ISSN 1471-0498

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

How Safe are Central Counterparties in Derivatives Markets?

Mark Paddrik and H. Peyton Young

Number 826
June, 2017

Manor Road Building, Oxford OX1 3UQ

How Safe are Central Counterparties in Derivatives Markets?*

Mark Paddrik†
H. Peyton Young‡

June 15, 2017

Abstract

We propose a general framework for estimating the likelihood of default by central counterparties (CCP) in derivatives markets. Unlike conventional stress testing approaches, which estimate the ability of a CCP to withstand nonpayment by its two largest counterparties, we study the direct and indirect effects of nonpayment by members and/or their clients through the full network of exposures. We illustrate the approach for the credit default swaps (CDS) market under shocks that are similar in magnitude to the Federal Reserve's 2015 CCAR trading book shock. The analysis indicates that the main U.S. CCP for this market (ICE Clear Credit) could be more vulnerable than conventional stress testing approaches suggest.

Keywords: Credit default swaps, central counterparties, stress testing, systemic risk, financial networks

JEL Classification Numbers: D85, G01, G17, L14

---

*We thank Randall Dodd, Marco Espinosa, Samim Ghamami, Dasol Kim, Stathis Tompaidis, and Julie Vorman for their valuable comments. Additionally we would like to thank OFR's ETL, Applications Development, Data Services, Legal, and Systems Engineering teams for collecting and organizing the data necessary to make this project possible.

†Office of Financial Research, U.S. Department of Treasury, email: Mark.Paddrik@ofr.treasury.gov.

‡Office of Financial Research, U.S. Department of Treasury, email: Hobart.Young@ofr.treasury.gov.

1 Overview

Central counterparties (CCPs) have assumed a key role in clearing over-the-counter (OTC) derivatives, thanks to regulatory reforms since the financial crisis (BCBS and IOSCO (2015)). In a centrally cleared market, parties to a derivatives contract enter into two back-to-back contracts with the CCP that offset one another. There are several important advantages to this arrangement: it creates greater transparency and standardization of contracts, it offers greater potential for the netting of positions, and it shortens the length of intermediation chains, which in principle can reduce contagion (Cont and Minca (2016)). A significant disadvantage is that it increases systemic vulnerability by introducing a critical node whose failure would have widespread consequences (Yellen (2013)). It is therefore crucial to understand whether CCPs can withstand large and sudden shocks to asset values, such as occurred in the crisis of 2007-09 and in the European debt crisis of 2011-12.

The conventional approach to stress testing CCPs is to examine whether they have sufficient funds on hand – in the form of initial margins and default fund contributions – to cover payment delinquencies if their two largest counterparties should fail to meet clearing obligations (CFTC (2016)). Several recent papers have argued that this standard is inadequate, because it considers only the direct impact of the two members' failure to pay, and does not take into account network spillover and contagion effects that can amplify the initial payment shortfalls (Nahai-Williamson et al. (2013), Poce et al. (2016)).

We examine this issue for the US market in credit default swaps (CDS) and its principal central counterparty, ICE Clear Credit. Although the paper is in the same spirit as Nahai-Williamson et al. (2013) and Poce et al. (2016), our approach differs in several key respects. First, our access to Depository Trust & Clearing Corporation data means that we have a nearly complete picture of the network of CDS exposures at different points in time. Indeed, we know the details of all contractual positions in which the reference entity and/or one of the counterparties is U.S.-based. Second, we model contagion using a variant of the Eisenberg-Noe model (2001) that incorporates behavioral responses to balance sheet stress that can be institution specific. We then estimate the total payment deficiencies that would result from a given financial shock to the system. In particular we consider the impact of shocks that are similar in magnitude to the Federal Reserve's

2015 Comprehensive Capital Analysis and Review (CCAR) shock, which was specifically designed to subject the financial markets to a severe but plausible market stress. Such a shock triggers a sudden drop in the value of credit instruments, which translates into large and sudden variation margin payments on CDS contracts. Firms that are large net sellers of protection may not be able to meet these variation margin payments, which puts increased stress on their counterparties and can lead to a systemwide cascade of payment delinquencies.[1]

When we take account of these network spillover effects we find that the CCP for this market is potentially more vulnerable to default than conventional stress tests would suggest. Indeed, under plausible estimates of firms' liquidity buffers, a significant number of member firms may be unable to meet their variation margin payment obligations, which could lead the CCP to default. This default risk can be mitigated by increasing the size of the default fund, charging higher initial margins, requiring greater liquidity by participants in the market, or a combination of these measures.

The plan of the paper is as follows. In the next section we summarize the protections that the CCP has in place to deal with defaults by its members (the default waterfall). Section 3 discusses the nature of the DTCC data and how we use it to derive variation margin payment demands and failure probabilities under the CCAR shock. Section 4 analyzes the Cover-2 standard in the context of the CCAR shock. Although this analysis suggests that the CCP's default fund is adequate to cover the defaults of its two largest members, we show that a different conclusion emerges when we take into account network contagion. In Section 5 we introduce the contagion model, which traces how payment delinquencies by some firms can escalate as they cascade through the network of CDS exposures. A novel feature of this model is the treatment of stress transmission, which depends on firms' liquidity buffers (which we treat as random variables) and on their risk management policies. We demonstrate the sensitivity of the model to different values for the shock and stress transmission parameters in Section 6.

Section 7 develops a probabilistic model of firm failure rates that takes into account positive correlation in the failure probabilities. The model is very general and allows us to estimate the

[1]Poce et al. (2016) study the Italian fixed income market instead of the derivatives market. They apply an exogenous shock to firms' equity, estimate the impact on the assets of their counterparties using a Merton model, and then examine the impact on the CCP for the market in Italian government bonds (Cassa di Compensazione e Garanzia).Unlike the present study they do not have direct knowledge of firms' network exposures, but must impute them. As in our study, however, they find that network contagion effects are substantial and imply a greater risk of CCP default than does the conventional Cover-2 standard.

probability of a CCP failure *relative* to the probability of a member's failure while making minimal assumptions about the degree of correlation among member failure rates. We use this model to estimate the probability that a CCP would default during a stress of similar magnitude to the CCAR shock. This estimate takes account of two distinct effects. First, the variation margin (VM) payments are sudden and large, which can lead to payment delinquencies (or default) due to *liquidity* constraints. Second, one or more parent firms may fail due to *insolvency*, which would lead them to default on their VM payments. We estimate the impact on a CCP of these two effects in combination. Our conclusion is that, in a severe financial crisis of a comparable magnitude to the CCAR shock, the CCP could be significantly more likely to fail than a typical member.

