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MULTIPRODUCT MONOPOLY MADE SIMPLE

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Abstract

We present a tractable class of multiproduct monopoly models that involve a generalized form of homothetic preferences. This class includes CES, linear and logit demand. Within the class, profit-maximizing quantities are proportional to efficient quantities. We discuss cost-passthrough, including cases where optimal prices do not depend on other products' costs. We show how the analysis can be extended to Cournot oligopoly. Finally, we discuss optimal monopoly regulation when the firm has private information about its vector of marginal costs, and show that if the probability distribution over costs satisfies an independence property, then optimal regulation leaves relative price decisions to the firm.

Keywords: Multiproduct pricing, homothetic preferences, cost passthrough, monopoly regulation, multidimensional screening.

JEL Classification: D42, D82, L12, L51.

1   Introduction

The theory of monopoly is a large and diverse subject. Classical questions include the characterization of optimal pricing by a multiproduct monopolist seeking to maximize profit or social welfare or, as with Ramsey pricing, a combination of both. Monopoly questions have also been central to the economics of asymmetric information, as for example in the analysis of nonlinear pricing (where the monopolist has incomplete information about the demands of heterogeneous consumers) and of optimal regulation (where the regulator has incomplete information about the monopolist's costs or demands).

In this paper we show how a range of issues in the theory of multiproduct monopoly can be addressed in a relatively simple manner in class of multiproduct demand systems defined by net consumer surplus being a homothetic function of quantities (section 2). As shown in section 3, this is quite a broad class of demand systems, and as well as the obvious

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case of homothetic preferences it includes linear demand and logit demand, just to name some familiar examples. The class encompasses a wide variety of forms of substitutability and complementarity between products.

The key implication of our demand structure is that, with constant returns to scale technology, profit-maximizing quantities, as well as quantities with Ramsey pricing, are proportional to efficient quantities—i.e., quantities with marginal cost pricing. We also find that price-cost margins in the Ramsey problem are proportional to the margins chosen by an unregulated monopolist.

The proportionality property greatly simplifies multiproduct monopoly analysis, and the problem of profit maximization can be solved in two easy steps (section 4). The first is to calculate the efficient pattern of quantities which correspond to marginal cost pricing. The second is to solve for the scalar factor by which to reduce the efficient quantities to achieve maximum profit. The proportionality property also simplifies comparative static analysis, such as how monopoly prices vary with cost parameters. In some leading examples there is zero "cross-cost" passthrough—i.e., the profit maximizing price of each product depends only on its own cost. In such cases, a product's price is the price the firm would choose if it had only that product to sell. Section 5 departs from monopoly to show how these techniques can extend to multiproduct Cournot oligopoly, when firms again choose the efficient pattern of relative quantities in symmetric equilibrium.

The fact that, within our demand structure, a profit-maximizing firm has efficient incentives with respect to the pattern, though of course not the level, of quantities has implications for regulation of multiproduct monopoly (section 6). It suggests that, in our class of demand systems, it might be optimal for regulation to allow the monopolist considerable discretion over the pattern of relative quantities (or prices). If the probability distribution over costs satisfies a particular kind of independence property, this intuition is precise and the choice of relative quantities (or prices) is delegated to the firm.

The desirability of proportional quantity reductions has roots in the theory of optimal taxation. Thus Diamond and Mirrlees (1971) show that, in the absence of redistributive concerns, if optimal commodity taxes are small, then they entail an approximately equal percentage reduction in compensated demands relative to marginal cost pricing. For demand systems in our class, this property holds exactly and not just locally.

The optimal regulation of multi-product monopoly is analyzed by Laffont and Tirole

(1993, chapter 3). In their main model, cost outturns are observable but the regulator cannot observe cost-reducing effort or the firm's underlying cost type. If the cost function is separable between quantities on the one hand and the firm's effort and type on the other, then the "incentive-pricing dichotomy" holds—pricing should not be used to provide effort incentives. If there is a social cost of public funds, Ramsey pricing is therefore optimal, as characterized by "super-elasticity" formulas for markups. In our family of demand systems such expressions are considerably simplified. The analysis of regulation in section 6 below does not consider effort incentives, but is for the situation with unobservable and multi-dimensional costs. The analysis, which builds on the approach followed by Armstrong (1996) and Armstrong and Vickers (2001), contributes to the theory of mechanism design by exhibiting a tractable multi-dimensional screening problem.

2   A family of demand systems

Suppose a monopolist supplies n products, where the quantity of product i is denoted x_i and the vector of quantities is $x = (x_1, ..., x_n)$. Consumers have quasi-linear utility, which implies that demand can be considered to be generated by a single representative consumer with gross utility function $u(x)$ defined on $x \in \mathbb{R}^n_+$, where $u(0) = 0$ and u is concave.[1] We suppose that u is differentiable throughout the interior of $\mathbb{R}^n_+$, although demand might be unbounded when some prices are zero. The consumer chooses quantities $x \in \mathbb{R}^n_+$ in order to maximize $u(x) - p \cdot x$, where $a \cdot b \equiv \sum_{i=1}^n a_i b_i$ denotes the dot product of two vectors a and b. The price vector which induces interior quantity vector x to be demanded, i.e., the inverse demand function $p(x)$, is

$$p(x) \equiv \nabla u(x) \ ,$$

where we use the gradient notation $\nabla f(x) \equiv (\partial f(x)/\partial x_1, ..., \partial f(x)/\partial x_1)$ for the vector of partial derivatives of a function f. We suppose that u is increasing in x over the relevant range, so that prices $p(x)$ are positive. The revenue generated by selling quantities x is

$$r(x) = x \cdot \nabla u(x)$$

and the consumer's net surplus from consuming quantities x is

$$s(x) \equiv u(x) - r(x) \ ,$$

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[1] For some demand systems, utility is defined only for quantities which lie in a convex subset of $\mathbb{R}^n_+$. (For instance, the logit system we discuss later has utility defined on only for $\sum_i x_i < 1$.) The following analysis is easily adapted to such cases.

where the concavity of u implies $s(x) \geq 0$.

