Equity Pricing New Keynesian Models with Nominal Rigidities and Investment

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Abstract

This paper derives explicitly an equity pricing relationship in a simple New Keynesian model. This relationship is used to study the equity pricing implications of New Keynesian models. I find that New Keynesian models suffer from the same asset pricing shortcomings as more traditional RBC versions and that this can be attributed to the presence of nominal rigidities. I then add capital adjustment costs to study how the interaction of both investment adjustment costs and capital adjustment costs affect the results.

Keywords: Asset Pricing, New Keynesian, Nominal Rigidities, Investment Adjustment Costs, Capital Adjustment Costs

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How does monetary policy affect financial markets? The recent financial crisis and policy responses to it have highlighted how little is understood, particularly in a theoretical sense, about how macroeconomic policy affects asset prices. Coming out of the financial crisis much ink has been spilled, both academic and non-academic, in answering this question. There has been an apparently single-minded focus in trying to understand how policy makers should react to mitigate such crises. In the context of a prolonged period of crises, as in the past decade, this is understandable. But, how are assets fundamentally priced in the models that frame our current understanding of the macroeconomy - the New Keynesian Model? This question has received much less attention. Attempting to answer the former at the expense of the latter is a little like putting the cart before the horse. This paper begins to fill this gap and addresses the latter question as a way to better understand the former. That is, this paper is an exploration of asset pricing questions in macroeconomics with a view to provide a framework that permits one to understand how macroeconomic policy affects financial markets.

By its very nature monetary policy is the most relevant macroeconomic policy for the study of asset prices since it reacts much faster and more frequently to real economic disturbances than fiscal policy. Furthermore monetary policy is seen as controlling the nominal interest rate which has a much more direct link to financial variables than fiscal policy instruments. The current paper serves as a first step towards the ultimate goal of understanding the asset pricing implications of monetary policy by providing the necessary extension of asset pricing models into a New Keynesian setting. The monetary policy effect on asset prices has been studied in the empirical macroeconomics literature, e.g. the empirical impact of monetary policy shocks on asset prices in the tradition of Rigobon and Sack (2004). There is also a significant empirical literature in the opposite direction, i.e. on how monetary policy reacts to asset prices. Bernanke and Gertler (2000), Rigobon and Sack (2003) and more recently Christiano et al. (2010) are examples of this literature.

De Paoli et al. (2010) stress that the importance of New Keynesian models in policy analysis requires one to at least understand the asset pricing implications of these models. There is, however, only a limited literature looking at how asset prices behave in New Keynesian models and all of this work has only been published very recently; Wei (2009), De Paoli et al. (2010), Challe and Giannitsarou (2014) and Swanson (2015) being the most relevant literature for the central aims of this paper. While they provide a good starting point for the analysis of asset prices in New Keynesian models none delve into the explicit
relationship linking the underlying economic variables to asset prices. That is, if we let \( R_{t+1}^e \) be the return on equity, then none derive an explicit relationship of the form,

\[
R_{t+1}^e = f(\text{Real Variables}, \text{Nominal Variables}, \text{Parameters})
\] (0.1)

The objective of this paper is to fill this gap in the literature. Such a relationship is crucial to any such attempt because the mechanisms by which policy affects variables on the right hand side\(^1\) are well understood. So by finding \( f(\cdot) \) one can understand, in an equilibrium sense, how policy affects the return on equity.

This missing link between asset prices and the real economy arises for two reasons. First, macroeconomics, until recently, has treated the real and financial sectors of the economy as separate. Second, the vast majority of New Keynesian models ignore endogenous capital accumulation. Work by Boldrin et al. (2001), albeit in a Real Business Cycle model, show that the capital gains channel drives the return on equity, and that the price of capital is the main driver of this channel. Thus the absence of endogenous capital accumulation, and the resulting investment dynamics, have prevented a complete analysis/understanding of asset prices. Despite the tendency to ignore capital accumulation there are various papers that do allow capital accumulation, and a variable Tobin’s Q, to play a role.\(^2\) In all such models it is possible to report equity returns\(^3\) but one cannot delve deeper into the drivers of this return due the lack of any relationship that links the equity return to the underlying economy. The objective of this paper is to demonstrate that such a relationship can be derived in a simple New Keynesian model. This allows one to gain finer insights into the drivers of the equity return in New Keynesian models than hitherto explored in the literature.

The central result of this paper shows that an equity return relationship can be derived for New Keynesian models under very general conditions. I find that the return on equity is made up of two components (i) the return on capital and (ii) the expectation of future profit streams of the firm. This relationship is standard and expected. I do not contend that such a mechanism is not at work in the background of other models that allow for variable Tobin’s Q in

\(^1\) e.g. consumption, investment, capital, output, inflation, etc.


\(^3\) In models that ignore capital accumulation it is still possible to study asset prices but they miss an important driver of equity prices.
New Keynesian models. Rather, this is the first time that this relationship been explicitly derived and explored. Furthermore, the result derived is independent of the solution method used to solve the model, i.e. it can be applied models solved using global or local approximation methods. In this sense, the relationship derived is considered more general and the results more robust than previously studied in literature.

I find that the simple model studied in this paper cannot achieve any degree of quantitative replication for asset pricing. As a result I conclude that New Keynesian models suffer from the same drawbacks with regards to asset pricing as their RBC counterparts. This is unsurprising given the simplicity of the model and more generally misses the point of the paper. This paper serves as a ‘proof of concept’ that asset prices can be derived in New Keynesian models rather than attempting quantitative replication. Despite the poor quantitative replication I use the results to gain qualitative insights into how New Keynesian features affect the asset pricing mechanisms. Such insights provide clues as to the features needed to improve the asset pricing implications of such models.\footnote{Indeed this is very much akin to the early papers in asset pricing in the Real Business Cycle literature that used simple models that couldn’t dream of quantitative replication to better understand the mechanisms at work and to provide clues for that literature about how best to proceed in order to achieve better quantitative replication.}

This paper is organised as follows. Section 1 details the model set up. Section 2 proceeds to derive the equity pricing relationship. Section 3 provides the calibration and solution method used to obtain the quantitative results discussed in Section 4. Section 5 provides an extension of the model studied in Section 1, while Section 6 concludes.

1 Model Set-Up

The model studied in this chapter is a standard New Keynesian model with sticky prices, sticky wages and investment. The model consists of a continuum of households, a continuum of intermediate goods producers, an employment agency, a final goods producer, a capital investment goods producer and a central bank.

In particular I employ a multi-sector set-up for the production of capital investment that closely follows that of Justiniano et al. (2011). This multi-sector approach allows one to clearly understand the process by which output is converted to capital. This process is as follows. Perfectly competitive investment good producers purchase aggregate output and use this to produce the invest-
ment good. This investment good is purchased by monopolistically competitive intermediate goods firms who use the investment good to add to the productive capital stock in the subsequent period. The productive capital is then combined with labour to produce the differentiated intermediate good and sold to a perfectly competitive final goods production firm. This process is crucial for the understanding of equity prices as it allows one to clearly understand how the price of capital is determined in the model. Fluctuations in the price of capital will drive investment decisions, which in turn drive profits and dividends, and ultimately the price of equity.

1.1 Households

1.1.1 Employment Agencies

Perfectly competitive employment agencies buy the labour supply of each household, \( n_s^t (i) \). This is costlessly aggregated into aggregate labour which is used as an input by the intermediate goods firms.

The employment agencies aggregate labour supply using a Dixit-Stiglitz aggregator,

\[
n_s^t (i) = \left[ \int_0^1 n_s^t (i) \frac{\varepsilon^w - 1}{\varepsilon^w} \, di \right]^{\frac{1}{\varepsilon^w - 1}}, \tag{1.1}
\]

where \( \varepsilon^w > 0 \) measures the degree of substitutability between different labour inputs. Employment agencies seek to maximise their profits leading to the demand function for labour,

\[
n_s^t (i) = \left[ \frac{w_t (i)}{W_t} \right]^{1 - \varepsilon^w} n_s^t, \tag{1.2}
\]

where \( n_s^t \) is the aggregate labour supply and \( w_t \) is the aggregate wage level defined as,

\[
w_t = \left[ \int_0^1 w_t (i)^{1 - \varepsilon^w} \, di \right]^{\frac{1}{1 - \varepsilon^w}}. \tag{1.3}
\]

1.1.2 Households

There exist a continuum of households indexed on the unit interval \( i \in (0, 1) \). Households seek to maximise lifetime utility,

\[
U_t (i) = E_t \left\{ \sum_{h=0}^{\infty} \beta^h u \left( c_{t+h} (i) - \nu c_{t+h-1} (i), n_s^{t+h} (i) \right) \right\} \tag{1.4}
\]
where $\beta$ is the time discount factor.

The period utility of the household depends on consumption, $c$, and labour supply $n$. It has the same form across all households. It exhibits internal habits in consumption, the strength/intensity of which is governed by the parameter $\nu$.

