ISSN 1471-0498

DEPARTMENT OF ECONOMICS

DISCUSSION PAPER SERIES

THE DYNAMICS OF SOCIAL INFLUENCE

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Number 742
February 2015

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February 04, 2015
This version: February 21, 2015

Abstract

Individual behaviors such as smoking, fashion, and the adoption of new products is influenced by taking account of others' actions in one's decisions. We study social influence in a heterogeneous population and analyze the long-run behavior of the dynamics. We distinguish between cases in which social influence arises from responding to the number of current adopters, and cases in which social influence arises from responding to the cumulative usage. We identify the equilibria of the dynamics and show which equilibrium is observed in the long-run. We find that the models exhibit different behavior and hence this differentiation is of importance. We also provide an intuition for the different outcomes.

JEL classifications: C62, C70, D70, G00

Keywords: social influence, imitation, equilibrium selection

*I thank Peyton Young for his guidance and support throughout this research. I thank Johannes Ruf for framing discussions at an early stage of this project. Further, I thank Sergiu Hart, Dirk Helbing, Michael Maes, Heinrich Nax, Pierre Tarrès, Willemien Kets and the participants at the 25th International Conference on Game Theory at Stony Brook, the Transatlantic Theory Workshop, Paris (2014), and the seminar at the Department of Computational Social Science at ETH Zürich for helpful comments. I would like to acknowledge the generous support of the Oxford-Man Institute of Quantitative Finance, the Office of Naval Research grant # N00014-09-1-0751, and the Air Force Office of Scientific Research grant # FA9550-09-1-0538.

1 Introduction

Social influence describes a process in which individual opinions or behaviors are affected by the opinions or behaviors of others in a group. In this paper we study a general family of dynamic models of social influence. Agents update their action at random points in time. Their decisions are influenced by the actions previously taken by other members of the society, where the strength of this influence varies across agents. We distinguish between two cases: On the one hand, we consider the case where social influence arises from responding to the number of *current* adopters (*Adoption*). On the other hand, we consider the case where social influence arises from the *cumulative* impact of prior adoptions (*Usage*). In *Adoption* each player's action is only counted once (current adoption). In *Usage* each player's action is counted by the number of periods in which he took that action. Thus the repeated action by a single player may be counted several times. We first identify the equilibria of these models and then study their stability in stochastic environments. We characterize the stable states of the dynamics and find that the behavior differs for the two models.

The discussion of herding behavior and social influence has a long history. For example Trotter (1916) identifies herd instinct as one of the primary instincts along with self-preservation, nutrition, and reproduction. Freud (1921a,b) points to an individual's tendency to follow the masses ("Herabsinken zum Massenindividuum"). For other early discussions of social influence see, for example, Le Bon (1895). Subsequently the study of the influence of a group on the decision or action of an agent entered the disciplines of economics, sociology, and psychology (see, for example, Keynes 1930, 1936, Hamilton 1971, Schelling 1971, Shiller 1984, 2000).

In the next section we discuss the related literature. In section 3 we introduce the model and the different types of social influence. Section 4 contains all the results. After some initial observations we introduce our guiding example. We then analyze the social influence models *Adoption* and *Usage* and compare them. Section 5 concludes.

2 Related literature

Before discussing the literature on social influence in more detail we shall make an important remark regarding the difference between social learning and social influence (see, for example, Montgomery and Casterline 1996, Young 2009). *Social learning* takes into account other players' signals and information in evaluating alternative actions.[1] *Social influence* "refers to the effects of interpersonal interactions that derive their power from factors that are intrinsically 'social' "(Montgomery and Casterline 1996).[2] In this paper, we study the dynamics of social influence. Nevertheless, *Usage* is also a key ingredient of social learning models. The cumulative usage of, for example, an innovation may have a 'real' impact on the utility function beyond the propensity of a player to conform with

[1] See, for example, Banerjee (1992) and Ellison and Fudenberg (1995).

[2] This could be the desire to conform, avoid conflict, mitigate uncertainty, or a more direct utility from a shared use of an innovation such as mobile phones.

society (see Young 2009). For example, if a taxi driver frequently uses a mobile phone application this is likely an indicator that it actually helps him to gain more work.[3]

Models of social influence have numerous applications, for example, financial herding (e.g., Scharfstein and Stein 1990, Welch 1992, Shiller 2000), political and social movements (Schelling 1978, Lohmann 1994, Cabinet Office 2012), and diffusion processes such as innovation adoption (Rogers 1962, Bass 1969, Valente 1996, Meade and Islam 2006, Young 2009). There is broad experimental evidence for social influence. Asch (1955) conducted a series of enlightening experiments showing that a considerable proportion of subjects trust the majority over their own senses. More recently Salganik et al. (2006) show the effects of social influence in a study on music taste. Other experimental studies include voting and opinion polls (Cukierman 1991), human fertility (Bongaarts and Watkins 1996), diffusion of information technologies (Teng et al. 2002), stock market participation (Hong et al. 2004), household energy consumption (Schultz et al. 2007), mobile phones (de Silva et al. 2011), and mobile banking (Yu 2012).

Considering the above examples there are cases where social influences takes the form of *Adoption* and others where it takes the form of *Usage*. We shall use innovation adoption as our guiding motivation and example.[4] For example, consider the spread of mobile phones and its associated applications in developing countries.[5] On the one hand, from a 'hardware perspective' the act of buying a mobile phone is a one-off act and thus the relevant information to other potential buyers is the number of current adopters (*Adoption*). On the other hand, from a 'software perspective' (e.g., mobile money, WhatsApp) the cumulative usage is likely to be a more important driver of social influence than the mere installation of the product (*Usage*).[6]

The key novelty of our framework is that we differentiate between social influence arising from responding to the number of adopters versus social influence arising from responding to the cumulative usage. We consider noisy best-reply models and show a selection within the set of fixpoints (independent of the starting state). Interestingly the two seemingly similar models often exhibit different long-run behavior.

