ISSN 1471-0498

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

TESTING THE INVARIANCE OF EXPECTATIONS MODELS OF INFLATION

Jennifer L. Castle, Jurgen A. Doornik, David F. Hendry and Ragnar Nymoen

Number 510
November 2010

Manor Road Building, Oxford OX1 3UQ

Testing the Invariance of Expectations Models of Inflation

Jennifer L. Castle†, Jurgen A. Doornik♦, David F. Hendry♦ and Ragnar Nymoen*
†Magdalen College and Institute for Economic Modelling, Oxford Martin School, University of Oxford, UK
♦Economics Department and Institute for Economic Modelling, Oxford Martin School, University of Oxford, UK
⋆Economics Department, Oslo University, Norway

November 9, 2010

Abstract

The new-Keynesian Phillips curve (NKPC) includes expected future inflation as a major feedforward variable to explain current inflation. Models of this type are regularly estimated by replacing the expected value by the actual future outcome, then using Instrumental Variables or Generalized Method of Moments methods to estimate the parameters. However, the underlying theory does not allow for various forms of non-stationarity in the data–despite the fact that crises, breaks and regimes shifts are relatively common. We investigate the consequences for NKPC estimation of breaks in data processes using the new technique of impulse-indicator saturation, and apply the resulting methods to salient published studies to check their viability.

JEL classifications: C51, C22.
KEYWORDS: New-Keynesian Phillips curve; Inflation expectations; Structural breaks; Impulse-indicator saturation.

Contents

*We are grateful to Neil Ericsson, Katarina Juselius, Søren Johansen, and Bent Nielsen for helpful comments and suggestions. This research was supported in part by grants from the Open Society Institute and the Oxford Martin School.

1 Introduction

Expectations play an important role in most financial markets and in many economic theories. Central Banks use interest rates for inflation 'targets' based on expected, or forecast, inflation one or two years ahead. Nevertheless, it is unclear how accurate agents' expectations of future variables are, even considering sophisticated agents. For example, although exchange rates are a key financial price, Nickell (2008) shows the 2-year ahead consensus for £ERI systematically mis-forecasting by a large margin over the prolonged time period, 1996–2002. Chart 1.16 in Bank of England (2009) shows similar mis-forecasting from 2008. Moreover, despite a substantial investment in modelling and forecasting, and a committee of experts to help it (namely, the Monetary Policy Committee), the Bank of England still significantly mis-forecast CPI inflation as seen in figure 2, from Bank of England (2008): the CPI then rose above 5% in September 2008, outside the entire range shown in their graph. Or consider consensus oil price forecasts for 2008, where none included any price near the $145 high, nor the $40 per barrel that eventuated. Finally, the collapse of many of the world's largest financial institutions reveals how inaccurate their expectations of asset values have proved to be.

Indeed, 'crises' occur with impressive frequency, but unimpressive anticipation: see e.g., Clements and Hendry (2001), and Barrell (2001). Consequently, forecast failures are all too common as e.g., Stock and Watson (1996), and Clements and Hendry (1998) document, and the primary causes of such failures seem to be location shifts, namely changes in previous unconditional means: see e.g., Clements and Hendry (1999), and Hendry (2000, 2006). These examples are not intended as criticisms of the agencies and procedures involved: rather, together they suggest that it is extremely difficult to form accurate expectations about future events, especially because the mean of the future distribution often differs in unanticipated ways from the present mean.

Nevertheless, the currently dominant model of agents' expectations assumes that they are rational, namely, they coincide with the conditional expectation, $\mathsf{E}\left[y_{t+1}|\mathcal{I}_t\right]$, of the unknown future value, y_{t+1}, given all relevant information, denoted $\mathcal{I}_t$. Indeed, $\mathsf{E}\left[y_{t+1}|\mathcal{I}_t\right]$ is often replaced in econometric models by the later outcome, with an error that is claimed to be unpredictable from present information:

$$\mathsf{E}\left[y_{t+1} \mid \mathcal{I}_t\right] = y_{t+1} + v_{t+1} \tag{1}$$

where $\mathsf{E}[v_{t+1}|\mathcal{I}_t] = 0$. The error v_{t+1} in (1) cannot be independent of y_{t+1}. Then equations of the form:

$$y_t = \beta_1 \mathsf{E}\left[y_{t+1} \mid \mathcal{I}_t\right] + \beta_2 y_{t-1} + \beta_3 x_t + u_t \tag{2}$$

(where x_t is assumed 'exogenous') are re-written as:

$$y_t = \beta_1 y_{t+1} + \beta_2 y_{t-1} + \beta_3 x_t + \epsilon_t \tag{3}$$

usually with the auxiliary assumption that $\epsilon_t \sim \mathsf{D}\left[0, \sigma_\epsilon^2\right]$.

The formulation in (3) is almost invariably used in new-Keynesian Phillips curve (NKPC) models of inflation. Estimating the parameters of such equations by Instrumental Variables (IV) or Generalized Method of Moments (GMM) methods usually reveals high inflation persistence (i.e., $\beta_1 + \beta_2$ close to unity), implying large costs of reducing inflation once it rises, and consequently entailing 'tough' interest rate policies to avoid such a scenario. Policy at the Bank of England during much of 2008 reflected such a belief, although other empirical models of inflation suggest persistence is partly an artifact of the model specification adopted: see e.g., Castle (2008). Most dynamic stochastic general equilibrium models (DSGEs) likewise impose rational expectations: see Smets and Wouters (2002), for a recent implementation.

This paper investigates the role and significance of expectations in the context of NKPCs when there are location shifts in the underlying processes. Previous tests of feedforward models include Hendry (1988) and Engle and Hendry (1993), both of which focused on non-constancy to differentiate between models. Here we consider using impulse-indicator saturation-based tests initiated in Hendry (1999) and analyzed by Hendry, Johansen and Santos (2008), Johansen and Nielsen (2009) and Castle, Doornik and Hendry (2009). The impulse indicators are selected in the 'forecasting' equation derived from (2), then tested for significance in (3), related to the test for super exogeneity in Hendry and Santos (2010). Under the null of correct specification, few such impulse indicators will be selected, and those that are should not be significant when added to (3); moreover, parameter estimates should not alter much. Under the alternative that there are unmodeled outliers or breaks, there will be significant impulse indicators in the 'forecasting' equation, and these will remain significant when added to (3). The properties of this class of automatic model selection procedures using *Autometrics* (see Doornik, 2009) are discussed in Castle, Doornik and Hendry (2010) and Hendry and Mizon (2010a).

The structure of the paper is as follows. Section 2 briefly describes the recent technique of impulse-indicator saturation, denoted by its acronym IIS, which will provide the tool for investigating both location shifts and their impacts on estimates of NKPCs. Subsection 2.1 investigates the consequences of adding instrumental variables from the reduced form of an endogenous variable to a structural equation it enters. Section 3 then reviews the requirements for forming conditional expectations of future values given all relevant information. Section 4 discusses the formulation of new-Keynesian Phillips curve models that embody (1), then section 4.1 considers the empirical evidence on NKPC estimation. Section 5 analyzes the impacts on feedforward models of ignoring breaks and section 6 provides some simulation findings on the application of IIS. Sections 7 and 8 respectively report new Euro-area and US NKPC estimates with and without IIS. Section 9 concludes.

2 Impulse-indicator saturation

Impulse-indicator saturation adds an indicator for every observation to the set of candidate regressors, entered (in the simplest case) in blocks of $T/2$ for T observations (when there are fewer than $T/2$ other regressors). The theory of IIS is derived under the null of no outliers, but with the aim of detecting and removing outliers and location shifts when they are present. We first describe the simplest form of 'split half' IIS, the case for which Hendry *et al.* (2008) and Johansen and Nielsen (2009) develop an analytic theory and derive the resulting distributions of estimators, then consider the more sophisticated algorithm used by *Autometrics*, an Ox Package implementing automatic model selection: see Doornik (1999, 2009).

First, add half the impulse indicators to the model, record the significant ones, then drop that first set of impulse indicators. Now add the other half, recording again. These first two steps correspond to 'dummying out' $T/2$ observations for estimation, noting that impulse indicators are mutually orthogonal. Finally combine the recorded impulse indicators and select the significant subset. Under the null of no outliers or location shifts, Hendry *et al.* (2008) derive the distribution of the mean after IIS in scalar IID processes, and also show that on average αT indicators will be retained adventitiously, where α is the chosen significance level. Johansen and Nielsen (2009) generalize the theory to more, and unequal, splits, as well as to dynamic models with possibly unit roots. Moreover, Johansen and Nielsen (2009) prove that under the null of no outliers or shifts, there is almost no loss of efficiency in testing for T impulse indicators when setting $\alpha \leq 1/T$, even in dynamic models, and relate IIS to robust estimation facing potential data contamination. While such high efficiency despite having more candidate regressors than observations is surprising at first sight, retaining an impulse indicator when it is not needed merely 'removes' one observation, which is all that will happen on average under the null. Thus, efficiency is of the order of $100(1-\alpha)\%$.

The algorithm used by *Autometrics* has several block divisions and does not rely on the impulse indicators being orthogonal. With IIS at the recommended tight significance levels, Monte Carlo experiments in Castle *et al.* (2009) have confirmed the null distribution, and shown that *Autometrics* selection has the appropriate properties: at nominal significance levels of $\alpha = 2.5\%$, 1%, and 0.1%, the probabilities of retaining any irrelevant dummies are close to α. Castle *et al.* (2009) also show that IIS is capable of detecting up to 20 outliers in 100 observations as well as multiple shifts, including breaks close to the start and end of the sample. They also compare its power to detect breaks in U.S. real interest rates with the procedure in Bai and Perron (1998) based on Garcia and Perron (1996): extending the sample to 1947:2–2009:3, covering most of the post-war period, revealed substantial benefits to IIS as there are breaks or outliers near the start and end of the sample as well as other shifts.

2.1 Adding instrumental variables to structural equations

The IIS tests involve adding the significant impulse indicators from the forecasting equation to the structural equation. Hendry (2010b) shows that adding valid over-identifying instruments from the reduced form of a second structural equation has almost no impact on the estimates of the parameters of a first structural equation. Finding instruments in the reduced form which are irrelevant in the structural equation, but nevertheless adding them to that structural equation, cannot improve efficiency, but leaves the population parameter values of that structural equation unaffected. The null hypotheses for the irrelevant variables in the first equation are rejected at their nominal significance levels, and there is little impact on the null rejection rates of the relevant variables. Adding the instruments does increase the spread of the t-distributions of estimated parameters relative to knowing the instruments enter the second equation's reduced form, but leaves the t-distribution essentially the same relative to when it is not known that they are additional instruments. The latter is the situation here with any selected impulse indicators from the forecasting equation. There should be almost no impact on the estimates of the parameters of the first structural equation, and the added instruments should be insignificant when the first structural equation is correctly specified, so the estimated equation standard error should also be nearly unaffected. Thus, it is legitimate to add over-identifying instruments to a structural equation, and if they are significant on doing so, that demonstrates the mis-specification of the structural equation.

3 Models of expectations

A 'rational' expectation (denoted RE, following Muth, 1961) is the conditional expectation of a variable, y_{t+1}, given available information $\mathcal{I}_t$, often written as:

$$y^{re}_{t+1} = \mathsf{E}\left[y_{t+1} \mid \mathcal{I}_t\right] \tag{4}$$

Agents are assumed to adopt such a minimum mean square error formulation as it avoids arbitrage and hence unnecessary losses. Since RE requires free information, unlimited computing power, and free discovery of the form of $\mathsf{E}[y_{t+1}|\mathcal{I}_t]$, such an approach has many critics (see e.g., Kirman, 1989, Frydman and Goldberg, 2007, and Juselius, 2006). Nevertheless, in stationary processes (including difference stationary and trend stationary), assuming that agents use $\mathsf{E}[y_{t+1}|\mathcal{I}_t]$ is not unreasonable, perhaps with learning (see e.g., Evans and Honkapohja, 2001).