2 The CCP waterfall

The major central counterparty for the CDS market in the United States is ICE Clear Credit, which is a privately held, for-profit company (OFR (2017)). It cleared more than 97 percent of the notional value of CDS contracts in the United States, as of the date of our study (October, 2014). The only other central counterparty of any consequence in this market is CME Clearing, which in 2014 cleared less than 3 percent of the contracts. In this paper we focus solely on ICE Clear Credit, which we shall refer to simply as "the" CCP for this market.

As of October 3, 2014, 30 member firms were empowered (though not required) to clear their CDS contracts through the CCP (see Table 1). Contracts with nonmembers are permitted, but in such cases there must be a member who acts as intermediary and fully guarantees all payments due from the nonmember to the CCP. Members' balance sheets are subject to scrutiny by the CCP, they must post initial margin against their contracts according to rigorous criteria established by the CCP, and they must contribute to a common guarantee fund that can be drawn on if some members default on their payments. Indeed, there is a whole series of procedures and safeguards designed to protect the CCP in case one or more members default. This waterfall structure is outlined in Table 2.

Initial margins posted by each member are held in a segregated account at the CCP and can only be used to cover losses generated by that member should it default. Nonmembers also post initial margin with the CCP, and any losses they generate (including those in excess of the initial

Table 1: Members of ICE Clear Credit as of December 2014.

ICE Members			
1.a	Bank of America, N.A.	8.a	HSBC Bank USA, N.A.
.b	Merrill Lynch, Pierce, Fenner & Smith Inc.	.b	HSBC Bank plc
.c	Merrill Lynch International	.c	HSBC Securities (USA) Inc.
2.a	Barclays Bank PLC	9.a	JPMorgan Chase Bank, National Association
.b	Barclays Capital Inc.	.b	J.P. Morgan Securities LLC
3.a	BNP Paribas	10.a	Morgan Stanley Capital Services LLC
.b	BNP Paribas Securities Corp.	.b	Morgan Stanley & Co. LLC
4.a	Citibank N.A.	11.a	Nomura International PLC
.b	Citigroup Global Markets Inc.	.b	Nomura Securities International, Inc.
5.a	Credit Suisse International	12.a	Société Générale
.b	Credit Suisse Securities (USA) LLC	.b	SG Americas Securities, LLC
6.a	Deutsche Bank AG, London Branch	13.a	The Bank of Nova Scotia
.b	Deutsche Bank Securities Inc.	14.a	UBS AG, London Branch
7.a	Goldman, Sachs & Co.	.b	UBS Securities LLC
.b	Goldman Sachs International	15.a	Wells Fargo Securities, LLC

Note: Members with the same numeric value belong to the same holding company and will be treated as a single defaulting firm in our data set.
Source: ICE Clear Credit

Table 2: Principal Elements of the Waterfall Structure of ICE Clear Credit as of December 2014.

Tranche	Total Amount
Initial Margins	$15 billion
Guarantee Fund	$1.65 billion
CCP Capital	$500 million
Member Assessments	Up to 3 times nondefaulting members' GF contributions

Note: Initial margins and the guarantee fund are made up of U.S. Treasuries and cash (USD, CAD, EUR, GBP, JPY).
Source: ICE Clear Credit (2016).

margin) are supposed to be fully covered by the member who acts as guarantor. The guarantee fund (GF) is funded solely by the members and is held in a common account to cover losses that exceed initial margins in the segregated accounts.

If a member defaults, the CCP auctions the member's portfolio of contracts, or it may transfer the contracts to non-defaulting members at mutually agreed prices. The initial margin (IM) posted by the defaulting member is applied to any losses that result from this process. In particular the

IM is applied to the delinquent VM payments plus any further losses that may result from the auction or transfer process. (The latter may take several days to complete and will typically occur in highly stressed market conditions, hence the losses incurred in this novation process may be substantial.) To the extent that the IM is insufficient to cover the losses, the CCP draws on the guarantee fund. If this also proves insufficient, it taps the shareholders' paid-in capital, which as Table 2 shows is very small relative to the other parts of the waterfall. If all of these sources are still insufficient, the CCP is empowered to assess the non-defaulting members by up to three times the original amount they contributed to the GF. Although this assessment power would appear to offer a substantial line of defense, in practice it is not very helpful. The difficulty is that when the GF is exhausted, many of the members will already be in default and unable to pay the assessments.

3 The DTCC data and the CCAR shock

We conduct our analysis using detailed data provided to the Office of Financial Research (OFR) by the Depository Trust & Clearing Corporation (DTCC). The data include all CDS transactions in which at least one of the counterparties or the reference entity is a U.S. entity. We have a detailed picture of counterparty exposures for a large segment of the CDS market, including exposures between banks, dealers, hedge funds, asset managers, and insurance companies. We can apply a hypothetical credit shock and compute the value of the payment and premium legs of each CDS position as a function of spread, duration, and underlying reference entity.[2]

We focus on the change in value of each contract, and the resulting VM payment owed to each counterparty, under of the Federal Reserve's 2015 Comprehensive Capital Analysis and Review (CCAR) shock. This shock was designed to test the robustness of the financial system under a large and sudden change in asset values. The date of the shock was October 3, 2014. The shock causes a sudden decrease in the value of corporate and sovereign debt instruments, which results in large and sudden demands for VM on CDS contracts.

