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OxCarre Research Paper 201

Race to Burn the Last Ton of Carbon and the Risk of Stranded Assets

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Abstract

A cap on global warming implies a tighter carbon budget which can be enforced with a credible second-best renewable energy subsidy designed to lock up fossil fuel and curb cumulative emissions. Such a subsidy brings forward the end of the fossil fuel era, but accelerates fossil fuel extraction and global warming in the short run. A weaker fossil fuel oligopoly implies that anticipation of a given global carbon budget induces fossil producers to deplete reserves more voraciously and accelerate global warming. This race to burn the last ton of carbon is more intensive for the feedback than open-loop Nash equilibrium, so that the Green Paradox effect of a renewable energy subsidy is stronger. There is an intermediate phase of limit pricing to keep renewable energy producers at bay, which becomes much more relevant when a cap on global warming is enforced. A stronger fossil fuel oligopoly lengthens the period of limit pricing and typically brings forward the carbon-free era. Finally, the mere risk of a cap on global warming being enforced at some unknown, future date makes fossil fuel extraction more voracious and accelerates global warming.

Keywords: Second-best climate policy, Green Paradox, carbon budget, stranded assets, oligopolistic resource markets, limit pricing, voracious extraction, regime shift.

JEL codes: H21, Q51, Q54

May 2016, revised September 2017

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² I acknowledge support from the ERC Advanced Grant ‘Political Economy of Green Paradoxes’ (FP7-IDEAS-ERC Grant No. 269788) and the BP funded OXCARRE. I am very grateful to Niko Jaakkola and Cees Withagen for their comments on the feedback Nash equilibrium and limit pricing, respectively. Helpful comments from Rabah Arezki, Cameron Hepburn, Jacob Janssen, Gerard van der Meijden, Armon Rezai and Gerhard Toews are also gratefully acknowledged.

1. Introduction

At the 2015 Paris United Nations Climate Change Conference governments have committed to put a cap on global warming of 2°C and strive to get keep temperature below 1.5°C. A cap of 2°C means that at most about three hundred Giga tons of carbon can still be burned. The International Panel of Climate Change, the Carbon Tracker Initiative (Carbon Tracker, 2013) and a host of academic studies (e.g., Allen et al., 2009; Allen, 2016; McGlade and Ekins, 2015; van der Ploeg and Rezai, 2016) have argued that such a 2°C cap on global warming requires a credible carbon budget for cumulative emissions. Recent debates and concerns about stranded assets among pension funds and other investors indicate that the markets might be anticipating the possibility that climate policy will be toughened and that not all fossil fuel reserves might be burned. Indeed, the Generation Foundation (2013) has argued:

‘The inevitable transition to a low-carbon economy will revolutionize financial markets at an unprecedented magnitude. Although we cannot, and should not, abandon the world’s current energy infrastructure overnight, investors who equate the transition with drawn-out, incremental change do so at their own peril as the stranding of carbon assets may occur at unforeseen rates and at an unpredictable scale.’

Governor Carney of the Bank of England and other presidents of central banks have also warned about the risk of stranded carbon assets for investors and economies. The problem is after all that proven and probable fossil fuel reserves of the major oil and gas companies alone are much higher than the carbon budget necessary to cap temperature at 2°C.^{3 4} Burning these reserves beyond the safe limit increases global warming to unacceptable levels. Furthermore, the problem of stranded assets exacerbates if the relentless discoveries of oil and gas including shale gas in recent years especially in developing economies (e.g., Arezki et al., 2017) continues in future years and is followed up by exploitation investments. Oil and gas companies are

³ The first one to point to the risk of stranded assets was McKibbin (2012), who argued that a 2°C cap implies that not more than 565 GtCO₂ can be burnt by mid-century whilst proven coal, oil and gas reserves held by fossil fuel producers and countries amount to 2,795 GtCO₂ and are thus a factor 5 bigger. These numbers are by now out of date, but the gist of the argument is even more valid today as demonstrated by the more recent figures released by the Carbon Tracker Initiative. They should be a source of concern for management and shareholders of companies like BP, Chevron, ExxonMobil, Royal Dutch Shell, Total SA, Eni and ConocoPhillips, and for countries such as Algeria, Russia and Venezuela with state-owned oil and gas companies.

⁴ It has been calculated for the electricity sector that the global stock of infrastructure, which, if operated to the end of its normal economic life, implies global mean temperature increases of 2°C or more (with 50% probability) will be reached as early as 2017 on current trends (Pfeiffer et al., 2016). Hence, no new emitting electricity infrastructure can be built after 2017 unless other electricity infrastructure is retired early or retrofitted with carbon capture technologies. These calculations suggest that the global economy is very close to the 2°C cap on peak global warming. Others have also offered reasons why the end of the fossil fuel era might be close despite substantial discoveries of new fossil fuel reserves (e.g., Helm, 2016; van der Ploeg, 2016).

therefore either overvalued by a substantial amount, so one can profit from shorting these companies on futures markets, or alternatively the cap of 2°C is not credible.

But with a strict global carbon budget would it not be rational for policy makers of oil- and gas-rich countries and for oil and gas producers to pump up their oil and gas reserves more vigorously before their competitors do so? Would such a fully credible cap on global warming encourage fossil fuel producers to engage in a race to burn the last ton of carbon and bring forward the carbon-free era where only renewable energies are used?⁵ Does such a race to burn the last ton of carbon still arise if the market merely anticipates a risk that the cap on global warming is enforced at some future date? Would the race to burn be less intense then?

At the same time monopolistic fossil fuel producers may use limit pricing to discourage entry of renewable energy producers (Hoel, 1978). Setting the price to maximise profits relatively high would induce more entry of renewable producers. Fossil fuel producers may thus set a price lower than that; at a level that still makes profits yet keeps out renewable energy producers. By flooding the market and depressing prices, fossil fuel producers can keep out renewable energy producers.⁶ If the substitute is supplied competitively and infinitely elastically at constant production cost, fossil fuel has constant extraction cost and fossil fuel producers are monopolistic, it has been shown that there is an initial phase with Hotelling pricing where the rent on fossil fuel rises at the market rate of interest and a subsequent limit pricing phase where the fossil price is set just below the renewable energy cost and the phase ends when reserves are fully exhausted (Hoel, 1978). Will such a phase of limit pricing still occur in oligopolistic resource markets where extraction costs rise as reserves fall?

In view of these questions, my first aim is to gain analytical understanding on the race to burn the last ton of carbon and how this race is affected by limit pricing if fossil fuel extraction costs rise as reserves are depleted. I therefore analyse the open-loop and feedback Nash equilibrium outcomes for an oligopolistic market of fossil fuel producers whose extraction costs rise as the cheapest oil, gas and coal deposits are depleted and who face a carbon-free perfect substitute (renewable energy) or the backstop for short. The condition that the cost of extracting the last ton of carbon must equal the cost of renewable energy then implies that the amount of fossil fuel

⁵ One should also bear in mind that the Saudis have in recent years for strategic and/or internal political reasons decided not cut back production in response to the increased supply of oil and gas from Iran and the new supplies of shale gas to the global market. Furthermore, the war on fracking and renewable energy might arise from continuous-time Cournot competitions in which firms produce alternative energy sources compete with each other setting quantities (Chan and Sircar, 2015). The collapse in oil prices may also come from strategic interactions by a limit-pricing cartel of oil producers and an importer producing substitutes for oil with production costs falling with R&D investment (Jaakkola, 2015).

⁶ They may also build excess capacity to discourage entry even further as this works like a threat that it can lower future prices even more.

to be left in the crust of the earth or stranded carbon assets decreases in the net cost of renewable energy.⁷ I show that for both outcomes an increase in the number of fossil fuel producers increases the speed of fossil fuel extraction and accelerates global warming in the short run, but cumulative carbon emissions are unaffected. More competition on the fossil fuel market induces a quicker end of the fossil fuel era. I show that this race to burn the last ton of carbon is more intense for the feedback than the open-loop Nash equilibrium outcome. I also show that the phase of Hotelling pricing is followed by an intermediate phase of limit pricing before the carbon-free era finally commences.⁸

The mechanism for rapacious extraction differs from that in dynamic common pool problems (e.g., Gordon, 1954; Lehviri and Mirman, 1980; Ostrom et al., 1994) or from the voracity effect (Lane and Tornell, 1996; Tornell and Lane, 1999), both of which arise from the natural resource being publicly rather than privately owned. In my framework voracious extraction takes place as fossil fuel owners anticipate that there is a strict budget for cumulative carbon emissions and thus deplete faster in order to not to get stuck with unsaleable carbon assets.

My second aim is to investigate how the race to burn the last ton of the carbon is affected by second-best climate policies. Governments prefer the stick to the carrot as it is politically tough to commit to a slow and rising path of carbon taxation and easier to postpone carbon taxation or subsidise renewable energy. A cap on peak global warming requires a credible carbon budget, which can be enforced by an appropriate subsidy on renewable energy. This induces faster fossil fuel extraction and accelerates global warming in the short run, which is the Green Paradox (e.g., Sinn, 2008), but locks up more fossil fuel reserves in the crust of the earth in the long run.⁹ Enforcing a cap on global warming with a renewable energy subsidy thus induces a short-run cost of accelerated heating of the planet and exacerbates the race to burn the last ton of carbon.

My final aim is to analyse the effects of the probability of a serious climate policy taking place at some unknown, future date on the speed of fossil fuel depletion and on stranded carbon assets. I apply a regime shift framework and show that an increased risk of the cap on peak

⁷ Benchekroun et al. (2017) study an economy with an oligopoly of oil producers (OPEC), a fringe of shale oil producers and a backstop supplied by competitive renewable energy producers. They investigate reversals in the order of extraction of the two types of fossil fuel given linear energy demand functions and limit pricing to keep renewable energy producers at bay, but focus at open-loop Nash-Cournot equilibrium, extraction costs that do not rise with cumulative extraction and asymptotic depletion. They show that OPEC can force relative expensive and dirty shale oil producers to produce before OPEC's oil reserves are fully exhausted, and this dominates the welfare losses from imperfect competition in the global oil market especially given the current shale oil boom. They also show that renewable energy subsidies boost supply of dirty shale oil but cut OPEC's oil supply.

