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RELATIONAL CONTRACTS AND SPECIFIC TRAINING

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Abstract

This paper explores the implications of specific training for relational contracts. A standard result for sustaining a relational contract is that the parties must jointly receive a surplus over what they can get by separating. This has been interpreted as employees with relational contracts having discretely higher pay and productivity than inherently equally productive, or near equally productive, employees without relational contracts. Investment in specific training relaxes the incentive constraints on relational contracts, so the optimal level of investment can be higher for those with a relational contract than for those without, adding further to the productivity of those employed under a relational contract. But the additional cost of optimal investment precisely offsets the post-investment surplus for marginal employees in relational contracts, which removes the discontinuity in the joint payoff from a relational contract. An example shows that with optimal investment there may not even be a discontinuity in productivity between those employed with a relational contract and those employed without one because the incentive constraints on the former result in lower effort despite their higher training.

*Keywords:* relational incentive contracts, investment, specific training, dual labour market

*JEL classification:* C73, D82, D86
1 Introduction

There is a long tradition in labour economics of labour markets being viewed as segmented in the sense that inherently equally productive, or near equally productive, workers have discretely different labour market outcomes. Early views of this segmentation are reflected in the concepts of dual labour markets used by Lewis (1954) and of separate primary and secondary labour markets discussed by Doeringer and Piore (1971). The reason for it has been a major concern of Dale Mortensen, whose contributions to labour economics this issue honours — see, for example, Mortensen (2005). This paper explores the role relational contracts and specific training might play in generating such outcomes.

The classic unemployment model of Shapiro and Stiglitz (1984) has been reinterpreted by Bulow and Summers (1986) as a model of a dual labour market. In that model, the employed are in a job in which performance is unverifiable in the sense that payment cannot be explicitly contracted on it. Instead, performance is sustained by the possibility that the employment will be ended or its terms made less advantageous in the future. Such arrangements have become known as relational contracts. The unemployed in the original model are reinterpreted by Bulow and Summers (1986) as being employed in less productive jobs in which performance is sufficiently well measured for pay to be explicitly conditioned on performance and so a relational contract is unnecessary to sustain it. In that reinterpretation, there is a discrete jump in productivity, and in the corresponding payoffs, between the two jobs even when there is no difference in the inherent productivity of the employees in them.

Investment in specific training might be expected to increase this jump in productivity. As Klein and Leffler (1981) note in the context of consumer product markets in which the quality of the product is not observable by consumers before purchase, sunk expenditures make it easier to sustain unverifiable performance because they reduce the benefits to firms of shading current quality at the expense of future business. There is, therefore, an incentive to incur such expenditures beyond the level that would occur if quality were observable in advance. In the context of employment, investment in specific training is a natural form for such expenditure to take. As long as this investment is to some extent productive, it will enhance the productivity of those employed under relational contracts relative to others.

Discontinuities in outcomes have also been attributed to the incentives for training that leave workers in a “low skill trap”. See, for example, Finegold and Soskice (1988) and the papers discussed in Keep and Mayhew (1999). Burdett and Smith (2002) model this formally in the context of a matching market. Workers choose their level of training before they enter the labour market. The training is general, so it is equally valuable in any match they make. Because of the matching framework, there can be
multiple equilibria. For a given specification, there can be equilibria with high average skills and high average productivity but also equilibria with low average skills and low average productivity. The primary concern in this literature is comparison between separate labour markets, particularly their relevance to differences in productivity between British and German firms, and the distinct equilibria in Burdett and Smith (2002) correspond to this. But these papers recognize that some employers establish high skill/high productivity jobs alongside low skill/low productivity ones.

The present paper explores formally the implications of specific training in the context of relational contracts. In such settings, specific training has the same role as the sunk expenditures in consumer markets discussed by Klein and Leffler (1981). In the classic treatment in Becker (1975), specific training becomes more valuable when a relationship is longer term in the sense that there is a lower probability of exogenous quits. In a relational contract setting, specific training makes longer term relationships easier to sustain because it increases the benefits of continuing the relationship relative to making an alternative match. That results in a return to specific training over and above the return in a job in which performance can be enforced without a relational contract. This paper demonstrates that formally. Thus, in contrast to the contract literature that has emphasised the under-investment in specific training that occurs because of unverifiable information, here there can be over-investment in specific training relative to that arising when performance is verifiable.