Our model is most closely related to Schelling (1978, Chapter 3) and Granovetter (1978). Schelling (1978, Chapter 3) describes the class of *critical mass models* of social interaction and gives several instructive examples. He notes that "though perhaps not in physical and chemical reactions, in social reactions it is typically the case that the 'critical number' for one person differs from another's" (Schelling 1978). He goes on to define the *tipping value* which, for each player, determines the critical mass of the aggregate action of the population for which a player will 'tip over' from playing one action to another.

[3]For example, HAILO_{TM} was the first provider of a mobile phone application for black cabs in London.

[4]'Innovation' and 'new product' are used interchangeably in the literature.

[5]See, for example, Kiringai and Fengler (2010) for a discussion of mobile money in Kenya. They find for both, mobile phones and mobile money an S-curved adoption curve supporting the claim that social influence is present (see Young 2009).

[6]Note that this differentiation has not gone unnoticed in marketing departments. Hardware providers such as Apple focus their reporting and marketing effort on the number of products sold, i.e., roughly the number of adopters. On the other hand, software providers, such as Skype, focus their analysis and messaging on the number of people who are online (or, more generally, use the product) at any point in time, i.e., the usage intensity.

The heterogeneity of social influence has been confirmed in recent work, suggesting that cognitive factors influence the propensity to herding behavior (Dohmen et al. 2012, Baddeley et al. 2007, Moussaid et al. 2013). In a similar vein Granovetter (1978) develops a threshold model for binary decision games and describes the nature of fixpoints given the individual players' thresholds or tipping values.

We shall adopt the assumption of heterogeneous players who repeatedly revise their action which is binary. As in Schelling (1978) and Granovetter (1978) we employ a threshold model with heterogeneous tipping values. We consider a continuous time model of asynchronous updating, thus guaranteeing the existence of equilibria (see Lopez-Pintado and Watts 2008). Granovetter (1978) notes that if misperception is random "the situation is more complex and the stability of the underlying equilibrium becomes particularly important". The study of this scenario is precisely the purpose of the current paper.

For further studies on social influence, see Macy (1991), Dodds and Watts (2004, 2005), Horst and Scheinkman (2006), Bramoulle (2007), Lopez-Pintado and Watts (2008), Scheinkman (2008), Baddeley (2010), Young (2011), Babichenko (2013), Moussaid et al. (2013). Another strand of the literature analyzes social influence under the assumption of Bayesian learning and is often termed *herding* and (Banerjee 1992, Bikhchandani et al. 1992, Scharfstein and Stein 1990, Welch 1992, Devenow and Welch 1996, Cont and Bouchaud 2000, Bouchaud 2013). These models are often applied in finance and explain with the help of information cascades the build up of bubbles. The literature on the evolution of social norms is also closely related (see, for example, Axelrod 1986, Young 1993, 1998). Further, the spread of infectious diseases in biology is often modeled in a similar way, but the models considered are mainly independent interaction models (see Kermack and McKendrick 1927 and Dodds and Watts 2005).[7]

As in much of the literature we consider a 'mean-field' approach, assuming that an agent observes every other agent with equal probability. A separate strand of the literature studies social influence given an underlying network of interactions (see, for example, Blume 1993, Valente 1996, Young 1998, Morris 2000, Jackson and Watts 2002, Centola and Macy 2007, Golub and Jackson 2010, Banerjee et al. 2013). The processes we consider are Markovian. We use the concept of *stochastic stability* (see Foster and Young 1990, Kandori et al. 1993, Young 1993). The idea is to study a perturbed version of the original process, such that the resulting Markov process is irreducible and ergodic and therefore the process has a unique stationary distribution. By letting the level of noise approach zero one can identify those states that will be observed in the long-run with arbitrarily high probability. We also make use of recent work on reinforced random walks (Pinsky 2013). Pinsky analyzes a random walk on $\mathbb{Z}$ whose probability of moving left or right depends on the recent history. By an appropriate translation we can use his results to find the long-run stable states of our dynamic.

[7]In an independent interaction model the probability of infection is independent of the number of contacts with infected agents. On the other hand in threshold models (as considered here) the probability of 'infection' depends on the number of currently 'infected' agents.

3 The model

We shall first introduce the general framework for analyzing social influence. Let $N = \{1, ..., n\}$, $n \in \mathbb{N}$ be a finite population of types. Let $P = \{1, ..., p\}$, $p \in \mathbb{N}$ be the set of players where each player is associated with exactly one type.[8] Let $q_i = \frac{\sum_{j=1}^{p} \mathbf{1}_{j \text{ is of type } i}}{p}$ (for $i = 1, \ldots, n$) be the ratio of players of type i. We shall, by abuse of notation, use types and players interchangeably since any two players of a certain type will always act the same (in expectation), that is, have the same utility and response function. Let $A = \{m, d\}$[9] be the actions available to each player $i \in P$.

Let $u_i : [0, 1] \times A \to \mathbb{R}$ be the utility of agent $i \in P$ when observing the signal about the society $s \in [0, 1]$ and playing action $a \in A$. Suppose that the utility of an action is separable into a component arising from a player's inherent preference for an action and a component specifying the utility he derives from social conformity. After normalizing, let $p_i \in \mathbb{R}$ be player i's direct utility difference when playing action d over action m. Further let $\rho_i \in \mathbb{R}^+$ be a player's *index of social conformity*. Finally, suppose that the impact of social influence is linear. A player's utility from playing action a is now given by

$$u_i(a) = \begin{cases} p_i + \rho_i s & \text{if } a = d, \\ \rho_i(1 - s) & \text{if } a = m. \end{cases} \tag{1}$$

Let $f_i : \mathbb{R}^2 \to [0, 1]$ be the *response function* for player i, specifying the probability to play action d given his utilities $u_i(s, d) \in \mathbb{R}$ and $u_i(s, m) \in \mathbb{R}$. Note that $1 - f_i(\cdot, \cdot)$ is the probability that i plays action m. We shall initially consider a best-response model:

$$f_i = \begin{cases} 1 & \text{if } u_i(s, d) > u_i(s, m), \\ 0.5 & \text{if } u_i(s, d) = u_i(s, m), \\ 0 & \text{else.} \end{cases} \tag{2}$$

We shall consider a continuous time process where each player is activated by iid Poisson arrival processes. Let $t = 0, 1, 2, \ldots$ be the time steps in which a (unique) player gets activated.[10,11] In a given time step t the activated player i will be called *active*. Define

$$\mathbf{1}_i^t = \begin{cases} 1 & i \text{ is active in } t, \\ 0 & \text{else.} \end{cases} \tag{3}$$

[8]More generally, we could think of a person comprising of several players, possibly of different types. This could be the case for multiple reasons. For example, one person might be more important (or observable) in the population than another person and therefore his adoption or usage might be 'counted' multiple times. Further, a person might simply use a product more than another person, or use a product more or less dependent on which type currently would like to use the product.

