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Present Bias, Temptation and Commitment
Over the Life-Cycle: Estimating and
Simulating Gul-Pesendorfer Preferences

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Abstract

This paper provides a quantitative assessment of the ‘temptation preferences’ of Gul and Pesendorfer (2001) for understanding consumer life-cycle choices. I first confirm the empirical relevance of these preferences. I then show that they provide rational and straightforward explanations for many life-cycle features that appear to be inconsistent with standard preferences. These include the puzzle of ‘excess sensitivity’ in consumption; the ‘retirement-consumption puzzle’; the demand for commitment devices; and the slow downsizing in housing towards the end of the life-cycle.

Keywords: life-cycle models; temptation preferences; housing; estimating Euler-Equations

JEL classification: D12; D91; E21; G11; R21


1 Introduction

‘Present bias’ has recently received much attention from economists, psychologists and even policy makers. The difficulties individuals might have to plan for the future, to delay gratification so as to access the returns on investments, or even to accumulate resources to finance consumption in the future, have been extensively discussed. It has been observed that people often procrastinate sound choices (such as dieting or exercise) and prefer immediate gratification. In the context of life-cycle saving decisions, it has been argued that immediate gratification might lead individuals to save much less than they planned to save (see for instance Bernheim (1995)). As an implication individuals who understand this tendency might have a demand for commitment devices - illiquid assets such as retirement plans or housing - to implement their optimal savings plans (see Strotz (1956), Laibson (1997)). These types of behaviour are inconsistent with the standard model of intertemporal choice where individuals discount the future geometrically and instantaneous utility depends only on chosen alternatives. As a consequence, two alternative preference structures that exhibit present bias have received considerable attention.

The ‘$\beta - \delta$’ model, that was formally introduced by Phelps and Pollak (1968) based on ideas proposed in Strotz (1956) and later studied by Laibson (1997) and Harris and Laibson (2001), relaxes the assumption of the standard model on discounting and introduces ‘hyperbolic discounting’. Under hyperbolic preferences the discount rate from today’s perspective between today and tomorrow is larger than the discount rate between any two consecutive dates in the future. The ‘temptation model’ proposed by Gul and Pesendorfer (2001) on the other hand relaxes the standard model’s assumption about instantaneous utility. These preferences highlight the possibility of temptation and the relevance for instantaneous utility of those feasible alternatives that are not chosen. For this reason, I refer to them as ‘temptation preferences’.

In this paper I provide a quantitative assessment of the temptation preferences introduced by Gul and Pesendorfer (2001). I first confirm the empirical relevance of these preferences. I then show that they provide rational and straightforward explanations for many life-cycle features that appear to be inconsistent with standard preferences. These include the puzzle of ‘excess sensitivity’ in consumption, the ‘retirement-consumption puzzle’, the demand for commitment devices (for long-term savings), and the slow downsizing in housing towards the end of the life-cycle.

The Gul-Pesendorfer framework exhibits present bias but does not imply that preferences change over time. Therefore temptation preferences are dynamically consistent while allowing for temptation resistance, hence self-control. Dynamic consistency is an important advantage of the temptation model over the $\beta - \delta$ model. The analysis of
time-inconsistent preferences poses a number of conceptual problems, as one needs to specify how individuals today perceive their future selves. In the literature, different selves are modelled as playing games the outcomes of which inform dynamic life-cycle choices. Within this context, welfare analysis of, say, different policies that change intertemporal prices and dynamic incentives, become intrinsically complicated by the problem of dealing with multiple agents: even when dealing with a single individual, one is forced to consider ‘distributional’ issues. The fact that temptation preferences allow a simple recursive formulation and induce dynamically consistent choices makes this approach very convenient, while the axiom-based representation, which generalises standard preferences, provides a strong theoretical foundation for this approach. However, until recently, very few contributions have explored the empirical plausibility of such an axiomatization or investigated its implications for life-cycle choices.

I develop a dynamic structural model of consumer demand for housing and consumption with temptation preferences. In particular, I derive the Euler equations for temptation preferences. An appropriate log-linearization then allows me to derive expressions that are linear in the parameters and can therefore be estimated using repeated cross-sectional data and synthetic panel techniques. I use these log-linearized Euler equations to estimate preference parameters on household-level U.S. data obtained from the Consumer Expenditure Survey (CEX). As a result I can identify the curvature of the utility function as well as the importance of the ‘temptation motive’.

I use simulations to compare the consumption life-cycle profiles induced by my estimated model to those observed in the data and to those induced by a model with standard preferences. I obtain very different life-cycle profiles under temptation preferences than under standard preferences. By contrast, as discussed in Angeletos et al. (2001), the model with $\beta - \delta$ preferences generates life-cycle profiles that are not very different from those obtained with standard preferences.

I find that the simulated consumption profile under temptation preferences tracks the income profile more closely than the simulated consumption profile under standard preferences. In the absence of liquidity constraints, predictable changes in income should not affect consumption under standard preferences. This no longer holds under temptation preferences. A high level of income at a given point in time induces a stronger temptation and therefore changes the consumption choices as well. As a consequence the puzzle of excess sensitivity of consumption, the reaction of consumption growth to predictable changes in income, is better explained by temptation preferences.

The model with temptation preferences also predicts a sharp drop of consumption after retirement. This prediction is consistent with the observed data and known in the literature as the retirement-consumption puzzle. I argue that in the temptation model
the sharp drop in consumption after retirement comes from the rational and forward-looking behaviour of optimising households.

Preferences that exhibit present bias also imply a demand for commitment. Commitment devices, such as illiquid assets, help individuals to implement their optimal consumption-saving plans. In the $\beta - \delta$ model individuals hold illiquid assets in order to restrict the consumption choice set of their future selves. In the Gul-Pesendorfer model by contrast individuals want to restrict their consumption choice set in order to reduce the cost of temptation resistance, they wish to exercise self-control. Individuals with temptation preferences hold illiquid assets that cannot be spent on current consumption (i.e. they apply self-imposed liquidity constraints) and hence do not induce temptation. Until recently, very few contributions have explored the plausibility of housing as a potential candidate for commitment in a life-cycle context. However, housing is of particular interest and plays an important role in intertemporal decision-making, since the largest portion of household wealth is held in this form. Also, given the high emotional and financial transaction costs of its adjustment, housing is a natural candidate for commitment.

In the last part of the paper, I compare the housing life-cycle profiles induced by my temptation model to those observed in the data and to those induced by a model with standard preferences. I also assess the relevance of the ‘commitment’ motive of housing, which in my model can be summarised by a single parameter.

There are a few papers in the literature analysing temptation preferences in different contexts. Bucciol (2012) estimates the importance of the temptation motive on Survey of Consumer Finances data using the Method of Simulated Moments technique and finds that it is significantly different from zero. Similarly, Huang, Liu, and Zhu (2013) use the Consumer Expenditure Survey data and obtain evidence supporting the existence of temptation preferences. Krussel, Kuruşçu, and Smith (2010) apply temptation preferences in a standard macroeconomic setting with taxation. They conclude that a savings subsidy improves welfare by making it less attractive to succumb to temptation. Schlafmann (2015) also applies temptation preferences in a macroeconomic model in order to understand the effects of temptation on housing and mortgage choices and the welfare consequences of mortgage regulations. Her results show that households with higher temptation are less likely to become home owners, while higher down-payment requirements could be beneficial to these households.

More studies are available when it comes to the question of whether people have

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1I have recently learnt about the paper by Huang, Liu, and Zhu (2013), which estimates the temptation parameter in a very similar way as I do in this paper. My paper is different from theirs since I focus on housing rather than illiquid assets in general. I also simulate my model using my estimated parameter values.
demand for commitment devices. Wartenbroch (1998) shows that people buy smaller quantities of tempting goods even when they are sold with quantity discounts. Thaler and Benartzi (2004) report evidence of people committing in advance to allocate a portion of their future salary increase towards retirement savings. Ashraf, Karlan, and Yin (2006) examine the impact of offering a commitment savings product in the Philippines and find a significant increase in savings by consumers who purchased it. Beshears et al. (2011) report evidence that people who have access to both liquid and less liquid accounts, allocate more savings to the less liquid, commitment account. Concerning the commitment role of housing, it is worth highlighting two recent works. Ghent (2015) uses a life-cycle framework with $\beta - \delta$ preferences to study households’ housing and mortgage choices under different down-payment requirements. One of her results is that the commitment role of housing is not a quantitatively important determinant of housing decisions in her framework. Angelini et al. (2013), on the other hand, build a life-cycle model with temptation preferences and conclude that housing is heavily used as a commitment device in later life.

The main technical contributions of my paper is to estimate the temptation parameter directly using the Euler equation and to simulate the model under temptation preferences. By doing so, the paper makes three substantive contributions to our understanding of life-cycle consumption. First, in a realistic life-cycle model of consumption and savings, it shows that the model with temptation preferences generates life-cycle profiles that are very different from those obtained with standard preferences. In particular, it is able to reconcile the puzzle of excess sensitivity in consumption with intertemporal optimization. Second, it shows that temptation preferences can explain the sharp drop after retirement in the life-cycle consumption profile by rational and forward-looking behaviour of optimising households. Third, it demonstrates by simulations that housing is an important commitment device for savings over the life-cycle. It shows that the model with temptation preferences generates a life-cycle housing profile which is very similar to what we observe in the data. In particular, it is able to explain the slow downsizing in housing towards the end of the life-cycle. Relative to other contributions, this paper focuses on the implications of temptation preferences for life-cycle choices and compares them to those implied by standard preferences and to those observed in the data. The main conclusion is that models with temptation preferences are able to explain many life-cycle consumption-savings anomalies, which a standard model finds hard to account for.

The rest of the paper is organised as follows. In Section 2, I describe the model with temptation preferences. In Section 3, I derive the log-linear form of the Euler equations for temptation preferences and describe the data I use. In Section 4, I report
the estimated Euler equations for standard and temptation preferences. In Section 5 I report results for my simulations and compare the properties of the estimated models with standard and with temptation preferences. In Section 6, I discuss the implications of my analysis and conclude the paper.

2 A Life-Cycle Model with Temptation Preferences

I start with a simple model of life-cycle consumption and savings in a dynamic stochastic framework. I modify this model so that it can capture the possible commitment motive of housing. Households live for $T$ periods as adults, of which $W$ periods are spent as workers and $T-W$ periods as retirees. They maximise their present discounted lifetime utility, which depends on nondurable consumption and the consumption of housing services. The key innovation in the model is the assumption that this utility may represent temptation preferences. Households can reallocate resources between periods by investing in a fully liquid asset or in a less liquid housing, which also provides housing services. They are only allowed to have collateralised debt, and only housing can serve as collateral. Households face uncertainty in two dimensions: idiosyncratic uncertainty over labor income and aggregate uncertainty over house prices.

2.1 Temptation Preferences

The period utility function follows the theoretical, axiomatic-based temptation preferences introduced by Gul and Pesendorfer (2001):

$$U(C_t, \tilde{C}_t, S_t) = u(C_t, S_t) - \left[ \nu(\tilde{C}_t, S_t) - \nu(C_t, S_t) \right]$$

where $C_t$ is the chosen level of nondurable consumption; $\tilde{C}_t$ is the most desirable nondurable consumption alternative, which is affordable; and $S_t$ is the flow of housing services. $u(\cdot)$ and $\nu(\cdot)$ are two von Neumann-Morgenstern utility functions representing two different rankings over alternatives. $u$ is the utility function under standard preferences, while $\nu$ is the utility function under temptation preferences. Households may be tempted to maximise their current period utility instead of maximising their discounted lifetime utility. In particular, they may wish to spend all of their available liquid resources on nondurable consumption since this alternative is the most tempting consumption alternative of all, $\tilde{C}_t$, which maximises their immediate utility:

$$\tilde{C}_t = \arg \max_{C_t \in S_t} (\nu(C_t; S_t)),$$

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2The formal description of the Gul-Pesendorfer model is in Appendix A.2.
where $\mathcal{A}_t$ represents the liquid budget set of the households for each period, to be defined later. The term in square brackets in equation (1) represents the temptation motive of the households. It is the utility cost of not choosing the most tempting consumption alternative: the difference between the temptation value of the most tempting and of the chosen consumption bundle. When exposed to temptation, households can decide to exercise self-control or to succumb to temptation. If they exercise self-control they have to pay the utility cost of temptation resistance, self-control. If, on the other hand, households succumb to temptation the cost of self-control becomes zero and the utility function simplifies to its standard form.\(^3\)

It is important to note that under my simplifying assumption, households can only be tempted by nondurable consumption and not by housing service flow.