For the purposes of quantitative results, the period utility function is of the GHH form (Greenwood et al., 1988) and has functional form,

$$u(c_t, c_{t-1}, n_t) = \frac{1}{1-\sigma} \left[ (c_t - \nu c_{t-1}) - \frac{\chi}{1+\varphi} n_t^{1+\varphi} \right]^{1-\sigma}$$

(1.5)

where $\sigma$ measures the degree of risk aversion, $\chi$ is the utility weight of the disutility of labour and $\varphi$ is the inverse of the Frisch elasticity of labour supply. This utility function is used as the economy exhibits shocks to the marginal efficiency of investment and GHH preferences have been shown to induce the correct co-movement between macroeconomic variables.

The period budget constraint for household $i$ is,

$$c_t(i)+ \frac{B_{t+1}(i)}{R_t P_t} + v_t S_t + (d_t + v_t) S_t = w_t(i) P_t + \chi w_t^{2}$$

(1.6)

Household income is used to purchase an aggregate consumption good, $c_t$, from the final goods firm and to purchase assets for the subsequent period. Household income is derived from the supply of labour and the return on asset holdings.

As each household is a monopolistic supplier of differentiated labour, $n^*_t(i)$, they can set their nominal wage, $w_t(i)$, in the labour market. Households face an intangible cost, measured as lost income, from changing their nominal wage across periods. This cost is modelled in the spirit of Rotemberg costs to ensure the wage Phillips curve has a closed form.\footnote{A closed non-linear form for either the wage or price Phillips curves is in general not possible under the Calvo style of price and wage inertia.} The parameter $\chi^w$ governs the degree of wage stickiness. Given the recent work of Born and Pfeifer (2016) one can move with significant ease between models with Calvo-style wage inertia and those with Rotemberg-style wage inertia by an appropriate parameterisation of $\chi^w$.

The intangible nature of the cost of changing wages is in the spirit of De Paoli et al. (2010) who similarly assume an intangible cost of adjusting prices in the
sticky price context. The intangible cost may be considered as modelling the psychological effect of changing prices for households. When changing wages households anticipate that there will be productive time lost in bargaining over wages which cannot be used to supply labour. This cost in terms of lower effective working hours is accounted for by the household via a lower income in the budget constraint. Importantly the cost is measured in lost aggregate income which the household has no control over. The intangible nature of the cost implies no impact on the aggregate resource constraint but rather affects the equilibrium by inducing households to make decisions based on lower effective income.

There are two broad classes of assets in which the household may invest: bonds and equity. Nominal bonds have a variable cost of $v^B_t = \frac{1}{R^B_t}$, where $R^B_t$ is the nominal return. The nominal bonds provide a known return in the subsequent period of one unit of the nominal consumption good. The nominal interest rate is known in advance and set by the central bank. This allows the central bank to influence real demand via monetary policy.

Equity has a variable cost and provides a real return comprised of a real dividend and capital return. Equity investment is in the form of shares in a broad equity index,

$$S_t = \int_0^1 q_t(i) S_t(i) \, di,$$

where $S_t(i)$ is the shares in the intermediate firms and $q_t(i)$ their market weights. The market weights are calculated as the proportion of output produced by each intermediate goods firm i.e.,

$$q_t(i) = \frac{y^i_t}{y_t}.$$  \hspace{1cm} (1.7)

This equity index provides an aggregate real dividend,

$$d_t = \int_0^1 q_t(i) d_t(i) \, di,$$

and real capital return at the prevailing price for the stock index, $v_t$.

It is further assumed that households have no productive activity at home so that all labour is supplied to firms. As suppliers of differentiated labour the household is demand constrained and therefore faces the demand curve for

\footnote{This is an un-modelled process in this set up}
labour,

\[ n^*_t (i) = \left[ \frac{w_t (i)}{w_t} \right]^{-\varepsilon^w} n^*_t \]  

(1.8)

Household optimisation of lifetime utility subject to their budget constraint and labour demand yields,

\[ \frac{w_t}{P_t} = -\frac{\varepsilon^w}{\varepsilon^w - 1} \frac{u_{n,t}}{\Lambda_t} + E_t \left[ m_{t,t+1} x_{t+1} \frac{w_{t+1} n_{t+1}}{P_{t+1}} \right] - x_t \frac{w_t}{P_t} \]  

(1.9)

\[ 1 = E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} P_t \frac{P_{t+1}}{\Lambda_{t+1}} \right\} \]  

(1.10)

\[ 1 = E_t \left\{ m_{t,t+1} \left( \frac{d_{t+1} + v_{t+1}}{v_t} \right) \right\} \]  

(1.11)

The auxiliary variable \( x_t \) has been introduced to simplify notation and is defined as,

\[ x_t = \frac{\chi}{\varepsilon^w - 1} \frac{w_t}{w_{t-1}} \left( \frac{w_t}{w_{t-1}} - 1 \right) + \frac{\chi}{\varepsilon^w - 1} \pi_t^w (\pi_t^w - 1) \]

where \( \pi_t^w = \frac{w_t}{w_{t-1}} \) is wage inflation. Finally under Rotemberg costs all households face the same problem and so the household indexation may be dropped once the first order conditions have been derived. The equation (1.9) is the wage Phillips curve, (1.10) defines the Fisher relationship and (1.11) is the Euler equation governing equity holdings. The stochastic discount factor is given by \( m_{t,t+1} \) and is defined as,

\[ m_{t,t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \]

\[ \Lambda_t = u_{c,t} - \beta \nu E_t [u_{c,t+1}] \]

1.2 Firms

1.2.1 Final Goods Firms

The output of intermediate goods firms, \( y_t (j) \), is bought by perfectly competitive final goods firms which costlessly aggregate the output. This aggregate output may be costlessly transformed into either consumption or investment goods.
The final goods firms aggregate output using the Dixit-Stiglitz aggregator,

\[ y_t = \left[ \int_0^1 y_t (j)^{\varepsilon_p - 1} \, dj \right]^{\frac{1}{\varepsilon_p}}, \quad (1.12) \]

where \( \varepsilon_p > 0 \) measures the degree of substitutability between different goods. The final goods firms maximise their profits leading to the standard demand function for the intermediate goods firms,

\[ y_t (j) = \left[ \frac{P_t (j)}{P_t} \right]^{-\varepsilon_p} y_t, \quad (1.13) \]

where \( y_t \) is the aggregate demand and \( P_t \) is the aggregate price level defined as,

\[ P_t = \left[ \int_0^1 P_t (j)^{1-\varepsilon_p} \, dj \right]^{\frac{1}{1-\varepsilon_p}}. \quad (1.14) \]

### 1.2.2 Intermediate Goods Firms

Monopolistically competitive intermediate goods firms seek to maximise lifetime profit. They dislike changing their price, \( P_t (j) \), between periods since doing so incurs a cost à la Rotemberg (1982). Following De Paoli et al. (2010) this cost is assumed to be an intangible cost that enters the firms optimisation problem as a form of ‘disutility’, i.e. it doesn’t affect cash flow. The intermediate goods firms own the capital in the economy and hence are responsible for making the capital investment decisions.

Intermediate goods firm \( j \) seeks to maximise lifetime real profit defined as,

\[ \Gamma = E_t \left\{ \sum_{h=0}^{\infty} m_{t,t+h} \left[ \frac{P_{t+h} (j) y_{t+h} (j)}{P_{t+h}} - \frac{w_{t+h}}{P_{t+h}} n_{t+h}^d (j) - \frac{\chi^P}{2} \left( \frac{P_{t+h} (j)}{P_{t+h-1} (j)} - 1 \right)^2 y_{t+h} \right] \right\}, \quad (1.15) \]

where \( y_t (j) \) is its output, \( n_{t+h}^d (j) \) is the labour demand, \( I_t (j) \) is capital investment made by the firm, and \( \frac{Q_t}{P_t} \) is the real cost of investment. The firm has no control over the cost of investment, which is set by the capital investment production sector. All variables without firm indexation are aggregate variables. Following the Rotemberg pricing literature the cost of price adjustment is assumed to be quadratic.\(^7\) The parameter \( \chi^P \) determines the strength of the disutility arising from adjusting prices, and hence the degree of price stickiness.

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\(^7\)The impact of non-symmetric functions are an interesting extension left for future work. Such non-symmetric functions would capture the idea that price changes in one direction is more palatable than the other. For example firms may prefer reducing prices over rising prices by the same amount.
Intermediate goods firms production must be feasible given the production technology available to the firm, characterised by the neo-classical production function \( f(\cdot) \) and takes the Cobb-Douglas form for each firm \( j \),

\[
z f(k(j), n(j)) = zk^\alpha(j) n^{1-\alpha}(j)
\]  

where \( \alpha \in (0, 1) \) is the capital share of production. This technology is assumed to be subject to non-idiosyncratic random shocks to productivity, \( z_t \), which follows an auto-regressive process, i.e.

\[
\ln z_{t+1} = \rho \ln z_t + \nu_t^y
\]  

where \( \rho \) measures the persistence of the shock and \( \nu_t^y \sim N(0, \sigma_z^2) \) is a random shock.