But as economic processes lack time invariance, (4) is far too demanding. Agents cannot do the entailed calculations, since (4) should be written formally as:

$$y^{re}_{t+1} = \mathsf{E}_{t+1}\left[y_{t+1} \mid \mathcal{I}_t\right] = \int y_{t+1} f_{t+1}\left(y_{t+1} \mid \mathcal{I}_t\right) \mathrm{d}y_{t+1} \tag{5}$$

The crucial feature is the explicit dating of the expectations operator, E_{t+1}, since (5) reveals that RE requires a 'crystal ball' to know in advance the entire future distribution $f_{t+1}(y_{t+1}|\mathcal{I}_t)$. This dating of the operator was deliberately omitted in both (1) and (4) to reflect widely-used conventions. Yet written as:

$$\widehat{y}_{t+1} = \mathsf{E}_t\left[y_{t+1} \mid \mathcal{I}_t\right] = \int y_{t+1} f_t\left(y_{t+1} \mid \mathcal{I}_t\right) \mathrm{d}y_{t+1} \tag{6}$$

$\widehat{y}_{t+1}$ will not even be unbiased for y_{t+1} unless $f_{t+1}(\cdot) = f_t(\cdot)$, so $\mathsf{E}_{t+1} = \mathsf{E}_t$. Simply writing (6) makes it evident why location shifts are so pernicious–one is integrating relative to the wrong mean.

In practice, the best an agent can do is to form a 'sensible expectation', y^{se}_{t+1}, which involves 'forecasting' $f_{t+1}(\cdot)$ by $\widehat{f}_{t+1}(\cdot)$:

$$y^{se}_{t+1} = \int y_{t+1} \widehat{f}_{t+1}\left(y_{t+1} \mid \mathcal{I}_t\right) \mathrm{d}y_{t+1}. \tag{7}$$

However, if the moments of $f_{t+1}(y_{t+1}|\mathcal{I}_t)$ alter, there are no good rules for $\widehat{f}_{t+1}(\cdot)$, except that $f_t(\cdot)$ is not a good choice when there are location shifts. Agents cannot know how $\mathcal{I}_t$ enters $f_{t+1}(\cdot)$ if there is a failure of time invariance. Since RE would be a more feasible approach when $f_{t+1}(\cdot) = f_t(\cdot)$, its viability depends on the extent and magnitude of location shifts in the underlying process, and IIS offers a method for evaluating that. Hendry and Mizon (2010b) provide a mathematical analysis of the implications of location shifts for models based on inter-temporal optimization assuming constant distributions, and prove that conditional expectations are biased and that the law of iterated expectations does not hold inter-temporally. Hendry (2010a) derives some implications of those findings for the analysis of climate change.

4 New-Keynesian Phillips curve

The 'hybrid' new-Keynesian Phillips curve (NKPC) is usually given by a model of the form:

$$\Delta p_t = \underset{\geq 0}{\gamma_f} \mathsf{E}_t\left[\Delta p_{t+1}\right] + \underset{\geq 0}{\gamma_b} \Delta p_{t-1} + \underset{\geq 0}{\lambda} s_t + u_t \tag{8}$$

where Δp_t is the rate of inflation, $\mathsf{E}_t\left[\Delta p_{t+1}\right]$ is expected inflation one-period ahead conditional on information available today, using the conventions of the literature, and s_t denotes firms' real marginal costs. The notation $\mathsf{E}_{t+1}\left[\Delta p_{t+1}|\mathcal{I}_t\right]$ in (5) distinguishes the timing for which the expectation is made (i.e., $t+1$), from the timing of the available information (i.e., t). For estimation, $\mathsf{E}_t\left[\Delta p_{t+1}\right]$ in (8) is usually replaced by Δp_{t+1} as in (1), leading to:

$$\Delta p_t = \gamma_f \Delta p_{t+1} + \boldsymbol{\beta}' \mathbf{x}_t + \epsilon_t \;\text{ where }\; \epsilon_t \sim \mathsf{D}\left[0, \sigma^2_\epsilon\right] \tag{9}$$

which includes Δp_{t+1} as a feedforward variable, where all other variables (including lags) are components of $\mathbf{x}_t$. Generally, Δp_{t+1} in (9) is instrumented by k variables $\mathbf{z}_t = (\mathbf{x}'_t : \mathbf{w}'_t)'$ using whole-sample estimates based on GMM, thereby implicitly postulating relationships of the form:

$$\Delta p_t = \boldsymbol{\kappa}' \mathbf{z}_t + v_t \tag{10}$$

as in Galí and Gertler (1999) and Galí, Gertler and Lopez-Salido (2001): compare Nymoen (2005) and Bjørnstad and Nymoen (2008). Mavroeidis (2004) discusses the potential problems of weak identification in such forward-looking models.

There are few studies of the properties of estimating forward-looking models like (9) when the null hypothesis, $\gamma_f = 0$, is true but the data processes like (10) involve location shifts. As with testing for super exogeneity (see Hendry and Santos, 2010), IIS is applied to the marginal model ('the forecasting equation' (10) for Δp_t) to check for location shifts, then the retained impulses are added to the structural equation and tested for significance. Impulses that matter in the marginal, or 'reduced form', model for Δp_t should nevertheless be insignificant in models like (9) when they are correctly specified. Consequently, the significance of added dummies refutes invariance. If estimates of γ_f also cease to be significant, that entails the potential spurious significance of the feedforward terms (9) as proxies for the unmodelled location shifts. As noted in section 2.1 and explained in Hendry (2010b), instrumental variables selected from the reduced form of an endogenous variable should be insignificant when added to a correctly-specified structural equation.

Intuitively, because Δp_{t+1} reflects breaks before they occur, as seen from time t, even instrumenting Δp_{t+1} could let it act as a proxy for those breaks, leading to γ_f being 'spuriously significant' in (9). As breaks are generally unanticipated, even by economic agents, precisely in a setting where (9) is an invalid representation, one would find $\gamma_f \neq 0$. Thus, we allow the DGP to be of the general form:

$$\Delta p_t = \boldsymbol{\phi}'\mathbf{z}_t + \sum_{i=1}^{r} \rho_i 1_{\{t_i \in \mathcal{T}\}} + \eta_t = \boldsymbol{\phi}'\mathbf{z}_t + \boldsymbol{\rho}'\mathbf{d}_t + \eta_t \tag{11}$$

where $\eta_t \sim \mathsf{IN}\left[0, \sigma_\eta^2\right]$ independently of the regressors, $1_{\{t_i \in \mathcal{T}\}}$ is unity when $t_i \in \mathcal{T}$, and zero otherwise, where $\mathcal{T}$ denotes the set of breaks, and $\mathbf{d}_t$ collects the $r \geq 0$ relevant indicators. We will investigate:
(a) the estimation biases that arise under the null $\gamma_f = 0$ in assumed feedforward models of the NKPC when breaks are not modelled;
(b) tests of the specification of feedforward models when the null $\gamma_f = 0$ holds;
(c) check the properties of the new procedures by simulation; and
(d) apply them to salient empirical models of the NKPC in the Euro-zone and USA.
Such an analysis is important as many Central Banks and policy agencies use models of the form (9), so a rigorous evaluation is urgently needed to discriminate cases where $\gamma_f \neq 0$ from when it is 'spuriously significant' due to unmodelled breaks. Indeed, the proposed tests have relevance to all empirical equations with leads, which now permeate empirical macromodels in monetary policy. Moreover, Euler equations have a similar hybrid form, so the tests are relevant for the demand side as well. Section 4.1 now discusses the existing empirical evidence on NKPC estimation.

4.1 Empirical evidence on NKPC estimation

The 'pure' NKPC is (8) with $\gamma_b = 0$ representing the case where all price-setting firms (that are aggregated over) form rational expectations. Both the pure and the hybrid form are usually presented in theory as 'exact', i.e., without an error term. When $\mathsf{E}_t[\Delta p_{t+1}]$ is replaced by Δp_{t+1} for estimation, a moving-average error term is implied. This has motivated 'robust' estimation with, for example, pre-whitening switched on in GMM, so many papers downplay the relevance of congruency for the evaluation of the NKPC.

The basis for the NKPC as a successful model of inflation is due to the results of Galí and Gertler (1999) (henceforth GG) on US data, and Galí *et al.* (2001) (henceforth GGL1) on Euro-area data. Others have discussed problems related to identification, inference and encompassing, but their critiques are claimed to be rebutted in Galí, Gertler and Lopez-Salido (2005) (GGL5), who re-assert that the NKPC, in particular the dominance of γ_f characterizing forward-looking behaviour, is robust to the choice of estimation procedure and specification bias. They assert that the following three results are proven characteristics of NKPC for all data sets:

1. The two null hypotheses $\gamma_f = 0$ and $\gamma_b = 0$ are rejected both individually and jointly.
2. The coefficient γ_f on expected inflation exceeds the coefficient γ_b on lagged inflation substantially. The hypothesis of $\gamma_f + \gamma_b = 1$ is typically not rejected at conventional levels of significance, although the estimated sum is usually a little less than unity numerically.
3. When real marginal costs are proxied by the log of the wage-share, the coefficient λ in (8) is positive and significantly different from zero at conventional levels of significance.

Critics of the NKPC have challenged the robustness of all three claims, but with different emphases and from different perspectives. The inference procedures and estimation techniques used by GG and GGL1 have been criticized by Rudd and Whelan (2005, 2007), and others, but GGL5 claim that their initial results #1 and #2 remain robust to these objections.

When it comes to #3, GGL5 overlook that several researchers have been unable to confirm their view that the wage-share is a robust explanatory variable in the NKPC. Bårdsen, Jansen and Nymoen (2004) showed that the significance of the wage share in the GGL1 model is fragile, as it depends on the exact implementation of the GMM estimation method used, thus refuting that result 3 is a robust feature of NKPC at least when estimated on Euro-area data.

Fanelli (2008) uses a vector autoregressive model on a Euro-area data set, but finds that the NKPC is a poor explanatory model. On US data, Mavroeidis (2005) has shown that real marginal costs appears to be an irrelevant determinant of inflation, confirming the view in Fuhrer (2006) about the difficulty of developing a sizeable coefficient on the forcing variable in the US NKPC. The studies cited represent evidence that clashes with the claim that #3 is robust. Instead, it is to be expected that depending on the operational definition of real marginal costs, the estimation method and the sample, the numerical and statistical significance of λ will vary across different studies.

Result 3 is just as important as #1 and #2 for the status of the NKPC as an adequate model, so if that part of the model is non-structural, it might be that #1 and #2 have another explanation than that intended of a good match between the NKPC and the inflation data generating process. Bårdsen *et al.* (2004) (Euro area), and Bjørnstad and Nymoen (2008) (OECD panel data) demonstrate that the significance of γ_f can be explained by a linear combination of better forcing variables that are a subset of the over-identifying instruments. Their presence is revealed by the significance of the Sargan (1964) specification test. Importantly, the re-specified models in these two studies lend themselves directly to interpretation either as conventional Phillips curves, or as an equilibrium-correction price equation consistent with the theory of monopolistic competition in product market and a certain element of coordination in wage bargaining, see Sargan (1980), Nymoen (1991), Bårdsen, Eitrheim, Jansen and Nymoen (2005, Ch4-6). Hence, the NKPC fails to parsimoniously encompass these models. However, like Russell, Banerjee, Malki and Ponomareva (2010) (US panel data), we are concerned with the impact of unmodelled location shifts on the NKPC, to which issue we now turn.