The functional form for utility, $u$, is assumed to be a CRRA function of the composite good, which in turn is a Cobb-Douglas aggregate of nondurable consumption and housing services. The temptation utility function, $\nu$, is simply a rescaled CRRA utility function, following Gul and Pesendorfer (2004). So the functional forms are

$$
\begin{align*}
  u(C_t, S_t) &= \frac{(C_t^\alpha S_t^{1-\alpha})^{1-\rho}}{1-\rho} \\
  \nu(C_t, S_t) &= \lambda \frac{(C_t^\alpha S_t^{1-\alpha})^{1-\rho}}{1-\rho}
\end{align*}
$$

where $0 \leq \alpha \leq 1$ is the weight of nondurable consumption in the composite good, $\rho \geq 0$ is the inverse of the elasticity of intertemporal substitution (for the composite good) and $\lambda \geq 0$ is the degree of absolute temptation. The degree of absolute temptation measures households’ sensitivity to the tempting alternative. Notice that preferences are standard when $\lambda = 0$, and, the larger is $\lambda$, the greater is the temptation, which households face and the higher is the utility cost of temptation resistance, self-control.

### 2.2 Budget Constraint

Following Deaton (1991), I write the standard intertemporal budget constraint for the household in terms of cash-on-hand. Households start any period $t$ with a given amount of liquid wealth, $LW_t$, and receive uncertain labor income, $Y_t$, that add up to cash-on-hand, $X_t$.\(^4\) Given the amount of cash-on-hand, households decide how much to consume, where

$$
LW_{t+1} = R_t X_{t+1} + \mathcal{A}_t.
$$

Cash-on-hand is the sum of liquid wealth and labor income: $X_{t+1} = LW_{t+1} + Y_{t+1}$. The resources available in the current period can be spent on consumption, liquid asset, housing

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\(^3\)In case of exercising self-control $C_t \neq \tilde{C}_t$, but in case of succumbing to temptation $C_t = \tilde{C}_t$ and accordingly $U(C_t, \tilde{C}_t, S_t) = u(\tilde{C}_t, S_t) - \left[\nu(\tilde{C}_t, S_t) - \nu(\tilde{C}_t, S_t)\right] = u(\tilde{C}_t, S_t)$

\(^4\)The liquid wealth next period is the current period liquid asset augmented by its rate of return: $LW_{t+1} = R_t X_{t+1} + \mathcal{A}_t$. Cash-on-hand is the sum of liquid wealth and labor income: $X_{t+1} = LW_{t+1} + Y_{t+1}$. The resources available in the current period can be spent on consumption, liquid asset, housing.
$C_t$, how much to invest in illiquid housing, $I_t$ at unit price $Q_t$, how much repayment to make on the existing mortgages, $\xi_t$ and how much new mortgage to take out, $\vartheta_t$.

$$X_{t+1} = R_{t+1}^X (X_t - C_t - Q_t I_t + \vartheta_t - \xi_t) + Y_{t+1}$$

(4)

By deciding on the amount of consumption, housing investment, mortgage repayment and new mortgage take-out, households determine how much to save in the form of liquid asset, which yields risk-free return, $R_{t+1}^X$.

The liquid wealth available next period is consequently the current period liquid asset augmented by its rate of return, while next period’s cash-on-hand is the sum of next period’s liquid asset and next period’s labor income.

Since I do not allow for other types of non-collateralised debt, the sum of consumption, the value of the investment in housing and the repayment on the existing mortgage has to be smaller or equal to the sum of current period cash-on-hand and the new mortgage takeout in each period.

$$C_t + Q_t I_t + \xi_t \leq X_t + \vartheta_t$$

(5)

### 2.3 Illiquid Housing

In addition to the liquid asset, households have access to illiquid housing. For simplicity, the amount of housing (e.g. the size of the house) is assumed to be adjustable at no cost in each period. I also disregard housing depreciation and maintenance costs. Housing investment, $I_t$ adds to the existing stock of housing, $H_t$. Hence the law of motion for housing is

$$H_{t+1} = H_t + I_t, \quad H_0 > 0$$

(6)

Housing generates housing services, $S_t$, which yield instantaneous utility. I assume that there is a linear technology between the stock of housing and housing services.

$$S_t = b H_t$$

(7)

There is no rental market in the model, so housing services can only be consumed by owning. Housing can be used as collateral for mortgage loans. Households can get collateralised debt at a constant price of $R^M$ up to a given fraction, $1 - \psi$, of the value of housing. So at the moment of the first mortgage take-out the following inequality has investment and mortgage repayment: $X_t + \vartheta_t = A_t + C_t + Q_t I_t + \xi_t$. Combining these three equation, I get back the law of motion for cash-on-hand.

$^5R_{t+1}^X$ represents gross real interest rate between periods $t$ and $t + 1$, $R_{t+1}^X = 1 + r_{t+1}^X$. 

8
to hold
\[ M_t \leq (1 - \psi)Q_t H_t, \quad (8) \]
where \( M_t \) is the mortgage, \( Q_t \) is the house price and \( \psi \) can be interpreted as the mortgage down-payment requirement.

Housing plays a special role in the model: it is assumed to be illiquid and as such it could serve as a commitment device. The illiquidity of housing is modelled by assuming that housing can be immediately liquidated only up to a given fraction, \( \delta \).

\[ -\delta H_t \leq I_t \quad (9) \]

Introducing illiquidity in this form is more tractable than including emotional or financial transaction cost of adjustment into the model. It also lets me alter housing liquidity only by changing one parameter of the model, which turns out to be useful later when I test the importance of housing as a commitment device. Notice, that at any given period of time the liquidation constraint only binds for those households who would like to get out more than the fraction \( \delta \) of their housing wealth. For those who do not want to liquidate more than that, housing asset is basically another liquid asset.

### 2.4 Financial Markets

Households are only allowed to have collateralised debt. Since there is no other type of debt available they may have an incentive to take out a mortgage even if the mortgage rate is higher than the risk-free rate. Households with an existing mortgage can apply for a new mortgage, \( \vartheta_t \), but have to keep repaying the existing mortgage, \( \xi_t \). The law of motion for the mortgage stock is as follows

\[ M_{t+1} = R^M M_t + \vartheta_t - \xi_t \quad (10) \]

Next period’s mortgage equals the existing mortgage with its interest, \( R^M \), plus the new mortgage taken out minus the repayment on the existing mortgage. Throughout the paper I only focus on the net new mortgage, which I define as the new mortgage net of repayment on the existing mortgage.

\[ \Upsilon_t = \vartheta_t - \xi_t \quad (11) \]

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\( R^M \) is the gross real mortgage rate, \( R^M = 1 + r^M \)
Households do not have to choose their new mortgage and repayment separately. It is enough to choose the path of the net new mortgage. Having solved for the optimal path of net new mortgage, \( \Upsilon_t \), one can always solve back for \( \vartheta_t \) and \( \xi_t \). This conclusion is based on a simple and straightforward assumption: whenever the household’s repayment is more than the minimum repayment, it does not apply for a new mortgage. Since the mortgage interest rate is fixed over time, there is no gain from paying back more from the old mortgage and applying for a new one at the same time. Analytically,

\[
\vartheta_t = \begin{cases} 
\Upsilon_t + \xi_{\min} & \text{if } \Upsilon_t \geq -\xi_{\min} \\
0 & \text{else}
\end{cases}
\]

\[
\xi_t = \begin{cases} 
\xi_{\min} & \text{if } \Upsilon_t \geq -\xi_{\min} \\
-\Upsilon_t & \text{else}
\end{cases}
\] (12)

I assume that repayment on the existing mortgage is bounded from below: households have to pay at least the interest on the mortgage in each period. The natural upper bound for repayment on the other hand is to pay back all the mortgage with its interest.

\[
\nu^M M_t \leq \xi_t \leq R^M M_t
\] (13)

As was highlighted earlier, households face a constraint on the level of their mortgage take-out: the maximum amount they can have is a constant fraction of the value of their housing.

\[
\vartheta = \begin{cases} 
(1 - \psi)Q_t H_t - R^M M_t & \text{if } (1 - \psi)Q_t H_t > R^M M_t \\
0 & \text{else}
\end{cases}
\] (14)

It is important to see that the restriction on mortgage take-out represented by equation (14) is only enforced at the moment of taking out a new mortgage. As house prices fluctuate, the constraint can be violated for households with an existing mortgage since there is no mechanism through which households could insure themselves against this uncertainty. As a result, whenever the existing mortgage exceeds the maximum possible mortgage take-out, households cannot apply for a new mortgage.

### 2.5 Sources of Uncertainty

In the model households face uncertainty in two dimensions: idiosyncratic uncertainty over labor income and aggregate uncertainty over house prices.

Following Zeldes (1989), labor income \( Y_t \) at any time before retirement is exogenously
described by a combination of deterministic and random components

\[ Y_t = Y_t^P Z_t \quad \log(Z_t) \sim N(-0.5\sigma_z^2, \sigma_z^2) \]  

where \( Y_t^P \) is the permanent component and \( Z_t \) is the transitory component. Furthermore I assume that the permanent component can be described as

\[ Y_t^P = G_t Y_{t-1}^P N_t \quad \log(N_t) \sim N(-0.5\sigma_n^2, \sigma_n^2) \]  

with \( G_t \) being a deterministic function of age and \( N_t \) is the innovation.\(^7\) I also assume that the shocks (\( N_t \) and \( Z_t \)) are independent.

Labor income \( Y_t \) at any time after retirement is a constant fraction \( a \) of the last working year’s permanent labor income. One can think of this as a pension that is wholly provided by the employer and/or the state.

\[ Y_t = a Y_{W}^P \]  

The log of the house price is assumed to be determined by a random walk process with drift. I show later that this assumption is consistent with the data.

\[ \log Q_t = d_0 + \log Q_{t-1} + \log \varepsilon_t \quad \log(\varepsilon_t) \sim N(-0.5\sigma_\varepsilon^2, \sigma_\varepsilon^2) \]  

Having all the details of the theoretical model specified, I can define the vector of state variables, \( \Omega_t = (X_t, H_t, M_t, Q_t, Y_t^P) \) and formulate the households’ value function in period \( t \) in recursive form as:

\[ V_t(\Omega_t) = \max_{C_t, S_t} U(C_t, \tilde{C}_t, S_t) + \beta \mathbb{E}_t V_{t+1}(\Omega_{t+1}) \]  

subject to the budget constraints, the income processes, the form of the utility functions specified earlier, and the definition of the most tempting consumption alternative. Defining the liquid budget set to be: \( \mathcal{A}_t = \{ x_t \in R^+ : x_t \leq X_t + \delta Q_t H_t \} \), the most tempting consumption alternative in this model is the sum of cash-on-hand and the part of housing that can be immediately liquidated (henceforth liquid resources).

\[ \tilde{C}_t = \arg \max_{C_t \in \mathcal{A}_t} (\nu(C_t, S_t)) = X_t + \delta Q_t H_t. \]  

Since there is no analytical solution for the problem, I characterise the solution of the

\(^7\)The assumption of log-normality with given parameters is a simplification. In this case the mean values of \( N_t \) and \( Z_t \) equal 1. The expected value of a log-normal variable with mean \( \mu \) and variance \( \sigma^2 \) is given by \( \exp(\mu + \frac{\sigma^2}{2}) \)
model with the corresponding Euler equations, which are derived in Appendix A.4.\textsuperscript{8} Having these optimality conditions at hand, I can estimate the crucial model parameters, the elasticity of intertemporal substitution and the temptation parameters. This model has three Euler equations, one for the liquid asset, one for housing and one for the mortgage. As all three Euler equations include these parameters of interest, I estimate them using only one of the Euler equations. For ease of estimation, I use the Euler equation for the liquid asset

$$\frac{\partial U_t}{\partial c_t} = E_t \left[ \beta_{t+1} R_{t+1}^X \left( \frac{\partial U_{t+1}}{\partial c_{t+1}} + \frac{\partial U_{t+1}}{\partial \tilde{c}_{t+1}} \right) \right]$$

where

$$\beta_{t+1} = \beta \left( \frac{Y_{t+1}^P}{Y_t^P} \right)^{-\rho}.$$ 

Following Carroll (1992), variables are normalised by permanent income and denoted by lower case letters (e.g. $c_t = C_t / Y_t^P$).\textsuperscript{9} Now using the functional form for the utility function, the Euler equation becomes

$$(1 + \lambda)^\alpha \left( \frac{c_t^{1-\alpha} s_t^{1-\alpha}}{c_t} \right)^{1-\rho} = E_t \left[ \beta_{t+1} R_{t+1}^X \left( (1 + \lambda)^\alpha \left( \frac{c_{t+1}^{1-\alpha} s_{t+1}^{1-\alpha}}{c_{t+1}} \right)^{1-\rho} - \lambda \alpha \left( \frac{c_{t+1}^{1-\alpha} s_{t+1}^{1-\alpha}}{\tilde{c}_{t+1}} \right)^{1-\rho} \right) \right]$$

Dividing both sides by $(1 + \lambda)^\alpha$, I get

$$(1 + \lambda)^\alpha \left( \frac{c_t^{1-\alpha} s_t^{1-\alpha}}{c_t} \right)^{1-\rho} = E_t \left[ \beta_{t+1} R_{t+1}^X \left( (1 + \lambda)^\alpha \left( \frac{c_{t+1}^{1-\alpha} s_{t+1}^{1-\alpha}}{c_{t+1}} \right)^{1-\rho} - \lambda \alpha \left( \frac{c_{t+1}^{1-\alpha} s_{t+1}^{1-\alpha}}{\tilde{c}_{t+1}} \right)^{1-\rho} \right) \right],$$

where I call $\lambda/(1 + \lambda)$ the degree of relative temptation as in Bucciol (2012), which measures the importance of temptation relative to consumption. This equilibrium condition shows that the marginal cost of giving up one unit of current consumption must be equal to the marginal benefit of consuming the proceeds of the extra liquid saving in the next period, minus the marginal cost of resisting the additional temptation in the next period, caused by the higher savings in liquid asset. Therefore the cost of saving is higher for tempted households than for non-tempted ones, everything else being equal.