Intermediate goods firms are demand constrained. The demand for output of intermediate goods firm \( j \) is defined by,

\[
y_t(j) = \left[ \frac{P_t(j)}{P_t} \right]^{-\epsilon_p} y_t.
\]  

Finally, as owners of capital the intermediate goods firms face the capital accumulation process,

\[
k_{t+1}(j) = (1 - \delta) k_t(j) + I_t(j),
\]  

where \( \delta \) is the fixed depreciation rate. Capital in the economy is identical, but the investment decision is firm specific. The presence of such a firm specific capital investment decision means that there is in general no closed form for the Phillips Curve under Calvo-style pricing. This is because the capital stock of each firm becomes heterogeneous thereby influencing pricing decisions. This is not an issue under Rotemberg-style pricing as firms are ex-post identical and so have identical capital stocks, thus make identical pricing decisions. This ensures a closed form Phillips Curve under Rotemberg costs - this closed form will be crucial for the equity price derivation. However, given the work of Keen and Wang (2007) and Ascari and Rossi (2009) one can also easily move between Calvo-style price rigidity and Rotemberg-style price rigidity easily by appropriate parameterisation of \( \chi^P \), so this pricing assumption is not restrictive.

The intermediate goods firm \( j \) maximises subject to the production function (1.16), the demand constraint (1.18), and the capital accumulation function (1.19).
(1.19). This leads to the following optimality conditions,

\[
\frac{w_t}{P_t} = \mu_t f_{n,t}, \quad \text{(1.20)}
\]

\[
E_t \left\{ m_{t,t+1} \chi^n P_{t+1} \left( \frac{P_{t+1}}{P_t} - 1 \right) y_{t+1} \right\} = (\varepsilon^n - 1) y_t + \chi^n P_t \left( \frac{P_t}{P_{t-1}} - 1 \right) y_{t-1} - \varepsilon^n \frac{w_t}{P_t} z_{t,f_{n,t}} \quad \text{(1.21)}
\]

\[
Q_t = E_t \left\{ m_{t,t+1} \frac{w_{t+1} f_{k,t+1}}{P_{t+1} f_{n,t+1}} + \frac{Q_{t+1}}{P_t} (1 - \delta) \right\}. \quad \text{(1.22)}
\]

Analogously to the household problem, under Rotemberg pricing all firms face the same problem so that once the first order conditions have been derived one can drop firm indexation. Thus the aggregate relationships have the same form as at the firm level. The relationship (1.20) is the labour demand condition, (1.21) is the non-linear Phillips Curve and (1.22) is the optimality condition for capital investment. The variable \( \mu_t \) is the Lagrange multiplier on (1.16) and is interpreted as a time varying mark-up.\(^8\)

Sveen and Weinke (2004) refer to \( MS_{t+1} = \frac{w_{t+1} f_{k,t+1}}{P_{t+1} f_{n,t+1}} \) in (1.22) as the marginal savings from capital investment. Since capital is owned by the firm, it is no longer the marginal product of capital that matters but rather \( MS_{t+1} \).

What matters now is not how productive capital is per se but rather its ability to reduce effective marginal cost. The effective marginal cost is given by

\[
\frac{w_{t+1}}{P_{t+1} z_{t+1} f_{n,t+1}} \text{ since each unit of labour is paid the real wage and produces } \frac{1}{z_{t+1} f_{n,t+1}} \text{ units of output. Consider the firms decision to invest in capital. This means that it can produce at least the same amount of output using less labour since it has more capital, so its effective marginal cost falls. Finally this saved effective marginal cost is invested in capital which can produce } z_{t+1} f_{k,t+1} \text{ units of output.}
\]

### 1.3 Capital Investment Production Firms

Perfectly competitive investment goods firms purchase the aggregate output from the final goods firm at the aggregate price level. This is converted into investment capital goods and sold to the intermediate goods providers as investment capital price \( Q_t \). The conversion of aggregate output into the investment good is imperfect and governed by,

\[
I_t = z_t \left[ 1 - s \left( \frac{y_t}{y_{t-1}} \right) \right] y_t, \quad \text{(1.23)}
\]

\(^8\)The time varying mark-up can be found by combining (1.20) and (1.21).
where \( y^I_t \) is the amount of final output bought by the capital production sector and \( z^I_t \) is a shock to the marginal efficiency of investment. The conversion of final output to the investment good is subject to adjustment costs which are captured by the function \( s(\cdot) \). As in the literature, I assume that at steady state the adjustment cost satisfies, \( s(\cdot) = s'(\cdot) = 0 \), and \( s''(\cdot) = \zeta > 0 \). The particular functional form used is,

\[
s \left( \frac{y^I_t}{y^I_{t-1}} \right) = \frac{\zeta}{2} \left( \frac{y^I_t}{y^I_{t-1}} - 1 \right)^2.
\]

(1.24)

The MEI shock follows an AR(1) process in logs,

\[
\ln z^I_{t+1} = \rho^I \ln z^I_t + \nu^I_t
\]

(1.25)

where \( \rho^I \) measures the persistence of the shock and \( \nu^I_t \sim N \left( 0, \sigma^2 \right) \) is a random shock.

The capital goods producer seeks to maximise profit subject to the capital production technology, i.e. it solves,

\[
\max E_t \left\{ \sum_{h=0}^{\infty} m_{t,t+h} \left[ \frac{Q_{t+h}}{P_{t+h}} I_{t+h} - y^I_{t+h} \right] \right\}
\]

s.t.

\[
I_t = z^I_t \left[ 1 - s \left( \frac{y^I_t}{y^I_{t-1}} \right) \right] y^I_t,
\]

which leads to the following optimality condition,

\[
\frac{Q_t}{P_t} = \frac{1}{z^I_t \mathcal{H}_t} E_t \left[ 1 - m_{t,t+1} \frac{Q_{t+1}}{P_{t+1}} z^I_{t+1} s' \left( g_{t+1} \right) \right],
\]

(1.26)

where, \( g_t = \frac{y^I_t}{y^I_{t-1}} \) and \( \mathcal{H}_t = 1 - s \left( g_t \right) - g_t s' \left( g_t \right) \).

### 1.4 Central Bank

The economy is closed by specifying how the central bank sets the nominal interest rate. The central bank sets the gross nominal interest rate \( R^n_t \) according to the Taylor rule,

\[
\frac{\mathcal{R}^n_t}{\mathcal{R}^n_t} = (\pi^t_t)^{\rho_{\pi}} \left( \frac{y^I_t}{\bar{Y}} \right)^{\rho_{\theta}} \eta_t,
\]

(1.27)

where the constants \( \rho_{\pi} \) and \( \rho_{\theta} \) control the degree to which the central bank responds to price inflation and output deviations. The central bank's rule is
subject to uncertainty via the nominal interest rate shock $\eta_r$ which follows an autoregressive process similar to the aggregate technology shocks,

$$\ln \eta_{t+1} = \iota \ln \eta_t + v_t^{R_n}$$

(1.28)

where $\iota$ is the degree of persistence of the shock and $v_t^{R_n} \sim N (0, \sigma_{R_n}^2)$ is a random shock.

### 1.5 Equilibrium

The goods market clearing condition requires that consumption and gross capital investment equal the aggregate output produced by intermediate firms, i.e.

$$c_t + y_t^I = y_t.$$  

(1.29)

Labour market clearing requires that the labour supplied by households equals the labour demand of firms, i.e.

$$n^s_t = n^d_t = n_t.$$  

(1.30)

Equilibrium in the bond markets requires that nominal bonds are in zero net supply,

$$B^n_t = 0.$$  

(1.31)

Finally equilibrium in the equity market requires that all shares are held, i.e.

$$S_t = S_t(j) = 1,$$  

(1.32)

where $S_t(j)$ is the outstanding shares in firm $j$. All firms solve the same problem which implies that in equilibrium the dividends are identical across firms, i.e.

$$d_t = d_t(j).$$  

(1.33)

### 2 Asset Pricing Implications

There are two assets in the economy: equity and the nominal bond. The nominal bond is priced by the central bank according to its policy function (1.27). It both compensates for aggregate uncertainty and protects the household from inflation risk by taking this into account. Thus we may use the central bank’s policy rule to understand movements in the nominal risk free rate.

The return on equity, however, does not have an obvious form related to the real
economy. It is the purpose of this section to derive a relationship between the return on equity and the variables of the real economy. Such a relationship, while operating in the background in many models, has not previously been brought to the fore and derived for a New Keynesian model. The relationship of interest is that which links $R_{t+1}$ in the equity Euler equation, $1 = E_t [m_{t+1} R_{t+1}]$, to the underlying economic variables. That is one is interested in a relationship of the form,

$$R_{t+1} = f \text{ (real economic variables, nominal economic variables, deep model parameters)}$$

which not only gives a value for $R_{t+1}$ but also informs one about the deeper mechanism by which agents price assets. By relating the real and financial sides of the economy this relationship allows one to understand how the underlying economic variables impact on asset prices and how policy, via an impact on the real economy, affects asset prices.