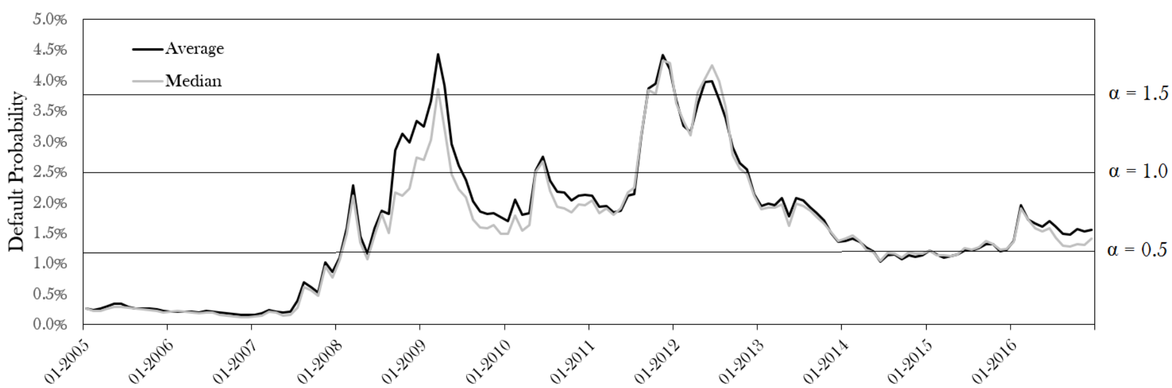
The shock is assumed to be large but not implausible. For example, Moody's estimates that a shock of this magnitude would occur with probability 4 percent per annum, that is, about once every 25 years (Zandi (2016)). The question this paper examines is how likely it is that the CCP

[2]For more detail on the methodology underpinning these computations see Paddrik et al. (2016).

could withstand a shock of approximately this magnitude. Specifically, we study the potential resilience of the CCP under shocks ranging from 0.5 to 1.5 times the magnitude of the CCAR shock.

The first step is to estimate the probability that each members would fail under the CCAR shock. We assume that subsidiaries of the same parent firm are likely to fail if and only if the parent fails so we group the members at the bank holding company (BHC) level and view these 15 firms as the entities that are subject to failure.[3] Such a shock implies a widening of credit spreads on the BHCs, from which we can infer the annual default probabilities using the methodology described in Luo (2005). On the target date of the shock the implied default probabilities among the 15 member BHCs ranged from 1.7 percent to 3.5 percent, with an average of 2.5 percent per annum. These numbers are comparable to the default probabilities implied by CDS spreads during the financial crisis of 2007-09, as shown in Figure 1.

Figure 1: Annual Default Probabilities Implied by CDS Spreads for the 15 BHCs.



Source: Authors' calculations using data provided by Markit Group Ltd.

4 Conventional risk analysis for the CCP

The usual way of evaluating potential risk to the CCP is to determine whether it has enough cash or highly liquid assets in its default fund to cover its obligations when two of its members default simultaneously (BCBS and IOSCO (2015)). This 'Cover 2' standard is typically applied to

[3]This approach is consistent with CFTC Regulation 39.33(a) on the implementation of the Cover 2 standard, which assumes that the two largest BHCs fail.

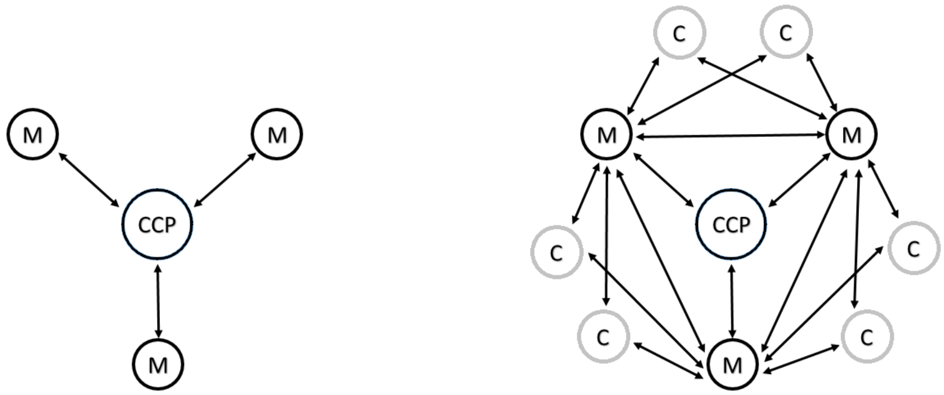
a scenario where the two defaulting members are assumed to be those with the largest net VM obligations to the CCP.[4]

Under the CCAR shock there are eight members that have non-negligible obligations to the CCP. Assuming that any one of them were to default, the IM collected from that member would be sufficient to cover the shortfall except in one case in which the default fund would be more than adequate to absorb the remaining shortfall. (It will be recalled that the default fund as of the CCAR shock date was about \$1.65 billion.) The same conclusion holds if the two largest members default simultaneously. It would therefore appear that the CCP is well-protected against defaults by its members even in a highly stressed environment.

We assert that this conclusion is overly optimistic, because it does not account for the amplification that can occur through network contagion. The left panel of Figure 2 depicts how conventional stress testing limits the channels of stress to the direct impact of a shock on the CCP. The right panel illustrates how stress can become amplified through the complete network of exposures. We shall show that when two members fail simultaneously and network effects are taken into account, there is a non-negligible probability that the CCP's default fund will be insufficient to cover delinquent payments by the members. We also argue that the CCP's ability to tap its members for additional assessments will be severely limited for two reasons. First, the funds will be needed in a very short time period (typically a few hours) and the assessments may be contested. Second, many of the members will already be under severe stress and unable to pay the additional assessments.

[4]A given member will owe VM to the CCP on some contracts and possibly be owed VM by the CCP on other contracts.

Figure 2: Network Depiction of CCP Stress Testing



(a) Limited **(b)** Complete

Note: M stands for member and C stands for client in the network relationships.
Source: Authors' analysis.