Suppose the monopolist has differentiable cost function $c(x)$, so that $\nabla c(x)$ is the vector of marginal costs. Without regulation, the firm chooses quantity vector x^π to maximize its profit $r(x) - c(x)$. Assuming an interior solution, this problem has first-order condition

$$0 = \nabla r(x) - \nabla c(x) = \nabla[u(x) - s(x)] - \nabla c(x) = p(x) - \nabla s(x) - \nabla c(x) \ ,$$

or

$$p(x^\pi) = \nabla c(x^\pi) + \nabla s(x^\pi) \ . \tag{1}$$

Thus, the profit-maximizing price for product i is above the associated marginal cost if and only if net consumer surplus $s(x)$ increases with x_i at the optimal quantities x^π. Of course, total welfare, $u(x) - c(x)$, is maximized at quantities x^w which involve price equal to marginal cost, so that $p(x^w) = \nabla c(x^w)$. A comparative statics argument shows that $s(x^\pi) \leq s(x^w)$, and consumer surplus is higher when total welfare is the objective.

In an interesting set of cases, the profit-maximizing quantity vector x^π is proportional to the welfare-maximizing quantity vector x^w. As will be shown shortly, this is the case whenever net consumer surplus $s(x)$ is a homothetic function of x, i.e., if $s(x)$ can be written as $s(x) = G(q(x))$ where $q(x)$ is homogeneous degree 1. We first characterize when this is the case:

Lemma 1 *Consumer surplus $s(x)$ is homothetic in x if and only if gross utility can be written in the form*

$$u(x) = h(x) + g(q(x)) \tag{2}$$

where $h(\cdot)$ and $q(\cdot)$ are homogeneous degree 1 functions.

Proof. To show sufficiency, note that (2) implies that inverse demand is

$$p(x) = \nabla u(x) = \nabla h(x) + g'(q(x))\nabla q(x) \ . \tag{3}$$

Revenue is

$$r(x) = x \cdot p(x) = h(x) + g'(q(x))q(x) \ , \tag{4}$$

where we used the fact that $x \cdot \nabla h(x) \equiv h(x)$ for a homogenous degree 1 function. Consumer surplus $s(x)$ is therefore

$$s(x) = g(q(x)) - g'(q(x))q(x) \ , \tag{5}$$

which is homothetic since $s(x)$ is a function of x via the homogenous function $q(x)$.

To show necessity, let $\tilde{x}$ be an arbitrary vector of quantities and observe that for any scalar $k > 0$ we have that consumer surplus

$$s(\tilde{x}/k) = \frac{d}{dk}ku(\tilde{x}/k) \ . \tag{6}$$

Suppose that consumer surplus $s(x)$ is homothetic, so that $s(x) \equiv G(q(x))$ for some function $q(x)$ which is homogeneous degree 1. It is more convenient to write $G(q) \equiv g(q) - qg'(q)$ for another function $g(\cdot)$.[2] Then

$$s(\tilde{x}/k) = g(q(\tilde{x})/k) - \frac{q(\tilde{x})}{k}g'(q(\tilde{x})/k) = \frac{d}{dk}kg(q(\tilde{x})/k) \ . \tag{7}$$

From (6) and (7) we have by integration that

$$ku(\tilde{x}/k) = h(\tilde{x}) + kg(q(\tilde{x})/k)$$

for some function h. Writing $x = \tilde{x}/k$ this becomes

$$u(x) = \frac{h(kx)}{k} + g(q(x)) \ .$$

Since this holds for all k we deduce that $u(x) = h(x) + g(q(x))$, where $h(x)$ is homogeneous degree 1 as desired. ∎

A natural question is how this family (2) relates to those demand systems where consumer surplus is homothetic in prices rather than quantities. Expressed as a function of prices, consumer surplus is the convex function $v(p) = \max_{x \geq 0}\{u(x) - p \cdot x\}$. Duality implies that $u(x)$ can be recovered from $v(p)$ using the procedure $u(x) = \min_{p \geq 0}\{v(p) + p \cdot x\}$, and it is straightforward to show that if $v(p)$ is homothetic in p then $u(x) = \min\{v(p) + p \cdot x\}$ is homothetic in x. Thus, the utility functions such that consumer surplus is homothetic in prices are simply the homothetic utility functions, i.e., where $h \equiv 0$ in (2), which is a strict subset of the family of preferences we study.

For the remainder of the paper we assume that utility can be written in the form (2). We refer to $q(x)$ as the "composite" quantity, and consumer surplus $s(x)$ depends only on

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[2]Given any function $G(\cdot)$, one can generate the corresponding $g(\cdot)$ using the procedure

$$g(q) = -q \int^q \frac{G(\tilde{q})}{\tilde{q}^2}d\tilde{q} \ .$$

this composite quantity. For a specific utility function $u(x)$ it may not be obvious *a priori* whether it can be written in the form (2). However, Lemma 1 implies that this is the case whenever consumer surplus, $s(x) \equiv u(x) - x \cdot \nabla u(x)$, is homothetic, which in practice is easy to check.

In (2) we must have $g(0) = 0$ given that $u(0) = 0$ and $g(\cdot)$ must be concave given that $u(\cdot)$ is concave.[3] The function $g(q) - g'(q)q$ in (5) is thus an increasing function, and so s increases with x_i—and so the most profitable price for product i is above marginal cost—if and only if q increases with x_i. A sufficient condition for u in (2) to be concave is that h and g be concave <u>and</u> either (i) q is concave and g is increasing or (ii) q is convex and g is decreasing. Note that if u satisfies (2), then the modified environment in which a subset of these products are removed also satisfies (2). That is, if a subset of products have quantities x_i set equal to zero, the utility function u defined on the remaining products continues to satisfy (2).[4]

The consumer with utility (2) can maximize her utility in a two-stage process: first maximize utility subject to buying a given composite quantity, and then choose the composite quantity. In the first stage, faced with price vector p the consumer maximizes

$$h(x) + g(q(x)) - p \cdot x \tag{8}$$

subject to $q(x) = Q$, say. Define the function

$$\phi(p) \equiv \min_{x \geq 0} : \{ p \cdot x - h(x) \mid q(x) = 1 \} , \tag{9}$$

which is a generalized "expenditure function" for buying a unit of composite quantity at prices p. The function ϕ, which might be negative, is increasing and concave in p, and (when differentiable) it differentiates to yield $x^*(p)$, the vector of quantities which solves problem (9). In particular, since ϕ is concave, increasing the price p_i will weakly decrease $x_i^*(p)$. Since $p \cdot x - h(x)$ and $q(x)$ are both functions that are homogeneous degree 1, if $x^*(p)$ solves the minimization problem (9) then $Qx^*(p)$ maximizes net surplus (8) subject to $q(x) = Q$. Thus, the consumer chooses the same pattern of relative quantities, $x^*(p)$, for each target level of composite output Q.