⁸ Van der Meijden and Withagen (2016) show a similar result for monopolistic fossil fuel producers.

⁹ Di Maria and van der Werff (2012) and van der Ploeg and Withagen (2015) survey the extensive literature on the Green Paradox.

global warming and on cumulative carbon emissions being enforced with a second-best climate policy accelerates global warming and reinforces the race to burn the last ton of carbon, but less so than if the cap on global warming is enforced immediately.

The outline of the paper is as follows. Section 2 sets up the model. Section 3 derives the oligopolistic market outcomes and shows the three phases for the energy market. Section 4 shows how the speed of extraction varies with the number of fossil fuel firms for both the open-loop and the feedback Nash equilibrium outcomes and further characterises the optimal outcomes. Section 5 analyses the short-run Green Paradox effects of using a renewable energy subsidy to put a cap on peak global warming. Section 6 gives numerical policy simulations to illustrate the results and gain further insights. Section 7 discusses the effects of the risk of a cap on global warming as a political regime shift. Section 8 concludes.

2. The model

To demonstrate the race to burn the last ton of carbon, I adopt a simple partial equilibrium framework of a global oligopolistic fossil fuel market with an exogenous interest rate. Each fossil fuel producer maximises the present value of its profits subject to a depletion equation. To get a grip on the issue of a budget for cumulative carbon emissions (the carbon budget for short) and stranded carbon assets, I suppose that extraction of fossil fuel becomes more difficult as more reserves have been depleted in the past. To highlight the effects of the Green Paradox, I suppose that for political reasons the first-best policy of a gradual rising price of carbon is not feasible. Instead, policy makers rely on the second-best policy of subsidising renewable energy which is assumed to be a perfect carbon-free substitute for fossil fuel (a backstop). The difference between business as usual and second-best climate policy is thus simply the imposition of a lump-sum financed subsidy for renewable energy production. Crucially, I suppose that each fossil fuel producer has a private stock of reserves and abstract from dynamic common pool problems. My rationale for voracious fossil fuel depletion thus does not depend on this.

Fossil fuel producers face competition from renewable energy producers, which produce a perfect carbon-free substitute at constant cost b per unit of energy (the backstop). Renewable energy producers only produce if the energy price is at least b and else they are out of the market. Aggregate energy demand depends negatively on the world energy price p , so that $R + F = D(p)$, where R denotes renewable energy demand and F fossil fuel demand. The inverse energy demand function is $p = D^{-1}(R + F) \equiv p(R + F)$. There are a fixed number of N

fossil fuel producers. Producer i produces fossil fuel F_i , so aggregate fossil fuel production is $F = \sum_{i=1}^N F_i$. Unit fossil fuel extraction cost for each fossil fuel producer is denoted by $G(S) > 0$ and declines in remaining aggregate reserves S , so that $G'(S) < 0$. Note that $S \equiv \sum_{i=1}^N S_i$, where S_i denotes the stock of remaining in-situ fossil fuel reserves of producer i .

Fossil producer i maximises its net worth:

$$(1) \quad V_i \equiv \int_0^{\infty} \left[p \left(R + \sum_{j=1}^N F_j \right) F_i - G(S) F_i \right] e^{-rt} dt$$

subject to its fossil fuel depletion equation

$$(2) \quad \dot{S}_i = -F_i \leq 0, \quad S_i(0) = S_{i0} \geq 0, \quad \int_0^{\infty} F_i(t) dt \leq S_{i0},$$

and the condition that the market is not overtaken by renewable energy producers,

$$(3) \quad p(t) \leq b, \quad R(t) \geq 0, \quad \text{c.s.}, \quad \forall t \geq 0,$$

where r denotes the constant market rate of interest. For analytical convenience, I adopt a linear energy demand function and assume that the extraction cost of one unit of fossil fuel is linear in reserves. Furthermore, I impose some parameter restrictions to rule out corner solutions.

Assumption 1: $D(p) = \delta_0 - \delta_1 p$ with $\delta_0 > \delta_1 b$ and $\delta_1 > 0$.

Assumption 2: $G(S) = \gamma_0 - \gamma_1 S$ with $0 < \gamma_0 - \gamma_1 S_0 < b < \gamma_0$ and $\gamma_1 > 0$.

To ensure positive demand at all prices below the price of the backstop, I assume $\delta_0 > \delta_1 b$. The price elasticity of global energy demand $\varepsilon(p) \equiv -pD'(p) / D(p) = \delta_1 p / (\delta_0 - \delta_1 p) > 0$ increases in the price of energy. The super-elasticity of global energy demand is positive, i.e., $\theta \equiv p\varepsilon'(p) / \varepsilon = \delta_0 / (\delta_0 - \delta_1 p) > 0$.¹⁰ I suppose that all fossil fuel producers face identical extraction costs and identical initial reserves, so that I can focus on symmetric outcomes. The global stock of initial fossil fuel reserves, S_0 , is fixed and initial reserves of each fossil fuel producer are $S_{0i} = S_0 / N$, $i = 1, \dots, N$. To highlight differences between a cartelised, oligopolistic and a competitive global fossil fuel market, I vary N from 1 to infinity. Since unit fossil fuel extraction cost is not affected by the number of fossil fuel producers, splitting up the world in more fossil fuel producers does not affect the outcome.

¹⁰ The sign of the super-elasticity is closely related to whether demand is sub- or super-convex (Mrázová and Neary, 2017). The super-elasticity is positive for all members of the class of HARA demand functions except that it is zero for iso-elastic demand functions (e.g., Kagan et al., 2015).

3. Stranded assets, market structure and the race to burn the last ton of carbon

I focus in the first instance at the open-loop Nash equilibrium (OLNE) outcome for the fossil fuel oligopoly and then compare this with the feedback or Markov perfect Nash equilibrium (FBNE). The Hamiltonian function for fossil fuel producer i is defined by

$$(4) \quad H_i \equiv p \left(R + \sum_{j=1}^N F_j \right) F_i - G \left(\sum_{j=1}^N S_j \right) F_i - s_i F_i + \mu_i \left(b - p \left(R + \sum_{j=1}^N F_j \right) \right),$$

where $s_i \geq 0$ denotes the shadow value of reserves or the scarcity (Hotelling) rent and $\mu_i \geq 0$ denotes the shadow cost of constraint (3) to producer i . The static optimality conditions for this problem are $\partial H_i / \partial F_i \leq 0$, $F_i \geq 0$, c.s., $i = 1, \dots, N$, and yield

$$(5) \quad \left(1 - \frac{1}{N\varepsilon} \right) p \leq G(S) + s_i - \frac{p}{R+F} \frac{\mu_i}{\varepsilon}, \quad F_i \geq 0, \quad \text{c.s.}, \quad i = 1, \dots, N.$$

If fossil fuel is extracted ($F_i > 0$) and the price is below the cost of the backstop ($\mu_i = 0$), producers set marginal revenue equal to marginal extraction cost plus the scarcity rent. If not, marginal revenue falls short of extraction cost plus scarcity rent (as $\mu_i > 0$). Conversely, if renewable energy is produced, marginal revenue equals the cost of the backstop. The co-state equations are $r s_i - \dot{s}_i = \partial H_i / \partial S_i = -G'(S) F_i$, $i = 1, \dots, N$. These equations can be solved to give the scarcity rent as the present discounted value of all future cuts in extraction costs from holding one extra unit of fossil fuel in the crust of the earth:

$$(6) \quad s_i(t) = - \int_t^\infty \exp(-r(t'-t)) G'(S(t')) F_i(t') dt'.$$

3.1. The Hotelling rules for optimal depletion

If $F_i > 0$ and $p < b$, the co-state equations can upon substituting $s_i = (1 - 1/N\varepsilon)p - G(S)$ from (5) and taking account of the endogeneity of ε be rewritten in terms of the price of fossil fuel:

$$(7) \quad \dot{p} = r \left[p - \frac{N\varepsilon}{N\varepsilon - 1} G(S) \right] \left(\frac{N\varepsilon - 1}{N\varepsilon - 1 + \theta} \right), \quad i = 1, \dots, N, \quad \text{if } F_i > 0 \text{ and } R = 0, \quad i = 1, \dots, N.$$

For a competitive fossil fuel market with infinitely elastic energy demand ($\varepsilon \rightarrow \infty$) the modified Hotelling rule (7) boils down to $\dot{p} = r[p - G(S)]$, $i = 1, \dots, N$, which equates the return on holding an extra unit of fossil fuel in the ground (the capital gains) to that on taking this unit out of the ground (return on investing the proceeds from selling it minus the cost of extracting it).

For a monopoly ($N = 1$) equation (7) becomes $\dot{p} = r \left[p - \frac{\varepsilon}{\varepsilon - 1} G(S) \right] \left(\frac{\varepsilon - 1}{\varepsilon - 1 + \theta} \right)$. If also energy demand is iso-elastic and extraction costs zero, extraction is efficient, $\dot{p} = rp$ (Stiglitz, 1976).

