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Simon Cowan

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Manor Road Building, Oxford OX1 3UQ

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Abstract

Monopoly third-degree price discrimination raises social welfare above the level with a uniform price when direct demand functions have constant curvatures that differ across markets and are below 1, and the maximum willingness to pay is identical across markets.

Keywords: third-degree price discrimination, monopoly, social welfare.

JEL Classification: D42, L12, L13.

1 Introduction

The effect on social welfare of third-degree price discrimination by a monopolist is positive for a simple class of demand functions. Each direct demand function has constant curvature (a measure of convexity) that is below 1, curvature differs across markets, and the maximum willingness to pay is the same in each market. It follows that a market with a more convex demand function has a lower discriminatory price.

Pigou (2013), first published in 1920, showed that if demand functions are linear, and all markets are served at the uniform price, social welfare is lower with discrimination than with uniform pricing. Total output is unchanged, but the output is inefficiently distributed across markets under discrimination as marginal utilities are unequal. If discrimination is to increase welfare some of the demand functions must be non-linear and there must be a large enough increase in total output to offset the inefficient distribution.

The model is the next section. Marginal cost is assumed there to be zero, both for simplicity and because the main result then holds for a broader class of

*Department of Economics, University of Oxford, Manor Road Building, Oxford OX1 3UQ, UK. Email: simon.cowan@economics.ox.ac.uk. I am grateful to Iñaki Aguirre, Peter Neary and John Vickers for helpful comments on an earlier version.

demand functions. Section 3 extends the model to allow for positive marginal cost. The general relationship between demand curvature and the price elasticity of demand is explored in Section 4. Section 5 concludes.

2 The Model

There are m markets, denoted by subscripts $i = 1, \ldots, m$. Demand in market i at price p_i is

$$q_i(p_i) = 1 - p_i^{\beta_i} \tag{1}$$

for $0 \leq p_i \leq 1$ and $\beta_i > 0$. The associated utility function,

$$u_i(q_i) = -\frac{\beta_i}{1+\beta_i}(1-q_i)^{\frac{1+\beta_i}{\beta_i}},$$

is increasing and strictly concave in q_i. Equivalently each market has a unit mass of consumers, who buy one unit or none each. Willingness to pay, v, has a power function distribution on $[0, 1]$. The cumulative distribution function is v^{β_i} and demand is the set of those with willingness to pay above the price.[1] The price elasticity of demand, $-p_i q_i'(p_i)/q_i(p_i) = \beta_i p_i^{\beta_i}/(1 - p_i^{\beta_i})$, is strictly increasing in the price. It ranges from 0 when $p_i = 0$ to infinity as the price goes towards 1. At a given price the elasticity falls as β_i increases.

The curvature of direct demand, $-p_i q_i''(p_i)/q_i'(p_i) = 1 - \beta_i$, is constant and below 1. Demand is concave when $\beta_i \geq 1$, convex when $\beta_i \leq 1$ and linear if $\beta_i = 1$. The markets differ in their β_i values and thus in their curvatures. Marginal cost is zero. Profit, $\pi_i(p_i) = p_i q_i = p_i - p_i^{1+\beta_i}$, is strictly concave in p_i. The discriminatory price that maximizes profit is $p_i^* = 1/(1 + \beta_i)^{1/\beta_i}$. Discriminatory output, $q_i^* = \beta_i/(1 + \beta_i)$, is increasing in β_i. If marginal cost were to increase above zero the effect on the discriminatory price—the cost pass-through coefficient—would be $1/(1+\beta_i)$. For this class of demands pass-through is strictly below 1.

The uniform price maximizes the aggregate profit function $\sum_i \pi_i(p)$, which is strictly concave. The unique solution for the uniform price, $\bar{p}$, is characterized by the first-order condition $\sum_i \pi'(\bar{p}) = 0$. All markets are served at the uniform price because the maximum willingness to pay is the same in all markets. After Robinson (1969) a market with $p_i^* < \bar{p}$ is called "weak", and one with a higher price than the uniform one is "strong". In a weak market marginal profitability at the uniform price, $\pi_i'(\bar{p})$, is negative, while in a strong market this term is positive. The uniform price lies strictly in between the highest and lowest discriminatory prices. If the uniform price was, say, above the highest discriminatory price then profit in each market would increase with a reduction

[1] See Bagnoli & Bergstrom (2005) for a discussion of the power function distribution.

in the uniform price so the original price would not have maximized profits. This is an application of Theorem 1 of Nahata et al. (1990).