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[3]Since u is concave, the function $k \to u(kx)$ is concave for given x, so when u takes the form (2) this implies that the function $kh(x) + g(kq(x))$ is concave in k, so that $g(\cdot)$ is itself concave.

[4]However, if $q = 0$ whenever one $x_i = 0$, which is a strong form of product complementarity, then the firm must supply all products to generate any consumer surplus.

In the second stage the consumer chooses composite quantity Q to maximize the concave function $g(Q) - Q\phi(p)$, and so faced with prices p the consumer's demand is $x(p) = Qx^*(p)$ where Q satisfies the first-order condition $g'(Q) = \phi(p)$. Price vectors which yield the same $\phi(p)$ induce the consumer to choose the same composite quantity Q, and so $\phi(p)$ can be considered to be the "composite" price which corresponds to composite quantity $q(x)$. In particular, inverse demand satisfies

$$\phi(p(x)) \equiv g'(q(x)) . \tag{10}$$

Expression (10) relates composite price to composite quantity, and $g'(Q)$ can therefore be regarded as the inverse demand function for composite quantity (which in our context can be positive or negative). From this perspective it is useful to write

$$\eta(Q) \equiv -\frac{Qg''(Q)}{g'(Q)} \tag{11}$$

for the elasticity of this inverse demand, and to note that the "marginal revenue" of composite output is decreasing if

$$g'(Q) + Qg''(Q) \text{ decreases with } Q . \tag{12}$$

Substituting $x(p) = Qx^*(p)$ into the identity $p \equiv \nabla u(x(p))$, where ∇u is given in (3), shows that the individual product prices satisfy

$$p \equiv \nabla h(x^*(p)) + \phi(p)\nabla q(x^*(p)) . \tag{13}$$

Finally, consider which price vectors induce the same pattern of relative demands, i.e., the same $x^*(p)$.[5] Notice that (3), together with the fact that ∇h and ∇q are homogeneous degree zero, implies that $p(kx) - p(x) = [g'(kq(x)) - g'(q(x))]\nabla q(x)$ is a vector proportional to $\nabla q(x)$. In geometric terms, quantity vectors on the ray joining x to the origin correspond in consumer demand to price vectors which lie on the straight line starting at $p(x)$ pointing in the direction $\nabla q(x)$. Putting this another way, for a given p all prices on the straight line $p + t\nabla q(x^*(p))$, where t is a scalar variable, induce the same pattern of relative demands.

We next list a number of familiar demand systems where utility takes the form (2).

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[5] This is reminscent of the question of linear income-expenditure paths (Engel curves) addressed by Gorman (1961). With preferences of the Gorman polar form, an increase in real income (the equivalent of equi-proportionate price decreases in our setting) causes demands relative to those at a baseline level of utility $x(p) - x_0(p)$ to increase equi-proportionately.

3 Special cases

It is useful here as well as later in the paper to consider special cases of the family (2) in which $h(x)$ or $q(x)$ is linear in x. Four examples of demand systems are discussed in this section, the first two of which have linear $h(x) = a \cdot x$ while the remaining two have linear $q(x) = b \cdot x$.

Homothetic utility: Clearly, if $u(\cdot)$ is itself homothetic then (2) is trivially satisfied by setting $h \equiv 0$. In this case, expression (9) implies that $\phi(p)$ is homogeneous degree 1, and $x^*(p) = \nabla\phi(p)$ is homogeneous degree zero. That is, price vectors which lie on the same ray from the origin (in price space) induce demands which lie on the same ray from the origin (in quantity space).

The leading example of a homothetic utility function is the CES demand system, where

$$u(x) = g\left(\left[\sum_{i=1}^{n} w_i x_i^\rho \right]^{\frac{1}{\rho}} \right) , \tag{14}$$

where $w_i \geq 0$ and $\sum_i w_i = 1$, where $\rho \leq 1$ (but $\rho \neq 0$), and where $g(\cdot)$ is an increasing, strictly concave function. Here, composite quantity $q(x) = (\sum_{i=1}^{n} w_i x_i^\rho)^{1/\rho}$ is an increasing, concave, homogeneous degree 1 function. The limiting case $\rho \to 0$ corresponds to Cobb-Douglas preferences $q(x) = \prod_{i=1}^{n} x_i^{w_i}$, and the limiting case $\rho \to -\infty$ corresponds to perfect complements $q(x) = \min\{x_1, ..., x_n\}$. With CES preferences, the consumer can obtain positive utility from a subset of products, i.e., when some $x_i = 0$, only when $\rho > 0$. One can check that

$$\phi(p) = \left(\sum_i w_i^{\frac{1}{1-\rho}} p_i^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} \quad ; \quad x_i^*(p) = \left(\frac{w_i}{p_i} \right)^{\frac{1}{1-\rho}} \left(\sum_i a_i^{\frac{1}{1-\rho}} p_i^{\frac{\rho}{\rho-1}} \right)^{-\frac{1}{\rho}} . \tag{15}$$

Linear demand: Suppose that $u(x)$ takes the quadratic form

$$u(x) = a \cdot x - \tfrac{1}{2} x^T M x \tag{16}$$

for constant vector $a > 0$ and (symmetric) positive-definite matrix M. Here, inverse demands are $p(x) = a - Mx$, and by writing $h(x) = a \cdot x$, $q(x) = \sqrt{x^T M x}$, $s(x) = \tfrac{1}{2} x^T M x$ and $g(q) = -\tfrac{1}{2} q^2$ (so that $\eta \equiv -1$), utility takes the form (2). Here, $\nabla q(x) = Mx$ and so the set of price vectors which correspond to those quantity vectors lying on a given ray from the origin takes the form $p = a - tMx$ for scalar t, which are rays originating from

the vector of choke prices a. We have

$$\phi(p) = -\sqrt{(a-p)^T M^{-1}(a-p)} \ , \ x^*(p) = \frac{M^{-1}(a-p)}{\sqrt{(a-p)^T M^{-1}(a-p)}} \ .$$

Note that $q(x)$ and $s(x)$ might decrease with x_i when off-diagonal elements of M are negative (which corresponds to products being complements).

Linear demand is an instance of the subclass of (2) where h takes the linear form $h(x) = a \cdot x$, when $\phi(p)$ in (9) is a function that is homogenous degree 1 in the adjusted price vector $(p-a)$.