In an oligopolistic fossil fuel market two adjustments have to be made to the Hotelling rule. First, the second term in the square brackets indicates that the marginal extraction cost is marked up for oligopoly power, especially if there are fewer competitors and the price elasticity of energy demand is small (low $N\varepsilon$). This curbs the expected appreciation of fossil fuel reserves and slows down the rate of extraction (especially if the number of producers and the price elasticity of global energy demand are small). Second, given that the super-elasticity for linear energy demand is positive, $\theta = \delta_0 / R > 0$, the final term in round brackets is less than one and corresponds to an additional negative adjustment in the Hotelling rules. This reflects that extracting an additional unit of fossil fuel depresses the global price of energy and thus pushes all producers further up the demand curve, thereby curbing the price elasticity $\varepsilon = \delta_1 p / R$, increasing oligopoly power and slowing down extraction. Both capturing rents from market power and strategic considerations thus tend to slow down fossil fuel extraction.

3.2. The three phases of optimal energy use

The optimum programme consists of three phases. The first one is the Hotelling phase and takes place during $0 \leq t < T_1$. During this phase fossil fuel extraction is positive, there is no renewable energy use, and the price and also marginal revenue from producing energy are below the cost of the backstop. The second one is the limit pricing phase which takes place during $T_1 \leq t < T_2$ and has constant positive fossil fuel extraction and zero renewable energy use. During this phase the marginal revenue from producing energy is below the cost of the backstop, but the price of energy is only an infinitesimal amount below the backstop cost. If marginal revenue would be higher, the price would exceed the cost of the backstop and fossil fuel producers would be put out of business by renewable energy producers. The transition from the first to the second place takes place when the price of energy has almost reached the cost of the backstop at which point the marginal revenue of energy is still below the cost of the backstop. Denoting an infinitesimal small amount by $1/\infty$, we thus have

$$(8) \quad p(t) = b - 1/\infty, \quad T_1 \leq t < T_2.$$

The third and final phase is the carbon-free phase which takes place for $\forall t \geq T_2$ and has zero fossil fuel and positive renewable energy use. The transition from the second to the third phase

occurs when the marginal revenue of energy hits the price of the backstop. Since at this point the scarcity rent must have dropped to zero (cf. Hoel, 1978), we have the boundary conditions

$$(9) \quad \left(1 - \frac{1}{N\varepsilon(p(T_2))}\right) p(T_2) = G(S(T_2)) = b, \quad i = 1, \dots, N.$$

The depletion equations (2), efficiency conditions (5), pricing rules (7) and boundary conditions $p(T_1) = b - 1/\infty$ and (9) fully determine the OLN solution for these three phases.

Phase 3: The carbon-free era, $t \geq T_2$

For this era we have $F = 0$ and $S(t) = (\gamma_0 - b) / \gamma_1 \equiv S_2(b)$, $\forall t \geq T_2$ from the second part of (7b) and Assumption 2, where $S(t) \equiv \sum_{i=1}^N S_i(t)$. Hence, abandoned fossil fuel reserves or stranded assets for short are higher if the price of renewable energy is lower and the cost of fossil fuel extraction is higher. From $p(R) = b$ and Assumption 1, we get renewable energy demand as a decreasing function of the cost of renewable energy, $R = \delta_0 - \delta_1 b$, $\forall t \geq T_2$. The end of this phase follows from adding the duration of the limit pricing phase (see equation (11) below) to the end of the Hotelling pricing phase (see equation (18) below).

Phase 2: The limit pricing phase, $T_1 \leq t < T_2$

Fossil fuel producers set the price just below the cost of the backstop, hence

$$(10) \quad F(t) = D(b) + 1/\infty \quad \text{and} \quad S(t) = S_2(b) + (T_2 - t)D(b) + 1/\infty, \quad T_1 \leq t \leq T_2.$$

During this phase the scarcity rent follows from (6). Clearly, $s_i(T_2) = 0$ and making use of (8)

$$s_i(t) = \gamma_1 D(b) (1 - e^{r(t-T_2)}) / (rN). \quad \text{At the end of phase 1} \quad \left(1 - \frac{1}{N\varepsilon}\right) b = G(S(T_1)) + s_i(T_1) \quad \text{must hold.}$$

Substituting $S(T_1) = S_2(b) + (T_2 - T_1)D(b)$ and $s_i(T_1)$, making use of Assumptions 1 and 2, using the definition of ε , and rewriting, I get the following duration of the limit pricing phase:

$$(11) \quad T_2 - T_1 - \frac{1}{rN} (1 - e^{-r(T_2-T_1)}) = \frac{1}{N\delta_1\gamma_1} > 0 \quad \Rightarrow \quad T_2 - T_1 \equiv T(\bar{N}, \bar{\delta}_1, \bar{\gamma}_1, r).$$

As the number of fossil producers becomes very large ($N \rightarrow \infty$), (9) gives $T_2 = T_1$ and the limit pricing phase becomes degenerate. For monopolistic or oligopolistic fossil fuel markets there is

a unique and strictly positive value of $T_2 - T_1 \equiv T(N, \delta_1, \gamma_1, r)$ that solves (11).¹¹ It follows that fewer fossil fuel producers increase the duration of limit pricing. The stock of fossil fuel reserves at the end of the Hotelling pricing phase is given by

$$(12) \quad S(T_1) = S_2(b) + T(N, \delta_1, \gamma_1, r)(\delta_0 - \delta_1 b) \equiv S_1(\bar{b}, \bar{N}, \bar{\delta}_0^+, \bar{\delta}_1^-, \bar{\gamma}_1^+, r).$$

This stock decreases in the cost of renewable energy. Since the duration of the limit pricing phase increases in the interest rate, this stock also increases in the interest rate. A smaller sensitivity of energy demand to the price of energy and a smaller sensitivity of extraction cost to remaining reserves (lower δ_1 and γ_1) lengthen the limit pricing phase and boosts the stock of reserves that is left at the end of Hotelling pricing. An autonomous increase in demand does not affect the duration of limit pricing but does increase the stock of reserves at the end of the Hotelling pricing phase. The size of the renewable energy subsidy does not affect the duration of the limit pricing phase.¹² Finally, fewer fossil fuel producers increases the period of limit pricing and increases the stock of reserves at the end of the Hotelling pricing phase.

Phase 1: The Hotelling pricing phase, $t \leq T_1$

The dynamics of this phase follow from (2) and (7). Using $F = \delta_0 - \delta_1 p$, substituting the price elasticity and the super-elasticity from Assumption 1 and using Assumption 2 and (12), we get:

$$(13a) \quad \dot{S} = -(\delta_0 - \delta_1 p), \quad S(0) = S_0, \quad S(T_1) = S_1(b, N, \delta_0, \delta_1, \gamma_1, r),$$

$$(13b) \quad \dot{p} = r \left[p - \frac{1}{N+1} \frac{\delta_0}{\delta_1} - \frac{N}{N+1} (\gamma_0 - \gamma_1 S) \right], \quad p(T_1) = b, \quad 0 \leq t \leq T_1.$$

The saddle-point system (13) can be solved for $(S(t), F(t), p(t), 0 \leq t \leq T_1)$ and T_1 . Once T_1 is known, the end of the fossil fuel era, $T_2 = T_1 + T(N, \delta_1, \gamma_1, r)$, can be calculated.

Result 1: *The global energy market has an initial phase of falling fossil fuel depletion with Hotelling pricing, an intermediate phase of constant fossil fuel depletion with limit pricing, and a final phase of only renewable energy. Cheaper renewable energy curbs stranded fossil fuel assets and peak warming. It also curbs fossil fuel use during limit pricing and curbs the stock of*

¹¹ Define the left-hand side of (11) by $f(T)$ with $T \equiv T_2 - T_1$. Note $f(0) = 0$. If $N > 1$, $f'(0) > 0$. The function $f(x)$ rises monotonically and $f(T) \rightarrow T$ from below as $T \rightarrow \infty$. There is thus a unique value that solves (11).

¹² With more general demand functions, this is not the case. Van der Meijden and Withagen (2016) focuses at the case of a monopolistic fossil fuel producer where extraction costs per unit of fossil fuel are constant. Their Proposition 3 gives conditions for which an increase in the renewable energy subsidy reduced or increases the duration of the limit pricing phase.

reserves at the end of Hotelling pricing. Limit pricing lasts longer if there are fewer fossil fuel producers, the interest rate is high, energy demand is less sensitive to the price and extraction costs react less to remaining reserves, but its length is unaffected by autonomous energy demand or the cost of renewable energy.

This result also holds for the feedback Nash equilibrium outcome. The solution concepts require that no individual fossil fuel producer has an incentive to depart from limit pricing. Although this is reasonable for the symmetric setup used here, this need not be so in an asymmetric setup.

4. Deriving the time paths of prices, demand and reserves during Hotelling pricing

Here the open-loop and feedback Nash equilibrium outcomes are derived and compared.

4.1. The open-loop Nash equilibrium

The state-space dynamics for fossil fuel prices and remaining reserves including the date of the transition to transition pricing follow from the two-point-boundary-value problem (13). Conjecturing the saddle-path for the fossil fuel price that satisfies (13),

$$(14) \quad p(t) = b - \pi[S(t) - S_1(b, N, \delta_0, \delta_1, \gamma_1, r)], \quad 0 \leq t \leq T_1 \quad \text{and} \quad p(T_1) = b,^{13}$$

and using (13a) and (13b), I get (ignoring constants) $\dot{p} = \pi R = \delta_1 \pi^2 S = -r\pi S + r \frac{N}{N+1} \gamma_1 S$. This

equation has to hold for every value of S , so π is the positive root of $\delta_1 \pi^2 + r\pi - r \frac{N}{N+1} \gamma_1 = 0$:

$$(15) \quad \pi = \frac{r}{2\delta_1} \left[\sqrt{1 + \frac{4N}{N+1} \frac{\gamma_1 \delta_1}{r}} - 1 \right] \equiv \pi^o > 0 \quad \text{and} \quad \frac{\partial \pi^o}{\partial N} > 0.$$

This is the value of π for the open-loop Nash equilibrium (denoted by O), which assumes that the different fossil fuel producers condition on all information available at time zero. The speed of adjustment during Hotelling pricing is higher if there are more fossil fuel producers.

