Flexible Labour, Income Effects, and Asset Prices

Rahul Nath

Number 851
May, 2018
Flexible Labour, Income Effects, and Asset Prices

Rahul Nath

May, 2018

Abstract

This paper studies how flexible labour decisions affect asset pricing in a Real Business Cycle model. It uses Jaimovich-Rebelo preferences with internal habits in consumption and distinguishes between two income effect channels (i) the ‘habit income effect’ channel and (ii) the ‘separability income effect’ channel. I find that asset prices are superior when the first channel is strong and the second is weak, this is the case of using GHH preferences with internal habits in consumption.

Keywords: Asset Pricing, Income Effects, Jaimovich-Rebelo Preferences

JEL Codes: E13, E32, E44, G12

*I would like to thank David Vines, Guido Ascari and Martin Weale for useful discussions, feedback and comments.
†Exeter College, University of Oxford, E-mail address: rahul.rb.nath@gmail.com
Macroeconomic asset pricing models simultaneously hold that (i) there are strong internal habits in consumption, and (ii) there are strong adjustment costs to capital or investment. In order to reconcile these beliefs with observed asset pricing facts, these models are also forced to hold that (iii) labour is somehow contemporaneously restricted in response to a neutral technology shock. The intuition here is simple. Consider a prototypical DSGE model, agents have three instruments available to achieve smooth consumption - labour, investment and consumption. Habits in consumption induce inertia in consumption, while capital adjustment costs (or similar) induce inertia in investment. This forces households to mercilessly exploit the only channel that remains frictionlessly open to them - labour. Thus agents achieve their consumption smoothing goal via income smoothing. By restricting the contemporaneous response of labour forces agents once again to face the costly inter-temporal trade-off between consumption and investment. Consequently one achieves an improvement in quantitative asset pricing results.

Jermann (1998) and Boldrin et al. (2001) provide salient examples of this in the literature. Jermann (1998) finds that in a model with inelastic labour, habits in consumption and capital adjustment costs joint replication results are superior to those without all three features. In their preferred two-sector model, Boldrin et al. (2001) require limited inter-sectoral mobility to achieve their superior asset pricing results.\(^1\) Indeed, Boldrin et al. (2001) state “inflexibilities in labour are a crucial ingredient for producing an equity premium.” (Boldrin et al., 2001) This prescription appears to have driven this literature ever since.

This paper has two objectives. First, it brings together the macroeconomic asset pricing literature with the recent macroeconomic literature that studies the income effect\(^2\) of labour supply in macroeconomic models. Second, it provides a study of how the cyclical behaviour of labour hours affects macroeconomic asset pricing via various income effect channels.

Under standard preferences, the above arguments would appear to doom any attempt at introducing flexible labour into a macroeconomic asset pricing model. However, there has been much recent work on Jaimovich-Rebelo (JR) preferences (Jaimovich and Rebelo, 2009) which allow flexible labour but permit one

\(^1\)This assumption requires that factors of production cannot move between sectors in the period of the shock, i.e. all reallocation occurs via inter-temporal channels, therefore making labour supply in each of their two sectors inelastic in the period of the shock.

\(^2\)This is also referred to as the wealth effect in the literature. The terms income effect and wealth effect will be used interchangeably in this paper, and should be read to mean the same concept.
to control the size of the income effect. JR preferences nest as polar cases the GHH specification of Greenwood et al. (1988), and the KPR specification of King et al. (1988), both of which have standard in macroeconomic models. This class of preferences has been used extensively in the literature on news effects on business cycles (e.g. Jaimovich and Rebelo (2009) and Schmitt-Groh and Uribe (2012)) however it’s use in the macroeconomic asset pricing literature seems appears missing.

In this paper I study JR preferences with internal consumption habits. JR preferences with habits are studied in Schmitt-Groh and Uribe (2012) in the context of news shocks and business cycles, and more recently by Holden et al. (2017) in the context of reconciling these preferences with estimates of the Frisch elasticity of labour supply. In this paper I show that in the presence of internal consumption habits there are two income effect channels induced under these general JR preferences. The first is habit induced, and is referred to as the ‘habit income effect’ channel. While the second is a result of the degree of linear separability between consumption and labour within the utility function. This second income effect channel is referred to loosely as the ‘separability income effect’ channel. I further show that quantitative asset pricing results are always superior under GHH preferences than KPR preferences at all habit intensities, and that this difference can be directly related to these two income effect channels. Finally I highlight that a non-zero income effect can be induced in GHH preferences via internal habits, a feature which allows one to induce any cyclical response of labour.

This paper shows that a strong ‘habit income effect’ with a weak ‘separability channel’ is the combination of the two income effect channels that performs best at asset price replication in a flexible labour setting. This result is encouraging given empirical estimates of Schmitt-Groh and Uribe (2012) also find this combination useful. On the other hand, Dey (2014) finds that an intermediate value for both is useful, however Dey (2014) does not allow for the ‘habit income effect’ in the GHH specification. It must be stressed that there is very limited empirical evidence on the size of the ‘separability channel.’ The preferred utility specification in this paper is the GHH specification augmented with internal habits, while the KPR analog is used primarily as a comparison device.