5 The network contagion model

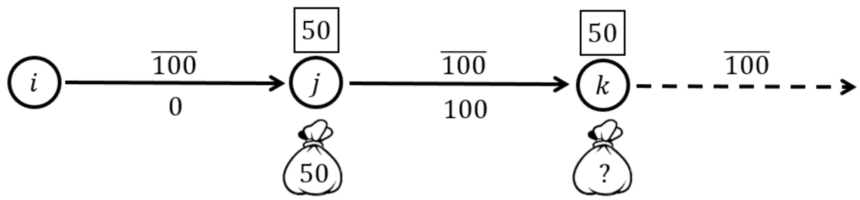
The network contagion model is a variant of Glasserman and Young (2015) that builds on the benchmark model of Eisenberg and Noe (2001).[5] The key contribution of Eisenberg-Noe is to show how to define a consistent set of payments when firms have credit obligations to one another through interlocking balance sheets. If the assets of some firms suffer an exogenous shock, the Eisenberg-Noe framework allows one to compute the extent to which the initial loss in asset values cascades through the system, possibly leading to further defaults.

In the present context, the set-up is somewhat different: an exogenous shock to credit instruments determines the payment obligations (VM payments) between firms on their CDS contracts. These payments must be made within a very short time frame – typically just a few hours. If the VM owed by a given firm exceeds the amount it is owed, the firm experiences short-term stress. This stress can be relieved by drawing on cash and cash equivalents held by the parent institution, but if the stress is large, these funds may be inadequate. In that case, the firm may either delay payments, make some payments in illiquid collateral instead of in cash, or default completely. Any of these responses will exacerbate the stress on its counterparties, leading to systemwide contagion.

To illustrate, consider a hypothetical situation involving three firms (i, j, k) as shown in Figure

[5]A similar model is used in Paddrik, Rajan and Young (2016) to analyze the extent to which the CCP contributes to network contagion.

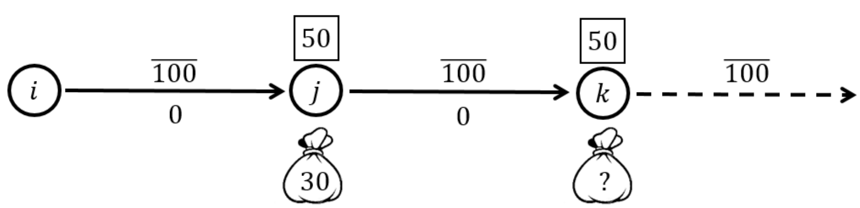
Figure 3: Payment Chain



Source: Authors' analysis.

3. Firm i owes 100 to firm j, which owes 100 to firm k, shown above the arrows. We suppose that these VM payments are triggered by a sudden exogenous shock and are due within a few hours. Suppose that i defaults completely, meaning the realized payment, shown below the arrow, is zero. Then j seizes the initial margin it collected from i (50 in the square box), but this is not enough to cover its obligations to k, which are due immediately. Hence j dips into the firm's treasury (50 in the safe bag) and meets its payment of 100. It could happen, however, that the treasury only contains 30 in liquid assets, as shown in Figure 4. In this case, j would default in its payment to k, which could cause k to default to its counterparties, depending on the amount in k's treasury.

Figure 4: Payment Deficit Contagion



Source: Authors' analysis.

This example shows that the transmission of payment shortfalls is subject to considerable uncertainty. It depends on the amount of cash available in a firm's treasury not claimed by other sources, its non-cash assets on hand, and its relationships with its counterparties. In our network model, we shall treat these factors as random variables. The approach differs from models based on Eisenberg and Noe (2001), which treat default as a deterministic event that is triggered when the default boundary is breached.

We now describe the network model in full. Given a shock to the reference entities on which CDS contracts are written, we shall represent the induced VM payment obligations by a matrix $[\bar{p}_{ij}]$, where $\bar{p}_{ij}$ is the *net* amount of VM owed by node i to node j in the aftermath of the shock. (Thus, not both $\bar{p}_{ij}$ and $\bar{p}_{ji}$ are positive because they are bilaterally netted.) Let $\bar{p}_i = \sum_{j \neq i} \bar{p}_{ij}$ be the total payment obligation of i to all other nodes. We shall restrict attention to the nodes i such that $\bar{p}_i > 0$; the others represent firms that are solely buyers of protection and have no VM obligations. Let $i = 0, 1, 2, ..., n$ index the nodes with positive payment obligations and let "0" represent the CCP.

The *relative liability* of node i to node j is

$$a_{ij} = \bar{p}_{ij} / \bar{p}_i. \tag{1}$$

The *relative liability matrix* $A = (a_{ij})$ is row substochastic, that is for every i, $\sum_{j \neq i} a_{ij} \leq 1$.

For each node i, let c_{ki} denote the amount of initial margin i collects from counterparty k. The purpose of the IM is to cover possible payment delinquencies. In particular, if counterparty k fails to pay VM to i in a timely manner, the position may be closed out and the IM will be applied to any losses that are incurred between the time of the counterparty's default and the time it takes to close out the position. Alternatively, i may accept partial payment by k and not close out the position, but seize the IM as security until the balance is paid. (Of course this is risky because the value of the contract to i might deteriorate further in the interim.)

Let $p_{ki} \leq \bar{p}_{ki}$ denote the realized current payment from k to i. If $p_{ki} < \bar{p}_{ki}$ the difference will be made up out of the initial margin sitting in k's account at firm i provided $\bar{p}_{ki} - p_{ki} \leq c_{ki}$. If $\bar{p}_{ik} - p_{ki} > c_{ki}$, then the difference $\bar{p}_{ik} - (p_{ki} + c_{ki})$ must be borne by i. We define the *stress* at i, s_i, to be the amount by which i's payment obligations exceed the incoming payments from i's counterparties buttressed by the initial margins, that is,

$$s_i = \left[\sum_{k \neq i} \bar{p}_{ik} - \sum_{k \neq i} ((p_{ki} + c_{ki}) \wedge \bar{p}_{ki}) \right]_+ .^6 \tag{2}$$

Note that when all of i's counterparties pay in full, that is $p_{ki} = \bar{p}_{ki}$ for all k, then there is no

[6] In general, $x \wedge y$ denotes the minimum of two real numbers x and y.

stress at i ($s_i = 0$).