The possibility of specific training also has implications for the discrete jump in the joint payoff to employer and employee at the start of employment between marginal employees in a relational contract and marginal employees not in a relational contract. In their context of consumer markets, Klein and Leffler (1981) argue that firms cannot earn super-normal profits in a market with free entry and so will incur expenditures that drive profits to zero. But the conclusion that firms would receive super-normal profits in the absence of such expenditures is a consequence of consumers paying a constant price for purchasing the product. More generally, as MacLeod and Malcolmson (1989) show, where (as in employment) bonuses and back-loading of payments can be used in addition to fixed wages, the gain from a relational contract can be divided in any way between the two parties, so there is no need for a particular party to receive more than it could get from alternatives available elsewhere. Which party receives the benefit then depends on market characteristics, see MacLeod and Malcolmson (1998). But at least one party must still receive a payoff from a relational contract discretely higher than available elsewhere.

The present paper shows that the possibility of specific training changes things.

1It is similarly a consequence of payment in Shapiro and Stiglitz (1984) being restricted to a constant wage that the benefit from the relational contract is captured by employees in the form of efficiency wages.
Provided all those employed under a relational contract receive some specific training, the discontinuity in the joint payoff to employer and employee at the start of employment between marginal employees with a relational contract and marginal employees without one is removed. Essentially, the amount of specific training under the relational contract is adjusted so that these joint payoffs are equated for the marginal employee. This result does not come from the free entry argument in Klein and Leffler (1981) — it is the outcome the parties choose independently of assumptions about entry. Perhaps even more starkly, an example shows that, when the only advantage of jobs with relational contracts is that specific training is valuable in them, there may not even be a discrete jump in productivity between marginal employees in the two types of jobs — the increase in productivity from investment is precisely offset by the lower effort resulting from the additional incentive constraints on a relational contract.

This paper is organized as follows. The next section sets out the model and the assumptions used for the analysis. Section 3 analyses jobs with relational contracts. Section 4 studies the interaction of jobs with relational contracts and those without. Section 5 illustrates the results with a specific example. Proofs of results are in an appendix.

2 The model

There are two types of jobs that differ in the extent to which specific training adds to productivity. Both can potentially continue indefinitely. In training (T) jobs, specific training is productive but an employee’s performance is unverifiable, so payment explicitly conditional on performance is not possible. In non-training (NT) jobs, specific training is not productive but an employee’s performance is verifiable, so payment can be made explicitly contingent on that performance. There are differences between employees that affect their productivity in the two types of jobs.

Output in a T job in period $t$ is $y(e_t, I, \theta)$, where $e_t \in [0, \bar{e}]$ is the employee’s effort at $t$, $I \in [0, T]$ is the investment in specific training for the employee at the start of the relationship and $\theta \in [\underline{\theta}, \bar{\theta}]$ is the employee’s inherent type that affects productivity, with higher $\theta$ resulting in lower output for given $(e_t, I)$. (An employee’s type could reflect an inherent general skill that is transferable between the two types of jobs.) An employee’s type is observed by both employer and employee before an employee is hired. Similarly, the amount of specific training is observed by both parties, so there is no asymmetric information. Specific training, however, has a one-off cost $C(I)$. The employee also has cost of effort $c(e_t)$ in period $t$. Although more explicit forms will be explored later, for the moment the joint payoff to the firm and an employee of type $\theta$ in an NT job for one period is specified simply as $s(\theta)$. These functions have the
following properties.

**Assumption 1** For all \((I, \theta) \in [0, \bar{T}] \times [\underline{\theta}, \bar{\theta}]\), \(y(0, I, \theta) = 0\) and, for \(e \in (0, \bar{e}]\), \(y(e, I, \theta) > 0\) and twice differentiable in all its arguments, with \(y_1(e, I, \theta) > 0\), \(y_2(e, I, \theta) > 0\), \(y_3(e, I, \theta) < 0\), \(y_{11}(e, I, \theta) \leq 0\), \(y_{13}(e, I, \theta) \geq 0\) and \(y_{23}(e, I, \theta) < 0\).

\(c(0) = 0\) and for all \(e \in (0, \bar{e}]\), \(c(e)\) is twice differentiable, with \(c'(e) > 0\) and \(c''(e) \geq 0\).

\(C(I)\) is strictly increasing and strictly convex, with \(C(0) = 0\) and \(\lim_{I \to 0+} C(I) \geq 0\).

\(\bar{g}(\theta)\) is continuous and both strictly positive and non-increasing for all \(\theta \in [\underline{\theta}, \bar{\theta}]\).

