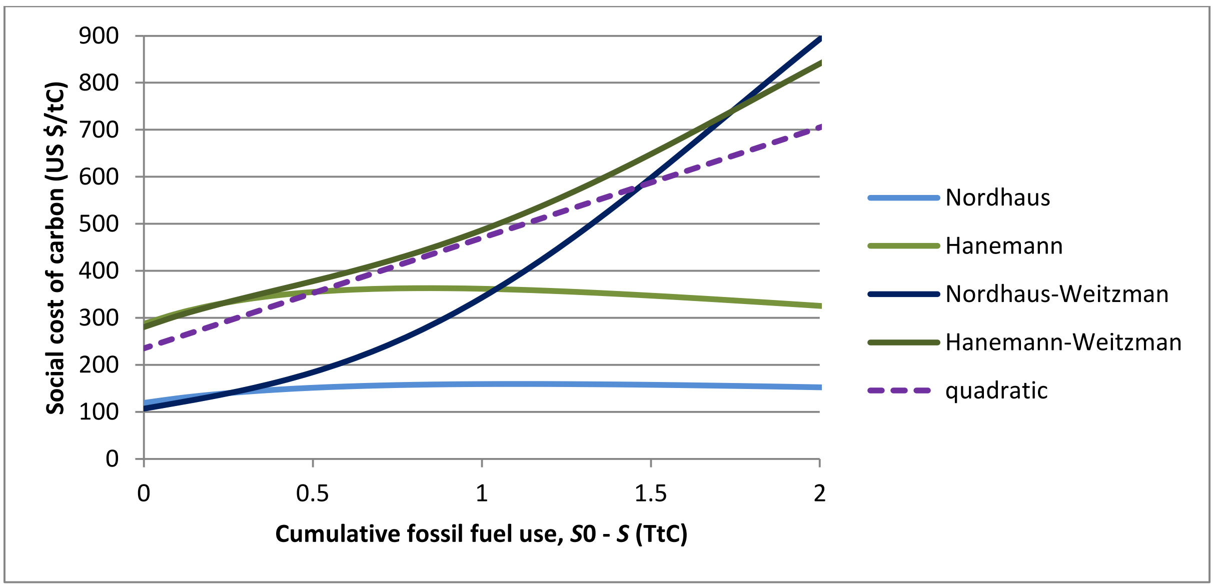
These estimates of the social cost of carbon are rather high, which is consequence of using a rather low growth-corrected discounted rate r. If a higher rate of time preference, more intergenerational inequality aversion and/or a lower growth rate of the economy (and thus of global warming damages) is used, these estimates would be rather smaller.[4] For example, if the growth-corrected discount rate is 3% instead of 1%, the social cost of carbon would be curbed by two thirds to 51 US$/tC for the Nordhaus damages and 113 US$/tC for the Haneman damages. These estimates of the social cost of carbon are more in line with the ballpark estimates used in popular discussions.

For the other damage functions the social cost of carbon varies with time as it depends on the time paths of the stock of atmospheric carbon or of cumulative fossil fuel use. However, as we abstract from decay of atmospheric carbon, the *SCC* is constant throughout the carbon-free era and only depends on the stock of atmospheric carbon at the end of the fossil fuel era, $E(t_F)$, or the stock of untapped fossil fuel, $S(t_F)$, where t_F denotes the moment of time when the fossil fuel era ends and the carbon-free era starts. For the Nordhaus-Weitzman and Hanemann-Weitzman models we substitute the temperature module (1) and the atmospheric carbon stock equation (2) into (4) to calculate the *SCC* at the end of the carbon-free era from which moment onwards the stock of untapped fossil fuel is constant:

$$SCC = \frac{R'(T)\times GDP\times(\partial T/\partial E)\times(\partial E/\partial S)}{r} = \frac{-R'(T)\times 70\times(3.683/E)\times 0.5}{0.01}\ \text{US\$/tC}.$$

The *SCC* for our specification (3) is $SCC^{quadratic} = 0.5D'(E)\times 70/r = 2.35(1+S_0-S)/r$ US$/tC.

Figure 3: The social cost of carbon versus the stock of remaining fossil fuel



[4] The Ramsey rule suggests that the appropriate interest rate to use is $r^* = \rho + \theta g_C$, where ρ is the pure rate of time preference, θ the coefficient of relative intergenerational inequality aversion and g_C the growth rate in aggregate consumption per capita. With high enough growth richer generations carry more of the burden and climate policy becomes less ambitious as reflected in a lower social cost of carbon resulting from a higher r^*, especially if intergenerational inequality aversion is large. We suppose a pure rate of time preference equal to $\rho = 0.01$, intergenerational inequality aversion equal to $\theta = 1$ and growth rates equal to $g_C = g = 2\%$. It follows that $r^* = 0.03$ and $r = 0.01$. With more inequality aversion, $\theta = 2$, $r^* = 0.05$ and $r = 0.03$.

Fig. 3 plots the social cost of carbon for the various global warming models discussed above. Our quadratic specification (3) captures the higher marginal costs of global warming stressed by Weitzman (2012) whereas the Nordhaus (2008) and Hanemann (2008) models do not capture these costs. Note that all expressions for the social costs of carbon and energy use are given in 2012 prices.

4. Effects of carbon taxes and renewables subsidies on market outcomes

We assume that there is a renewable backstop energy source which does not emit carbon and also assume that this is a perfect substitute for fossil fuel and has an infinitely elastic supply. For illustrative purposes, we suppose that the cost of renewables, *b*, is 50% more expensive than the current market price of fossil fuel to reflect that solar, wind and other forms of renewable energy are not competitive yet, so that $b = 1.5\bar{p}_0 = 705$ US$/etC. These are bold assumptions. For example, wind or solar energy suffer from intermittence problems and require energy storage which oil, gas or coal do not. Renewables are thus in practice imperfect substitutes for fossil fuel. Furthermore, the supply of renewables is not necessarily infinitely elastic, because a rising price of energy may induce an expanding supply of renewables. Still, these assumptions about the backstop allow us to get precise insights which will help us to understand the impacts of optimal climate policy on transition times and untapped fossil fuel.

We assume that fossil fuel extraction becomes more costly as more reserves have been depleted, because then the ‘low hanging fruit’ has been extracted from the earth and less accessible fields or mines have to be explored. Indeed, the so-called Herfindahl rule states that it is optimal to deplete the least-cost reserves first. Furthermore, we assume that extraction costs become infinitely large as reserves are fully depleted. Hence, if the cost of extracting 1 TtC of fossil fuel is given by $G(S)$, we have $G' < 0$ and $\lim_{S \to 0} G(S) = \infty$. For calibration purposes, we simply set $G(S) = \bar{p}_0 / S$ US$. The scarcity rent is the amount earned per unit of fossil fuel over and above the cost of extraction. The scarcity rent is thus $100(1 - 1/S_0) = 42\%$ of the current market price. This seems high, but this figure is meant to correspond to an average for oil, natural gas and coal. Consequently, the rent for relatively abundant coal is much lower and for scarce and limited oil and gas the rent is higher.