As previously noted, firms will react to stress in ways that depend on such factors as their liquidity buffers, non-CDS positions, and general risk management policies. We do not have enough information to model these factors explicitly. Instead, we shall adopt a reduced-form approach in which we posit a response function for each firm i that maps the amount of stress, s_i, to an *expected deficiency* in payments

$$d_i = \bar{p}_i - p_i = f_i(s_i). \tag{3}$$

It is natural to assume that $f_i(s_i)$ is monotone increasing in s_i, that is, the greater the stress the greater the expected deficiency in payments. Although f_i may not be known precisely, we can sometimes bound it from below a truncated linear function:

$$d_i = f_i(s_i) \geq \tau_i s_i \wedge \bar{p}_i, \quad \tau_i \geq 0. \tag{4}$$

The scalar τ_i is a *transmission factor*. When τ_i is small the firm is able to absorb most of the stress, for example by drawing on its own cash reserves. The larger the value of τ_i, the more the stress is passed on to i's counterparties in the form of reduced payments.

The linear form can be used to place a bound on various responses to stress. To illustrate, let b_i denote the liquid reserves available at firm i to handle VM payment obligations. Let us view b_i as the realization of a random variable B_i with density $g_i(b_i)$ on some interval of the nonnegative real numbers. This random aspect of b_i may reflect unobserved heterogeneity in the firm's risk management practices and fluctuations in the quantity of cash reserves available for financing CDS operations.

Assume that i defaults if its reserves, together with incoming payments and posted IM, are insufficient to cover its obligations, that is, $b_i < s_i$. The *expected deficiency* in i's VM payment is

$$d_i = \bar{p}_i \, \mathrm{P}(b_i < s_i). \tag{5}$$

To estimate the quantity $\mathrm{P}(b_i < s_i)$, note that the maximum demand on i's reserves equals i's total obligations $\bar{p}_i$, hence it is reasonable to assume that the support of B_i is $[0, \bar{p}_i]$. Second, let us suppose that $g_i(b_i)$ is nonincreasing, that is smaller reserves are at least as likely as larger reserves. Then for every realized level of stress s_i,

$$\mathrm{P}(b_i < s_i) \geq s_i/\bar{p}_i. \tag{6}$$

Together (5) and (6) imply

$$d_i \geq s_i, \tag{7}$$

that is, the transmission factor is at least 1.

There may be some situations in which i's counterparties are willing to accept partial payment, or payments in illiquid collateral whose cash value is less than $\bar{p}_i$. Let $r(\bar{p}_i)$ be the expected VM payment conditional on i's default, which we can write as

$$d_i = (\bar{p}_i - r(\bar{p}_i))\, \mathrm{P}(b_i < s_i). \tag{8}$$

The value of $r(\bar{p}_i)$ will depend on various factors that are specific to firm i, and also on i's relationships with its counterparties. A fairly conservative assumption is that $r(\bar{p}_i) \leq \bar{p}_i/2$, in which case (6) and (8) imply that

$$d_i \geq s_i/2. \tag{9}$$

This suggests that values in the interval $[1/2, 1]$ are plausible lower bounds on the transmission factors τ_i. Note, however, that values larger than 1 can result if a firm engages in precautionary hoarding, such as delaying payments to counterparties to preserve its liquidity. In what follows, we shall therefore consider values of τ_i that lie in the interval $[0.5, 1.5]$.[7]

In the case of the CCP we have additional information that allows us to estimate the transmission factor more precisely. Liquid reserves are held in a *guarantee fund* (GF), which is available to cover residual losses when the members' initial margins are depleted. Let c_{k0} be the initial margin collected by the CCP from each member $k \neq 0$, and let γ_0 be the total amount in the CCP's

[7] Similarly, Poce et al. (2016) posit a liquidity parameter $\rho \in [0, 1]$ that determines how much a firm can offset stress on its balance sheet by drawing on liquid reserves. They assume a value of ρ that corresponds to $\tau = 0.60$ in our model.

guarantee fund. Then the difference between the CCP's VM obligations and the resources it has to meet them is

$$s_0 = \left[\sum_{k \neq 0} \bar{p}_{0k} - \sum_{k \neq 0} ((p_{k0} + c_{k0}) \wedge \bar{p}_{k0}) - \gamma_0 \right]_+ , \tag{10}$$

where k ranges over the CCP's members.

In theory, the CCP can draw on additional resources to cover the amount s_0, namely members' paid-in capital and additional member assessments. In practice, however, the capital is negligibly small (about \$50 million compared to $\gamma_0 = \$1.6$ billion. It is unlikely that assessments could raise much cash on very short notice, especially because many of the members will already be under severe stress and their liquid reserves used up. We shall assume that $\tau_0 = 1$, that is, the CCP transmits any residual balance sheet stress directly to its members by applying a haircut to the net amount it owes them (a practice known as variation margin gains haircutting). It follows that the payment from the CCP to each counterparty j is

$$p_{0j} = [\bar{p}_{0j} - a_{0j} s_0]_+ . \tag{11}$$

Given any vector $p \in \mathbb{R}^{2n+2}$ such that $0 \leq p_{ij} \leq \bar{p}_{ij}$ for all $0 \leq i, j \leq n$, let $\Phi(p)$ be the mapping defined by expressions 2 - 11, that is,

$$\forall i \neq 0, \quad \Phi(p)_{ij} = \left[\bar{p}_{ij} - \tau_i a_{ij} \left[\sum_{k \neq i} \bar{p}_{ik} - \sum_{k \neq i} ((p_{ki} + c_{ki}) \wedge \bar{p}_{ki}) \right]_+ \right]_+ , \tag{12}$$

$$\Phi(p)_{0j} = \left[\bar{p}_{0j} - a_{0j} \left[\sum_{k \neq 0} \bar{p}_{0k} - \sum_{k \neq 0} ((p_{k0} + c_{k0}) \wedge \bar{p}_{k0}) - \gamma_0 \right]_+ \right]_+ . \tag{13}$$

