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Non-Cooperative and Cooperative Responses to Climate Catastrophes in the Global Economy: A North-South Perspective

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Non-Cooperative and Cooperative Responses to Climate Catastrophes in the Global Economy: A North-South Perspective*

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Abstract

The optimal response to a potential productivity shock which becomes more imminent with global warming is to have carbon taxes to curb the risk of a calamity and to accumulate precautionary capital to facilitate smoothing of consumption. This paper investigates how differences between regions in terms of their vulnerability to climate change and their stage of development affect the cooperative and non-cooperative responses to this aspect of climate change. It is shown that the cooperative response to these stochastic tipping points requires converging carbon taxes for developing and developed regions. The non-cooperative response leads to a bit more precautionary saving and diverging carbon taxes. We illustrate the various outcomes with a simple stylized North-South model of the global economy.

Key words: global warming, tipping point, precautionary capital, growth, risk avoidance, carbon tax, free riding, international cooperation, asymmetries.

JEL codes: D81, H20, O40, Q31, Q38.

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1. Introduction

Climatologists predict that important parts of climate change will be discontinuous and come in shocks, albeit that these shocks typically have a very slow and long onset. Examples are a reversal of the Gulf Stream or disappearance of the ice sheets in Greenland or the Arctic (Lenton and Ciscar, 2013). This implies that damage of climate change will come in shocks as well and that integrated assessment studies cannot only rely on the standard recipe of a carbon tax equal to the present value of all future marginal damages arising from emitting one ton of carbon (e.g., Tol, 2002; Nordhaus, 2008, 2014; Stern, 2007; Golosov et al., 2014). Recently the attention is growing for investigating the consequences of these potential tipping points for climate policy (e.g., Cai et al., 2012; Lemoine and Traeger, 2014; Pindyck, 2013; van der Ploeg and de Zeeuw, 2014). The calamities are modelled as a sudden structural increase in the release of carbon or a shift in the damage function or a sudden shock to productivity. The basic question is how climate policy is affected if there is a probability that such calamities occur.

In a standard growth model where the damage of such a discontinuous event is modelled as a shock to total factor productivity and where the probability that the event occurs increases with global warming, van der Ploeg and de Zeeuw (2014) show that the optimal response is twofold. First, a carbon tax is needed to curb the risk of the climate catastrophe and, second, precautionary capital accumulation is needed to cope with a downward drop in consumption after the calamity. The market may take care of the last response but if not, some form of capital subsidy is needed. They also show that this component of the carbon tax is big as compared to the standard carbon tax, in a setting where both the marginal and non-marginal global warming damages are present.

This paper extends this analysis to a North-South perspective where developed countries are in a further stage of development and less vulnerable to climate catastrophes. A transboundary externality occurs because the greenhouse gas emissions in each region add to the stock of atmospheric carbon and thus to the probability of climate shocks. It is to be expected that a non-cooperative response will lead to lower carbon taxes and therefore to more precautionary saving, although this last effect is not the consequence of a direct externality. Initially, the developing region faces the tradeoff of lowering the carbon tax to spur its development versus increasing the carbon tax to curb the risk of a calamity that will severely hit this region. We will show that the cooperative response is to have initially different carbon taxes in the North and the South that converge later, as long as the calamity has not occurred. However, in the non-cooperative response these carbon taxes in the North and the South are initially comparable but diverge later, as long as the calamity has not occurred.

The standard approach to the problem of international pollution control is to compare cooperative and non-cooperative approaches using convex damage functions of the stock of pollution (Dockner and Long, 1993; van der Ploeg and de Zeeuw, 1992). This paper considers the same transboundary externality but the damage is modelled differently, and more in accordance with what the climatologists predict, which leads to a different type of differential game. An important cost of non-cooperative climate policies is increasing the hazard of a climate calamity. To the extent that this is an irreversible catastrophe, the costs are significant. As a first step, the non-cooperative response is modelled as a so-called open-loop Nash equilibrium in this paper. Although multi-country versions of integrated assessment models have been developed to highlight different incentives for climate policy in the different blocks of countries (e.g., Nordhaus and Boyer, 2000; Hassler and Krusell, 2012), dynamic game theory is typically not used to assess the benefits of cooperation. This paper considers economic growth and climate tipping from a dynamic game perspective but as most of the other studies, still abstracts from international trade, migration and capital flows.

Although international lump-sum transfers can in principle ensure smoothing of consumption and a common carbon tax across the globe (Chichilniski and Heal, 1994), this is hard to achieve in international negotiations. Our approach focuses on internalizing the transboundary externalities of carbon emissions that increase the hazard of a climate catastrophe affecting everyone. Our core results are as follows. In general, the carbon taxes are lower in the absence of cooperation, as is to be expected. Since this gives higher carbon emissions, so that the probability of climate tipping goes up, precautionary saving increases. If the regions cooperate, they aim for a high common carbon tax but allow for a lower carbon tax initially in the developing region, in order to catch up first. If the regions do not cooperate, the developed region always has a low carbon tax because this region is less vulnerable to climate calamities. The developing region, which is more vulnerable to climate calamities, has a low carbon tax initially, giving higher priority to growth, but a high carbon tax later on, giving higher priority to preventing climate tipping.

Section 2 presents our two-region growth model with climate tipping points and discusses the outcomes after tipping has occurred. Section 3 derives the outcome under international climate policy cooperation and a non-cooperative outcome. Section 4 compares cooperative and non-cooperative responses with business-as-usual scenarios using a calibrated North-South model of the global economy. Section 5 concludes.

2. A two-region growth model with climate tipping

For each region, we consider a continuous-time Ramsey growth model with a potential shock to total factor productivity as a consequence of climate tipping. For example, this shock is due to flooding, sudden desertification of land or a sudden increased occurrence of storms and droughts. Production requires the input of fossil fuels and this leads to carbon emissions that accumulate in the atmosphere. The stock of atmospheric carbon increases the hazard of climate tipping which connects the regions. We abstract from other connections such as international trade, migration and capital flows. One region represents the developed countries and this materializes in a higher initial capital stock. The other region represents the developing countries and has a lower initial capital stock. We also assume that the developed region is less vulnerable to climate calamities than the developing region.

The conditional probability that climate tipping occurs at time T is given by the hazard rate $h(t) = \lim_{\Delta t \to 0} \frac{\Pr[T \in (t, t+\Delta t) \,|\, T \notin (0,t)]}{\Delta t}$, so that $h(t)\Delta t$ indicates approximately the probability that the event takes place between t and $t + \Delta t$, given that it has not occurred before t. Defining $h(t) = H\left(P(t)\right)$, $H'(P) > 0$, where P denotes the stock of atmospheric carbon, turns the hazard rate into an endogenous variable and captures the fact that a higher carbon stock increases the probability that the event takes place. More specifically, $1/H(P)$ indicates the average time until climate tipping occurs. The hazard rate model represents a way to capture the idea of tipping points and regime shifts that are frequently observed in natural systems and are characterized by uncertainty and structural changes in the dynamics of the system (e.g., Biggs et al., 2012). Notwithstanding that climate catastrophes often take many decades or even centuries to fully materialize, we assume for illustrative purposes that the full impact of the calamity when it strikes is felt immediately. This can be relaxed (e.g., Cai et al., 2012; van der Ploeg and de Zeeuw, 2014).

Fossil fuel use is denoted by E and has constant marginal cost $d > 0$. The resulting carbon emissions, also denoted by E, accumulate into the stock of atmospheric carbon P. The carbon stock depreciates at a slow rate γ, and only part ψ of the carbon emissions stays up in the atmosphere. In this paper we do not consider substitution possibilities to renewables (van der Ploeg and de Zeeuw, 2014) or a finite supply of fossil fuels (Engström and Gars, 2014). Utility is denoted by U, consumption by C, the production function by $AF(K,E)$, the depreciation rate of capital by $\delta > 0$, and the uniform rate of time preference by $\rho > 0$. For simplicity, we abstract from population growth and technical progress. Total factor productivity A drops to $(1-\pi_i)A, i=1,2,$ after climate tipping, where $0<\pi_i<1$ denotes the vulnerability to climate change in region i. We assume that the developed region, denoted by 1, is less vulnerable to climate change than the developing region, denoted by 2, so that $\pi_1 < \pi_2$.