5 Estimating feedforward models ignoring breaks

Once breaks occur, under the null that $\gamma_f = 0$, the DGP is (11) at time t, so at $t+1$:

$$\Delta p_{t+1} = \boldsymbol{\rho}'\mathbf{d}_{t+1} + \boldsymbol{\phi}'\mathbf{z}_{t+1} + \eta_{t+1} \tag{12}$$

Subtracting (12) from (11) to eliminate the breaks introduces the future value:

$$\Delta p_t = \Delta p_{t+1} - \boldsymbol{\rho}'\Delta\mathbf{d}_{t+1} - \boldsymbol{\phi}'\Delta\mathbf{z}_{t+1} - \Delta\eta_{t+1}. \tag{13}$$

5.1 Static DGP

In the simplest case where $\boldsymbol{\phi} = \mathbf{0}$, (13) becomes:

$$\Delta p_t = \Delta p_{t+1} - \boldsymbol{\rho}'\Delta\mathbf{d}_{t+1} - \Delta\eta_{t+1} \tag{14}$$

The differenced dummies in (14) are just 'blips' rather than impulses, or impulses rather than step shifts, so become an almost indistinguishable part of a composite error term when unmodelled, even if they are easily detected in (11). This suggests that a coefficient near unity may be obtained for γ_f when estimating (14) using instrumental variables (IVs) that are correlated with Δp_{t+1} and orthogonal to $\Delta\eta_{t+1}$. Indeed, since the break in (11) is also partly proxied by the lagged dependent variable, providing lagged values of Δp_t are used as instruments, even after instrumenting, Δp_{t+1} will 'pick up' a spurious effect and lead to a large coefficient in (14). For example, if Δp_{t-1} is the only IV used in estimation when the postulated model is, perhaps after application of the Frisch and Waugh (1933) theorem to remove any $\mathbf{z}_t$ regressors:

$$\Delta p_t = \theta\Delta p_{t+1} + e_t \tag{15}$$

then from (11):

$$\mathsf{E}\left[\widehat{\theta}\right] = \mathsf{E}\left[\frac{\sum_{t=2}^{T-1}\Delta p_{t-1}\Delta p_t}{\sum_{t=2}^{T-1}\Delta p_{t-1}\Delta p_{t+1}}\right] = \mathsf{E}\left[\frac{\sum\left(\boldsymbol{\rho}'\mathbf{d}_{t-1}+\eta_{t-1}\right)\left(\boldsymbol{\rho}'\mathbf{d}_t+\eta_t\right)}{\sum\left(\boldsymbol{\rho}'\mathbf{d}_{t-1}+\eta_{t-1}\right)\left(\boldsymbol{\rho}'\mathbf{d}_{t+1}+\eta_{t+1}\right)}\right] \simeq \frac{\boldsymbol{\rho}'\left(\sum\mathbf{d}_{t-1}\mathbf{d}_t'\right)\boldsymbol{\rho}}{\boldsymbol{\rho}'\left(\sum\mathbf{d}_{t-1}\mathbf{d}_{t+1}'\right)\boldsymbol{\rho}}.$$

If there are just a few outliers not in successive, or overlapping, periods, then $\widehat{\theta}$ would be near zero and ill-determined when $\gamma_f = 0$ (although that is mainly due to the simplicity of the specification in (12), since there is no persistence when $\boldsymbol{\phi} = \mathbf{0}$). However, even if there is just a single location shift of size δ from T_1 to $T_2 > T_1 + 2$ so:

$$\mathbf{d}_t' = \left(1_{\{T_1\}}\ 1_{\{T_1+1\}}\cdots 1_{\{T_2\}}\right)$$

as:

$$\sum_{j=T_1}^{T_2} 1_{\{j\}} = 1_{\{T_1\leq t\leq T_2\}}$$

then $\boldsymbol{\rho}'\mathbf{d}_t = \delta 1_{\{T_1\leq t\leq T_2\}}$ and hence:

$$\boldsymbol{\rho}'\left(\sum_{t=2}^{T-1}\mathbf{d}_{t-1}\mathbf{d}_t'\right)\boldsymbol{\rho} = \delta^2\sum 1_{\{T_1\leq t-1\leq T_2\}}1_{\{T_1\leq t\leq T_2\}} = \delta^2\left(T_2 - T_1\right)$$
$$\boldsymbol{\rho}'\left(\sum_{t=2}^{T-1}\mathbf{d}_{t-1}\mathbf{d}_{t+1}'\right)\boldsymbol{\rho} = \delta^2\sum 1_{\{T_1\leq t-1\leq T_2\}}1_{\{T_1\leq t+1\leq T_2\}} = \delta^2\left(T_2 - T_1 - 1\right)$$

leading to the estimate in (15):

$$\mathsf{E}\left[\widehat{\theta}\right] \simeq \frac{\left(T_2-T_1\right)\delta^2}{\left(T_2-T_1-1\right)\delta^2} = \frac{\left(T_2-T_1\right)}{\left(T_2-T_1-1\right)} \simeq 1. \tag{16}$$

Consequently, despite the complete irrelevance of Δp_{t+1} in the DGP, and the use of lagged values Δp_{t-i} as instruments, its estimated coefficient will be near unity when there are unmodelled location shifts.

Even if $T_2 - T_1 - 1$ is as small as 3, a notable coefficient will be obtained. The estimated standard error of $\widehat{\theta}$ will be approximately:

$$\mathsf{SE}\left[\widehat{\theta}\right] \simeq \frac{\sqrt{2\sigma_\eta^2}}{\sqrt{(T_2 - T_1 - 1)\,\delta^2}}$$

as:

$$e_t = \delta\left(1_{\{T_2\}} - 1_{\{T_1\}}\right) - \Delta\eta_{t+1}$$

so:

$$\mathsf{E}\left[\widehat{\sigma}_e^2\right] \simeq 2\left(\sigma_\eta^2 + T^{-1}\delta^2\right)$$

If there is a single location shift of $\delta = r\sigma_\eta$:

$$\mathsf{SE}\left[\widehat{\theta}\right] \simeq \frac{\sqrt{2\,(1+r^2)}\sigma_\eta}{r\sigma_\eta\sqrt{(T_2 - T_1 - 1)}} = \frac{\sqrt{2\,(1+r^{-2})}}{\sqrt{(T_2 - T_1 - 1)}} \tag{17}$$

which will be less than $1/2$ for even small and relatively short breaks (e.g., $r = 3$ and $T_2 - T_1 = 7$) leading to a 'significant' $\widehat{\theta}$.

5.2 Dynamic DGP, dynamic model

We now allow for a dynamic DGP:

$$\Delta p_t = \kappa \Delta p_{t-1} + \boldsymbol{\rho}'\mathbf{d}_t + \eta_t \tag{18}$$

The model is:

$$\Delta p_t = \theta_1 \Delta p_{t+1} + \theta_2 \Delta p_{t-1} + \boldsymbol{\theta}_3'\mathbf{z}_t + e_t \tag{19}$$

We use the Frisch–Waugh theorem to remove $\mathbf{z}_t$, equivalent to $\boldsymbol{\theta}_3 = \mathbf{0}$, so do not change notation. Section 10 details the required calculations, approximating by neglecting powers of κ greater than squared. Then estimation of (19) using Δp_{t-2} as the identifying instrument yields:

$$\mathsf{E}\left[\begin{pmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_2 \end{pmatrix}\right] \simeq \frac{1}{(1-\kappa-\kappa^2)} \begin{pmatrix} (1-\kappa)^2 \\ \kappa\left(1 - 2\kappa + \kappa^2\right) \end{pmatrix}$$

where $\delta \neq 0$. For example when $\kappa = 0.35$:

$$\mathsf{E}\left[\begin{pmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_2 \end{pmatrix}\right] \simeq \frac{1}{(1-0.35-0.35^2)} \begin{pmatrix} (1-0.35)^2 \\ 0.35 \times \left(1 - 2 \times 0.35 + 0.35^2\right) \end{pmatrix} = \begin{pmatrix} 0.81 \\ 0.28 \end{pmatrix}$$

so there would be a root just outside the unit circle. Because of the approximation that $\kappa^3 \simeq 0$, values of κ have to be less than 0.5. If $\kappa = 0$ as well:

$$\mathsf{E}\left[\begin{pmatrix} \widetilde{\theta}_1 \\ \widetilde{\theta}_2 \end{pmatrix}\right] \simeq \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Consequently, expectations are estimated to be important, even when they are in fact irrelevant, and persistence is thought to be high.

	mean	MCSD	mean	MCSD
y_{+1}	0.359	0.321	0.526	0.173
y_{-1}	0.580	0.214	0.481	0.132
z	0.539	0.421	0.327	0.215
ESE$[y_{+1}]$	0.375	0.735	0.198	0.117
ESE$[y_{-1}]$	0.254	0.443	0.151	0.090
ESE$[z]$	0.484	0.950	0.261	0.147

Table 1: Moments of estimates with and without IIS

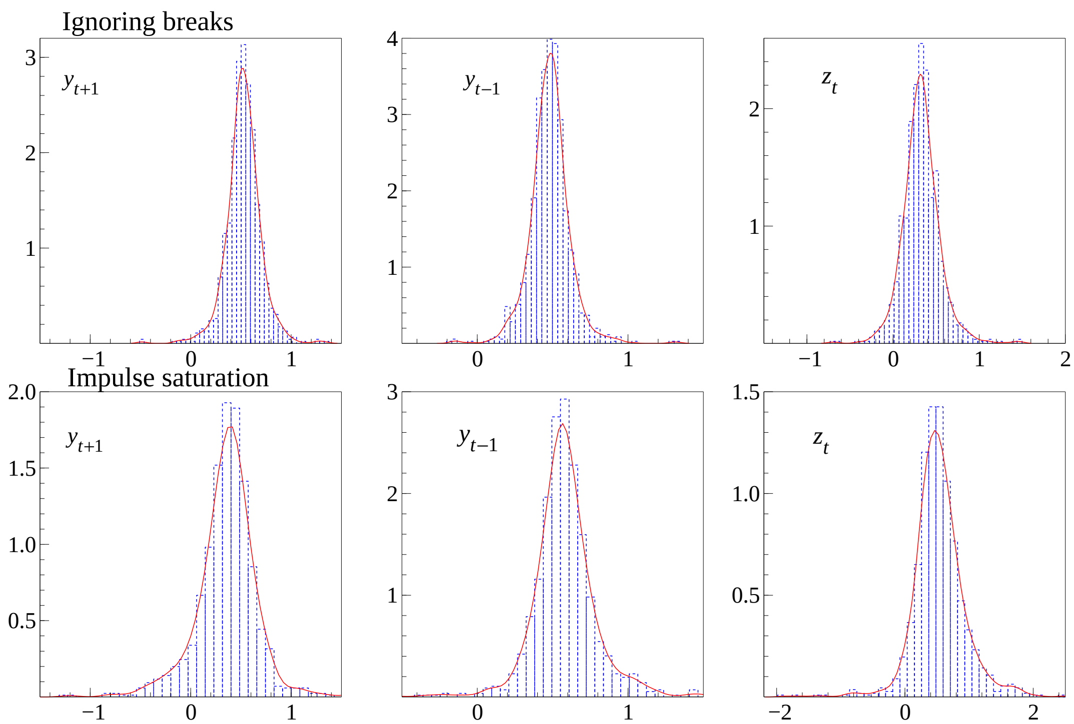

Figure 1: Estimation with and without IIS

5.3 Monte Carlo illustration

Simulating this case yields the results in Table 1. Without IIS, the coefficient of $\widehat{y}_{t+1}$ is highly significant and exceeds $1/2$ even though the population value is zero; with IIS, it is insignificant and ill-determined. Also, without IIS, the sum of the two y coefficients is unity, whereas it drops to 0.94, albeit still larger than the DGP persistence of 0.8. The graphical output in figure 1 compares the distributions of NKPC estimation with and without IIS, showing the shift to the origin in the coefficient of $\widehat{y}_{t+1}$.