The discriminatory price and $p_i^*/(1+\beta_i)$ (which will be called the pass-through-adjusted margin) change with β_i monotonically and in opposite ways.

Lemma. *The discriminatory price is strictly increasing in β_i. The pass-through-adjusted margin is strictly decreasing in β_i.*

Proof. From the formula for the discriminatory price $\ln(p_i^*) = -\ln(1+\beta_i)/\beta_i$, so

$$\frac{dp_i^*}{d\beta_i} = \frac{p_i^*}{\beta_i^2}\left(\ln(1+\beta_i) - \frac{\beta_i}{1+\beta_i}\right).$$

It follows that

$$\frac{d}{d\beta_i}\left(\frac{p_i^*}{1+\beta_i}\right) = \frac{p_i^*}{(1+\beta_i)\beta_i^2}\left(\ln(1+\beta_i) - \beta_i\right).$$

To sign the two derivatives consider the function $f(x) \equiv \ln(1+x)$ for $x > -1$. This has derivative $f'(x) = 1/(1+x)$ and is strictly concave. As x changes from 0 to $\beta_i \neq 0$ the change in the function is thus bounded above and below:

$$f'(0)\beta_i > f(\beta_i) - f(0) > f'(\beta_i)\beta_i$$

i.e.

$$\beta_i > \ln(1+\beta_i) > \frac{\beta_i}{1+\beta_i}.$$

The signs of the derivatives follow. □

More convex demand (a reduction in β_i) generates a lower discriminatory price and a higher pass-through-adjusted margin. The welfare bounds analysis of Varian (1985) is now used. Social welfare is consumer surplus plus producer surplus, which equals utility as marginal cost is zero. Because utility is concave in output, with derivative equal to the price, the change in aggregate welfare, ΔW, in the move to discriminatory prices is bounded below,

$$\Delta W \geq \sum_i p_i^* \Delta q_i$$

where $\Delta q_i = q_i^* - q_i(\bar{p})$ is the change in output. If the lower bound is positive then discrimination increases social welfare.

Proposition 1. *Suppose demand functions differ in their constant curvatures, which are below 1, and have the same maximum willingness to pay, and that marginal cost is zero. Social welfare is higher with discrimination.*

Proof. The change in output is proportional to marginal profitability at the uniform price:

$$\Delta q_i = \frac{\beta_i}{1+\beta_i} - 1 + \bar{p}_i^{\beta} = \frac{-1 + (1+\beta_i)\bar{p}_i^{\beta}}{1+\beta_i} = \frac{-\pi_i'(\bar{p})}{1+\beta_i}.$$

The lower bound to the change in welfare is thus the weighted sum of the pass-through-adjusted margins,

$$\sum_i p_i^* \Delta q_i = \sum_i \frac{p_i^*}{1+\beta_i}(-\pi_i'(\bar{p})).$$

By the Lemma $p_i^*/(1+\beta_i)$ is higher in the weak markets than in the strong markets. Let X be the highest value of $p_i^*/(1+\beta_i)$ in the strong markets. Subtracting $X\sum_i(-\pi_i'(\bar{p})) = 0$ gives

$$\sum_i \frac{p_i^*}{1+\beta_i}(-\pi_i'(\bar{p})) = \sum_i \left(\frac{p_i^*}{1+\beta_i} - X\right)(-\pi_i'(\bar{p})) > 0.$$

The strict inequality holds because $\left(\frac{p_i^*}{1+\beta_i} - X\right)(-\pi_i'(\bar{p})) > 0$ in each weak market (a positive term times a positive term), while for a strong market this expression is either positive—the product of two negative terms—or zero. □

Robinson (1969)— first published in 1933—showed the importance of differing curvatures of demand for the effect on total output but without establishing when welfare would increase.[2] Figure 1 illustrates the positive welfare effect of discrimination for two markets using inverse demand curves. The weak market has $\beta_1 = 0.5$, with inverse demand $p_1(q) = (1-q)^2$, and $\beta_2 = 5$ in the strong market (with inverse demand $p_2(q) = (1-q)^{0.2}$). The discriminatory prices are $p_1^* = 0.444$ and $p_2^* = 0.699$. The uniform price is $\bar{p} = 0.664$. The reduction in social welfare in the strong market, where demand is highly concave, is small because both the price increase and the quantity reduction are small. This welfare loss is the increase in the deadweight triangle, represented by the small triangle c and the rectangle d. Meanwhile in the weak market the price reduction is large, as is the quantity increase because demand is relatively convex. The welfare gain in that market is a plus b.