Throughout this paper I study asset prices in a Real Business Cycle (RBC) model. This is not because I believe that nominal rigidities are not impor-
tant for asset pricing, but rather because the literature on asset pricing in New Keynesian models is very young. In contrast the macroeconomic asset pricing literature in RBC models is sufficiently mature to permit comparability of results across various papers in the literature.

This paper is organised as follows. Section 1 provides a broader motivation for this paper. Section 2 presents the general RBC model studied. Section 3 introduces the labour market in detail and presents the key theoretical results. Section 4 discusses the parametrisation and solution method. Section 5 presents results on the labour dynamics and proposes an alternative calibration scheme. Section 6 studies the asset pricing implications of flexible endogenous labour, and Section 7 concludes.

1 Motivation

In the years since the global financial crisis there has been significant renewed interest in the links between the real economy and financial economy in modern macroeconomic models. Given the circumstances of policy making during the crisis, where monetary policy was tasked with both macroeconomic stability and financial stability, of particular interest is the transmission mechanism via which monetary policy seeking to stabilise the real economy affects the financial economy. Going the other way, one is also interested in how macroeconomic policy seeking financial stability might affect real variables, and hence interact with policy that is simultaneously seeking some degree of stability in macroeconomic variables, via the conduct of monetary policy.

The study of monetary policy mechanisms in modern macroeconomic models requires one to posit the presence of nominal rigidities. Such nominal rigidities introduce inefficiencies into allocation decisions, and hence move the economy to a second best equilibrium that can only be improved on through policy intervention. The labour market plays a central role in such New Keynesian DSGE models, and it is via labour market that the presence of nominal rigidities are generally justified. However, as discussed above, macroeconomic asset pricing results are undermined by the presence of an endogenous labour market that responds contemporaneously to a shock. Consequently there has been little coherent treatment and development of asset pricing models in the presence of nominal rigidities. This study considers specifically labour movements along

\footnote{\text{e.g. one could argue that price change is sluggish because firms need to continue paying wage contracts that have already been negotiated}}\footnote{\text{Notable exceptions are Wei (2009), De Paoli et al. (2010), Challe and Giannitsarou (2014) and Swanson (2015). However, none of these papers explicitly studies the impact of labour}}
the intensive margin, i.e. the number of hours worked. This is the margin along which nominal rigidities are usually assumed to operate. That is, nominal rigidities affect decisions about how many hours to work rather than the employment decision. The analysis along the extensive margin, i.e. the employment margin, as has been studied recently in Kuehn et al. (2017) who use search and matching frictions to study how this margin affects asset prices.

The motivation behind this paper is that it provides an initial, and necessary, step in the path towards a more mature asset pricing literature in models with nominal rigidities. It seeks to provide an analysis of asset prices with flexible labour markets in the absence of nominal rigidities. Thus it permits one to disentangle the impact nominal rigidities have on asset prices via features that limit labour market flexibility once such rigidities are introduced. For this reason the objective is not to match quantitative asset pricing facts. Indeed, one expects that the quantitative results will differ from the data due to the simplicity of the model studied. Rather, more modestly, I seek to study an important mechanism that has been hitherto discounted in the macroeconomic asset pricing literature and is of central importance if one seeks to understand how macroeconomic policy affects asset prices.

2 The Model

The model is a Real Business Cycle (RBC) model augmented with internal consumption habits and capital adjustment costs. It consists of infinitely lived, homogeneous households and identical perfectly competitive firms. Households own all of the factors of production and allocate resources over time to maximise lifetime utility. Firms use the available resources and production technology to produce output. Finally the economy is subject to aggregate uncertainty via random shocks to the productivity of technology.

2.1 Households

Households seek to maximise lifetime utility,

$$U_t = E_t \left\{ \sum_{j=0}^{\infty} \beta^j u \left( c_{t+j} - \nu c_{t+j-1}, 1 - n_{t+j}^s \right) \right\}$$

(2.1)

where $\beta$ is the time discount factor. The period utility of the household depends on consumption, $c$, and labour supply $n^s$. It exhibits internal habits in market decisions on asset pricing.
consumption, the strength/intensity of which is governed by the parameter $\nu$.

The period budget constraint for the household is,

$$c_t + i_t \leq w_t n_t + r_t k_t$$

(2.2)

Household income is derived from rental of the factors of production to firms via perfectly competitive factor markets. They supply labour at the competitive wage rate $w_t$ and rent capital to firms at the competitive rental rate $r_t$. This income is used to purchase the consumption good, $c_t$ and invest in capital, $i_t$.