The user cost of fossil fuel is $q \equiv p + \tau$, where p is the market price of fossil fuel and τ the specific carbon tax. For the time being we consider exogenous changes in τ. In section 6 we derive the optimal social cost of carbon and global carbon tax. We assume that in both cases the carbon tax revenues are rebated in lump-sum fashion to the private sector. We also consider subsidies υ on the use of renewables, which are financed by levying lump-sum taxes which do not distort behaviour of households and firms.

How much fossil fuel does the market leave untapped in the crust of the earth?

Since current fossil fuel prices are lower than the cost of renewables, the fossil fuel era ends at the point in time t_F where the user cost of fossil fuel (q) reaches the cost of renewables (b) net of the subsidy (υ). Since at the end of the fossil fuel era fossil fuel is obsolete, the scarcity rent on fossil fuel must be zero. Hence, the user cost of fossil fuel at the end of the fossil fuel era must equal the extraction cost plus the carbon tax: $q(t_F) = G\left(S(t_F)\right) + \tau(t_F) = \dfrac{\overline{p}_0}{S(t_F)} + \tau(t_F).$ To be indifferent between fossil fuel and renewables at that point of time, this user cost of fossil fuel must equal the cost of renewables, $b - \upsilon$. We thus find that the optimal amount of fossil fuel to be left in the crust of the earth at the end of the fossil fuel era is:

(5) $$S(t_F) = \frac{\overline{p}_0}{b - \upsilon - \tau(t_F)} \text{ TtC}, \quad b = 1.5\overline{p}_0 = 705 \text{ US\$/etC}.$$

Under "laissez faire" there are no taxes or subsidies, hence (5) indicates that the market outcome locks up 0.67 TtC of fossil fuel in the crust of the earth. Cumulative fossil fuel use is 1.05 TtC which is three to four times bigger than the carbon budget of 0.3 TtC discussed above. The corresponding stock of atmospheric carbon is from equation (2) given by 1.37 TtC, which yields from equation (1) ultimate global warming of 3.2^{o} Celsius above pre-industrial temperature.

Introducing a renewables subsidy of 20% (i.e., $\upsilon = 0.2\,b$) locks up more fossil fuel in the crust of the earth, namely 0.83 TtC, so that cumulative fossil fuel use is 0.88 TtC and global warming is ultimately curbed to 2.94^{o} Celsius. Doubling the renewables subsidy to 40% locks up 1.12 TtC of fossil fuel and thus limits cumulative fossil fuel use to 0.60 TtC and curbs global warming to 2.5^{o} Celsius above the pre-industrial global mean temperature.

Demand for fossil fuel and renewables

To understand more about the dynamic adjustment paths and to be able to calculate the timing of the advent of the carbon-free era, we need to make additional assumptions about fossil fuel demand. Suppose therefore that the elasticity of market demand is constant and given by $\varepsilon = 0.85$, so that demand for fossil is given by $F = (A/q)^{\varepsilon}$ and the inverse demand function by $q = p + \tau = AF^{-1/\varepsilon}$. We calibrate this demand function given a current market price of fossil fuel of 470 US\$/tC and global fossil fuel use of 9.3 GtC in 2011, hence we have that $A = 470 \times \left(\dfrac{9.3}{1000}\right)^{1/0.85} = 1.9.$ If renewables use is strictly positive, fossil fuel use must be zero which follows from these two sources of energy being perfect substitutes. Hence, we have that the rates of fossil fuel and renewables use are given by:

(6) $F(t)=0$ and $R(t)=\left[A/(b-\upsilon)\right]^{\varepsilon}=6.6$ eGtC = 370 G millions BTU, $\forall t \geq t_F$.

Calibration of initial fossil fuel reserves and "laissez faire" outcome

The fossil fuel phase of the market outcome follows from the two additional differential equations:

(7) $\dot{S}=-F=-(A/q)^{\varepsilon}$, $S_0=1.72$ TtC, $A=1.9$, $\varepsilon=0.85$,

(8) $\dot{q}=\left[r-G(S)\right]q+\dot{\tau}-r\tau=(r-\overline{p}_0/S)q+\dot{\tau}-r\tau$, $q(t_F)=b=1.5\overline{p}_0=705$ US\$/etC,

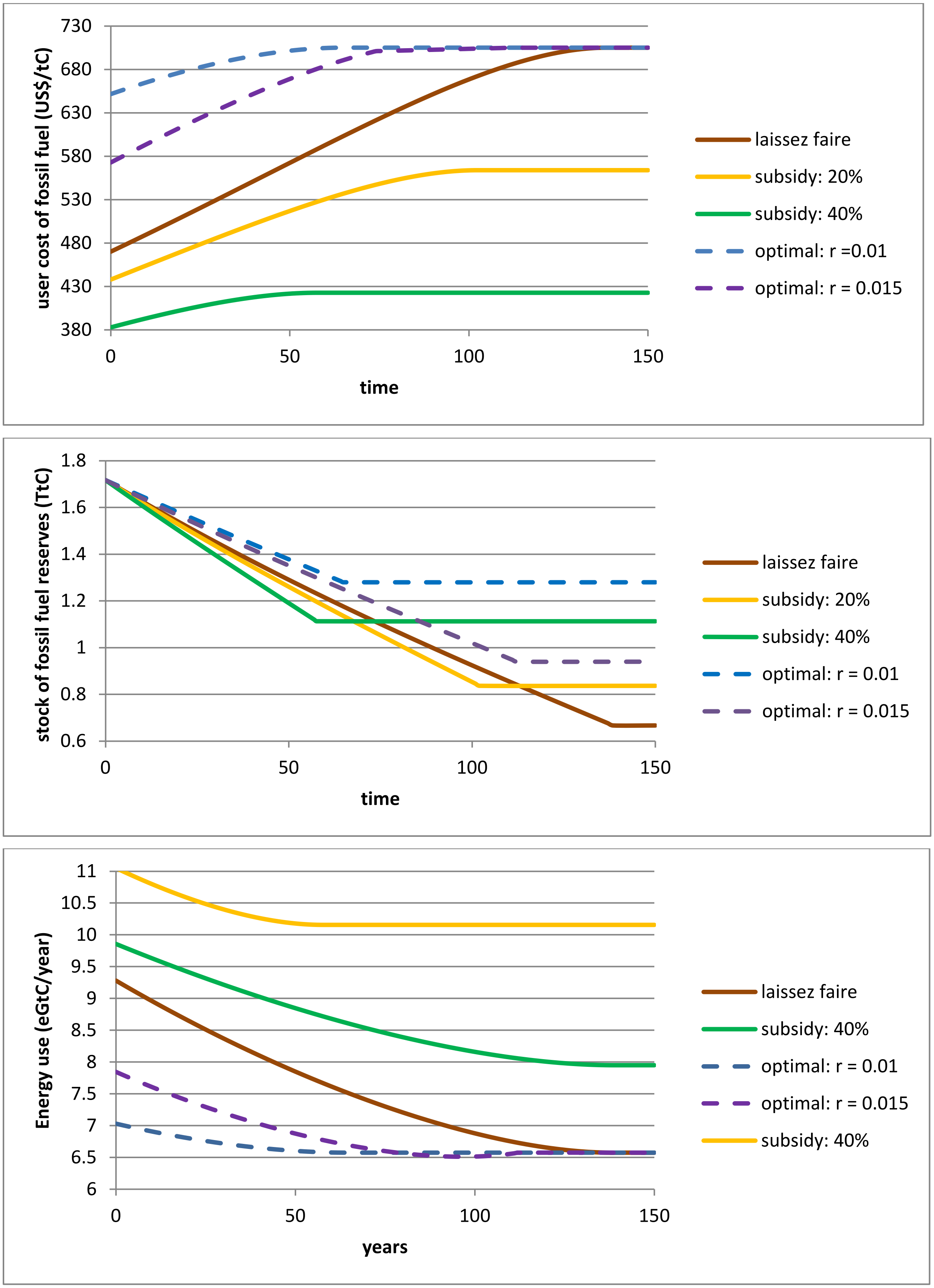
$R(t)=0, 0\leq t\leq T.$ Equation (7) is the fossil fuel depletion equation. Since the fossil fuel phase ends at time t_F, cumulative fossil fuel use equals $\int_0^{t_F} F(t)dt=S_0-S(t_F).$ The Hotelling rule for efficient resource extraction states that the return on taking fossil fuel out of the ground, selling it and investing it, $r\left[p-G(S)\right]$, must equal the expected return on leaving it in the ground, $\dot{p}$. This arbitrage equation for the market price of fossil fuel can be rewritten as equation (8) in terms of the user cost of fossil fuel.