Both employer and employee are risk neutral and discount the future with the same discount factor \(\delta \in (0, 1)\). The payoff to the employer from employment in the \(T\) job in period \(t\) can then be written \(y(e_t, I, \theta) - W_t\), where \(W_t\) is the payment to the employee. The payoff to the employee can similarly be written as \(W_t - c(e_t)\). The joint payoff to them both from being matched is just the sum of their individual payoffs. Conditional on \((e_t, I, \theta)\), the joint payoff from a \(T\) job in period \(t\) is \(s(e_t, I, \theta) = y(e_t, I, \theta) - c(e_t)\). The discounted future joint payoff \(gain\) from filling a \(T\) job with stationary effort \(e\) is

\[
S(e, I, \theta) = \frac{1}{1 - \delta} [y(e, I, \theta) - c(e) - \bar{g}(\theta)] - C(I) .
\] (1)

**Assumption 2** For all \(\theta \in [\underline{\theta}, \bar{\theta}]\), \(S(e, I, \theta)\) is

1. strictly increasing in \(e\) for \(e = 0\) and strictly decreasing in \(e\) for \(e = \bar{e}\) for all \(I \in [0, \bar{T}]\);
2. strictly increasing in \(I\) for \(I > 0\) sufficiently small and strictly decreasing in \(I\) for \(I = \bar{T}\) for all \(e \in (0, \bar{e}]\);
3. strictly concave in \((e, I)\) for all \((e, I) \in (0, \bar{e}] \times [0, \bar{T}]\).

First-best effort for given \((I, \theta)\), denoted \(e^*(I, \theta)\), maximizes \(s(e, I, \theta)\) and is given by

\[
y_1(e^*(I, \theta), I, \theta) = c'(e^*(I, \theta)) .
\] (2)

In a \(T\) job, specific training is productive, so it may be optimal to have \(I > 0\). First-best investment for an employee in the \(T\) job is given by

\[
I^*(\theta) \in \arg \max_{I \in [0,\bar{T}]} \frac{1}{1 - \delta} [y(e^*(I, \theta), I, \theta) - c(e^*(I, \theta)) - \bar{g}(\theta)] - C(I) , \text{ for } \theta \in [\underline{\theta}, \bar{\theta}] .
\] (3)

Conditional on strictly positive investment, by the envelope theorem \(I^*(\theta)\) satisfies

\[
\frac{1}{1 - \delta} y_2(e^*(I^*(\theta), \theta), I^*(\theta), \theta) = C'(I^*(\theta)) .
\] (4)
Optimal investment $I^*(\theta)$ may be zero either because type $\theta$ is employed in the NT job or because investment is not worthwhile even in the T job. It is convenient to define

$$\theta^* = \sup \theta \in [\underline{\theta}, \overline{\theta}] \text{ such that } I^*(\theta) > 0.$$  \hspace{1cm} (5)

In the first best, types $\theta \in [\underline{\theta}, \theta^*)$ are employed in the T job.

Because performance in a T job is unverifiable, performance above the minimum level $e = 0$ can be sustained only by a relational contract. The relational contract model here is that in MacLeod and Malcomson (1989) with the addition of differences in employee types and the possibility of investment in specific training. Because employee types are observed by both parties, the analysis there carries over to each type of employee separately for given $I$, with $s(\theta)$ the joint payoff each period if the relational contract is discontinued. Payment $W_t$ to the employee in the relational contract has two components, a fixed component $w_t$ that is guaranteed conditional on employment taking place but independent of performance and a bonus component $b_t$ conditional on performance in period $t$. The bonus cannot be legally enforced because performance in the T job is unverifiable, so the relational contract must be such that it is in the employer’s interest to pay it. Added to the analysis in MacLeod and Malcomson (1989) is an initial period in which the parties, having observed $\theta$ for the employee, decide on the specific investment $I$ to be made in the relationship at one-off cost $C(I)$. The next section analyses effort and investment in specific training in jobs with a relational contract.

3 Relational contract jobs

A relational contract cannot be enforced by going to court. It will, therefore, be carried out only if it is in the interest of both parties to do that, a characteristic referred to in the literature as self-enforcing. Levin (2003) formally defines a relational contract as “self-enforcing if it describes a perfect public equilibrium of the repeated game.” The set of self-enforcing contracts is largest when the punishments for deviation are, as in Abreu (1988), the most severe available, which here corresponds to the deviating party receiving the future payoff that would result from the relationship ending. Because the parties are risk neutral and use the same discount factor, they can redistribute the joint gain from their relationship in any way they choose by an upfront payment at the start of the relationship. It is thus optimal for them to select an equilibrium contract that maximizes that joint gain at the start of the relationship. Moreover, by a general argument in Levin (2003, Theorem 2) that applies to the model here, if an optimal contract exists, there are stationary contracts that are optimal. An optimal stationary contract depends only on current payoff-relevant information, so optimal effort and
payment functions have the form \( e_t = e(I, \theta), w_t = w(I, \theta) \) and \( b_t = b(e_t, I, \theta) \).