This Euler equation differs from the standard one in two respects: the traditional Euler equation is derived from a model without housing services in the utility function i.e. with $\alpha = 1$, and without temptation, i.e. with $\lambda = 0$. Setting $\alpha = 1$ and $\lambda = 0$, equation (23) simplifies to the traditional Euler equation, which is extensively used for

\textsuperscript{8}The Euler equation approach does not require a full structural specification: as a result, I can afford to be agnostic about a number of model details such as income or house price processes.

\textsuperscript{9}See standardisation of the model in Appendix A.3.
estimating the elasticity of intertemporal substitution (EIS) parameter:

\[ c_t^{-\rho} = \mathbb{E}_t \left[ \beta_{t+1} R^X_{t+1} c_{t+1}^{-\rho} \right]. \]  

(24)

Note that equation (23) - similar to equation (24) - is not a consumption function. It is an equilibrium condition, hence it can be used to derive orthogonality conditions in order to estimate the parameters of the utility function.

3 Bringing the Model to Data

There are at least three reasons why I do not estimate the nonlinear equation (23) directly, which are extensively discussed in Attanasio and Low (2004)\(^{10}\): the finite sample properties of the nonlinear GMM estimates\(^{11}\); the potential measurement error in the data; and the fact that I do not have a real panel dataset, but use instead the synthetic panel technique. Because of these I need the model to be linear in parameters in order to be able to estimate them satisfactorily.

The linearisation of the Euler equation in the presence of temptation is not straightforward. For better understanding, I present the main steps of the derivation here, while the detailed derivation of the linear approximation can be found in Appendix A.5. First I rewrite the Euler equation (23) for the liquid asset in terms of the pricing kernel as follows

\[ 1 = \mathbb{E}_t k_{t+1} R^X_{t+1}, \]

where the pricing kernel is

\[ k_{t+1} = \beta_{t+1} \left( \frac{c_{t+1}}{c_t} \right)^{-\kappa} \left( \frac{s_{t+1}}{s_t} \right)^{\kappa-\rho} \left[ 1 - \frac{\lambda}{1 + \lambda} \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right)^{-\kappa} \right], \]

(25)

with \( \kappa = 1 - \alpha (1 - \rho) \). The term in the square brackets of equation (25) shows up under temptation preferences only and depends on the liquid resource-consumption ratio. Lettau and Ludvigson (2001) show that consumption and wealth share a common stochastic trend, hence they are cointegrated and their ratio is stationary. Consequently, all the variables in the pricing kernel are stationary and one can take the log-linear approximation of the stochastic discount factor around its steady state value. The resulting

\(^{10}\)Attanasio and Low (2004) show in Monte Carlo simulations that for all parameter specifications, the performance of the non-linear GMM estimator is considerably worse than that of the estimators based on the log-linearized equations.

\(^{11}\)The finite sample distribution is not well described by the asymptotic distribution.
log-linear pricing kernel is given by equation (A.11) from Appendix A.5

\[ \ln k_{t+1} = \ln \beta_{t+1} - \ln (1 + \phi) - \kappa \ln \left( \frac{c_{t+1}}{c_t} \right) + (\kappa - \rho) \ln \left( \frac{s_{t+1}}{s_t} \right) + \kappa \phi \left[ \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) - \ln \left( \frac{\tilde{c}}{c} \right) \right] + \eta_{t+1}, \]

where \( \eta_{t+1} \) includes second and higher moments of consumption growth, the growth of housing service flow, and the log of the liquid resource-consumption ratio. The parameter \( \phi \) is a function of other model parameters:

\[ \phi = \frac{\lambda}{(1 + \lambda)^{\chi \kappa} - \lambda}, \]

where \( \chi = \tilde{c}/c \) is the steady state ratio of liquid resources to nondurable consumption.\(^{12}\)

Using the linearised Euler equation

\[ 0 = \ln R^X_{t+1} + \ln k_{t+1} \]

I can derive the estimable version of the Euler equation under temptation preferences

\[ \ln \left( \frac{c_{t+1}}{c_t} \right) = \theta_0 + \theta_1 \ln R^X_{t+1} + \theta_2 \ln \left( \frac{s_{t+1}}{s_t} \right) + \theta_3 \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) + \epsilon_{t+1}, \quad (26) \]

where \( \theta_0 \) contains constants and the unconditional means of second and higher moments of the relevant variables, \( \epsilon_{t+1} \) summarises expectation errors, possible measurement errors and deviations of second and higher moments from their unconditional means. The regression coefficients are related to the model parameters as follows:

\[ \begin{align*}
\theta_1 &= \frac{1}{\kappa} \\
\theta_2 &= \frac{\kappa - \rho}{\kappa} \\
\theta_3 &= \phi = \frac{\lambda}{(1 + \lambda)^{\chi \kappa} - \lambda}.
\end{align*} \quad (27) \]

Equation (26) differs from the traditional Euler equation used in the empirical literature: it additionally includes the growth rate of the housing service flow, and the log of the ratio of liquid resources to consumption. The growth of housing service flow plays a role because housing service flow is in the utility function, while the log of liquid resources over consumption enters the equation because of the presence of temptation.

\(^{12}\)From CEX, I take the median value of liquid resources over nondurable consumption, which equals to 9.
Notice, that setting the temptation parameter, \( \lambda \) to zero, equation (26) simplifies to the standard Euler equation with housing services in the utility function

\[
\ln \left( \frac{c_{t+1}}{c_t} \right) = \theta_0 + \theta_1 \ln R^X_{t+1} + \theta_2 \ln \left( \frac{s_{t+1}}{s_t} \right) + \epsilon_{t+1} \quad (28)
\]

Alternatively, setting the weight of nondurable consumption in the composite good parameter, \( \alpha \) to one, reduces \( \kappa \) to \( \rho \) and \( \theta_2 \) to zero. Then equation (26) simplifies to the Euler equation derived from a model with temptation preferences but no housing service in the utility function

\[
\ln \left( \frac{c_{t+1}}{c_t} \right) = \theta_0 + \theta_1 \ln R^X_{t+1} + \theta_3 \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) + \epsilon_{t+1} \quad (29)
\]

In case of setting both \( \lambda \) to zero and \( \alpha \) to one, equation (26) becomes the traditional Euler equation with consumption growth on the left-hand side and the log of the gross real interest rate on the right-hand side.

\[
\ln \left( \frac{c_{t+1}}{c_t} \right) = \theta_0 + \theta_1 \ln R^X_{t+1} + \epsilon_{t+1} \quad (30)
\]

In the next section, I estimate all four specification of the Euler equations, equations (26) and (28)-(30) on U.S. micro data.

### 3.1 CEX Data

I use the Consumer Expenditure Survey (CEX), which is a household-level micro dataset collected by the Bureau of Labor Statistics (BLS). BLS interviews about 5000 households each quarter, 80 percent of them are then reinterviewed the following quarter, but the remaining 20 percent are replaced by a new, random group. Hence, each household is interviewed at most four times over a period of a year. The sample is representative of the U.S. population.

During the interviews, a number of questions are asked concerning household characteristics and detailed expenditures over the three months prior to the interview. Household characteristic variables I consider are family size, the number of children by age groups, the marital status of the household head and the number of hours worked by the spouse. Non-durable consumption expenditure data is available on a monthly basis for each household.

Besides household characteristics and expenditures, CEX also collects detailed information on household income and wealth status. Most importantly it provides rich information on different types of savings, and on rented and owned housing. Homeown-
ers report the approximate value of their houses while renters report the rental price of their homes. In order to use as many observations as possible, I incorporate both house owners and renters in the estimation. Information on financial assets and dwellings are only gathered in the final interview, leading to one observation per year per household, therefore I impute the report on rent and house value to the earlier quarters.

I work with quarterly data. I exclude non-urban households, as well as those households who have incomplete information on savings and/or housing from the sample. Furthermore I only keep households of which the head\textsuperscript{13} is at least 21 and no more than 60. I end up with 148,000 observations (interviews), for around 46,000 households for the sample period 1994q1-2010q4.

In the previous section, I derived an estimable Euler equation, equation (26). Here I construct the variables corresponding to the model variables from the CEX dataset. As equation (26) shows I need data on four variables in order to estimate the parameters of the model. These are the nondurable consumption growth, the growth of housing service flow, liquid resources over consumption and the real interest rate.

Consumption.
I collect the available monthly expenditures data from the \textit{Detailed Expenditures Files (EXPN)} of CEX. I define consumption as all expenditures on nondurable goods and services, except spending on education and health care. I then create quarterly consumption by aggregating monthly expenditures. To avoid the complicated error structure that the timing of the interviews would imply on quarterly data, I take the spending in the month closest to the interview and multiply it by three. I deflate the nominal consumption by the consumer price index for nondurables with base-period 1982-1984.

Housing Service Flow.
Under \textit{Detailed Expenditures Files (EXPN)}, there are two separate Files, which contains information on rented and owned living quarters. These are the so-called \textit{Rented Living Quarters (RNT)} and \textit{Owned Living Quarters (OPB)} files in the survey. Monthly rent is available for rented quarters, while the approximated value of the house is available for the owned living quarters. For renters, I define quarterly housing service flow to be three times the reported monthly rent. For homeowners, I approximate quarterly housing service with 2 percent of the value of housing.\textsuperscript{14} Again, I deflate the housing service flow by the consumer price index for nondurables.

Liquid Resources.
Liquid resources represent the maximum amount available for consumption in any given

\textsuperscript{13}The CEX defines the head as the male in a male-female couple and as the reference person otherwise
\textsuperscript{14}Own calculation based on Bureau of Economics Analysis (BEA) suggests housing service to be around 8 percents of the value of housing per year
period. In the model it is the sum of liquid wealth, quarterly labor income and a fraction \( \delta \) of the value of housing. The CEX survey collects financial and income information in the last interview, which can be found in the \textit{CU Characteristics and Income File (FMLY)}. The asset categories I incorporate in liquid wealth are “savings accounts”, “securities as stocks, mutual funds, private bonds, government bonds or Treasury notes” and “U.S. Savings bonds”. In the CEX there is information on the earned after tax income in the past 12 months, so I can easily calculate the quarterly flow of labor income by dividing that reported amount by four. The value of housing that can be liquidated is a fraction \( \delta \) of the reported, approximate value of the house for the home-owners and zero for renters. I deflate all variables by the consumer price index for nondurables.

**Real Interest Rate and Inflation.**

I take series of 3-month Treasury Bill and consumer price index for nondurables with base-period 1982-1984 from the Federal Reserve Bank of St. Louis. I then subtract ex post inflation between quarter \( t-1 \) and quarter \( t \) from the nominal interest rate between quarter \( t-1 \) and quarter \( t \) to get the ex post real interest rate.

### 3.2 Synthetic Panel

As highlighted in the previous section CEX lacks long time series for the same households. Hence, instead of using the short panel dimension of the dataset, I use the synthetic panel approach to estimate the Euler equation first proposed by Deaton (1985) and Browning, Deaton, and Irish (1985). I create a quasi-panel by identifying groups or cohorts of households with similar characteristics of the household head and follow average values of the variables of interest for these homogenous groups over time as they age. Hence if there are \( N \) cohorts observed for \( T \) quarters, this method gives us \( NT \) observations. In reality different cohorts are observed over different time horizons, hence the available synthetic panel is not balanced. Groups or cohorts are defined by the year of birth of the household head. Cohort definition is summarised in Table 1.