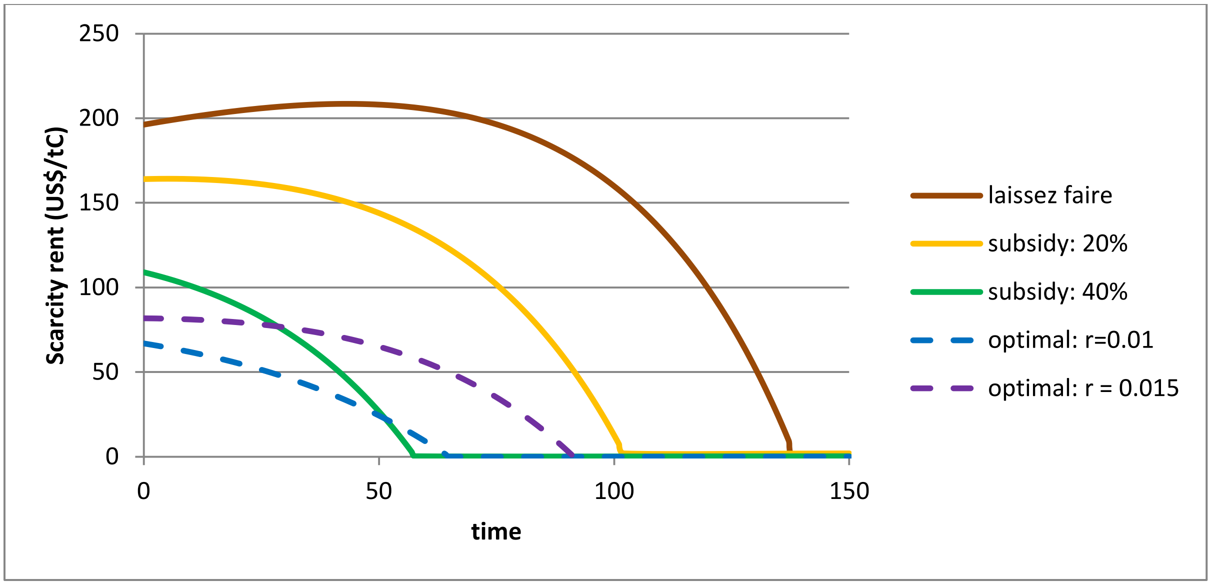
The differential equations (7) and (8) can be solved over the time interval [0, t_F] with reserves pinned down at the start by S_0 and the user cost of fossil fuel pinned down at the end by $q(t_F)$ = 705 US\$/tC. The time of the fossil fuel phase t_F is determined in such a way that the end condition (5) is satisfied.[5] We first solved this system under "laissez faire" to calibrate the initial stock of fossil fuel reserves. We did this by choosing S_0 and t_F given $q(0) = p(0) = 470$ US\$/tC, $q(t_F) = 705$ US\$/etC and $S(t_F) = 0.67$ TtC. This yielded $S_0 = 1.72$ TtC, which we then used as our calibrated estimate of initial reserves as already mentioned when discussing the carbon accumulation equation (1). This calibration procedure also yielded a length of the fossil fuel era of $t_F = 138$ years under "laissez faire".

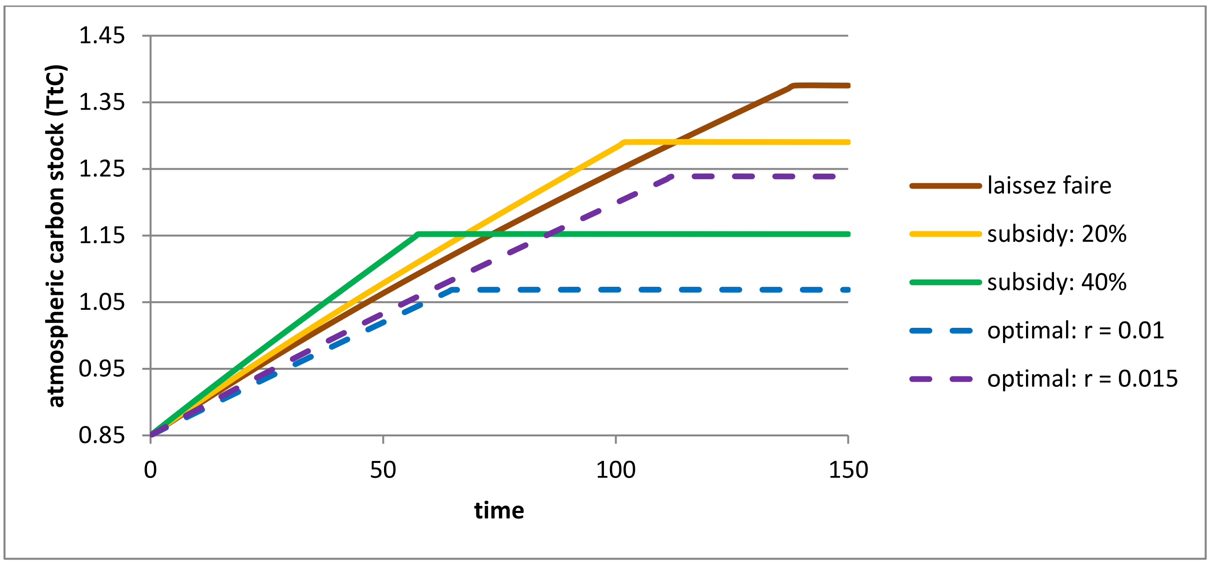
The solid brown lines in fig. 4 below give the dynamic adjustment paths under "laissez faire". Since there are no carbon taxes, the user cost of fossil fuel equals the market price of fossil fuel. As fossil fuel prices rise, fossil fuel use and carbon emissions fall, global warming increases less rapidly from the current level of 1.4^{o} Celsius. At the end of the fossil fuel era, renewables take over and the global mean temperature stays at 3.2^{o} Celsius. Of course, in practice the stock of carbon will decay slowly and the global mean temperature will slowly fall again but we abstract from this.