6 Simulation of feedforward and feedback mechanisms

Monte Carlo simulation helps assess the properies of feed-forward versus feedback mechanisms when there are location shifts by comparing selection and estimation with and without IIS under both null and alternative. The experimental design covers two cases of the NKPC: the backward-looking DGP with no future expectations term (the null), and the DGP model with forward- and backward-looking mechanisms (the alternative). The DGP is given by:

$$y_t = \gamma_f y^e_{t+1} + \gamma_b y_{t-1} + \beta z_t + \psi d_{81-100,t} + \epsilon_t, \quad \epsilon_t \sim \mathsf{IN}\left[0, \sigma^2_\epsilon\right] \tag{20}$$

$$z_t = \lambda_0 + \lambda_1 z_{t-1} + \lambda_2 z_{t-2} + \eta_t, \quad \eta_t \sim \mathsf{IN}\left[0, \sigma^2_\eta\right], \tag{21}$$

for $t = 1, \ldots, T$, where $y^e_{t+1} = \mathsf{E}\left[y_{t+1}|\mathcal{I}_t\right]$ is the rational expectation when $\mathcal{I}_t = (z_t, \mathcal{I}_{t-1})$ is the information set available at t. Furthermore, $\mathsf{E}\left[\epsilon_t|z_t, \mathcal{I}_{t-1}\right] = 0$ when z_t is exogenous and observed. An AR(2) process (at least) for the exogenous variable is required for identification, see Pesaran (1981).

There are two possible forms of location shift, either through the intercept of the exogenous variable or directly into the equation for y_t. We examine the latter as the break is then internal and unmodelled. There is a location shift of 5 standard deviations at $T_1 = 81$, $(I_{81,t} + \cdots + I_{100,t})$, denoted $d_{81-100,t}$, designed to proxy the location shift observed in UK wage growth data since 1946 (see Castle and Hendry, 2009). Parameter values are $\psi = 5$, $\beta = 1$, $\lambda_0 = 0$, $\lambda_1 = 1.5$, $\lambda_2 = -0.7$, $\sigma^2_\epsilon = 1$, $\sigma^2_\eta = 1$, $T = 100$ and $M = 1000$ replications are undertaken.

Six models are considered:

(i) feed-forward: $\gamma_f \neq 0$, $\gamma_b = 0$;

(ii) hybrid: $\gamma_f \neq 0$, $\gamma_b \neq 0$; and

(iii) feedback: $\gamma_f = 0$, $\gamma_b \neq 0$;

and these three models augmented by IIS with selection of the indicators at $\alpha = 0.01$. Selection is undertaken in two stages for the forward and hybrid models. First, the reduced form is estimated with IIS to obtain a set of indicators:

$$y_t = \rho y_{t-1} + \gamma_0 z_t + \gamma_1 z_{t-1} + \sum_{i=1}^{T} \delta_i 1_i + u_t \tag{22}$$

where y_{t-1}, z_t and z_{t-1} are forced to enter the regression (so only the indicators are selected over).[1] The retained indicators, $\mathbf{d}_t$, are then included in the forward/hybrid model (8), which is estimated using 2SLS with the set of instruments including the constant, z_{t-1} and z_{t-2}. For the backward-looking model, IIS is applied directly at the first stage as (8) can be estimated using OLS when $\gamma_f = 0$.

6.1 Backward-looking DGP

We first consider the case where the DGP is generated as a feedback mechanism ($\gamma_f = 0$ and $\gamma_b = 0.8$). Table 2 reports the unconditional parameter estimates, retention probabilities and diagnostic test rejection frequencies for the feedback DGP with a break. Retention probabilities refer to the number of replications in which $|\mathsf{t}| > c_\alpha$ for IV estimation, as selection is not undertaken.

The shaded columns refer to the case where the model coincides with the DGP. When the break is unmodelled, the estimate of γ_b is higher at 0.93, suggesting more persistence: to closer fit the break requires a near unit root. Diagnostic test rejection frequencies all highlight mis-specification due to the unmodelled break. When IIS is applied, 20 impulse-indicators are retained on average successfully capturing the step shift. This results in coefficient estimates close to their DGP parameter values, with slight over-rejection of the diagnostic tests.

When the data are generated under the null of no forward-lookingness, but a hybrid model is nevertheless estimated, then an unmodelled break results in a highly significant $\widehat{\gamma}_f$ of 0.44 despite its irrelevance in the DGP, matching the above theory. The exogenous variable has a small impact, and is close to insignificant, corresponding to previous empirical findings (see §4.1). Applying IIS to the mis-specified

[1]Diagnostic tests are switched off when undertaking selection here as the reduced form errors are necessarily autocorrelated, to avoid retaining additional indicators which could be kept to ensure congruency.

model reduces the magnitude and significance of $\widehat{\gamma}_f$, and $\widehat{\gamma}_b$ is much closer to its DGP value. Hence, unmodelled breaks lead to more forward-lookingness than the DGP contains, but modelling the breaks by IIS mitigates that erroneous conclusion. When the model is purely forward-looking, so does not nest the DGP, under-specification is evident in the diagnostic test rejection frequencies, with the Sargan test rejecting most of the time. An empirical researcher is unlikely to set $\gamma_b = 0$ in the face of such mis-specification.

Model:	Backward		Hybrid		Forward	
IIS:	No	Yes	No	Yes	No	Yes
$\widehat{\gamma}_f$			0.438	0.109	1.230	1.459
			(0.065)	(0.157)	(0.091)	(0.090)
			7.20	**1.59**	**15.61**	**17.55**
$\widehat{\beta}$	0.835	0.993	0.198	0.807	−1.017	−1.612
	(0.068)	(0.043)	(0.110)	(0.284)	(0.269)	(0.253)
	12.58	**24.18**	**1.93**	**3.27**	**−4.11**	**−6.79**
$\widehat{\gamma}_b$	0.932	0.808	0.586	0.744		
	(0.016)	(0.014)	(0.053)	(0.090)		
	59.42	**60.46**	**12.01**	**10.13**		
$\widehat{\sigma}_\epsilon$	1.857	0.989	1.463	1.070	4.297	2.971
No. impulses	-	19.68	-	17.72	-	17.72
Retention probability						
$\widehat{\gamma}_f$			1.00	0.29	1.00	1.00
			1.00	0.25	1.00	0.99
$\widehat{\beta}$	1.00	1.00	0.46	0.83	0.96	1.00
	1.00	1.00	0.23	0.69	0.87	0.97
$\widehat{\gamma}_b$	1.00	1.00	1.00	0.99		
	1.00	1.00	1.00	0.99		
Diagnostic test rejection frequencies						
$\chi^2_{\mathsf{nd}}(2)$	0.342	0.062	0.963	0.269	0.963	0.736
	0.189	0.013	0.953	0.220	0.953	0.620
Portmanteau	1.000	0.143	0.031	0.167	0.031	0.453
	0.991	0.058	0.014	0.080	0.014	0.312
$\mathsf{F}_{\mathsf{ar}}(1)$	1.000	0.090	0.287	0.189	0.287	0.799
	1.000	0.034	0.138	0.084	0.138	0.635
$\mathsf{F}_{\mathsf{ar}}(5)$	1.000	0.072	0.240	0.133	0.240	0.605
	0.998	0.026	0.134	0.063	0.134	0.358
$\mathsf{F}_{\mathsf{het}}$	0.240	0.080				
	0.110	0.020				
$\mathsf{F}_{\mathsf{arch}}(1)$	0.893	0.085	0.113	0.108	0.113	0.233
	0.816	0.019	0.075	0.040	0.075	0.156
χ^2_S			0.237	0.113	1.000	0.899
			0.093	0.030	1.000	0.768

Table 2: The backward looking DGP with break of $5\sigma_\epsilon$ at $T_1 = 81$. Monte Carlo standard errors in parentheses, Monte Carlo t-statistics in bold. Retention probabilities report frequencies at 5% (top) and 1% (bottom). Test statistics report rejection frequencies at 5% (top) and 1% (bottom).

In Table 2, $\chi^2_{\mathsf{nd}}(2)$ is the normality test in Doornik and Hansen (2008). The Portmanteau test is the Box–Pierce test with a degrees-of-freedom correction given by Ljung and Box (1978). Let $\mathsf{F}_{\mathsf{name}}$ denote an approximate F-test. Then F_{ar} tests are Lagrange-multiplier tests for autocorrelation of order k: see Godfrey (1978), and Pagan (1984) for an exposition. The heteroscedasticity test, $\mathsf{F}_{\mathsf{het}}$, computed only for OLS estimation, uses squares of the original regressors: see White (1980). Engle (1982) provides the $\mathsf{F}_{\mathsf{arch}}$ test for k^{th}-order autoregressive conditional heteroskedasticity (ARCH); and the χ^2_S test for the validity of instruments is given by Sargan (1958).

6.2 Hybrid DGP

Data for the hybrid DGP simulations are obtained by first generating the exogenous process given by (21), then generating the endogenous process from the reduced form. The location shift is added to the reduced form DGP, translated back to the structural model as a step shift. To obtain the reduced form parameterization, first set $\psi = 0$ in (20) and solve for the constant parameter reduced form:

$$y_t = \rho_0 + \rho_1 y_{t-1} + \varphi_0 z_t + \varphi_1 z_{t-1} + u_t \tag{23}$$

so:

$$y_{t+1} = \rho_0 + \rho_1 y_t + \varphi_0 z_{t+1} + \varphi_1 z_t + u_{t+1} \tag{24}$$

and hence:

$$\begin{aligned} y_{t+1} &= \rho_0 + \rho_1 y_t + \varphi_0 z_{t+1} + \varphi_1 z_t + u_{t+1} \\ &= \rho_0 + \rho_1 \left(\rho_0 + \rho_1 y_{t-1} + \varphi_0 z_t + \varphi_1 z_{t-1} + u_t\right) + \varphi_0 \left(\lambda_0 + \lambda_1 z_t + \lambda_2 z_{t-1} + \eta_{t+1}\right) \\ &\quad + \varphi_1 z_t + u_{t+1} \\ &= \rho_1^2 y_{t-1} + \left(\rho_0 \left(1+\rho_1\right) + \varphi_0 \lambda_0\right) + \left(\varphi_0 \left(\rho_1 + \lambda_1\right) + \varphi_1\right) z_t + \left(\rho_1 \varphi_1 + \varphi_0 \lambda_2\right) z_{t-1} \\ &\quad + \varphi_0 \eta_{t+1} + u_{t+1} + \rho_1 u_t \end{aligned} \tag{25}$$