Formally, social welfare in each of the two regions is defined as the expected present discounted value of the utility of consumption:

(1) $$W_i \equiv \mathrm{E}\left[\int_0^\infty e^{-\rho t}U(C_i(t))dt\right], \quad i=1,2.$$

Capital accumulation in each of the two regions is given by

(2) $$\dot{K}_i(t) = \tilde{A}_i(t)F\left(K_i(t),E_i(t)\right) - dE_i(t) - C_i(t) - \delta K_i(t), \;\; K_i(0)=K_{i0}, \;\; i=1,2,$$

with total factor productivity in each of the two regions given by

(3) $$\tilde{A}_i(t) = A, \quad 0 \le t < T, \qquad \tilde{A}_i(t) = (1-\pi_i)A < A, \quad t \ge T, \quad i=1,2,$$

where the tipping point T is subject to the hazard rate $H(P)$, and the dynamics of the stock of atmospheric carbon is given by

(4) $$\dot{P}(t) = \psi\left(E_1(t)+E_2(t)\right) - \gamma P(t), \quad P(0)=P_0.$$

The maximizations of social welfare (1) with respect to C_i and E_i, subject to (2)–(4), constitute a differential game (Başar and Olsder, 1982) because the carbon emissions in one region affect the carbon stock and thus the hazard rate in both regions. International lump-sum transfers would lead to a uniform carbon tax and ensure consumption smoothing across the globe. These transfers may be desirable from a global welfare perspective but they are hard if not impossible to realise in international negotiations. Therefore, we restrict the cooperative response to agreements that only internalize the emission externality.

It is well known that the non-cooperative Nash equilibrium of a differential game depends on the information structure. Usually two Nash equilibria are considered. If the decision variables C_i and E_i depend on time and the initial state (i.e., the initial stocks K_{0i} and P_0), the open-loop Nash equilibrium results. If these decision variables depend on time t and the current state (i.e., the current stocks $K_i(t)$ and $P(t)$), and if the solution is derived in a dynamic programming framework, the feedback Nash or Markov-perfect equilibrium results. The feedback Nash equilibrium is usually seen as a more realistic equilibrium concept but the solution is very difficult to find for the differential game (1)–(4). However, if we assume that each region takes the emission path of the other region as given, the differential game splits into two dynamic optimization problems and becomes tractable. Consistency of the solutions to these dynamic optimization problems yields the open-loop Nash equilibrium (see the next section).

After climate tipping has occurred, the differential game defined by (1)–(4) breaks down in two separate standard Ramsey growth models with total factor productivity $(1-\pi_i)A, i=1,2$, respectively. In order to analyse the differential game before climate tipping has occurred, we need the optimal social welfare levels for any value of the capital stock. The Hamilton-Jacobi-Bellman (HJB) equations in the value function V^A are

$$(5) \qquad \rho V^A(K_i,\pi_i) = \underset{C_i}{\text{Max}}\left[U(C_i) + V^A_{K_i}(K_i,\pi_i)\left\{Y(K_i,\pi_i) - C_i\right\}\right], \quad i=1,2,$$

where superscript A denotes after-tip values and where the optimal levels of output Y net of fossil-fuel costs and capital depreciation are given by

$$(6) \qquad Y(K_i,\pi_i) \equiv \underset{E_i}{\text{Max}}\left[(1-\pi_i)AF(K_i,E_i) - dE_i - \delta K_i\right], \quad i=1,2.$$

The optimality condition for consumption implies that the marginal utility of consumption equals the marginal value of capital, $U'(C_i)=V_{K_i}^A(K_i,\pi_i)$, which yields the optimal consumption $C^A(K_i,\pi_i)$, so that equations (5) become

$$\rho V^A(K_i,\pi_i)=U\left(C^A(K_i,\pi_i)\right)+U'\left(C^A(K_i,\pi_i)\right)\left[Y(K_i,\pi_i)-C^A(K_i,\pi_i)\right],\quad i=1,2. \tag{7}$$

Using $U'(C_i)=V_{K_i}^A(K_i,\pi_i)$, differentiating equation (7) with respect to K_i yields

$$-V_{K_iK_i}^A(K_i,\pi_i)\left[Y(K_i,\pi_i)-C^A(K_i,\pi_i)\right]=V_{K_i}^A(K_i,\pi_i)\left[Y_{K_i}(K_i,\pi_i)-\rho\right],\quad i=1,2. \tag{8}$$

The left-hand side of equation (8) is the time-derivative of the marginal value of capital (since the term between brackets represents net capital accumulation) and thus the time-derivative of the marginal utility of consumption. Assuming that the utility function U is a power utility function with constant elasticity of intertemporal substitution σ, equation (8) is equivalent to the following set of differential equations in the time domain:

$$\begin{gathered}\dot{C}_i(t)=\sigma\left[Y_{K_i}\left(K_i(t),\pi_i\right)-\rho\right]C_i(t)\ \text{ and}\\ \dot{K}_i(t)=Y\left(K_i(t),\pi_i\right)-C_i(t),\quad K_i(T)=K_{iT},\quad t\geq T,\quad i=1,2,\end{gathered} \tag{9}$$

which are the Keynes-Ramsey rule and the dynamics of the capital stock. The system (9) would also result from applying Pontryagin's maximum principle and rewriting the co-state equation into the Keynes-Ramsey rule for optimal consumption. The reason we have chosen to start with the HJB equations (5) is that this approach proves to be convenient for analysing the problem before climate tipping has occurred (see the next section). Steady-state capital stocks $\bar{K}_i^A(\pi_i)$ follow from the modified golden rules $Y_{K_i}(\bar{K}_i^A,\pi_i)=\rho, i=1,2$. In the numerical simulations in Section 4, we will fix the shocks π_i and calculate the optimal consumption functions $C_i^A(K_i)$ (where the first subscript i refers to the different shocks π_i) as the log-linear approximations to the stable manifolds of the system (9) (see appendix 3). Using (7) we get the optimal social welfare levels $V_i^A(K_i), i=1,2$, as functions of the capital stocks, after climate tipping has occurred.

3. Cooperative and non-cooperative responses

Before climate tipping has occurred, the North and the South have to solve the problems (1)–(4), either in a cooperative or in a non-cooperative setting. After climate tipping has occurred, the externality is not present anymore and the regions are faced with independent Ramsey growth models. The solutions to these problems were derived in the previous section. It follows that the social welfare indicators (1) can be written as

$$(10)\qquad W_i \equiv \mathrm{E}\left[\int_0^T e^{-\rho t} U(C_i(t))dt + e^{-\rho T} V_i^A\left(K_i(T)\right)\right], \quad i = 1, 2.$$

subject to (2)–(4) and the after-tip value functions (7). Since the tipping point is uncertain, these are stochastic dynamic optimization problems. However, the Hamilton-Jacobi-Bellman (HJB) equations for these type of problems are deterministic (Polasky et al., 2011), which simplifies the analysis. This implies that in case of a social planner or cooperation, the analysis can follow the same procedure as in the previous section for the after-tip problems (van der Ploeg and de Zeeuw, 2014).

In the non-cooperative case the problem becomes more complicated. Starting with the set of the HJB equations implicitly assumes that the decision variables C_i and E_i depend on the state of the system (K_1,K_2,P), which would yield the feedback Nash or Markov perfect equilibrium. However, as we will see in the next section, the analysis becomes rather cumbersome. We could switch to numerical methods,[1] but we prefer to focus on the open-loop Nash equilibrium. In that case, the time path of emissions $E_j(t)$ of region j is given and is in fact an exogenous input to the state transition of the optimal control problem of region i, and vice versa. Consequently, the decision variables C_i and E_i in the HJB equations do not depend on K_j anymore. The HJB equations are not stationary in this case but the respective analyses of the optimal policies for the two regions can still follow the same procedure as in the cooperative case. At the end, we find for each region the dynamic system (cf. (9)) that yields the optimal time paths for consumption and for capital and thus for carbon emissions. In fact these systems describe the rational reaction of the policy $\left(C_1(t), E_1(t)\right)$ to $\left(C_2(t), E_2(t)\right)$ and the rational reaction of the

[1] In general, one needs to solve two simultaneous fixed points with a numerical method based on Chebyshev polynomials and the collocation method (e.g., Judd, 1998), as has been done in the industrial organization literature (e.g., Doraszelski, 2003; Saini, 2012; Jaakkola, 2015).

policy $\left(C_2(t), E_2(t)\right)$ to $\left(C_1(t), E_1(t)\right)$. Consistency of the two systems yields the open-loop Nash equilibrium. A disadvantage of the open-loop Nash equilibrium is that not all dynamic strategic effects are into account. For example, both regions will emit more when they know that the other region observes the stock of atmospheric carbon, reacts to a higher stock by decreasing emissions, and in this way partly offsets the higher emissions (van der Ploeg and de Zeeuw, 1992). Therefore, it is to be expected that the open-loop Nash equilibrium under-estimates the level of emissions, as compared to the feedback Nash or Markov perfect equilibrium, but a precise analysis is left for further research. For the purpose of this paper, the open-loop Nash equilibrium is transparent and intuitive, as will be seen in section 3.2, and allows us to draw conclusions on the differences between cooperative and non-cooperative responses to climate tipping.