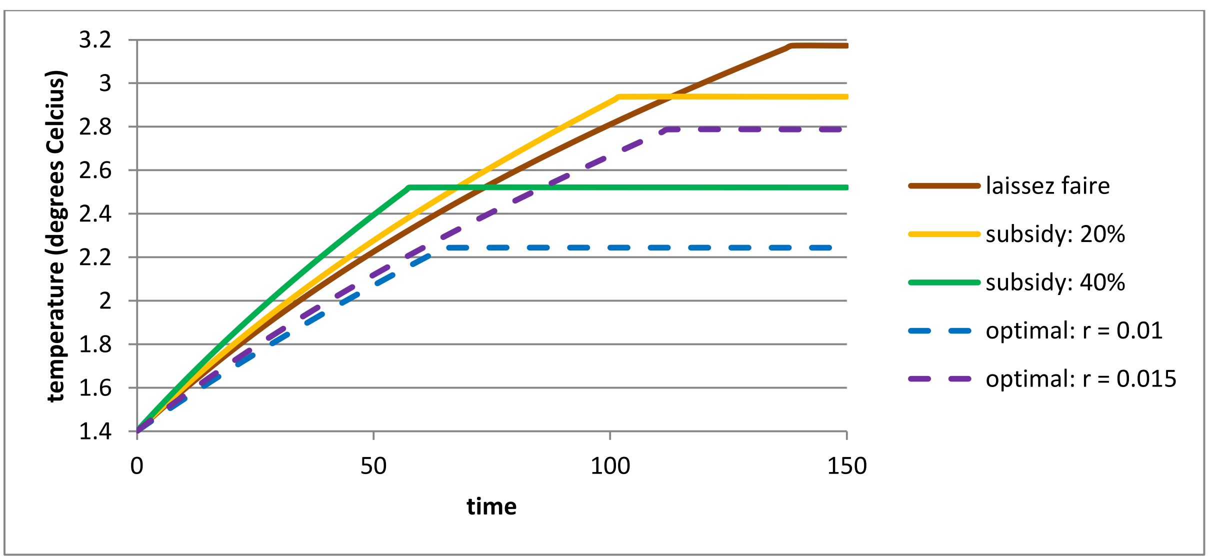
[5] We solve this two-point-boundary-value problem (TPBVP) computationally by nesting a 4th-order Runge-Kutta algorithm for solving equations (7) and (8) into a Gauss-Newton algorithm for solving for t_F and $q(0)$ (or for t_F and $S(0)$ for the calibration simulation) to ensure that the boundary conditions at times 0 and t_F are satisfied. We do this in EXCEL by nesting the 4th-order Runge-Kutta algorithm from the add-on POPTOOLS in the SOLVER routine.

Figure 4: Renewables subsidies and the Green Paradox versus optimal policies (S_0 = 1.72 TtC)




Scarcity rent (US$/tC)
250
200
150
100
50
0
0
50
100
150
time
laissez faire
subsidy: 20%
subsidy: 40%
optimal: r=0.01
optimal: r = 0.015


atmospheric carbon stock (TtC)
1.45
1.35
1.25
1.15
1.05
0.95
0.85
0
50
100
150
time
laissez faire
subsidy: 20%
subsidy: 40%
optimal: r = 0.01
optimal: r = 0.015


temperature (degrees Celcius)
3.2
3
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
0
50
100
150
time
laissez faire
subsidy: 20%
subsidy: 40%
optimal: r = 0.01
optimal: r = 0.015

Anticipated carbon taxes

It is a well known result that, if extraction costs are zero and all fossil fuel reserves are fully used up, introducing a specific carbon tax τ that rises over time at the rate of interest r leaves fossil fuel extraction and reserves paths completely unaffected. This can be seen immediately from the last two terms in equation (8) dropping out. In this case, the user price of fossil fuel including the carbon tax, the producer price of fossil fuel excluding the carbon tax and the carbon tax itself all increase at the rate of interest.

However, we have assumed that extraction costs are non-zero and stock-dependent. Furthermore, we have assumed that costs become infinitely large as reserves get fully depleted so that we get *partial* exhaustion as can be seen from the strictly positive solution for $S(t_F)$ from equation (5). A carbon tax that grows at the interest rate r is then not neutral, since it reduces the amount of fossil fuel that is locked up at the end of the fossil fuel era $S(t_F)$ and thus curbs cumulative emissions and global warming.

A credible future carbon tax that is to be implemented in the near or distant future or a carbon tax that rises at a faster rate than the interest rate induces fossil fuel owners to extract fossil fuel more rapidly than under "laissez faire" and thus accelerates global warming. This is known as the Green Paradox (Sinn, 2008). However, such a rapidly rising carbon tax will also lead to more fossil fuel being locked up in the crust of the earth at the end of the fossil fuel phase which mitigates global warming and might reverse the Green Paradox. For the sake of brevity we will illustrate such effects with a renewables subsidy.

Renewables subsidies and Green Paradox effects

Before we discuss our simulation results for renewables subsidies, we briefly discuss their effects for the case where extraction costs are zero and fossil fuel reserves are fully exhausted. The fossil fuel price, fossil fuel use, the stock of remaining fossil fuel and the duration of the fossil fuel phase are then:

$$q(t) = (b-\upsilon)e^{r(t-t_F)}, \quad F(t) = \left(\frac{A}{b-\upsilon}\right)^{\varepsilon} e^{\varepsilon r(t_F - t)}, \quad S(t) = S_0 - \frac{1}{\varepsilon r}\left(\frac{A}{b-\upsilon}\right)^{\varepsilon} e^{\varepsilon r t_F}\left(1-e^{-\varepsilon r t}\right), \quad 0 \le t \le t_F,$$

$$t_F = \frac{1}{\varepsilon r}\ln\left(\frac{\varepsilon r S_0 + \left(\frac{A}{b-\upsilon}\right)^{\varepsilon}}{\left(\frac{A}{b-\upsilon}\right)^{\varepsilon}}\right) > 0.$$

A renewables subsidy ($\upsilon > 0$) thus depresses the rising time path for the user cost of fossil fuel, lifts up the declining time path for fossil fuel use and cuts the duration of the fossil fuel phase.[6] We thus see that a renewables subsidy increases carbon emissions and accelerates global warming in line with the so-called

[6] For our calibration the length of the fossil fuel phase is cut from 137.5 to 123 years by a 20% renewables subsidy. These lengths are not that different from when we allow for stock-dependent extraction costs (see table 1 below).

Green Paradox. Once we allow for stock-dependent extraction cost, this adverse effect on global warming may be offset by a beneficial effect on global warming of locking up more fossil fuel in the crust of the earth at the end of the fossil fuel era.

