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Direct welfare analysis of relative price regulation

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Abstract

The paper synthesizes and develops the welfare analysis of regulating relative prices, for example price differences, of which banning price discrimination is a special case. Welfare results are derived directly by convexity arguments using functions of welfare levels. The method is also used to obtain results about effects on consumer surplus.

Keywords: Price discrimination

JEL Classification: D42, L12.

1 Introduction

How do constraints on the *relative* prices charged by a profit-maximizing monopolist – in particular price differences – affect social welfare and consumer surplus? The literature on price discrimination address this question by comparing laissez-faire with the case where no price differences are allowed – see, for example, Varian (1985), Aguirre, Cowan and Vickers (2010) (henceforth 'ACV'), and the subsequent contributions by Cowan (2012, 2016). The present paper shows how the results from that literature, and some new ones, can be derived economically by defining market variables directly as functions of welfare or consumer surplus.[1]

The starting point for the analysis is the observation that for any form of price *difference* regulation, the monopolist will not wish to change all its prices by the same absolute amount at its (constrained) optimum. That is because price difference regulation places no constraint on parallel price shifts. The firm's marginal incentives to raise prices in

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[1]This approach is in the spirit of the 'competition in utility space' analysis of Armstrong and Vickers (2001), who model firms as supplying utility directly to consumers.

parallel therefore sum to zero across markets even when price difference constraints bind. The convexity properties of those price incentives considered as functions of *welfare* then yield direct welfare comparisons not only between uniform pricing and laissez-faire, but also between those outcomes and situations when price difference constraints bind but do not require price uniformity. The method of analysis works just as easily for n markets as for two, and covariance conditions are shown to be important.

Further results on the comparison between uniform pricing and laissez-faire are then obtained by the use of auxiliary functions that again exploit the fact that the firm's marginal incentives to change prices sum to zero with uniform pricing. Finally, the method of analysis is adapted to yield results on how relative price regulation affects consumer surplus.

2 Welfare analysis with price difference regulation

A product with constant unit cost $c > 0$ is supplied in n markets by a profit-maximizing monopolist. Demand in market i is $x_i(p_i)$, which is assumed to be a smooth function of price p_i, and inverse demand is $p_i(x_i)$. Profit from that market is $\pi_i(x_i) = (p_i(x_i) - c)x_i$. The firm's total profit is $\Pi = \sum_i \pi_i$. The firm chooses the vector of quantities $x = (x_1, ..., x_n)$ to maximize Π subject to the $(n-1)n$ price difference constraints

$$p_i(x_i) - p_j(x_j) \leq r_{ij} \tag{1}$$

for $i \neq j$, where all constants $r_{ij} \geq 0$. At most half of the constraints (1) will bind. Price discrimination is banned when all $r_{ij} = 0$. Let p^0 be the profit-maximizing uniform price in that case and assume that demand is strictly positive in all markets at price p^0. There is no binding regulatory constraint if all $r_{ij} > p_i^* - p_j^*$, where $x_i^* = \arg\max \pi_i(x_i)$ and $p_i^* \equiv p_i(x_i^*)$. To save on notation, superscripts $*$ and 0 respectively denote functions evaluated at x_i^* and x_i^0.

Define $\eta_i(x_i) \equiv -\frac{p_i(x_i)}{x_i p_i'(x_i)} > 0$ as demand elasticity in market i and $\mu_i(x_i) \equiv \frac{p_i(x_i)-c}{p_i(x_i)}$ as the mark-up. A useful term is $\xi_i(x_i) \equiv \eta_i(x_i)\mu_i(x_i)$, and let $\sigma_i(x_i) \equiv -\frac{x_i p_i''(x_i)}{p_i'(x_i)}$ be the elasticity of the slope of inverse demand, a familiar measure of (inverse) demand curvature, which is related to ξ_i by

$$x_i \xi_i' = -[1 + (1 - \sigma_i)\xi_i] \ . \tag{2}$$

It is assumed that $\sigma_i(x_i) < 2$ for all i, so profit is each market is single-peaked in price, but unlike in ACV profit need not be strictly concave in price. The very mild further assumption is made that $\frac{1}{n}\sum_i \sigma_i^0 \xi_i^0 < 2$.[2] Note that $\eta_i(x_i(p_i))\sigma_i(x_i(p_i)) = -\frac{p_i x_i''(p_i)}{x_i'(p_i)}$, which is (minus) the elasticity of the slope of demand.