**Proposition 1** A stationary relational contract with an employee of type \( \theta \in [\theta, \bar{\theta}] \) with investment in specific training \( I \in [0, T] \) can be sustained if and only if

\[
\max_{e \in [0, \bar{e}]} \delta y(e, I, \theta) - c(e) - \delta z(\theta) \geq 0. \tag{6}
\]

For \((I, \theta)\) satisfying (6), stationary effort \( e(I, \theta) \) can be implemented if and only if it satisfies

\[
\delta y(e(I, \theta), I, \theta) - c(e(I, \theta)) - \delta z(\theta) \geq 0. \tag{7}
\]

Proposition 1 is based on a result in MacLeod and Malcomson (1989). The incentive compatibility requirement (7) implies

\[
\frac{1}{1 - \delta} [y(e, I, \theta) - c(e) - z(\theta)] \geq \frac{c(e(I, \theta))}{\delta}. \tag{8}
\]

The left-hand side of (8) is the difference in the joint payoff to the employer and a type \( \theta \) employee from a \( T \) job over an \( NT \) job, gross of the cost of investment. This is the surplus required to sustain the relational contract. Condition (8) implies that this surplus has to be strictly positive to sustain any relational contract with \( e(I, \theta) > 0 \) (which any \( T \) job must have because zero effort in a \( T \) job produces zero joint payoff which is less than the strictly positive joint payoff \( z(\theta) \) in an \( NT \) job). Thus, in the absence of investment in specific training, as a result of which \( C(I) = 0 \), there must be a discrete difference in the joint payoff between a \( T \) job and an \( NT \) job. With the functions all continuous, there must, therefore, be a discrete difference in outcome between the marginal type \( \theta \) in a \( T \) job and the marginal type \( \theta \) in an \( NT \) job.

Let \( \Theta(I) \) denote the set of \( \theta \) for which (6) is satisfied for given \( I \). In view of Proposition 1, effort in an optimal relational contract for employee type \( \theta \) conditional on \( I \) is a solution to

\[
\max_{e \in [0, \bar{e}]} \frac{1}{1 - \delta} [y(e, I, \theta) - c(e) - z(\theta)] \quad \text{subject to (7), for } \theta \in \Theta(I), I \in [0, T].
\]

**Proposition 2** For \( \theta \in \Theta(I) \) and \( I \in [0, T] \), an optimal effort schedule \( \hat{e}(I, \theta) \) for a relational contract has \( \hat{e}(I, \theta) \leq e^*(I, \theta) \) for all \((I, \theta)\) and the form

\[
\hat{e}(I, \theta) = \begin{cases} 
 e^*(I, \theta), & \text{if } e^*(I, \theta) \text{ satisfies (7)}; \\
 \max e \in [0, \bar{e}] \text{ that satisfies (7) with equality, otherwise}. \end{cases} \tag{9}
\]

The following additional properties apply when \( y(e, I, \theta) \) is additively separable in \( e \) and \( \theta \) and \( z(\theta) \) is independent of \( \theta \).

**Corollary 1** Suppose \( y(e, I, \theta) \) is additively separable in \( e \) and \( \theta \), \( z(\theta) \) is independent of \( \theta \) and there exists \( \theta \in \Theta(I) \) such that \( e^*(I, \theta) \) satisfies (7). For \( \bar{\theta}(I) \) the highest such \( \theta \), optimal
stationary effort takes the form

\[
\hat{e}(I, \theta) = \begin{cases} 
e^*(I, \theta), & \text{for } \theta \in \left[\bar{\theta}, \tilde{\theta}(I)\right]; \\
\max e \in [0, \overline{e}] & \text{that satisfies (7) with equality, for } \theta \in \left[\bar{\theta}(I), \sup \Theta(I)\right].
\end{cases}
\]

Under the conditions of this corollary, employee types can be partitioned such that \(\theta \leq \bar{\theta}(I)\) have effort that is first best conditional on \(I\), whereas \(\theta > \bar{\theta}(I)\) have effort below first best conditional on \(I\).