[9]*Dextre* and *maldextre* are the terms for right and left in Esperanto.

[10]Note that by the assumption of iid Poisson arrival processes we have that almost surely no two players are activated at the same moment.

[11]When unambiguous we shall sometimes omit the specification of the time period.

Let $s(t) \in [0,1]$ be agent i's observation of society at time t. For each player i, let a_i^t be the action he takes at time t. The state at the end of a given period t is given by the action profile $\mathbf{a}^t = (a_i^t)_{i \in P}$. Let $\mathbf{a}^0$ be any permissible initial configuration. Then

$$a_i^t = \mathbf{1}_i^t \cdot B^t[f_i(u_i(s(t), d), u_i(s(t), m))] + (1 - \mathbf{1}_i^t) \cdot a_i^{t-1} \tag{4}$$

for all $t \geq 1$, where $(B^t)_{t \in \mathbb{N}}$ is a family of independent Bernoulli random variables taking values in A.

Let $\bar{a}^t = \sum_{i=1}^p \mathbf{1}_{a_i^t = d}/p \in [0, \frac{1}{p}, \ldots, 1]$ be the *population's average action* in period t. We consider two models of social influence, arising from responding to different observations about society:

- **Adoption.** An active player responds to the current number of adopters:

$$s^{adoption}(t) = \bar{a}^{t-1} = \frac{\sum_{j \in P} \mathbf{1}_{a_j^{t-1} = d}}{p} \tag{5}$$

- **Usage.** An active player responds to the cumulative usage in the past k periods:[12,13]

$$s^{usage}(t) = \frac{\sum_{v=t_0}^{t-1} \sum_{i=1}^{p} \mathbf{1}_{a_i^v = d} \mathbf{1}_i^v}{k} \quad \text{for} \quad t_0 = t - k \tag{6}$$

When unambiguous we shall write $s(t)$ for $s^{adoption}(t)$ or $s^{usage}(t)$ respectively.

To illustrate, suppose there are four players. Player 1 initially plays d and all other players play m. Suppose we are in time step $t = 7$ and play unfolded as shown in table 1. For *Adoption* the relevant observation in period $t = 7$ of current adopters of action

Table 1: Actions up to $t = 6$

time step	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$
active player	–	player 2	player 1	player 4	player 1	player 2	player 2
player 1	d		$\boxed{d}$		$\boxed{ⓓ}$		
player 2	m	d				$\boxed{d}$	$\boxed{ⓓ}$
player 3	ⓜ						
player 4	m			$\boxed{ⓜ}$			

Relevant actions for $s^{adoption}(7)$ circled, relevant actions for $s^{usage}(7)$ boxed ($k = 5$).

d is $s^{adoption}(7) = 50\%$ (the relevant actions are circled in Table 1). For *Usage* suppose,

[12] In general we assume that k is fixed. The only exception we shall consider is $k = t$, that is, the sample comprises all actions taken in the past.

[13] To be precise we need to define $s(t)$ differently for $t < k$. We can simply assume the average of the past t actions.

for example, that $k = 5$. Then the observation in period $t = 7$ of the cumulative usage of action d is $s^{usage}(7) = 80\%$ (the relevant actions are boxed in Table 1). Note that in our example player 3's previous actions have no influence on the observed cumulative usage.

We shall analyze the two models for several regimes of sampling and errors. Initially we will study the unperturbed dynamics as defined above. That is, in the case of *Usage*, $k = t$. We then turn to study perturbed dynamics. For *Usage* the natural perturbation is to limit the 'look-back' window to a constant k. For the *Adoption* regime we shall consider a uniform action tremble. That is, there exists a small probability $\varepsilon > 0$ such that a player picks an action uniformly at random. The response function for player i then is

$$f_i = \begin{cases} 1 - \frac{\varepsilon}{2} & \text{if } u_i(s(t), d) > u_i(s(t), m), \\ 0.5 & \text{if } u_i(s(t), d) = u_i(s(t), m), \\ \frac{\varepsilon}{2} & \text{else.} \end{cases} \tag{7}$$

4 Analysis

Definition 1. *Let each player i's* tipping value $\mu_i \in \mathbb{R}$ *be such that he wants to play d if*

$$\mu_i < s(t), \tag{8}$$

and he wants to play m if

$$\mu_i > s(t), \tag{9}$$

and if $\mu_i = s(t)$ he is indifferent. Note that μ_i is the (unique) zero of the function $u_i(s(t), d) - u_i(s(t), m)$.

It immediately follows that a player with $\mu_i \in (-\infty, 0)$ always prefers to play action d and a player with $\mu_i \in (1, \infty)$ always prefers action m. Note that players of the same type have the same tipping value.

We shall make the simplifying assumption that for all players i, μ_i is not a multiple of $1/p$. Thus a player always has a unique best response. Note that this assumption is generic if the parameters of the utility functions are drawn from continuous independent distributions.