Define

$$\theta_i(x_i) \equiv x_i + (p_i(x_i) - c)x_i'(p_i(x_i)) = x_i[1 - \xi_i(x_i)] \tag{3}$$

as the marginal effect on π_i of a small increase in the price of product i. Thus $\theta_i(x_i^*) = 0$ and from $\xi_i(x_i^*) = 1$ we have that $p_i^* - c = \frac{c}{\eta_i^* - 1}$. If relative price constraints bind, the vector x that solves the firm's maximization problem also satisfies

$$\textstyle\sum_i \theta_i(x_i) = 0 \tag{4}$$

because if price vector p satisfies constraints (1) then so does p plus scalar t added to each price. Let $H \equiv \{i : p_i^* > p_i^0\}$ be the set of markets in which the firm would like to charge more than the profit-maximizing uniform price, and let L be the set of other markets. For all constraints r_{ij} the firm will choose $x_i \in [x_i^*, x_i^0]$ in H-markets and $x_i \in [x_i^0, x_i^*]$ in L-markets. Thus we restrict attention to the set of quantity vectors $\mathfrak{X} \equiv \{x : x_i \in [\min(x_i^*, x_i^0), \max(x_i^*, x_i^0)]\}$ and the associated prices.

Since for all $x \in \mathfrak{X}$ we have that $p_i(x_i) \geq c$, welfare $w_i(x_i)$ in market i is a monotonically increasing function with $w_i' = p_i - c$. The inverse function $X_i(w)$ can be defined by $w_i(X_i(w)) \equiv w$. Thus define

$$\phi_i(w_i) \equiv \theta_i(X_i(w_i)) \tag{5}$$

and since

$$\theta_i'(x_i) = 2 - \sigma_i(x_i)\xi_i(x_i) \tag{6}$$

we have that

$$\phi_i'(w_i) = \theta_i'(X_i(w_i))X_i'(w_i) = \frac{2 - \sigma_i \xi_i}{p_i - c}\ . \tag{7}$$

Call market i *regular* if $\phi_i(w_i)$ is convex. This is equivalent to ACV's increasing ratio condition that the RHS term in (7), which equals $[\frac{2}{p_i - c} + \frac{x_i''(p_i)}{x_i'(p_i)}]$, is decreasing in p_i.

Assume initially that all markets are regular. Then for any welfare levels w_i^A and w_i^B we have

$$(w_i^A - w_i^B)\phi_i'(w_i^A) \geq \phi_i(w_i^A) - \phi_i(w_i^B)\ . \tag{8}$$

[2] This last condition, which ensures that $E[\phi_i'(w_i^0)] > 0$ in Proposition 1(i) below, is closely related to the second-order condition for p^0 that $\sum_i [2 - \sigma_i^0 \xi_i^0] x_i'(p^0) < 0$.

Corresponding to (4) say that a welfare vector w is *difference-compatible* if $\sum_i \phi_i(w_i) = 0$. So if w^A and w^B are difference-compatible we have from (8) that

$$\textstyle\sum_i (w_i^A - w_i^B)\phi_i'(w_i^A) \geq \sum_i [\phi_i(w_i^A) - \phi_i(w_i^B)] = 0 \ . \tag{9}$$

Special cases: From (9) we see that if $\phi_i'(w_i^A)$ is constant and positive across markets, then w^A is the difference-compatible outcome that maximizes total welfare $W \equiv \sum_i w_i$. Thus the welfare vector w^0 with uniform pricing maximizes W with linear demands (when all $\sigma_i = 0$) because $\phi_i'(w_i^0) = \frac{2}{p^0 - c}$ is constant. Indeed it is constant if demands have the form $x_i(p_i) = a_i - k_i f(p_i)$, with f the same for all i and $f' > 0$, because $\phi_i'(w_i^0) = \frac{2}{p^0-c} + \frac{f''(p^0)}{f'(p^0)}$.[3] On the other hand, if

$$\phi_i'(w_i^*) = \frac{2 - \sigma_i^*}{p_i^* - c} = (2 - \sigma_i^*)\frac{\eta_i^*}{p_i^*} = \frac{1}{c}(2 - \sigma_i^*)(\eta_i^* - 1) > 0 \tag{10}$$

is constant across i, then w_i^* maximizes W subject to (9). Following Cowan (2016), for logistic demands

$$x_i(p_i) = \frac{1}{a_i e^{b_i p_i} + k_i} \ , \tag{11}$$

where $a_i > 0$ and $b_i > 0$, we have

$$[2 - \sigma_i(x_i)]\frac{\eta_i(x_i)}{p_i(x_i)} = b_i \tag{12}$$

for all x_i, and so for x_i^* in particular. Hence $\phi_i'(w_i^*)$ is constant if all $b_i = b$, when laissez-faire is the best difference-compatible outcome.[4] These special cases can be generalized as follows, where $E[w_i] \equiv \frac{1}{n}\sum_i w_i$.