The result in Proposition 1 is unchanged if there are different numbers of consumers in each market. Demand in i is now $n_i q_i(p_i)$ where n_i is the number of consumers. The discriminatory price and the pass-through-adjusted margin do not depend on n_i. Marginal profitability, $\pi_i'(\bar{p})$, is proportional to n_i for a given price, and the relative numbers will affect the uniform price, $\bar{p}$. The size, but not the sign, of the lower bound to the welfare change will be affected.

[2]Shih et al. (1988) extended Robinson's analysis and focussed on the relationship between inverse demand curvature (which equals direct demand curvature divided by the price elasticity of demand) and the total output effect.

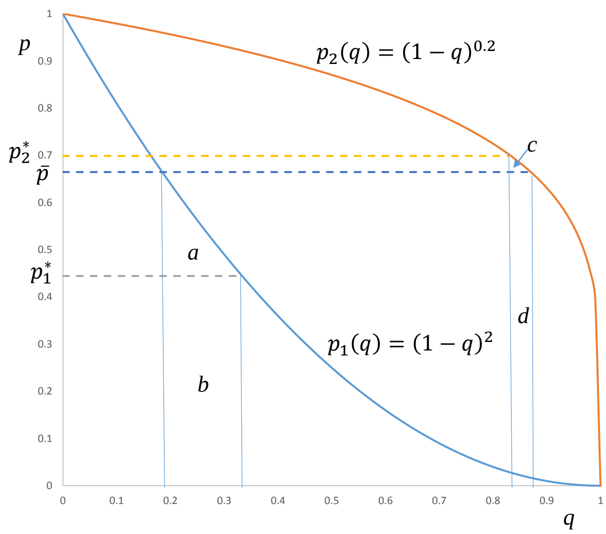

Figure 1: Discrimination raises welfare when curvature differs

It is important that the maximum willingness to pay is identical across markets. To see what can happen otherwise, consider the generalization to the demand function in (1) given by $q_i(p_i) = 1 - (p_i/a_i)^{\beta_i}$ where $a_i > 0$ is the maximum willingness to pay in market i. The discriminatory price becomes $p_i^* = a_i/(1+\beta_i)^{1/\beta_i}$, which is proportional to a_i. Thus a market can have a lower value of β_i, which pushes down the price, but a higher a_i, which increases the price. Differences in demand curvature are no longer sufficient, in general, to order the markets in terms of discriminatory prices. If the demand functions share a common β and differ only in a_i discrimination reduces social welfare provided all markets are served at the uniform price. This is an application of Proposition 1 of Aguirre et al. (2010).[3]

The result in Proposition 1 extends to a class of demand functions which are more convex than those defined in (1), provided marginal cost is zero. Now the demand function is

$$q_i(p_i) = p_i^{\beta_i} - 1 \tag{2}$$

for $0 < p_i \leq 1$ and $\beta_i \in (-1, 0)$. When $\beta_i = 0$ demand is $q_i(p_i) = -\ln(p_i)$. The function in (2) is a constant-elasticity demand, $p_i^{\beta_i}$, that is shifted so that there

[3] The change in output in i is $\Delta q_i = -\pi_i'(\bar{p})/(1+\beta)$, so total output is unchanged with discrimination, as in the linear demands case of Pigou (2013).

is a maximum willingness to pay of 1.[4] The utility functions are

$$u_i(q_i) = \frac{\beta_i}{1+\beta_i}(1+q_i)^{\frac{1+\beta_i}{\beta_i}}$$

for $\beta_i < 0$ and $u_i(q_i) = -e^{-q_i}$ for $\beta_i = 0$. Both are increasing and strictly concave.

The price elasticity of demand increases as the price rises. The curvature of demand is still $1 - \beta_i$, which now lies in $[1, 2)$. Demand is strictly convex, and more so than with the demand in (1). The cost pass-through coefficient, $1/(1 + \beta_i)$, exceeds 1.