For our model with stock-dependent extraction costs, the solid yellow lines in fig. 4 give the time paths if the cost of renewables is reduced from 705 to 564 US$/etC with a 20% subsidy ($\upsilon = 0.2$). We notice five effects. First, the time it takes to transition to the carbon-free era t_F shortens from 138.5 to 102 years. Second, as mentioned above, the stock of fossil fuel that is left forever locked up in the crust of the earth $S(t_F)$ increases from 0.67 to 0.82 TtC. This curbs the long-run stock of atmospheric carbon from 1.375 to 1.29 TtC, so that long-run global warming is reduced from 3.2^o to 2.9^o Celsius. Third, despite curbing global warming in the long run, fossil fuel use is ramped up during the fossil fuel phase which accelerates global warming before the carbon-free era commences. This effect is known as the Green Paradox: owners of fossil fuel reserves pump their fossil fuel up more quickly for fear of their reserves becoming less worth as a result of the cheaper renewables. Fossil fuel prices are lower due to the induced fossil fuel glut. The price of fossil fuel jumps down on impact from 470 to 438 US$/tC and converges at the end of the fossil fuel era to 564 US$/tC instead of 705 US$/tC. Fourth, the scarcity rent on fossil fuel is obviously lower as a result of the renewables subsidy and the induced faster pumping of fossil fuel and ends up being zero at the end of the fossil-fuel era. Finally, fossil fuel use at the end of the fossil fuel phase must equal renewables use (from the continuity of the path for energy prices). As a result of the subsidy, final energy use jumps up from 6.76 to 7.95 eGtC.

The effects of doubling the renewables subsidy to 40% can be seen from the green lines in fig. 4. Green Paradox effects are amplified, but cumulative fossil fuel use and global warming are curbed by more.

A renewables subsidy is often advocated as second-best alternative to an optimal carbon tax, because electorates prefer the 'carrot' to the 'stick'. There are three problems with such a strategy. First, global warming first gets worse before it gets better. Second, although green welfare may increase, overall welfare may fall (see section 5). Third, if electorates notice that global warming may worsen during the fossil fuel phase, they might undermine the credible announcement of offering renewables subsidies and thereby destroy the effectiveness of climate policy altogether.

5. Welfare: Are large renewables subsidies counterproductive?

The global planner maximizes global social welfare $W(0)$, which is defined by the present discounted value of the difference between, on the one hand, the consumer surplus, and, on the other hand, the sum of fossil fuel extraction costs, renewables costs and global warming damages:

$$W(0) \equiv \int_0^\infty \left[U\left(F(t)+R(t)\right) - G\left(S(t)\right) - bR(t) - D\left(E(t)\right) \right] e^{-rt} dt =$$

(9)
$$\int_0^\infty \left\{ \frac{A\left(F(t)+R(t)\right)^{1-1/\varepsilon}}{1-1/\varepsilon} - \left[\frac{F(t)}{S(t)} + bR(t) + 0.005\left(1+S_0-S(t)\right)^2 \right] \bar{p}_0 \right\} e^{-rt} dt \text{ trillion US\$.}$$

The consumer surplus is defined as the area under the demand curve, so $U'(F+R) = q = A(F+R)^{-1/\varepsilon}$. Since carbon taxes are rebated in lump-sum fashion, they do not appear in this expression. The same is true for renewables subsidies, which are financed by lump-sum taxes. Substituting the demand curve, the carbon accumulation equation (1) and the expression for renewables use (6), we rewrite the expression for global social welfare (9) as:

(9′)
$$W(0) = \int_0^{t_F} \left\{ \frac{A^\varepsilon q(t)^{1-\varepsilon}}{1-1/\varepsilon} - \left[\frac{\left(A/q(t)\right)^\varepsilon}{S} + 0.0025\left(1+S_0-S(t)\right)^2 \right] \bar{p}_0 \right\} e^{-rt} dt$$
$$+ \left[\frac{A^\varepsilon (b-\upsilon)^{1-\varepsilon}}{\varepsilon-1} - 0.0025\left(1+S_0-S(t_F)\right)^2 \bar{p}_0 \right] \frac{e^{-rt_F}}{r} \text{ trillion US\$.}$$

Does a renewables subsidy hurt social welfare?

To convert the change in welfare under the 20% subsidy from the "laissez-faire" outcome into monetary units, we divide the welfare change by the marginal utility of initial consumption of fossil fuel (which equals one under quasi-linear preferences) and express it as a percentage of initial world GDP. This gives a present value welfare gain for the 20% renewables subsidy of 7.3% of world GDP, so that for this case the welfare gains from the ultimate reduction in global warming from 3.2^{o} to 2.9^{o} Celsius dominate the transient welfare losses of the Green Paradox effects.

However, the benefits of larger renewables subsidies taper off quickly. Indeed, doubling of the renewables subsidy to 40% is counterproductive because the economic costs of the subsidy rise more than proportionally with the size of the subsidy. Compared with "laissez faire" there is now a global social welfare *loss* of 21.0% of world GDP. The adverse short-run welfare implications of both lower utility from fossil fuel production and acceleration of global warming associated with Green Paradox effects now dominate the beneficial long-run welfare implications of a lower carbon stock (1.15 TtC instead of 1.29 or 1.38 TtC under a 20% subsidy or "laissez faire", respectively) and less global warming (2.5^{o} Celsius instead of 2.9^{o} or 3.2^{o} Celsius). The negative welfare effects of faster running down of fossil fuel reserves thus outweigh the positive welfare effects of a shorter length of the fossil fuel era (58 years versus 102 or 138.5 years under a 20% subsidy or "laissez faire", respectively) and locking up more fossil fuel forever in the crust of the earth (1.12 GtC versus 0.84 or 0.67 TtC). Hence, renewables subsidies need not improve global social welfare, especially for large subsidies. It is also easy to see that these subsidies are more likely to be counterproductive if the interest rate is larger, because then the welfare gains from the ultimate curbing of global warming are discounted more heavily. Table 1 summarizes these impact and long-run effects under "laissez faire" and under a 20% and 40% renewables subsidy. We now consider the social optimum.

Table 1: Effects of renewables subsidies and optimal carbon tax (S_0 = 1.72 TtC)

	"laissez faire"	20% subsidy	40% subsidy	optimum $r = 0.01$	optimum $r = 0.015$
Duration of fossil fuel era, t_F	138	102	58	65	112
Locked up fossil fuel, $S(t_F)$ (TtC)	0.67	0.84	1.12	1.28	0.94
Cumulative fossil fuel use, $S_0 - S(t_F)$ (GtC)	1050	883	604	437	777
Average fossil fuel use, $[S_0 - S(t_F)]/t_F$ (GtC)	7.6	8.7	10.5	6.7	6.9
Ultimate carbon stock in atmosphere, $E(t_F)$ (TtC)	1.375	1.29	1.15	1.07	1.24
Global warming, T (^{o} Celsius)	3.17	2.94	2.52	2.24	2.79
Welfare gain (% GDP)	-	7.3	–21.0	71.7	-
$p(0)$ (US$/tC)	470	435	383	341	356
$p(T)$ (US$/tC)	705	564	423	367	427

6. Social optimum and the optimal carbon tax

The social optimum follows from maximizing expression (9) or (9′) for global social welfare $W(0)$ subject to the fossil depletion equation (7) and the carbon accumulation equation $\dot{E} = 0.5F,\, E(0) = 0.85$ TtC (see van der Ploeg and Withagen (2012a) and the appendix). The social optimum has an initial phase where fossil fuel is used exclusively followed from time t_F onwards by a final phase where renewables are used exclusively. As before, this result derives from the assumptions that fossil fuel and renewables are perfect substitutes and renewables supply is infinitely elastic. Furthermore, it can be established that the time path of the optimal carbon tax always slopes upwards and is concave. This result requires no decay of the stock of atmospheric carbon. Intuitively, individual owners of fossil fuel reserves internalize that further depletion of their reserves forces them to go to less accessible fields and therefore as the fossil fuel phase continues their extraction costs rise and their use of fossil fuel is curbed. This is the reason why the rise in the carbon tax flattens off as reserves diminish. Finally, the social optimum can be realized in a market economy by setting the specific carbon tax equal to the social cost of carbon. There is no role for a renewables subsidy in the social optimum. However, if there are learning by doing effects in renewables use, a renewables subsidy is required to internalize these effects (e.g., Rezai and van der Ploeg, 2013).