Proposition 1 *If all markets are regular, among difference-compatible outcomes total welfare (i) is maximized by uniform pricing if $\sigma_i^0\eta_i^0$ is higher in H-markets in the sense that $Cov[\sigma_i^0\eta_i^0, (w_i^0 - w_i)] \geq 0$, but (ii) is maximized by laissez-faire if $(2 - \sigma_i^*)(\eta_i^* - 1)$ is higher in H-markets in the sense that $Cov[(2 - \sigma_i^*)(\eta_i^* - 1), (w_i - w_i^*)] \geq 0$.*

Proof. (i) Under the stated condition $Cov[\phi_i'(w_i^0), (w_i - w_i^0)] \geq 0$ because $\phi_i'(w_i^0) = \frac{2}{p^0-c} - \frac{\sigma_i^0\eta_i^0}{p^0}$. So from (9)

$$\begin{aligned} 0 &\geq \textstyle\sum_i (w_i - w_i^0)\phi_i'(w_i^0) \\ &= E[w_i - w_i^0]E[\phi_i'(w_i^0)] + Cov[\phi_i'(w_i^0), (w_i - w_i^0)] \\ &\geq E[w_i - w_i^0]E[\phi_i'(w_i^0)] \ , \end{aligned}$$

[3] Regularity is the condition that this last expression be decreasing in p^0.

[4] This generalizes Cowan's (2016) Proposition 2 by allowing k_i to vary across markets.

with $E[\phi_i'(w_i^0)] > 0$, proving that uniform pricing maximizes total welfare.

(ii) In this case $Cov[\phi_i'(w_i^*), (w_i - w_i^*)] \geq 0$ from (10), so from (9)

$$\begin{aligned} 0 &\leq \textstyle\sum_i (w_i^* - w_i)\phi_i'(w_i^*) \\ &= E(w_i^* - w_i)E[\phi_i'(w_i^*)] + Cov[\phi_i'(w_i^*), (w_i^* - w_i)] \\ &\leq E(w_i^* - w_i)E[\phi_i'(w_i^*)] \ , \end{aligned}$$

proving that laissez-faire maximizes welfare. ■

Parts (i) and (ii) of Proposition 1 respectively generalize Propositions 1 and 2 of ACV to the multi-market case, without assuming concavity of profit in price, and by comparing w^0 and w^* with all difference-compatible w.

This method of analysis for price difference regulation can be extended by generalising the constraints (1) to

$$F(p_i(x_i)) - F(p_j(x_j)) \leq r_{ij} \ , \tag{13}$$

where $F(p)$ is a strictly increasing function. If all $r_{ij} = 0$, then (1) requires uniform pricing. For instance, with $F(p) = \log(p - c)$, constraints (13) take the form $\frac{p_i(x_i)-c}{p_j(x_j)-c} \leq e^{r_{ij}}$, and equi-proportionate expansions of $(p_i - c)$ are unconstrained, as in Example 4 below.

3 Welfare comparison between laissez faire and uniform pricing

Now we focus on the comparison between w^* and w^0, rather than all difference-compatible outcomes. The method of comparison uses smooth auxiliary functions $g_i(x_i)$ such that $g_i(x_i^0) = x_i^0$. From (3), (4) and the fact that all $\xi_i(x_i^*) = 1$ we have for all such auxiliary functions that

$$\textstyle\sum_i g_i(x_i^*)[1 - \xi_i(x_i^*)] = \sum_i g_i(x_i^0)[1 - \xi_i(x_i^0)] = 0 \ .$$