3.1. International cooperation

When the North and the South cooperate, the social welfare indicator becomes $W \equiv W_1 + W_2$ where W_1 and W_2 are given in (10). Since the hazard rate $H(P)$ of a climate catastrophe depends on the stock of atmospheric carbon P, the value function $V^C(K_1, K_2, P)$ (with superscript C denoting cooperation) is a function of the capital stocks K_1 and K_2 and the global carbon stock P. Furthermore, it is easy to show that in this problem the value function can be split in separated value functions for the two regions: $V^C(K_1, K_2, P) = V_1^C(K_1, P) + V_2^C(K_2, P)$. Due to the hazard rate *H(P)*, the HJB equation includes an extra term that captures the expected capitalized losses from a climate catastrophe (van der Ploeg and de Zeeuw, 2014) and becomes

$$\begin{aligned}(11)\qquad \rho\sum_{i=1}^{2} V_i^C(K_i, P) = \underset{C_1, C_2, E_1, E_2}{\text{Max}} \sum_{i=1}^{2} \Big\{ U(C_i) + V_{iK_i}^C(K_i, P)\left[AF(K_i, E_i) - dE_i - C_i - \delta K_i\right] + \\ + V_{iP}^C(K_i, P)\left(\psi \sum_{j=1}^{2} E_j - \gamma P\right) - H(P)\left[V_i^C(K_i, P) - V_i^A(K_i)\right] \Big\}\end{aligned}$$

with the optimality conditions

$$(12)\qquad U'(C_i) = V_{iK_i}^C, \quad AF_{E_i}(K_i, E_i) = d + \tau_i^C \;\text{ and }\; \tau_i^C \equiv \psi \frac{-V_{1P}^C - V_{2P}^C}{V_{iK_i}^C}, \quad i = 1, 2,$$

where τ_i^C is region *i*'s *cooperative social cost of carbon*.

With τ_i^C as carbon taxes or additional costs of fossil-fuel input, we define Y^C as the optimal output levels net of all costs and capital depreciation:

$$(13)\qquad Y^C(K_i,\tau_i^C) = \underset{E_i}{\text{Max}}\left[AF(K_i,E_i) - (d+\tau_i^C)E_i - \delta K_i \right],\quad i=1,2.$$

Differentiating (11) with respect to K_i and P, using (12)–(13) and using equality of cross-derivatives, yields a set of differential equations for the first-order derivatives of V_i^C as functions of time (Pontryagin conditions). This leads to (omitting the dependence on time t):

$$(14)\qquad \begin{aligned} &-\dot{V}_{iK_i}^C = \left[Y_{K_i}^C(K_i,\tau_i^C) - \rho - H(P) \right] V_{iK_i}^C + H(P) V_{iK_i}^A(K_i) \text{ and} \\ &\dot{V}_{iP}^C = \left[\rho + \gamma + H(P) \right] V_{iP}^C + H'(P)\left[V_i^C - V_i^A(K_i) \right],\quad i=1,2, \end{aligned}$$

where $\dot{V}_{iK_i}^C = V_{iK_iK_i}^C \dot{K}_i + V_{iK_iP}^C \dot{P}$ and $\dot{V}_{iP}^C = V_{iPK_i}^C \dot{K}_i + V_{iPP}^C \dot{P}$. From the first part of (14), using (12), we get the Keynes-Ramsey rules:

$$(15)\qquad \dot{C}_i^C = \sigma\left[Y_{K_i}^C(K_i^C,\tau_i^C) - \rho + \theta_i^C \right] C_i^C \text{ with } \theta_i^C \equiv H(P^C)\left[\frac{V_{iK_i}^A(K_i^C)}{U'(C_i^C)} - 1 \right],\quad i=1,2,$$

where θ_i^C are the precautionary returns on capital accumulation (Smulders et al., 2014, van der Ploeg and de Zeeuw, 2014). The growth rate of aggregate consumption is thus proportional to the marginal net product of capital minus the pure rate of time preference plus the precautionary return. This leads to precautionary saving with the purpose to mitigate a downward drop in consumption after the calamity and to smooth consumption over time. The terms between brackets in the expressions for the precautionary returns θ_i^C in (15) are positive, because the marginal utility of consumption is larger after the calamity than before the calamity (note that the marginal utility of consumption equals the marginal value of capital). Since the drop in consumption in the South is larger than in the North, the upward jump in the marginal utility of consumption in the South is larger. It follows that the precautionary return in the South is larger than in the North.

Using (12)–(14), we get the dynamics for the cooperative social costs of carbon:

(16) $$\dot{\tau}_i^C = r_i^C \tau_i^C - \psi H'(P^C)\left(\frac{\sum_{j=1}^{2}\left[V_j^C(K_j^C, P^C) - V_j^A(K_j^C) \right]}{U'(C_i^C)} \right), \quad i = 1, 2.$$

The cooperative social costs of carbon are the present discounted values of all future marginal damages of a climate calamity from burning one ton of carbon fuel today. It is the *marginal* hazard rate times the *non-marginal* future catastrophic damage that gives effectively a marginal future damage from burning one unit of carbon today. The discount rate $r_i^C \equiv Y_{K_i}^C(K_i^C, \tau_i^C) + \gamma + H(P^C) + \theta_i^C$ is the sum of the interest rate, the rate of carbon decay in the atmosphere, the hazard rate, and the precautionary return. It follows that under international cooperation, the social costs of carbon are the present discounted values of the global sum of expected non-marginal damages from a calamity:

(17) $$\tau_i^C(t) = \int_t^\infty e^{-\int_t^s r_i^C(u)du} \psi H'(P^C(s)) \frac{\sum_{j=1}^{2}\left\{ V_j^C(K_j^C(s), P^C(s)) - V_j^A(K_j^C(s)) \right\}}{U'(C_i^C(s))} ds, \quad i = 1, 2.$$

Hence, the carbon taxes are large if the drops in future welfare from climate calamities and the marginal hazard rate are large. Note that the marginal hazard rate pushes up the carbon tax, whereas the level of the hazard rate depresses it via the higher discount rate.

It is difficult to generally characterize the interplay between precautionary savings and the carbon taxes but we will show what happens in a numerical simulation of a calibrated model for the world economy in the next section. However, ceteris paribus, the carbon taxes τ_i^C for the two regions, given by (17), will converge in the long run because the regions are concerned about the *joint* welfare loss when climate tipping occurs. In the short run these taxes will be different. The developing region starts with a lower capital stock and a higher marginal product of capital and therefore uses a higher discount rate. Furthermore, the developing region has a lower level of consumption in the beginning and therefore a higher marginal utility of consumption. As can be seen from (17), both these effects imply that the South initially has a lower carbon tax than the North.

The cooperative response to potential climate tipping is described by the set of differential equations consisting of the equations for the carbon taxes (16), the Keynes-Ramsey rules (15), the equations for capital accumulation

$$(2') \qquad \dot{K}_i^C = Y^C(K_i^C, \tau_i^C) + \tau_i^C E_i^C - C_i^C, \quad K_i^C(0) = K_{i0}, \quad i = 1, 2,$$

and the equation for the accumulation of the stock of atmospheric carbon

$$(4') \qquad \dot{P}^C = \psi\left(E_1^C + E_2^C\right) - \gamma P^C, \quad P^C(0) = P_0, \qquad 0 \le t \le T.$$

Given the expression for the optimal output levels Y^C in (13), we can write the optimal emission rates as $E_i^C = -Y_{\tau_i^C}^C(K_i^C, \tau_i^C), i = 1, 2$. The second terms in the right-hand side of the capital dynamics in (2′) are the lump-sum rebates of the tax revenues if the social costs of carbon are implemented as a carbon tax in a decentralized market economy.

The resulting system (2′), (4′), (15) and (16) is a boundary-value problem, with initial conditions for the capital stocks and the carbon stock and transversality conditions for the consumption levels and the carbon taxes. The steady state is a target steady state before the calamity has occurred. After the calamity has occurred, the system moves to the steady states of the after-tip growth models. The target steady states for the capital stocks $\bar{K}_i^C$ follow from the modified golden rules $Y_{K_i}^C(\bar{K}_i^C, \bar{\tau}_i^C) = \rho - \bar{\theta}_i^C, i = 1, 2$, and the target steady states for the carbon taxes follow from (16), using these modified golden rules:

$$(18) \qquad \bar{\tau}_i^C = \frac{\psi H'(\bar{P}^C)\sum_{j=1}^{2}\left[U(\bar{C}_j^C) - \rho V_j^A(\bar{K}_j^C)\right]}{\left[\rho + \gamma + H(\bar{P}^C)\right]\left[\rho + H(\bar{P}^C)\right]U'(\bar{C}_i^C)}, \quad i = 1, 2.$$

The other target steady states $\bar{C}_i^C$ and $\bar{P}^C$ follow immediately, using (2′) and (4′).