In the estimation I only use cohorts which have average cell size by quarter higher than 200. This restriction is used in order to reduce the sampling noise. I also impose an age limit on the cohorts: I exclude observations for cohorts whose head on average is younger than 21 years or older than 60 years. Having defined many cohorts, I can estimate a consistently aggregated version of equation (26) for all cohorts simultaneously, using appropriate instrumental variable techniques.

\[
\Delta \ln(c_{t+1}^g) = \theta_0 + \theta_1 \ln(1 + r_{t+1}^X) + \theta_2 \Delta \ln(s_{t+1}^g) + \theta_3 \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right)^g + \gamma' \Delta z_{t+1} + u_{t+1} \tag{31}
\]

where superscript \( g \) denotes cohort averages. To make the model more realistic, I assume
that the household’s utility is shifted by a number of demographic variables, following Attanasio and Weber (1995). The vector $z_{t+1}$ includes all of these demographic and labor supply variables, as well as seasonal dummies.

<table>
<thead>
<tr>
<th>Cohort</th>
<th>Year of Birth</th>
<th>Age in 1994</th>
<th>Average Cell Size</th>
<th>Used in Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1975-79</td>
<td></td>
<td></td>
<td>no</td>
</tr>
<tr>
<td>2</td>
<td>1970-74</td>
<td>20-24</td>
<td>477</td>
<td>yes</td>
</tr>
<tr>
<td>3</td>
<td>1965-69</td>
<td>25-29</td>
<td>527</td>
<td>yes</td>
</tr>
<tr>
<td>4</td>
<td>1960-64</td>
<td>30-34</td>
<td>599</td>
<td>yes</td>
</tr>
<tr>
<td>5</td>
<td>1955-59</td>
<td>35-39</td>
<td>579</td>
<td>yes</td>
</tr>
<tr>
<td>6</td>
<td>1950-54</td>
<td>40-44</td>
<td>529</td>
<td>yes</td>
</tr>
<tr>
<td>7</td>
<td>1945-49</td>
<td>44-49</td>
<td>447</td>
<td>yes</td>
</tr>
<tr>
<td>8</td>
<td>1940-44</td>
<td>50-54</td>
<td>358</td>
<td>yes</td>
</tr>
<tr>
<td>9</td>
<td>1935-39</td>
<td>55-59</td>
<td>289</td>
<td>yes</td>
</tr>
<tr>
<td>10</td>
<td>1930-34</td>
<td></td>
<td></td>
<td>no</td>
</tr>
</tbody>
</table>

Table 1: COHORT DEFINITION

3.3 Instruments

I use an instrumental variable estimation technique for several cohorts simultaneously. Caution is necessary when choosing the appropriate instruments.

In case there is measurement error in the levels of variables, taking first differences creates an $MA(1)$ structure of the residuals in equation (31). But even without the presence of measurement error, taking first differences between cohort means of different subsamples leads to an $MA(1)$ process in the residuals. Consequently and by the construction of the data, I always have to take account of the $MA(1)$ error structure. Hence, the full residual in equation (31) is the sum of the white noise (expectational error and the deviation of second and higher moments of variables from their unconditional means) and the $MA(1)$ component. Checking the first-order autocorrelations of the residuals, I conclude that the residuals are dominated by the $MA(1)$ part. As a result, I cannot use one-period lagged variables as instruments, however instruments lagged two or more periods give consistent estimates. When aggregate variables such as the real interest rate or the inflation rate are used as instruments, they do not need to be lagged more than one period if the measurement errors in these variables are serially uncorrelated.

---

15Instantaneous utility function is $U_t = U(C_t)f(Z_t, \gamma)$, where I assume $f(Z_t, \gamma) = \exp(\gamma'Z_t)$

16See Appendix A.6 for details on the effect of this on the variance-covariance matrix of residuals
The panel dimension of CEX also implies that adjacent cells do not include completely different households. This fact also needs careful consideration for the following reason. Households at their first interview in time period $t$ appear also at time $t + 1$, $t + 2$ and $t + 3$. Those at their fourth interview in time period $t$ appear also at time $t - 1$, $t - 2$ and $t - 3$. In the presence of household-specific fixed effects, I get inconsistent estimates if I use all the households both in the construction of the relevant variables and the instruments. Hence I follow Attanasio and Weber (1995) and manipulate the sample such that there is no overlap between households used in the construction of the instruments and those used in the construction of the variables that enter the estimated equation. I use all observations when I construct the variables entering my regression, but select subsamples when I construct the instruments. Specifically, in construction of lag 2 instruments, I use only households at the fourth interview, for lag 3 I only use households at the fourth and third interview and for lag 4 instruments, I exclude households at their first interview. Using this method, one can be sure that there is no overlap between households used in the construction of variables and the construction of instruments.

Because of the presence of MA(1) residuals for each cohort and because I estimate equation (31) for nine cohorts simultaneously, the error structure of this Euler equation is quite complicated. This has to be taken into account in the construction of an efficient estimator. I provide details on the standard error correction strategy in Appendix A.6. I present estimates of parameters in the next section that are already corrected for the complicated error structure.

4 Estimation Results

In Table 2, I report the estimation results for four different log-linearized Euler equations using GMM estimation for several cohorts simultaneously. Instruments are the different lags of consumption growth, nominal interest rates and household characteristics. Household characteristics are the number of family members, number of family members who are younger than 2, who are between 2 and 15, a dummy for single households, and a dummy for labor participation of the spouse. The validity of instruments cannot be rejected by the values of Sargan’s $p$-statistic. Variables that are meant to capture the effects of changing family composition on the discount factor are added to each equation. In my chosen specification these are seasonal dummies, the log of the family size (famsize) and the number of children below age 2 (num2).

The first two columns in Table 2 present the results from estimating the Euler equation with standard preferences. The two models only differ in the presence of housing
services in the utility function in the second column. The last two columns in Table 2 present the results from estimating the Euler equation with temptation preferences. These two models again differ in the presence of housing services in the utility function in the last column. Besides presenting the direct parameter estimates of equation (31), Table 2 also shows the approximations for the derived parameters of interest, the elasticity of intertemporal substitution and the degree of relative temptation, $\lambda/(1 + \lambda)$.

The EIS parameter measures the responsiveness of the growth rate of consumption to the real interest rate. Note that in columns 1 and 3 of Table 2, EIS shows the responsiveness of nondurable consumption, while in columns 2 and 4 it shows the responsiveness of composite consumption. The degree of relative temptation parameter measures the importance of temptation relative to consumption: in column 3 the importance of temptation is measured relative to nondurable consumption, while in column 4 it is measured relative to the composite consumption. The standard errors for both parameters are

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>Coeff. in eq.(31)</th>
<th>$\alpha = 1$</th>
<th>$\alpha \neq 1$</th>
<th>$\alpha = 1$</th>
<th>$\alpha \neq 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\log(1 + r^X)$</td>
<td>$\theta_1$</td>
<td>0.928**</td>
<td>0.805**</td>
<td>1.901**</td>
<td>1.697**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.313)</td>
<td>(0.330)</td>
<td>(0.815)</td>
<td>(0.794)</td>
</tr>
<tr>
<td>$\Delta \log (s)$</td>
<td>$\theta_2$</td>
<td>0.376</td>
<td>0.580*</td>
<td>(0.235)</td>
<td>(0.315)</td>
</tr>
<tr>
<td>$\log(c/c)$</td>
<td>$\theta_3$</td>
<td>0.056*</td>
<td>0.049</td>
<td>(0.030)</td>
<td>(0.030)</td>
</tr>
<tr>
<td>$\Delta \log (\text{famsize})$</td>
<td></td>
<td>0.478***</td>
<td>0.457***</td>
<td>0.669***</td>
<td>0.612***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.073)</td>
<td>(0.0749)</td>
<td>(0.120)</td>
<td>(0.117)</td>
</tr>
<tr>
<td>$\Delta (\text{num2})$</td>
<td></td>
<td>-0.527***</td>
<td>-0.591***</td>
<td>-0.630***</td>
<td>-0.716***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.120)</td>
<td>(0.130)</td>
<td>(0.153)</td>
<td>(0.185)</td>
</tr>
<tr>
<td>E.I.S</td>
<td></td>
<td>0.928**</td>
<td>0.775**</td>
<td>1.901**</td>
<td>1.980</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.313)</td>
<td>(0.305)</td>
<td>(0.815)</td>
<td>(1.303)</td>
</tr>
<tr>
<td>$\lambda/(1 + \lambda)$</td>
<td></td>
<td>0.222**</td>
<td>0.229**</td>
<td>(0.088)</td>
<td>(0.104)</td>
</tr>
<tr>
<td>Sargan p-stat</td>
<td></td>
<td>0.53</td>
<td>0.51</td>
<td>0.94</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Standard errors are in parenthesis. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$
All specifications include a constant and three seasonal dummies. The instrument set is different across columns. Standard errors for the derived structural parameters are estimated by the Delta method. $\alpha$ is the weight on nondurable consumption in the composite basket, and $\lambda/(1 + \lambda)$ is the relative temptation.
approximated by applying the delta method.\textsuperscript{17}

### 4.1 The Elasticity of Intertemporal Substitution

Under models with standard preferences presented in column 1 and 2 of Table 2, I find no significant difference in the point estimates of the elasticity of intertemporal substitution parameter. Also, the estimated values are very similar to what others estimated in the literature (see for example Attanasio and Weber (1993), Blundell, Browning, and Meghir (1994) or Bucciol (2012)).

Under models with temptation preferences presented in column 3 and 4 of Table 2, I estimate the EIS parameters to be above 1. In the third model specification the EIS parameter is 1.90 for nondurable consumption, while in the last specification the EIS parameter is 1.98 for composite consumption. Despite the fact that the coefficients are not precisely estimated, the high values for the EIS parameter are striking compared to the results of the traditional Euler estimations.

To get an intuition why the responsiveness of consumption growth differs so much between standard and temptation preferences, further consideration is in order. As we see in the derivation below, the interest rate has a strong indirect effect on consumption growth in the temptation model through liquid resources in addition to its direct effect. To see this, let us first take the derivative of both the standard Euler equation, equation (30), and the Euler equation with temptation preferences, equation (29) with respect to the real interest rate,

\[
\frac{\partial \Delta \ln(c_{t+1})}{\partial r_{t+1}^X} \approx \theta_1 \frac{\partial \ln(1 + r_{t+1}^X)}{\partial r_{t+1}^X} \tag{32}
\]

\[
\frac{\partial \Delta \ln(c_{t+1})}{\partial r_{t+1}^X} \approx \theta_1 \frac{\partial \ln(1 + r_{t+1}^X)}{\partial r_{t+1}^X} + \theta_3 \left( \frac{\partial \ln \tilde{c}_{t+1}}{\partial r_{t+1}^X} - \frac{\partial \ln c_{t+1}}{\partial r_{t+1}^X} \right) \tag{33}
\]

The only difference between equation (33) and its standard preference counterpart, equation (32), is the term attached to the coefficient \(\theta_3\). Therefore this term has to account for the estimation result of a doubled EIS parameter. Recall that \(\tilde{c}_t\) is the wealth that is available for liquidation in period \(t\)

\[\tilde{c}_t = x_t + \delta Q_t h_t,\]

which is the sum of the available cash-on-hand and an exogenous fraction of the value of housing. It is important to note that the value of housing is determined by the

\textsuperscript{17}Note that the estimation results from the Euler equation can only be taken seriously if the liquidity constraint does not bind for households, who are in the sample. In order to test this, I run the same regressions only for those households, who report positive liquid savings. The results are robust for this change.
present discounted value of the future stream of housing services. As such, the value of housing crucially depends on the real interest rate and is very sensitive to any of its changes. Also, by definition the value of housing affects the value of liquid resources, which is an important element of the Euler equation under temptation preferences. Consequently, under temptation preferences the real interest rate has a strong indirect effect on consumption growth through liquid resources in addition to its direct effect. An increase in the real interest rate implies that future housing service flows are discounted at a higher rate, hence current value of housing becomes lower. Therefore, the indirect effect of the real interest rate is negative.

Table 2 shows that to explain the variation in consumption growth under standard preferences the EIS parameter has to be around 0.8-0.9. Under temptation preferences, the real interest rate has a strong negative indirect effect through liquid resources. Consequently, to explain the same variation of consumption growth with the same variation of the real interest rate the EIS parameter has to be higher under temptation preferences than under standard preferences to compensate for the negative indirect effect. This is why in columns 3 and 4 of Table 2 I find the estimated EIS parameter to be around 1.9.

4.2 The Temptation Parameter

I present estimates of the relative temptation parameter in the last two columns of Table 2, which correspond to the models with temptation preferences. Testing the empirical existence of temptation is equivalent to test the null hypothesis that the parameter of relative temptation is zero, \( \frac{\lambda}{1 + \lambda} = 0 \). As seen in Table 2, both of the models with temptation preferences indicate rejection of the null hypothesis that \( \frac{\lambda}{1 + \lambda} = 0 \), providing support for the empirical presence of temptation.