2.1 Equity Pricing

I adapt the methodology of Altug and Labadie (2008, Ch. 10) to derive an equilibrium pricing relationship for equity in the model. Altug and Labadie (2008) only consider the simplest case of a perfectly competitive firm without investment adjustment costs. I extend this analysis to the more general case considered in the present paper.

The derivation of the relationship consists of three distinct steps. First the financial structure of the firm as outlined by Altug and Labadie (2008, Ch. 10) is derived. Second, a profit aggregation result needs to be derived in order to move from the individual firm to that of the equity index, which is an aggregate of the individual firms. Finally, the households Euler Equation for equity and the firms investment first order condition are invoked to arrive at a stochastic difference equation which is solved to get the desired relationship between the return on equity and the underlying economic variables.

2.1.1 The Financial Structure of the Firm

We begin by outlining the financial structure of the intermediate goods firms as it is shares in these firms which are held by households. This financial structure is sufficiently general but it does assume that the Modigliani-Miller theorem holds, i.e. that the form of financing does not matter so that one can consider the firm as completely equity financed. Indeed, in the present set-up there is no scope for firms to access debt financing of any sort. Gross real profits of the
firm are given by,

\[ \frac{\Xi_t(j)}{P_t} = \frac{P_t(j) y_t(j)}{P_t} - \frac{w_t n_t(j)}{P_t} \] (2.1)

The firm can disburse real profits as dividends to stock, \( d_t(j) \), or hold them as retained earnings, \( RE_t(j) \), i.e.

\[ \frac{\Xi_t(j)}{P_t} = d_t(j) s_t(j) + RE_t(j) \] (2.2)

Similarly real investment is financed via retained earnings, \( RE_t(j) \), or new equity issue, \( v_t(S_{t+1}(j) - S_t(j)) \), i.e.

\[ \frac{Q_t}{P_t} I_t(j) = RE_t(j) + v_t(S_{t+1}(j) - S_t(j)) \] (2.3)

We can combine (2.2) and (2.3) to yield,

\[ \frac{\Xi_t(j)}{P_t} = d_t(j) + \frac{Q_t}{P_t} I_t(j) \] (2.4)

where we have used the equilibrium condition \( S_t = 1 \) \( \forall j, t \) to get to the relationship above. According to (2.4) real profits are either used to pay real dividends or used to finance real investment. In particular note that the firm does not retain any earnings.

### 2.1.2 Profit Aggregation

In order to consider the return on the share index we derive a form for aggregate real profit. Let us define this as

\[ \frac{\Xi_t}{P_t} = \int_0^1 \frac{\Xi_t(j)}{P_t} dj. \] (2.5)

The aggregate profit is defined as\(^9\),

\[ \Xi_t = \int_0^1 P_t(j) y_t(j) dj - \int_0^1 w_t n_t^d(j) dj \]
\[ = \int_0^1 P_t(j) y_t(j) dj - w_t n_t^d \]
\[ = P_t y_t - w_t n_t \]

where we have used the intermediate goods firms demand condition, (1.13), and the equilibrium labour condition. However, note that here \( y_t \) is given by the

\(^9\)For profit aggregation it does not matter if we use real or nominal since the aggregate price level can be moved outside the integral
Dixit-Stiglitz aggregate of intermediate goods given by,

\[ Y_t = \left[ \int_0^1 \left( F(k_t(j), n_t(j)) \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \frac{d}{d} \right]^{\frac{1}{\varepsilon_p - 1}}. \]  

(2.6)

In general this is not useful for linking aggregate output to aggregate factors in a simple, and mathematically tractable, manner in the presence of price dispersion and firm specific capital. In the presence of price dispersion, firm specific capital leads to complete heterogeneity between firms. This is because when only some fraction of firms can reset prices, hence leading to price dispersion, heterogeneity will very quickly appear as previous pricing decisions will impact capital investment decisions via an impact on marginal costs. In such a case all firms will eventually have different capital stocks. However, under Rotemberg pricing all firms are, in equilibrium, ex-post identical and so charge the same price in all periods, so that there is no price dispersion. Thus all firms will, ex-post and in equilibrium, make identical capital investment decisions. If firms are ex-post identical then for measure \( \Omega \) of intermediate firms one has,

\[ k_t(j) = \frac{k_t}{\Omega} \quad \text{and} \quad n_t(j) = \frac{n_t}{\Omega} \]

where \( k_t \) and \( n_t \) are aggregate capital and labour respectively. The production function exhibits constant returns to scale so one gets,

\[ F(k_t(j), n_t(j)) = \frac{1}{\Omega} F(k_t, n_t). \]

Thus under Rotemberg pricing one gets,

\[ y_t = \int_0^1 \left( \frac{1}{\Omega} F(k_t, n_t) \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} d j \]

\[ = F(k_t, n_t), \]

where the final equality comes from the fact that there are measure \( \Omega \) of intermediate firms on the unit interval \( j \in (0, 1) \).

Under Rotemberg pricing one can therefore relate the aggregate output to aggregate factors which is not possible under Calvo pricing without the need to resort to alternative aggregation schemes or additional restricting assumptions. In this sense the result presented here is sufficiently general as it only relies on well known characteristics that appear with the Rotemberg pricing assumption.
So one can write the relationship between output and the marginal products as,

\[ y_t = z_t f_{k,t} k_t + z_t f_{n,t} n_t. \]

So we get,

\[ \Xi_t = P_t z_t f_{k,t} k_t + P_t z_t f_{n,t} n_t - w_t n_t. \] (2.7)

In real terms this becomes,

\[ \frac{\Xi_t}{\bar{P}_t} = z_t f_{k,t} k_t + \left[ z_t f_{n,t} - \frac{w_t}{\bar{P}_t} \right] n_t. \] (2.8)

From the intermediate firm’s first order condition, (1.21), we have,

\[ \frac{w_t}{\bar{P}_t} = \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_t \right) z_t f_{n,t}. \] (2.9)

where,

\[ \Theta_t = \frac{\chi_p}{\varepsilon_p} \pi_t (\pi_t - 1) - E_t \left[ m_{t,t+1} \frac{\chi_p}{\varepsilon_p} \pi_{t+1} (\pi_{t+1} - 1) \frac{y_{t+1}}{y_t} \right], \] (2.10)

has been introduced for notational simplicity. Thus the aggregate profit may be written as,

\[ \frac{\Xi_t}{\bar{P}_t} = z_t f_{k,t} k_t + \left( \frac{1}{\varepsilon_p} - \Theta_t \right) z_t f_{n,t} n_t. \] (2.11)

This relationship states that aggregate profit in this case is given by the revenue product of capital plus that portion of the revenue product of labour all firms retain by virtue of being able to set prices. Re-writing (2.11) as,

\[ \frac{\Xi_t}{\bar{P}_t} = \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_t \right) z_t f_{k,t} k_t + \left( \frac{1}{\varepsilon_p} - \Theta_t \right) y_t. \]

In this form, the first term is the the saving that all firms make from owning capital. The second term is the pure profit all firms makes by being able to set prices, i.e. due to the presence of market power.

2.1.3 The Return on Equity Relationship

Aggregating (2.4) and combining with (2.11) yields the following relationship for aggregate dividends,

\[ d_t = z_t f_{k,t} k_t + \left( \frac{1}{\varepsilon_p} - \Theta_t \right) z_t f_{n,t} n_t - \frac{Q_t}{\bar{P}_t} I_t. \] (2.12)
This may be substituted into the households Euler equation for equity to yield,

$$v_t = E_t \left\{ m_{t,t+1} \left[ v_{t+1} + \frac{1}{\varepsilon_p} \frac{Q_{t+1}}{P_{t+1}} \right] z_t f_{n,t+1} - \Theta_{t+1} \right\} + \frac{1}{\varepsilon_p} \frac{Q_{t+1}}{P_{t+1}} I_{t+1} \right\}. \quad (2.13)$$

Using the capital accumulation equation we can remove investment to get,

$$v_t = E_t \left\{ m_{t,t+1} \left[ v_{t+1} + \frac{1}{\varepsilon_p} \frac{Q_{t+1}}{P_{t+1}} \right] z_t f_{n,t+1} - \Theta_{t+1} \right\} - \frac{Q_{t+1}}{P_{t+1}} k_{t+2} + (1 - \delta) \frac{Q_{t+1}}{P_{t+1}} k_{t+1} \right\}. \quad (2.14)$$

We now use the investment optimality condition (reference the equation) to establish that,

$$E_t \left\{ m_{t,t+1} \left[ z_t f_{k,t+1} k_{t+1} + (1 - \delta) \frac{Q_{t+1}}{P_{t+1}} k_{t+1} \right] \right\} = \frac{Q_{t}}{P_{t}} k_{t+1} + E_t \left\{ m_{t,t+1} \left[ \frac{1}{\varepsilon_p} - \Theta_{t+1} \right] z_t f_{k,t+1} k_{t+1} \right\}$$

We first establish that,

$$E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) z_t f_{k,t+1} + (1 - \delta) \frac{Q_{t+1}}{P_{t+1}} k_{t+1} \right] \right\} = \frac{Q_{t}}{P_{t}} E_t [k_{t+1}] \quad (2.15)$$

where we have used the the price Phillips Curve, (1.21), re-write the $\frac{Q_{t}}{P_{t}} f_{k,t+1}$ term.