As owners of capital, households face the capital accumulation process,

$$k_{t+1} = (1 - \delta)k_t + \phi \left( \frac{i_t}{k_t} \right) k_t$$

(2.3)

where $\delta$ is the fixed depreciation rate. Capital accumulation consists of two components; undepreciated capital and investment. Investment is subject to capital adjustment costs, governed by $\phi(\cdot)$, which assumes that resources are required to transform investment into capital. This introduces inertia into the accumulation of capital since it takes several periods to achieve the desired levels of capital. Non-linearity of the function $\phi(\cdot)$ leads to non-linearity in the accumulation of capital. In particular there is a non-linearity in how the relative proportion of investment, captured by the investment-capital ratio, translates into additional capital.\(^6\) The functional form for the adjustment cost function is given by the following split function,

$$\phi \left( \frac{i}{k} \right) = \begin{cases} 
    b_1 \left( \frac{i}{k} \right)^{-\frac{1}{\xi}} + b_2 & \text{if } i > 0 \\
    0 & \text{if } i = 0
\end{cases}$$

(2.6)

This general functional form is used in both Jermann (1998) and Boldrin et al. (2001).\(^7\) It separates the conversion of investment into a fixed component and a variable component. The parameter $\xi$ governs the curvature of the adjustment cost function, and hence the penalty imposed on investment. As $\xi$ falls there is greater curvature and hence stronger adjustment costs. Finally, as $\xi \to \infty$ the

\(^6\)The function $\phi(\cdot)$ is assumed to have the following properties,

$$\phi'(\cdot) > 0, \quad \phi''(\cdot) < 0,$$

$$\phi(0) = 0, \quad \phi(\delta) = \delta.$$  

(2.4)

(2.5)

\(^7\)The lower branch of this split function ensures that if there is no investment then there are no adjustment costs, so that it satisfies the regulatory conditions in (2.5). In the RBC model considered here consumption smoothing arguments ensure that there will always be positive investment. As such one can safely ignore the lower branch as it will never be invoked.
functional form collapses to a one without capital adjustment costs.\(^8\)

Households optimisation of lifetime utility subject to the budget constraint and capital accumulation process yields the standard conditions.

### 2.2 Firms

Firms seek to maximise lifetime profit,

\[
\Omega_t = E_t \left\{ \sum_{j=0}^{\infty} m_{t,t+j} \left[ y_{t+j} - r_{t+j}k_{t+j} + w_{t+j}n_{t+j}^d \right] \right\}
\]

(2.7)

where \( y \) is output and \( n^d \) is labour demand. The firm must pay wages, \( w \), for each unit of labour employed, \( n^d \), and rental rate, \( r \), on each unit of capital employed, \( k \). The factor prices are determined in perfectly competitive factor markets. Firms are owned by households so discount future profits at the household’s stochastic discount factor.

The only restriction on the firm is that production be feasible given the production technology available to the firm, characterised by the neo-classical production function \( f(\cdot) \). The production function takes the Cobb-Douglas form,

\[
y_t = z_t f(k_t, n_t) = z_t k_{t}^{\alpha} n_{t}^{1-\alpha}
\]

(2.8)

where \( \alpha \in (0, 1) \) is the capital share of production. This technology is assumed to be subject to random shocks to total factor productivity, \( z_t \), which follows an auto-regressive process, i.e.

\[
\ln z_{t+1} = \rho \ln z_t + \varepsilon_t
\]

(2.9)

where \( \rho \) measures the persistence of the shock and \( \varepsilon_t \) is a random shock. Households, as owners of capital, face the intertemporal decisions in the economy. Thus firms, freed of all intertemporal decisions, maximise lifetime profit, (2.7),

\(^8\)Steady state equations imply that \( b_1 = \delta \frac{\xi}{1-\xi} \) and \( b_2 = \frac{\delta}{1-\xi} \). So as \( \xi \to \infty \) one gets the following,

\[
b_1 = \delta \frac{\xi}{1-\xi} \to 1,
\]

\[
b_2 = \frac{\delta}{1-\xi} \to 0,
\]

\[
\frac{1}{1-\xi} \left( \frac{i}{k} \right)^{1-\frac{\xi}{1}} \to \frac{i}{K},
\]

so that the adjustment cost function approaches a linear function. Thus the capital accumulation function approaches one that does not exhibit capital adjustment costs.
by maximising the period profit subject to output being feasible. This yields the standard first order conditions that the factor prices are equal to their marginal products.

2.3 Equilibrium

In equilibrium all markets clear, i.e.

\[ y_t = c_t + i_t, \]
\[ n_t^s = n_t^d = n_t \]

The full macroeconomic system under equilibrium is provided in Appendix A.

3 Endogenous Labour Supply and Preference Structures

This paper seeks to understand how the presence of endogenous labour supply decisions affects the asset pricing results. The labour market in the model is characterised on the supply side by,

\[ w_t = -\frac{u_{n,t}}{\Lambda_t}, \quad (3.1) \]

and on the demand side by,

\[ w_t = z_t f_{n,t} \quad (3.2) \]

where \( f_{n,t} \) is the marginal product of labour, \( u_{n,t} \) is the marginal utility of labour, and \( \Lambda_t \) is the marginal utility of consumption. In the absence of habits one has \( \Lambda_t = u_{c,t} \). While in the presence of internal habits it takes form,

\[ \Lambda_t = u_{c,t} - \beta \nu E_t [u_{c,t+1}], \quad (3.3) \]

i.e. it accounts for the impact a change in contemporaneous consumption has on future marginal utility. In equilibrium the labour market clears so that the two equations are equal.