So by defining

$$\gamma_i(w_i(x_i)) \equiv g_i(x_i)[1 - \xi_i(x_i)] \tag{14}$$

we have that

$$\textstyle\sum_i \gamma_i(w_i^*) = \sum_i \gamma_i(w_i^0) = 0 \ , \tag{15}$$

and for suitable choices of $g_i(x_i)$ we can use further convexity arguments to sign $\sum_i (w_i^* - w_i^0)$. As to the economic interpretation of the $g_i(x_i)$ functions, note from (3) that $\gamma_i = \frac{g_i}{x_i}\theta_i$

and recall that θ_i is the marginal incentive to raise price in market i. Thus (15) says that there is no gain from increasing all prices in such a way that p_i increases locally in proportion to $\frac{g_i}{x_i}$. For some specifications the g_i functions can be related to price constraints of the form (13). In particluar, if $\frac{g_i}{x_i} = f(p_i)$ is the same function of price for all i, then $\gamma_i = f(p_i)\frac{\partial \pi_i}{\partial p_i}$, and if $\sum_i \gamma_i = 0$ so price changes proportional to $f(p_i)$ are unprofitable. With $F(p_i) \equiv \log f(p_i)$ in (13) such price changes are unconstrained, as is illustrated by Example 4 below, which has $\frac{g_i}{x_i}$ proportional to $(p_i - c)$. Examples 1 and 2 are chosen to yield the results in ACV and in Cowan (2012, 2016), while Example 3 gives a new result.

For a given g_i function, from (14) we have

$$(p_i - c)\gamma_i'(w_i) = g_i'(1 - \xi_i) + \frac{g_i}{x_i}[1 + (1 - \sigma_i)\xi_i] \tag{16}$$

and in particular

$$(p^0 - c)\gamma_i'(w_i^0) = g_i'(x_i^0)(1 - \xi_i^0) + [1 + (1 - \sigma_i^0)\xi_i^0] \ . \tag{17}$$

Note that $\xi_i^0 = \mu^0\eta_i(x_i^0) = \eta_i(x_i^0)/\eta^0$, where $\eta^0 \equiv \frac{E[x_i^0\eta_i(x_i^0)]}{E[x_i^0]}$ is the average elasticity weighted by quantities at uniform pricing. Now consider some illustrations of the method, starting with the simplest.

Example 1: Let $g_i(x_i) \equiv x_i^0$. Then from (16)

$$\gamma_i'(w_i) = \frac{x_i^0}{\pi_i}[1 + (1 - \sigma_i)\xi_i] \ . \tag{18}$$

This example is well-suited to the case of constant elasticities because $\sigma_i = 1 + \frac{1}{\eta_i}$ and so (18) implies that

$$\gamma_i'(w_i) = \frac{x_i^0}{\pi_i}(1 - \mu_i) = \frac{cx_i^0}{p_i\pi_i}$$

and in particular $\gamma_i'(w_i^0) = \frac{c}{p^0(p^0-c)}$ independently of i. (More generally demands of the form $x_i(p_i) = a_i e^{-h_i f(p_i)}$ imply that $\gamma_i'(w_i^0) = \frac{1}{p^0-c} + \frac{f''(p^0)}{f'(p^0)}$ for all i. Constant elasticities has $f(p) = \ln p$ and $h_i = \eta_i$.) And $\gamma_i(w_i)$ is convex if $p_i\pi_i$ is increasing in p_i, that is if

$$0 < \pi_i + p_i\theta_i = \pi_i[1 + \frac{1}{\mu_i} - \eta_i] \ ,$$

which always holds if all $\eta_i - 1 \leq \frac{1}{\mu^0} = \eta^0$. With convex $\gamma_i(w_i)$ we have

$$0 \geq \textstyle\sum_i(w_i^* - w_i^0)\gamma_i'(w_i^0) = \dfrac{c}{p^0(p^0 - c)} \textstyle\sum_i(w_i^* - w_i^0) \tag{19}$$

and $W^0 \geq W^*$, confirming Proposition 6(ii) of ACV.

Example 2: If $g_i(x_i) = \frac{x_i p_i'(x_i)}{p_i'(x_i^0)}$, then $g_i' = \frac{g_i}{x_i}(1-\sigma_i)$ and so from (17)

$$-p_i'(x_i^0)\gamma_i'(w_i) = \frac{2-\sigma_i}{x_i \xi_i} \ . \tag{20}$$

So if the $\gamma_i(w_i)$ are convex, then

$$\begin{aligned} 0 &= \sum_i[\gamma_i(w_i^0) - \gamma_i(w_i^*)] \leq \sum_i(w_i^0 - w_i^*)\gamma_i'(w_i^0) \\ &= \frac{1}{p^0 - c}\sum_i(w_i^0 - w_i^*)(2-\sigma_i^0) \ , \end{aligned}$$