Definition 2. *Let Agg be the* Aggregate Dynamic. *That is, given the population's average action $\bar{a}$, Agg gives the share of players who would play d, given they observe this state.*

$$Agg : [0, 1] \to \left\{0, \frac{1}{p}, \frac{2}{p}, \ldots, 1\right\} \tag{10}$$

$$Agg(\bar{a}) = \frac{1}{p} \sum_{i=1}^{p} \mathbf{1}_{\mu_i < \bar{a}} \tag{11}$$

Lemma 3. *Agg has at least one fixpoint. If x^* is a fixpoint of Agg all players of the same type play the same action, that is*

$$x^* \in \{\bar{q}_i = \sum_{j=1}^{i} q_j \,:\, \mu_i < \bar{q}_i < \mu_{i+1}\}_{i=1,\dots,n}.^{14} \tag{12}$$

Proof. We shall first show that there exists a fixpoint. If $Agg(1) = 1$ then 1 is a fixpoint and similarly if $Agg(0) = 0$, 0 is a fixpoint. Thus we still have to consider the case where $0 < Agg(1) < 1$. Define the function:

$$g:\ [0,1] \ \rightarrow \ \left\{-1, ..., \frac{-1}{p}, 0, \frac{1}{p}, ..., 1\right\} \tag{13}$$

$$g(x) \ \mapsto \ Agg(x) - x \tag{14}$$

It suffices to prove that there exists x such that $g(x) = 0$. Since $Agg(0) > 0$ we have $g(0) \neq 0$ and thus $g(0) > 0$. Also $Agg(1) < 1$ and thus $g(1) < 0$. Hence there must eventually be a change from positive to negative sign with increasing x. Remember that g is defined on $\left\{-1, ..., \frac{-1}{p}, 0, \frac{1}{p}, ..., 1\right\}$. We have by monotonicity of Agg that for every $x < 1$, $g\left(x + \frac{1}{p}\right) \geq g(x) - \frac{1}{p}$, that is the function g decreases at most in steps of $\frac{1}{p}$. Therefore there exists x^* such that $g(x^*) = 0$ and thus $f(x^*) = x^*$.

For the second part of the lemma note that if x^* is a fixpoint of Agg all players of the same type necessarily play the same action. It remains to show that for any fixpoint $\bar{q}_i$ we have $\mu_i < \bar{q}_i < \mu_{i+1}$. By contradiction, suppose there exists a fixpoint $\bar{q}_i < \mu_i$. Then for the share of players q_i it is optimal to play m when observing $\bar{q}_i$, given that their tipping value is μ_i and thus $\bar{q}_i$ is not a fixpoint. A similar argument applies for $\bar{q}_i > \mu_{i+1}$. □

We shall now introduce an example of innovation diffusion which will guide us throughout the paper. Given a population as proposed by Rogers (1962), suppose there are

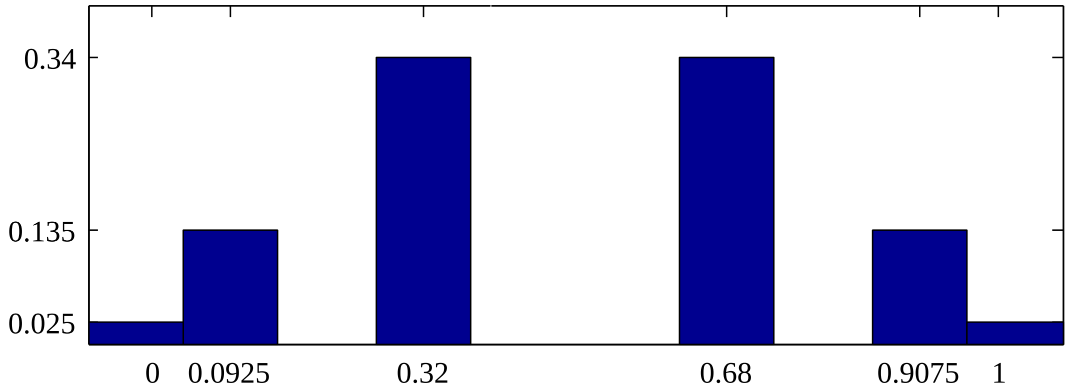
- 2.5% *innovators*, who always play the innovation action d, that is, they play the innovation independent of social influence and hence their tipping value is ‘negative’ ($\mu_{\text{innovators}} < 0$),
- 13.5% *early adopters*, who play the innovation if at least ‘few’ play the innovation ($\mu_{\text{early adopters}} = 9.25\%$),
- 34% *early majority*, who play the innovation if at least an ‘intermediate proportion’ play the innovation ($\mu_{\text{early majority}} = 32\%$),
- 34% *late majority*, who play the innovation if at least ‘many’ play the innovation ($\mu_{\text{late majority}} = 68\%$),
- 13.5% *laggards*, who play the innovation if at least ‘almost everybody’ play the innovation ($\mu_{\text{laggards}} = 90.75\%$),

[14] Set $\mu_{n+1} = 1$ for completeness.

- 2.5% *non-adopters*, who never play the innovation ($\mu_{\text{non-adopters}} > 1$).[15]

Note that the broad categories (i.e., few, intermediate proportion, etc.) represent a 'coarse' mapping from observations to beliefs.[16] Figure 1 shows the distribution of tipping values. Figure 2 shows the function Agg and the fixpoints $x_1^*, \ldots, x_5^*$.