3.2. Non-cooperative Nash equilibrium

When the North and the South do not cooperate, the respective social welfare indicators are W_1 and W_2, given by (10). The two regions do not internalize the transboundary externalities arising from the carbon emissions E_i. However, as we have argued above, for reasons of tractability, we assume that each region takes the time path of carbon emissions $E_j(t)$ of the other region as given. Each region therefore has to solve an optimal control problem with value function $V_i^N(K_i, P, t), i = 1, 2$ (with superscript N denoting non-cooperation). We start with the HJB equations for the respective optimal control problems but we derive the solutions in the time domain. Consistency of these solutions yields the open-loop Nash equilibrium. The HJB equations become

$$(19)\quad \begin{aligned} &\rho V_i^N - V_{it}^N = \underset{C_i, E_i}{\text{Max}} \left\{ U(C_i) + V_{iK_i}^N \left[AF(K_i, E_i) - dE_i - C_i - \delta K_i \right] \right. \\ &\left. + V_{iP}^N \left(\psi E_i + \psi E_j(t) - \gamma P \right) - H(P) \left[V_i^N - V_i^A(K_i) \right] \right\}, \quad i = 1, 2, \end{aligned}$$

with the Nash equilibrium conditions for optimal consumption and energy use:

$$(20)\quad U'(C_i) = V_{iK_i}^N, \quad AF_{E_i}(K_i, E_i) = d + \tau_i^N \text{ with } \tau_i^N \equiv \psi \frac{-V_{iP}^N}{V_{iK_i}^N}, \quad i = 1, 2,$$

where τ_i^N is region i's *non-cooperative social cost of carbon*. These social costs of carbon are lower than in the case of cooperation because the global warming externalities are not internalized. They do internalize the domestic parts of the global warming externalities. Under business as usual the global warming externalities are not internalized at all.

Since E_j in the HJB equation (19) for V_i^N only depends on time t, it follows that V_i^N, C_i and E_i only depend on (K_i,P,t). If all these variables would depend on the state of the system (K_1,K_2,P), in order to derive the feedback Nash or Markov perfect equilibrium, the remaining steps become much more difficult. Following the same steps as in section 3.1, we get the Pontryagin conditions (omitting the dependence on time t):

$$(21)\quad \begin{aligned} &-\dot{V}_{iK_i}^N = \left[Y_{K_i}^N(K_i, \tau_i^N) - \rho - H(P) \right] V_{iK_i}^N + H(P) V_{iK_i}^A(K_i) \text{ and} \\ &\dot{V}_{iP}^N = \left[\rho + \gamma + H(P) \right] V_{iP}^N + H'(P) \left[V_i^N - V_i^A(K_i) \right], \quad i = 1, 2, \end{aligned}$$

where $\dot{V}_{iK_i}^N = V_{iK_it}^N + V_{iK_iK_i}^N \dot{K}_i + V_{iK_iP}^N \dot{P}$ and $\dot{V}_{iP}^N = V_{iPt}^N + V_{iPK_i}^N \dot{K}_i + V_{iPP}^N \dot{P}$. Equations (21) lead to the set of differential equations describing the non-cooperative response to a potential climate tipping point. The system consists of the Keynes-Ramsey rules

$$(22)\quad \dot{C}_i^N = \sigma \left[Y_{K_i}^N(K_i^N, \tau_i^N) - \rho + \theta_i^N \right] C_i^N \text{ with } \theta_i^N \equiv H(P^N) \left[\frac{V_{iK_i}^A(K_i^N)}{U'(C_i^N)} - 1 \right], \; i = 1, 2,$$

the dynamics for the non-cooperative social costs of carbon

(23) $\dot{\tau}_i^N = r_i^N \tau_i^N - \psi H'(P^N)\left[\frac{V_i^N(K_i^N, P^N) - V_i^A(K_i^N)}{U'(C_i^N)}\right], \quad i = 1, 2,$

where $r_i^N \equiv Y_{K_i}^N(K_i^N, \tau_i^N) + \gamma + H(P^N) + \theta_i^N$, the equations for capital accumulation

(2″) $\dot{K}_i^N = Y^N(K_i^N, \tau_i^N) + \tau_i^N E_i^N - C_i^N, \quad K_i^N(0) = K_{i0}, \quad i = 1, 2,$

and the equation for the accumulation of the stock of atmospheric carbon

(4″) $\dot{P}^N = \psi\left(E_1^N + E_2^N\right) - \gamma P^N, \quad P^N(0) = P_0, \qquad 0 \le t \le T,$

where fossil fuel uses in the two regions are given by $E_i^N = -Y_{\tau_i^N}^N(K_i^N, \tau_i^N), i = 1, 2$.

As in the case of international cooperation, the precautionary return (see (22)) in the South is larger than in the North, because the drop in consumption in the South is larger which leads to a larger upward jump in the marginal utility of consumption. The precautionary returns depend on the hazard rate and thus on the stock of atmospheric carbon. In the absence of international cooperation, this stock will be higher and therefore the precautionary returns will be pushed up. However, the precautionary returns also depend on the capital stocks and on consumption. We will illustrate the total effect in a numerical simulation of a calibrated model for the world economy in the next section.

It follows from (23) that the non-cooperative social costs of carbon become

(24) $\tau_i^N(t) = \int_t^\infty e^{-\int_t^s r_i^N(u)du} \psi H'(P^N(s)) \frac{V_i^N(K_i^N(s), P^N(s)) - V_i^A(K_i^N(s))}{U'(C_i^N(s))} ds, \quad i = 1, 2.$

The conclusions in the non-cooperative case are opposite to the conclusions in the cooperative case from the previous sub-section. Ceteris paribus, the carbon taxes τ_i^N for the two regions will diverge in the long run because the regions are only concerned about their *own* welfare loss when climate tipping occurs. The carbon taxes in the South will then be higher than the carbon taxes in the North. However, in the short run these taxes will be closer together, since the South still has an incentive to lower the taxes initially. The developing region starts with a lower capital stock and therefore has a higher marginal product of capital and a higher marginal utility of consumption in the beginning.

The resulting system (2″)–(4″)–(22)–(23) is a boundary-value problem, with initial conditions for the capital stocks and the carbon stock and transversality conditions for the consumption levels and the carbon taxes. The target steady states follow in the same way as under international cooperation, given in the previous sub-section. The differences are driven by the target steady states for the carbon taxes, which become

$$(25) \qquad \bar{\tau}_i^N = \frac{\psi H'(\bar{P}^N)\left[U(\bar{C}_i^N) - \rho V_i^A(\bar{K}_i^N)\right]}{\left[\rho + \gamma + H(\bar{P}^N)\right]\left[\rho + H(\bar{P}^N)\right]U'(\bar{C}_i^N)}, \quad i = 1, 2.$$

Comparing equations (25) and (18), it is clear that total carbon taxes in the steady state will be lower in the absence of cooperation, increasing the carbon stock and the hazard rate and in this way pushing up the precautionary returns and the capital stocks. Changes in the capital stocks and the consumption levels affect the carbon taxes and the precautionary returns and it is not so clear how these general equilibrium effects will eventually work out. In the next section we quantify these effects in a stylized calibrated two-region model of growth and development for the global economy.

4. Illustrative calculations for a North-South model of the global economy

In this section we present a numerical illustration. We use a simple stylized North-South model of the global economy and base our calibration on data from the BP Statistical Review and the World Bank Development Indicators (see appendix 1). Suitable expressions for the crucial variables follow when using a CES utility function and a Cobb-Douglas production function (also see appendix 1). Region 1 is the developed region (the "North") and starts out with an initial capital stock that is 9 times larger than the initial capital stock of region 2 (the "South"). We assume the same total factor productivity and a relatively high capital share. This captures that institutions evolve over time as the economy develops and institutional quality is subsumed in the capital stock of each region. Human capital to the extent that it develops by investments is also included in this broad measure of capital. The other key asymmetry is that we assume catastrophic drops of 10 and 30 percent in total factor productivity for, respectively, the North and South, i.e., π_1 = 0.1 and π_2 = 0.3. We suppose that at the initial carbon stock, P_0 = 826 GtC (or 388 ppm by vol. CO_2), the hazard rate is $H(826)$ = 0.02 and that it rises linearly to 0.033 if the carbon stock rises to 1652 GtC. This gives $H(P) = 0.02 + 1.614 \times 10^{-5}(P - 826)$. It means that we assume that if the carbon stock doubles, the average time it takes for tipping to occur drops from 50 to 30 years. With a climate sensitivity of 3, doubling of the carbon stock induces an additional 3 degrees Celsius.[2]

We first discuss the target steady-state outcomes and then the transient paths of the cooperative, non-cooperative and business-as-usual (BAU) scenarios.

4.1. Steady states

The business-as-usual scenario has zero carbon taxes and zero precautionary savings ($\tau_1 = \tau_2 = \theta_1 = \theta_2 = 0$) and is the same as the after-tip system (9) but without the shocks π_i to the total factor productivity. These systems give rise to the steady states reported in table 1 (see appendix 2 for the functional forms of the steady states). As a result of climate tipping, we see that the capital stock, economic activity and consumption are lower, especially in the South where the calamity strikes the hardest. Furthermore, the carbon emissions decrease and the carbon stock is therefore considerable lower after the climate tipping (1414 instead of 1992 GtC). Consequently, global warming is lower after the tip (3.7 instead of 5.2 degrees Celsius). Table 1 also reports the target steady states in case potential climate tipping is taken into account, comparing the cooperative and a non-cooperative response. We make the following observations.