Logit demand: Suppose that consumer demand takes the logit form

$$x_i(p) = \frac{e^{a_i - p_i}}{1 + \sum_j e^{a_j - p_j}} \ , \tag{17}$$

where $a = (a_1, ..., a_n)$ is a constant vector. It follows that inverse demand is

$$p_i(x) = a_i - \log \frac{x_i}{1 - q(x)} \ , \tag{18}$$

where $q(x) \equiv \sum_j x_j$ is total quantity. This inverse demand function (18) integrates to give the utility function

$$u(x) = a \cdot x - \sum_j x_j \log x_j - (1 - q(x)) \log (1 - q(x)) \ .$$

(As with any demand system resulting from discrete choice, the utility function is only defined on the domain $\Sigma_i x_i \leq 1$.) This utility can be written in the required form (2) as

$$u(x) = \underbrace{a \cdot x + \sum_i x_i \log \frac{q(x)}{x_i}}_{h(x)} + \ g(q(x)) \ .$$

Here, $h(x)$ as labelled is homogenous degree 1, as is total output $q(x)$, while $g(q)$ is equal to the entropy function $g(q) = -q \log q - (1-q) \log(1-q)$, which is concave in $0 \leq q \leq 1$. Since $g'(q) = \log(1-q) - \log q$, demand for composite quantity as a function of composite price takes the logistic form. Consumer surplus is $s(x) = -\log(1 - q(x))$. Thus with logit, as with homogeneous goods, consumer surplus is a function only of total quantity, and product differentiation is reflected separately in the $h(x)$ term. One can check that

$$\phi(p) = -\log \left(\sum_i e^{a_i - p_i} \right) \ , \ x_i^*(p) = \frac{e^{a_i - p_i}}{\sum_j e^{a_j - p_j}} \ . \tag{19}$$

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Since $\nabla q(x) \equiv (1, ..., 1)$, the set of prices which correspond to quantity vectors on a given ray from the origin takes the form $p + (t, ..., t)$, as can be seen directly from $x^*(p)$ in (17).

Independent demands with constant curvature: Suppose inverse demand for product i depends only on x_i, and the price which induces demand x_i for product i is $p_i(x_i)$. Suppose that the curvature of inverse demand, $-x_i p_i''(x_i)/p_i'(x_i)$, does not depend either on x_i or on i. An instance of this is exponential demand, so that direct demand for product i takes the form $x_i(p_i) = e^{(a_i - p_i)/b_i}$ with b_i positive. Inverse demand is therefore $p_i(x_i) = a_i - b_i \log x_i$, where $x_i \leq e^{a_i/b_i}$, and inverse demand curvature is $-x_i p_i''(x_i)/p_i'(x_i) \equiv 1$. In this case, utility is $u(x) = \sum_i x_i(a_i + b_i - b_i \log x_i)$, and if we define $q(x) \equiv b \cdot x$, we can write this utility in the form (2) as

$$u(x) = \underbrace{a \cdot x + \sum_{i=1}^{n} b_i x_i \log \frac{q(x)}{x_i}}_{h(x)} + \; g(q(x))$$

where, as labelled, $h(x)$ is homogenous degree 1 and $g(q) = q(1 - \log q)$.[6] Since $g'(q) = -\log q$, the demand for composite quantity given composite price is also exponential. In this special case consumer surplus is simply the linear function $s(x) = q(x)$. Finally, we have

$$\phi(p) = -\log \left(\sum_i b_i e^{(a_i - p_i)/b_i} \right) - 1 \; , \; x_i^*(p) = \frac{e^{(a_i - p_i)/b_i}}{\sum_j b_j e^{(a_j - p_j)/b_j}} \; .$$

Thus, as with logit demand, exponential demand falls into the subclass of (2) where composite quantity $q(x)$ is linear function of the individual quantities. With any demand system in this subclass, the set of prices which correspond to quantity vectors on a given ray from the origin take the form of parallel straight lines. This contrasts with the earlier subclass involving linear $h(x)$, where these lines were not parallel but emanated from a point.

4 Maximizing profit

In this section we discuss how to maximize profit and welfare when the demand system satisfies (2). Consider the Ramsey problem of maximizing a weighted sum of profit and

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[6] As discussed in Bulow and Pfleiderer (1983), inverse demand curves with constant curvature exhibit a constant rate of cost passthrough. Similar analysis applies when the constant curvature differs from 1, which corresponds to demand functions of the form $x_i(p_i) = (a_i - b_i p_i)^k$. In these cases utility has linear $h(x)$ rather than linear $q(x)$.

consumer surplus. If $0 \leq \alpha \leq 1$ is the relative weight on consumer interests, the Ramsey objective is

$$[r(x) - c(x)] + \alpha s(x) = \alpha g(q(x)) + (1 - \alpha)g'(q(x))q(x) - [c(x) - h(x)] . \qquad (20)$$

Here, we used the formulas (4)-(5). This Ramsey problem nests profit-maximization ($\alpha = 0$) and welfare-maximization ($\alpha = 1$) as polar cases.

We assume that[7]

$$c(x) \text{ is homogeneous degree 1 and convex.} \qquad (21)$$

As with the consumer's problem in section 2, the Ramsey problem can be solved in two stages: first maximize the objective subject to supplying a given composite quantity, and then choose the composite quantity. For given Q, we wish therefore to minimize $c(x) - h(x)$ in (20) subject to $q(x) = Q$. As in (9), write

$$\kappa = \min_{x \geq 0} : \{c(x) - h(x) \mid q(x) = 1\} , \qquad (22)$$

which is solved by choosing $x = x^*$, say. Since $c(x) - h(x)$ is homogeneous degree 1, it follows that Qx^* minimizes $c(x) - h(x)$ subject to $q(x) = Q$. Then the solution to the Ramsey problem involves choosing composite quantity Q to maximize

$$\alpha g(Q) + (1 - \alpha)g'(Q)Q - \kappa Q , \qquad (23)$$

and then choosing the individual quantities to be Qx^*. Here, composite quantity which maximizes (23) increases with α and decreases with κ.

We deduce that maximizing any Ramsey objective involves choosing quantities proportional to x^*. In particular, profit-maximizing quantities ($\alpha = 0$) are proportional to the efficient quantities corresponding to marginal-cost pricing ($\alpha = 1$). That is, the firm has an incentive to choose the pattern of relative quantities in an efficient manner, and the sole inefficiency arises from it choosing too little composite quantity.