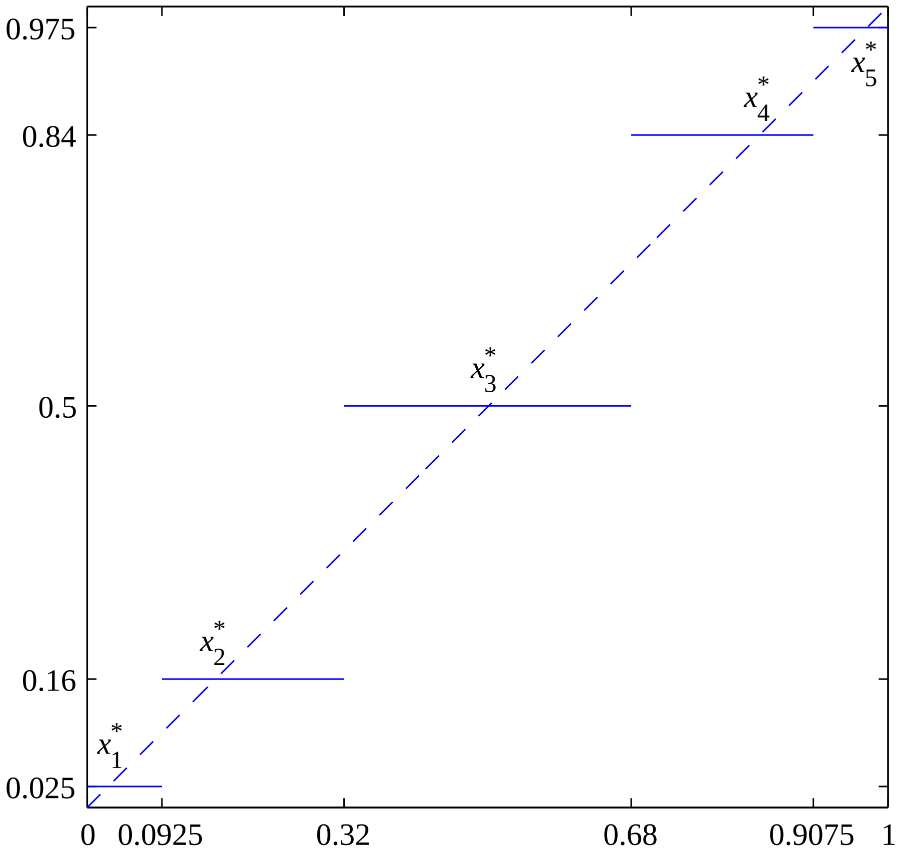
Figure 1: Tipping value distribution – x-axis shows μ_i, y-axis shows q_i



[15]Note that in Rogers (1962) the two last categories are considered as one. He mentions the possibility to split them in order to create a symmetric distribution. Rogers argues that adopter distributions are well approximated by the normal distribution; for example innovators are 2 standard deviations or more above the mean level of innovativeness.

[16]In political philosophy and psychology it is understood that some beliefs may be coarse-grained while others are fine-grained (see, for example, Sturgeon 2009).

Figure 2: Agg – x-axis shows μ_i, y-axis shows Agg



4.1 Adoption

In this section we shall consider the social influence model *Adoption*. An active player bases his decision on the number of current adopters (see equation 5):

$$s(t) = \bar{a}^{t-1} = \frac{\sum_{j \in P} \mathbf{1}_{a_j^{t-1}=d}}{p}$$

Theorem 4. *The dynamic has at least one absorbing state. The absorbing states of the dynamic process coincide with the fixpoints of Agg and each absorbing state is associated with exactly one fixpoint of Agg and vice versa. The set of absorbing states which can be reached is dependent on the initial state.*

Proof. By contradiction, suppose there exists an absorbing state $\mathbf{a}^*$ with $x^* \in [0, 1]$ such that x^* is not a fixpoint of Agg. Suppose $x^* > Agg(x^*)$ (the other case is analogous. That is, the number of players playing d is x^*, however the number of players for whom it is desirable to play d is $Agg(x^*)$ which is strictly smaller than x^*. Hence there exists at least one player i playing d for whom $\mu_i > x^*$ and if i is activated in a given period he will change his action from d to m. Thus the state $\mathbf{a}^*$ is not an absorbing state. Since by

Lemma 3 Agg has at least one fixpoint, for a given fixpoint x^* we can assign an action to each player such that the action profile is an absorbing state. We simply order the players by their tipping values (from small to large) and assign action d successively to players until the ratio x^* is reached. Finally note that the dynamic is not ergodic and therefore the absorbing states attainable depend on the initial state $\mathbf{a}^0$. □

Note that Theorem 4 is closely related to the convergence result in Babichenko (2013). Their model, in our language, allows for negative social influence. They show convergence to approximate Nash equilibrium if the observation of society is discretized, that is $\lfloor s^{adoption} \rfloor_\delta$ for some $\delta > 0$. Note that Babichenko (2013) also show that their dynamic converges in $O(n \log n)$ time steps.

4.1.1 Perturbed dynamics

We now consider the perturbed process with a uniform error rate. We shall first introduce the concept of stochastic stability (using our process as an example) and then move on to analyze the process.

Stochastic stability

Consider the stochastic process governing the change of $\bar{a}$. The process is Markovian and its recurrent classes are characterized by the fixpoints of Agg. It is a regular perturbed Markov process and we can therefore use stochastic stability analysis (Foster and Young 1990, Kandori et al. 1993, Young 1993). We consider the long-run behavior of the process when ε becomes small. Note that the perturbed process is ergodic for $\varepsilon > 0$ and thus has a unique stationary distribution, say Π_ϵ over the state space $[0, \frac{1}{p}, \ldots, 1]$. We thus study $\lim_{\varepsilon \to 0} \Pi_\varepsilon = \Pi_0$.

Definition 5. *A state $\bar{a} \in [0, \frac{1}{p}, \ldots, 1]$ is* stochastically stable *if $\Pi_0(\bar{a}) > 0$. Denote the set of stochastically stable states by* $\mathbf{S}$.

For a given parameter ε denote the probability of transiting from $\bar{a}$ to $\bar{a}'$ in one period by $\mathbb{P}_\varepsilon[\bar{a}, \bar{a}']$. The *resistance* of a transition $\bar{a} \to \bar{a}'$ is the unique real number $r(\bar{a}, \bar{a}') \geq 0$ such that $0 < \lim_{\varepsilon \to 0} \mathbb{P}_\varepsilon[\bar{a}, \bar{a}']/\varepsilon^{r(\bar{a}, \bar{a}')} < \infty$. For completeness let $r(\bar{a}, \bar{a}') = \infty$ if $\mathbb{P}_\varepsilon[\bar{a}, \bar{a}'] = 0$. Hence a transition with resistance r has probability of the order $O(\varepsilon^r)$. We shall call a transition (possibly in multiple periods) $\bar{a} \to \bar{a}'$ a *least cost transition* if it exhibits the lowest order of resistance. That is, let $\bar{a}, \bar{a}_1, \ldots, \bar{a}_k = \bar{a}'$ (k finite) be a path of one-period transitions from $\bar{a}$ to $\bar{a}'$. Then a least-cost transition minimizes $\sum_{l=0}^{k-1} r(\bar{a}_l, \bar{a}_{l+1})$ over all such paths.