First, because the calamity will strike the hardest in the South, precautionary savings and the targeted capital stocks before tipping are considerably higher in the South than in the North. The precautionary returns are a little bit higher in the absence of international cooperation (0.47% instead of 0.46% per annum in the South and 1.54% instead of 1.50% per annum in the North). The reason is that lower carbon taxes in the non-cooperative outcome increase the probability of climate tipping and thus some additional precautionary saving occurs. However, this effect is small.

Second, in case of international cooperation, both regions aim for a carbon tax of around \$80 per ton of carbon emitted (\$80.0 in the North and \$81.7 in the South). Because the regions cooperate, they take account of the potential *joint* welfare loss and thus have

[2] We use $3\ln(P_t / 596.4)/\ln(2)$ for the temperature compared to pre-industrial temperature.

comparable social costs of carbon in the long run. In the absence of cooperation, however, the regions are on their own and their carbon taxes differ considerably ($14.8 in the North and $58.5 in the South), since the South will be hit the hardest by the calamity. Moreover, the carbon taxes are considerably lower than in the cooperative case, because the climate externality is not internalized. As a result, the carbon stock will become much higher (2005 instead of 1836 GtC) and there will be more global warming (5.3 instead of 4.9 degrees Celsius) which brings forward the expected time of the climate calamity.

Table 1: Before- and after-catastrophe steady states

	Steady states		Target steady states (before tipping)	
	Business-as-usual	After tipping	Cooperative	Non-cooperative
$\bar{K}_1$ (T$)	226.8	192.3	249.6	252.7
$\bar{K}_2$ (T$)	226.8	129.6	330.9	336.4
$\bar{C}_1$ (T$)	34.03	28.85	34.27	34.33
$\bar{C}_2$ (T$)	34.03	19.45	34.62	34.63
$\overline{GDP}_1$ (T$)	45.37	38.46	46.33	46.88
$\overline{GDP}_2$ (T$)	45.37	25.93	50.69	51.09
$\bar{P}$ (GtC)	1992	1414	1836	2005
Temp (Celsius)	5.22	3.74	4.87	5.25
$\bar{\theta}_1$ (%/year)	0	0	0.46	0.47
$\bar{\theta}_2$ (%/year)	0	0	1.50	1.54
$\bar{\tau}_1$ ($/tC)	0	0	80.0	14.8
$\bar{\tau}_2$ ($/tC)	0	0	81.7	58.5

Finally, economic activity and thus consumption and fossil use are in the long run a bit higher in the South, due to more precautionary capital accumulation. This also explains why, in the cooperative case, the carbon tax in the South is slightly larger than in the North, because the marginal utility of consumption is a bit lower in the South (cf. (17)). In the short run, however, economic activity and consumption will be lower in the

South, because this region needs to catch up. The carbon tax will initially be lower in the South as well, for the same reason. We will illustrate this by presenting the transient dynamics towards the target steady states in the next sub-section.

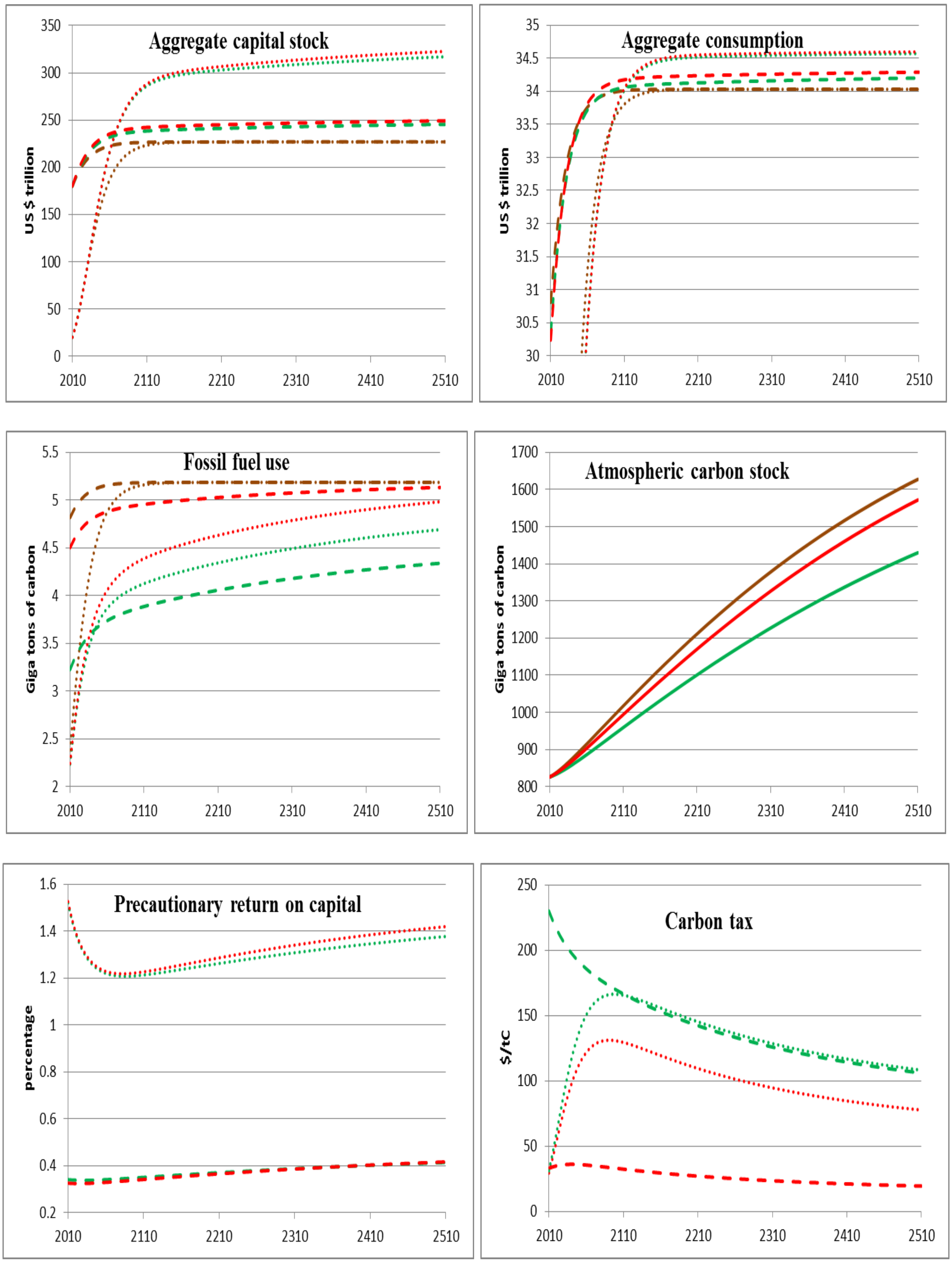
4.2. Transient dynamics

Focusing on the steady states of the systems for business-as-usual, cooperation and non-cooperation can be misleading. These steady states are not affected by the different stages of development in the North and the South, but we have $K_1(0) > K_2(0)$. Therefore we discuss here the transient dynamics. The four-dimensional boundary-value problems (2′)–(4′)–(15)–(16), in case of cooperation, and (2″)–(4″)–(22)–(23), in the absence of cooperation, are not easy to solve. We log-linearize these two systems around the steady states and determine the stable sub-systems. In this way, we find suitable approximations of the stable before-catastrophe manifolds as discussed in more detail in appendix 4. We use the calibration details discussed in appendix 1. In Figure 1, we show the time paths for the capital stocks, consumption, fossil-fuel use, the carbon stock, the precautionary returns and the carbon taxes. The brown lines denote business-as-usual, the green lines denote the cooperative outcome, and the red lines denote the non-cooperative Nash equilibrium. Dashed lines indicate outcomes for the North and dotted lines for the South. We make the following observations.

First, the South has to catch up with the North, so that consumption is in the beginning substantially lower in the South. If climate tipping does not occur for a long time, the consumption levels converge close together in all cases. In the long run, the business-as-usual, cooperative and non-cooperative outcomes lead to approximately the same levels of consumption.

Second, the capital stock in the South starts low but if climate tipping does not occur for a long time, the capital stock in the South will eventually be larger than the capital stock in the North. After all, the South has to prepare for a larger climate catastrophe. Both capital stocks are always higher than in the business-as-usual scenario if climate tipping can occur. In the non-cooperative outcome, the capital stocks in both the North and the South are slightly larger than under cooperation. The reason is that the hazard rate is a bit higher under non-cooperation, because the stock of atmospheric carbon is a bit higher, and thus the precautionary savings must rise.