At the switch from the fossil fuel phase to the renewable phase, we have $R(t_F) = 6.6$ eTtC from equation (6) and thus we also have $F(t_F) = 6.6$ GtC. Hence, the social price of fossil fuel at the time of the switch must equal $q(t_F) = b = 705$ US\$/tC (from continuity in the time path of the social price of energy). The optimal amount of fossil fuel to lock up in the earth at the start of the carbon-free era follows from:

$$(5') \qquad q(t_F) = G\big(S(t_F)\big) + \frac{0.5D'\big(E(t_F)\big)}{r} = \frac{\bar{p}_0}{S(t_F)} + \frac{0.005\big[1 + S_0 - 0.5S(t_F)\big]\bar{p}_0}{r} = b, \quad b = 705 \text{ US\$/etC}.$$

The solution to equation (5′) is $S(t_F) = 1.28$ TtC with our calibrates values for the interest rate, $r = 0.01$, and the initial stock of fossil fuel reserves, $S_0 = 1.72$ TtC. Hence, the optimal ultimate stock of atmospheric carbon is $E(t_F) = 1.07$ TtC and the corresponding level of global warming follows from the temperature module (2) and is given by 2.24^{o} Celsius. The social cost of carbon when the carbon-free era commences is $\theta(t_F) = 0.5D'\big(E(t_F)\big)/r = 338$ US\$/tC, which is constant from then on. Since the scarcity rent of fossil fuel is zero at the time of the switch, the market price of oil excluding the carbon tax at the time of the switch must equal extraction costs, $p(t_F) = 470/S(t_F) = 367$ US\$/tC.

Fossil fuel reserves, the social price of fossil fuel and the social cost of carbon for the carbon phase and the optimal switch time follow from solving for $t \in [0, t_F]$ the dynamic system defined by equation (7) with $S(t_F)$ from equation (5′) and

$$(8') \quad \dot{q} = \big[r - G(S)\big]q - 0.5D'\big(E_0 + 0.5(S_0 - S)\big) = r\big[q - \bar{p}_0 / S\big] - 0.005(1 + S_0 - S)\bar{p}_0, \; q(t_F) = 705 \text{ US\$/etC},$$

where $q \equiv p + \tau$ is the social price of fossil fuel and $\lambda = p - G(S)$ the scarcity rent on fossil fuel (see appendix).[7] This results in an initial user cost of energy of 652 US\$/tC and a duration of the fossil fuel phase of $t_F = 65$ years. The social cost of carbon θ follows from:

$$(10) \qquad \dot{\theta} = r\theta - 0.5D'\big(E_0 + 0.5(S_0 - S)\big) = r\theta - 0.005(1 + S_0 - S)\bar{p}_0, \quad 0 \le t \le t_F, \quad \theta(t_F) = 338 \text{ US\$/tC}.$$

---

[7] We solve this again by nesting a Runge-Kutta for integrating the ordinary differential equations (7) and (8′) into a Gauss-Newton algorithm for solving the three terminal boundary conditions for the appropriate values of $q(0)$, $\tau(0)$ and t_F. Using the addins POPTOOLS and Solver this can be done within a simple Excel sheet.

If the specific carbon tax τ is set to the social cost of carbon determined by equation (10), the Hotelling rule for the market economy (8) becomes the Hotelling rule for the social optimum (8′). Hence, with this carbon tax the market economy exactly replicates the social optimum. Using this result and integrating equation (10) with (5′) forward in time, we get the optimal carbon tax as the present value of future marginal global warming damages:

$$(4') \qquad \tau(0) = \theta(0) = \int_0^{t_F} 0.005\left[1 + S_0 - S(t)\right] \bar{p}_0 e^{-rt} dt + 0.005\left[1 + S_0 - S(t_F)\right] \bar{p}_0 \frac{e^{-rt_F}}{r} \text{ US\$/tC.}$$

This yields an initial carbon tax of 311 US$/tC which rises monotonically at a decreasing rate towards a tax of 338 US$/tC at the switch time 65 years later. It does so at a decreasing rate, since fossil fuel producers curb their rates of extraction as reserves are depleted and become less accessible. This carbon tax corresponds to roughly 85 $-cents on a gallon of gasoline or 17 Euro-cents on a litre of petrol.

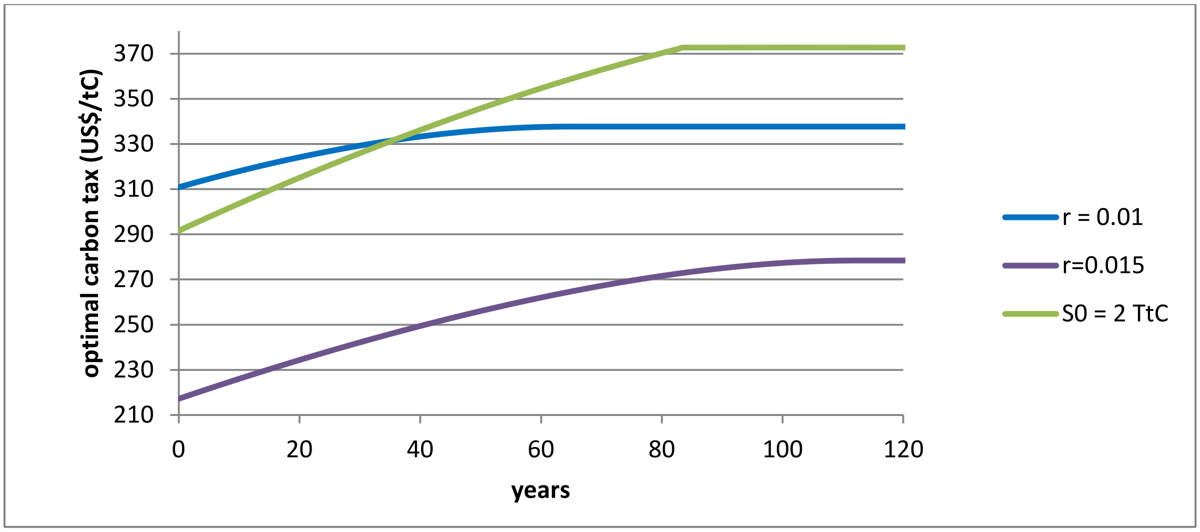
The dotted blue lines in fig. 4 plot the resulting time paths and the optimal carbon tax (i.e., the social cost of carbon) and the components of the optimal user cost of energy are plotted in fig. 5. Table 1 above also summarizes the impact and long-run effects under the optimal carbon tax.