4.2. The feedback Nash equilibrium

In contrast, the subgame-perfect or feedback Nash equilibrium (denoted by F) solution with linear strategies supposes that each fossil fuel producer conditions extraction rates on the current stock of reserves. This solution is obtained using the principle of dynamic programming rather

¹³ This is a feedback realisation of an open-loop Nash equilibrium solution.

than Pontryagin's Maximum Principle and is from an informational standpoint more attractive (e.g., Başar and Olsder, 1982; van der Ploeg and de Zeeuw, 1992). Appendix 1 shows that the adjustment speed for the open-loop Nash equilibrium during limit pricing (15) then becomes

$$(15') \quad \pi = \frac{r(N+1)}{4\delta_1} \left[\sqrt{1 + \frac{8N}{(N+1)^2} \frac{\gamma_1 \delta_1}{r}} - 1 \right] \equiv \pi^F > \pi^O \text{ for } \forall N > 1 \text{ and } \frac{\partial \pi^F}{\partial N} > 0.$$

Equation (15') indicates that a larger number of fossil fuel producers (N) speeds up the rate of extraction and brings forward the transitions to the limit-pricing phase and the carbon-free era. Compared with the oligopolistic OLNE outcome, for a given number of fossil fuel producers the oligopolistic FBNE outcome speeds up extraction and leads to an earlier onset of the limit-pricing phase and carbon-free era. However, the number of fossil fuel producers leaves the proportion of stranded assets unaffected in both the OLNE and the FBNE outcomes.

4.3. Remaining reserves and fossil fuel demand during Hotelling pricing

Upon substitution of (14) into the depletion equation (13a), I solve for remaining reserves

$$(16) \quad S(t) = \left[S_1(b, N, \delta_0, \delta_1, \gamma_1, r) + \frac{b}{\pi} - \frac{\delta_0}{\delta_1 \pi} \right] (1 - e^{-\delta_1 \pi t}) + e^{-\delta_1 \pi t} S_0, \quad 0 \leq t \leq T_1,$$

Fossil fuel demand then follows from

$$(17) \quad \begin{aligned} F(t) &= \delta_0 - \delta_1 [b + \pi S_1(b, N, \delta_0, \delta_1, \gamma_1, r)] + \delta_1 \pi S(t) \\ &= \delta_1 \pi \left[S_0 - S_1(b, N, \delta_0, \delta_1, \gamma_1, r) - \frac{b}{\pi} + \frac{\delta_0}{\delta_1 \pi} \right] e^{-\delta_1 \pi t}, \quad 0 \leq t \leq T_1. \end{aligned}$$

4.4. End of the phase of Hotelling pricing

Over time as reserves are depleted and become scarcer, the price of fossil fuel rises and demand for fossil fuel drops until the limit pricing phase commences (from (15)). The time of transition

to limit pricing T_1 follows from (16), i.e., $\left(S_1 + \frac{b}{\pi} - \frac{\delta_0}{\delta_1 \pi} \right) (1 - e^{-\delta_1 \pi T_1}) + e^{-\delta_1 \pi T_1} S_0 = S_1$. Hence,

$$(18) \quad T_1 = \frac{1}{\delta_1 \pi} \ln \left(1 + \frac{\delta_1 \pi}{\delta_0 - \delta_1 b} [S_0 - S_1(b, N, \delta_0, \delta_1, \gamma_1, r)] \right) > 0.$$

Equation (18) (and the comparative statics indicated in (12)) imply that the Hotelling price phase lasts longer if there are new discoveries of fossil fuel (higher S_0), but is brought forward if global fossil fuel demand rises (higher δ_0) or technological progress or subsidies bring down the cost of renewable energy (lower b). With fewer fossil fuel producers (smaller N), the phase of Hotelling pricing takes longer due to the slower adjustment during the phase of Hotelling

pricing (lower π) and due to the longer period of limit pricing and thus higher stock at the end of the phase of Hotelling pricing (higher S_1). Still, the effect of smaller N is ambiguous as there is an offsetting negative effect on the duration of Hotelling pricing due to the higher steady-state stock of reserves as can be seen from the effect of π on the expression inside the log of (18).

Result 2: *The feedback Nash equilibrium leads to faster depletion of fossil fuel and an earlier onset of limit pricing and the end of the carbon-free era than the open-loop Nash equilibrium if the number of fossil fuel producers exceeds one. More fossil fuel producers speeds up extraction and brings forward limit pricing, but has an ambiguous effect on the duration of Hotelling pricing and leaves the stock of stranded assets unaffected. Oligopolistic fossil fuel markets delay emergence of limit pricing and the carbon-free era relative to a perfectly competitive fossil fuel market.*

4.5. Diagrammatic illustration

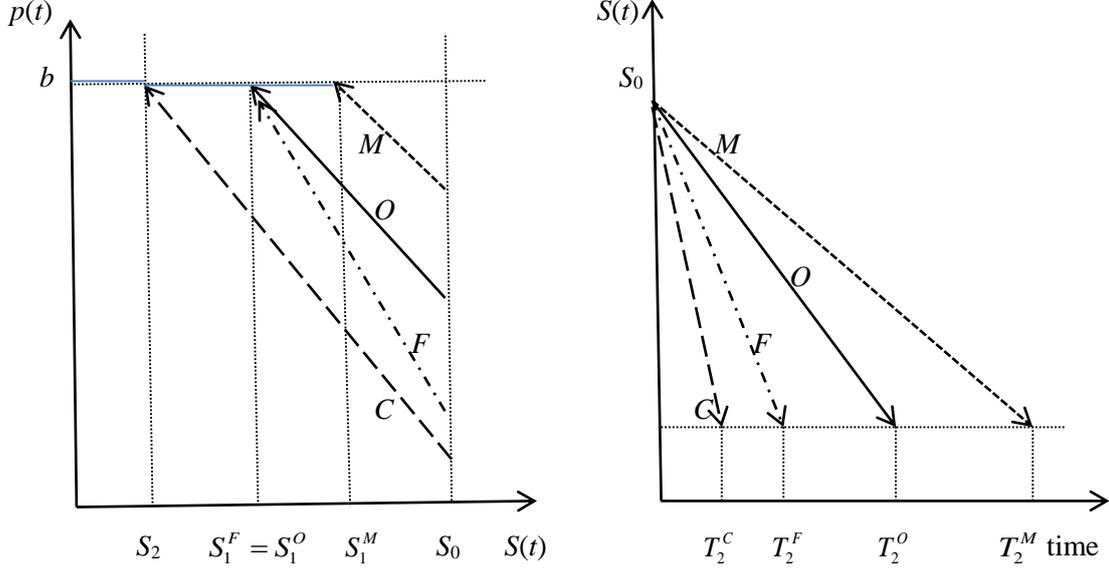
Figure 1 illustrates Results 1 and 2 and the analytical solution graphically. The left panel portrays the phase-plane dynamics (13) and the right panel gives the time paths for fossil fuel reserves. A fossil fuel cartel corresponds to $N = 1$ and extracts the slowest, so that the Hotelling pricing phase takes longest.¹⁴ A perfectly competitive fossil fuel market corresponds to $N \rightarrow \infty$ and extracts the quickest and thus brings on limit pricing and the carbon-free era most quickly. The speed of extraction of oligopolistic fossil fuel markets with finite values of $N > 1$ takes on intermediate values. Since limit pricing lasts longer if there are fewer fossil fuel producers, we have $0 = T^C < T^F = T^O < T^M$. We also have $S_0 > S_1^M > S_1^O = S_1^F > S_1^C = S_2$. In contrast to monopolies of final good producers that use fossil fuel in their production process, monopolistic fossil fuel producers are not the environmentalists' best friend. In fact, oligopolistic and competitive fossil fuel markets induce a race to burn the last ton of carbon.

As illustrated by the dashed lines in Figure 1, compared with the open-loop Nash equilibrium, the feedback Nash equilibrium intensifies the race to burn the last ton of carbon. The intuition for the faster speed of extraction under an oligopolistic FBNE is as follows. The burning of an extra ton of carbon by fossil fuel producer i implies that the stock of in-situ fossil fuel reserves is reduced by a ton. This pushes up extraction costs of all rival fossil fuel producers, who consequently cut back production of fossil fuel a little. As a result, the global price of fossil fuel is somewhat higher, which gives producer i an incentive to extract a bit more than it would have

¹⁴ Appendix 2 shows that a cartel that maximises joint profits indeed corresponds to $N = 1$.

done otherwise. This *strategic* effect boosts the speed of fossil fuel extraction.¹⁵ The race to burn the last ton of carbon is thus more intense under the FBNE than the OLNE outcome.

Figure 1: Market structure and the race to burn the last ton of carbon



Key: The short-dashed, solid, dashed-dotted and dashed-dotted lines indicate the outcomes under a monopolistic (M), oligopolistic open-loop Nash equilibrium (O), oligopolistic feedback Nash equilibrium (F) and perfectly competitive (C) fossil fuel market, respectively.