The firm's maximum profit given it supplies composite quantity Q is

$$(g'(Q) - \kappa)Q , \qquad (24)$$

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[7]When u is homothetic, so that $h \equiv 0$, the following analysis applies, with small adjustments, when the cost function $c(x)$ is homothetic rather than homogenous degree 1. The assumption that c is convex can be relaxed but it avoids the possibility that the firm might wish to operate multiple plants.

and so this multiproduct firm acts like a single-product firm choosing quantity Q with unit cost κ and inverse demand curve $g'(Q)$. For the firm to be profitable we require that $g'(0) > \kappa$. When marginal costs are constant, so that $c(x) \equiv c \cdot x$ for a vector c, κ in (22) is simply $\phi(c)$ where the function $\phi(\cdot)$ is defined in (9), while $x^* = x^*(c)$. In particular, the firm's most profitable composite quantity (or composite price) is a function only of its "composite" cost, $\phi(c)$, and its maximum profit is a decreasing function of $\phi(c)$. Since the profit-maximizing firm's choice of composite quantity falls with $\phi(c)$, and since consumer surplus, s, is an increasing function of composite quantity, we deduce that the firm necessarily chooses a lower level of consumer surplus when any unit cost c_i increases.[8]

Just as the pattern of relative quantities is the same in all Ramsey problems, so too are relative price-cost margins. To see this, note that (3) implies

$$p(x) - \nabla c(x) = g'(q(x))\nabla q(x) - [\nabla c(x) - \nabla h(x)] \ .$$

In a Ramsey problem, the optimal quantity vector is Qx^* when composite quantity is Q. Similarly to (13), we have

$$\nabla c(x^*) = \nabla h(x^*) + \kappa \nabla q(x^*) \ .$$

It follows that when the firm supplies composite quantity Q its most profitable price-cost margins are

$$p - \nabla c(x^*) = [g'(Q) - \kappa]\nabla q(x^*) \ . \tag{25}$$

These margins are proportional to $\nabla q(x^*)$, and shrink equi-proportionately when Q is larger. (In the limit when $\alpha = 1$ we have $g'(Q) = \kappa$ and there is marginal-cost pricing.) Thus, as composite quantity Q falls the optimal price vector traces out a straight line starting at the efficient prices $\nabla c(x^*)$ in the direction $\nabla q(x^*)$.

To summarise:

Proposition 1 *Suppose that utility takes the form (2). As more weight is placed on consumer surplus in the Ramsey problem, the composite quantity increases, the composite price decreases, each individual quantity increases equi-proportionately, and each price-cost margin decreases equi-proportionately.*

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[8]Our family of demand systems therefore excludes the possibility explored by Edgeworth (1925) that imposing a linear tax on a product supplied by a multiproduct monopolist could reduce all of its prices.

From now on we focus on the case of profit maximization ($\alpha = 0$). Within the family of utility functions (2), the procedure for finding the most profitable quantity or price vector is straightforward. First, calculate the efficient quantities which correspond to marginal cost pricing, which typically is an easy task. The most profitable quantities are proportional to these efficient quantities, and the problem reduces to choosing the optimal scaling factor for these quantities (a scalar problem). In particular the firm chooses Q to maximize $(g'(Q) - \kappa)Q$ so that the Lerner index

$$\frac{g'(Q) - \kappa}{g'(Q)} = \eta(Q) \tag{26}$$

is satisfied. (Recall that $g'(Q)$ is the composite price corresponding to Q, and that η is given in (11). There is a unique solution to (26) if marginal revenue is decreasing so that (12) holds.) Just as profit-maximizing quantities are proportional to efficient quantities, the profit-maximizing price vector lies on the straight line starting at $\nabla c(x^*)$ pointing in the direction $\nabla q(x^*)$, which again is just a scalar problem.

The factor by which the unregulated firm contracts its quantities relative to the efficient quantities is closely connected with the fraction of available surplus it can extract as profit. Indeed, in the iso-elastic case where $\eta(Q)$ is constant this connection is exact. To illustrate, consider the case of linear demand (16), where κ is negative, $g(Q) = -\frac{1}{2}Q^2$ and $\eta \equiv -1$. Then one can check that the profit-maximizing and welfare-maximizing composite quantities in (23) are $-\frac{1}{2}\kappa$ and $-\kappa$ respectively, while the corresponding levels of maximum profit and maximum welfare are respectively $\frac{1}{4}\kappa^2$ and $\frac{1}{2}\kappa^2$. Thus quantities are halved with unregulated monopoly compared to efficient quantities, and the firm can extract half of the maximum available surplus as profit.

We next discuss how the most profitable prices relate to the firm's costs.[9] To do this, suppose that $c(x) \equiv c \cdot x$, in which case expression (25) becomes

$$p = c - Qg''(Q)\nabla q(x^*(c)) . \tag{27}$$

Consider first the subclass where h takes the linear form $h(x) = a \cdot x$. (This includes cases where utility is homothetic and where demand is linear.) Expressions (10) and (13) imply

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[9]Weyl and Fabinger (2013) discuss the passthrough of costs to prices in various settings of monopoly and imperfect competition with single-product firms. Our demand system (2) yields some relatively simple multiproduct passthrough relationships.

that $p = a + g'(Q)\nabla q(x^*(c))$, and so expression (27) implies that

$$p = \frac{c - \eta(Q)a}{1 - \eta(Q)} . \tag{28}$$

When η is constant, expression (28) implies that the price for product i depends only on c_i, not on any other product's cost, and so there is no "cross-cost" passthrough in prices, even though there may be substantial cross-price effects in the demand system. Moreover, provided that the consumer can obtain positive utility with a subset of products (e.g., if $\rho > 0$ in the CES specification), the most profitable price for one product is unaffected if the firm is restricted to offer a subset of products (or even just that product).[10]

For instance, if u is homothetic and $g(Q) = \frac{1}{\gamma}Q^\gamma$, where $0 < \gamma < 1$, then $\eta \equiv 1 - \gamma$ and (28) implies

$$p = \frac{1}{\gamma}c .$$

Here, these prices do not depend on the form of the aggregation function $q(x)$ in the homothetic utility function. Likewise, with linear demand we have $\eta \equiv -1$ and expression (28) implies that the profit-maximizing prices are[11]

$$p = \tfrac{1}{2}(c + a) . \tag{29}$$

More generally within this subclass with linear h, expression (28) and the fact that the most profitable Q decreases with each cost implies that all cross-cost passthrough terms for p_i have the same sign as $(a_i - c_i)\eta'(Q)$.