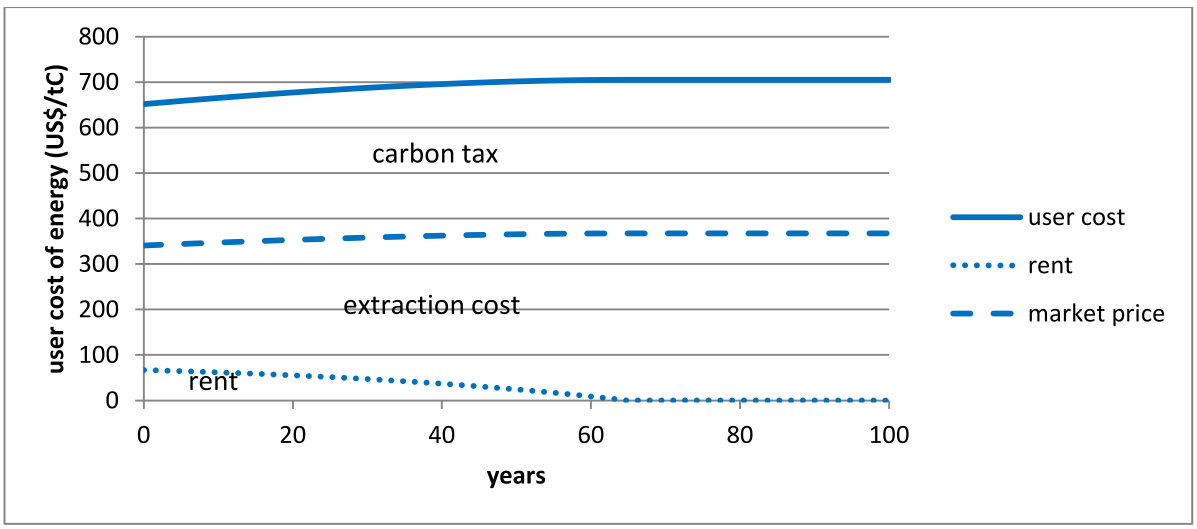
Compared with “laissez faire”, the social optimum leads to a present-value welfare gain of 71.7% of world GDP. In annuity terms this is a gain of 0.7% of world GDP. The duration of the optimal fossil fuel phase (65 years) is substantially shorter than under “laissez faire” (138 years). The relatively flat time path for the optimal carbon tax manages to curb cumulative fossil use from 1050 to 437 GtC which corresponds to an average annual use of 6.7 GtC instead of 7.6 GtC under “laissez faire”. The social optimum thus does not suffer from the Green Paradox as can be seen from the time paths for fossil fuel depletion in the third panel of fig. 4.

If policy makers are less precautionary and adopt a higher interest rate (say, r = 0.015), the time path for the optimal carbon tax is lowered throughout. The carbon tax thus rises from 217 to 278 US$/TC as can be seen from the purple line in fig. 5. Hence, carbon emissions are higher during the fossil fuel phase. Furthermore, the duration of the fossil fuel phase increases from 65 to 112 years. Cumulative carbon emissions are therefore much higher, namely 777 TtC instead of 437 TtC. Average yearly emissions are only a bit higher, namely 6.9 GtC instead of 6.7 GtC. Ultimate global warming is higher also.

If initial reserves are 10% higher which is probably most of what can be expected from the shale gas revolution, the time path for the optimal carbon tax has to be steepened to slow down extraction. The optimal policy is thus to have initially a lower carbon tax (292 US$/tC instead of 311 US$/tC) but end up with a larger carbon tax (373 US$/tC instead of 338 US$/tC). The time it takes before the fossil fuel era is taken over by the renewables is longer (84 instead of 65 years). Due to the abundance of fossil fuel reserves, cumulative fossil use is higher (586 GtC instead of 437 GtC) and thus global warming rises to 2.49° C which is larger than the 2.24° Celsius under our benchmark estimate for S_0 of 1.72TtC. Cumulative fossil use and ultimate global warming would have been even more if it were not for the fact that more fossil fuel is locked up at the start of the carbon-free era (1.41 TtC instead of 1.28 TtC).

Figure 5: Optimal cost of carbon and components of user cost of fossil fuel (S_0 = 1.72 TtC)





Renewables subsidies of 20 and 40% lead to higher cumulative fossil fuel use than the social optimum but less than under "laissez faire", namely 883 and 604 GtC, respectively. However, these renewables subsidies induce higher average fossil fuel use than under "laissez faire", namely 8.7 and 10.5 GtC per year, respectively. This confirms that renewables subsidies suffer, in contrast to the optimal carbon tax, from Green Paradox effects. Global warming is curbed more under the optimal carbon tax (2.2° Celsius) than with renewables subsidies (2.9° Cand 2.5° C) and thus more than under "laissez faire" (3.2° Celsius).

7. Concluding remarks

We have offered an illustrative calibration of a simple model of global warming and of what might be done about it. We suppose that to limit global warming to 2^o Celsius, we must keep the stock of atmospheric carbon below 1 TtC. This corresponds to a ballpark climate sensitivity of 2.55^o C for doubling of the atmospheric carbon stock. Half of emitted carbon stays in the atmosphere forever and the other half returns to the surface of the oceans and the earth. We thus abstract from natural decay of atmospheric carbon and from positive feedback effects in the carbon cycle. We let global warming damages increase steeply when cumulative fossil use increases, which is roughly calibrated from a loss of 4.2% of world GDP at 2.5^o Celsius, a loss of 50% of world GDP at 6^o Celsius and a loss of 99% of world GDP at 12^o Celsius. We calibrate initial fossil fuel reserves, 1.72 TtC, so that the "laissez-faire" outcome of the model corresponds to the current market price of fossil fuel (470 US$/etC). To limit global warming to 2^o C, we have a carbon budget of fossil fuel that we can burn of 0.3 TtC. Fossil fuel and renewables are perfect substitutes in consumption and production. We suppose that fossil fuel extraction costs are currently 58% of the market price and that they vary inversely with the stock of remaining fossil fuel. Renewables are not competitive yet, since we suppose that their costs are 50% higher than the current market price. Finally, we adopt a partial equilibrium framework with a constant growth-corrected world interest rate of 1% per annum in the benchmark.

The first-best policy is to have a carbon tax rising from 311 US$/tC to 338 US$/tC. This amounts to about 140 US$/month per US family or 80 US$/month per UK family.[8] The policy of optimally pricing carbon brings forward the carbon-free era by 74 years and locks up 613 GtC more carbon in the crust of the earth than under "laissez faire". As a result of this and of lower energy use during the fossil fuel phase, global warming is ultimately curbed from 3.17^o Celsius to 2.24^o Celsius. The annuity gain in welfare is 0.72% of world GDP, which seems quite significant. These are, of course, illustrative figures. Introducing a lower equilibrium climate sensitivity, allowing for the substantial lag between global mean temperature and the stock of atmospheric carbon and using Nordhaus-Weitzman instead of Hanemann-Weitzman damages would all reduce the social cost of carbon and the optimal global carbon tax. A more short-sighted policy with a higher interest rate leads to more fossil fuel use, postpones the advent of the carbon-free era and leaves less fossil fuel locked up in the earth, hence leads ultimately to more global warming. The resulting global carbon tax is lower, both in the short and higher in the long run. More intergenerational inequality aversion also means that there is less willingness of current generations to sacrifice consumption for a less

[8] Of course, the revenue from the carbon tax is rebated in lump-sum fashion. Figures are based on an annual footprint of 19 tCO_2 per US family and 11 tCO_2 per UK family (see UK Carbon Trust, www.carbontrust.com). Poorer countries typically have a smaller CO_2 footprint and thus face smaller carbon tax payments.

future global warming. Hence, the optimal global carbon tax is smaller and there is more fossil fuel use, a later introduction of renewables and more global warming.