5. Enforcing the carbon budget, stranded assets and the Green Paradox

Peak global warming PW increases with cumulative emissions or the carbon budget,

$$(19) \quad PW = PW_0 + TCRE \times [S_0 - S(T_2)],$$

where PW_0 is initial global warming and $TCRE$ is the transient climate response (e.g., Allen, 2016). At the 2015 United Nations Climate Change Conference in Paris, governments around the globe have committed to keep global warming below 2°C and strive to a cap of 1.5°C relative to pre-industrial temperatures. A target of 2°C then corresponds to a safe carbon budget of $B \equiv S_0 - S(T_2) = (PW - PW_0) / TCRE$, where PW now indicates the cap on temperature. Let the price of renewable energy be $b = \bar{b} - \nu$, where \bar{b} is the market price of renewable energy (measured in terms of equivalent tons of carbon) and ν the specific subsidy on renewable energy. Then $S(t) = (\gamma_0 - b) / \gamma_1 \equiv S_2(b)$, $\forall t \geq T_2$ and (19) give the subsidy (financed by lump-sum taxes) that is needed to enforce the cap on peak global warming:

¹⁵ With linear demand the FBNE outcome with $N \rightarrow \infty$ gives a race with immediate exhaustion of all fossil fuel reserves in almost no time, $\pi^F \rightarrow \infty$ from (15'), so the scarcity rent is immediately driven to zero.

$$(20) \quad v = \bar{b} - \gamma_0 + \gamma_1(S_0 - B) = \bar{b} - \gamma_0 + \gamma_1 \left(S_0 - \frac{PW - PW_0}{TCRE} \right).$$

Hence, the required renewable energy subsidy has to be higher if the market price of renewable energy, the stock of initial fossil fuel reserves and the transient climate response are high (large \bar{b} , S_0 and $TCRE$), the cost of fossil fuel extraction is small, and the cap on peak warming is tight (low γ_0 and PW). The carbon budget and cap on peak global warming can be attained by pricing carbon via an emissions tax or permits market, but here I assume that politicians prefer the carrot to the stick and thus use the second-best policy of subsidising renewable energy.

Figure 2 illustrates graphically for a given number of fossil fuel producers how a renewable energy subsidy designed to cut the carbon budget and put a cap on peak global warming induces Green Paradox effects in the short run. The left panel shows that the price path during the Hotelling price phase lies completely below the business-as-usual price path provided the initial price falls as a result of the subsidy. Hence, the rate of extraction and emissions during Hotelling pricing are always higher than under business as usual. This is the Green Paradox.

Result 3: *Credible enforcement of a maximum carbon budget by subsidising renewable energy implies $\partial p(0) / \partial v < 0$ and accelerates fossil fuel use and carbon emissions in the short run, but locks up more carbon in the long run by speeding up the transition to the carbon-free era. These results hold for both the open-loop and the feedback Nash equilibrium outcome.*

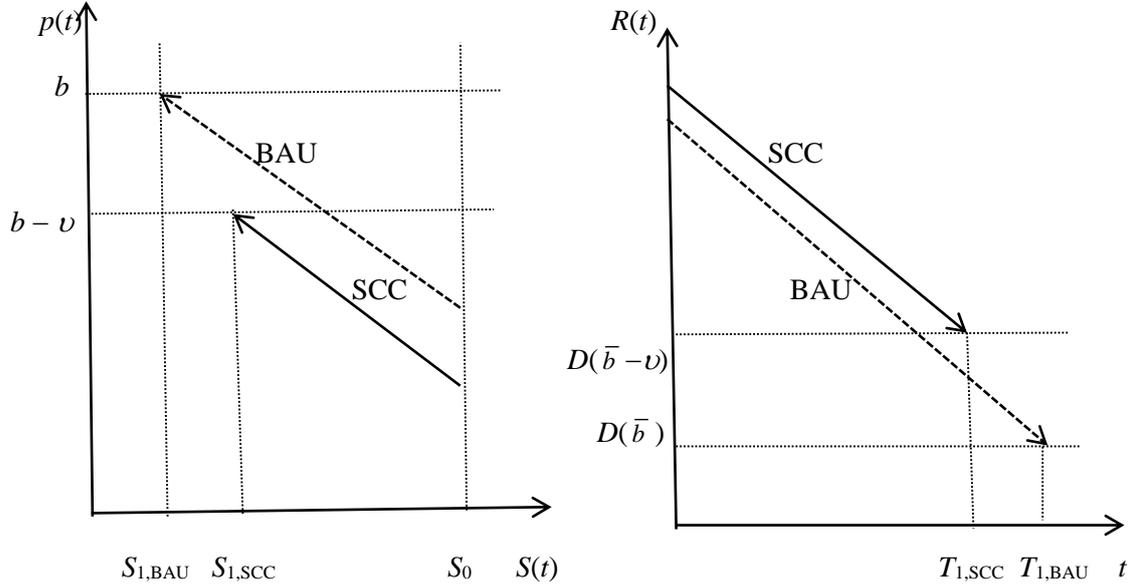
Proof: See Appendix 3.

The right panel of Figure 2 indicates that throughout the Hotelling pricing phase extraction and carbon emissions with a cap on peak warming are higher than under business as usual (the Green Paradox effect) and that the fossil fuel era is shortened. Hence, cumulative extraction and emissions and peak global warming are less than under business as usual. The acceleration of global warming caused by more voracious extraction in a more competitive fossil fuel market is reinforced by Green Paradox effects stemming from second-best climate policies.

Result 3: *Credible enforcement of a maximum carbon budget by subsidising renewable energy implies $\partial p(0) / \partial v < 0$ and accelerates fossil fuel use and carbon emissions in the short run, but locks up more carbon in the long run by speeding up the transition to the carbon-free era. These results hold for both the open-loop and the feedback Nash equilibrium outcome.*

Proof: See Appendix 3.

Figure 2: Enforcing the safe carbon budget and the Green Paradox



Key: The dashed and solid indicate the outcomes under business as usual (BAU) and under enforcement of the safe carbon budget (SCC), respectively. The renewable subsidy needed to enforce the SCC accelerates extraction, emissions and global warming in the short run (the Green Paradox), but ends the fossil fuel era more quickly and locks up more carbon.

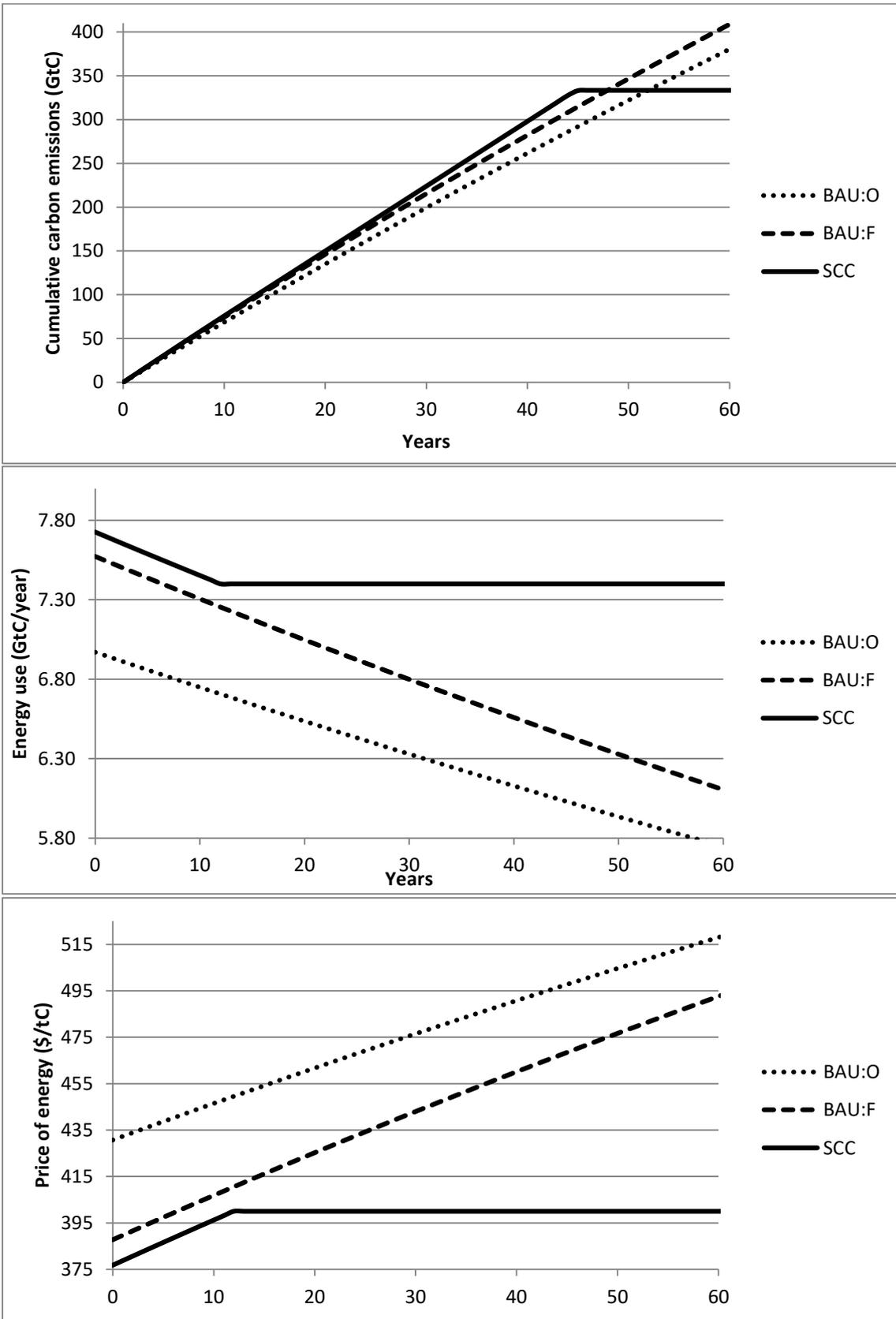
6. Climate policy with oligopolistic fossil fuel markets: numerical illustrations

Here a rough calibration is given not for numerical realism, but to illustrate the interplay between oligopolistic fossil fuel markets, energy transitions and second-best climate policies. I suppose a $TCRE$ of $2^\circ\text{C}/\text{TtC}$ and PW_0 of 1.33°C . To calibrate fossil fuel costs, I suppose that $\gamma_0 = \$1500/\text{tC}$ and $\gamma_1 = 0.3$. At initial fossil fuel reserves of $S_0 = 4000 \text{ GtC}$, this implies that current extraction cost is $\$300/\text{tC}$ and that this cost doubles if a further 1000 GtC is extracted. I suppose that renewable energy is initially less competitive than fossil fuel and has a production cost b of $\$800/\text{tC}$. I suppose a linear energy demand function with autonomous demand δ_0 equal to $13 \text{ GtC}/\text{year}$ and a price sensitivity δ_1 of 0.014 . For the initial year of the simulations reported in Figure 2 for business as usual, this implies a price elasticity of 0.72 . I suppose that the interest rate r is 2% per annum. For the base case discussed in section 6.1 I assume $N = 8$, but section 6.2 considers the effects of different number of fossil fuel producers.