As both the degree of absolute and the degree of relative temptation parameters are nonlinear functions of \( \theta_3 \), I apply the delta-method to approximate them and present the corresponding results in Table 2. The estimates for the degree of relative temptation is around 0.22 with reasonable precision and there is no significant difference in the estimates between the two models with temptation preferences, i.e. columns 3 and 4 of Table 2. The validity of instruments cannot be rejected by the values of Sargan’s \( p \)-statistic.

The parameter of relative temptation can be interpreted as the contribution of temptation to utility relative to the contribution of consumption. The value of 0.22 for the temptation parameter suggests that temptation plays an important role in households’ intertemporal decision making. The weight on the utility cost of temptation is about one-fifth of the weight on the utility of consumption.
5 Life-Cycle Simulations

So far I have estimated some of the key model parameters and showed that the temptation model cannot be empirically rejected. In fact, the estimation results suggests that temptation is a crucial element of intertemporal decision-making. In order to understand the main mechanisms of the structural model with temptation preferences I simulate the life-cycle choices for a large number of households. Simulation needs optimal policy functions, which I cannot derive analytically, therefore I need to apply numerical techniques.

I start this section with the calibration of the remaining model parameters. Some of them are estimated from different data sources, while other parameter values are adapted from elsewhere in the literature. After setting the parameter values, I use backward induction technique in order to approximate the optimal policy functions. Having calculated the optimal policy functions, I draw realisations for the idiosyncratic income shocks for each household, and for the aggregate house price shock. Then I simulate the entire life-cycle of consumption and housing decisions for a large number of households and for many different evolutions of the house price. Altogether I run 100 simulations, with 2000 households each.

In the second part of this section I present the simulation results and compare life-cycle profiles induced by my model to those induced by a model with standard preferences and to those observed in the data.

5.1 Calibration

Income Process. To obtain the age-specific component of the life-cycle income profiles (G), I fit a third-order age polynomial to the logarithm of cohort income data gathered from the CEX.

\[
\ln(y_{g,t})^g = \beta_0 + \beta_1 \text{age}_{g,t} + \beta_2 \frac{\text{age}_{g,t}^2}{10} + \beta_3 \frac{\text{age}_{g,t}^3}{100} + u_{g,t}
\]  

(34)

where \(g\) stands for group/cohort averages. The age of the cohort, \(\text{age}_{g}\), is calculated by taking average age over those household heads who belong to the same group. The regression results for the whole sample are presented in Table 3.

The replacement rate, \(a\), is calculated on the same dataset as the ratio of the before- and after-retirement average labor income. Since the normal retirement age was 67 over the sample period in the U.S., I consider 8-year intervals around age 67 in order to calculate these averages. I set the before-retirement age band to between 59-66, while the after-retirement period to between ages 67-74. The estimated value for \(a\) is reported.
in Table 3. I do not estimate the variances of the components of the income process ($\sigma_n$, $\sigma_z$), but use the estimates of Carroll and Samwick (1997).

<table>
<thead>
<tr>
<th></th>
<th>log $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>0.283*** (0.026)</td>
</tr>
<tr>
<td>Age$^2$/10</td>
<td>-0.049*** (0.006)</td>
</tr>
<tr>
<td>Age$^3$/100</td>
<td>0.003*** (0.000)</td>
</tr>
<tr>
<td>Constant</td>
<td>5.077*** (0.343)</td>
</tr>
</tbody>
</table>

Observations 171  
R-squared 0.86  
Replacement rate 0.80

Table 3: INCOME PROCESS

**House Price Process.** Estimation of the parameters of the house price process is based on the Case-Shiller real home price index series for the U.S., for quarters between 1984q1-2014q1. I estimate an AR(1) process with linear trend for the logarithm of the real Case-Shiller index.

$$\log Q_{t+1} = d_0 + d_1 t + \rho_h \log Q_t + \log \varepsilon_t$$

The results of the estimation are in Table 4. The persistence parameter of the log house price $\rho_h$ is estimated to be very close to 1, and unit root tests do not reject the null hypothesis that this parameter is actually 1. This implies that the log house price process can be approximated by a random walk. The estimated quarterly variance of the house price shock, $\sigma^2_{\varepsilon}$, equals 0.0005, which corresponds to 0.002 at an annual frequency.

**Housing Service.** I estimate the annual housing service flow over housing stock, $b$, using data from the Bureau of Economic Analysis (BEA). I use housing gross value added at current dollars to approximate the housing service flow and use residential fixed assets at current dollars to approximate the housing stock.\(^{18}\) The average of gross housing

\(^{18}\)Gross housing value added can be found in Table 7.4.5, "Housing Sector Output, Gross Value Added and Net Value Added" in National Income and Product Accounts (NIPA) of the BEA. Residential fixed assets can be found in Table 1.1, "Current-Cost Net Stock of Fixed Assets and Consumer Durable Goods" of the Fixed Asset Tables of the BEA.
value added over residential fixed assets over the period 1994-2011 is 8.6%. In order to calculate the net value added over residential fixed assets, I take the depreciation rate for residential capital from the BEA.\textsuperscript{19} The depreciation rate is calculated as depreciation divided by housing fixed assets, which is about 2.5% for the period 1994-2011. Consequently, the net value added over residential fixed assets is calculated to be 6.1% over the same period.

**Housing Share in the Composite Good.** I calculate the share of housing services in total consumption from aggregate data in the same way as I calculate housing service flow from the BEA data. I divide housing services by total consumption net of health and educational expenditures.\textsuperscript{20} The average share over the period 1994-2011 is 0.18, thus the share of non-housing consumption is 0.82.

**Number of Equivalent Adults.** Since I compare my model with household level data, I have to take into account demographic changes within households over the life-cycle. Similarly to the OECD equivalence scale, I use weight 1 for the first adult in the households, weight 0.7 for all the other adults in the households and weight 0.5 for each child under age 18.

The benchmark model parameters are collected in Table 5.

\textsuperscript{19}Depreciation for residential capital is taken from the Table 1.3, "Current-Cost Depreciation of Fixed Assets and Consumer Durable Goods" of the BEA Fixed Asset Tables

\textsuperscript{20}I take all the data from Table 2.4.5., "Personal Consumption Expenditures by Type of Product" of the Bureau of Economic Analysis.
<table>
<thead>
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<th>Parameter</th>
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<tr>
<td>T</td>
<td>Number of years as adult</td>
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<tr>
<td>W</td>
<td>Number of years as worker</td>
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<tr>
<td>Age</td>
<td>Age-specific income, linear trend</td>
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<tr>
<td>$Age^2/10$</td>
<td>Age-specific income, quadratic trend</td>
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<td>$Age^3/100$</td>
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<td>$a$</td>
<td>Replacement rate</td>
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<td>Std.dev.transitory income shock</td>
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<td>$R^M$</td>
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<td>Weight of durables in composite good</td>
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<td>b</td>
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<td>$\delta$</td>
<td>Liquidity parameter</td>
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<td>$\psi$</td>
<td>Down-payment requirement</td>
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Table 5: ANNUAL PARAMETERS FOR THE TEMPTATION MODEL
5.2 Simulation Results

5.2.1 Baseline

In this section, I compare the life-cycle profiles for consumption and assets that I obtain simulating the model under standard and temptation preferences.

Figure 1 plots the average consumption life-cycle profiles up to age 75 for the standard and the temptation models together with average household income. Both models generate smooth, hump-shaped consumption profiles that peak at around age 50 and have a sizeable drop after retirement. In both models consumption tracks income. At the beginning of the life-cycle households are credit-constrained since they have not accumulated much illiquid housing to borrow against. Therefore, they increase their consumption slowly as their labour income increases. Since in the benchmark parametrisation the interest rate is lower than the inverse of the discount factor, $\beta R^X < 1$, households prefer a downward-sloping consumption profile in the absence of liquidity constraints. Towards the end of the life-cycle the consumption paths are decreasing.

![Figure 1: SIMULATED CONSUMPTION and INCOME](image)

The most noticeable feature of the figure, however, is the substantial difference between the profiles obtained under standard and temptation preferences. During working ages households with temptation preferences consume much more than those with standard preferences and consequently save much less when labor income is high. This result arises because saving is more costly for tempted households because they have to pay the cost of self-control.

After retirement, on the other hand, tempted households face a much bigger drop
in consumption compared to standard households. In order to understand why, it is worth taking a look at the Euler equations that have to hold in the absence of liquidity constraints. As shown before, the Euler equation under temptation preferences can be written as in equation 21,

$$\frac{\partial U_t}{\partial c_t} = E_t \left[ \beta_{t+1} R^X \left( \frac{\partial U_{t+1}}{\partial c_{t+1}} + \frac{\partial U_{t+1}}{\partial c_t} \right) \right],$$

while the Euler equation under standard preferences is

$$\frac{\partial U_t}{\partial c_t} = E_t \left[ \beta_{t+1} R^X \frac{\partial U_{t+1}}{\partial c_{t+1}} \right].$$

Notice that the Euler equation for temptation preferences includes the derivative of the utility function with respect to the most tempting consumption alternative. As long as households do not succumb to temptation, this derivative is always negative, representing the cost of self-control. Consequently, households with temptation preferences prefer larger intertemporal changes in consumption than those with standard preferences. Assuming that $\beta_{t+1} R^X < 1$, tempted households reduce their consumption at a faster rate later in life, when they are less likely to be liquidity constrained, than standard households.

Figure 1 also suggests that temptation preferences could provide a rational and straightforward explanation for some life-cycle features that appear to be inconsistent with standard preferences. The excess-sensitivity puzzle and the retirement puzzle can be easily rationalised under temptation preferences.

**Excess-Sensitivity Puzzle.**

The prediction of the standard life cycle model is that households do not change their consumption in response to changes in predictable income. Since Hall (1978), many papers in the empirical consumption literature have highlighted the importance of the ‘excess sensitivity’ of consumption to predictable income. Most of these papers interpret the excess sensitivity as a rejection of the standard life-cycle model (see for instance Throow (1969) Campbell and Mankiw (1991) and Carroll and Summers (1991)). Others claim that by using a more flexible version of the life-cycle model one can reconcile excess sensitivity with intertemporal optimization, for instance allowing for non-separability between consumption and leisure; or considering changes in the demographic composition of the household (Heckman (1974), Attanasio et al. (1999) or Browning and Ejrnæs (2002)).

In the Gul-Pesendorfer framework, the excess sensitivity of consumption growth to income growth is due to the rational behaviour of tempted households. A high level of
income at a given point in time induces stronger temptation, a higher cost of self-control and therefore higher actual consumption. As a result, the observed excess sensitivity of consumption to predictable income, which is a puzzle for the standard life-cycle models, is consistent with the life-cycle model under temptation preferences.\footnote{I estimate the sensitivity of consumption growth on predictable income growth using my simulated households under standard and temptation preferences.}

**Retirement-Consumption Puzzle.**

The largest predictable change in household income is clearly the considerable drop at retirement, after exiting the labor market. The prediction of the standard life cycle model is that households, knowing about this sizeable drop in income later in their life, accumulate wealth in order to support consumption after retirement. Hamermesh (1984) pointed out that households’ inadequate savings at retirement observed in the data may not be consistent with the standard life-cycle model. Since then, many papers have tried to reconcile this observation with the standard life-cycle model (see Aguiar and Hurst (2005), Aguiar and Hurst (2007), Battistin et al. (2009), Hurst (2008) and Aguila, Attanasio, and Meghir (2011)).

Under temptation preferences, as we can see in Figure 1, optimal behaviour is to decrease consumption at a faster rate than under standard preferences. This is because saving is much more costly for tempted households: besides the cost of giving up consumption in the current period they also have to pay the utility cost of self-control caused by higher savings next period. As a consequence, faster decumulation of wealth can be perfectly rational when households are no longer liquidity constrained. The observed drop in consumption later in life is consistent with a life-cycle model under temptation preferences.

Figure 2a shows that there is a large housing accumulation up until age 50 according to both models. Households with temptation preferences accumulate much less housing for two reasons: they have to exercise self-control in order to sacrifice current consumption and buy housing but they also have to face the cost of self-control caused by higher savings in housing (note that a fraction, $\delta$, of housing being liquid can tempt households). After retirement, housing starts to decrease in order to support consumption after the significant permanent income drop. Housing decumulation is much slower

\[ \Delta \ln C_t = a + \beta \Delta \ln (E_{t-1}Y_t) + \varsigma_t \]

where using equation (15)-(17)

\[
E_{t-1}Y_t = \begin{cases} 
G_t Y_{t-1}^P & \text{if } t \leq W \\
\frac{a Y_{t-1}^P}{Y_W} & \text{if } t > W .
\end{cases}
\]

The marginal propensity to consume out of predictable income changes, $\beta$, is estimated to be 0.12 under standard preferences, while 0.25 under temptation preferences.
under temptation preferences then under standard preferences.