Φ is monotone and bounded, hence by Tarski's Theorem it has at least one fixed point (Tarski et al. (1955)). In the empirical applications we shall specify values for the parameters $\tau_0, \tau_1 ..., \tau_n$, and then recursively compute the greatest fixed point of this system by taking the limit of the sequence $p^1 = \Phi(\bar{p}), p^2 = \Phi(p^1), ...$

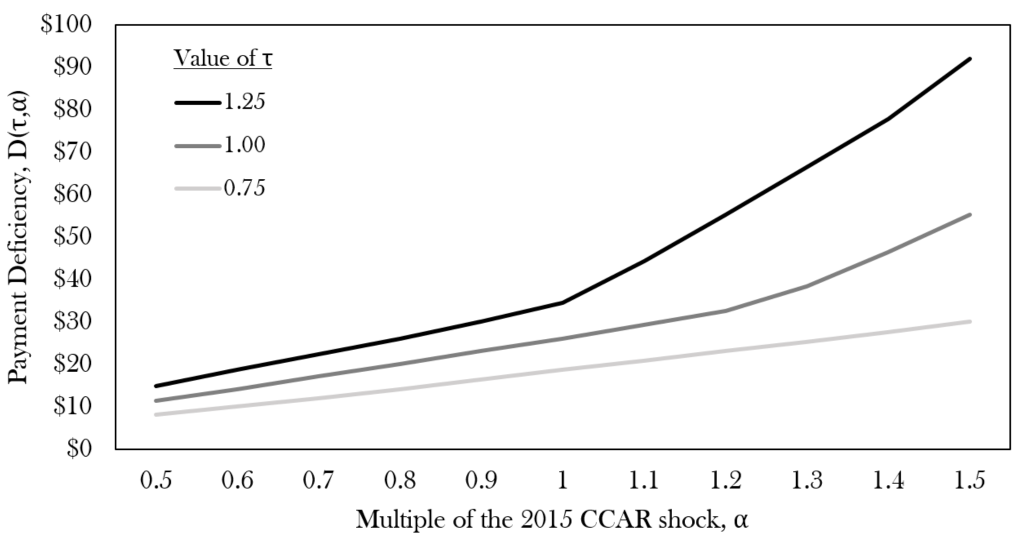
6 Sensitivity analysis

We now apply this framework to evaluate the potential amount of contagion in the CDS market that would be produced by a shock over a range of values for τ and α. We define the total *impact* of the shock to be the total deficiency in VM payments summed over all directed edges in the network.[8] In our notation the total deficiency can be expressed as follows

$$D = D(\tau, \alpha) = \sum_i d_i, \tag{14}$$

where $d_i = \bar{p}_i - p_i$ and $(p_0, ..., p_n)$ is the equilibrium payment vector representing the greatest fixed point of the mapping Φ. For simplicity, let us assume that $\tau_1 = \tau_2 = ... = \tau_n = \tau$ for some scalar $\tau \geq 0$ and $\tau_0 = 1$ for the CCP. For various values of τ and α we compute the payment deficiency $D = D(\tau, \alpha)$; the results are shown in Figure 5.

Figure 5: Payment Deficiency, $D(\tau, \alpha)$



Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit Group Ltd.

Figure 5 shows that the total deficiency $D(\tau, \alpha)$ is convex and increasing in both τ and α. For values of α greater than 1, the shock becomes large enough to overwhelm both the IM posted by

[8] A deficiency in payment from a given counterparty i to another counterparty j leads j to tap the IM (if any) that it collected from i to deal with such contingencies. This represents a loss to firm i.

members of the CCP and the guarantee fund held by the CCP. The ensuing default by the CCP leads to large payment deficiencies to its members that cascade through the system.

The application of a common value τ to all nodes in the system is purely for illustrative purposes. In reality, the values τ_i will depend on the risk management policies and liquidity buffers of particular firms. We do not have enough information to pin down these values precisely but the curves in Figure 5 give some idea of the magnitude of the impact for different levels of τ and α.

7 Estimating the probability of CCP failure

We now apply our model to the situation where VM payments are determined by a shock, and one or more members fail completely due to other (unmodelled) stresses on their balance sheets. The magnitude of the shocks are assumed to be similar to the 2015 CCAR shock and are fairly severe but not implausible.

The first step is to specify values of the transmission parameters τ_i. As previously noted these values will depend on characteristics of individual firms that are mostly unobservable. For illustrative purposes we shall assume they are all equal to some target value τ that lies in the interval $[0.5, 1.5]$. Fix a value for τ and assume that a subset of k members of the CCP fail simultaneously, where $0 \leq k \leq 15$. The failed members cannot meet any of their VM obligations, which are determined by a shock. We shall explore values of α in the range $[0.5, 1.5]$, where $\alpha = 1$ corresponds to the 2015 CCAR shock itself.[9]

Given values α and τ, and given a specific subset of k members that fail, one computes the greatest fixed point of the mapping Φ in (12) - (14) to find the equilibrium payments. The CCP *fails* if in equilibrium it has a deficit, because in this event it has exhausted the stock of IM posted by defaulting members and also the guarantee fund.[10]