6.1. Climate policy simulation for case of oligopolistic fossil fuel markets

Figure 3 plots two scenarios for when there is an oligopoly with 8 fossil fuel producers. The first one is business as usual (BAU) for which the subsidy ν is set to zero. The Hotelling pricing

Figure 3: Imposing the safe carbon budget



phase then ends after 421 years for the OLNE and 400 years for the FBNE outcome. Since limit pricing lasts for 33 years, the carbon-free era starts after 454 or 433 years. At that point a total of 2,333 GtC of fossil fuel reserves is stranded. This corresponds to a total of 1,667 GtC of cumulative emissions, of which 1,608 GtC is emitted during Hotelling pricing and a mere 59 GtC during limit pricing. Since the speed of adjustment during the Hotelling pricing phase π is faster under FBNE than under OLNE (25.6% per year instead of 23.0% per year), the initial fossil fuel price under FBNE must be lower than under OLNE (\$388 instead of \$431 per ton of carbon). As soon as limit pricing starts, the fossil fuel price has to be just below the price of the carbon-free backstop, \$800 per ton of carbon. The Hotelling pricing phase is the relevant one under business as usual as limit pricing only takes place in the very distant future.

The second scenario corresponds to the one that enforces the safe carbon budget (SCC). Enforcing a target of 2°C corresponds to a safe carbon budget of $B = 333$ GtC or 1222 GtCO₂. Since emissions since pre-industrial times are 650 GtC, the carbon budget from pre-industrial times onwards corresponding to a peak warming cap of 2°C is almost 1 TtC. To enforce this target, policy makers that shy away from carbon taxation can use a second-best renewable energy subsidy ν of \$400/GtCe (a 50% subsidy). Total cumulative emissions thus drop from 1,667 to 333 GtC. Of this 91 GtC is burnt during the 12 years of Hotelling pricing and 243 GtC is burnt during the 33 years of limit pricing. The carbon-free era thus commences in 45 years.

The initial price of fossil fuel drops from \$431 to \$379 per ton of carbon for the OLNE outcome and \$388 from to \$377 per ton of carbon for the FBNE outcome. In both outcomes using the renewable energy subsidy to implement the safe carbon budget thus induces a Green Paradox which is confirmed by fossil fuel use and emissions under this policy being higher than under business as usual as can be seen from Figure 3. Interestingly, the Green Paradox effect is stronger for the OLNE than the FBNE outcome. The reason is that, due to the additional strategic effect in the FBNE outcome, fossil fuel use is already higher than under the OLNE outcome. Although the renewable energy subsidy accelerates global warming in the short run, it brings forward the carbon-free era and thus limits cumulative emissions and peak warming.

6.2. Effect of market structure on energy transitions and cumulative emissions

Table 1 shows the effects of different types of fossil fuel producers on cumulative emissions during Hotelling pricing ($S_0 - S_1$) and limit pricing ($S_1 - S_2$), the speeds of adjustment during Hotelling pricing under the open-loop and feedback Nash equilibrium (π^O and π^F), the end of Hotelling pricing and the end of limit pricing for these two equilibria (T_1 and T_2), and the initial

energy price under these two equilibria ($p(0)$). Roman letters indicate outcomes under business as usual. Italics indicate (if different) outcomes if policy makers enforce the safe carbon budget.

Table 1: Market structure, energy transitions and cumulative emissions

	$N = 1$	$N = 4$	$N = 8$	$N = 20$	$N = 50$	$N = 100$
$S_0 - S_1$ (GtC)	1148 <i>0</i>	1543 <i>0</i>	1608 <i>91</i>	1644 <i>241</i>	1658 <i>298</i>	1662 <i>316</i>
$S_1 - S_2$ (GtC)	518 <i>333</i>	124 <i>333</i>	59 <i>243</i>	22 <i>92</i>	9 <i>36</i>	4 <i>18</i>
π OLNE	0.137	0.209	0.230	0.244	0.250	0.252
π FBNE	0.137	0.226	0.256	0.280	0.290	0.292
T_1 OLNE (years)	417 <i>0</i>	429 <i>0</i>	421 <i>12</i>	414 <i>31</i>	411 <i>38</i>	410 <i>40</i>
T_1 FBNE (years)	417 <i>0</i>	415 <i>0</i>	400 <i>12</i>	388 <i>31</i>	382 <i>37</i>	380 <i>39</i>
T_2 OLNE (years)	705 <i>45</i>	498 <i>45</i>	454 <i>45</i>	427 <i>43</i>	416 <i>42</i>	413 <i>42</i>
T_2 FBNE (years)	705 <i>45</i>	484 <i>45</i>	433 <i>45</i>	400 <i>43</i>	387 <i>42</i>	382 <i>42</i>
$p(0)$ OLNE (\$/tC)	643 <i>400</i>	477 <i>400</i>	431 <i>379</i>	399 <i>341</i>	385 <i>325</i>	380 <i>320</i>
$p(0)$ FBNE (\$/tC)	643 <i>400</i>	452 <i>400</i>	388 <i>377</i>	339 <i>332</i>	316 <i>313</i>	308 <i>307</i>

Key: Romans indicate business as usual and italics indicate the safe carbon budget scenario with a 50% subsidy and a budget of 333 GtC. N is the number of fossil fuel producers. $S_0 - S_1$ and $S_1 - S_2$ are cumulative emissions during the Hotelling and limit pricing phases, respectively. π is the adjustment speed during the Hotelling pricing phase. T_1 and T_2 are the starts of limit pricing and the carbon-free era, respectively. $p(0)$ is the initial energy price.

The duration of limit pricing is the same for business as usual as for the safe carbon budget scenario. It is highest for a monopoly (288 years), lower for small and powerful oligopolies (e.g., 69 years if $N = 4$ or 33 years if $N = 8$) and dropping to zero as the number of fossil fuel producers becomes very large (e.g., 2.4 years for $N = 100$). For business as usual and open-loop Nash equilibrium, the duration of Hotelling pricing rises from 417 years for a monopoly ($N = 1$) to 435 years for a duopoly ($N = 2$). The duration then falls monotonically as the number of fossil fuel producers increases (e.g., to 410 years for $N = 100$). For business as usual and feedback Nash equilibrium, the pattern is the same albeit that the Hotelling pricing lasts phase is shorter now (especially for larger N). The total duration of the fossil fuel phase falls monotonically for

business as usual from 705 years for a monopoly to 413 or 382 years for 100 fossil fuel producers. The initial energy price is highest for a monopoly and then falls as the number of fossil fuel producers increases, more so for feedback than for open-loop Nash equilibrium. Initial fossil fuel use thus rises monotonically from 4 GtC to 7.7 GtC and 8.7 GtC per year for the open-loop and the feedback Nash equilibrium.

For the safe carbon budget scenario, the durations of limit pricing are unaffected but those for Hotelling pricing shorten drastically to ensure that cumulative emissions do not exceed 333 GtC. For monopolies and strong oligopolies (for N from 1 to 5) the Hotelling price phase becomes degenerate and the fossil fuel producers economy immediately switches to 33 years of limit pricing. Hence, the price of energy immediately drops to just below \$800/GtCe. For weaker oligopolies ($N > 5$), there is a non-degenerate phase of Hotelling pricing.

As the number of fossil fuel producers increases the safe carbon budget is spent more during the Hotelling pricing phase and less during the limit pricing phase. As was the case in section 6.1 for $N = 8$, the initial drop in fossil fuel prices relative to business as usual is smaller for the feedback than the open-loop Nash equilibrium for all other values of N too. This implies that the Green Paradox effects are smaller for the feedback than for the open-loop Nash equilibrium, especially for larger values of N . The point is that the strategic effect taken account of in the feedback Nash equilibrium already leads to more rapacious depletion.

7. Risk of credible enforcement of a cap on peak global warming

The premise so far was that the cap on peak global warming is *immediately* enforced. An alternative is that markets anticipate that at some unknown, future date the cap is enforced and deal with this political risk accordingly.¹⁶ I therefore now use a regime shift framework, where h is the constant probability or more precisely the hazard rate of the regime shift. The expected date of the future regime shift, i.e., the date at which renewable energy is finally subsidised sufficiently to just enforce the required carbon budget and cap on peak global warming, t^* , thus equals $1/h$. Formally, the conditional probability that the political tip towards an ambitious climate policy occurs at t^* is given by the hazard rate $h = \lim_{\Delta t \rightarrow 0} \Pr[t^* \in (t, t + \Delta t) | t^* \notin (0, t)] \Delta t$, so

$h \Delta t$ is the probability that the tip takes place between t and $t + \Delta t$, given that it has not occurred before t . To add dynamics to the risk of a political tip, one could let the hazard rate increase with global warming but I abstract from this here. I suppose that both before and after the

¹⁶ Van der Ploeg (2017) also uses regime shift analysis to discuss ongoing rather than one-off political regime changes, where the hazard of the political regime change is endogenous and depends on relative fighting efforts by government and rebel factions.

carbon budget is enforced the outcome of the oligopolistic fossil fuel markets is described by the feedback Nash equilibrium. The post-tip value function of producer i for the oligopolistic feedback Nash equilibrium once the renewable subsidy is implemented and the cap on peak warming are enforced is $V_i(S_1, \dots, S_N) = A_0 + A_1 S_i - 0.5 N A_2 S_i^2$. I denote the pre-tip value function of fossil fuel producer i by $W_i(S_1, \dots, S_N)$. The pre-tip problem solves for the FBNE outcome from the modified Hamilton-Jacob-Bellman equations:

$$(21) \quad rW_i(S_1, \dots, S_N) = \text{Max}_{R_i} \left[p \left(\sum_{j=1}^N R_j \right) R_i - G \left(\sum_{j=1}^N S_j \right) R_i - \sum_{j=1}^N W_{is_j} R_j \right] - h [W_i(S_1, \dots, S_N) - V_i(S_1, \dots, S_N)].$$

The last term on the right-hand side of (21) indicates the expected loss in instantaneous profits to fossil producer i resulting from the risk of policy makers starting to implement a tough climate policy from some future, unknown date onwards. The risk of a political tip is thus modelled as a regime shift. To simplify, I assume that the market assumes that the tip resulting from a political change in climate policy occurs before the market in the pre-tip, business-as-usual phase would have phased out fossil fuel entirely in favour of renewable energies.