Proof.

$$E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) z_t f_{k,t+1} + (1 - \delta) \frac{Q_{t+1}}{P_{t+1}} k_{t+1} \right] \right\} = E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) z_t f_{k,t+1} + (1 - \delta) \frac{Q_{t+1}}{P_{t+1}} k_{t+1} \right] \right\}$$

$$- \text{cov}_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) z_t f_{k,t+1} + (1 - \delta) \frac{Q_{t+1}}{P_{t+1}} k_{t+1} \right] , k_{t+1} \right\}$$

$$= \frac{Q_{t}}{P_{t}} E_t [k_{t+1}]$$

where we have used the firm’s first order condition in the final step to get to the result and that in equilibrium we have $E_t [k_{t+1}] = k_{t+1}$ because capital is predetermined via the capital accumulation equation. Rearranging we arrive at the desired result. Substituting back into households Euler equation for equity yields,

$$v_t = \frac{Q_{t}}{P_{t}} k_{t+1} + E_t \left\{ m_{t,t+1} \left[ v_{t+1} + \frac{1}{\varepsilon_p} \frac{Q_{t+1}}{P_{t+1}} \right] z_t f_{n,t+1} - \Theta_{t+1} \right\} + \frac{1}{\varepsilon_p} \frac{Q_{t+1}}{P_{t+1}} k_{t+2} \right\}$$

$$E_t \left\{ m_{t,t+1} \left[ v_{t+1} + \frac{1}{\varepsilon_p} \frac{Q_{t+1}}{P_{t+1}} \right] z_t f_{n,t+1} - \Theta_{t+1} \right\} + \frac{1}{\varepsilon_p} \frac{Q_{t+1}}{P_{t+1}} k_{t+2} \right\}$$

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This is a stochastic difference equation which may be solved forward to yield,

\[ v_t = \frac{Q_t}{P_t} k_{t+1} + E_t \left\{ \sum_{i=1}^{\infty} m_{t,t+i} \left( \frac{1}{\varepsilon^P} - \Theta_{t+i} \right) y_{t+i} \right\}. \quad (2.16) \]

Where we have used the transversality condition which guarantees that the value of capital falls to zero as \( t \to \infty \).

The inefficiency in the investment sector arises in the conversion of final output into investment capital. However, once this transformation has taken place there is no inefficiency in converting investment goods into capital. That is the firm can incorporate new investment purchases costlessly into capital. This costless transformation of investment goods into capital allows one to interpret the price of capital investment, \( \frac{Q_t}{P_t} \), as the price of capital.\(^{10}\) Hence, the first term, \( \frac{Q_t}{P_t} k_{t+1} \), is the value of capital holdings at the end of the period evaluated at the current price of capital.

The first part of the conditional expectation is the aggregate ‘pure profit’ intermediate goods firms earn in the given period by virtue of being able to set prices. To see this note that under flexible prices, i.e. when \( \chi^F = 0, \Theta_t = 0 \) in all periods so that the above solution becomes,

\[ v_t = \frac{Q_t}{P_t} k_{t+1} + E_t \left\{ \sum_{i=1}^{\infty} m_{t,t+i} \left( \frac{1}{\varepsilon^P} \right) y_{t+i} \right\}. \quad (2.17) \]

Under flexible prices the real profit is given by,

\[ \frac{\Xi_t}{P_t} = f_{k,t} k_t + \left( \frac{1}{\varepsilon^P} \right) f_{n,t} n_t = \left( \frac{\varepsilon^P - 1}{\varepsilon^P} \right) f_{k,t} k_t + \frac{1}{\varepsilon^P} y_t \]

Real profit under flexible prices is therefore composed of two components. The first is the savings the firm makes by virtue of owning capital and hence not having to pay rent to capital and the second is the ‘pure profit’ firms earn by virtue of being able to set prices. If firms did not own capital then the first term would disappear as the saving on rental costs no longer exists however the second term remains hence its definition as the ‘pure profit’. Having noted that the first term of the expectation is the ‘pure profit’ and is present regardless of sticky prices then we can say that the term \( \Theta_t \) is the loss of ‘pure profit’ due to the presence of sticky prices. Since prices are sticky firms lose some of their

\(^{10}\)The incorporation of adjustment costs in the transformation of investment into capital is studied later in the paper.
potential ‘pure profit’ as they choose not to reset their price to the flexible level price in any given period and hence does not realise all of the potential ‘pure profit’. The price of equity (A.4) therefore is comprised of two channels - a capital channel and an expectations channel. This is in contrast to the RBC model where equity prices only depend on the capital channel.

The return on equity is defined as,

$$R_{t+1}^e = \frac{v_{t+1} + d_{t+1}}{v_t},$$

so we have found a relationship between the return on equity and variables in the real economy. The advantage of such an equilibrium relationship is that it is independent of the method used to solve the model, since there is no feedback to the economy via the asset markets.\textsuperscript{11} That is, it can be implemented on the solution of the model to recover the equity prices implied by the model. This feature of the derivation allows one to understand how policy that affects real variables, e.g. monetary policy, affects asset pricing fundamentals once the model has been solved using a given policy specification.

### 3 Benchmark Calibration and Solution Method

The benchmark calibration of the parameters are taken from values that are standard in the literature. Preference and technology parameters are standard, below I focus the discussion on the parameters that govern the degree of nominal rigidity in the benchmark calibration.

The price degree of substitutability parameter is calibrated by making an assumption on the mark-up charged by firms, $\frac{\varepsilon_p}{\kappa - 1}$. In accordance with other literature, I assume a mark-up of $\frac{1}{2}$ which implies a parameter value of $\varepsilon_p = 6$. This assumption is standard in the literature given its empirically plausibility as shown in Christiano et al. (2005), among others.

I follow De Paoli et al. (2010) and Ireland (2001) in setting $\chi^P = 77$. Keen and Wang (2007) provide a correspondence between mark-up and the percentage of re-optimising firms under Calvo pricing and the resulting appropriate calibration for $\chi^P$ under Rotemberg pricing. Using a mark-up of $\frac{1}{2}$, the calibration $\chi^P = 77$ implies that under Calvo pricing somewhere between 20% – 25% of

\textsuperscript{11}By this I mean that in equilibrium bonds are in zero net supply and all equity is held. Thus the asset market returns do not form a feedback loop to the real economy. It is exactly this lack of a feedback that allows the results above to be obtained. Current work is looking at models where this is no longer the case, e.g. where firms engage in costly borrowing.
firms re-optimise their prices. This is reasonable given the literature using Calvo pricing often yields point estimates of price re-optimisation frequency around 18 months which implies that 20% – 25% of firms re-optimise their prices under Calvo. This range is commonly found in the empirical literature as shown in Christiano et al. (2005) and Altig et al. (2011) among others and is relatively standard. In addition, Ascari and Rossi (2009) show that in the case of zero trend inflation the following relationship exists between frequency of adjustment in Calvo models and the parameter governing the adjustment cost in Rotemberg,

\[ \chi^p = \frac{(\varepsilon^p - 1) \theta^p}{(1 - \theta^p)(1 - \beta \theta^p)}, \]  

(3.1)

where \( \theta^p \) is the proportion of firms that cannot change prices under Calvo, \( \beta \) is the discount factor and \( \varepsilon^p \) governs the degree of substitutability between intermediate goods. This relationship is in line with the empirical estimates provided by Keen and Wang (2007).

The wage degree of substitutability parameter is calibrated in a similar fashion to that for intermediate goods, i.e. an assumption is made about the wage mark-up of households \( \left( \frac{\varepsilon^w - 1}{\varepsilon^w - 1} \right) \). I assume a mark-up of 1.05 which implies a parameter value of \( \varepsilon^w = 21 \). Born and Pfeifer (2016) establish the following relationship between the frequency of adjustment in Calvo models and the parameter governing the adjustment cost in Rotemberg,

\[ \chi^w = \frac{(\varepsilon^w - 1) \theta^w}{(1 - \theta^w)(1 - \beta \theta^w)} N, \]  

(3.2)

where \( \theta^w \) is the proportion of households that cannot change prices under Calvo , \( \varepsilon^w \) governs the degree of substitutability between different labour types and \( N \) is the share of the wage change in the adjustment cost base. Under Calvo pricing, where measure \((1 - \theta^w)\) of households may change wages, the average wage duration is given by \( \frac{1}{1-\theta^w} \). There is some variation in estimates of average wage duration in the literature, varying from 3 quarters (Christiano et al., 2005; Chugh, 2006) to 5.6 quarters (Barattieri et al., 2014). I assume average wage duration of 4 quarters as assumed in (Gali, 2008, Ch. 6, p.129) and Furlanetto et al. (2013), indeed this is similar to Altig et al. (2011) and Smets and Wouters (2007) both of whom assume average wage duration of 3.6 quarters. The adjustment cost base for goods is \( y_t \) and for wages it is \( \frac{N}{\varepsilon^w} \). So the wage share of the adjustment cost base is \( N \approx \frac{1-\alpha}{\varepsilon^w} \). The assumption of average wage duration of 4 quarters implies a parameter value of \( \chi^w = 92.689 \) when \( \varepsilon^w = 21 \). 