Figure 1: Pre-tip simulations



Key: green = cooperative outcome; red = non-cooperative outcome; brown = business as usual; solid = global; dashed = North; dotted = South. For example, cooperative and non-cooperative outcomes for the South's capital stock are indicated by green and red dotted lines, respectively. The cooperative outcomes for aggregate consumption for North and South are indicated by the dashed and dotted green lines, respectively.

Third, the carbon taxes are, of course, higher in the case of cooperation than in case that the North and the South fail to cooperate. In the previous sub-section, we have already seen that the carbon taxes under cooperation in the North and the South will converge in the long run, if tipping does not occur for a long time. However, in the short run the carbon taxes in the North are much higher than in the South. In the previous sub-section, we have seen that the carbon taxes in the non-cooperative outcome in the North and the South will be different in the long run, if tipping does not occur for a long time. However, in the short run the figure 1 indicates that the carbon taxes in the North and the South start at the same level. Hence, in the cooperative outcome the carbon taxes in the North and the South are initially different but converge in the long run, whereas in the non-cooperative outcome the taxes are initially similar but diverge in the long run.

Finally, convergence of the stock of atmospheric carbon and the global mean temperature is very slow. It is clear, however, that the use of fossil fuel and the stock of atmospheric carbon are the highest when business is as usual and the lowest when the North and the South cooperate.

5. Conclusion

The future costs of global warming may to a large extent result from climate tipping points (Lenton and Ciscar, 2013). The catastrophic damages as such are an important driver of the carbon taxes. Another important driver is how much more imminent the climate tipping point becomes with global warming. The most striking result in analysing these climate tipping points is that both precautionary capital accumulation and a carbon tax are needed. More capital is required to prepare for the shock and to smooth consumption over time, and less fossil-fuel use is required to curb the risk of climate tipping. In this paper we consider a North-South model where the North is less vulnerable to climate catastrophes than the South.

Despite the difficulties in establishing international cooperation, countries may be willing to aim for internalizing the transboundary externalities of carbon emissions leading to a higher carbon stock and therefore a higher hazard of a climate catastrophe. In this case, carbon taxes will converge in the long run, if climate tipping does not occur for a long time, but will initially be higher in the developed than in the developing parts of the world economy, as poorer countries need to catch up with richer countries. If countries do not cooperate on their climate policies, carbon taxes are of course lower. Furthermore, these taxes diverge instead of converge in the long run, but start at the same level. Consequently, the use of fossil fuel and thus the carbon stock and the hazard rate are higher, so that global warming becomes more severe. It follows that precautionary savings are higher if the countries do not cooperate, although this effect proves to be small. Moreover, because the South is more vulnerable to climate calamities, it needs to engage much more in precautionary capital accumulation than the North.

We envisage a number of future directions for research. First, following van der Ploeg and de Zeeuw (2014) one can allow for more conventional, marginal global warming damages in addition to the damages resulting from stochastic tipping points. Second, one can allow for catastrophic shocks in the carbon cycle as in van der Ploeg (2014). Third, one can allow for the scarcity of fossil fuel and the scarcity rents that this implies. Engström and Gars (2014) have investigated this issue and the Green Paradox effects that might result in the tractable discrete-time model of global growth and climate change developed by Golosov et al. (2014). It is important to see how this plays out in a multi-region world. Fourth, our two-region model of the global economy is highly stylized and designed for illustrative purposes. It is important to study the effects of technological development, international trade, capital mobility and mitigation options, for example. Fifth, one might try to solve for a feedback Nash or Markov perfect equilibrium and see what this implies for the gains of international policy cooperation. Finally, one might investigate the international political economy of dealing with climate tipping points when different parts of the globe are affected by global warming in a different way.

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Appendix 1: Calibration and functional forms

Table 1 summarizes our calibration.

The utility functions $U(C_i) = C_i^{1-1/\sigma} / (1-1/\sigma), i = 1,2$, have a constant elasticity of intertemporal substitution of σ = 0.5 (implying a constant coefficient of intergenerational inequality aversion of 2) and a pure rate of time preference of ρ = 0.014. The Cobb-Douglas production functions $F(K_i, E_i) = K_i^{\alpha} E_i^{\beta}, i = 1,2$, have a capital share of α = 0.3 and an energy share of β = 0.0623. The depreciation rate is set to δ = 0.05.

We calibrate to the business-as-usual (i.e., negligible carbon taxes and no precautionary capital accumulation) outcome for the world economy for the year 2010. Data sources are the BP Statistical Review and the World Bank Development Indicators.

Table A1: Calibration of the two-region model with a tipping point

Variable/parameter	
Pure rate of time preference, ρ	0.014
Elasticity of substitution, σ	0.5
Depreciation rate of capital, δ	0.05
Share of capital in value added, α	0.3
Share of fossil fuel in value added, β	0.0623
Total factor productivity, A	8.5044
Eventual climate shock, π_i	0.1, 0.3
Fraction of carbon staying up, ψ	0.5
Natural decay of carbon, γ	0.003
Cost of fossil fuel, d	9 US \$/million BTU = 504.3 US \$/tC
2010 levels of GDP	42.1 and 20.9 trillion US \$
2010 levels of capital, K_{i0}	180 and 20 trillion US \$
2010 fossil fuel use, E_{i0}	5.551 and 2.749 GtC
Initial stock of carbon, P_0	826 GtC = 388 ppm by vol. CO2

The initial 2010 capital stocks are set to $K_1(0) = 180$ and $K_2(0) = 20$ trillion US dollars, and the 2010 level of world GDP is 63 trillion US dollars. We measure fossil fuel in GtC, so the emission-input ratio equals one. We use a market price for fossil fuel of d = 504.3 US\$ per ton of carbon (or 9 US\$ per million BTU). Global fossil fuel use in 2010 is 8.3 GtC (or 468.3 million GBTU). Using $\frac{E_1(0)}{E_2(0)} = \left(\frac{K_1(0)}{K_2(0)}\right)^{\frac{\alpha}{1-\beta}} = 9^{\frac{0.3}{1-0.0623}} = 2.02$, we get $E_1(0) = 5.551$ and $E_2(0) = 2.749$ GtC in 2010. We thus have 42.1 and 20.9 trillion US dollars for GDP in the developed and developing part of the global economy. The level of total factor productivity that matches these levels of output and inputs in both regions is $A = 8.5044$.

The initial capital stocks of 180 and 20 trillion US dollars are below the steady-state levels of 211 and 104 trillion US dollars to reflect that the developed region is still catching up. The fraction of carbon staying in the atmosphere is set equal to $\psi = 0.5$ and the rate of decay of atmospheric carbon is set equal to $\gamma = 0.003$.

Appendix 2: Before- and after-catastrophe steady states

Our specification with CES utility and Cobb-Douglas production functions yields tractable forms for the crucial variables in the analysis. Combining (3), (6), (13) and the equivalent expression for the non-cooperative case, it follows that optimal fossil-fuel use is generally given by $E_i = \beta \tilde{A}_i F / (d + \tau_i)$, so that output net of fossil fuel costs and capital depreciation becomes

$$\text{(A1)} \quad Y_i(K_i, \tau_i) = (1-\beta)\left[\tilde{A}_i \left(\frac{\beta}{d+\tau_i}\right)^{\beta}\right]^{\frac{1}{1-\beta}} K_i^{\frac{\alpha}{1-\beta}} - \delta K_i, \quad i = 1, 2.$$

The modified golden rules $Y_{iK_i}(\bar{K}_i, \bar{\tau}_i) = \rho - \bar{\theta}_i,\ i = 1, 2$, yield

$$\text{(A2)} \quad \bar{K}_i = \frac{\alpha}{\rho + \delta - \bar{\theta}_i} f_i(\bar{K}_i, \bar{\tau}_i), \quad f_i(K_i, \tau_i) \equiv \left[\tilde{A}_i \left(\frac{\beta}{d+\tau_i}\right)^{\beta}\right]^{\frac{1}{1-\beta}} K_i^{\frac{\alpha}{1-\beta}}, \quad i = 1, 2,$$

so that the (target) steady-state capital stocks become

$$\text{(A3)} \quad \bar{K}_i = \left[\tilde{A}_i \left(\frac{\alpha}{\rho + \delta - \bar{\theta}_i}\right)^{1-\beta} \left(\frac{\beta}{d+\bar{\tau}_i}\right)^{\beta}\right]^{\frac{1}{1-\alpha-\beta}}, \quad i = 1, 2.$$

Since $\bar{E}_i = \beta f_i(\bar{K}_i, \bar{\tau}_i) / (d + \bar{\tau})$, the other (target) steady states are given by

$$\bar{P} = \sum_{j=1}^{2}\left[\frac{\psi}{\gamma}\left(\frac{\beta}{d+\bar{\tau}_j}\right)\left(\frac{\rho+\delta-\bar{\theta}_j}{\alpha}\right)\bar{K}_j\right], \qquad \bar{\theta}_i = H(\bar{P})\left[\frac{\bar{C}_i^{1/\sigma}}{C_i^A(\bar{K}_i)^{1/\sigma}} - 1\right],$$

$$\text{(A4)} \qquad \bar{C}_i = \left[\left(1 - \beta + \frac{\beta \bar{\tau}_i}{d + \bar{\tau}_i}\right)\left(\frac{\rho + \delta - \bar{\theta}_i}{\alpha}\right) - \delta\right]\bar{K}_i,$$

$$\bar{\tau}_i = \frac{\psi H'(\bar{P}) \bar{C}_i^{1/\sigma} \sum_{j=1}^{2}\left[\frac{\bar{C}_j^{\,1-1/\sigma}}{1-1/\sigma} - \rho V_j^A(\bar{K}_j)\right]}{\left[\rho + \gamma + H(\bar{P})\right]\left[\rho + H(\bar{P})\right]}, \quad i = 1, 2,$$

where in the non-cooperative case the summation in the expression for the carbon tax (the last steady state) reduces to only the $j = i$ term.