Taking expectations:

$$\begin{aligned} \mathsf{E}_t \left[y_{t+1} | z_t, \mathcal{I}_{t-1}\right] &= \mathsf{E}_t \left[\rho_1^2 y_{t-1} + \left(\rho_0 \left(1+\rho_1\right) + \varphi_0 \lambda_0\right) + \left(\varphi_0 \left(\rho_1 + \lambda_1\right) + \varphi_1\right) z_t \right. \\ &\qquad \left. + \left(\rho_1 \varphi_1 + \varphi_0 \lambda_2\right) z_{t-1} | z_t, \mathcal{I}_{t-1}\right] \\ &= \rho_1^2 y_{t-1} + \left(\rho_0 \left(1+\rho_1\right) + \varphi_0 \lambda_0\right) + \left(\varphi_0 \left(\rho_1 + \lambda_1\right) + \varphi_1\right) z_t + \left(\rho_1 \varphi_1 + \varphi_0 \lambda_2\right) z_{t-1} \end{aligned} \tag{26}$$

Using $y_{t+1}^e = \mathsf{E}_t \left[y_{t+1} | z_t, \mathcal{I}_{t-1}\right]$ and substituting (26) in (20):

$$\begin{aligned} y_t &= \gamma_f y_{t+1}^e + \gamma_b y_{t-1} + \beta z_t + \epsilon_t \\ &= \gamma_f \left(\rho_1^2 y_{t-1} + \left(\rho_0 \left(1+\rho_1\right) + \varphi_0 \lambda_0\right) + \left(\varphi_0 \left(\rho_1 + \lambda_1\right) + \varphi_1\right) z_t + \left(\rho_1 \varphi_1 + \varphi_0 \lambda_2\right) z_{t-1}\right) \\ &\quad + \gamma_b y_{t-1} + \beta z_t + \epsilon_t \\ &= \left(\gamma_f \rho_1^2 + \gamma_b\right) y_{t-1} + \gamma_f \left(\rho_0 \left(1+\rho_1\right) + \varphi_0 \lambda_0\right) + \left(\gamma_f \varphi_0 \left(\rho_1 + \lambda_1\right) + \gamma_f \varphi_1 + \beta\right) z_t \\ &\quad + \gamma_f \left(\rho_1 \varphi_1 + \varphi_0 \lambda_2\right) z_{t-1} + \epsilon_t \end{aligned} \tag{27}$$

Comparing coefficients in (23) and (27) leads to the following set of restrictions:

$$\begin{aligned} \rho_0 &= \varphi_0 \frac{\lambda_0}{\left(\rho_2 - 1\right)} \\ \rho_1 &= \frac{\left(1 - \sqrt{1 - 4\gamma_f \gamma_b}\right)}{2\gamma_f} \\ \rho_2 &= \frac{\left(1 + \sqrt{1 - 4\gamma_f \gamma_b}\right)}{2\gamma_f} \\ \varphi_0 &= \frac{\beta}{\gamma_f} \left(\rho_2 - \lambda_1 - \lambda_2 \rho_2^{-1}\right)^{-1} \\ \varphi_1 &= \varphi_0 \frac{\lambda_2}{\rho_2} \end{aligned}$$

using $1 - \gamma_f \rho_1 = \gamma_f \rho_2$.

The difference between y_{t+1} and $\mathsf{E}_t\left[y_{t+1}|z_t, \mathcal{I}_{t-1}\right]$ is:

$$\varphi_0 \eta_{t+1} + u_{t+1} + \rho_1 u_t \tag{28}$$

which has a variance:

$$\sigma_e^2 = \varphi_0^2 \sigma_\eta^2 + \left(1 + \rho_1^2\right) \sigma_\epsilon^2 \tag{29}$$

as against σ_ϵ^2 when $\mathsf{E}_t\left[y_{t+1}|z_t, \mathcal{I}_{t-1}\right]$ is known. The coefficient in (20) is γ_f so:

$$y_t = \gamma_f y_{t+1} + \gamma_b y_{t-1} + \beta z_t + \epsilon_t - \gamma_f \left(\varphi_0 \eta_{t+1} + \epsilon_{t+1} + \rho_1 \epsilon_t\right) \tag{30}$$

so the error variance is:

$$\sigma_\nu^2 = \sigma_\epsilon^2 + \gamma_f^2 \left(\varphi_0^2 \sigma_\eta^2 + \left(1 + \rho_1^2\right) \sigma_\epsilon^2\right) - 2\gamma_f \rho_1 \sigma_\epsilon^2 \tag{31}$$

The location shift, $\psi d_{81-100,t}$ is added to (23). We set $\gamma_f = 0.45$ and $\gamma_b = 0.45$, with all other parameters as above.[2] Many theoretical papers set $\gamma_f + \gamma_b = 1$, thereby imposing a unit root in order to preclude the existence of a long-run level tradeoff between inflation and real activity, see e.g. Rudd and Whelan (2006). Although we consider a stationary process, the process looks like a unit root when the location shift is unmodelled.

Model:	Backward		Hybrid		Forward	
IIS:	No	Yes	No	Yes	No	Yes
$\widehat{\gamma}_f$			0.554 (0.051) **11.24**	0.442 (0.120) **5.00**	0.078 (0.205) **0.74**	-0.725 (0.279) **-2.52**
$\widehat{\beta}$	2.961 (0.172) **17.45**	3.599 (0.078) **47.57**	0.446 (0.270) **1.83**	1.077 (0.692) **2.37**	4.731 (1.054) **4.54**	8.872 (1.437) **6.70**
$\widehat{\gamma}_b$	0.529 (0.030) **17.60**	0.378 (0.014) **26.82**	0.470 (0.026) **19.17**	0.447 (0.032) **16.45**		
$\widehat{\sigma}_\epsilon$	3.326	1.222	2.679	2.092	6.737	5.821
No. impulses	-	21.31	-	17.81	-	17.81
Retention probability						
$\widehat{\gamma}_f$			1.00 1.00	0.95 0.87	0.32 0.20	0.76 0.63
$\widehat{\beta}$	1.00 1.00	1.00 1.00	0.42 0.24	0.55 0.40	0.95 0.91	0.98 0.97
$\widehat{\gamma}_b$	1.00 1.00	1.00 1.00	1.00 1.00	1.00 1.00		
Diagnostic test rejection frequencies						
$\chi^2_{\mathsf{nd}}(2)$	0.799 0.653	0.048 0.009	0.437 0.366	0.312 0.187	0.666 0.545	0.453 0.327
Portmanteau	1.000 1.000	0.412 0.225	0.020 0.004	0.119 0.043	0.999 0.998	0.895 0.825
$\mathsf{F}_{\mathsf{ar}}(1)$	1.000 1.000	0.710 0.426	0.077 0.020	0.304 0.174	1.000 1.000	0.884 0.762
$\mathsf{F}_{\mathsf{ar}}(5)$	1.000 1.000	0.329 0.103	0.117 0.051	0.378 0.261	1.000 1.000	0.588 0.466
$\mathsf{F}_{\mathsf{het}}$	0.374 0.200	0.039 0.005				
$\mathsf{F}_{\mathsf{arch}}(1)$	0.999 0.998	0.186 0.074	0.060 0.033	0.104 0.040	0.986 0.974	0.672 0.565
χ^2_S			0.082 0.017	0.120 0.055	0.994 0.981	0.972 0.937

Table 3: The hybrid DGP with break. Legend as Table 2.

[2]The reduced form parameters are $\rho_0 = 0$, $\rho_1 = 0.627$, $\rho_2 = 1.595$, $\varphi_0 = 4.16$, and $\varphi_1 = -1.825$, with $\sigma_\nu = 2.055$.

Table 3 records the results, with the shaded columns highlighting the case where the DGP and model coincide once the break is modelled. When the break is unmodelled, the hybrid model places more weight on the forward-looking component and a unit root is estimated in most replications. $\widehat{\beta}$ is much lower than β, and all the dynamics are captured through the endogenous variables. Applying IIS detects 18 impulse-indicators on average, and returns the parameter estimates close to their DGP parameters, including the equation standard error. The mis-specified feedforward and feedback models deliver poor parameter estimates as neither model nests the DGP, so under-specification results in omitted variable bias. IIS in the feedback model does mitigate the effect of the location shift: Castle and Hendry (2010) discuss under-specified models facing breaks.

Thus, IIS helps to distinguish between unit roots and structural breaks in the feedback model, and analogously, provides more accurate estimates of the forward-looking component in the hybrid model. Not accounting for breaks results in more forward-looking behaviour and persistence than there really is.

7 Euro-area NKPC estimation with impulse-indicator saturation

As proposed above, a separate source of the sizeable coefficient estimates of the forward term in NKPC models reviewed in section 4 may be a 'forward knowledge bias' in estimates of γ_f, since (8) does not take into account non-stationarities in the form of location shifts. Instrumenting by Δp_{t-j} will 'pick up' the effects of location shifts (which are in fact unpredictable) and contribute to a positive bias in the estimated γ_f. As we propose, a test of this hypothesis can be based on invariance tests similar to those proposed for testing exogeneity by Hendry and Santos (2010). Dummies from the marginal model of Δp_t (from which $\mathsf{E}_t[\Delta p_{t+1}]$ is obtained) should be insignificant in (8), and significance of the added dummies refutes invariance whereas insignificance of the estimates of γ_f is inconsistent with the claimed forward-looking formulation.

As a reference, we estimate the 'pure' NKPC similar to equation (13) in GGL1, but with IV estimation instead of GMM.[3] Both Δp_{t+1} and s_t are treated as endogenous. We use the same set of instruments: five lags of inflation, two lags of s_t, detrended output and wage inflation, for $T = 102$ (1972(2) to 1998(1)):

$$\begin{aligned} \widehat{\Delta p}_t &= \underset{(0.083)}{0.925}\ \widehat{\Delta p}_{t+1} + \underset{(0.016)}{0.0142}\ s_t + \underset{(0.011)}{0.010} \\ &\quad \widehat{\sigma} = 0.32\% \quad \chi^2_S(9) = 14.57 \end{aligned} \tag{32}$$

Significance at 5% and 1% are respectively denoted by * and **. The estimated γ_f is less than unity, so formally a stable forward solution applies (even for strongly exogenous s_t). But $\gamma_f = 1$ cannot be rejected, so dynamic stability hinges on equilibrium correction in s_t.

The Euro-area hybrid NKPC over the same sample is:

$$\begin{aligned} \widehat{\Delta p}_t &= \underset{(0.135)}{0.655}\ \widehat{\Delta p}_{t+1} + \underset{(0.117)}{0.280}\ \Delta p_{t-1} + \underset{(0.014)}{0.012}\ s_t + \underset{(0.010)}{0.009} \\ &\quad \widehat{\sigma} = 0.28\% \quad \chi^2_S(6) = 11.88 \end{aligned} \tag{33}$$

The dominance of Δp_{t+1} over Δp_{t-1} is confirmed (#2 above), and the elasticities sum to 0.94. The 0.66 estimate of γ_f is comparable to, and only a little lower than, the GMM estimates in Table 2 in GGL1 who report four estimates: $0.77, 0.69, 0.87$, and 0.60.

[3]Bårdsen *et al.* (2004) uses GMM. The results are similar to the IV results here. Changes in the GMM estimation method affect the point estimates just as much as the change to IV does. For example, there is a sign change in the estimated coefficient of the wage-share coefficient as a result of a change in pre-whitening method.