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Table 1: Calibrated Benchmark Parameters (Quarterly Frequency)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Share</td>
<td>$\alpha = 0.36$</td>
</tr>
<tr>
<td>Depreciation Rate</td>
<td>$\delta = 0.025$</td>
</tr>
<tr>
<td>Goods Degree of Substitutability</td>
<td>$\varepsilon^p = 6$</td>
</tr>
<tr>
<td>Price 'Disutility' Parameter</td>
<td>$\chi^p = 77$</td>
</tr>
<tr>
<td>Technology Shock Autocorrelation Coefficient</td>
<td>$\sigma_z = 0.007$</td>
</tr>
<tr>
<td>Time Preference</td>
<td>$\beta = 0.9963$</td>
</tr>
<tr>
<td>Frisch Elasticity of Labour Supply</td>
<td>$\frac{1}{\phi} = 0.75$</td>
</tr>
<tr>
<td>Investment Adjustment Cost</td>
<td>$\zeta^I = 3.86$</td>
</tr>
<tr>
<td>Standard Deviation of MEI Shock</td>
<td>$\sigma_I = 0.0756$</td>
</tr>
<tr>
<td>Curvature Parameter</td>
<td>$\sigma = 2$</td>
</tr>
<tr>
<td>Habit Intensity</td>
<td>$\nu = 0.65$</td>
</tr>
<tr>
<td>Labour Degree of Substitutability</td>
<td>$\varepsilon^w = 21$</td>
</tr>
<tr>
<td>Nominal Wage Adjustment Parameter</td>
<td>$\chi^w = 92.689$</td>
</tr>
<tr>
<td>Inflation Response</td>
<td>$\rho_\pi = 1.5$</td>
</tr>
<tr>
<td>Output Response</td>
<td>$\rho_y = 0.6$</td>
</tr>
<tr>
<td>Policy Shock Autocorrelation Coefficient</td>
<td>$\iota = 0.65$</td>
</tr>
<tr>
<td>Standard Deviation Monetary Policy Shock</td>
<td>$\sigma_R^\pi = 0.0028$</td>
</tr>
<tr>
<td>MEI Shock Autocorrelation Coefficient</td>
<td>$\rho^I = 0.72$</td>
</tr>
</tbody>
</table>

I calibrate the habit intensity parameter at $\nu = 0.65$ in line with the estimates obtained in Christiano et al. (2005). I note that this calibration for the habit intensity parameter is at the low end of estimates. The asset pricing literature highlights that strengthening the habit intensity should lead to improved asset pricing results. As such, by using this low end calibration I am actually biasing the results against better asset pricing replication.

The investment adjustment cost parameters are taken from Justiniano et al. (2011) and I follow Challe and Giannitsarou (2014) in the calibration of the central bank parameters. All of which are relatively standard in the literature.

### 3.1 Solution Methodology

The model is solved to the fifth order in Dynare++. I take fifth order solutions in line with Swanson (2015). It is argued in Swanson (2015) that one must solve the model at least a third order approximation in order to have time varying risk premia. Furthermore, Swanson (2015) shows that fourth and sixth order approximations are very similar to the fifth order suggesting convergence in the Taylor series for the state variables.

I use non-linear methods to find the equity and risk free returns from the simulated data. The particular algorithm used is the Parameterised Expectations Algorithm (PEA) of den Haan and Marcet (1990); Marcet and Marshall (1994); Marcet and Lorenzoni (1998). I employ non-linear methods to recover the asset prices from the simulated data as the conditional expectation is key to these calculations and PEA provides a method of evaluating this numerically.
Table 2: Asset Pricing Results

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Flex Price Model</th>
<th>Sticky Price Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean (% p.a)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk Free</td>
<td>1.49</td>
<td>1.04</td>
<td>1.17</td>
</tr>
<tr>
<td>Equity</td>
<td>7.87</td>
<td>3.13</td>
<td>2.91</td>
</tr>
<tr>
<td>Equity Premium</td>
<td>6.52</td>
<td>2.09</td>
<td>1.73</td>
</tr>
</tbody>
</table>

|                  |      |                  |                    |
| **Standard Deviations (% p.a)** |      |                  |                    |
| Risk Free        | 2.62 | 2.88             | 2.26               |
| Equity           | 16.72| 9.77             | 9.01               |
| Equity Premium   | 15.69| 9.54             | 8.78               |

4 Quantitative Results

The results in 3 highlight that the simple New Keynesian model studied here cannot achieve quantitative replication of asset pricing facts. Thus one cannot resolve asset pricing puzzles inherent in DSGE macroeconomic models simply by moving to a model with New Keynesian features.

In the prototypical model studied here, the poor quantitative asset pricing performance is due to two factors. First, there is a direct effect as nominal price rigidities lead to smaller profits. Secondly, the presence of nominal wage rigidities requires an endogenous labour market, but the forward looking behaviour induced in the labour market has detrimental indirect effects on equity prices. The operation of both of these mechanisms in concert leads to the observed quantitative asset pricing results where one sees a deterioration of quantitative equity returns in the presence of nominal rigidities.

The presence of sticky prices leads to smaller profits as already discussed. This means that profits play a smaller effect on the equity price and consequently on the equity return than if prices were flexible.

4.1 The Labour Channel - Twists and Shifts

It has long been held as dogma in the macroeconomic asset pricing literature that flexible labour is anathema to quantitative replication of asset pricing facts. So much so that Boldrin et al. (2001) state “inflexibilities in labour are a crucial ingredient for producing an equity premium.” All of these models restrict both consumption and investment by some combination of frictions.\(^{12}\) The reasoning

\(^{12}\)The most common combination is internal habits in consumption and capital adjustment costs. Internal habits in consumption introduce inertia in consumption and capital adjustment costs introduce inertia in investment.
here is simple - by restricting labour flexibility agents are forced to use a costly intertemporal channel (i.e. the consumption-investment tradeoff) in order to achieve consumption smoothing goals rather than the (previously) costless intratemporal labour/income channel to smooth consumption.

The presence of nominal rigidities, while helping explain real variables, actively work against asset pricing results. Nominal rigidities make the labour market forward looking and it is this ‘forward looking-ness’ that makes all the difference. Nominal rigidities in wages mean that equilibrium labour decisions are less responsive to contemporaneous shocks since they are forward looking. Consequently, households have smoother income which allows for smoother consumption profiles. Thus nominal rigidities allow households to achieve their consumption smoothing goal via income smoothing, rather than the relatively more costly intra-temporal investment channel.

Intuitively the presence of nominal rigidities leads to the real wage being less responsive to a shock. In response to this households and firms utilise the hours channel more intensively than in the case of no nominal rigidities. Thus agents use the labour hours channel, i.e. the channel that is not affected by nominal rigidities, to effectively circumvent the presence of nominal rigidities. This can be understood as the presence of ‘twists’ as well as ‘shifts’ in the labour market.

The labour market equilibrium occurs when labour supply equals labour demand. Labour supply is given by rearranging the wage Philips curve equation,

\[
\frac{w_t}{P_t} = \left( \frac{1}{\Theta^w_t + 1} \right) \left\{ -\frac{\epsilon^w}{\epsilon^w - 1} \frac{u_{n,t}}{\lambda_t} + \frac{1}{n_t} \frac{E_t}{1} \left[ m_{t,t+1} \Theta^w_{t+1} \frac{u_{t+1}}{P_{t+1}} - \Lambda_t \right] \right\}, \tag{4.1}
\]

where,

\[
\Theta^w_t = \frac{\chi^w}{\epsilon^w - 1} \pi^w_t \left( \pi^w_t - 1 \right).
\]

Labour demand is given by rearranging the price Philips curve equation,

\[
\frac{w_t}{P_t} = \left( \frac{\epsilon^p - 1}{\epsilon^p} + \Theta^p_t \right) z_t (1 - \alpha) k^\alpha_t n^{-\alpha}_t - \frac{1 - \alpha}{n_t} E_t \left[ m_{t,t+1} \Theta^p_{t+1} y_{t+1} \right], \tag{4.2}
\]

where,

\[
\Theta^p_t = \frac{\chi^p}{\epsilon^p} \pi^p_t \left( \pi^p_t - 1 \right).
\]

In flex price model both \(\chi^w = 0\) and \(\chi^p = 0\) so that any shock leads to shifts in both curves. However, in the presence of nominal rigidities one encounters the expectation terms which are both multiplied by \(\frac{1}{n_t}\), which allows the curves to ‘twist’. The degree of this twisting is strongest at low values of hours so that is
causes, all else equal, a flattening of both curves. The twisting serves to make both labour demand and labour supply less responsive to the real wage - this is as the intuition above would suggest. This twisting captures the fact that the labour market wedge\(^\text{13}\) is endogenous allowing agents to remain away from the flexible price labour market equilibrium. A detailed analysis of this mechanism is the focus of concurrent working paper. For the analysis here it suffices to observe that the twisting of the curves in the labour market due to the presence of nominal rigidities forces the hours channel to work more intensively, as evidenced in the impulse responses presented in Figure: 1.