Define the substitution effect as the shift of the labour demand curve, (3.2), in response to the technology shock. The income effect is defined as the shift of the labour supply curve due to non-labour factors. Since capital is predetermined it will not respond contemporaneously to the shock. Thus labour demand does not respond contemporaneously to any non labour factors. Consequently the income effect is solely a labour supply phenomenon while the substitution
effect is solely a labour demand phenomenon. Taken together these shifts in labour demand and labour supply determine equilibrium in the labour market.

3.1 Preference Structures and Income Effects

As the income effect is a labour supply phenomenon, the specification of the preference structure is of crucial importance. In this section I show that, for a very general class of preferences, an income effect is absent only if (i) there are no internal consumption habits and (ii) consumption enters the utility function in a linearly additive manner.

Let us consider preference structures where the period utility takes the following general form,

\[ u(c_t, c_{t-1}, n_t) = U(c_t - \nu c_{t-1} - G(n_t) X_t) \] (3.4)

\[ X_t = (c_t - \nu c_{t-1})^{1-\gamma} X_{t-1}^{\gamma} \] (3.5)

This is the general form for JR preferences with internal habits in consumption. This general form nests, as extremes, the general form for GHH preferences with internal habits when \( \gamma = 0 \), and the general form for KPR preferences with internal habits when \( \gamma = 1 \).

Under this general form the labour supply function is,

\[ w_t = \frac{G'(n_t) X_t}{\left(1 - \gamma \frac{X_t}{c_t - \nu c_{t-1}} G(n_t)\right) - \beta \nu E_t \left(1 - \gamma \frac{X_{t+1}}{c_{t+1} - \nu c_t} G(n_t)\right) U_{t+1}} \] (3.6)

where \( U_t' \) denotes the derivative of the function \( U(\cdot) \) in period \( t \). Hence we see that the labour supply function can be written solely as a function of labour if and only if both (i) there are no internal consumption habits and (ii) consumption enters the utility function in a linearly additive manner. That is,

\[ w_t = G'(n_t) \iff \nu = 0 \quad \gamma = 0 \] (3.7)

so that non-labour factors do not affect labour supply and hence there is no income effect.\(^9\) If either of these conditions is not met then an income effect will be present. As \( \nu \) or \( \gamma \) vary, an income effect emerges. The income effect can be decomposed into two channels (i) that due to habits - the ‘habit income effect’ channel, and (ii) that due to non-separability of consumption and labour within the utility function - the ‘separability income effect’ channel. Thus calibration

\(^9\)This is briefly alluded to in Schmitt-Groh and Uribe (2012) but not pursued in that paper.
of \(\nu\) and \(\gamma\) will have implications for the cyclical behaviour of labour. One notes that an income effect will always be present in KPR preferences. However, in the case of GHH preferences \((\gamma = 0)\) the income effect is a consequence of the presence of internal habits and is therefore habit induced.\(^{10}\)

The minimum combination of frictions in asset pricing models attempting to achieve quantitative replication are internal consumption habits together with capital adjustment costs. In such models one will necessarily have an income effect resulting from the presence of internal habits. This ‘habit income effect’ could also possibly be augmented by some degree of non-separability between labour and consumption depending on the specific preferences used, resulting in an additional ‘separability income effect.’ Thus the behaviour of labour will play an important role in the dynamics of the economy and consequently asset prices. The treatment of labour in the asset pricing literature seeks to marginalise this income effect channel induced by internal habits.

I exploit the presence of the habit induced income effect in GHH preferences to study how the endogenous movement of labour affects asset pricing results. In particular the use of GHH preferences allows one to study how changes in the cyclical behaviour of labour brought about by changes in the income effect channel affects quantitative asset pricing results. On the other hand, KPR preferences allow one to keep habit constant in order to study the varying income effect induced by changing \(\gamma\). Furthermore, by looking at the polar extremes allows one to fully characterise the range of responses for JR preferences without having to explicitly compute the results for all values of \(\gamma\), this is especially useful given the empirical uncertainty surrounding \(\gamma\).

I consider GHH preferences of the following functional form,

\[
u(c_t, c_{t-1}, n_t) = \frac{1}{1 - \sigma} \left[ (c_t - \nu c_{t-1}) - \chi n_t^{1 + \varphi} \right]^{1 - \sigma} \tag{3.8}
\]

where \(\sigma\) measures the degree of risk aversion, \(\chi\) is the utility weight of the disutility of labour, \(\varphi\) is the inverse of the Frisch elasticity of labour supply and

\(^{10}\)The income effect only emerges for GHH preferences in the presence of internal habits. Were the habits external then the equilibrium labour condition would be unchanged from the no habit case. Thus \(\gamma = 0\) would be a sufficient condition to eliminate the income effect. External habit models were considered but have not been reported as they induce an unacceptably high level of volatility in the risk free rate. Furthermore, preferences with external habits are not able to jointly account for moderate levels of risk aversion and moderate levels of risk aversion in wealth. (Boldrin et al., 1997)
\( \nu \) is the habit intensity. The KPR analog to the GHH functional form is,

\[
u(c_t, c_{t-1}, n_t) = \frac{1}{1 - \sigma} \left( c_t - \nu c_{t-1} \right) \left( 1 - \chi \frac{n_t^{1+\varphi}}{1 + \varphi} \right)^{1-\sigma} \tag{3.9}
\]

4 Parameterisation and Solution Method

The preference parameters requiring calibration are the rate of time preference \((\beta)\), the Frisch elasticity of labour supply \((\varphi)\), the degree of risk aversion \((\sigma)\), the measure of labour disutility \((\chi)\), the habit intensity \((\nu)\), and the degree of separability \((\gamma)\).