After tipping where $\bar{\theta}_i = \bar{\tau}_i = 0,\ i = 1, 2$, and $\tilde{A}_i = (1 - \pi_i) A,\ i = 1, 2$, this reduces to

(A5)
$$\bar{K}_i^A = \left[(1-\pi_i) A \left(\frac{\alpha}{\rho+\delta} \right)^{1-\beta} \left(\frac{\beta}{d} \right)^{\beta} \right]^{\frac{1}{1-\alpha-\beta}},$$
$$\bar{C}_i^A = \left[(1-\beta)\left(\frac{\rho+\delta}{\alpha} \right) - \delta \right] \bar{K}_i^A, \quad i = 1, 2.$$

The figures for the before- and after-catastrophe steady states corresponding to the calibration of appendix 1 are reported in table 1.

Appendix 3: Approximation to the after-catastrophe stable manifold

The solution trajectories for the after-catastrophe outcomes are well approximated by a log-linear approximation of the stable manifold relating aggregate consumption to only the aggregate capital stock in each country. The reason for not having to relate the manifold to the atmospheric carbon stock is that, with our specification of one-off catastrophic damages, changes in the degree of global warming do not affect the after-catastrophe value functions and consumption manifolds. Furthermore, since the North and the South are isolated after the catastrophe, the cooperative and non-cooperative outcomes coincide. It is relatively easy to calculate the optimal consumption functions $C_i^A(K_i)$ as the log-linear approximations to the stable manifolds of the after-catastrophe system (9) because we can use l'Hôpital's rule to determine the slopes $\dot{C}_i / \dot{K}_i$ of the stable manifolds in these steady states:

(A6)
$$C_{iK_i}^A(\bar{K}_i^A) = \lim_{K_i \to \bar{K}_i^A} \frac{\sigma\left[Y_{iK_i}(K_i) - \rho \right] C_{iK_i}^A(K_i) + \sigma Y_{iK_iK_i}(K_i) C_i^A(K_i)}{Y_{iK_i}(K_i) - C_{iK_i}^A(K_i)}, \quad i = 1, 2.$$

This yields the simple quadratic equation

(A7)
$$\left(C_{iK}^A(\bar{K}_i^A) \right)^2 - \rho C_{iK}^A(\bar{K}_i^A) + \sigma Y_{iK_iK_i}(\bar{K}_i^A) \bar{C}_i^A = 0, \quad i = 1, 2.$$

The positive solutions to (A7) exceed the value $\rho > 0$ and are the slopes of the stable manifolds in the steady states. Using logarithmic differentiation of the state equations, we thus obtain the following expressions for the approximations of the after-catastrophe stable manifolds:

(A8)
$$C_i^A(K_i) \cong \bar{C}_i^A \left(\frac{K_i}{\bar{K}_i^A} \right)^{\phi_i}, \quad \phi_i \equiv \frac{C_{iK}^A(\bar{K}_i^A) \bar{K}_i^A}{\bar{C}_i^A} > 0, \quad i = 1, 2.$$

For the calibration discussed in appendix 1, the log-linear approximations $C_i^A(K_i)$ to the stable manifolds of the systems (9) are given by $C_1^A(K_1) \cong 3.001 K_1^{0.4303}$ and $C_2^A(K_2) \cong 2.398 K_2^{0.4303}$. One can demonstrate that the speed of convergence of the after-catastrophe Ramsey growth systems increase with the rate of discount ρ and with the elasticity of intertemporal substitution σ, but decrease with the share of capital in value added.

Equations (7) gives the after-tipping value functions:

$$\text{(A9)} \quad V_i^A(K_i) = \frac{U\left(C_i^A(K_i)\right) + U'\left(C_i^A(K_i)\right)\left[Y_i(K_i) - C_i^A(K_i)\right]}{\rho}, \quad i = 1, 2.$$

With $g_i(K_i) \equiv f_i(K_i, 0)$ we can calculate the after-tipping values as

$$\text{(A10)} \quad V_i^A(K_i) = \frac{C_i^A(K_i)^{1-1/\sigma}}{\rho(\sigma - 1)} + \frac{C_i^A(K_i)^{-1/\sigma}\left[(1-\beta)g_i(K_i) - \delta K_i\right]}{\rho}, \quad i = 1, 2,$$

and from the first-order conditions we have that $V_{iK_i}^A(K_i) = C_i^A(K_i)^{-1/\sigma}$.

Appendix 4: Approximation of the before-catastrophe stable manifolds

A4.1.The state-space system for the specific functional forms

Before tipping we have a higher-dimensional dynamic system: (2′), (4′), (15) and (16) in the cooperative case and (2″), (4″), (22) and (23) in the non-cooperative case. These systems can for our specification with CES utility and Cobb-Douglas production functions be given by (omitting some dependencies to save space):

$$\text{(A11a)} \qquad \dot{K}_i = Y_i + \tau_i E_i - C_i, \quad K_i(0) = K_{i0}, \quad i = 1, 2,$$

$$\text{(A11b)} \qquad \dot{P} = \psi(E_1 + E_2) - \gamma P, \quad P(0) = P_0,$$

$$\text{(A11c)} \qquad \dot{C}_i = \sigma\left(Y_{iK_i} - \rho + H(P)\left[\frac{C_i^{1/\sigma}}{C_i^A(K_i)^{1/\sigma}} - 1\right]\right)C_i, \quad i = 1, 2,$$

$$\text{(A11d)} \quad \dot{\tau}_i = \left[Y_{iK_i} + \gamma + H(P)\frac{C_i^{1/\sigma}}{C_i^A(K_i)^{1/\sigma}}\right]\tau_i - \psi H'(P) C_i^{1/\sigma} \sum_{j=1}^{2}\left[V_j - V_j^A(K_j)\right], \quad i = 1, 2,$$

where in the non-cooperative case the summation in (A11d) reduces to only the $j = i$ term. The steady state of the system (A11) is given by (A2) and (A4). Note that the functional forms for Y_i, f_i and E_i are the same for $i = 1$ and $i = 2$ because total factor productivity A before tipping is the same in both regions (see equations (A1) and (A2)).

From (11), (12) and (13), and using the CES utility function, we can rewrite the before-catastrophe value functions in the cooperative case as

$$\sum_{i=1}^{2} V_i(K_i,P) = \frac{\sum_{i=1}^{2}\left\{ \frac{C_i^{1-1/\sigma}}{\sigma-1} + C_i^{-1/\sigma}(Y_i+\tau_i E_i) + H(P)V_i^A(K_i)\right\} + (V_{1P}+V_{2P})\left(\psi\sum_{j=1}^{2}E_j-\gamma P\right)}{\rho+H(P)}$$

or using $V_{1P}+V_{2P} = -C_1^{-1/\sigma}\tau_1/\psi = -C_2^{-1/\sigma}\tau_2/\psi$, we get

$$\text{(A12)}\quad \sum_{i=1}^{2} V_i(K_i,P) = \frac{\sum_{i=1}^{2}\left\{ \frac{C_i^{1-1/\sigma}}{\sigma-1} + C_i^{-1/\sigma}\left(Y_i+\frac{1}{2}\tau_i\frac{\gamma P}{\psi}\right) + H(P)V_i^A(K_i)\right\}}{\rho+H(P)},$$

and from the first-order conditions we have that $V_{iK_i} = C_i^{-1/\sigma}, \sum_{j=1}^{2} V_{jP} = -\tau_i C_i^{-1/\sigma}/\psi$.

From (19) and (20) and using the CES utility function, we can write the before-catastrophe value functions in the non-cooperative case as

$$\text{(A13)}\quad V_i = \frac{C_i^{1-1/\sigma}}{[\rho+H(P)](\sigma-1)} + \frac{C_i^{-1/\sigma}}{\rho+H(P)}\left(Y_i-\tau_i E_j+\tau_i\frac{\gamma P}{\psi}\right) + \frac{H(P)V_i^A(K_i)}{\rho+H(P)},$$
$$i=1,2,\ j\neq i,$$

and from the first-order conditions we have $V_{iK_i} = C_i^{-1/\sigma}, V_{iP} = -\tau_i C_i^{-1/\sigma}/\psi$. Note that V_{iC_i} boils down to zero in steady state.