A renewables subsidy 20% shortens the fossil fuel phase by 36 years and locks up 167 GtC more fossil than under “laissez faire”. This contributes to mitigating global warming, but the subsidy also elicits the market to extract fossil fuel more rapidly which causes acceleration of global warming during the fossil fuel phase. These Green Paradox effects worsen welfare. But with a 20% subsidy the overall effect is a small annuity gain in welfare of 0.07% of world GDP. However, with larger renewables subsidies the combined negative welfare effects from lower utility of fossil fuel consumption and higher global warming damages associated with the Green Paradox increase by more than the potential green welfare gains of bringing the carbon-free era forward and locking up more fossil fuel. For example, with a subsidy of 40% limits global warming to 2.52^o Celsius but nevertheless leads to an annuity welfare loss of 0.02% of world GDP as the Green Paradox effects have started to dominate. Renewables subsidies thus reduce global warming damages in the long run, but they will harm overall global social welfare if they are too large. This is more likely for higher interest rates arising from, say, a higher rate of time preference or more intergenerational inequality aversion. In general, renewables subsidies are not a helpful second-best climate policy. The optimal carbon tax does a better job at flattening the time path for the market price of fossil fuel whilst not lowering the entire price path and thereby causing Green Paradox effects. Of course, renewables subsidies are warranted if there are failures in R&D into renewables or there are learning-by-doing effects in the use of renewables.

Our calibrations and calculations of the effects of the optimal carbon tax and various renewables subsides are purely illustrative and designed to be used in the classroom. They are meant to highlight the various effects at play as clearly as possible in the simplest possible model with endogenous timing of the advent of the carbon-free era and the optimal amount of fossil fuel reserves to leave untapped. More realistic models of the optimal global carbon tax will have to allow for general equilibrium effects and the role of capital markets. For example, Green Paradox effects will typically be mitigated as the interest rate is pushed down as oil producers invest the proceeds from faster resource extraction. More general models need to examine the effects on global warming of an endogenous interest rate under Ramsey growth as in Golosov et al. (2013), van der Ploeg and Withagen (2013), Gerlagh and Liski (2012), Rezai et al. (2012) and Rezai and van der Ploeg (2013) or under endogenous growth with directed technical change as in Acemoglu et al. (2012). They also have to allow for different types of fossil fuel (oil, natural gas, coal and unconventional sources such as shale gas and tar sands) and renewables. Using shale gas and switching from coal to gas gives a window of opportunity to develop cost-effective renewables before global warming becomes intolerable as discussed in van der Ploeg and Withagen (2012b). They must allow the various sources of energy to be imperfect substitutes for the capital-labour aggregate as discussed in Hassler et al. (2011). It is also important to take account of gas being a good substitute for coal in electricity generation and a good substitute for oil in transport as discussed in Helm (2012) and to allow for an upward-sloping supply of renewables. Finally, it is important to allow for different national jurisdictions and consider the conflict and cooperation that might evolve between different types of carbon-emitting countries in a multi-region world as, for example, in Hassler and Krusell (2012) and to allow for the strategic conflict arising from oil importers putting a tariff component in their carbon tax to cream off part of the scarcity rent enjoyed by oil producers and oil producers trying to cream off part of the climate rent of oil importers as discussed in Liski and Tahvonen (2004).

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Appendix: Social optimum and optimal carbon tax trajectory

The social optimum follows from choosing the time paths of F and R to maximize social welfare,

(A1) $$\int_0^\infty \left[U(F+R)-G(S)F-bR-D(E)\right]e^{-rt}dt,$$

subject to

(A2) $$\dot{S}=-F \text{ and } \dot{E}=0.5F.$$

This yields the following optimality conditions:

(A3) $$\left.\begin{matrix} U'(F+R)=A(F+R)^{-1/\varepsilon}\le b \\ R\ge 0 \end{matrix}\right\}\text{c.s.,} \qquad \left.\begin{matrix} U'(F+R)=A(F+R)^{-1/\varepsilon}\le G(S)+\lambda+0.5\mu \\ F\ge 0 \end{matrix}\right\}\text{c.s.,}$$

(A4) $$\dot{\lambda}=r\lambda+G'(S)F,$$

(A5) $\dot{\mu} = r\mu - D'(E),$

where λ is the scarcity rent of an extra TC of fossil fuel in the ground and μ is the marginal cost of having an extra TC in the atmosphere. The optimum has a distinct fossil-fuel phase and a carbon-free phase:

(A6) $\dot{S} = -(A/q)^{\varepsilon}, \quad \dot{q} = r\left[q - G(S)\right] - D'\left(E_0 + 0.5(S_0 - S)\right), \quad \dot{\tau} = r\tau - 0.5D'(E), \quad 0 \le t \le t_F,$

(A7) $F(t) = 0$ and $R(t) = (A/b)^{\varepsilon}, \forall t > t_F.$

where $q = p + \theta$ is the social price of carbon, $\theta = 0.5\,\mu$ is the social cost of carbon, and p is the market price of fossil fuel. The scarcity rent of fossil fuel follows from $\lambda = q - G(S) - \theta = p - G(S).$ The fossil fuel phase is described by a three-dimensional TPBVP with the following boundary conditions:

(A8)
$$S(0) = S_0, \quad q(t_F) = b, \quad \theta(t_F) = \frac{D'\left(E_0 + 0.5(S_0 - S(t_F))\right)}{2r},$$
$$G\left(S(t_F)\right) + \frac{D'\left(E_0 + 0.5(S_0 - S(t_F))\right)}{2r} = b.$$

We use the first three boundary conditions to simulate the TPBPV by nesting a 4^{th}-order Runge-Kutta algorithm within a Gauss-Newton algorithm for ensuring that these boundary conditions are satisfied by varying $q(0)$ and $\tau(0)$. We then determine the switch time t_F so that the simulated value of $S(t_F)$ satisfies the fourth boundary condition. To calculate welfare $W(0)$, we solve:

$$\dot{W} = rW - \left[U(F+R) - G(S)F - bR - D(E)\right], 0 \le t \le t_F, \quad W(t_F) = \frac{U\left(\Psi(b)\right) - b\Psi(b) - D\left(E_0 + 0.5(S_0 - S(t_F))\right)}{r}.$$

In van der Ploeg and Withagen (2012a) and (2012b) we extend this framework to allow for an upward-sloping supply of renewables and for different types of fossil fuel (oil/gas and coal).