Result 4: *Anticipation of a small political risk of a credible enforcement of a future cap on peak global warming at some unknown, future date implies that the price of fossil fuel during the Hotelling pricing phase adjusts faster than when there is no such political risk:*

$$(22) \quad p(t) = \pi_0 - \pi^{TIP} S(t) \text{ with } \pi_0 > 0, \pi^{TIP} > \pi^F > \pi^O > 0, 0 \leq t \leq T_1 \text{ and } p(T_1) = b.$$

The political risk accelerates fossil fuel extraction and global warming during the Hotelling pricing phase, and more so if the political risk of a toughening of climate policy is higher and the expected date when the carbon cap is enforced is earlier.

Proof: See Appendix 4.

A small risk of a future political regime shift that puts a cap on peak global warming speeds up the rate of fossil fuel extraction and accelerates global warming before the climate policy has even been put into place. This anticipation effect is called the Green Paradox effect and is bigger the bigger the magnitude of this political risk and the more competition there is on the global fossil fuel market.

8. Concluding remarks

The focus has been on symmetric oligopolistic fossil fuel markets with linear energy demand and linear extraction cost functions. Fossil fuel eventual becomes obsolete when an initially uncompetitive carbon-free perfect substitute for fossil fuel energy finally kicks in. Such a global energy market has an initial phase of falling fossil fuel depletion with Hotelling pricing, an intermediate phase of constant fossil fuel depletion with limit pricing, and a final phase where only renewable energy is used. Making renewable energy cheaper to credibly enforce a cap on global warming depresses cumulative emissions and increases stranded fossil fuel assets. Such a policy also cuts fossil fuel use during limit pricing and ensures that more fossil fuel reserves and carbon are locked up at the end of Hotelling pricing. Limit pricing lasts longer if there are fewer fossil fuel producers, the interest rate is high, energy demand is less sensitive to the price and extraction costs react less to remaining reserves, but its length is unaffected by autonomous energy demand or the cost of renewable energy. However, during Hotelling pricing the market reacts by pumping oil and gas more vigorously. This race to burn the last ton of carbon is more intense if the global fossil fuel market is more competitive.¹⁷ Weaker oligopolies speed up extraction and bring forward limit pricing and the onset of the carbon era, and more so for the feedback than the open-loop Nash equilibrium. Put differently, more competitive fossil fuel markets bring forward limit pricing and the carbon-free era.

There is unwanted heating of the planet in the short run due to the race to burn the last ton of carbon. This Green Paradox effect is stronger for the open-loop than for the feedback Nash equilibrium, since the latter allows for a strategic effect that speeds up extraction already. The phase of Hotelling pricing in the business-as-usual scenario is very long indeed, so that limit pricing is for practical intents and purposes irrelevant. However, for the safe carbon budget scenario where cumulative emissions must be slashed considerably, the duration of Hotelling pricing shortens drastically and therefore the phase of limit pricing and keeping renewable energy producers at bay is in the foreseeable future. In fact, I find that for monopolies and very strong oligopolies the Hotelling pricing phase becomes degenerate and enforcing the safe carbon budget leads to limit pricing from the beginning. This is not the case for weak oligopolies or competitive markets.

The mere risk of a political regime shift towards enforcement of a cap on peak warming at some unknown, future date also induces a race to burn the last ton of carbon, and especially so if the perceived risk of a political regime shift is high. The political risk accelerates fossil fuel

¹⁷ In practice, increased competition may engender more technological innovation and thus lower extraction costs. This would boost the rate of fossil fuel extraction also.

extraction and global warming during Hotelling pricing, especially if the expected date when the carbon cap is enforced is earlier. Furthermore, if the risk of a political regime shifts rises with global temperature, I conjecture that the race to burn the last of carbon is further intensified. Although the planet will heat in the short run, peak warming will be curbed provided the cap is credible and enforced.

A crucial proviso is that I have abstracted from irreversible exploration and exploitation investments and from internal adjustment costs. Such features carry the political risk that oil and gas producers end up holding carbon assets that will be worthless and substantial sunk costs must be written off when it is anticipated that there is a risk that policy makers will take credible action to combat planetary heating.¹⁸ A related issue is that the risk of stranded assets only starts to bite when certain green investments have been made and this takes time.¹⁹ In future research it is important to address these issues and how these affect behaviour of companies and other private agents as well as that of institutional investors and policy makers.²⁰

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¹⁸ Offsetting factors are that certain investments of oil and gas producers such as pipelines can also be used for transport of electricity, abandoned oil and gas platforms can also be used for the storage of hydrogen and synthetic gas made from energy from seawater and offshore windfarms (as happens in the North Sea), and intimate knowledge of foreign countries, etc. can have value for other business too.

¹⁹ For example, to speed up the transition to green energy in northern Europe and New England an alternative is needed consisting of perhaps a combination of electricity cables from to the southern part of Europe and the US, algae, windmills and energy storage via hydrogen production. Only once such a cost-effective alternative is anticipated by the market will a race to burn the last ton of carbon take place.

²⁰ Now it is cheap to hedge against the risk of climate policy being tightened in the future (Anderssen et al., 2016). As more agents hedge against climate risk, such hedges will become more expensive.

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Appendix 1: Derivation of the subgame-perfect Nash equilibrium outcome

Focusing at the Hotelling pricing phase, the FBNE solution requires solving the Hamilton-

Jacobi-Bellman (HJB) equations $rV_i(S_1, \dots, S_N) = \text{Max}_{R_i} \left[p \left(\sum_{j=1}^N F_j \right) F_i - G(S_i) F_i - \sum_{j=1}^N V_{is_j} F_j \right]$ for

each of the fossil fuel producers. The optimality condition is $\left(1 - \frac{1}{N\varepsilon}\right)p = G(S_i) + s_i$, $i = 1, \dots, N$,

where $s_i = V_{is_i}$. Conjecturing the quadratic value functions

$$(A1) \quad V_i(S_1, \dots, S_N) = A_0 + A_1 S_i - 0.5 N A_2 S_i^2 + \sum_{j=1, j \neq i}^N [A_3 S_j - 0.5 A_4 S_j^2 + A_5 S_i S_j],$$

the rents are $s_i = A_1 - A_2 N S_i + A_5 \sum_{j=1, j \neq i}^N S_j$ and thus from the optimality condition

$$(A2) \quad \frac{N\varepsilon - 1}{N\varepsilon} p = \frac{N+1}{N} p - \frac{\delta_0}{N\delta_1} = \gamma_0 + A_1 - (\gamma_1 + A_2) N S_i + A_5 \sum_{j=1, j \neq i}^N S_j.$$

This gives the fossil fuel price rule

$$(A3) \quad p = \frac{N}{N+1} \left(\frac{\delta_0}{N\delta_1} + \gamma_0 + A_1 \right) - \frac{N}{N+1} \left[(\gamma_1 + A_2) N S_i - A_5 \sum_{j=1, j \neq i}^N S_j \right].$$

with $\pi = \frac{N}{N+1} (A_2 + \gamma_1)$. Defining $\pi_0 \equiv b + \pi S(T)$, the aggregate HJB equation in terms of

$V = \sum_{i=1}^N V_i(S_1, \dots, S_N) = A_0 + A_1 S - \frac{1}{2} A_2 S^2$ boils down to

$$(A4) \quad r(A_0 + A_1 S - \frac{1}{2} A_2 S^2) = (\pi_0 - \pi S - \gamma_0 + \gamma_1 S - A_1 + A_2 S) [\delta_0 - \delta_1 (\pi_0 - \pi S)].$$

Equating coefficients on S^2 , one gets $-\frac{r}{2} A_2 = (\gamma_1 + A_2 - \pi) \delta_1 \pi$ or $A_2 = \frac{2\delta_1 \pi}{r + 2\delta_1 \pi} (\pi - \gamma_1)$.

Putting this into the definition of π , one obtains $\pi = \frac{N}{N+1} \left[\frac{2\delta_1 \pi}{r + 2\delta_1 \pi} (\pi - \gamma_1) + \gamma_1 \right]$ or

$2\delta_1 \pi^2 + r(N+1)\pi - rN\gamma_1 = 0$. Hence, the FBNE solution is (14) as for the OLN solution

except that π is replaced by $\pi = \frac{r(N+1)}{4\delta_1} \left[\sqrt{1 + \frac{8N}{(N+1)^2} \frac{\gamma_1 \delta_1}{r}} - 1 \right] \equiv \pi^F$. This proves (15').