The analysis follows that used for Proposition 1. The profit function $\pi_i(p_i) = p_i^{1+\beta_i} - p_i$ is strictly concave, as is $\pi_i(p_i) = -p_i \ln(p_i)$ for $\beta_i = 0$. The discriminatory prices are $p_i^* = 1/(1 + \beta_i)^{1/\beta_i}$ for negative β_i, as before, and $p_i^* = 1/e$ when $\beta_i = 0$. The discriminatory quantities are $q_i^* = -\frac{\beta_i}{1+\beta_i}$ for $\beta_i \in (-1, 0)$ and $q_i^* = 1$ for $\beta_i = 0$. The change in the quantity as the price changes from the uniform price is $\Delta q_i = -\pi_i'(\bar{p})/(1 + \beta_i)$. Varian's lower bound is thus given by the same formula as for Proposition 1. The Lemma holds when $\beta_i \in (-1, 0)$ as well as when $\beta_i > 0$. The next result is immediate.

Proposition 2. *Suppose demand functions differ in their constant curvatures, which are in* $[1, 2)$*, and have the same maximum willingness to pay, and that marginal cost is zero. Social welfare is higher with discrimination.*

The two propositions can be combined, provided the maximum willingness to pay is common to all markets. Suppose there are two demand functions, one with curvature equal to 1.5 and the other with curvature of -1, and in both the maximum willingness to pay is 1. Social welfare will be higher with discrimination.

3 Extending the main result

When marginal cost is positive the technique of Aguirre et al. (2010) needs to be used. The effect on aggregate welfare of allowing a small amount of discrimination, and then increasing this amount, can be determined. Social welfare is concave in the amount of discrimination, and the lower bound to the change in welfare is the weighted sum of the pass-through-adjusted margins. The generalization applies only to Proposition 1 and shares its two main features: as demand curvature increases (i) the discriminatory profit margin falls and (ii) the cost pass-through coefficient rises sufficiently that the product of these two—the pass-through-adjusted margin—rises.

[4] This demand function, with a multiplicative factor, can also be derived from a truncated Pareto distribution of valuations.

Proposition 3. *Suppose that demand functions differ in their constant curvatures, which are below 1, and have the same maximum willingness to pay, and that marginal cost is positive. Social welfare is higher with discrimination.*

Proof. See the Appendix. □

This nests Proposition 1 but has a longer proof.[5] Proposition 2, which applies for curvature in between 1 and 2, does not generalize to strictly positive marginal cost. Higher curvature leads to a lower discriminatory margin, but the pass-through-adjusted margin can rise or fall.

What happens when curvature is above 2? Consider the demand function $q_i(p_i) = p_i^{-\epsilon_i}$ for $\epsilon_i > 1$. This is a constant-elasticity demand function whose elasticity, ϵ_i, is above 1, so that the monopoly price is finite. With elastic demand each price cut increases revenue, so marginal cost must be strictly positive to ensure that output is finite. Direct demand curvature is $1 + \epsilon_i > 2$. Again a higher value of demand curvature entails a lower discriminatory price. The pass-through-adjusted margin, however, falls as demand curvature rises, because cost pass-through, $\epsilon_i/(\epsilon_i - 1)$, is decreasing in ϵ_i. The effect of discrimination on welfare when constant price elasticities differ across markets can be positive or negative.[6]

4 Curvature and the price elasticity

A feature of the model is that a market with a higher curvature of direct demand also has a higher price elasticity of demand. Here this relationship is explored more generally. Suppose that demand is a function of the price and of a parameter, θ_i, which is the only exogenous source of variation across markets, so demand is $q(p_i, \theta_i)$. Without loss of generality the market size parameter, or the number of consumers, n_i, is 1. Consider two demand functions in this family with strictly negative price derivatives and which are twice continuously differentiable in p_i. The ratio of the price derivatives of demand is $r(p) = q_p(p, \theta_2)/q_p(p, \theta_1)$ where the subscript denotes the partial derivative. In the statistical interpretation of demand minus 1 times the price derivative equals the density function. The demand slope ratio is, therefore, the likelihood ratio. Differentiating the likelihood ratio with respect to price gives

$$r'(p) = r(p)\left(-\frac{q_{pp}}{q_p}(p, \theta_1) + \frac{q_{pp}}{q_p}(p, \theta_2)\right),$$

[5] The result depends on the fact that $\ln(1 + x) - x$, which is negative, is larger in absolute value than $\ln(1 + x) - x/(1 + x)$ for positive x, so that the sum of these two expressions is negative.