with the inequality reversed if the $\gamma_i(w_i)$ are concave. Thus with convex γ_i, if σ_i^0 is greater in H-markets than L-markets in the sense that $Cov[\sigma_i^0, (w_i^0 - w_i^*)] \geq 0$, then $W^0 \geq W^*$. With constant σ_i we have from (20) that $\gamma_i''(w_i)$ has the sign of $1 - \sigma_i\xi_i$. Then if $\max(\sigma_i, \sigma_i\xi_i^0) \leq 1$ for all i this term is always positive, and the $\gamma_i(w_i)$ are indeed convex. (We have that $\max(\sigma_i, \sigma_i\xi_i^0) \leq 1$ if $\sigma_h \leq 1$ for $h \in H$ and $\sigma_l\frac{\eta_l^0}{\eta^0} \leq 1$ for $l \in L$.) But with concave γ_i, if σ_i^0 is smaller in H-markets than L-markets, then $W^* \geq W^0$. With constant σ_i the $\gamma_i(w_i)$ are concave if $\min(\sigma_i, \sigma_i\xi_i^0) \geq 1$ for all i. These considerations yield ACV's Proposition 5.

Relating Propositions 5 and 6(ii) of ACV to the multi-market case, these implications of Examples 1 and 2 can be summarised as follows.

Proposition 2 *(i) With constant η_i uniform pricing is better for welfare than laissez-faire if all $\eta_i - 1 \leq \eta^0$. (ii) With constant σ_i uniform pricing is (a) better for welfare than laissez-faire if σ_i is higher in H-markets in the sense that $Cov[\sigma_i, (w_i^0 - w_i^*)] \geq 0$, all $\sigma_i \leq 1$ and $\sigma_l\frac{\eta_l^0}{\eta^0} \leq 1$ for all $l \in L$, but (b) worse for welfare than laissez-faire if σ_i is lower in H-markets in the sense that $Cov[\sigma_i, (w_i^0 - w_i^*)] \leq 0$, all $\sigma_i \geq 1$ and $\sigma_h\frac{\eta_h^0}{\eta^0} \geq 1$ for all $h \in H$.*

4 Consumer surplus analysis

Similar methods to those used in sections 2 and 3 yield results on the effect of price discrimination on consumer surplus. Clearly price restrictions benefit consumers whenever they increase welfare because they reduce profit. But discrimination might be bad for consumers when positive (or ambiguous) for welfare, or good for consumers as well as for welfare. To explore these possibilities, define $s_i(x_i) \equiv w_i(x_i) - \pi_i(x_i)$ as surplus in market

i, and note that $s_i'(x_i) = -x_i p_i'(x_i) > 0$, so there exists an inverse function $\tilde{X}_i(s)$ defined by $s_i(\tilde{X}_i(s)) \equiv s$. Parallel to (5) let

$$\psi_i(s_i) \equiv \theta_i(\tilde{X}_i(s_i)) \ . \tag{21}$$

Since $w_i'(x_i)/s_i'(x_i) = \xi_i(x_i)$ we have that

$$\psi_i'(s_i(x_i)) = \frac{\theta_i'(x_i)}{s_i'(x_i)} = \phi_i'(w_i(x_i))\xi_i(x_i) = (2 - \sigma_i \xi_i)\frac{\eta_i}{p_i} \ ,$$

so $\psi_i'(s_i^*) = \phi_i'(w_i^*)$, and $\psi_i''(s_i)$ has the sign of

$$\frac{x_i \theta_i''(x_i)}{\theta_i'(x_i)} - \frac{x_i s_i''(x_i)}{s_i'(x_i)} = \frac{x_i \theta_i''(x_i)}{\theta_i'(x_i)} - [1 - \sigma_i(x_i)] \ ,$$

which has the sign of

$$\sigma_i - 2(1 - \sigma_i)(1 - \sigma_i \xi_i) - x_i \sigma_i' \xi_i \ .$$

This is ambiguous – for example, negative with linear demand, when $\sigma_i = 0$, but positive with exponential demand, when $\sigma_i = 1$. (By contrast it was natural to focus on $\phi_i(w_i)$ being convex.)