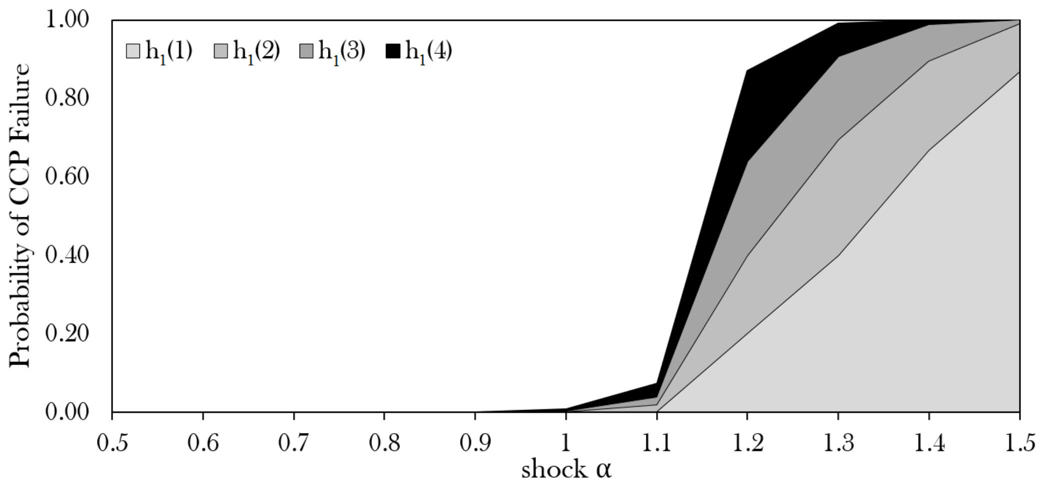
The next step is to determine how often the CCP fails, that is for a given integer k, $1 \leq k \leq 15$, what proportion of draws of k members from 15 lead to the CCP's failure? Denote this proportion by $h_{\alpha,\tau}(k)$. This is the *conditional probability* that the CCP fails given that exactly k members fail

[9] As remarked earlier, Zandi (2016) estimates that a shock at least as large as the 2015 CCAR shock has a probability of about 4 percent. Assuming the shocks are normally distributed, a shock that is 1.5 times as large would have a probability of about 0.7 percent.

[10] As mentioned earlier, the CCP can draw on two further sources of funds to stay afloat, namely the CCP's capital and assessment powers. However the first is of negligible size (see Table 2) and the second has limited scope because if the CCP fails, most of its members will already be in default.

for exogenous reasons, the shock is of size α, and the transmission factor for all firms equals τ.

Figure 6: Probability of CCP Default when $\tau = 1$.



Note: Probability of the CCP suffering a default given one, two, three or four BHC members fail, assuming the transmission factor $\tau = 1$.
Source: Authors' calculations using data provided by Depository Trust & Clearing Corporation and Markit Group Ltd.

The final step is to estimate the probability that k members would fail simultaneously in a period of severe financial stress. This is difficult to do with any precision because severe financial crises are very infrequent. One of the few attempts to estimate these probabilities was done by Giglio (2011). Based on historical CDS spreads, he estimates the joint failure probabilities for a group of 15 BHCs, using the fact that some BHCs wrote CDS contracts on the failure of others in the group. His estimates suggest that there was significant positive correlation in the market-implied default rates among members of the group.

Given the considerable uncertainty surrounding these estimates, however, we adopt a different and more general estimation strategy. Namely, we shall show how to bound the probability that the CCP defaults *relative* to the probability that a typical or average member defaults under minimal assumptions about the degree of correlation in the default rates. Consider a set of n member firms, and let Q_k denote the probability that exactly k of them fail in a period of stress. For given values of α and τ, let $h_{\alpha,\tau}(k)$ be the conditional probability that the CCP fails given that exactly k of its n members fail. The average probability that a given member fails can be written

$$p = \sum_{k=0}^{n} kQ_k/n. \tag{15}$$

The probability that the CCP fails is

$$q = \sum_{k=0}^{n} h_{\alpha,\tau}(k)Q_k. \tag{16}$$

We wish to estimate the ratio q/p; in particular we shall bound it from above and below under mild restrictions on the probabilities Q_k. Specifically let us assume that they are nonincreasing in k ($Q_1 \geq Q_2 \geq ... \geq Q_n$), and for sufficiently large k the Q_k's are close to zero.[11]

To bound q/p from below we solve the optimization problem

$$\min q/p = \frac{n\sum_{k=0}^{n} h_{\alpha,\tau}(k)Q_k}{\sum_{k=0}^{n} kQ_k}, \tag{17}$$

subject to

$$\begin{aligned} 1 \geq Q_0 \geq Q_1 \geq ... \geq Q_n \geq 0 \\ \forall k \geq \bar{k},\ Q_k = 0 \end{aligned} \tag{18}$$

An upper bound on q/p is found by solving the corresponding maximum problem.

We estimate the conditional failure probabilities $h_{\alpha,\tau}(k)$ by successively choosing singles, pairs, triples, and quadruples at random from the 15 BHC members and computing the proportion of cases in which the CCP fails. The results are shown in Figure 6 for the value $\tau = 1$. To illustrate the method for estimating q/p, consider the case $\alpha = 1.25$. The conditional failure probabilities are

$$h_{1.25,1}(0) = 0.00, \quad h_{1.25,1}(1) = 0.27, \quad h_{1.25,1}(2) = 0.70, \quad h_{1.25,1}(3) = 0.91, \quad h_{1.25,1}(4) = 0.99. \tag{19}$$

Let us further assume that $Q_k = 0$ for all $k \geq \bar{k} = 4$. (This is a conservative assumption; in fact the U.S. authorities did not allow more than two large institutions to fail during the recent

[11] These assumptions are consistent with Giglio's estimates. His data suggest that during the 2007-09 crisis, Q_2 was almost an order of magnitude smaller than Q_1, and Q_3 was about half the size of Q_2. (These estimates are based on Figure 10 in Giglio (2011) over the period of July 2008 through June 2009.)

financial crisis.)[12] By solving the corresponding optimization problems with n = 15 we find that q/p satisfies the bounds

$$4.05 \leq q/p \leq 4.85. \tag{20}$$

These bounds are not particularly sensitive to the cut-off value $\bar{k}$. For example, suppose that $Q_k = 0$ for all $k \geq 5$. It can be shown that in this case q/p satisfies precisely the same bounds as in (20). If we assume that $Q_k = 0$ for all $k \geq 6$, we obtain the bounds $3.86 \leq q/p \leq 4.85$. The implication is that, under a shock that is somewhat larger than the 2015 CCAR shock and assuming that the transmission factor $\tau = 1$, the CCP would be about four to five times more likely to fail than a typical member.