In view of Proposition 2, optimal investment in specific training for \(\theta\) employed in the \(T\) job is \(\hat{I}(\theta)\) that solves

\[
\max_{I \in [0, T]} \frac{1}{1 - \delta} \left[ y(\hat{e}(I, \theta), I, \theta) - c(\hat{e}(I, \theta)) - \varepsilon(\theta) \right] - C(I), \text{ for } \theta \in \left[\bar{\theta}, \bar{\theta}\right].
\tag{10}
\]

The first-order condition for an interior solution to the problem in (10) for \(\theta\) such that (7) is a binding constraint is

\[
\frac{1}{1 - \delta} \left\{ \left[ y_1(\hat{e}(\hat{I}(\theta), \theta), \hat{I}(\theta), \theta) - \frac{c'(\hat{e}(\hat{I}(\theta), \theta))}{\delta} \right] \hat{e}_1(\hat{I}(\theta), \theta) \\
+ y_2(\hat{e}(\hat{I}(\theta), \theta), \hat{I}(\theta), \theta) \right\} - C'(\hat{I}(\theta)) = 0.
\tag{11}
\]

Moreover, from the definition of \(\hat{e}(I, \theta)\), it follows that, for \(\hat{e}(I, \theta) < e^*(I, \theta)\),

\[
\left[ y_1(\hat{e}(I, \theta), I, \theta) - \frac{c'(\hat{e}(I, \theta))}{\delta} \right] \hat{e}_1(I, \theta) + y_2(\hat{e}(I, \theta), I, \theta) = 0.
\]

Thus, the first-order condition can be written

\[
\frac{1}{\delta} c'(\hat{e}(\hat{I}(\theta), \theta)) \hat{e}_1(\hat{I}(\theta), \theta) = C'(\hat{I}(\theta)).
\tag{12}
\]

This is essentially an application of the envelope theorem.

**Proposition 3** Suppose \(y_{12}(e, I, \theta) \leq 0\) for all \((I, \theta)\). Then, conditional on \(\hat{I}(\theta) > 0\) for \(\theta\), \(\hat{I}(\theta) \geq I^*(\theta)\), with the inequality strict for all \(\theta\) such that \(\hat{e}(\hat{I}(\theta), \theta) < e^*(\hat{I}(\theta), \theta)\).

Proposition 3 gives a condition under which investment in specific training for employees in the \(T\) job is never below first best and is strictly above first best for those whose effort is strictly below first best. The condition specified is that effort and investment are substitutes. The essential reason for the result is that investment in specific training relaxes the incentive compatibility constraint (7) because it increases the joint payoff from continuing the \(T\) job over that from the \(NT\) job for given effort. If effort in the relational contract is below first-best, investment thus allows effort to be increased closer to first best, so it has a marginal benefit over and above its direct effect on productivity. Thus, provided the lower effort does not reduce the direct marginal
product of investment too much, it is optimal to invest more than if performance were verifiable (in which case no relational contract would be required to sustain effort). With effort and investment (at least weak) substitutes, lower effort does not reduce the direct marginal product of investment. So this assumption is sufficient, but not necessary, to ensure the result holds. It will certainly continue to hold as long as effort and investment are not too strong complements.

In the first best, types $\theta \in [\theta^*, \theta^+]$ receive specific training. Let

$$\hat{\theta} = \sup \theta \in [\theta, \theta^+] \text{ such that } \hat{I}(\theta) > 0,$$

(13)

so that, with unverifiable effort, types $\theta \in [\theta, \hat{\theta}]$ receive specific training. The next result compares $\hat{\theta}$ with $\theta^*$.

**Proposition 4** If $\hat{\theta} < \theta^$, then $\hat{\theta} \leq \theta^*$, with the inequality strict for $\hat{I}(\hat{\theta}) \neq I^*(\hat{\theta})$.

Proposition 4 establishes that, if some types do not receive specific training (even if only because they are not employed in the $T$ job), the marginal employee type in that job who receives specific training is never higher than first best and is strictly less than first best if receiving specific training different from first best. The essential point here is that having the incentive compatibility constraint (7) bind, although increasing the marginal payoff to investment in specific training for given effort, reduces the total payoff because it results in effort below first best. Thus, for sufficiently unproductive types $\theta$, investment ceases to be worthwhile if there is a fixed cost to investment. Even if there is no fixed cost to investment, it makes it more attractive to allocate less productive employees to the $NT$ job.

4 Interaction between relational and non-relational contracts

This section studies the interaction between $T$ jobs and $NT$ jobs. For convenience, let $\hat{S}(I, \theta) = S(\hat{e}(I, \theta), I, \theta)$. The primary result is the following.