**No Downsizing Puzzle.**

From available CEX data, one can notice that housing decumulation after retirement is in fact extremely slow, if there is any. This phenomena is called the ‘no downsizing puzzle’ in the literature. No downsizing in housing is not just a feature of U.S. households. A nice documentation of housing stock holding on European data is given by Angelini, Brugiavini, and Weber (2014), where the authors observe that many households are simultaneously liquidity constrained and long on housing, but they do not use housing to increase their nondurable consumption after retirement.

A standard life-cycle model with no bequest motive or uncertainty finds it hard to explain the no downsizing puzzle. This is because the standard framework gives a straightforward implication for wealth accumulation: it should follow the pattern of the income. Accordingly, having observed a hump-shaped income profile, we should predict a hump-shaped savings profile as well.

![Housing](a) Housing  ![Net Financial Liabilities](b) Net Financial Liabilities

**Figure 2: SIMULATED ASSET HOLDINGS**

In the framework of temptation preferences though, slow downsizing of illiquid housing can be explained by the rational behaviour of tempted households. These households try to find ways to tie their hands and not to succumb to tempting overconsumption. One obvious way is to keep their savings in non-liquid housing which is harder to spend on current consumption immediately. In this way they are basically able to lower their cost of self-control and implement their consumption plans.

Figure 2b shows the evolution of net financial liabilities up to age 75 under standard and temptation preferences. Net financial liabilities are defined as liquid asset minus the mortgage stock. They are negative and U-shaped over the life-cycle under each preference specification. Given their higher housing accumulation standard households
can benefit from higher mortgage levels throughout their lives, which, according to Figure 2b, they take advantage of. It is also worth noticing that there is a short period when liabilities are decreasing again after retirement due to the discrete drop in labor income.

5.2.2 Housing as a Commitment Device

Households with standard preferences have no demand for commitment devices since they are committed to their choices by definition. Preferences that exhibit present bias on the other hand imply a demand for commitment devices. These commitment devices, such as illiquid assets, help households to implement their optimal savings plans. Under temptation preferences households hold illiquid assets in order to restrict their own consumption choice set and to reduce their cost of temptation resistance. In this section I shed light on the importance of housing as a commitment device over the life-cycle.

As we have seen, the liquidity of housing can be summarized by a single model parameter, \( \delta \), which I call the liquidity parameter. It is a crucial parameter that affects the optimal choices of households, and hence the quantitative performance of the model through the budget constraint.

On the one hand changes in \( \delta \) changes the restrictions on the speed at which households can decumulate their housing. It becomes important towards the end of the life-cycle but may not be a binding constraint, since it only restricts households when they want to decumulate housing at a faster rate. The lower is the liquidity parameter the harder it is to decumulate housing, which could lead households to hold less housing. I call this decumulation channel of \( \delta \).

On the other hand changes in \( \delta \) change the usefulness of housing as a commitment device. The commitment role of housing is only important for households with preferences that exhibit present bias, standard households do not value commitment devices. The lower is the liquidity parameter, the better housing serves as a commitment device, which could lead tempted households to hold more housing. I call this commitment channel of \( \delta \). Therefore, the aggregate effect of the parameter \( \delta \) on housing demand is the sum of the decumulation channel and the commitment channel, which work in opposite directions.

In what follows I compare simulation results of standard and temptation preferences under different liquidity parameters, \( \delta \), and quantify the importance of housing as a commitment device. The benchmark liquidity parameter shown in Table 5 is 0.75, while I also show the quantitative performance of the models for lower liquidity of housing, setting the liquidity parameter to be \( \delta = 0.5 \).

In Figure 3, I plot the implications of these two values of the liquidity parameter for
the life-cycle paths of housing demand under standard and temptation preferences. The solid lines represent the benchmark case with $\delta = 0.75$, while the dashed lines correspond to the lower value of the liquidity parameter, $\delta = 0.5$.

In Figure 3a I plot the effect of the liquidity parameter change on housing demand for standard preferences. The life-cycle paths of housing demand are very similar under both parametrisation. A change in $\delta$ affects households’ decisions because it restricts the speed at which they can deccumulate housing, hence only through the deccumulation channel of $\delta$. As we discussed before standard households are always committed to their consumption choices, never tempted to consume more, hence they do not have a demand for commitment devices. Consequently they are not affected by the commitment channel of $\delta$. On aggregate, standard households are not much affected by the decrease in housing liquidity.

![Graph](a) Standard Preferences  
![Graph](b) Temptation Preferences

**Figure 3: THE EFFECT OF THE LIQUIDITY PARAMETER**

In Figure 3b I plot the effect of the liquidity parameter change on the housing demand for temptation preferences. Making housing less liquid by decreasing parameter $\delta$ from 0.75 to 0.5 increases the demand for housing by about 50% at the peak of the life-cycle. Higher demand for housing is clearly explained by the higher demand for commitment devices through the commitment channel: the lower is the fraction of housing that can be liquidated and spent immediately on nondurable consumption, the more useful is housing as a commitment device. Keeping savings in more illiquid housing decreases households’ cost of self-control. Note that the positive effect of the commitment channel on housing demand could be even higher when the deccumulation channel has a role (which has an opposite effect on housing demand).

The model suggests that housing is quantitatively important as a commitment device. Tempted households wish to hold their savings in illiquid housing in order to decrease
their cost of temptation resistance, self-control. As a result and as Figure 3b suggests, a significant fraction of housing demand can be explained by the demand for commitment devices under temptation preferences.

6 Conclusions

In this paper I provide a quantitative assessment of temptation preferences proposed by Gul and Pesendorfer (2001). The aim of the paper is to explore the empirical plausibility of the temptation preference approach and to discuss its implications for life-cycle choices. I find that the consumption-savings behaviour of households is better explained by temptation than by standard preferences.

The major technical contribution of the paper is to identify the importance of the temptation motive of housing by estimating the temptation parameter. Using the Consumer Expenditure Survey data I find that the temptation motive plays an important role in households’ decision-making. The weight on the utility cost of temptation resistance is about one-fifth of the weight on the utility of consumption. Also, in contrast with findings under standard preferences, I estimate the elasticity of intertemporal substitution parameter to be well above one under temptation preferences.

The major substantive contribution of the paper is to simulate households’ life-cycle profiles under temptation and under standard preferences. This allows me to compare these simulated life-cycle profiles with each other and with those observed in the data. In particular I show that many life-cycle features that appear to be inconsistent with standard preferences can be rationalised and understood under temptation preferences. These include the puzzle of excess sensitivity in consumption, the retirement-consumption puzzle, the demand for commitment devices, and the slow downsizing in housing towards the end of the life-cycle.

The results of this paper are very suggestive and lead me to conclude that temptation preferences are more powerful in explaining life-cycle choices than standard preferences. However, there are at least two important issues that I have ignored: the possibility of temptation in housing service flow and the fact the households make decisions on housing ownership as well. Both of these features can be very important in households’ consumption-savings choices, hence it would be worthwhile to incorporate these issues in future work that extends the model presented here.
A Appendix

A.1 The Household’s Problem

\[ V_t(\Omega_t) = \max_{\{C_t, \hat{C}_t, S_t\}} U(C_t, \hat{C}_t, S_t) + \beta \mathbb{E}_t V_{t+1}(\Omega_{t+1}) \]

\[ \Omega_t = (X_t, H_t, M_t, Q_t, Y^P_t) \]

subject to a set of constraints

\[ X_{t+1} = R^X_t(X_t - C_t - Q_t I_t + \vartheta_t - \xi_t) + Y_{t+1} \]
\[ H_{t+1} = H_t + I_t \]
\[ M_{t+1} = R^M M_t + \vartheta_t - \xi_t \]
\[ S_t = b H_t \]
\[ -\delta H_t \leq I_t \]
\[ \Upsilon_t = \vartheta_t - \xi_t \]
\[ r^M M_t \leq \xi_t(M_t) \leq R^M M_t \]
\[ \vartheta_t \leq (1 - \psi) Q_t H_t - R^M M_t \quad \text{if} \quad (1 - \psi) Q_t H_t > R^M M_t \]
\[ \vartheta_t = 0 \quad \text{if} \quad (1 - \psi) Q_t H_t \leq R^M M_t \]
\[ \hat{C}_t = X_t + \delta Q_t H_t \]

A.2 The Gul-Pesendorfer Framework

The main difference between the standard assumptions of dynamic decision theory and Gul and Pesendorfer’s framework can be summarized as follows. According to the standard theory, preferences are defined over objects. Adding objects to a set cannot make the set less preferred. By contrast, in Gul-Pesendorfer’s model, preferences are defined over sets of objects. Adding objects to a set can make the set itself worse through the temptation of the new object added to the set. Gul and Pesendorfer derive temptation preferences with self control based on four axioms. Three of them are standard axioms of consumer choice.

**AXIOM 1 (Preference Relation):** \( \preceq \) is a complete and transitive binary relation

**AXIOM 2 (Strong Continuity):** The sets \( \{ B : B \preceq A \} \) and \( \{ B : A \preceq B \} \) are closed
**AXIOM 3** (Independence): \( A \succ B \) and \( \alpha \in (0,1) \) implies \( \alpha A + (1 - \alpha) C \succ \alpha B + (1 - \alpha) C \)

**AXIOM 4** (Set Betweenness): \( A \succeq B \) implies \( A \succeq A \cup B \succeq B \)

The first three axioms are standard, but the last axiom, *Set Betweenness*, is the one that allows household to be tempted by the additional alternative in the set and to exercise self control. The fact that \( A \) is weakly preferred to \( A \cup B \) shows that an alternative (in set \( B \)) which is not chosen may affect the utility of the decision maker because it causes temptation. The assumptions are that temptation is utility decreasing, all the alternatives can be ranked according to how tempting they are, and only the most tempting alternative available affects the decision-maker’s utility. The fact that \( A \cup B \) is weakly preferred to \( B \) indicates the possibility of self control.

Gul and Pesendorfer show that the binary relation \( \succeq \) satisfies Axioms 1-4 if and only if there are continuous linear functions \( U, u, v \) such that

\[
U(A) := \max_{x \in A} [u(x) - (\max_{y \in A} v(y) - v(x))]
\]

when self-control is exercised and

\[
U(A) := \max_{x \in A} u(x) \text{ subject to } v(x) \geq v(y) \text{ for all } y \in A
\]

when no self-control is exercised. Here \( u \) represents long run/commitment utility over alternatives; \( v \) represents temptation utility over alternatives, so one can interpret \( \max_{y \in A} v(y) - v(x) \) as the utility cost of self control. In other words, one can think about alternatives as having two types of ranking, the commitment and the temptation ranking. When the household decides on which alternative she would like to consume, she takes a look at both rankings and maximizes the possible utilities. After the decision, she enjoys the commitment and temptation utility of the alternative she did choose, but she suffers the loss of the temptation utility of the best alternative which she could have chosen as well.