This behaviour allows labour to be more flexible in the sense that it can respond more vigorously to a shock. Thus agents achieve consumption smoothing via income smoothing and not via the investment channel. Hence aggregate output devoted to production of investment goods are also smoother. This reduces the efficacy of the investment adjustment cost. This means that firms experience both smoother investment prices and less variation in their purchase of investment goods than if nominal wage rigidities were absent. This leads to a

\[ \text{MRS}_t = \frac{\frac{\pi_t}{x_w t} + x_p t - E_t \left\{ m_{t+1} + 1 x_p t + 1 \frac{\pi_{t+1}}{\pi_{t+1}} \right\}}{\frac{\pi_t}{x_w t} + E_t \left\{ m_{t+1} + 1 x_p t + 1 \frac{\pi_{t+1}}{\pi_{t+1}} \right\}} \]  
\[ \text{MPN}_t = \frac{\pi_t}{x_w t} + E_t \left\{ m_{t+1} + 1 x_p t + 1 \frac{\pi_{t+1}}{\pi_{t+1}} \right\} \]  

where \( x_p t = \frac{\pi_t}{x_w t} \pi_t \left( \pi_t \frac{\pi_{t+1}}{\pi_{t+1}} - 1 \right) \) and \( x_w t = \frac{\pi_t}{x_w t} \pi_t \left( \pi_t - 1 \right) \).

---

\(^{13}\)The labour market wedge is defined as the ratio of the marginal rate of substitution and the marginal product of labour, i.e. \( \frac{\text{MRS}_t}{\text{MPN}_t} \). Which in the general case is given by,
smoother dividend process, leading to both smaller and smoother equity returns. Thus nominal wage rigidities, introduced in New Keynesian models to improve the behaviour of real variables, undermines the asset pricing mechanism.

5 Imperfect Investment Transformation

In the benchmark model once investment is purchased by the firm it can be transformed perfectly into productive capital in the subsequent period. One can envision that converting investment capital into productive capital entails costs, which include costs of installation and possible firm reorganisation. As such not all investment is converted into productive capital. Within the macroeconomic asset pricing literature this reality is measured by the introduction of capital adjustment costs that introduce curvature into the transformation of investment into capital. Indeed, the asset pricing literature seems to have settled on the need for internal habits in consumption coupled with strong capital adjustment costs to generate realistic quantitative asset pricing results. In this subsection I study how the introduction of such costs to the transformation of investment into capital affect both the theoretical and quantitative results of the benchmark model.

The multi-sector modelling of the transformation of final output into productive capital allows one to introduce capital adjustment costs in a very simple manner. Capital adjustment costs occur at the firm level once investment has been purchased. This results in a modification of the capital accumulation process to,

$$k_{t+1} = (1 - \delta) k_t + \phi \left( \frac{I_t}{k_t} \right) k_t$$

where $\phi \left( \frac{I_t}{k_t} \right)$ is a non-linear function which allows for resources being required to transform investment into capital. This introduces inertia into the accumulation of capital since it takes several periods to achieve the desired levels of capital. The economy can no longer achieve any feasible level of capital both costlessly and immediately, as is the case when adjustment costs are absent (Abel, 1981). In particular there is a non-linearity in how the relative proportion of investment, captured by the investment-capital ratio, translates into additional capital.\(^{14}\)

The function $\phi(\cdot)$ is assumed to satisfy $\phi'(\cdot) > 0$, $\phi''(\cdot) < 0$, $\phi(0) = 0$ and $\phi(\delta) = \delta$. The specific functional form for the adjustment cost function is given

\(^{14}\)Since $\frac{I_t}{k_t} \in [0, 1]$ this means $\phi \left( \frac{I_t}{k_t} \right) \in [0, 1]$ so that not all investment is converted into capital hence introducing inertia into capital accumulation.
by the following split function,
\[
\phi \left( \frac{i}{k} \right) = \begin{cases} 
\frac{b_1}{1-\xi} \left( \frac{i}{k} \right)^{1-\frac{i}{\xi}} + b_2 & \text{if } i > 0 \\
0 & \text{if } i = 0
\end{cases}
\] (5.2)

This general functional form is used in both Jermann (1998) and Boldrin et al. (2001), although both only report the upper branch of the split function. This function separates the conversion of investment into a fixed component and a variable component. The parameter $b_2$ governs the fixed amount of investment that flows through to capital, it captures the idea that there is a certain minimum proportion of all investment that will be converted into capital. The variable component is governed by the parameters $b_1$ and $\xi$, where $b_1$ is the additional investment converted into capital and $\xi$ governs the curvature of the adjustment cost function, hence the penalty imposed on investment. As $\xi$ fall there is greater curvature and hence stronger adjustment costs. Finally, as $\xi \to \infty$ the functional form collapses to a one without capital adjustment costs.

This change to the capital accumulation process changes the investment first order condition to,
\[
\frac{Q_t}{P_t} \frac{1}{\phi'(g_t)} = E_t \left\{ m_{t+1} \frac{w_{t+1}f_{k,t+1}}{P_{t+1}f_{n,t+1}} + \frac{Q_{t+1}}{P_{t+1}} \frac{1}{\phi'(g_{t+1})} \left( 1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1}) \right) \right\}
\] (5.3)

where $g_t = \frac{I_t}{k_t}$ is the investment-capital ratio.

The lower branch of this split function ensures that if there is no investment then there are no adjustment costs, so that it satisfies the regulatory conditions in (??). In the DSGE models considered in this thesis consumption smoothing arguments ensure that there will always be positive investment. As such one can safely ignore the lower branch as it will never be invoked. Simulations were conducted with the full split function, i.e. with the possibility of zero investment, and I find that the lower branch is never invoked, i.e. investment is always positive.

Steady state equations imply that $b_1 = \delta \frac{i}{\xi}$ and $b_2 = \delta \frac{i}{1-\xi}$. So as $\xi \to \infty$ one gets the following,
\[
b_1 = \delta \frac{i}{\xi} \to 1, \\
b_2 = \delta \frac{i}{1-\xi} \to 0,
\]
so that the adjustment cost function approaches a linear function. Thus the capital accumulation function approaches one that does not exhibit capital adjustment costs.
5.1 Equity Pricing Result

The presence of capital adjustment costs impacts the equity pricing result of the benchmark model since it leads to a change in the investment first order condition to (5.3). This leaves Sections 2.1.1 and 2.1.2 of the derivation of the equity price unchanged, the impact of the change in the investment first order condition appears only in the derivation in the final section 2.1.3. The equity price under the assumption of capital adjustment costs takes form,

\[ v_t = \frac{Q_t}{P_t} \frac{k_{t+1}}{\phi'(g_t)} + E_t \left\{ \sum_{i=1}^{\infty} m_{t,i+1} \left( \frac{1}{\epsilon_p} - \Theta_{t+i} \right) y_{t+i} \right\}. \quad (5.4) \]

The detailed derivation is presented in Appendix A. Thus the presence of capital adjustment costs lead to a magnification of the value of capital if \( \phi'(g_t) < 1 \) and a muting of the value of capital if \( \phi'(g_t) > 1 \). Importantly this only affects the capital component of the equity price so bypassing the impact of nominal rigidities discussed in the previous section. Furthermore, as it only depends on the contemporaneous investment-capital ratio it will also add volatility to the value of capital as the smoothing of the investment series due to sticky wages will be less pronounced.

5.2 Quantitative Results

The quantitative results are obtained in the same manner as for the benchmark, with an identical calibration used. The parameter \( \xi \), governing the strength of the capital adjustment cost, is calibrated to a benchmark value of 0.45. This calibration is approximately double that used in both Jermann (1998) and Boldrin et al. (2001) so that the adjustment cost is much weaker than assumed in those papers. The weaker adjustment cost is possible because of the presence of adjustment costs in the investment goods sector. This calibration is within the empirically plausible range, however there is no empirical work to my knowledge that estimates a model with both adjustment costs in the investment goods sector and the capital accumulation process.