In line with the literature I calibrate \(\beta = \frac{1}{1.0037}\) to match the average quarterly return on U.S. T-Bills over the period 1970, Q1 to 1998, Q4. The parameter \(\chi\) measures the utility weight of the disutility of from labour and is parametrised so that one-third of time is spent working in the deterministic steady state.

Monacelli and Perotti (2008) provides a mapping of the Frisch elasticity of substitution into a GHH setting from a number of authors. Mapping these values across to our functional form implies that \(\varphi = \zeta - 1\), where \(\zeta\) is the inverse of the Frisch elasticity in Monacelli and Perotti (2008). Under this transform the parametrisations of \(\varphi\) range from \(\varphi = \frac{1}{1.55}\) (Smets and Wouters, 2007) to \(\varphi = \frac{1}{1.4}\) (Schmitt-Groh and Uribe, 2012; Jaimovich and Rebelo, 2009), while Monacelli and Perotti (2008) use an intermediate value of \(\varphi = \frac{1}{1.8}\), which I also use here. This calibration is supported by Chetty et al. (2011) who suggest that Frisch elasticities of aggregate labour supply above 1 are inconsistent with micro evidence.

GHH preferences lead to a decoupling of IES from the coefficient of risk aversion.\(^{11}\) (Jahan-Parvar et al., 2013). The ability of GHH to decouple IES from risk aversion permits one to study how results change as just the habit intensity varies without consequences for the degree of risk aversion.\(^{12}\) As a benchmark I use \(\sigma = 2\), which is a common assumption.

The habit intensity \((\nu)\) and degree of separability \((\gamma)\) are the parameters of interest in this study. I present results for a continuum of values, \(\nu = [0, 0.9]\), for habit intensity. The upper limit is the usual value for the habit intensity taken

\(^{11}\) A proof of this can be found in the technical appendix to Jahan-Parvar et al. (2013) on the authors’ websites.

\(^{12}\) e.g. given the strict relationship between IES and \(\sigma\) with CRRA preferences the calibration of \(\sigma\) changes as the habit intensity is varied to ensure that the relationship is maintained.
in most studies. As noted in the introduction, and in previous sections, there are very few macroeconomic studies that estimate $\gamma$. Estimates by Schmitt-Groh and Uribe (2012) suggest a value of $\gamma = 0$ in an RBC model with news shocks, while Dey (2014) and Khan and Tsoukalas (2011) both suggest that $\gamma$ may be higher in models with nominal rigidities. Dey (2014) does not allow for habits in the GHH specification of his model, while Khan and Tsoukalas (2011) do not allow for habits at all so that the higher estimates for $\gamma$ in these papers may also be picking up the ‘habit income effect’ to some extent. Given the extreme uncertainty around $\gamma$ I study the two polar cases, noting that for any habit intensity these polar span the entire range for $\gamma$.

The technology parameters govern the production function, the capital accumulation process and the total factor productivity shock process. The numerical values for these parameters are relatively standard across the literature. The calibrated parameter values are taken from Kehoe and Perri (2002). A similar parametrisation is used in Boldrin et al. (2001) and Christiano and Eichenbaum (1992), except the standard deviation of innovations differs as they assume labour augmenting technology shocks. The standard deviation of technology shocks is the same as that assumed in Cooley et al. (1995).

The functional form for the adjustment cost function is calibrated so that the model is invariant to $\xi$ in the long run, capturing the idea that adjustment costs are a short term phenomenon.\(^\text{13}\) The parameter $\xi$ is a free parameter and cannot be derived from the deep parameters of the model. The baseline parametrisation of $\xi$ is 0.23, which is used in both Jermann (1998) and Boldrin et al. (2001) facilitating comparison of results. Boldrin et al. (2001) cite that this value for $\xi$ is near the lower end of possible parametrisations. Thus the calibration of the capital adjustment cost parameter implies very strong adjustment costs.

\[^{13}\text{The steady state equations imply that the adjustment cost function parameters satisfy,}\]

\[b_1 = \delta \xi,\]

\[b_2 = \frac{\delta}{1 - \xi}.\]
4.1 Solution Method

The model is solved using non-linear methods; the particular algorithm used is the Parameterised Expectations Algorithm (PEA) of den Haan and Marcet (1990); Marcet and Marshall (1994); Marcet and Lorenzoni (1998). The PEA solution procedure is a projection method\(^{14}\) that allows one to compute global solutions.\(^{15}\) For the study of asset prices, as is the case in this paper, one requires global solutions so as to capture the higher order moments that are ignored by local approximation techniques, including their higher order approximations.