We next investigate the marginal model ('the forecasting equation'). We model Δp_t by the variables that are in the instrument set for the NKPC estimation, and then investigate structural breaks using impulse-indicator saturation in *Autometrics.* With the significance level set at 0.025, *Autometrics* finds 11 dummies. When the hybrid NKPC is augmented by these dummies, the model is not congruent, with $\chi^2(6) = 17.83^{**}$. Following earlier analysis (in Bårdsen *et al.*, 2004), an interpretation is that some of the variables in the instrument set have separate explanatory power for Δp_t, consistent with (earlier) standard models of inflation.

Adding $gap_{t-1}$to the equation as an explanatory variable makes the dummy augmented NKPC congruent $\chi^2_S(5) = 3.45$, with no significant tests of residual mis-specification. Using *Autometrics* with significance level 0.05 gives (coefficients of dummies are multiplied by 100):

$$\begin{aligned}
\widehat{\Delta p}_t \;=\; & -\underset{(0.264)}{0.298}\ \widehat{\Delta p}_{t+1} + \underset{(0.029)}{0.115}\ s_t + \underset{(0.126)}{0.505}\ \Delta p_{t-1} + \underset{(0.021)}{0.086} + \underset{(0.0004}{0.0015}\ gap_{t-1} \\
& + \underset{(0.30)}{1.09}\ I_{73(1),t} + \underset{(0.38)}{1.09}\ I_{73(3),t} + \underset{(0.31)}{0.73}\ I_{73(4),t} + \underset{(0.34)}{0.85}\ I_{74(2),t} \\
& + \underset{(0.33)}{0.80}\ I_{74(3),t} + \underset{(0.38)}{0.98}\ I_{76(2),t} + \underset{(0.29)}{0.57}\ I_{76(3),t} - \underset{(0.28)}{0.66}\ I_{78(4),t} + \underset{(0.28)}{0.69}\ I_{83(1),t} \\
& \widehat{\sigma} = 0.27\% \;\; \chi^2_S(5) = 5.06 \;\; \mathsf{F}_{\mathsf{ar}}(5,85) = 1.55 \\
& \mathsf{F}_{\mathsf{arch}}(4,96) = 1.49 \;\; \mathsf{F}_{\mathsf{het}}(14,80) = 1.41 \;\; \chi^2_{\mathsf{nd}}(2) = 1.04
\end{aligned} \tag{34}$$

The diagnostic tests are described following Table 2. Nine of the 11 'reduced form' dummies are retained, so represent evidence for lack of invariance in the feedforward NKPC. Further, the coefficient of the forward term is substantially affected: it is no longer significantly different from zero, and indeed is negative. The coefficient of the wage-share is also affected and is now sizeable, allowing the wage-share to serve as a statistically and substantively important equilibrating mechanism.

Re-estimation of the augmented model on the shorter, post-break, sample which starts in 1983(2) yields:

$$\begin{aligned}
\widehat{\Delta p}_t \;=\; & \underset{(0.390)}{0.082}\ \widehat{\Delta p}_{t+1} + \underset{(0.038)}{0.066}\ s_t + \underset{(0.1372)}{0.350}\ \Delta p_{t-1} + \underset{(0.029)}{0.050} + \underset{(0.0004}{0.00063}\ gap_{t-1} \\
& \widehat{\sigma} = 0.20\% \;\; \chi^2_S(6) = 1.79 \;\; \mathsf{F}_{\mathsf{ar}}(4,51) = 0.15 \;\; \mathsf{F}_{\mathsf{arch}}(4,52) = 1.10 \\
& \mathsf{F}_{\mathsf{het}}(14,45) = 1.08 \;\; \chi^2_{\mathsf{nd}}(2) = 0.07
\end{aligned} \tag{35}$$

The estimated coefficient of the forward term is now practically zero. Of the other coefficients, only the lagged inflation term is significant at the 5% level, and all coefficients are scaled down compared to (34). Both these results confirm that the significance of the feedforward term in (32) and (33) depended on its being a proxy for unmodelled location shifts.

8 US NKPC estimation with impulse-indicator saturation

The pure NKPC on the same sample period used by GG, and their instruments, but with IV instead of GMM gives for $T = 152$ (1960(1) to 1997(4)):

$$\begin{aligned}
\widehat{\Delta p}_t \;=\; & \underset{(0.048)}{0.992}\ \Delta p_{t+1} + \underset{(0.018)}{0.011}\ s_t + \underset{(0.00052)}{5.12e^{-005}} \\
& \widehat{\sigma} = 0.20\% \;\; \chi^2_S(8) = 17.7^{*}
\end{aligned} \tag{36}$$

which can be compared to the GMM estimate at the top of page 207 in GG which are $0.95(0.045)$ and $0.023(0.012)$. GG's equation is without intercept, and clearly that is also zero in (36). Without the intercept, the standard error of the wage-share is reduced to 0.011, the point estimates being unaffected.[4][5] Hence, in the same way as above, the significance and size of the wage-share coefficient seems to depend on 'technicalities' in NKPC estimation, so can hardly be said to be robust, or to represent a structural feature of the NKPC, despite #3 in GGL5's list.

The hybrid NKPC for the US is:

$$\begin{aligned}\widehat{\Delta p}_t &= \underset{(0.092)}{0.623}\ \Delta p_{t+1} + \underset{(0.081)}{0.357}\ \Delta p_{t-1} + \underset{(0.014)}{0.014}\ s_t + \underset{(0.00041)}{0.00016} \\ &\quad \widehat{\sigma} = 0.23\% \quad \chi^2_S(7) = 7.60\end{aligned} \tag{37}$$

The estimates of γ_b and γ_f are similar to the Euro-area hybrid in equation (33), and they are representative of the GMM estimates found in Table 2 in GG. $\widehat{\gamma}_f$ dominates, and they sum almost to unity, so #1 and #2 are confirmed by the estimation. Sargan's χ^2_S test which is significant in (36), is insignificant in (37), which is direct evidence that Δp_{t-1} is misplaced as an instrument and belongs to the category of explanatory variables. However, in terms of mis-specification tests, (37) fails badly, as all are significant.

Autometrics finds nine location-shift dummies in the marginal model for the US inflation rate at a 0.025 significance level. When they are added to (37), they are significant, the diagnostics improve, except for the residual autocorrelation which is still highly significant. It has not been straightforward to find a congruent model from this information set, but moving Δp_{t-2} and gap_{t-1} from being instruments to explanatory variable at least helps (the autocorrelation and heteroskedasticity statistics now have significance levels 0.023 and 0.013). Estimation of the augmented hybrid US NKPC yields (coefficients of dummies are multiplied by 100):

$$\begin{aligned}\widehat{\Delta p}_t &= \underset{(0.168)}{0.253}\ \Delta p_{t+1} + \underset{(0.083)}{0.502}\ \Delta p_{t-1} + \underset{(0.085)}{0.196}\ \Delta p_{t-3} + \underset{(0.013)}{0.022}\ s_t + \underset{(0.018)}{0.028}\ gap_{t-1} \\ &\quad + \underset{(0.00041)}{0.00032} + \underset{(0.18)}{0.51}\ I_{63(4),t} + \underset{(0.19)}{0.66}\ I_{72(1),t} - \underset{(0.19)}{0.62}\ I_{72(2),t} + \underset{(0.24)}{0.73}\ I_{74(3),t} \\ &\quad - \underset{(0.20)}{0.63}\ I_{75(2),t} + \underset{(0.21)}{0.44}\ I_{76(4),t} + \underset{(0.18)}{0.59}\ I_{77(4),t} + \underset{(0.19)}{0.46}\ I_{78(2),t} - \underset{(0.20)}{0.44}\ I_{81(2),t} \\ &\quad \widehat{\sigma} = 0.18\% \ \ \chi^2_S(5) = 3.87 \ \ \mathsf{F}_{\mathsf{ar}}(5, 132) = 2.71^* \\ &\quad \mathsf{F}_{\mathsf{arch}}(4, 144) = 0.79 \ \ \mathsf{F}_{\mathsf{het}}(20, 122) = 1.93^* \ \ \chi^2_{\mathsf{nd}}(2) = 1.04\end{aligned} \tag{38}$$

All the shift-dummies from the 'reduced form' are statistically significant at the 5% level (and most at lower levels). The estimate of the feed-forward term has been reduced from 0.62 to 0.25 while the standard error of the estimate has increased, so the t-value is just 1.5. The coefficient of the wage-share improves statistically: compared to (37), the point estimate has increased somewhat, and the standard error has been reduced in (38). So this endogenous variable does not 'suffer' in the same way as the lead in inflation does.

When the 'post-break' sample 1981(3) to 1997(4) is used to estimate the augmented model, the results are similar to (38) although all variables are less significant on the short sample, see (39).

[4]The mean of s_t is not exactly zero over the sample period, despite being described as 'a deviation from steady-state' in the text.

[5]GGL5 compare Euro and US results. The pure NKPC is reported as $\widehat{\lambda} = 0.25$, with no comments about the difference from GG.

$$\widehat{\Delta p_t} = \underset{(0.296)}{0.277}\ \Delta p_{t+1} + \underset{(0.110)}{0.239}\ \Delta p_{t-1} + \underset{(0.119)}{0.271}\ \Delta p_{t-3} + \underset{(0.037)}{0.033}\ s_t + \underset{(0.022)}{0.029}\ gap_{t-1} + \underset{(0.0015)}{0.0015} \tag{39}$$

$$\widehat{\sigma} = 0.16\%\ \ \chi^2_S(5) = 6.84\ \ \mathsf{F_{ar}}(5,55) = 3.66^{**}$$
$$\mathsf{F_{arch}}(4,58) = 0.48\ \ \mathsf{F_{het}}(20,45) = 0.73\ \ \chi^2_{\mathsf{nd}}(2) = 1.42$$

Specifically, the coefficient of the feedforward term is 0.28 with t-value 0.94. The lagged inflation terms are significant, but note that they sum to only 0.51, against 0.70 in (38). The results show a significant test of autocorrelation, $\mathsf{F_{ar}}(5,55)$, which further output reveals is due to negative first- and second-order autocorrelation.

9 Conclusion

The new-Keynesian Phillips curve (NKPC) includes expected future inflation as a major feedforward variable to explain current inflation. Models of this type are regularly estimated by replacing the expected value by the actual future outcome, then using Instrumental Variables (IV) or Generalized Method of Moments (GMM) methods to estimate the parameters. However, the underlying theory does not allow for various forms of non-stationarity in the data–despite the fact that crises, breaks and regimes shifts are relatively common.

We have shown the serious consequences for NKPC estimation of unmodelled breaks in data processes using the new technique of impulse-indicator saturation: a failure to model location shifts, or blocks of outliers, can induce spurious significance of the feedforward measure when expectations do not in fact matter. Applying the resulting methods to two salient empirical studies of Euro-area and US NKPCs radically alters the results. In the former, the future variable had a negative, insignificant coefficient; and in the latter, its value was more than halved and again was insignificant. All of the features of adding instrument described in §2.1 are violated in these empirical studies:
(a) the estimated structural parameters change substantially in magnitude, and sometimes in sign;
(b) the added instruments are highly significant;
(c) the fit is substantively improved.
Thus, the previous models were clearly mis-specified by failing to account for breaks. Until a theory of NKPCs is formulated that accommodates breaks of the kind commonly observed in economics, or more robust methods are developed, *caveat emptor* forceably applies to using estimated NKPCs.

References

Bai, J., and Perron, P. (1998). Estimating and testing linear models with multiple structural changes. *Econometrica*, **66**, 47–78.

Bank of England (2008). *August Inflation Report*. London: Bank of England Monetary Policy Committee.

Bank of England (2009). *February Inflation Report*. London: Bank of England Monetary Policy Committee.