[6] See Ippolito (1980) and Aguirre & Cowan (2015) for analyses of the welfare effects of discrimination with constant elasticity demands.

which is positive if direct demand curvature in market 1, $-pq_{pp}(p,\theta_1)/q_p(p,\theta_1)$, exceeds that in market 2. In statistical terms when $r'(p) \geq 0$ the monotone likelihood ratio property holds. An implication of the likelihood ratio property is that the hazard rate in market 1 is above that in market 2, which is equivalent to

$$-\frac{q_p}{q}(p,\theta_1) \geq -\frac{q_p}{q}(p,\theta_2).$$

See Wolfstetter (1999), p. 139, for a proof that the monotone likelihood ratio property implies the monotone hazard rate condition. It follows that the price elasticity of demand (which is the price times the hazard rate) is higher in market 1. The second inequality is strict if the first one is, and this is the interesting case.

Proposition 4. *Suppose direct demand is $q(p_i,\theta_i)$. If, at a given price, a change in θ_i implies direct demand curvature is higher then the price elasticity is also larger.*

Proposition 4, in its strict form, has several applications. First, it applies to the demand functions in (1) and (2), and to the constant-elasticity demands discussed in the previous sub-section. Second, suppose that the demand functions are exponential, with $q(p,\lambda_i) = e^{-\lambda_i p}$ for $\lambda_i > 0$. Direct demand curvature is $p\lambda_i$, which also is the price elasticity of demand. Third, consider demands derived from logistic distributions of valuations, where only the means of the distributions differ across markets. Demand is $q(p,\mu_i) = 1/\left(1+e^{(p-\mu_i)/b}\right)$, where μ_i is the mean of the distribution and $b > 0$. Direct demand curvature is $p(1-2q_i)/b$ and the price elasticity is $p(1-q_i)/b$. Both increase as μ_i falls. This generalizes to families of demand functions based on log-concave distributions of valuations, such as the normal, which differ only in their means.[7]

In Cowan (2016) I show that discrimination increases social welfare for demand functions derived from logistic distributions that differ only in their means. With this class of demands Varian's lower bound to the welfare change is positive, and the proof uses the fact that the weighted sum of the pass-through-adjusted margins is zero.

5 Conclusion

The demand functions in this paper are simple and show directly the linkage between direct demand curvature and social welfare. It is relatively straightforward to show that welfare is higher with discrimination when marginal cost

[7] Log-concavity of the density means that $\frac{d}{dp}\left(-\frac{q_{pp}}{q_p}\right) \geq 0$. Because demand depends on the difference between p_i and μ_i, a fall in μ_i has the same effect on curvature as a rise in the price.

is zero. The proof for positive marginal cost is much longer, which may explain why this has not been discovered previously. The main result provides a contrast to that of Pigou, and appears to capture the spirit of the analysis of Robinson.

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Appendix: Proof of Proposition 3

The demand function in equation (1) applies. Let marginal cost be $c \in [0, 1)$. The profit function is strictly concave for all prices that exceed marginal cost, and as the price rises $z_i(p_i) \equiv (p_i - c)q_i'(p_i)/\pi_i''(p_i)$ increases. These two facts allow the technique of Aguirre et al. (2010) to be used. The method is generalized here to allow for more than two markets. Suppose that in each market marginal profitability is proportional to marginal profitability at the uniform price: $\pi_i'(p_i) = (1-\lambda)\pi_i'(\bar{p})$ where $\lambda \in [0,1]$ is interpreted as the amount of discrimination. As λ increases the effect on the price is $p_i'(\lambda) = -\pi_i'(\bar{p})/\pi_i''(p_i)$, which has the same sign as $\pi_i'(\bar{p})$. The marginal effect on aggregate social welfare, W, is

$$W'(\lambda) = \sum_i (p_i - c)q_i'(p_i)p_i'(\lambda) = \sum_i z_i(p_i)(-\pi_i'(\bar{p})).$$

Social welfare is strictly concave in the amount of discrimination

$$W''(\lambda) = \sum_i z_i'(p_i)(-\pi_i'(\bar{p}))p_i'(\lambda) < 0$$

because $z_i'(p_i) > 0$, and $p_i'(\lambda)$ and $-\pi_i'(\bar{p})$ have opposite signs. It follows that there is a lower bound to the change in welfare as λ changes from 0 to 1

$$\Delta W > W'(1) = \sum_i z_i(p_i^*)(-\pi_i(\bar{p})).$$

Note that $z_i(p_i^*)$ is the pass-through-adjusted margin, so this lower bound is the weighted sum of the pass-through-adjusted margins with the weights summing to zero.