**Proposition 5** Suppose $\hat{S}(\hat{I}(\theta), \theta) \geq 0$ for some $\theta \in [\theta, \theta^+]$, all types $\theta$ in the $T$ job have $\hat{I}(\theta) > 0$, $\hat{\theta} < \theta^+$ and a relational contract in the $T$ job is feasible for $\hat{\theta} + d\theta$ with $\hat{I}(\hat{\theta} + d\theta) > 0$ as $d\theta \to 0+$. Formally, $\hat{I}(\hat{\theta} + d\theta)$ and $\hat{e}(\hat{I}(\hat{\theta} + d\theta), \hat{\theta} + d\theta)$ satisfy (7) as $d\theta \to 0+$. Then

$$\lim_{\theta \to \hat{\theta}} \hat{S}(\hat{I}(\theta), \theta) = 0.$$

Proposition 5 gives conditions under which the joint gain in the $T$ job over that in the $NT$ job is zero for the marginal type in the $T$ job and, hence, the joint payoff is continuous in $\theta$. There is then no discrete jump in payoffs between the marginal
types in the $T$ and the $NT$ jobs. This may seem surprising in the light of the results in Shapiro and Stiglitz (1984), as reinterpreted by Bulow and Summers (1986), and MacLeod and Malcomson (1989) that, for a relational contract to be sustained, there must be a discrete jump between the joint payoff from the relational contract and that from the outside alternative. The reason for the difference in the results is the investment in specific training. That creates a discrete jump between the joint payoff from the relational contract and that from the outside alternative after the investment has been made (and is, therefore, a sunk cost) which is sufficient to sustain the relational contract even when there is no jump before the investment is made. Under the conditions of Proposition 5, the optimal investment results in the same joint payoff from starting the two jobs for the employee type on the margin between them.

The conditions in Proposition 5 that $\hat{S}(\hat{I}(\theta), \theta) \geq 0$ for some $\theta \in [\theta, \bar{\theta}]$ and $\hat{\theta} < \theta$ merely ensure that there is some type for which the $T$ job with a relational contract is feasible and some type for which it is not chosen. A substantive condition is that a relational contract with positive investment is feasible for some $\theta$ marginally above $\hat{\theta}$. For such $\theta$, the payoff gain to setting up a relational contract must be non-positive. With $\hat{S}(\hat{I}(\theta), \theta)$ continuous in $\theta$, it must then be that the payoff gain in the $T$ job approaches zero as $\theta$ approaches $\hat{\theta}$. The same result holds even if the feasible relational contract for $\theta$ marginally above $\hat{\theta}$ has no investment as long as there is no fixed cost to investment. In either case, it is important that all types in the relational contract have some specific investment. If there were some that did not, Proposition 1 would imply a discretely higher joint payoff to marginal types in the $T$ job over that in the $NT$ job.

The following assumption about $NT$ jobs ensures that the conditions of Proposition 5 are satisfied.

**Assumption 3** In an $NT$ job, the output of a type $\theta$ employee is $y(e_t, 0, \theta)$ and the cost of effort $c(e_t)$.

Under Assumption 3, output in an $NT$ job is the same as that in a $T$ job without specific training and the cost of effort is the same as in a $T$ job. Thus

$$s(\theta) = y(e^*(0, \theta), 0, \theta) - c(e^*(0, \theta)).$$

By the envelope theorem and Assumption 1, $s(\theta)$ is then strictly decreasing in $\theta$, thus satisfying Assumption 1.

Assumption 3 focuses attention on the implications of investment in specific training because there would be no benefit from using the $T$ job if there were no such investment. Because performance in an $NT$ job is verifiable, the employer can make payment conditional on a specified performance. (Because both parties observe $\theta$, it is immaterial whether it is output or effort that is verifiable.) It is then in both parties’
interests for effort to be set at the first-best level $e^*(0, \theta)$ and straightforward to enforce this by appropriate choice of payment. Thus, there is no benefit to using the T job unless there is investment in specific training and a potential cost to using the T job that effort may be below first best. It is then without loss of generality that all types $\theta$ in the T job have $\hat{I}(\theta) > 0$, as assumed in Proposition 5. With Assumption 3, therefore, there is no discrete difference in the joint payoff between the marginal type $\theta$ in a T job and the marginal type $\theta$ in an NT job.

That does not, however, itself rule out a discrete difference in productivity between the marginal type $\theta$ in a T job and the marginal type $\theta$ in an NT job. The following example, however, shows that such a discrete difference in productivity does not necessarily occur.

5 An example

An analytically tractable example is that with\(^2\)

$$y(e, I, \theta) = e + I/\theta \quad \text{for } e > 0, \quad \text{with } \theta > 0; \quad c(e) = e^2/2; \quad C(I) = \gamma I^2/2, \quad \text{with } \gamma > 0,$$

with productivity in the NT job just $e$, consistent with Assumption 3. In this case, the employee’s type affects only the benefit of investment in specific training in the T job, not the productivity of effort in the absence of specific training. For this example, from (2), (3) and (14)

$$e^*(I, \theta) = 1; \quad I^*(\theta) = \frac{1}{\theta \gamma (1 - \delta)}; \quad \hat{z}(\theta) = \frac{1}{2}; \quad \theta^* = \bar{\theta}. \quad (16)$$

The last of these arises because, with no fixed cost to investment in specific training, it would always be worthwhile investing for all types and employing them in the T job if performance in that job were verifiable.