We can however obtain a natural concavity condition by way of an auxiliary function. For given $g_i(x_i)$ functions define $\beta_i(s_i)$ by

$$\beta_i(s_i(x_i)) \equiv \gamma_i(w_i(x_i)) \ ,$$

and then

$$\beta_i'(s_i(x_i)) \equiv \gamma_i'(w_i(x_i))\xi_i(x_i) \ . \tag{22}$$

With $g_i(x_i)$ as in Example 2 above, we therefore have from (20) that

$$-p_i'(x_i^0)\beta_i'(s_i(x_i)) = \frac{2 - \sigma_i}{x_i} \ , \tag{23}$$

which – equivalently to Cowan's (2012) 'passthrough assumption' – is decreasing x_i for a wide range of demand specifications. Then $\beta_i''(s_i)$ is concave, and so

$$\begin{aligned} 0 &= \textstyle\sum_i [\beta_i(s_i^0) - \beta_i(s_i^*)] \geq \sum_i (s_i^0 - s_i^*)\beta_i'(s_i^0) \\ &= \frac{1}{p^0} \textstyle\sum_i (s_i^0 - s_i^*)(2 - \sigma_i^0)\eta_i^0 \ . \end{aligned}$$

If $(2 - \sigma_i^0)\eta_i^0$ is higher in H-markets in the sense that $Cov[(2 - \sigma_i^0)\eta_i^0, (s_i^0 - s_i^*)] \geq 0$, then consumer surplus is greater with laissez-faire than uniform pricing, as in Proposition 1(i) of Cowan (2012). With logistic demands (11) and all $b_i = b$, from (12) and (23) we have $-p_i'(x_i^0)\beta_i' = -bp_i'(x_i)$, so $\beta_i'(s_i^0) = b$. Thus β_i'' has the sign of $-p_i''(x_i)$, which in turn has the sign of $(2k_i x_i - 1)$. If that is negative for the relevant range of x_i for all i then consumer surplus higher under laissez-faire than uniform pricing.[5] But the opposite holds if $x_i > \frac{1}{2k_i} > 0$ for all x_i in range.

Example 3: If $g_i(x_i) = \frac{\xi_i^0}{\xi_i(x_i)}x_i^0$, then $\beta_i(s_i(x_i)) = x_i^0\xi_i^0[\frac{1}{\xi_i(x_i)} - 1]$ so

$$\beta_i'(s_i) = -\frac{x_i^0\xi_i^0}{-x_i p_i'\xi_i^2}\xi_i' = [1 + (1-\sigma_i)\xi_i]\frac{x_i^0\xi_i^0}{\pi_i\xi_i} .$$

In particular

$$\beta_i'(s_i^0) = \frac{1 + (1-\sigma_i^0)\xi_i^0}{p^0 - c} ,$$

which equals $\frac{c}{p^0(p^0-c)}$ for all i with constant elasticities, So total surplus is greater with uniform pricing than with laissez-faire if the $\beta_i(s_i)$ functions are convex, for which the condition is that $(p_i - c)\pi_i$ is increasing in p_i, that is if

$$0 < \pi_i + (p_i - c)\theta_i = \pi_i(2 - \mu_i\eta_i) ,$$

which always holds if all $\eta_i \leq 2\eta^0$.[6]

Example 4: If $g_i(x_i) = \frac{\pi_i(x_i)}{p^0-c}$, then from (16) and (22) it follows that

$$(p^0 - c)\beta_i' = 3\xi_i - \sigma_i\xi_i^2 - 1 ,$$

and if the σ_i are constant, β_i'' has the sign of $(2\sigma_i\xi_i - 3)$ if $\xi_i' < 0$, which holds if $(\sigma_l - 1)\xi_l^0 < 1$ for all $l \in L$. Then the $\beta_i(s_i)$ are concave if $\min(\sigma_i, \sigma_i\xi_i^0) \leq \frac{3}{2}$ for all i, so

$$0 \leq \textstyle\sum_i(s_i^0 - s_i^*)\beta_i'(s_i^*) = \frac{1}{p^0 - c}\textstyle\sum_i(s_i^0 - s_i^*)(2 - \sigma_i) ,$$

and consumer surplus is greater with uniform pricing than laissez-faire if σ_i is greater in H-markets than L-markets. But if the $\beta_i(s_i)$ are convex, as with constant σ_i such that $\min(\sigma_i, \sigma_i\xi_i^0) \geq \frac{3}{2}$ for all i, then consumer surplus is higher with laissez-faire if σ_i is smaller in H-markets than L-markets. Exponential demand $x_i(p_i) = a_i e^{-b_i p_i}$, an instance of (11), has all $\sigma_i = 1$ and the $\beta_i(s_i)$ are concave – so uniform pricing is better for consumers

[5] This yields and extends Cowan's (2016) Proposition 2, which has logistic demands and $k_i = k < 0$.