The quantitative results are presented for the Sticky Price model. The same trend is seen in the Flex Price Model as noted in the previous section, i.e. the presence of nominal rigidities leads to a deterioration of the asset pricing results. Here I focus on the impact that the introduction of capital adjustment costs have on the sticky price specification. One notes that the presence of the capital adjustment cost leads to a significant improvement in the mean equity return and the volatility of equity. This improvement in equity results is at the expense of the risk free return statistics, indeed the risk free rate both falls and
Table 3: Asset Pricing Results - Sticky Price Model

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Benchmark Model</th>
<th>Capital Adjustment Cost Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean (% p.a)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk Free</td>
<td>1.49</td>
<td>1.17</td>
<td>1.04</td>
</tr>
<tr>
<td>Equity</td>
<td>7.87</td>
<td>2.91</td>
<td>5.57</td>
</tr>
<tr>
<td>Equity Premium</td>
<td>6.52</td>
<td>1.73</td>
<td>4.52</td>
</tr>
<tr>
<td><strong>Standard Deviations (% p.a)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Risk Free</td>
<td>2.62</td>
<td>2.26</td>
<td>3.99</td>
</tr>
<tr>
<td>Equity</td>
<td>16.72</td>
<td>9.01</td>
<td>13.05</td>
</tr>
<tr>
<td>Equity Premium</td>
<td>15.69</td>
<td>8.78</td>
<td>12.33</td>
</tr>
</tbody>
</table>

becomes too volatile.

The presence of capital adjustment costs leads to an even smoother investment series as firms build up capital stock over a number of periods. Consequently the demand for the investment good falls leading to a smoother gross investment series. This therefore forces consumption to become even more volatile. The equilibrium response is for the risk free rate to fall, with added volatility due to consumption taking on ever increasing exposure to shocks that hit the economy.

The improvement in the equity return statistics is the result of both a higher level of dividends and multiplier effect of the adjustment cost on the value of capital. The presence of capital adjustment costs leads to a smoother investment demand, as explained above, which leads to a larger response of dividends to shocks. This helps improve the equity return. The added volatility is the result of the investment adjustment shock adding to the volatility of the capital value via $\phi'(g_t)$.

6 Conclusion

This paper has studied the equity prices implied by the standard New Keynesian DSGE model. I have shown that an equity pricing result can be derived for these models that is independent of the solution technique used. Using this result I show that the benchmark New Keynesian model suffers from the same asset pricing shortcomings as their RBC brethren - i.e. the equity premium is both too small and too smooth. This result is attributed to the presence of a labour market that becomes more flexible along the labour hours dimension once nominal rigidities are introduced. Thus the first major conclusion of this paper is that features that allow macroeconomists to better explain the behaviour of real and nominal variables leads to a deterioration of asset pricing results.
I proceeded to extend the model to include capital adjustment costs, a friction that is prevalent in the RBC macroeconomic asset pricing literature. I show that this friction, when used in addition to the investment adjustment cost, can lead to a significant improvement of the equity return. However, this is at the expense of the risk free return.

Labour market rigidities are generally used as the justification for the presence of nominal rigidities. As a result the labour market is a cornerstone of New Keynesian DSGE models and cannot be ignored as is the case in RBC asset pricing models. The results in this paper therefore provide a first step towards a theory of asset prices in the face of realistic labour markets. Future research could focus on how to make labour less flexible, one possibility that comes to mind (and is the focus of current research) is to use a heterogeneous agent approach where agents differ in their access to capital markets. Given the lack of empirical work on New Keynesian models with both investment adjustment costs and capital adjustment costs, estimation of these models is also an avenue for future research. Such empirical work could shed light not only on the parameter estimates, but also the relative importance of these two frictions in the economy.

Finally, this paper allows one to begin to understand how monetary policy affects asset prices. Given the poor quantitative replication such analysis will necessarily be of a qualitative nature. For example, one could see how different policy specifications affect asset pricing results and the mechanisms at work.

A Appendix: Equity Price under Capital Adjustment Costs

The household Euler equation for equity is given by,

\[ v_t = E_t \left\{ m_{t+1} \left[ v_{t+1} + z_t f_{k,t+1} k_{t+1} + \left( \frac{1}{\varepsilon^p} - \Theta_{t+1} \right) z_t f_n n_{t+1} - \frac{Q_{t+1}}{P_{t+1}} I_{t+1} \right] \right\} \]

(A.1)

We now establish that,

\[ E_t \left\{ m_{t+1} \left[ f_{k,t+1} k_{t+1} - \frac{Q_{t+1}}{P_{t+1}} I_{t+1} \right] \right\} = \frac{Q_t}{P_t} \frac{k_{t+1}}{\phi' (g_t)} + E_t \left\{ m_{t+1} \left( \frac{1}{\varepsilon^p} - \Theta_{t+1} \right) f_{k,t+1} k_{t+1} \right\} \]

\[ - E_t \left\{ m_{t+1} \frac{Q_{t+1}}{P_{t+1}} \frac{k_{t+2}}{\phi' (g_{t+1})} \right\} \]
This removes the investment term and expresses the terms in terms of the capital stock. We first establish that,

\[ E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) f_{k,t+1} + \frac{Q_{t+1}}{P_{t+1}} \frac{1}{\phi'(g_{t+1})} (1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1})) \right] k_{t+1} \right\} = \frac{Q_t}{P_t} E_t \left\{ k_{t+1} \right\} \]

(A.2)

Proof.

\[ E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) f_{k,t+1} + \frac{Q_{t+1}}{P_{t+1}} \frac{1}{\phi'(g_{t+1})} (1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1})) \right] k_{t+1} \right\} = E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) f_{k,t+1} + \frac{Q_{t+1}}{P_{t+1}} \phi'(g_{t+1}) (1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1})) \right] k_{t+1} \right\} - \text{cov}_t \left( m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) f_{k,t+1} + \frac{Q_{t+1}}{P_{t+1}} \phi'(g_{t+1}) (1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1})) \right] k_{t+1} \right) = \frac{Q_t}{P_t} E_t \left\{ k_{t+1} \right\} \]

where we have used (5.3) to arrive at the solution. We next establish that,

\[ E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) f_{k,t+1} + \frac{Q_{t+1}}{P_{t+1}} \frac{1}{\phi'(g_{t+1})} (1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1})) \right] k_{t+1} \right\} = E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) f_{k,t+1} + \frac{Q_{t+1}}{P_{t+1}} \frac{k_{t+2}}{\phi'(g_{t+1})} \right] k_{t+1} \right\} + E_t \left\{ m_{t+1} \frac{Q_{t+1}}{P_{t+1}} \frac{k_{t+2}}{\phi'(g_{t+1})} \right\} \]

Proof.

\[ E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) f_{k,t+1} + \frac{Q_{t+1}}{P_{t+1}} \frac{1}{\phi'(g_{t+1})} (1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1})) \right] k_{t+1} \right\} = E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) f_{k,t+1} + \frac{Q_{t+1}}{P_{t+1}} \frac{1}{\phi'(g_{t+1})} (1 - \delta + \phi(g_{t+1}) - g_{t+1} \phi'(g_{t+1})) \right] k_{t+1} \right\} + E_t \left\{ m_{t+1} \frac{Q_{t+1}}{P_{t+1}} (1 - \delta) k_{t+1} + \phi(g_{t+1}) k_{t+1} \right\} = E_t \left\{ m_{t+1} \frac{Q_{t+1}}{P_{t+1}} \frac{k_{t+2}}{\phi'(g_{t+1})} \right\} + E_t \left\{ m_{t+1} \frac{Q_{t+1}}{P_{t+1}} \frac{k_{t+2}}{\phi'(g_{t+1})} \right\} \]

Where we get to the second step by expanding expanding the expectation and noting that \( \frac{g_{t+1} \phi'(g_{t+1}) k_{t+1}}{\phi'(g_{t+1})} = I_{t+1} \) and that \( k_{t+2} = (1 - \delta) k_{t+1} + \phi(g_{t+1}) k_{t+1} \). So that we have established that,

\[ E_t \left\{ m_{t+1} \left[ \left( \frac{\varepsilon_p - 1}{\varepsilon_p} + \Theta_{t+1} \right) f_{k,t+1} + \frac{Q_{t+1}}{P_{t+1}} I_{t+1} \right] \right\} = \frac{Q_t}{P_t} E_t \left\{ \frac{k_{t+1}}{\phi'(g_{t+1})} \right\} - E_t \left\{ m_{t+1} \frac{Q_{t+1}}{P_{t+1}} \frac{k_{t+2}}{\phi'(g_{t+1})} \right\} \]

(À.3)
The final result comes from rearranging the left hand side to get the desired result. Substituting this back into the households Euler equation yields,
\[
v_t = \frac{Q_t}{P_t} \frac{k_{t+1}}{\phi'(g_t)} + E_t \left\{ m_{t,t+1} \left[ v_{t+1} + \left( \frac{1}{\varepsilon_p} - \Theta_{t+1} \right) y_{t+1} - \frac{Q_{t+1}}{P_{t+1}} \frac{k_{t+2}}{\phi'(g_{t+1})} \right] \right\}
\]
This is once again a stochastic difference equation that may be solved forward to yield,
\[
v_t = \frac{Q_t}{P_t} \frac{k_{t+1}}{\phi'(g_t)} + E_t \left\{ \sum_{i=1}^{\infty} m_{t,t+i} \left( \frac{1}{\varepsilon_p} - \Theta_{t+i} \right) y_{t+i} \right\}, \quad (A.4)
\]

References