5 Labour Dynamics

The response of the labour impulse to the strength of the habit parameter is presented Figure 1. The response is always negative under KPR preferences, i.e. the income effect always dominates the substitution effect. In contrast GHH preferences permit a region, \(\nu \in [0, 0.48]\) for this specific calibration, where the labour response is positive. Within this region the income effect is present but sufficiently weak that it cannot dominate the substitution effect resulting in a procyclical movement of labour. Thus the labour market mechanism is operating as one would expect in a DSGE model with flexible prices.\(^{16}\)

Figure 1 is used as the primary reason for the focus on GHH preferences in this paper. Indeed, the KPR model is hindered in this study as it is only reached by varying \(\gamma\), while all other parameters are calibrated using the previous discussion.\(^{17}\) This allows one to study how the introduction of the ‘separability channel’ for the income effect impacts on results when holding the habit intensity constant. But it does not permit the KPR model to serve as anything more than a comparison device as it’s other parameters are constrained so as to be comparable to the GHH specification. The full dynamics of the economy under GHH preferences, as evidenced by the impulse response functions, are relatively standard. The impulse responses and quantitative results are presented in Appendix B for a range of habit intensities.

\(^{14}\)See Judd (1992); Maliar and Maliar (2014) for a review of projection methods used in macroeconomics

\(^{15}\)Simulations were also carried out using the Generalised Stochastic Simulation Approach of Judd et al. (2011) for comparison. The results were almost identical, and more accurate using PEA hence the reporting of these simulation results.

\(^{16}\)Cantore et al. (2014) provide a good discussion of the expected labour market operation in flexible price vs. sticky price models.

\(^{17}\)The parameter \(\chi\) is calibrated in the KPR case so that one-third of time is spent working in the deterministic steady state.
5.1 An Alternative Calibration Scheme for Habit Intensity

As the income effect under GHH preferences is habit induced this permits an alternative calibration scheme for the habit intensity parameter under GHH preferences. Under the alternate calibration scheme the habit parameter is calibrated to induce the appropriate cyclical movement in labour for the DSGE model of concern - i.e. strictly procyclical in RBC models or possibly countercyclical in New Keynesian models with sticky prices. Thus for GHH preferences we can use the habit intensity to control the strength of the income effect and hence the cyclical behaviour of labour.

Within the range $\nu \in [0, 0.48]$ there also exists a range for $\gamma$ that allows for the substitution effect to dominate the income effect. This range of permissible values of $\gamma$ is decreasing in the habit intensity. As previously discussed, $\gamma$ controls the degree of separability between labour and consumption. Thus one doesn’t need the assumption of perfect linear separability between labour and consumption to generate a positive response of labour. However, given most estimates place $\gamma$ close to zero there is only a limited amount of movement away from GHH preferences that are empirically plausible.

There is a large, and continuing, debate surrounding the effect of a productivity shock on labour (e.g. see discussion in Gali (1999) and Smets and Wouters (2007)). The advantage of using this alternative calibration scheme is that one can take a relatively neutral approach to the impact of a productivity shock on
labour. Under this scheme one could take the labour response to a technology shock from an empirical impulse response function and vary $\nu$ until the initial response matches the empirical one.\footnote{I do not take this approach in this paper as the aim is not to quantitatively match the data but rather understand the mechanism. This empirical exercise it is left for future work.}

The alternative calibration scheme has the added advantage of being able to study asset pricing results without calibrating parameters to maximise the ability to match asset pricing facts. This is in contrast to the calibration methodology adopted in the literature where the habit parameter is calibrated to maximise the ability of the model to match asset pricing facts. Thus, under this alternative approach to calibrating the habit intensity under GHH preferences, one can more objectively study the transmission mechanism between the real economy and asset prices.

6 Asset Pricing Implications

The assets considered are one period risk free bonds and one period equity. The gross risk free rate satisfies,

$$R_{t+1}^f = \frac{1}{E_t[\beta^{\Lambda_{t+1}/\lambda_t}]} ,$$

where $\Lambda_t$ is the marginal utility of consumption. The gross equity return comes from the investment first order condition,

$$R_{t+1}^e = \frac{\phi'(g_t)}{\phi'(g_{t+1})} \left[ z_{t+1} f_{k,t+1} \phi'(g_{t+1}) + 1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1}) \right] ,$$

where $g_t = \frac{i_t}{k_t}$ is the investment-capital ratio. The quantitative asset pricing results are presented for a selection of habit intensities for the GHH functional form, KPR results for comparison are presented in parentheses below their respective figures. The quantitative results are very far from the data, especially for equity. This is a priori expected given the simplicity of the model and the presence of only a single shock. The volatilities under KPR preferences are uniformly below that of their GHH counterparts as the KPR economy is in general less volatile than the GHH economy. The rest of this section will focus on the impact on mean returns.

The analysis that follows for both risk free and equity investment are specific examples of a more general link between consumption and investment in models...
with habits first discussed in Boldrin et al. (1995). In the presence of habits there is a positive feedback between consumption and investment since agents have a strong desire to smooth consumption. Consider a shock that raises income, this will see agents seek to increase investment. This simultaneously allows agents to slowly build up their habit stock and reduces variation in consumption. Superior asset pricing results are possible since increases in investment are costly. The analysis will highlight that this mechanism is weakened in models with sufficiently strong income effects as permits agents to achieve smooth consumption without the cost of changing investment. Therefore, by restricting the strength of the income effect one restricts this channel, thereby moving agents back the costly investment channel. Indeed this is apparent from the quantitative results for the mean risk free return and equity return between GHH and KPR models. The income effect is, ceteris paribus, stronger in KPR models compared to GHH models and this leads to the pattern of returns observed between the two models.