It remains to be shown that as β_i increases the discriminatory price increases and the pass-through-adjusted margin falls. Profit is $\pi_i(p_i) = (p_i - c)(1 - p_i^{\beta_i})$. The first-order condition is

$$1 - (1+\beta_i)p_i^{\beta_i} + c\beta_i p_i^{\beta_i - 1} = 0,$$

where the star superscript on the price is understood. This can be written as

$$p_i^{\beta_i}(1 + \beta_i L_i^*) = 1$$

where $L_i^* \equiv (p_i^* - c)/p_i^*$ is the Lerner Index, or the mark-up, at the discriminatory price. Taking logarithms yields

$$\beta_i \ln(p_i^*) + \ln(1 + \beta_i L_i^*) = 0.$$

The cost pass-through coefficient is $dp_i^*/dc = 1/(1 + \beta_i + (1-\beta_i)c/p_i^*)$, which is strictly below 1 as $\beta_i > 0$. Differentiating the above equation with respect to β_i gives the effect on the discriminatory price:

$$\frac{dp_i^*}{d\beta_i} = \frac{dp_i^*}{dc}\frac{p_i^*(1+\beta_i L_i^*)}{\beta_i^2}\left(\ln(1+\beta_i L_i^*) - \frac{\beta_i L_i^*}{1+\beta_i L_i^*}\right) > 0.$$

The sign obtains because the expression in large brackets is positive for the same reason that the parallel expression in the Lemma is positive. The pass-through-adjusted margin is

$$z_i(p_i^*) = \frac{p_i^* - c}{1 + \beta_i + (1 - \beta_i)c/p_i^*},$$

and

$$\frac{d}{d\beta_i} z_i(p_i^*) = \left(\frac{dp_i^*}{dc}\right)^2 \frac{p_i^*(1 + \beta_i L_i^*)}{\beta_i^2} ((\ln(1 + \beta_i L_i^*)A - \beta_i L_i^* B)$$

where $A \equiv 1 + (1 - \beta_i)\frac{c}{p_i^*} L_i^* \frac{dp_i^*}{dc}$ and $B \equiv 1 + (1 - \beta_i)\frac{c}{p_i^*} L_i^* \frac{dp_i^*}{dc} \frac{1}{1+\beta_i L_i^*}$.

Thus $dz_i(p_i^*)/d\beta_i$ has the sign of

$$\ln(1 + \beta_i L_i^*) - \beta_i L_i^* + (1 - \beta_i)\frac{c}{p_i^*} L_i^* \frac{dp_i^*}{dc}\left(\ln(1 + \beta_i L_i^*) - \frac{\beta_i L_i^*}{1 + \beta_i L_i^*}\right).$$

The Lemma implies $\ln(1 + \beta_i L_i^*) - \beta_i L_i^* < 0$. The remaining expression is negative or zero if demand is concave, i.e. $\beta_i \geq 1$, because, by the Lemma, the large-bracketed expression is positive. When $0 < \beta_i < 1$ the large-bracketed expression is multiplied by the product of four positive terms, $1 - \beta_i$, c/p_i^*, L_i^*, and dp_i^*/dc, each of which is bounded above by 1. Thus the expression to be signed is bounded above by

$$2\ln(1 + \beta_i L_i^*) - \beta_i L_i^* - \frac{\beta_i L_i^*}{1 + \beta_i L_i^*}.$$

For $x \geq 0$, define $g(x) = 2\ln(1 + x) - x - x/(1 + x)$. Note that $g(0) = 0$, and $g'(x) = -x^2/(1+x)^2 \leq 0$, with strict inequality for $x > 0$. So $g(x) < 0$ for $x > 0$. It follows that the upper bound is negative, and thus so is $dz_i(p_i^*)/d\beta_i$. The proof of Proposition 1 showed that if weak markets have higher pass-through-adjusted margins the weighted sum of the latter is positive.