Consider next the subclass where q takes the linear form $q(x) = b \cdot x$. Then (27) implies that

$$p = c - bQg''(Q) . \tag{30}$$

Since the most profitable Q decreases with each cost, all cross-cost passthrough terms are negative (positive) if $-Qg''(Q)$ is increasing (decreasing) in Q, and there is zero cross-cost passthrough if $Qg''(Q)$ is constant, i.e., when demand for composite quantity in terms

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[10]In the context of product line pricing, Johnson and Myatt (2015) explore when it is that a firm's optimal price for one product variant can be calculated by supposing that the firm only supplies that variant. (They consider both monopoly and Cournot settings.)

[11]It usually makes sense only to consider non-negative quantities, in which case (29) is only valid if a and c are such that the optimal quantities, $x = \frac{1}{2}M^{-1}(a - c)$, are positive. In the case of substitutes, where the matrix M necessarily has all non-negative elements, a necessary condition for this is that each $a_i \geq c_i$. (When M has all non-negative elements, the operation $x \mapsto Mx$ takes $\mathbb{R}^n_+$ into itself.) However, with complements, it is possible to have all x_i positive and some $a_i - c_i$ negative. In such cases, (29) indicates that $p_i < c_i$ for those products with $a_i < c_i$.

of composite price takes the exponential form. For example, with logit demand we have $b = (1, ..., 1)$, so the price-cost margin $p_i - c_i$ is the same for each product i, and the common markup $-Qg''(Q) = 1/(1 - Q)$, increases with $Q < 1$. It follows that cross-cost passthrough is negative and one product's price decreases with another product's cost.

5 Cournot competition

We now show how monopoly analysis can extend to oligopoly when the demand system satisfies (2) by considering Cournot competition. With Cournot behaviour each firm optimizes as a residual monopolist taking as given the quantities of other firms. Consider first the symmetric setting in which each of $m \geq 2$ firms offers the full range of n products and has the same homogeneous degree 1 cost function $c(x)$. A question of interest is whether it is an equilibrium for firms to produce with the efficient pattern of relative quantities, i.e., with quantities proportional to x^* which solves (22). Supposing that other firms do so, we can again analyze the problem facing say firm 1 in two stages: (i) choose the optimal pattern of relative quantities subject to supplying a given composite quantity, and (ii) choose the optimal level of the composite quantity.

For step (i), assume that other firms supply composite quantity $\bar{q}$ in aggregate, all using the efficient pattern x^*. Then the best pattern for firm 1 is also the one that maximizes industry profit, namely x^*. To see this, suppose that it produces so as to make total (i.e., industry-wide) composite quantity $Q > \bar{q}$. Doing so with pattern x^* leads to market prices $p^* = \nabla h(x^*) + g'(Q)\nabla q(x^*)$. Let $\hat{p}$ be market prices if firm 1 instead induces total composite quantity Q by pattern $\hat{x} \neq x^*$. From the monopoly analysis above we know that pattern x^* is the most efficient and profitable way for the industry to produce Q. So if $\hat{x}$ were more profitable than x^* for firm 1, other firms would make less profit at prices $\hat{p}$ than at p^*, in which case, given that their costs are the same in either scenario, their combined revenues would satisfy the inequality $\bar{q}\hat{p} \cdot x^* < \bar{q}p^* \cdot x^*$. In that case consumers could buy quantity vector Qx^* for less at prices $\hat{p}$ than at p^*, and so get strictly greater surplus. But consumer surplus, being a function of Q in our demand system, must be the same at $\hat{p}$ and p^*. It is therefore a best-response for a firm to produce with the efficient pattern x^* if its rivals do so.

Step (ii) then involves the scalar problem for each firm of how much composite quantity to supply. An individual firm chooses its $\tilde{q}$, given that all its rivals choose the same q, in

order to maximize $\tilde{q} \times (g'((m-1)q + \tilde{q}) - \kappa)$, so that total composite quantity, $Q = mq$, satisfies $g'(Q) = \kappa - \frac{1}{m}Qg''(Q)$. (The second-order condition is ensured if (12) holds.) From (23) we see that this outcome coincides with the solution to the Ramsey problem with weight $\alpha = \frac{m-1}{m}$ on consumer surplus.[12] Thus composite output increases, and composite price decreases, as the number of competitors increases. In sum:

Proposition 2 *Suppose that utility takes the form (2). Then with symmetric firms it is a Cournot equilibrium for firms to choose the efficient pattern of relative quantities, and relative to monopoly supply each individual quantity is expanded equi-proportionately and each price-cost margin is contracted equi-proportionately.*

When $c(x) \equiv c \cdot x$, expression (27) for prices still holds, but the higher composite quantity implies that equilibrium prices are now closer to costs. Consider for instance the subclass where h takes the linear form $h(x) = a \cdot x$. Expression (13) implies that $c = a + \phi(c)\nabla q(x^*(c))$, and since equilibrium composite output satisfies $g'(Q) = \phi(c) - \frac{1}{m}Qg''(Q)$, expression (27) implies that

$$p = \frac{mc - \eta(Q)a}{m - \eta(Q)} \ .$$

As with monopoly supply, cross-cost passthrough terms for product i have the same sign as $(a_i - c_i)\eta'(Q)$. So if $\eta(Q)$ is constant (for example with linear demand where $\eta \equiv -1$), then the equilibrium price for one product does not depend on the costs of others, and nor is one product's price affected when only a subject of products is supplied by the industry. Likewise in the subclass when q has the linear form $q(x) = b \cdot x$ it is straightforward to show that

$$p = c - \frac{b}{m}Qg''(Q)$$

and hence that, as with monopoly, cross-cost passthrough terms are negative (positive) if $-Qg''(Q)$ is increasing (decreasing) in Q.

An alternative setting for oligopolistic competition is where the n products are partitioned into m disjoint subsets, one for each firm. Assume that $q(x)$ is "sub-homogeneous" in the sense that it can be expressed as a homogeneous degree 1 function $q(..., q^k(x^k), ...)$ of composite quantity functions for each firm, each of which is itself homogeneous degree 1. (Our CES and logit examples meet this condition.) If moreover $h(x)$ is the sum of

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[12]On the question of whether an oligopoly can be said to maximize an objective function see Slade (1994).

homogeneous degree 1 functions, one for each firm, then it is a dominant strategy for each firm to choose its own pattern of relative quantities efficiently.[13] (This intra-firm efficiency property does not depend on inter-firm competition being Cournot, and is true also of, for example, Bertrand competition.) However, the market-wide pattern of relative quantities at oligopoly equilibrium is not generally efficient in this case unless firms are symmetric.