6.1 The Risk Free Rate and Income Effects

Figure 2 presents the results for the mean returns over the entire habit range for both GHH and KPR preferences. This figure allows one to study both the change in asset prices as \( \nu \) and \( \gamma \) vary. Thus one can think of Figure 2 as decomposing the asset returns into their two component income effects, i.e. that attributable to internal habits and that attributable to the degree of separability between habit adjusted consumption and labour.
Figure 2 shows that the risk free rate falls as $\nu$ increases. As habits are strengthened agents prefer smoother consumption paths and consequently their demand for the risk free bond increases leading to a fall in the return. That is agents are willing to accept lower return on consumption in subsequent periods in order to achieve their desire for smoother consumption paths as habits are increased. The income effect induced by internal habits therefore leads to a fall in the risk free rate.

Secondly the risk free rate falls as the income effect induced by $\gamma$ falls, i.e. as labour and habit adjusted consumption move towards a greater degree of separability. In this case agents face greater consumption volatility resulting from income that is less smooth. Consequently agents are willing to accept a lower rate to transfer this extra income into future consumption at each habit intensity.

### 6.2 Equity Return and Income Effects

The impact of the two different income effects on mean equity prices are presented in Figure 3. There is a hump shaped response as habit intensity increases with the return rising with increased habit intensity until $\nu = 0.7$ and falling thereafter. The return rises with a fall in $\gamma$ for any given habit intensity. As neither of these terms enter the return on equity form explicitly, the impact of both $\gamma$ and $\nu$ on the equity return occurs through ‘general equilibrium’ channels. It is also worth noting that the average higher price of equity in KPR models leads to the scaling down of the equity return in these models leading to the average return on equity lying below that for GHH models.
In order to understand these ‘general equilibrium’ channels I decompose the equity return into dividend and equity price channels using,

\[ R_{t+1}^e = \frac{d_{t+1} + v_{t+1}}{v_t} \]  

(6.1)

where \( d_{t+1} \) is the dividend paid by equity at end of period \( t \) and \( v_t \) is the price of equity in period \( t \). This decomposition yields,

\[ v_t = \frac{k_{t+1}}{\phi'(g_t)} \]  

(6.2)

\[ d_{t+1} = \alpha y_{t+1} - I_{t+1} \]  

(6.3)

so that the dividend channel is given by \( \frac{d_{t+1}}{v_t} \) and the capital gains channel is \( \frac{v_{t+1}}{v_t} \). The proportion of the equity return for each of these channels is very stable at 0.37% and 99.63% for the dividend and capital gains channels respectively for both GHH and KPR models. From this one concludes that the key channel for understanding the equity return is how the capital gains channel behaves over the range of habit intensities as this constitutes the vast majority of the return.\(^{19}\) The key to understanding this channel is the impact of the various income effects on investment and how these translate into the capital gain.

First I establish the link between changes in investment in different periods.

\(^{19}\)Since the capital gains channel is such a large proportion of the return the corresponding figure for the capital gains channel will be almost identical to Figure 3.
Figure 4: Capital Gain Impulse Response at Impulse

on the capital gain,

$$\frac{\partial}{\partial t} \frac{v_{t+1}}{v_t} = \frac{k_{t+2}}{\phi'(g_{t+1})} \frac{\phi''(g_t)}{k_t k_{t+1}} < 0,$$

$$\frac{\partial}{\partial t} \frac{v_{t+1}}{v_t} = \frac{-\phi''(g_t)}{[k_{t+1} \phi'(g_{t+1})]^2} \phi'(g_t) k_{t+2} > 0,$$

so that an increase in contemporaneous investment will lead to a fall in the capital gain. This makes intuitive sense as an increase in contemporaneous investment increases the demand for capital, thereby driving up the price of capital and causing a fall in the capital gain. The impact of an increase in the subsequent period investment is to increase the capital gain today as it drives up the price at which capital can be sold.

Figure 4 presents the impulse response (at impulse) for the capital gain. It highlights that here are two offsetting ‘general equilibrium’ effects on investment as the habit intensity increases. The first is that investment rises as the habit intensity increases, i.e. as agents prefer ever smoother consumption then investment will need to take up more of the changes in output. However, increases in the habit intensity also lead to an income effect that dominates leading to a fall in labour and hence a smaller increase in output. This second effect, via the labour channel, works in opposition to the first effect so that the overall impact on investment depends on their interaction. Given any habit intensity, investment rises as the income effect induced by $\gamma$ falls, i.e. as one moves towards GHH preferences. This is due to the fact that this secondary income
effect induced by $\gamma$ allows for smoother income, so as $\gamma$ falls agents have more
income and thus they will increase investment spending, driving the capital gain
down. Finally, this figure shows that at very high habit intensities the impact
of $\gamma$ disappears and only the habit intensity effect remains.