Bårdsen, G., Eitrheim, Ø., Jansen, E. S., and Nymoen, R. (2005). *The Econometrics of Macroeconomic Modelling*. Oxford: Oxford University Press.

Bårdsen, G., Jansen, E. S., and Nymoen, R. (2004). Econometric evaluation of the new Keynesian Phillips curve. *Oxford Bulletin of Economics and Statistics*, **66**, 671–686.

Barrell, R. (2001). Forecasting the world economy. In Hendry, D. F., and Ericsson, N. R. (eds.), *Understanding Economic Forecasts*, pp. 149–169. Cambridge, Mass.: MIT Press.

Bjørnstad, R., and Nymoen, R. (2008). The New Keynesian Phillips curve tested on OECD panel data. *Economics-The Open-Access, Open-Assessment E-Journal*, **2**, 1–18. http://www.economics-ejournal.org.

Castle, J. L. (2008). Checking the robustness and validity of model selection: An application to UK wages. Mimeo, Economics Department, Oxford University.

Castle, J. L., Doornik, J. A., and Hendry, D. F. (2009). Model selection when there are multiple breaks. Working paper 472, Economics Department, University of Oxford.

Castle, J. L., Doornik, J. A., and Hendry, D. F. (2010). Evaluating automatic model selection. *Journal of Time Series Econometrics*, forthcoming.

Castle, J. L., and Hendry, D. F. (2009). The long-run determinants of UK wages, 1860–2004. *Journal of Macroeconomics*, **31**, 5–28.

Castle, J. L., and Hendry, D. F. (2010). Model Selection in Under-specified Equations Facing Breaks. Working paper 509, Economics Department, University of Oxford.

Castle, J. L., and Shephard, N. (eds.)(2009). *The Methodology and Practice of Econometrics*. Oxford: Oxford University Press.

Clements, M. P., and Hendry, D. F. (1998). *Forecasting Economic Time Series*. Cambridge: Cambridge University Press.

Clements, M. P., and Hendry, D. F. (1999). On winning forecasting competitions in economics. *Spanish Economic Review*, **1**, 123–160.

Clements, M. P., and Hendry, D. F. (2001). An historical perspective on forecast errors. *National Institute Economic Review*, **177**, 100–112.

Doornik, J. A. (1999). *Object-Oriented Matrix Programming using Ox*. London: Timberlake Consultants Press. 3rd edition.

Doornik, J. A. (2009). Autometrics. In Castle, and Shephard (2009), pp. 88–121.

Doornik, J. A., and Hansen, H. (2008). An omnibus test for univariate and multivariate normality. *Oxford Bulletin of Economics and Statistics*, **70**, 927–939.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity, with estimates of the variance of United Kingdom inflation. *Econometrica*, **50**, 987–1007.

Engle, R. F., and Hendry, D. F. (1993). Testing super exogeneity and invariance in regression models. *Journal of Econometrics*, **56**, 119–139.

Evans, G. W., and Honkapohja, S. (2001). *Learning and Expectations in Macroeconomics*. Princeton: Princeton University Press.

Fanelli, L. (2008). Testing the new Keynesian Phillips curve through vector autoregressive models: Results from the Euro area. *Oxford Bulletin of Economics and Statistics*, **70**, 53—66.

Frisch, R., and Waugh, F. V. (1933). Partial time regression as compared with individual trends. *Econometrica*, **1**, 221–223.

Frydman, R., and Goldberg, M. D. (eds.)(2007). *Imperfect Knowledge Economics: Exchange Rates and Risk*. Princeton, New Jersey: Princeton University Press.

Fuhrer, J. C. (2006). Intrinsic and inherited inflation persistence. *International Journal of Central Banking*, **2**, 49–86.

Galí, J., and Gertler, M. (1999). Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics*, **44**, 195–222.

Galí, J., Gertler, M., and Lopez-Salido, J. D. (2001). European inflation dynamics. *European Economic Review*, **45**, 1237–1270.

Galí, J., Gertler, M., and Lopez-Salido, J. D. (2005). Robustness of the estimates of the hybrid new Keynesian Phillips curve. *Journal of Monetary Economics*, **21**, 1107–1118.

Garcia, R., and Perron, P. (1996). An analysis of the real interest rate under regime shifts. *Review of Economics and Statistics*, **78**, 111–125.

Godfrey, L. G. (1978). Testing for higher order serial correlation in regression equations when the regressors include lagged dependent variables. *Econometrica*, **46**, 1303–1313.

Hendry, D. F. (1988). The encompassing implications of feedback versus feedforward mechanisms in econometrics. *Oxford Economic Papers*, **40**, 132–149.

Hendry, D. F. (1999). An econometric analysis of US food expenditure, 1931–1989. In Magnus, J. R., and Morgan, M. S. (eds.), *Methodology and Tacit Knowledge: Two Experiments in Econometrics*, pp. 341–361. Chichester: John Wiley and Sons.

Hendry, D. F. (2000). On detectable and non-detectable structural change. *Structural Change and Economic Dynamics*, **11**, 45–65.

Hendry, D. F. (2006). Robustifying forecasts from equilibrium-correction models. *Journal of Econometrics*, **135**, 399–426.

Hendry, D. F. (2010a). Climate change: Possible lessons for our future from the distant past. In Dietz, S., Michie, J., and Oughton, C. (eds.), *The Political Economy of the Environment*, forthcoming. London: Routledge.

Hendry, D. F. (2010b). On adding over-identifying instrumental variables to simultaneous equations. Unpublished paper, Economics Department, University of Oxford.

Hendry, D. F., Johansen, S., and Santos, C. (2008). Automatic selection of indicators in a fully saturated regression. *Computational Statistics*, **33**, 317–335. Erratum, 337–339.

Hendry, D. F., and Mizon, G. E. (2010a). Econometric modelling of time series with outlying observations. *Journal of Time Series Econometrics*, forthcoming.

Hendry, D. F., and Mizon, G. E. (2010b). On the mathematical basis of inter-temporal optimization. Working paper 497, Economics Department, Oxford.

Hendry, D. F., and Santos, C. (2010). An automatic test of super exogeneity. In Watson, M. W., Bollerslev, T., and Russell, J. (eds.), *Volatility and Time Series Econometrics*, pp. 164–193. Oxford: Oxford University Press.

Johansen, S., and Nielsen, B. (2009). An analysis of the indicator saturation estimator as a robust regression estimator. In Castle, and Shephard (2009), pp. 1–36.

Juselius, K. (2006). *The Cointegrated VAR Model: Methodology and Applications*. Oxford: Oxford University Press.

Kirman, A. (1989). The intrinsic limits of economic theory: The emperor has no clothes. *Economic Journal*, **99**, 126–139.

Ljung, G. M., and Box, G. E. P. (1978). On a measure of lack of fit in time series models. *Biometrika*, **66**, 67–72.

Mavroeidis, S. (2004). Weak identification of forward-looking models in monetary economics. *Oxford Bulletin of Economics and Statistics*, **66**, 609–635.

Mavroeidis, S. (2005). Identification issues in forward-looking models estimated by GMM, with an application to the Phillips curve. *Journal of Money, Credit, and Banking*, **37**, 421–448.

Muth, J. F. (1961). Rational expectations and the theory of price movements. *Econometrica*, **29**, 315–

335.

Nagar, A. L. (1959). The bias and moment matrix of the general k-class estimators of the parameters in simultaneous equations. *Econometrica*, **27**, 575–595.

Nickell, S. J. (2008). Aspects of monetary policy in the UK. Mimeo, Nuffield College, Oxford.

Nymoen, R. (1991). A small linear model of wage- and price-inflation in the Norwegian economy. *Journal of Applied Econometrics*, **6**, 255–269.

Nymoen, R. (2005). Evaluating a central bank's recent forecast failure. Memorandum 22/2005, Department of Economics, University of Oslo.

Pagan, A. R. (1984). Model evaluation by variable addition. In Hendry, D. F., and Wallis, K. F. (eds.), *Econometrics and Quantitative Economics*, pp. 103–135. Oxford: Basil Blackwell.

Pesaran, M. H. (1981). Identification of rational expectations models. *Journal of Econometrics*, **16**, 375–398.

Rudd, J., and Whelan, K. (2005). Does labor's share drive inflation?. *Journal of Credit and Banking*, **37**, 297–312.

Rudd, J., and Whelan, K. (2006). Can Rational Expectations Sticky-Price Models Explain Inflation Dynamics?. *American Economic Review*, **96**, 303–320.

Rudd, J., and Whelan, K. (2007). Modeling inflation dynamics: A critical review of recent work. *Journal of Credit and Banking*, **39**, 155–170.

Russell, B., Banerjee, A., Malki, I., and Ponomareva, N. (2010). A multiple break panel approach to estimating United States Phillips curves. Unpublished paper, Economic Studies, University of Dundee, Dundee, UK.

Sargan, J. D. (1958). The estimation of economic relationships using instrumental variables. *Econometrica*, **26**, 393–415.

Sargan, J. D. (1964). Wages and prices in the United Kingdom: A study in econometric methodology (with discussion). In Hart, P. E., Mills, G., and Whitaker, J. K. (eds.), *Econometric Analysis for National Economic Planning*, Vol. 16 of *Colston Papers*, pp. 25–63. London: Butterworth Co.

Sargan, J. D. (1980). A model of wage-price inflation. *Review of Economic Studies*, **47**, 113–135.

Smets, F., and Wouters, R. (2002). An estimated stochastic dynamic general equilibrium model of the Euro Area. ECB Working Paper, 171, European Central Bank, Frankfurt.

Stock, J. H., and Watson, M. W. (1996). Evidence on structural instability in macroeconomic time series relations. *Journal of Business and Economic Statistics*, **14**, 11–30.

Wallis, K. F. (1980). Econometric implications of the rational expectations hypothesis. *Econometrica*, **48**, 49–73.

White, H. (1980). A heteroskedastic-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica*, **48**, 817–838.