6 Regulation

When the demand system falls within the family (2), we have shown that the unregulated monopolist can be trusted to choose an efficient pattern of relative quantities (or relative price-cost margins), even though it chooses to supply too little composite quantity. This suggests that, in some circumstances at least, the best way to control this firm's market power when the firm has private information about its costs is to control only its composite quantity, and to leave it free to choose relative quantities to reflect its private information.

To study this possibility, consider a multiproduct version of Baron and Myerson's (1982) classic analysis of how to regulate a monopolist with an unknown cost function when a transfer payment can be paid to the firm in return for greater output. To illustrate how optimal regulation might only control composite quantity, suppose that preferences are homothetic so that gross utility $u(x)$ can be written as $g(q(x))$, where g and q are increasing and concave and q is homogeneous degree 1. Suppose the firm has the vector of constant marginal costs, $c = (c_1, ..., c_n)$. As in (22), let

$$\phi(c) = \min_{x \geq 0} : \{c \cdot x \mid q(x) = 1\}$$

be the least costly way to supply one unit of composite quantity when the firm has costs c, which is a concave, homogeneous degree 1 function. The quantities which solve this problem are $x^*(c) \equiv \nabla \phi(c)$. We can decompose the cost vector as

$$c = \kappa \times \omega \ ,$$

where $\kappa = \phi(c)$ is the firm's composite cost and $\omega = c/\phi(c)$ is a vector which lies on the $(n-1)$-dimensional surface $\phi(c) = 1$ and which represents the firm's pattern of relative

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[13]This is also true in the logit example although $h(x)$ is not a sum of homogeneous degree 1 sub-functions. In the linear demand case, although composite output $q(x)$ is not sub-homogeneous, the maximization problem facing one firm, taking as given the ouputs of others, nevertheless belongs to our family (2).

costs. Thus, ω represents which ray from the origin on which c lies, while κ measures how far along that ray c is.

The regulator cannot observe the firm's costs and believes that c comes from a specified distribution, which then induces a distribution on the transformed variables (κ, ω). (The consumer demand function is common knowledge.) Suppose hypothetically that the regulator can observe the firm's relative cost parameter ω, but not its composite cost κ, and offers each iso-ω group of firms a tailored regulatory scheme.[14]

We can derive the optimal regulatory scheme conditional on the firm being known to have relative cost parameter ω using standard arguments from Baron and Myerson (1982, section 3). Conditional on ω, suppose the cumulative distribution function for κ is $F(\kappa \mid \omega)$ with associated density function $f(\kappa \mid \omega)$. Given ω, suppose that the regulatory scheme is such that if the firm supplies a vector of quantities x there is a total transfer (including prices paid) from consumers to the firm equal to $t(x \mid \omega)$. The type-κ firm's maximum profit from participating in this scheme is therefore

$$\Pi(\kappa) = \max_{x \geq 0} : t(x \mid \omega) - \kappa\omega \cdot x \ ,$$

and it is willing to participate provided that $\Pi \geq 0$. This profit satisfies the envelope condition $\Pi'(\kappa) = -\omega \cdot x(\kappa)$, where $x(\kappa)$ is the type-κ firm's optimal choice of quantities under the scheme. The function Π is necessarily convex in κ, and so in any scheme we must have $\omega \cdot x(\kappa)$ decreasing in κ.

If $0 \leq \beta \leq 1$ is the weight the regulator places on profit relative to consumer surplus, expected weighted welfare from the iso-ω firms under the scheme is

$$\int_0^\infty \left(u(x(\kappa)) - t(x(\kappa) \mid \omega) + \beta\Pi(\kappa) \right) f(\kappa \mid \omega)d\kappa$$

$$= \int_0^\infty \left(u(x(\kappa)) - \kappa\omega \cdot x(\kappa) - (1-\beta)\Pi(\kappa) \right) f(\kappa \mid \omega)d\kappa$$

$$= \int_0^\infty \left(u(x(\kappa)) - \left[\kappa + (1-\beta)\frac{F(\kappa \mid \omega)}{f(\kappa \mid \omega)} \right] \omega \cdot x(\kappa) \right) f(\kappa \mid \omega)d\kappa \ . \tag{31}$$

Here, the first equality follows from the identity $\Pi(\kappa) \equiv t(x(\kappa) \mid \omega) - \kappa\omega \cdot x(\kappa)$, while the second follows after integrating $\int \Pi f d\kappa$ by parts using $\Pi'(\kappa) = -\omega \cdot x(\kappa)$ and the fact that

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[14]This approach is similar to that in Armstrong (1996, section 4.4) and Armstrong and Vickers (2001, Proposition 5), where a multidimensional screening problem is solved by supposing that the principal can observe all-but-one dimension of the agent's private information, and then finding conditions which ensure that the incentive scheme offered to these group of agents does not actually depend on the observed parameters.

it is optimal to leave the highest-κ firm with no profit. Provided that it results in $\omega \cdot x(\kappa)$ being decreasing in κ, the regulator's problem is solved by choosing the vector $x(\kappa) \geq 0$ to maximize the objective (31) pointwise. Since $u(x) = g(q(x))$, this is achieved by inducing the type-κ firm to choose composite quantity $Q(\kappa \mid \omega)$, where

$$Q(\kappa \mid \omega) \text{ maximizes}_{Q \geq 0} : g(Q) - \left[\kappa + (1 - \beta) \frac{F(\kappa \mid \omega)}{f(\kappa \mid \omega)} \right] Q \ , \qquad (32)$$

and to choose individual quantities $x^*(\omega) \times Q(\kappa \mid \omega)$. Here, composite quantity, $Q(\kappa \mid \omega)$, decreases with κ if the term $[\cdot]$ in (32) increases with κ, a sufficient condition for which is that F/f increases with κ. Along the ray ω, it it optimal to induce the firm to supply the same efficient pattern of relative quantities, $x^*(\omega)$. Conditional on ω, this optimal scheme is implemented by offering the firm a transfer schedule which depends only on its composite output, say $T(Q \mid \omega)$, in which case the type-κ firm chooses its composite output to maximize $T(Q \mid \omega) - \kappa Q$. When the firm participates, expression (32) implies that the type-κ firm's composite price, $g'(Q)$, under the scheme is $\kappa + (1 - \beta) F(\kappa \mid \omega)/f(\kappa \mid \omega)$.