Note that equating coefficients on constants and S gives $A_0 = -\delta_1 \pi_0 (\pi_0 - \gamma_0 - A_1) / r$ and

$$A_1 = [(\pi_0 - \gamma_0) \delta_1 \pi - (\pi_1 - \gamma - A_2) (\delta_0 - \delta_1 \pi_0)] / (r + \delta_1 \pi).$$

Subtracting the two algebraic equations that yield (15) and (15'), respectively, gives

$$(A5) \quad 2\delta_1 \left[(\pi^F)^2 - (\pi^O)^2 \right] + r(N+1)(\pi^F - \pi^O) = (N+1-2)\delta_1 (\pi^{OLNE})^2.$$

The right-hand side of this equation is positive for all $N > 1$, hence the left-hand side must be positive too and thus $\pi^F > \pi^O > 0$ for all $N > 1$. For $N = 1$, $\pi^F = \pi^O$. Note that $\frac{\partial \pi^F}{\partial N} > 0$ if

$$(N+1) \left[\sqrt{1 + \Gamma \frac{N}{(N+1)^2}} - 1 \right] \text{ rises with } N, \text{ where } \Gamma \equiv \frac{8\gamma_1\delta_1}{r}. \text{ This is the case if}$$

$$(N+2)(N+1) + (\Gamma-2)N > \sqrt{(N+2)^2 + \Gamma N} \quad \text{or after squaring both sides if}$$

$$(N+2)N^2 + 4N + \Gamma^2 N + \Gamma(2N^2 + 3 + 2N) > 0. \text{ Hence, } \pi^F \text{ rises with } N.$$

Appendix 2: Cartel of fossil fuel producers

A cartel of fossil fuel producers maximises joint net worth

$$(A6) \quad V \equiv \sum_{i=1}^N V_i = \int_0^{\infty} [p(R+F)F - G(S_i)F] e^{-rt} dt$$

subject to the fossil fuel depletion equations

$$(A7) \quad \dot{S}_i = -F_i, \quad \int_0^{\infty} F_i(t) dt \leq S_{0i}, \quad i = 1, \dots, N,$$

and $p(t) \leq b$, $R(t) \geq 0$, c.s., $\forall t \geq 0$. The Hamiltonian function for the cartel boils down under Assumptions 1 and 2 to $H \equiv \delta_1^{-1}(\delta_0 - F)F - (\gamma_0 - \gamma_1 S)F - sF$, where s is the scarcity rent for the aggregate stock of fossil fuel. This yields the optimality conditions

$$\frac{\partial H}{\partial F} = \delta_1^{-1}(\delta_0 - 2R) - (\gamma_0 - \gamma_1 S) - s \leq 0, \quad F \geq 0, \text{ c.s.}, \quad \text{and} \quad rs - \dot{s} = \frac{\partial H}{\partial S} = \gamma_1 F.$$

Hotelling pricing regime, the first condition gives $F = \frac{\delta_0 - \delta_1(\gamma_0 - \gamma_1 S + s)}{2}$ or

$s = 2p - \gamma_0 + \gamma_1 S - \delta_0 / \delta_1$. The second condition gives $\dot{s} = rs - \gamma_1 F$. Hence, one gets

$$(A8) \quad \dot{S} = \delta_1 p - \delta_0, \quad \dot{p} = rp - \frac{r}{2}(\delta_0 \delta_1^{-1} + \gamma_0 - \gamma_1 S).$$

For $N = 1$, (13) boils down to (A8). Hence, the cartel outcome corresponds to the case $N = 1$.

Appendix 3: Proof of Result 2

The price paths in the left panel of Figure 2 with and without the renewable subsidy have the same slope π , so the Green Paradox emerges from (14) and (13) if $\partial p(0)/\partial v = -1 + \pi/\gamma_1 < 0$ or $\pi < \gamma_1$. Using expression (15) for the open-loop Nash equilibrium, it must be the case that

$$1 + \frac{4N}{N+1} \frac{\gamma_1 \delta_1}{r} < \left(1 + 2 \frac{\gamma_1 \delta_1}{r}\right)^2. \quad \text{As} \quad 1 + \frac{4N}{N+1} \frac{\gamma_1 \delta_1}{r} < 1 + \frac{4\gamma_1 \delta_1}{r} < \left(1 + 2 \frac{\gamma_1 \delta_1}{r}\right)^2, \quad \partial p(0)/\partial v < 0.$$

Using (15') for the feedback Nash equilibrium, it can be shown that

$$\frac{r(N+1)}{4\delta_1} \left[\sqrt{1 + \frac{8N}{(N+1)^2} \frac{\gamma_1 \delta_1}{r}} - 1 \right] < \gamma_1 \quad \text{or} \quad 1 + \frac{8N}{(N+1)^2} \frac{\gamma_1 \delta_1}{r} < 1 + \frac{8}{N+1} \frac{\gamma_1 \delta_1}{r} < \left(1 + \frac{4}{N+1} \frac{\gamma_1 \delta_1}{r}\right)^2.$$

Hence, $\partial p(0)/\partial v < 0$ again. \square

Appendix 4: Proof of Result 3

The optimality condition is (5), where $s_i = W_{is_i}$. Conjecturing the pre-tip value functions

$W_i(S_1, \dots, S_N) = B_0 + B_1 S_i - 0.5 N B_2 S_i^2$, one gets $s_i = B_1 - B_2 N S_i$ and thus from (5) also

$$\frac{N\varepsilon - 1}{N\varepsilon} p = \frac{N+1}{N} p - \frac{\delta_0}{N\delta_1} = B_1 + \gamma_0 - (B_2 + \gamma_1) N S_i \quad \text{and thus the feedback rule for the fossil fuel}$$

price is (22) with $\pi^{TIP} = \frac{N}{N+1} (B_2 + \gamma_1)$ and $\pi_0 = \frac{N}{N+1} (B_1 + \gamma_0) + \frac{1}{N+1} \frac{\delta_0}{\delta_1}$. The aggregate HJB

equation corresponding to (21) in terms of $W = \sum_{i=1}^N W_i(S_1, \dots, S_N) = B_0 + B_1 S - \frac{1}{2} B_2 S^2$ becomes

$$(A9) \quad (r+h)(B_0 + B_1 S - \frac{1}{2} B_2 S^2) = (\theta_0 - \theta S - \gamma_0 + \gamma_1 S - N B_1 + B_2 S) \left[\delta_0 - \delta_1 (\pi_0 - \pi^{TIP} S) \right] + h(A_0 + A_1 S - \frac{1}{2} A_2 S^2).$$

Equating coefficients on S^2 , it follows that $-\frac{1}{2}(r+h)B_2 = (\gamma_1 + B_2 - \pi^{TIP})\delta_1\pi^{TIP} - \frac{1}{2}hA_2$ or

$$(A10) \quad B_2 = \frac{2\delta_1\pi^{TIP}(\pi^{TIP} - \gamma_1) + hA_2}{r+h+2\delta_1\pi^{TIP}}.$$

Putting this into the definition of π^{TIP} , I get $\pi^{TIP} = \frac{N}{N+1} \left[\frac{2\delta_1\pi^{TIP}(\pi^{TIP} - \gamma_1) + hA_2}{r+h+2\delta_1\pi^{TIP}} + \gamma_1 \right]$ and

thus $2\delta_1(\pi^{TIP})^2 + (r+h)(N+1)\pi^{TIP} - N[(r+h)\gamma_1 + 2hA_2] = 0$. The positive solution of this quadratic equation is

$$(A11) \quad \pi^{TIP} = \frac{(r+h)(N+1)}{4\delta_1} \left[\sqrt{1 + \frac{8N\delta_1}{(N+1)^2(r+h)^2} [(r+h)\gamma_1 + 2hA_2]} - 1 \right].$$

It follows that $\pi^{TIP} = \frac{r(N+1)}{4\delta_1} \left[\sqrt{1 + \frac{8N\delta_1}{(N+1)^2} \frac{\gamma_1}{r}} - 1 \right] \rightarrow \pi^F$ as $h \rightarrow 0$. Furthermore, define

$$X \equiv 1 + \frac{8N\delta_1}{(N+1)^2(r+h)^2} [(r+h)\gamma_1 + 2hA_2] > 0, \text{ so } \frac{\partial \pi^{TIP}}{\partial h} = \frac{N+1}{4\delta_1} \left[\sqrt{X} - 1 + (r+h) \frac{1}{2\sqrt{X}} \frac{\partial X}{\partial h} \right] \text{ with}$$

$$\frac{\partial X}{\partial h} = \frac{8N\delta_1}{(N+1)^2(r+h)^3} \left((\gamma_1 + 2A_2)(r+h) - 2[(r+h)\gamma_1 + 2hA_2] \right) = \frac{8N\delta_1}{(N+1)^2(r+h)^3} [2A_2(r-h) - (r+h)\gamma_1].$$

Hence, $\partial X / \partial h > 0$ and thus $\partial \pi^{TIP} / \partial h > 0$ as $h \rightarrow 0$, so that $\pi^{TIP} > \pi^F$ for $h > 0$. In view of Result 2 and (15'), $\pi^{TIP} > \pi^F > \pi^O$. Finally, equating coefficients on S , it follows that

$$(A11) \quad B_1 = \frac{(\pi_0 - \gamma_0)\delta_1\pi^{TIP} + (\gamma_1 + B_2 - \pi^{TIP})(\delta_0 - \delta_1\pi_0) + hA_1}{r+h+N\delta_1}.$$

So $\pi_0 = \frac{N}{N+1}(B_1 + \gamma_0) + \frac{1}{N+1} \frac{\delta_0}{\delta_1}$ and the feedback rule for the price (22) follows.

The price rule (22) implies that initial prices are lower and initial fossil fuel extraction rates are higher with a positive risk of a political regime shift and therefore initially carbon emissions are higher too. \square