### A.3 Standardization

At terminal age \( t = T \) the value function becomes

\[
V_T(\Omega_T) = \frac{(C_{T-T}^\alpha S_{T-T}^{1-\alpha})^{1-\rho}}{1 - \rho} \lambda \frac{(C_{T-T}^\alpha S_{T-T}^{1-\alpha})^{1-\rho}}{1 - \rho} + \lambda \frac{(C_{T-T}^\alpha S_{T-T}^{1-\alpha})^{1-\rho}}{1 - \rho},
\]
where

$$\Omega_T = (X_T, H_T, M_T, Q_T, Y_T^p).$$

With standardized variables, using notation $\omega_T = (x_T, h_T, m_T, Q_T)$, the value function is

$$V_T(\omega_T) = U(c_T, s_T, \tilde{c}_T)$$

$$= U\left(\frac{C_T}{Y_T^p}, \frac{S_T}{Y_T^p}, \tilde{C}_T\right)$$

$$= (1 + \lambda) \frac{((\frac{C_T}{Y_T^p})^\alpha (\frac{S_T}{Y_T^p})^{1-\alpha})^{1-\rho}}{1-\rho} - \lambda \frac{(\tilde{C}_T (\frac{X_T}{Y_T^p}, \frac{H_T}{Y_T^p})(\frac{S_T}{Y_T^p})^{1-\alpha})^{1-\rho}}{1-\rho}$$

First I should get the relationship between $\tilde{C}_T (\frac{X_T}{Y_T^p}, \frac{H_T}{Y_T^p})$ and $\tilde{C}_T (X_T, H_T)$

$$\tilde{C}_T \left(\frac{X_T}{Y_T^p}, \frac{H_T}{Y_T^p}\right) = \frac{X_T}{Y_T^p} + \delta Q_T \frac{H_T}{Y_T^p}$$

$$= \frac{1}{Y_T^p} (X_T + \delta Q_T H_T)$$

So

$$\tilde{c}_T = \tilde{C}_T \left(\frac{X_T}{Y_T^p}, \frac{H_T}{Y_T^p}\right) = \frac{\tilde{C}_T (X_T, H_T)}{Y_T^p}$$

Hence the value function can be rewritten as

$$V_T(\omega_T) = (1 + \lambda) \frac{((\frac{C_T}{Y_T^p})^\alpha (\frac{S_T}{Y_T^p})^{1-\alpha})^{1-\rho}}{1-\rho} - \lambda \frac{(\tilde{C}_T (\frac{X_T}{Y_T^p}, \frac{H_T}{Y_T^p})(\frac{S_T}{Y_T^p})^{1-\alpha})^{1-\rho}}{1-\rho}$$

$$= \frac{1}{(Y_T^p)^{1-\rho}} \left\{ (1 + \lambda) (\frac{C_T S_T^{1-\alpha}}{1-\rho})^{1-\rho} - \lambda \frac{(\tilde{C}_T S_T^{1-\alpha})^{1-\rho}}{1-\rho} \right\}$$

Hence I get

$$V_T(\Omega_T) = (Y_T^p)^{1-\rho} V_T(\omega_T)$$

The value function at age $t = T - 1$ is

$$V_{T-1}(\Omega_{T-1}) = (Y_{T-1}^p)^{1-\rho} \max_{c_{T-1}, s_{T-1}, \tilde{c}_{T-1}} \left\{ U(c_{T-1}, s_{T-1}, \tilde{c}_{T-1}) + E_{T-1} \left[ \beta \left(\frac{Y_{T-1}^p}{Y_{T-1}^p}\right)^{1-\rho} V_T(\omega_T) \right] \right\}$$
And similarly to the previous result, the simple relationship I get is

\[ V_{T-1}(\Omega_{T-1}) = (Y_{T-1}^P)^{1-\rho}V_{T-1}(\omega_{T-1}) \]

It can be shown that this relationship holds at a generic time \( t \), hence the value function and the standardized value function at any point in time only differ by a scale factor. It is equivalent to maximize either function.

**A.4 Solution**

The stationary value function for the problem takes the following form

\[ V_t(\omega_t) = \max_{c_t, i_t, v_t} \left\{ U(c_t, s_t, \tilde{c}_t) + \mathbb{E}_t[\beta_{t+1}^*V_{t+1}(\omega_{t+1})] \right\} \]

with

\[ \beta_{t+1}^* = \beta \left( \frac{Y_{t+1}^P}{Y_t^P} \right)^{1-\rho} \]

subject to the standardized budget constraints

\[ x_{t+1} \leq R_X x_t - c_t - Q_i t + v_t \frac{Y_t^P}{Y_{t+1}^P} + \frac{Y_{t+1}}{Y_{t+1}^P} \]

\[ h_{t+1} \leq (h_t + i_t) \frac{Y_{t+1}^P}{Y_{t+1}} \]

\[ m_{t+1} \leq (R_M m_t + v_t) \frac{Y_{t+1}^P}{Y_{t+1}} \]

From equations (15)-(17), I know that

\[ \frac{Y_t^P}{Y_{t+1}^P} = \begin{cases} \frac{1}{\bar{G}_{t+1} N_{t+1}} & \text{if } t \leq W \\ 1 & \text{if } t > W \end{cases} \]

and

\[ \frac{Y_{t+1}^P}{Y_{t+1}} = \begin{cases} Z_{t+1} & \text{if } t \leq W \\ a & \text{if } t = W + 1 \\ 1 & \text{if } t > W + 1 \end{cases} \]

The first order condition of the derived value function \( V_t(\omega_t) \) with respect to \( c_t \) is:

\[ \frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial c_t} = \mathbb{E}_t \left[ \beta_{t+1}^* \frac{Y_{t+1}^P}{Y_{t+1}} R_X \frac{\partial V_{t+1}(\omega_{t+1})}{\partial x_{t+1}} \right] \]
Let us define from now on $\beta_{t+1} = \beta_t \frac{Y_t}{t+1} = \beta \left( \frac{Y_{t+1}^{Y_t}}{Y_{t+1}} \right)^{-p}$. 

$$\frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial c_t} = \mathbb{E}_t \left[ \beta_{t+1} X_t \frac{\partial V_{t+1}(\omega_{t+1})}{\partial x_{t+1}} \right]$$ \hspace{1cm} (A.1)

The first order condition with respect to $i_t$ is:

$$\mathbb{E}_t \left[ R_{t+1} X_t \frac{\partial V_{t+1}(\omega_{t+1})}{\partial x_{t+1}} \right] = \mathbb{E}_t \left[ \frac{1}{Q_t} \frac{\partial V_{t+1}(\omega_{t+1})}{\partial h_{t+1}} \right]$$ \hspace{1cm} (A.2)

The first order condition with respect to $\Upsilon_t$ is:

$$\mathbb{E}_t \left[ R_{t+1} X_t \frac{\partial V_{t+1}(\omega_{t+1})}{\partial x_{t+1}} \right] = -\mathbb{E}_t \left[ \frac{\partial V_{t+1}(\omega_{t+1})}{\partial m_{t+1}} \right]$$ \hspace{1cm} (A.3)

The envelope condition with respect to $x_t$ is

$$\frac{\partial V_t(\omega_t)}{\partial x_t} = \frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial c_t} + \mathbb{E}_t \left[ \beta_{t+1} X_t \frac{\partial V_{t+1}(\omega_{t+1})}{\partial x_{t+1}} \right]$$ \hspace{1cm} (A.4)

The envelope condition with respect to $h_t$ is

$$\frac{\partial V_t(\omega_t)}{\partial h_t} = b \frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial s_t} + \delta Q_t \frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial \tilde{c}_t} + \mathbb{E}_t \left[ \beta_{t+1} X_t \frac{\partial V_{t+1}(\omega_{t+1})}{\partial h_{t+1}} \right]$$ \hspace{1cm} (A.5)

The envelope condition with respect to $m_t$ is

$$\frac{\partial V_t(\omega_t)}{\partial m_t} = \mathbb{E}_t \left[ \beta_{t+1} R_{t+1} X_t \frac{\partial V_{t+1}(\omega_{t+1})}{\partial h_{t+1}} \right]$$ \hspace{1cm} (A.6)

Using equations (A.1) and (A.4) it turns out that

$$\frac{\partial V_t(\omega_t)}{\partial x_t} = \frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial c_t} + \frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial \tilde{c}_t}$$ \hspace{1cm} (A.7)

Using equations (A.1)-(A.3) and (A.5)

$$\frac{\partial V_t(\omega_t)}{\partial h_t} = b \frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial s_t} + \delta Q_t \frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial \tilde{c}_t} + Q_t \frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial c_t}$$ \hspace{1cm} (A.8)
Using equations (A.1), (A.2) and (A.6)

\[
\frac{\partial V_t(\omega_t)}{\partial m_t} = -R_M \frac{\partial U_t(c_t, s_t, \tilde{c}_t)}{\partial c_t} \tag{A.9}
\]

Hence I can rewrite the first order conditions using (A.7)-(A.9)

\[
\frac{\partial U_t}{\partial c_t} = E_t \left[ \beta_t+1 \frac{\partial U_{t+1}}{\partial c_t} + \frac{\partial U_{t+1}}{\partial \tilde{c}_{t+1}} \right]
\]

\[
\frac{\partial U_t}{\partial c_t} = E_t \left[ \beta_t+1 \frac{1}{Q_t} \left( b \frac{\partial U_{t+1}}{\partial s_{t+1}} + \delta Q_t \frac{\partial U_{t+1}}{\partial \tilde{c}_{t+1}} + Q_t \frac{\partial U_{t+1}}{\partial c_{t+1}} \right) \right]
\]

Now using the utility function defined by equations (1) and (3):

\[
U_t(c_t, s_t, \tilde{c}_t) = \left( c_t^{\alpha} s_t^{1-\alpha} \right)^{1-\rho} - \lambda \left( \left( c_t^{\alpha} s_t^{1-\alpha} \right)^{1-\rho} - \lambda \left( c_t^{\alpha} s_t^{1-\alpha} \right)^{1-\rho} \right)
\]

where

\[
\tilde{c}_t = x_t + \delta Q_t h_t
\]

\[
s_t = bh_t
\]

I can derive Euler equations for the different assets.

The **Euler equation for cash-on-hand**

\[
(1 + \lambda)\alpha c_t^{\alpha} s_t^{1-\alpha} = E_t \left[ \beta_t+1 R_{t+1} \left( (1 + \lambda)\alpha \frac{c_{t+1}^{\alpha} s_{t+1}^{1-\alpha}}{c_t} - \lambda \frac{c_{t+1}^{\alpha} s_{t+1}^{1-\alpha}}{\tilde{c}_{t+1}} \right) \right]
\]

The **Euler equation for housing**

\[
(1 + \alpha)\alpha c_t^{\alpha} s_t^{1-\alpha} = E_t \left[ \beta_t+1 \left( (1 + \lambda)\alpha \frac{c_{t+1}^{\alpha} s_{t+1}^{1-\alpha}}{c_t} + \frac{(1 - \alpha)(1 + \lambda)b c_{t+1}^{\alpha} s_{t+1}^{1-\alpha}}{Q_t s_{t+1}} \right) \right.
\]

\[\left. - \delta \lambda \alpha \frac{c_{t+1}^{\alpha} s_{t+1}^{1-\alpha}}{\tilde{c}_{t+1}} - \frac{(1 - \alpha)\lambda b c_{t+1}^{\alpha} s_{t+1}^{1-\alpha}}{Q_t s_{t+1}} \right]
\]

And the **Euler equation for mortgage**
The Euler equation for the liquid asset shows that the marginal cost of giving up one unit
of current consumption - invested in the liquid asset - must be equal to the marginal ben-
efit of consuming the proceeds of the extra saving in the next period, minus the marginal
cost of resisting the additional temptation in the next period, caused by the higher sav-
ings in the liquid asset. Hence the cost of saving is higher for tempted households than
for non-tempted ones, everything else being equal. Note that the marginal benefit of
consumption for next period has two terms in this case: the utility from consuming the
extra consumption and the temptation value of this extra consumption.

The Euler equation for housing shows that the marginal cost of giving up one unit of
current consumption - invested in housing - must be equal to the marginal benefit of the
resulting extra savings next period minus the marginal cost of resisting the additional
temptation in the next period caused by the higher savings in housing stock.
The marginal benefit here comes not only from the consumption of the resulting extra savings and its extra temptation value next period - as in the liquid asset case - but
also from the extra utility of enjoying more housing service by having the extra housing
stock. The marginal cost has two sources. On the one hand extra saving in housing
stock increases the next period temptation value of the best available alternative itself (as
$\tilde{C}_t = \tilde{C}_t(X_t, H_t)$). On the other hand, because of higher housing stock, the temptation
value of a given best alternative gets higher next period.
The first term of the marginal cost depend on the liquidity of housing (parameter $\delta$).
The temptation cost is mitigated more, the more illiquid is housing. When housing is
completely illiquid ($\delta = 0$) this cost term is eliminated.