Table 3 shows the estimated bounds for q/p under different combinations of τ and α, assuming that $Q_k = 0$ for all $k \geq 4$. Note that when either τ or α is 0.75 or less, the CCP is unlikely to fail. However if the shock is somewhat larger than the CCP shock ($\alpha \geq 1.25$) and if $\tau \geq 1.25$, the CCP is at considerably greater risk of failure than is the average member firm.

Table 3: Estimated Bounds of q/p

		α				
		0.50	**0.75**	**1.00**	**1.25**	**1.50**
τ	**0.50**	0.0	0.0	0.0	0.0	0.0
	0.75	0.0	0.0	0.0	0.0	0.0-0.2
	1.00	0.0	0.0	0.0	4.1-4.9	7.1-13
	1.25	0.0	0.0	1.0-1.3	7.5-15	7.5-15
	1.50	0.0	0.0-0.2	7.5-15	7.5-15	7.5-15

8 Conclusion

We have proposed a general framework for assessing the ability of a CCP to withstand a severe credit shock. The framework differs from conventional stress testing of CCPs in several key respects. First, we consider the possible impact of one or more members failing, not just the two members with the largest net exposures to the CCP. The failure of the largest two members is an event that may

[12] Two large firms defaulted (Lehman and Bear Stearns) and two others (Merrill Lynch and AIG) came close to default but were rescued by the authorities. The evidence from the crisis suggests that it is highly unlikely that three or more large firms would be allowed to fail. Moreover the market implied default rates during the crisis also assigned a low probability to four or more failures (Giglio (2011)).

have a large direct impact but a smaller probability than an event in which one or more members fail at random, hence the expected failure rate may be underestimated. Second, we consider the impact of the failing member(s) on all their counterparties, including other members of the CCP as well as non-members. This is crucial because payment deficiencies by the failing member will be transmitted and amplified through the network of exposures, increasing the ultimate impact on the CCP. Third, we propose a novel estimation methodology that allows us to place a lower bound on the amount of network contagion in the absence of detailed information about the liquid reserves of individual firms. In this respect the model differs markedly from conventional contagion models such as Eisenberg and Noe, which require detailed knowledge of the firms' balance sheets to determine the ultimate impact of a shock. Finally, we show how to estimate bounds on the probability that the CCP will fail *relative* to the probability that an individual member will fail, with minimal information about the degree of correlation among member failures, which in practice is difficult to estimate empirically.

Overall, our results suggest that the CCP is more vulnerable than conventional stress tests would indicate. The results do not include several channels that could further increase the amount of contagion and the concomitant risk of CCP default. One such channel is increased demand for IM in times of financial stress when firms may demand that their counterparties increase the amount of IM they post. These demands have the effect of placing the counterparties under even greater stress. A second channel consists of single-name CDS contracts in which the underlying is a bank holding company(BHC). The stress induced by demands for large VM payments may push some of these BHCs closer to (or into) default, which will trigger large VM payments by the sellers of CDS on these companies. A third channel is fire sales: when a seller of protection defaults on its VM payments, the counterparty will try to find another (solvent) firm that is willing to assume the position of protection seller. This expansion of supply in the face of weak demand will tend to increase the cost of novating the contracts and lead to potential losses that are not covered by the defaulting party's IM. All of these effects can, in principle, be incorporated into the model, and will exacerbate the amount of contagion in the system. The framework we have presented is a simple way of estimating a *lower* bound on the amount of contagion and its potential impact on the CCP.

References

BCBS and IOSCO (2015). Margin requirements for non-centrally-cleared derivatives. Technical report, BIS and OICU-IOSCO, Basil, Switzerland.

CFTC (2016). Supervisory stress test of clearinghouses. A report by staff of the U.S. Commodity Futures Trading Commission.

Cont, R. and Minca, A. (2016). Credit default swaps and systemic risk. *Annals of Operations Research*, 247(2):523–547.

Eisenberg, L. and Noe, T. (2001). Systemic risk in financial systems. *Management Science*, 47(2):236–249.

Giglio, S. (2011). Credit default swap spreads and systemic financial risk. *Proceedings, Federal Reserve Bank of Chicago*, 10(9):104–141.

Glasserman, P. and Young, H. P. (2015). How likely is contagion in financial networks? *Journal of Banking & Finance*, 50:383–399.

ICE (2016). ICE Clear Credit LLC: Financial statements: Years ended December 31, 2015 and 2014. Technical report, Intercontinental Exchange.

Luo, L. S. (2005). Bootstrapping default probability curves. *Journal of Credit Risk*, 1(4):169–179.

Nahai-Williamson, P., Ota, T., Vital, M., and Wetherilt, A. (2013). Central counterparties and their financial resources–a numerical approach. Bank of England Financial Stability Paper 19.

OFR (2017). New public disclosures shed light on central counterparties. Office of Financial Research Viewpoint 17-02.

Paddrik, M., Rajan, S., and Young, H. P. (2016). Contagion in the cds market. Office of Financial Research Working Paper 16-12.

Poce, G., Cimini, G., Gabrielli, A., Zaccaria, A., Baldacci, G., Polito, M., Rizzo, M., and Sabatini, S. (2016). What do central counterparties default funds really cover? a network-based stress test answer. *arXiv preprint arXiv:1611.03782*.

Tarski, A. et al. (1955). A lattice-theoretical fixpoint theorem and its applications. *Pacific Journal of Mathematics*, 5(2):285–309.

Yellen, J. (2013). Interconnectedness and systemic risk: Lessons from the financial crisis and policy implications. *Board of Governors of the Federal Reserve System, Washington, DC*.

Zandi, M. (2016). A straight-up stress test. Moody's Analytics analysis.