[6] However, if constant elasticities differ enough, consumer surplus may be greater with discrimination than with uniform pricing – see Aguirre and Cowan (2015). For example, with two markets and elasticities of 2 and 6, consumer surplus is higher with discrimination if the more elastic market is small enough in scale.

in aggregate – unless markets differ substantially.[7] It may be noted that Example 4 for consumer surplus mirrors Example 1 for welfare, but with the critical level for σ_i being around $\frac{3}{2}$ rather than 1.

Parallel with Proposition 2 these implications of Examples 3 and 4 can be summarised as follows.

Proposition 3 *(i) With constant σ_i uniform pricing is (a) better for consumer surplus than laissez-faire if σ_i is higher in H-markets in the sense that $Cov[\sigma_i, (s_i^0 - s_i^*)] \geq 0$, all $\sigma_i \leq 1$ and $\sigma_l \frac{\eta_l^0}{\eta^0} \leq \frac{3}{2}$ for all $l \in L$, but (b) worse for consumers surplus than laissez-faire if σ_i is lower in H-markets in the sense that $Cov[\sigma_i, (s_i^0 - s_i^*)] \leq 0$, all $\sigma_i \geq 1$, $\frac{\eta_l^0}{\eta^0} < \frac{1}{\sigma_l - 1}$ for all $l \in L$ and $\sigma_h \frac{\eta_h^0}{\eta^0} \geq \frac{3}{2}$ for all $h \in H$. (ii) With constant η_i uniform pricing is better for consumer surplus than laissez-faire if all $\eta_i \leq 2\eta^0$.*

5 Conclusion

These results show the usefulness of analysing price discrimination policy, and price difference regulation more generally, by defining market variables directly as functions of levels of welfare and consumer surplus. The methods could be tightened further, albeit with some loss of simplicity. For example, a condition such as (19), which implies that price discrimination is worse for welfare than uniform pricing with constant elasticities, does not require that $\gamma_i(w_i)$ be convex for all i and all relevant w_i. First, the key property that $(w_i^* - w_i^0)\gamma_i'(w_i^0) \leq \gamma_i(w_i^*) - \gamma_i(w_i^0)$ could hold even if $\gamma_i(w_i)$ is concave for some w_i. Second, that property might fail for some i but be more than offset by the equivalent condition for other markets so that it holds in aggregate, thus satisfying (19).

The numerical findings of Aguirre and Cowan (2015) suggest however that there is not a lot of slack in the analysis, at least in terms of what may be concluded on the basis of the *shape* of demands (as reflected notably by η_i and σ_i) independently of the relative *sizes* of demands in different markets. For example, discrimination can be good for welfare with constant elasticities of 1.5 and 3 – a difference not much greater than the analytical bound of 1 for the contrary result. And it can be good for consumers in aggregate with constant elasticities of 4 and 11 – a ratio of 2.75 compared with the analytical bound of 2 for the

[7] But as Simon Cowan has noted (by email), consumer surplus can be higher with price discrimination than uniform pricing with exponential demands if markets differ enough. An example of this is with demands $x_1(p) = e^{-p}$ and $x_2(p) = e^{-p/4}$ and $c = 0$. The concavity condition fails in such cases.

opposite always to hold.

References

Aguirre, Iñaki and Simon Cowan (2015), 'Monopoly price discrimination with constant elasticity demand', *Economic Theory Bulletin*, 3, 329–340.

Aguirre, Iñaki, Simon Cowan and John Vickers (2010), 'Monopoly price discrimination and demand curvature', *American Economic Review*, 100, 1601–1615.

Armstrong, Mark and John Vickers (2001), 'Competitive price discrimination', *RAND Journal of Economics*, 32, 1-27.

Cowan, Simon (2012), 'Third-degree price discrimination and consumer surplus', *Journal of Industrial Economics*, 60, 333-345.

Cowan, Simon (2016), 'Welfare-increasing third-degree price discrimination', *RAND Journal of Economics*, 47, 326-340.

Varian, Hal (1985), 'Price discrimination and social welfare', *American Economic Review*, 75, 870-875.