Young (1993) shows that the computation of the stochastically stable states can be reduced to an analysis of rooted trees on the set of recurrent classes of the unperturbed dynamic. Define the *resistance* between two recurrent classes $\bar{a}$ and $\bar{a}'$, $r(\bar{a}, \bar{a}')$ to be the sum of resistances of a least resistant path that starts in $\bar{a}$ and ends in $\bar{a}'$. Now identify the recurrent classes with the nodes of a graph. Given a node $\bar{a}$, a collection of directed edges T forms an *$\bar{a}$-tree* if from every node $\bar{a}' \neq \bar{a}$ there exists a unique outgoing edge in T, the graph has no cycles and $\bar{a}$ has no outgoing edge.

Definition 6. *The resistance $r(T)$ of a $\bar{a}$-tree T is the sum of the resistances of its edges. The* stochastic potential *of $\bar{a}$, $\gamma(\bar{a})$, is given by*

$$\gamma(\bar{a}) = \min\{r(T) : T \textit{ is an } \bar{a}\textit{-tree}\}. \tag{15}$$

Theorem 4 in Young (1993) states that the stochastically stable states are precisely those states where ρ is minimized.

Analysis

We can now turn to analyze the perturbed *Adoption* model. The process governing the change of $\bar{a}$ has a linear transition structure, namely, to go from state $\frac{i}{p}$ to $\frac{j}{p}$ one has to pass through all the states $\frac{i+1}{p}, \ldots, \frac{j-1}{p}$ for $i < j$ and similarly through all the states $\frac{i-1}{p}, \ldots, \frac{j+1}{p}$ for $i > j$. Hence to find the least resistant paths between any two recurrent classes it suffices to calculate the resistance between any two *neighboring* recurrent classes.

Lemma 7. *The resistances of the paths between two neighboring fixpoints of Agg (recurrent classes) x_i^*, x_{i+1}^* ($x_i^* < x_{i+1}^*$) are given by:*

$$r_{x_i^*, x_{i+1}^*} = \max_{x \in [x_i^*, x_{i+1}^*]} \{x - Agg(x)\} \tag{16}$$

$$r_{x_{i+1}^*, x_i^*} = \max_{x \in [x_i^*, x_{i+1}^*]} \{Agg(x) - x\} \tag{17}$$

Proof. We shall give the argument for $r_{x_i^*, x_{i+1}^*}$, the one for $r_{x_{i+1}^*, x_i^*}$ is analogous. We define several groups of players. Let P_d^+ be the set of players who played d in the last period and for whom d is currently a best reply and let P_d^- be the set of players who played d in the last period and for whom m is currently the best reply. Define P_m^+ and P_m^- analogously. Since we consider the resistance between two absorbing states we have for the starting state that $P_d^- = P_m^- = 0$. Given that $x_i^* < x_{i+1}^*$ we have that in x_{i+1}^* more players play d. Suppose that we construct a path such that only players who currently play m switch to d. Then any such switch is erroneous behavior as long as $x > Agg(x)$. Once $x < Agg(x)$ further transitions have resistance zero. For any given x with $x > Agg(x)$ one needs at least $x - Agg(x)$ errors to enter a region where x_{i+1}^* becomes an attractor. Since this must hold for all x with $x > Agg(x)$ we have

$$r_{x_i^*, x_{i+1}^*} = \max_{x \in [x_i^*, x_{i+1}^*]} \{x - Agg(x)\}. \tag{18}$$

This is sufficient since after this many trembles there exists a zero resistance path moving to the neighboring absorbing state associated with x_{i+1}^*. □

Theorem 8. *Suppose players have* uniform action trembles. *The stochastic potential* γ_i *of a recurrent class* $\boldsymbol{a}_i^* \in \{\boldsymbol{a}_1^*, \ldots, \boldsymbol{a}_l^*\}$ *associated with fixpoint* $x_i^* \in \{x_1^*, ..., x_l^*\}$ *is given by*

$$\gamma_{\boldsymbol{a}_i^*} = \sum_{\alpha=1}^{i-1} r_{x_\alpha, x_{\alpha+1}} + \sum_{\alpha=i+1}^{l} r_{x_\alpha, x_{\alpha-1}}. \tag{19}$$

The stochastically stable states are the states associated with fixpoints of Agg that minimize stochastic potential, that is:

$$\lim_{\varepsilon \to 0} \lim_{t \to \infty} \quad \bar{a}^t = \{\boldsymbol{a}_i^* : \ \gamma_{\boldsymbol{a}_i^*} = \min_{j=1,\ldots,l} \gamma_{\boldsymbol{a}_j^*}\}. \tag{20}$$

For generic games there exists a unique long-term stable state.

Proof. Let $x_1^*, ..., x_l^*$ (in increasing order) be the fixpoints of *Agg*. In order to pass from one to another one needs to pass through all the fixpoints in between. It follows that the stochastic potential of a fixpoint is given by

$$\gamma_{x_i^*} = \sum_{\alpha=1}^{i-1} r_{x_\alpha, x_{\alpha+1}} + \sum_{\alpha=i+1}^{l} r_{x_\alpha, x_{\alpha-1}}. \tag{21}$$

The first summand gives the resistances of passing from the rightmost fixpoint to x_i^*, the second the resistance from passing from the leftmost fixpoint to x_i^*. Now by Young (1993) we have that the stochastically stable states are precisely those states which have minimal stochastic potential. □

4.2 Usage

In this section we shall consider the social influence model *Usage*. An active player bases his decision on the cumulative usage (see equation 6):

$$s(t) = \frac{\sum_{v=t_0}^{t-1} \sum_{i=1}^{p} \mathbf{1}_{a_i^v = d} \mathbf{1}_i^v}{k} \quad \text{for} \quad t_0 = t - k$$

Theorem 9. *The dynamic has absorbing states if and only if* 0 *and/or* 1 *are fixpoints of Agg or the best response of any player is independent of social influence. In the former case all* $-$ *m and/or all* $-$ *d are the unique absorbing states. If initially players play heterogeneous actions both absorbing states are reached with positive probability. This holds for both* k *constant and* $k = t$.