With this formulation, the incentive compatibility condition (7) becomes

$$\delta \left( e(I, \theta) + \frac{1}{\bar{\theta}} \right) - \frac{e(I, \theta)^2}{2} - \frac{\delta}{2} \geq 0. \quad (17)$$

The highest effort consistent with this is the largest root of the quadratic equation for $e(I, \theta)$ given by setting the left-hand side of (17) equal to zero. That root is

$$\delta \left\{ 1 + \left[ 1 + \frac{2}{\delta} \left( \frac{I}{\theta} - \frac{1}{2} \right) \right]^{1/2} \right\} = \delta \left\{ 1 + \left[ \frac{2}{\delta} \left( \frac{I}{\theta} - \frac{1}{2} - \delta \right) \right]^{1/2} \right\}. \quad (18)$$

By Proposition 2, this effort is the optimal effort $\hat{e}(I, \theta)$ for the T job as long as it is less

\(^2\)The complete specification for $y(e, I, \theta)$ includes $y(0, I, \theta) = 0$ for all $(I, \theta)$.  

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than \(e^*(I, \theta)\), so

\[
\hat{e}(I, \theta) = \begin{cases} 
1, & \text{if } \delta \left\{ 1 + \left[ \frac{2}{\delta} \left( \frac{1}{\theta} - \frac{1-\delta}{2} \right) \right]^{1/2} \right\} > 1; \\
\delta \left\{ 1 + \left[ \frac{2}{\delta} \left( \frac{1}{\theta} - \frac{1-\delta}{2} \right) \right]^{1/2} \right\}, & \text{otherwise}. 
\end{cases}
\] (18)

If hired in a \(T\) job, investment in specific training for a type \(\theta\) employee when \(\hat{e}(I, \theta) < e^*(I, \theta)\) is given by (12) which, in the formulation in the example, becomes

\[
\gamma \hat{I}(\theta) = \frac{1}{\theta} \left\{ 1 + \frac{1}{\left[ \frac{2}{\delta} \left( \frac{1}{\theta} - \frac{1-\delta}{2} \right) \right]^{1/2}} \right\}. 
\] (19)

A solution for \(\hat{I}(\theta)\) can be obtained by finding a real root of the cubic equation in \(z := \left( \frac{1}{\theta} - \frac{1-\delta}{2} \right)^{1/2}\). An employee of type \(\theta\) will be hired for a \(T\) job as long as the joint payoff gain from that is greater than from an \(NT\) job; that is, as long as

\[
\frac{1}{1-\delta} \left[ \hat{e}(\theta, \hat{I}(\theta)) + \hat{I}(\theta) - \frac{\hat{e}(\theta, \hat{I}(\theta))^2}{2} - \frac{\gamma \hat{I}(\theta)^2}{2} \right] \geq \frac{1}{1-\delta^2}. 
\]

The example satisfies the conditions of Proposition 5, so this holds with equality for the cutoff type \(\hat{\theta}\). Productivity of type \(\theta\) in the \(T\) job is

\[
\hat{e}(\theta, \hat{I}(\theta)) + \frac{\hat{I}(\theta)}{\theta}
\]

and of type \(\theta\) in the \(NT\) job is 1.

The following figures illustrate the example for \(\delta = 0.9, \gamma = 10\) and \(\theta \in [1, 10]\). (The value of \(\delta\) needs to allow for the probability a \(T\) job comes to an end for reasons outside the model in addition to the pure time discount rate.) In Figure 1, the solid line plots the expression \(\delta \left\{ 1 + \left[ \frac{2}{\delta} \left( \frac{1}{\theta} - \frac{1-\delta}{2} \right) \right]^{1/2} \right\}\) that appears in (18) for optimal investment \(\hat{I}(\theta)\). The dashed line plots \(e^*(I, \theta) = 1\). In the \(T\) job with effort unverifiable, the solid line is optimal effort \(\hat{e}(\hat{I}(\theta), \theta)\) provided it is less than first-best effort \(e^*(I^*(\theta), \theta)\). Optimal effort is thus the lower envelope of the two lines. So, for type \(\theta\) employed in the \(T\) job, optimal effort \(\hat{e}(\hat{I}(\theta), \theta)\) is first-best up to \(\theta \approx 4.25\) (more precisely, 4.2426) and below first-best for higher \(\theta\).