If κ and ω are *independent* random variables then the conditional distribution $F(\kappa \mid \omega)$ does not depend on ω, and hence neither the optimal composite schedule $Q(\kappa \mid \omega)$ in (32) nor the associated optimal transfer schedule $T(Q \mid \omega)$, depend on ω. In this case, the optimal regulatory scheme contingent on observing ω does not depend on ω, and we have derived the optimal regulatory scheme when the regulator cannot observe ω. This optimal scheme gives an incentive for the firm to supply higher composite quantity, but does not attempt to influence its pattern of relative quantities.[15]

Intuitively, when κ and ω are stochastically independent the firm is happy for the regulator to observe its relative cost parameter ω but not its composite cost κ. That is, for a given composite quantity, the firm and the regulator have aligned preferences with respect to the choice of relative quantities. The regulator gains by delegating to the firm the choice of relative quantities, to enable the firm to make use of its private information about relative costs, and it does this by using a transfer scheme which depends only on the composite quantity supplied. However, if κ and ω were correlated, observing the firm's choice of relative quantities is informative about the firm's composite cost κ, which gives the firm an incentive to mis-report its relative costs as well as its composite cost.

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[15]Note that when $\beta = 1$, so that the regulator cares only about unweighted surplus, expression (32) implies that Q does not depend on ω regardless of the distribution. In this case, it is optimal to set prices equal to marginal costs for all firms, as discussed in Loeb and Magat (1979).

To illustrate the method, suppose that there are two products and utility takes the additively separable homothetic form $u(x) = \sqrt{x_1} + \sqrt{x_2}$, which can be written as $u(x) = g(q(x))$ where $q(x) = \left(\sqrt{x_1} + \sqrt{x_2}\right)^2$ and $g(Q) = \sqrt{Q}$. In this case we have

$$\kappa = \phi(c) = \frac{1}{\frac{1}{c_1} + \frac{1}{c_2}} \ . \tag{33}$$

Our method works when the distribution for (c_1, c_2) is such that κ and ω are independently distributed, i.e., if $1/c_1 + 1/c_2$ and c_1/c_2 are independent. It is well-known that the sum and ratio of two i.i.d. exponential variables are independent, and so the method works if each c_i is an independent draw from a distribution such that $1/c$ is exponentially distributed.[16] Specifically, suppose that each c_i has CDF $\Pr\{c \leq t\} = e^{-\frac{1}{t}}$. Then $1/\kappa$ is the sum of two exponential variables and so κ has CDF $F(\kappa) = (1 + \frac{1}{\kappa})e^{-\frac{1}{\kappa}}$ and corresponding density $f(\kappa) = \frac{1}{\kappa^3}e^{-\frac{1}{\kappa}}$, where F/f increases with κ. Expression (32) then implies that optimal composite price for the type-κ firm is $\kappa + (1 - \beta)\kappa^2(1 + \kappa)$, and so the optimal individual prices are

$$p_i = c_i \left[1 + (1 - \beta)\kappa(1 + \kappa)\right] \tag{34}$$

where κ is the function of c given by (33).

In expression (34) there is marginal-cost pricing for those firms with $\kappa = 0$, and so $p_i = c_i$ whenever the other product has minimum cost $c_j = 0$. The regulated price for one product is an increasing function of the other product's cost, even though this demand system has no cross-price effects and there is no statistical correlation in costs across products. Note also that all types of firm participate in the mechanism, and unlike in Armstrong (1996, section 3) there is no "exclusion" in the optimal scheme. However, so long as $\beta < 1$, when the firm has high costs the prices in (34) are above the unregulated profit-maximizing prices (which with this demand system are $p_i = 2c_i$), a possibility that was noted in the single-product context by Baron and Myerson (1982, p.292).[17]

Similar analysis can be applied in the alternative situation with "price-cap" regulation, when transfer payments to the firm are not feasible and instead the regulator specifies the set of quantity (or price) vectors from which the firm is permitted to choose. The simplest forms of price-cap regulation are "monotonic", in the sense that if x is a permitted quantity

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[16] Another way to describe this is that c_i comes from an inverse-χ^2 distribution with 2 degrees of freedom.

[17] Another example which works nicely is when composite quantity takes the Cobb-Douglas form $q(x) = \sqrt{x_1 x_2}$, so that $\kappa = 2\sqrt{c_1 c_2}$. In this case the method works when the joint distribution for (c_1, c_2) is such that the product $c_1 c_2$ and the ratio c_1/c_2 are stochastically independent, which is the case when each c_i is an independent draw from a log-normal distribution.

vector then any vector $\tilde{x} \geq x$ is also permitted.[18] As above, suppose that the regulator can observe the firm's ω parameter but not its κ parameter. Given that the regulator considers monotonic price-cap regulation, one can check that it will wish firms to choose an efficient pattern of relative quantities, so that these firms are permitted choose any composite quantity no lower than some threshold $\bar{Q}(\omega)$. In those cases where κ and ω are stochastically independent, it follows that the minimum composite quantity $\bar{Q}$ does not depend on ω, and this then constitutes the optimal monotonic price-cap scheme.

7 Conclusions

The main aim of this paper has been to show how multiproduct monopoly analysis is made relatively simple when consumer surplus is a homothetic function of quantities. Whether the firm's objective is profit or a Ramsey combination of profit and consumer surplus, it optimizes by selecting efficient (i.e., welfare-maximizing) quantities scaled back equi-proportionately. The resulting optimal markups yield, for example, transparent results on multiproduct cost passthrough, including instances of zero cross-cost passthrough. The method of analysis can extend, moreover, to Cournot oligopoly settings.

The family of demand systems with consumer surplus a homothetic function of quantities is of course restrictive. But it includes a number of familiar yet apparently diverse special cases, including CES and linear demands and discrete choice models such as logit. Moreover, it shows how those special cases are themselves instances of much wider sub-classes of demand system, involving $h(x) \equiv 0$, linear $h(x)$ and linear $q(x)$ respectively.

Finally, the family of demand systems analyzed in this paper appears to be a natural basis on which to explore the intuition that regulation of multiproduct monopoly should focus on the general level of prices and not the pattern of relative prices. Indeed we showed how that intuition can be precisely correct, thereby contributing to the theory of multi-dimensional screening.

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[18]Even in the single-product case, Alonso and Matouschek (2008) and Amador and Bagwell (2014) show how the regulator can sometimes do better than this by leaving gaps in the set of permitted prices, and they derive conditions which ensure that monotonic regulation is optimal.

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