To get the Jacobian of the cooperative and non-cooperative system (A11) with (A12) and (A13), respectively, we need some intermediate results (omitting dependencies):

$$\text{(A14)}\quad Y=(1-\beta)f-\delta K,\quad E=\frac{\beta}{d+\tau}f,\quad f_K=\frac{\alpha}{(1-\beta)K}f \quad\text{and}\quad f_\tau=\frac{-E}{1-\beta}.$$

It follows that

$$Y_K = \frac{\alpha f}{K} - \delta, \quad Y_{KK} = \frac{-\alpha(1-\alpha-\beta) f}{(1-\beta)K^2}, \quad Y_\tau = -E, \quad Y_{K\tau} = \frac{\alpha f_\tau}{K},$$

(A15)

$$E_K = \frac{\beta}{d+\tau} f_K \quad \text{and} \quad E_\tau = \frac{-E}{(d+\tau)(1-\beta)}.$$

A4.2. Linearization of the state-space system

The cooperative and non-cooperative systems (A11) for the before-catastrophe outcomes are boundary-value problems with initial conditions on the states (K_1, K_2, P), transversality conditions on the co-states $(C_1, C_2, \tau_1, \tau_2)$ and a saddle-point stable steady state. We present the Jacobian matrix of the linearized cooperative system, with the changes that occur in the non-cooperative case. This follows from equations (A12)-(A15) and is given by

(A16) $$B = \begin{pmatrix} Y_{1K_1} + \tau_1 E_{1K_1} & 0 & 0 & -1 & 0 & \tau_1 E_{1\tau_1} & 0 \\ 0 & Y_{2K_2} + \tau_2 E_{2K_2} & 0 & 0 & -1 & 0 & \tau_2 E_{2\tau_2} \\ \psi E_{1K_1} & \psi E_{2K_2} & -\gamma & 0 & 0 & \psi E_{1\tau_1} & \psi E_{2\tau} \\ \xi_{1,1} & 0 & \xi_{2,1} & \xi_{3,1} & 0 & \sigma Y_{1K_1\tau_1} C_1 & 0 \\ 0 & \xi_{1,2} & \xi_{2,2} & 0 & \xi_{3,2} & 0 & \sigma Y_{2K_2\tau_2} C_2 \\ \xi_{4,1} & \xi_{5,1} & \xi_{6,1} & \xi_{7,1} & 0 & \xi_{8,1} & \xi_{9,1} \\ \xi_{5,2} & \xi_{4,2} & \xi_{6,2} & 0 & \xi_{7,2} & \xi_{9,2} & \xi_{8,2} \end{pmatrix},$$

where $\xi_{1,i} \equiv \sigma\left[Y_{iK_iK_i} - H(P)\frac{C_i^{1/\sigma} C_{iK_i}^A(K_i)}{\sigma C_i^A(K_i)^{1+1/\sigma}} \right] C_i$, $\xi_{2,i} \equiv \sigma H'(P)\left[\frac{C_i^{1/\sigma}}{C_i^A(K_i)^{1/\sigma}} - 1 \right] C_i$,

$$\xi_{3,i} \equiv H(P)\left[\frac{C_i^{1/\sigma}}{C_i^A(K_i)^{1/\sigma}} \right],$$

$$\xi_{4,i} \equiv \left[Y_{iK_iK_i} - H(P)\frac{C_i^{1/\sigma} C_{iK_i}^A(K_i)}{\sigma C_i^A(K_i)^{1+1/\sigma}} \right] \tau_i - \psi H'(P)\left[1 - \frac{C_i^{1/\sigma}}{C_i^A(K_i)^{1/\sigma}} \right],$$

$\xi_{5,i} \equiv -\psi H'(P) C_i^{1/\sigma}[C_j^{-1/\sigma} - C_j^A(K_j)^{-1/\sigma}], \quad j \neq i, \quad i = 1,2,$ which in the non-cooperative case becomes $\xi_{5,i} \equiv 0$, $\xi_{6,i} \equiv H'(P)\frac{C_i^{1/\sigma}}{C_i^A(K_i)^{1/\sigma}}\tau_1 + H'(P)\tau_i$,

$$\xi_{7,i} \equiv H(P)\frac{C_i^{-1+1/\sigma}}{\sigma C_i^A(K_i)^{1/\sigma}}\tau_i - \psi H'(P)\frac{C_i^{-1+1/\sigma}}{\sigma}\sum_{j=1}^{2}\left[V_j - V_j^A(K_j) \right],$$

where in the non-cooperative case the summation reduces to only the $j = i$ term,

$$\xi_{8,i} \equiv \rho + \gamma + H(P)\frac{C_i^{1/\sigma}}{C_i^A (K_i)^{1/\sigma}} + Y_{iK_i\tau_i}\tau_i - \frac{\psi H'(P)}{\rho + H(P)}\left(-E_i + \frac{\gamma P}{2\psi}\right),$$

which in the non-cooperative case becomes $\xi_{8,i} \equiv \rho + \gamma + H(P)\dfrac{C_i^{1/\sigma}}{C_i^A (K_i)^{1/\sigma}} + Y_{iK_i\tau_i}\tau_i$,

$$\xi_{9,i} \equiv \frac{-\psi H'(P) C_i^{1/\sigma} C_j^{-1/\sigma}}{\rho + H(P)}\left(-E_j + \frac{\gamma P}{2\psi}\right), \quad j \neq i,$$

which in the non-cooperative case becomes $\xi_{9,i} \equiv \dfrac{\psi H'(P)\tau_i E_{j\tau_j}}{\rho + H(P)}, \quad j \neq i.$

A.4.3. Numerical spectral decomposition algorithm

We can write the log-linear approximation around the steady state as follows:

(A17) $$\dot{x} = Ax = \begin{pmatrix} A_{pp} & A_{pn} \\ A_{np} & A_{nn} \end{pmatrix}\begin{pmatrix} x_p \\ x_n \end{pmatrix} \text{ with } x_p \equiv \begin{pmatrix} \ln(K_1/\bar{K}_1) \\ \ln(K_2/\bar{K}_2) \\ \ln(P/\bar{P}) \end{pmatrix} \text{ and } x_n \equiv \begin{pmatrix} \ln(C/\bar{C}_1) \\ \ln(C_1/\bar{C}_2) \\ \ln(\tau_1/\bar{\tau}_1) \\ \ln(\tau_2/\bar{\tau}_2) \end{pmatrix},$$

where x_p denotes the vector of predetermined variables with initial conditions and x_n denotes the vector of non-predetermined variables. The matrix A is the state transition matrix of the log-linearized system (taking due care of the additional terms in (15) and (22) for the cooperative and non-cooperative case to allow for the precautionary returns) and follows from the Jacobian of the linearized system B in (A16):

(A18) $$\left[A_{ij}\right] = \left[B_{ij}\right]\bar{x}_j / \bar{x}_i, \quad i = 1,..,3, \quad j = 1,..,4.$$

The saddle-point property requires that the matrix A has three eigenvalues with negative real parts corresponding to the predetermined state variables and four eigenvalues with positive real parts corresponding to the non-predetermined state variables. Spectral decomposition gives $A = M\Lambda M^{-1} = N^{-1}\Lambda N$, where the diagonal matrix $\Lambda = \begin{pmatrix} \Lambda_p & 0 \\ 0 & \Lambda_n \end{pmatrix}$ has the eigenvalues of A on its diagonal. The eigenvalues associated with the predetermined variables are collected in the diagonal sub-matrix Λ_p and the others are collected in the diagonal sub-matrix Λ_n. Diagonalization of (A17) gives

(A19) $$\dot{y} = \Lambda y \text{ for } y = Nx,$$

which has the stable solution

(A20) $y_{p,i}(t) = e^{\lambda_{p,i} t} y_{p,i}(0), \quad i = 1,..,3$ and $y_{n,j}(t) = 0, \quad j = 1,..,4.$

The solution to (A17) is $x(t) = My(t).$ The stable manifold is $x_n(t) = M_{np} M_{pp}{}^{-1} x_p(t).$ With this log-linear approximation of the stable manifolds, we can readily calculate the solution trajectories for the log-linear deviations from the steady state and thus the trajectories for the state variables themselves.

Our spectral decomposition algorithm of the log-linearized state-space system yields answers that make sense for theoretically oriented *continuous-time* problems. The more frontier numerical infinite-horizon optimization methods for *discrete-time* problems use efficient discretization methods with ad-hoc terminal values, and maximize welfare directly rather than solving the first-order conditions (e.g., Cai et al., 2015b).