10 Appendix calculations

Section 5.2 used the following formulae, where the break $\boldsymbol{\rho}'\mathbf{d}_t = \delta 1_{\{T_1 \leq t \leq T_2\}}$ is completely internal to the sample, namely $4 \leq T_1 \leq T_2 \leq T-1$. The DGP is:

$$\Delta p_t = \kappa \Delta p_{t-1} + \boldsymbol{\rho}'\mathbf{d}_t + \eta_t = \left[\boldsymbol{\rho}'\left(\mathbf{d}_t + \kappa \mathbf{d}_{t-1}\right) + \kappa^2 \Delta p_{t-2} + \left(\eta_t + \kappa \eta_{t-1}\right)\right]$$

where the second expression is used as an approximation by neglecting powers of κ greater than squared. In the special case of $\delta = 0$, the feedforward model will be unidentified, having used the Frisch–Waugh theorem to remove $\mathbf{z}_t$, equivalent to $\boldsymbol{\theta}_3 = \mathbf{0}$, so we do not change notation. The following terms in break cross products occur in most formulae:

$$\boldsymbol{\rho}' \left(\sum_{t=3}^{T-1} \mathbf{d}_{t-k} \mathbf{d}'_{t+j} \right) \boldsymbol{\rho} = \delta^2 \sum 1_{\{T_1 \le t-k \le T_2\}} 1_{\{T_1 \le t+j \le T_2\}} = \delta^2 \left(T_2 - T_1 + 1 - k - j \right)$$

where all calculations are for a common sample of $t = 3, \ldots, T-1$. Hence:

$$\begin{aligned} \mathsf{E}\left[\sum_{t=3}^{T-1} \Delta p_t \mathbf{d}'_t \boldsymbol{\rho} \right] &= \kappa \mathsf{E}\left[\sum_{t=3}^{T-1} \Delta p_{t-1} \mathbf{d}'_t \boldsymbol{\rho} \right] + \boldsymbol{\rho}' \sum_{t=3}^{T-1} \mathbf{d}_t \mathbf{d}'_t \boldsymbol{\rho} \\ &\simeq \boldsymbol{\rho}' \sum_{t=3}^{T-1} \left(\mathbf{d}_t \mathbf{d}'_t + \kappa \mathbf{d}_{t-1} \mathbf{d}'_t + \kappa^2 \mathbf{d}_{t-2} \mathbf{d}'_t \right) \boldsymbol{\rho} \\ &\simeq \delta^2 \left(T_2 - T_1 + 1 \right) + \kappa \delta^2 \left(T_2 - T_1 \right) + \kappa^2 \delta^2 \left(T_2 - T_1 - 1 \right) \\ &= \delta^2 \left[\left(1 + \kappa + \kappa^2 \right) \left(T_2 - T_1 + 1 \right) - \kappa \left(1 + 2\kappa \right) \right]. \end{aligned}$$

Similarly:

$$\begin{aligned} \mathsf{E}\left[\sum_{t=3}^{T-1} \Delta p_{t-1} \mathbf{d}'_t \boldsymbol{\rho} \right] &\simeq \sum_{t=3}^{T-1} \left(\boldsymbol{\rho}' \mathbf{d}_{t-1} \mathbf{d}'_t \boldsymbol{\rho} + \kappa \boldsymbol{\rho}' \mathbf{d}_{t-2} \mathbf{d}'_t \boldsymbol{\rho} + \kappa^2 \boldsymbol{\rho}' \mathbf{d}_{t-3} \mathbf{d}'_t \boldsymbol{\rho} \right) \\ &= \delta^2 \left(T_2 - T_1 \right) + \kappa \delta^2 \left(T_2 - T_1 - 1 \right) + \kappa^2 \delta^2 \left(T_2 - T_1 - 2 \right) \\ &= \delta^2 \left[\left(1 + \kappa + \kappa^2 \right) \left(T_2 - T_1 + 1 \right) - \left(1 + 2\kappa + 3\kappa^2 \right) \right]. \end{aligned}$$

Next, letting $R = \left(T_2 - T_1 + 1 \right) / \left(T - 3 \right)$ with:

$$\sigma^2_{\Delta p} = \frac{1}{(T-3)} \mathsf{E}\left[\sum_{t=3}^{T-1} \left(\Delta p_t \right)^2 \right] \simeq \frac{1}{(T-3)} \mathsf{E}\left[\sum_{t=3}^{T-1} \left(\Delta p_{t-1} \right)^2 \right]$$

then assuming $\left(1 + 2\kappa + 3\kappa^2 \right) / \left(T - 3 \right) \simeq 0$:

$$\begin{aligned} \frac{1}{(T-3)} \mathsf{E}\left[\sum_{t=3}^{T-1} \left(\Delta p_t \right)^2 \right] &= \frac{1}{(T-3)} \mathsf{E}\left[\sum_{t=3}^{T-1} \left(\kappa \Delta p_{t-1} + \boldsymbol{\rho}' \mathbf{d}_t + \eta_t \right)^2 \right] \\ &\simeq \frac{\delta^2 R \left(1 + 2\kappa + 2\kappa^2 \right) + \sigma^2_\eta}{1 - \kappa^2}. \end{aligned}$$

Also:

$$\begin{aligned} \frac{1}{(T-3)} \mathsf{E}\left[\sum_{t=3}^{T-1} \Delta p_{t-1} \Delta p_t \right] &= \frac{1}{(T-3)} \mathsf{E}\left[\sum_{t=3}^{T-1} \Delta p_{t-1} \left(\kappa \Delta p_{t-1} + \boldsymbol{\rho}' \mathbf{d}_t \right) \right] \\ &\simeq \frac{\delta^2 R \left(1 + 2\kappa + 2\kappa^2 \right) + \kappa \sigma^2_\eta}{1 - \kappa^2} \end{aligned}$$

and similarly:

$$\frac{1}{(T-3)} \mathsf{E}\left[\sum_{t=3}^{T-1} \Delta p_{t-2} \Delta p_{t-1} \right] \simeq \frac{\delta^2 R \left(1 + 2\kappa + 2\kappa^2 \right) + \kappa \sigma^2_\eta}{1 - \kappa^2}.$$

Thirdly:

$$\frac{1}{(T-3)}\mathsf{E}\left[\sum_{t=3}^{T-1}\Delta p_{t-1}\Delta p_{t+1}\right]\simeq\frac{1}{(T-3)}\mathsf{E}\left[\sum_{t=3}^{T-1}\Delta p_{t-2}\Delta p_{t}\right]\simeq\frac{\delta^2 R\left(1+2\kappa+2\kappa^2\right)+\kappa^2\sigma_\eta^2}{(1-\kappa^2)}$$

having dropped $-\left(1+2\kappa+3\kappa^2\right)/(T-3)\simeq 0$ as before and using:

$$\begin{aligned}\frac{1}{(T-3)}\mathsf{E}\left[\sum_{t=3}^{T-1}\Delta p_{t-2}\mathbf{d}_t'\boldsymbol{\rho}\right]&\simeq\frac{1}{(T-3)}\sum_{t=3}^{T-1}\left(\boldsymbol{\rho}'\mathbf{d}_{t-2}\mathbf{d}_t'\boldsymbol{\rho}+\kappa\boldsymbol{\rho}'\mathbf{d}_{t-3}\mathbf{d}_t'\boldsymbol{\rho}+\kappa^2\boldsymbol{\rho}'\mathbf{d}_{t-4}\mathbf{d}_t'\boldsymbol{\rho}\right)\\&\simeq\delta^2 R\left(1+\kappa+\kappa^2\right).\end{aligned}$$

Finally:

$$\frac{1}{(T-3)}\mathsf{E}\left[\sum_{t=3}^{T-1}\Delta p_{t-2}\Delta p_{t+1}\right]\simeq\frac{1}{(T-3)}\mathsf{E}\left[\sum_{t=3}^{T-1}\Delta p_{t-3}\Delta p_{t}\right]\simeq\frac{\delta^2 R\left(2+3\kappa+2\kappa^2\right)+\kappa^3\sigma_\eta^2}{(1-\kappa^2)}$$

where:

$$\begin{aligned}\frac{1}{(T-3)}\mathsf{E}\left[\sum_{t=3}^{T-1}\boldsymbol{\rho}'\mathbf{d}_t\Delta p_{t-3}\right]&\simeq\frac{1}{(T-3)}\sum_{t=3}^{T-1}\left(\boldsymbol{\rho}'\mathbf{d}_{t-3}\mathbf{d}_t'\boldsymbol{\rho}+\kappa\boldsymbol{\rho}'\mathbf{d}_{t-4}\mathbf{d}_t'\boldsymbol{\rho}+\kappa^2\boldsymbol{\rho}'\mathbf{d}_{t-5}\mathbf{d}_t'\boldsymbol{\rho}\right)\\&=\delta^2 R\left(1+\kappa+\kappa^2\right)\end{aligned}$$

having dropped $-\left(2+3\kappa+4\kappa^2\right)/(T-3)\simeq 0$.

The postulated equation is:

$$\Delta p_t=\theta_1\Delta p_{t+1}+\theta_2\Delta p_{t-1}+e_t$$

where Δp_{t-1}, Δp_{t-2} are the IVs. The over-identifying IV is now Δp_{t-2} (in practice, $\mathbf{z}_{t-1}$ might be used). Then for $\mathbf{y}=\Delta\mathbf{p}$, $\mathbf{X}=(\Delta\mathbf{p}_{+1}:\Delta\mathbf{p}_{-1})$ and $\mathbf{Z}=(\Delta\mathbf{p}_{-1}:\Delta\mathbf{p}_{-2})$, as the equation is just identified:

$$\widetilde{\boldsymbol{\theta}}=\left(\mathbf{Z}'\mathbf{X}\right)^{-1}\left(\mathbf{Z}'\mathbf{y}\right)=\begin{pmatrix}\Delta\mathbf{p}_{-1}'\Delta\mathbf{p}_{+1} & \Delta\mathbf{p}_{-1}'\Delta\mathbf{p}_{-1}\\ \Delta\mathbf{p}_{-2}'\Delta\mathbf{p}_{+1} & \Delta\mathbf{p}_{-2}'\Delta\mathbf{p}_{-1}\end{pmatrix}^{-1}\begin{pmatrix}\Delta\mathbf{p}_{-1}'\Delta\mathbf{p}\\ \Delta\mathbf{p}_{-2}'\Delta\mathbf{p}\end{pmatrix}.$$

Then, using a Nagar (1959) approximation:

$$\begin{aligned}\mathsf{E}\left[\widetilde{\boldsymbol{\theta}}\right]&\simeq\left(\mathsf{E}\left[\begin{pmatrix}\Delta\mathbf{p}_{-1}'\Delta\mathbf{p}_{+1} & \Delta\mathbf{p}_{-1}'\Delta\mathbf{p}_{-1}\\ \Delta\mathbf{p}_{-2}'\Delta\mathbf{p}_{+1} & \Delta\mathbf{p}_{-2}'\Delta\mathbf{p}_{-1}\end{pmatrix}\right]\right)^{-1}\mathsf{E}\left[\begin{pmatrix}\Delta\mathbf{p}_{-1}'\Delta\mathbf{p}\\ \Delta\mathbf{p}_{-2}'\Delta\mathbf{p}\end{pmatrix}\right]\\&=\left[\frac{1}{1-\kappa^2}\begin{pmatrix}G+\kappa^2\sigma_\eta^2 & G+\sigma_\eta^2\\ G+\kappa^3\sigma_\eta^2 & G+\kappa\sigma_\eta^2\end{pmatrix}\right]^{-1}\frac{1}{1-\kappa^2}\begin{pmatrix}G+\kappa\sigma_\eta^2\\ G+\kappa^2\sigma_\eta^2\end{pmatrix}\\&\simeq\frac{1}{H}\begin{pmatrix}\left(G+\kappa\sigma_\eta^2\right)\left(G+\kappa\sigma_\eta^2\right)-\left(G+\sigma_\eta^2\right)\left(G+\kappa^2\sigma_\eta^2\right)\\ \left(G+\kappa^2\sigma_\eta^2\right)\left(G+\kappa^2\sigma_\eta^2\right)-\left(G+\kappa^3\sigma_\eta^2\right)\left(G+\kappa\sigma_\eta^2\right)\end{pmatrix}\\&\simeq\frac{1}{(1-\kappa-\kappa^2)}\begin{pmatrix}(1-\kappa)^2\\ \kappa\left(1-2\kappa+\kappa^2\right)\end{pmatrix}\end{aligned}$$

where:

$$\begin{aligned}H&=\left(G+\kappa^2\sigma_\eta^2\right)\left(G+\kappa\sigma_\eta^2\right)-\left(G+\kappa^3\sigma_\eta^2\right)\left(G+\sigma_\eta^2\right)\\&=G^2+G\kappa\sigma_\eta^2+G\kappa^2\sigma_\eta^2+\kappa^3\sigma_\eta^4-G^2-G\sigma_\eta^2-G\kappa^3\sigma_\eta^2-\kappa^3\sigma_\eta^4\\&\simeq -G\left(1-\kappa-\kappa^2\right)\sigma_\eta^2.\end{aligned}$$