Proof. We shall first show that if 0 (or 1) is a fixpoint of *Agg* the corresponding state is absorbing. Suppose that 0 is a fixpoint. Then there exists an observation s^* below which all players want to play m. Suppose that $s(t) < s^*$ in period t. Now independent of who is selected in subsequent periods he plays action m and thus for all $T \geq 0$, $s(t+T) \leq s(t) < s^*$. Hence *all* $-$ *m* is an absorbing state of the dynamic. If 1 is a fixpoint a similar argument applies.

Now if the best response of any player is independent of social influence there clearly exists an absorbing state, namely the state where every player plays the action he prefers (independent of social influence).

Next suppose that there exists some player for whom social influence may change the best response. Suppose there exists an absorbing state where some players play m and some play d. Suppose a player currently playing m, say i, will be convinced to play d if a high enough proportion of the population plays d (social influence may change his best response). Then, with positive probability enough players who currently wish to play d are selected successively changing the historic action profile such that when i is next selected his best response is d. This shows that there is no mixed-action absorbing state.

By a similar argument one can see that if $all - m$ and $all - d$ are absorbing states of the dynamic either state is attainable when starting with a heterogeneous action profile. Finally note that above arguments hold whether k is fixed or $k = t$. □

4.2.1 Perturbed dynamics

We now consider the perturbed *Usage* model with a fixed 'look-back' window.

Theorem 10. *Suppose players have finite 'look-back' of constant size k. Further suppose that the dynamic has no absorbing states. Let $\bar{q}_i = \sum_{j=1}^{i} q_j$. For $i = 1, \ldots, n$ let*

$$\zeta_i := \begin{cases} \frac{1}{\bar{q}_i^{\mu_i}(1-\bar{q}_i)^{1-\mu_i}} \prod_{j=2}^{i} \left(\frac{\bar{q}_{j-1}}{1-\bar{q}_{j-1}}\right)^{\mu_j - \mu_{j-1}} & \text{if } \mu_i < \bar{q}_i < \mu_{i+1}, \\ 0 & \text{else.} \end{cases} \tag{22}$$

Let $I^* \subseteq \{1, \ldots, n\}$ be such that for $i^* \in I^*$

$$\zeta_{i^*} = \max_{i \in \{1,\ldots,n\}} \zeta_i, \tag{23}$$

then

$$\lim_{k\to\infty} \lim_{t\to\infty} \frac{\sum_{v=t-k+1}^{t} \sum_{i=1}^{p} \mathbf{1}_{a_i^v = d} \mathbf{1}_i^v}{t} = \{\bar{q}_{i^*}\}_{i^* \in I^*}. \tag{24}$$

$\{\bar{q}_{i^*}\}_{i^* \in I^*}$ is a subset of the fixpoints of Agg. Further, for generic games there exists a unique long-term stable state.

Proof. This result follows from Pinsky (2013, Theorem 4). He studies a random walk on $\mathbb{Z}$ which at each point in time either takes one step to the right or one step to the left. Initially the probability of jumping one step to the right is $\bar{q}_1$ and of jumping to the left $1 - \bar{q}_1$. If the ratio of jumps to the right in the last k moves is greater or equal to μ_i (and smaller than μ_{i+1}) then the probability of jumping to the right is $\bar{q}_i$ and of jumping to the left $1 - \bar{q}_i$. Pinsky studies the 'speed' of the process $\tilde{s} = \lim_{t\to\infty} \frac{X_t}{t}$ where X_t is the position on $\mathbb{Z}$ of the random walk at time t when starting at zero. He finds results according to the theorem above. We shall argue that

$$\frac{\tilde{s}(\cdot) + 1}{2} = \lim_{t\to\infty} \frac{\sum_{v=0}^{t} \sum_{i=1}^{p} \mathbf{1}_{a_i^v = d} \mathbf{1}_i^v}{t}. \tag{25}$$

At time t, $\tilde{s}(\cdot) \cdot t$ is the number of times the process stepped to the right minus the number of times the process stepped to the left. Thus in $t - t \cdot \tilde{s}(\cdot)$ steps the process stepped equally often right as left. We note that the sum $\sum_{v=0}^{t} \sum_{i=1}^{p} \mathbf{1}_{a_i^v = d} \mathbf{1}_i^v$ is equivalent to stepping right when Pinsky's process is stepping right and remaining as is when Pinsky's process is stepping left. Thus the transformation follows. □

4.3 Comparison: Adoption versus Usage

We shall now compare the two different regimes from section 4.1.1 and 4.2.1.[17] We find that, in general, the long-run stable states may differ. Consider the example introduced earlier in this section.

We invite the reader to verify that the fixpoints of *Agg* are $x_1^* = 2.5\%$, $x_2^* = 16\%$, $x_3^* = 50\%$, $x_4^* = 84\%$, $x_5^* = 97.5\%$. (See Figure 2 for an illustration.)

We compute the long-run stable state under *Adoption* according to Theorem 8. One finds the following resistances (up to scaling)

$$r_{x_1^*,x_2^*} = 0.0675,\ r_{x_2^*,x_3^*} = 0.16,\ r_{x_3^*,x_4^*} = 0.18,\ r_{x_4^*,x_5^*} = 0.0675, \tag{26}$$

$$r_{x_2^*,x_1^*} = 0.0675,\ r_{x_3^*,x_2^*} = 0.18,\ r_{x_4^*,x_3^*} = 0.16,\ r_{x_5^*,x_4^*} = 0.0675, \tag{27}$$

and the stochastic potentials

$$\gamma_{x_1^*} = 0.475,\ \gamma_{x_2^*} = 0.475,\ \gamma_{x_3^*} = 0.455,\ \gamma_{x_4^*} = 0.475,\ \gamma_{x_5^*} = 0.475. \tag{28}$$

Thus the (unique) stochastically stable state is x_3^*.