7 Conclusion

In this paper I have shown that flexible labour is not anathema to the ability
of models to explain asset prices. Rather, in the presence of flexible labour
one needs to consider the types of income effect that the preference structure is
introducing. In particular this paper highlights the need to study not just the
direct channel but also the general equilibrium effects induced by flexible labour.
I show that a combination of a strong ‘habit income effect’ together with a weak
‘separability channel’ for the income effect allow one to achieve superior asset
pricing results. The results in this paper suggest that GHH preferences together
with strong internal habits are a good first step towards generating quantita-
tively realistic asset pricing results in a model with flexible labour. Given the
importance of the labour market in New Keynesian models this paper is a step
towards a model of asset pricing in the face of nominal rigidities. The extension
of the model into one with New Keynesian features is the subject of current work.

Finally, this paper has only studied the movement of labour along the intensive
margin. This is due to the fact that nominal rigidities, the crucial building block
of New Keynesian DSGE models, act only on this margin. An extension could
combine the analysis of this paper with that of Kuehn et al. (2017), who con-
sider how variation along the extensive margin affects asset prices. While such
an analysis is outside the scope of this paper, it could provide an interesting
study around how these labour market margins interact and differentially affect
asset prices.
A Appendix: Full Macroeconomic System

The full macroeconomic system in equilibrium is presented below, specific functional forms are presented in the main text of the paper.

\[
\Lambda_t = u_{c,t} - \beta \nu E_t [u_{c,t+1}] \quad (A.1)
\]

\[
w_t = -\frac{u_{n,t}}{\Lambda_t} \quad (A.2)
\]

\[
1 = E_t \left[ \beta \frac{\Lambda_{t+1}}{\Lambda_t} R_{t+1} \right] \quad (A.3)
\]

\[
R_{t+1} = \frac{\phi'(g_t)}{\phi'(g_{t+1})} \left[ z_{t+1} f_{k,t+1} \phi'(g_{t+1}) + 1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1}) \right] \quad (A.4)
\]

\[
g_t = \frac{i_t}{k_t} \quad (A.5)
\]

\[
k_{t+1} = (1 - \delta) k_t + \phi \left( \frac{i_t}{k_t} \right) k_t \quad (A.6)
\]

\[
c_t + i_t = w_t n_t^a + r_t k_t \quad (A.7)
\]

\[
y_t = c_t + i_t \quad (A.8)
\]

\[
y_t = z_t f(k_t, n_t) \quad (A.9)
\]

\[
w_t = z_t f_{n,t} \quad (A.10)
\]

\[
R^f_{t+1} = \frac{1}{E_t \left[ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \right]} \quad (A.11)
\]

\[
\ln z_{t+1} = \rho \ln z_t + \varepsilon_t \quad (A.12)
\]
## B Appendix: Business Cycle Results - GHH Preferences

### B.1 Quantitative Business Cycle Results

Table 3: Business Cycle Results

<table>
<thead>
<tr>
<th>Data</th>
<th>$\nu = 0$</th>
<th>$\nu = 0.25$</th>
<th>$\nu = 0.45$</th>
<th>$\nu = 0.5$</th>
<th>$\nu = 0.75$</th>
<th>$\nu = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Deviation (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output</td>
<td>1.72</td>
<td>1.31</td>
<td>1.26</td>
<td>1.15</td>
<td>1.11</td>
<td>0.78</td>
</tr>
<tr>
<td>Standard Deviation Relative to Output</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.79</td>
<td>1.19</td>
<td>1.15</td>
<td>1.09</td>
<td>1.07</td>
<td>0.85</td>
</tr>
<tr>
<td>Hours Worked</td>
<td>0.63</td>
<td>0.44</td>
<td>0.40</td>
<td>0.35</td>
<td>0.35</td>
<td>0.62</td>
</tr>
<tr>
<td>Investment</td>
<td>3.24</td>
<td>0.58</td>
<td>0.70</td>
<td>0.92</td>
<td>1.00</td>
<td>1.71</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.47</td>
<td>0.56</td>
<td>0.61</td>
<td>0.73</td>
<td>0.78</td>
<td>1.36</td>
</tr>
<tr>
<td>Contemporaneous Correlation with Output</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.87</td>
<td>1.00</td>
<td>1.00</td>
<td>0.99</td>
<td>0.98</td>
<td>0.90</td>
</tr>
<tr>
<td>Hours Worked</td>
<td>0.93</td>
<td>1.00</td>
<td>0.99</td>
<td>0.84</td>
<td>0.73</td>
<td>-0.37</td>
</tr>
<tr>
<td>Investment</td>
<td>0.86</td>
<td>1.00</td>
<td>0.97</td>
<td>0.91</td>
<td>0.89</td>
<td>0.88</td>
</tr>
<tr>
<td>Real Wage</td>
<td>0.21</td>
<td>1.00</td>
<td>0.99</td>
<td>0.96</td>
<td>0.95</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Values for KPR analog are shown in parentheses below the GHH values.
B.2 Impulse Responses

Figure 5: GHH Impulse Responses for various values of \( \nu \)

References


and asset returns in an exchange economy. *Macroeconomic Dynamics* 1(02), 312–332.