The Euler equation for mortgage shows that the marginal benefit of using an addi-
tional unit of mortgage for consumption today must be equal to the marginal cost of
the this

A.5 Log-linearizing the Euler Equation for Cash-on-Hand

I express the Euler equation (22) in the following fashion

$$1 = \mathbb{E}_t k_{t+1} R_{t+1}$$

where $k_{t+1}$ is the pricing kernel.

$$k_{t+1} = \beta_{t+1} \frac{c_t}{(c_{t+1} s_{t+1}^{1-\alpha})^{1-\rho}} \left( (c_{t+1} s_{t+1}^{1-\alpha})^{1-\rho} \right) 
- \frac{\lambda}{1 + \lambda} \left( \frac{c_{t+1}}{\tilde{c}_{t+1}} \right)$$
In my case, the pricing kernel simplifies to

\[ k_{t+1} = \beta_{t+1} \left( \frac{c_t}{c_{t+1}} \right)^{\alpha(1-\rho)} \left[ \frac{1 - \lambda}{1 + \lambda} \left( \frac{c_{t+1}}{c_t} \right)^{\alpha(1-\rho)} \right] \]

\[ = \beta_{t+1} \left( \frac{c_{t+1}}{c_t} \right)^{-1+\alpha(1-\rho)} \left( \frac{s_{t+1}}{s_t} \right)^{(1-\alpha)(1-\rho)} \left[ 1 - \frac{\lambda}{1 + \lambda} \left( \frac{\hat{c}_{t+1}}{c_{t+1}} \right)^{1+\alpha(1-\rho)} \right] \]

Let us denote now \( \kappa = 1 - \alpha(1-\rho) \) to get

\[ k_{t+1} = \beta_{t+1} \left( \frac{c_{t+1}}{c_t} \right)^{\kappa} \left( \frac{s_{t+1}}{s_t} \right)^{(1-\alpha)(1-\rho)} \left[ 1 - \frac{\lambda}{1 + \lambda} \left( \frac{\hat{c}_{t+1}}{c_{t+1}} \right)^{\kappa} \right] \]

Since all the variables involved in the pricing kernel are assumed to be stationary, one can take a log-linear approximation of that around the steady state. First let us take the log of the pricing kernel

\[ \ln k_{t+1} = \ln \beta_{t+1} - \kappa \ln \left( \frac{c_{t+1}}{c_t} \right) + (\kappa - \rho) \ln \left( \frac{s_{t+1}}{s_t} \right) \]

\[ + \ln \left[ 1 - \frac{\lambda}{1 + \lambda} \left( \frac{\hat{c}_{t+1}}{c_{t+1}} \right)^{-\kappa} \right] \]

Now taking the first order Taylor approximation around the steady state

\[ \frac{1}{k} (k_{t+1} - k) = \frac{1}{\beta} (\beta_{t+1} - \beta) - \kappa \left[ \frac{c_{t+1}}{c_t} - 1 \right] + (\kappa - \rho) \left[ \frac{s_{t+1}}{s_t} - 1 \right] \]

\[ + \frac{\kappa}{1 + \lambda} \left( \frac{\hat{c}}{c} \right)^{-\kappa} \left[ \frac{\hat{c}}{c} \left( \frac{\hat{c}_{t+1}}{c_{t+1}} - \frac{\hat{c}}{c} \right) \right] \]

If I denote the percentage deviation from the steady state by \( \hat{x} = \frac{x_t - x}{x} \), then this relationship becomes

\[ \hat{k}_{t+1} = \hat{\beta}_{t+1} - \kappa \left( \frac{c_{t+1}}{c_t} \right) + (\kappa - \rho) \left( \frac{s_{t+1}}{s_t} \right) + \kappa \left[ 1 - \frac{\lambda}{1 + \lambda} \left( \frac{\hat{c}}{c} \right)^{-\kappa} \right] \left( \frac{\hat{c}_{t+1}}{c_{t+1}} \right) \]

where I can use

\[ \phi = \frac{\lambda}{1 + \lambda} \left( \frac{\hat{c}}{c} \right)^{-\kappa} = \frac{\lambda}{\frac{1}{1 + \lambda} \left( \frac{\hat{c}}{c} \right)^{-\kappa} - \lambda} = \frac{\lambda}{(1 + \lambda) \left( \frac{\hat{c}}{c} \right)^{\kappa} - \lambda} \]

41
So the log-linearized pricing kernel becomes

\[ \hat{k}_{t+1} = \hat{\beta}_{t+1} - \kappa \left( \frac{c_{t+1}}{c_t} \right) + (\kappa - \rho) \left( \frac{s_{t+1}}{s_t} \right) + \kappa \phi \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) \]

In what follows, I use the approximation:

\[ \hat{x}_t \approx \ln x_t - \ln k_t + 1 - \ln \beta + \kappa \ln \left( \frac{c_{t+1}}{c_t} \right) + (\kappa - \rho) \ln \left( \frac{s_{t+1}}{s_t} \right) + \kappa \phi \left[ \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) - \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) \right] + \eta_{t+1} \]

Now in the steady state

\[ \ln k = \ln \beta + \ln \left[ 1 - \frac{\lambda}{1 + \lambda} \left( \frac{\tilde{c}}{c} \right)^{-\kappa} \right] \]

Substituting this steady state relationship into the previous equation

\[ \ln k_{t+1} - \ln k = \ln \beta_{t+1} - \ln \beta - \kappa \ln \left( \frac{c_{t+1}}{c_t} \right) + (\kappa - \rho) \ln \left( \frac{s_{t+1}}{s_t} \right) + \kappa \phi \left[ \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) - \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) \right] + \eta_{t+1} \quad (A.10) \]

I can rewrite

\[ -\ln \left[ 1 - \frac{\lambda}{1 + \lambda} \left( \frac{\tilde{c}}{c} \right)^{-\kappa} \right] = \ln \left[ 1 - \frac{\lambda}{1 + \lambda} \left( \frac{\tilde{c}}{c} \right)^{-\kappa} \right]^{-1} = \ln \left( \frac{1}{1 - \frac{\lambda}{1 + \lambda} \left( \frac{\tilde{c}}{c} \right)^{-\kappa}} \right) \]

Which equals \( \ln(1 + \phi) \).

\[ \ln(1 + \phi) = \ln \left( \frac{\frac{\lambda}{1 + \lambda} \left( \frac{\tilde{c}}{c} \right)^{-\kappa}}{1 - \frac{\lambda}{1 + \lambda} \left( \frac{\tilde{c}}{c} \right)^{-\kappa}} \right) = \ln \left( \frac{1}{1 - \frac{\lambda}{1 + \lambda} \left( \frac{\tilde{c}}{c} \right)^{-\kappa}} \right) \]
Now equation (A.10) can be rewritten

\[
\ln k_{t+1} = \ln \beta_{t+1} - \ln(1 + \phi) - \kappa \ln \left( \frac{c_{t+1}}{c_t} \right) + (\kappa - \rho) \ln \left( \frac{s_{t+1}}{s_t} \right)
\]

\[
+ \kappa \phi \left[ \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) - \ln \left( \frac{\tilde{c}}{c} \right) \right] + \eta_{t+1}
\]

(A.11)

And the linearized Euler equation

\[
0 = \ln R_{t+1}^X + E_t \ln k_{t+1}
\]

becomes

\[
\kappa \ln \left( \frac{c_{t+1}}{c_t} \right) = \ln R_{t+1}^X + \ln \beta_{t+1} - \ln(1 + \phi) + (\kappa - \rho) \ln \left( \frac{s_{t+1}}{s_t} \right)
\]

\[
+ \kappa \phi \left[ \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) - \ln \left( \frac{\tilde{c}}{c} \right) \right] + \eta_{t+1}
\]

where \( \eta_{t+1} \) already contains expectation errors, measurement errors and deviations of second and higher moments from their unconditional means. Hence I can obtain an empirical version of the consumption Euler equation in the presence of temptation and housing in the utility function:

\[
\ln \left( \frac{c_{t+1}}{c_t} \right) = \frac{1}{\kappa} \ln R_{t+1}^X + \frac{1}{\kappa} \ln \beta_{t+1} - \frac{1}{\kappa} \ln(1 + \phi) + \frac{\kappa - \rho}{\kappa} \ln \left( \frac{s_{t+1}}{s_t} \right)
\]

\[
+ \phi \ln \left( \frac{\tilde{c}_{t+1}}{c_{t+1}} \right) - \phi \ln \left( \frac{\tilde{c}}{c} \right) + \eta_{t+1}
\]

This equation can be further simplified to get equation (13), which I use in my estimation.

\section*{A.6 MA(1) Structure of the Error Terms}

I write the estimatable equation as \( y = X\beta + u \), where \( X \) denotes the matrix of \( k \) explanatory variables and \( u \) is the error term. Given the previously detailed MA(1) error structure, valid instruments are the exogenous variables in \( X \), deterministic contemporaneous variables and second and further lags of remaining variables. Denoting the matrix of instruments by \( Z \) (with more instruments, \( m \) than explanatory variables, \( k \)), the instrumental estimator I use in the paper is given by the following expression:

\[
\widehat{\beta}_{GMM} = \left[ X'Z(Z'Z)^{-1}Z'X \right]^{-1} \left[ X'Z(Z'Z)^{-1}Z'y \right]
\]

(A.12)
and the asymptotic variance-covariance matrix by:

\[
\hat{V}_{\beta_{GMM}} = [X'Z'(Z'Z)^{-1}Z'X]^{-1}[X'Z'(Z'Z)^{-1}S(Z'Z)^{-1}Z'X][X'Z'(Z'Z)^{-1}Z'X]^{-1} \tag{A.13}
\]

where \( S \) is an \( NT \times NT \) block matrix, \( N \) denoting the number of cohorts and \( T \) the number of time periods the cohorts are observed. Each block on the main diagonal is a \( T \times T \) matrix given by the variance-covariance matrix of the residuals of one cohort. Because of the presence of \( MA(1) \) structure in the residuals for a particular cohort with parameter \(-1\), these matrices have nonzero elements in the main diagonal and the first off-diagonals. The off-diagonal blocks of \( S \) represent the correlation of residuals of different cohorts. I assume that only contemporaneous correlation is possible, hence these matrices are diagonal.

For each cohort I can find the first-off diagonal elements if I calculate the covariances between the error in given period and the period before/after. The error term in equation (31) has two elements: the \( MA(1) \) term which represents the measurement errors plus the white noise component.

\[
u_{t+1} = me_{t+1} - me_t + v_{t+1}
\]

where \( me \) refers to the measurement in period \( t \) and \( v \) to the white noise component (with given variance \( \sigma_v^2 \)). I assume that the measurement error is also a random variable with zero mean and given variance \( (\sigma_{me}^2) \). Hence

\[
E(u_{t+1}) = 0
\]

the variance is

\[
Var(u_{t+1}) = E[(u_{t+1} - 0)(u_{t+1} - 0)]
= E[(me_{t+1} - me_t + v_{t+1})(me_{t+1} - me_t + v_{t+1})]
= \sigma_v^2 + 2\sigma_{me}^2
\]

and the covariance becomes

\[
Cov(u_{t+1}, u_t) = E[(me_{t+1} - me_t + v_{t+1})(me_t - me_{t-1} + v_t)]
= -\sigma_{me}^2
\]
A.7 EIS under Temptation Preferences

It needs a bit of derivation to prove that the response of consumption growth to the real interest rate change indeed has to be greater under temptation preferences. Let’s first take the total derivative of the Euler equation (31) with respect to the real interest rate.

\[
\frac{\partial \Delta \ln(c_{t+1})}{\partial r_{t+1}^X} \approx \theta_1 \frac{\partial \ln(1 + r_{t+1}^X)}{\partial r_{t+1}^X} + \theta_2 \frac{\partial \Delta \ln(s_{t+1})}{\partial r_{t+1}^X} + \theta_3 \left( \frac{\partial \ln \tilde{c}_{t+1}}{\partial r_{t+1}^X} - \frac{\partial \ln c_{t+1}}{\partial r_{t+1}^X} \right) + \gamma \frac{\partial \Delta z_{t+1}}{\partial r_{t+1}^X}
\]

The only difference here, compared to the same relationship derived from a model with standard preferences is the term corresponds to parameter \(\theta_3\). Therefore this is the term which leads to the result of doubled elasticity of intertemporal substitution. Let’s now restrict our attention to the effect of the real interest rate and the temptation on change in consumption growth. In Table 6 I show the exercise of explaining the observed data with the two different models. In case I observe 1% increase in the real interest rate and 0.8% increase in the consumption growth for example, the standard model could explain this with an elasticity of substitution parameter of 0.8. The same observation under temptation preferences though would predict an EIS of around 1.5. In this case though, there is the additional term capturing temptation, with estimated parameter (\(\theta_3\)) of 0.05. These estimated parameters predict that the temptation term has to has to decrease by 14% in order to be able to match the data.

<table>
<thead>
<tr>
<th>Model</th>
<th>(\Delta \ln(c_{t+1}))</th>
<th>(\Delta \ln(c_{t+1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard</td>
<td>0.8%</td>
<td>(\approx 0.8 \cdot 1%)</td>
</tr>
<tr>
<td>Temptation</td>
<td>0.8%</td>
<td>(\approx 1.5 \cdot 1%) + 0.05 \cdot (-14%)</td>
</tr>
</tbody>
</table>

Table 6: ESTIMATES OF PARAMETERS

Recall

\[
\tilde{c}_{t+1} = x_{t+1} + \delta Q_{t+1} h_{t+1}
\]

Now I write the unit house price as the present discounted value of future stream of housing service in terms of consumption goods, assuming that the interest rate is persistent.

\[
Q_{t+1} = \sum_{j=0}^{\infty} \left( \frac{1}{1 + r_{t+1}^X} \right)^j bp
\]

(A.14)

where \(p\) is the relative price of housing service compared to nondurable consumption. The relative price can be written as the fraction of the marginal utility of housing service and the marginal utility of consumption. For the ease of derivation, I assume that the
relative price is constant over time.

\[ p = \frac{\partial u_s}{\partial u_c} \]

Now using the formula for the sum of geometric series, I get the following simple expression for the unit house price

\[ Q_{t+1} = \frac{1 + r_{t+1}^X}{r_{t+1}^X} bp \] (A.15)
References