Next, we compute the long-run stable state under *Usage* according to Theorem 10. The rounded results of equation 22 are:

$$x_1^* :\ 1.026,\ x_2^* :\ 0.989,\ x_3^* :\ 0.977,\ x_4^* :\ 0.989,\ x_5^* :\ 1.026. \tag{29}$$

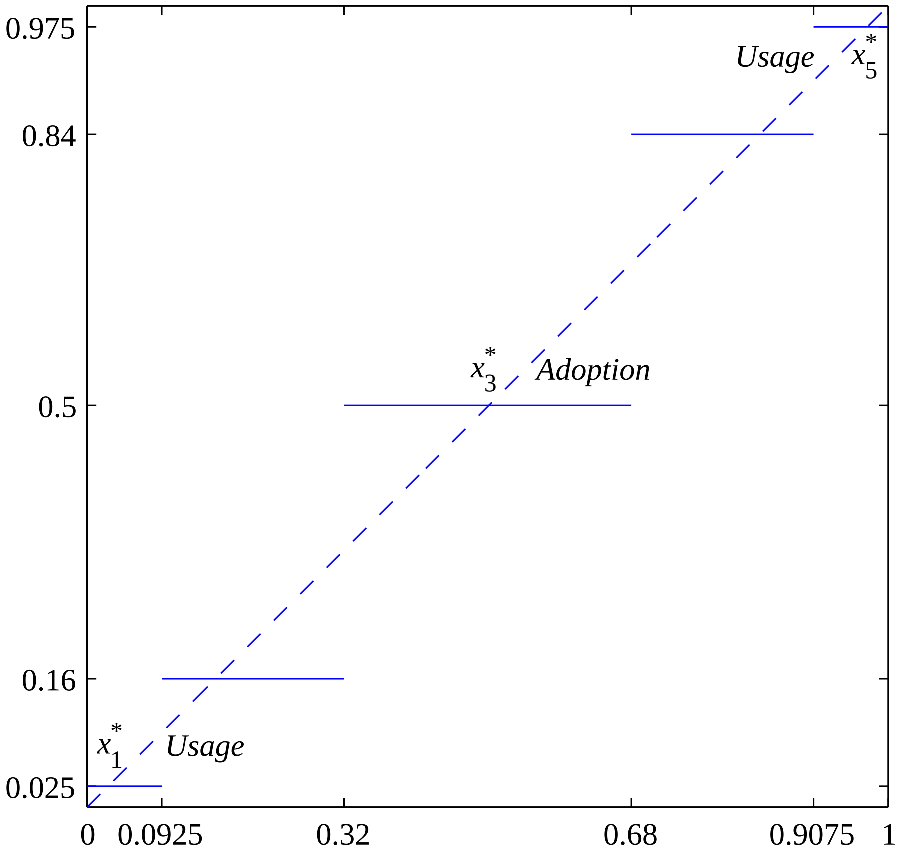
Thus the long-run stable states are x_1^* and x_5^*.

This shows, by example, that the two different models *Adoption* and *Usage* may yield significantly different outcomes (see figure 3).

Recall our example of mobile phone adoption versus usage of a mobile phone application. Under *Adoption* the shift from one fixpoint to another, say x_i^* to x_{i+1}^*, is governed by the erroneous behavior of players currently *not* playing the innovation (d). In contrast, under *Usage* the shift from one fixpoint to another , say x_i^* to x_{i+1}^*, is governed by higher usage intensity of players currently playing the innovation. More generally, on the one hand, the fluctuation of the number of adopters (away from a fixpoint) does *not* depend on the number of current adopters (given that the fluctuation depends on new adopters). On the other hand, the fluctuation of the cumulative usage depends on the number of current adopters, given that only current adopters have a positive usage.

[17] Note that the comparison of the unperturbed models immediately follows from Theorems 4 and 9 and reveals that the two models exhibit different behavior.

Figure 3: *Agg* with long-run stable states – x-axis shows μ_i, y-axis shows *Agg*



For two fixpoints, x_i^*, x_j^*, say that x_i^* is *more mixed* (*less mixed*) than x_j^* if $|x_i^* - 0.5| < |x_j^* - 0.5|$ ($|x_i^* - 0.5| > |x_j^* - 0.5|$). Then, on the one hand, the stability of *Adoption* depends on idiosyncratic errors and thus the stability of a fixpoint is independent of whether it is more or less mixed than another fixpoint. On the other hand, the stability under *Usage* is higher for less mixed states, when all else is equal. This is the case since it is less likely for a very small number of users to use a product often enough to 'skew' the observation compared to a more mixed state. In particular suppose we are currently in $x_2^* = 16\%$. In order to reach the basin of attraction of x_3^* the cumulative usage (over the last k periods) needs to be at least 32%. That is, in the last k periods, players for whom d is currently the best response (16% of the population) need to be activated at least $32\% \cdot k$ times. That is, on average such a player needs to be activated at least twice as often as players whose best response is currently m.[18] On the other hand, suppose we are currently in $x_3^* = 50\%$. In order to reach the basin of attraction of x_2^* the cumulative usage (over the last k periods) needs to be at most 32%. That is, in the last k periods, players for whom m is currently the best response (50% of the population) need to be activated at least $(100\% - 32\%) \cdot k$ times. That is, on average such a player needs to be activated at least 1.36 as often as players whose best response is currently d.[19] Since 1.36 is less than 2 it follows that the latter transition is more likely than the former.

[18]This follows from the simple calculation $32\%/16\% = 2$.

[19]This follows from the simple calculation $(100\% - 32\%)/50\% = 1.36$.

5 Conclusion

In this paper we have studied the dynamics of social influence. We considered two different models of social influence. On the one hand, social influence arises from the number of current adopters (of an innovation). On the other hand, social influence arises from the cumulative usage (of an innovation). We identified the long-run stable states under stochastic dynamics for the two models and found that the outcomes may be very different. The reason being that one model relies on fluctuations by any player while the other model relies on fluctuations by adopters. This suggests that one needs to carefully examine the case of social influence at hand in order to make valuable statements. For example, for technology adoption the driver of social influence in situations of hardware purchases (such as mobile phones) may be *Adoption*. But when considering communication software for mobile phones (such as mobile money) the cumulative usage (*Usage*) is more relevant to understand the process of social influence. Our predictions lend themselves to be tested in an experimental setting or on data which is both left for future research.

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