Type \(\theta\) is employed in the \(T\) job only if this effort and investment yield a higher joint payoff than the \(NT\) job. Figure 2 plots the joint payoff gain for \(\theta\) employed in the \(T\) job with effort given by (18) over being employed in the \(NT\) job, the solid line, and also the first-best payoff gain in the \(T\) job over the \(NT\) job, the dashed line. As to be expected from Figure 1, the two coincide at the same value of \(\theta\) at which effort given by (18) becomes first-best effort, \(\theta \approx 4.25\). For \(\theta\) less than this, first-best effort and
investment are attainable, so there is no loss of joint gain from effort being unverifiable. An employee with $\theta$ greater than this will be employed in the $T$ job as long as that has higher joint payoff than the $NT$ job but with effort below first-best. That applies to $\theta$ up to approximately 6.2 (more precisely, 6.1623), where the solid line in Figure 2 crosses the horizontal axis. An employee with $\theta$ greater than this is employed in the $NT$ job. Note that, if effort were verifiable in the $T$ job, the joint payoff gain would be given by the dashed line in Figure 2 and so would always be positive. Thus all employee types would be employed in the $T$ job.

The amount of investment in specific training for employees in the $T$ job as a function of type is plotted in Figure 3. For the formulation in this example, the quadratic equation for $e$ given by setting the left-hand side of (17) equal to zero has no real root for $I = 0$, so types are employed in the $T$ job only if there is strictly positive investment in specific training. It is only the possibility of specific training that enables a relational contract to be used. For this reason, Figure 3 is plotted only for $\theta \leq 6.2$.

The solid line is investment when effort in the $T$ job is at the highest level that satisfies (17), the dashed line the first-best investment. The former coincides with the latter at precisely the same value of $\theta$ that optimal effort first coincides with first-best effort as $\theta$ is decreased, $\theta \simeq 4.25$. Consistent with Proposition 3, optimal investment is above first-best investment for $\theta$ such that the incentive compatibility constraint (17) binds. As explained above, the reason is that increasing investment relaxes that constraint, so the marginal return on investment is higher than if effort were verifiable and the first-best attained. Thus, optimal investment is the upper envelope of the two curves in Figure 3. An interesting characteristic is that optimal investment is increasing for $\theta$ above 4.25 for those employed in the $T$ job, even though first-best investment is de-
Investment in specific training affects the productivity of employees in the $T$ job. In the example, productivity in the $T$ job is given by

$$
\hat{e}(\hat{I}(\theta), \theta) + \frac{\hat{I}(\theta)}{\theta} = \delta \left\{ 1 + \left[ 1 + \frac{2}{\delta} \left( \frac{\hat{I}(\theta)}{\theta} - \frac{1}{2} \right) \right]^{1/2} \right\} + \frac{\hat{I}(\theta)}{\theta}.
$$

Productivity of employees in the $NT$ job is just 1. These are plotted in Figure 4. The solid line is productivity in the $T$ job when effort is at the highest level consistent with the incentive compatibility constraint (17), which applies for $\theta$ above approximately 4.25. The dashed line is first-best productivity in the $T$ job, which applies for $\theta$ less that approximately 4.25. The dotted line is productivity in the $NT$ job. That coincides precisely with productivity in the $T$ job at the cutoff value of $\theta$ ($\theta \approx 6.2$) between the $T$ and $NT$ jobs. Thus productivity is continuous despite the discretely lower effort, and the discretely higher investment, in the $T$ job. The investment in specific training precisely offsets the effect of the lower effort on productivity. The continuity in productivity contrasts with the reinterpretation of the Shapiro and Stiglitz (1984) model of unemployment as a model of a dual labour market by Bulow and Summers (1986) for which, in the absence of investment in specific training, Proposition 1 implies that productivity must be discretely lower in the secondary labour market. In the example here, because effort in the jobs is determined endogenously and is equally productive.
in the absence of investment in specific training, effort in the $T$ job is reduced below that in the $NT$ job for types in the former for which the incentive compatibility constraint (17) binds. Investment in specific training precisely offsets that for marginal employee types in the two jobs.

6 Conclusion

This paper has extended a standard relational contract model with unverifiable effort to incorporate investment in specific training of employees. Such investment increases the joint payoff to the parties from continuing their relationship. In this way, the investment relaxes the incentive compatibility constraint that must be satisfied for a relational contract to be sustained and so increases the marginal benefit to investment in specific training. Provided effort and specific training are not too strongly complementary, this results in those employees who receive specific training receiving more than the first-best level that would occur with verifiable effort. Indeed, an example shows that this may result in specific training higher for employees for whom it is inherently less productive (and thus for whom first-best specific training is lower) because the additional marginal benefit from relaxing the incentive compatibility constraint is so large for those types. But the incentive compatibility constraint results in effort below the first-best level except for highly productive types and that reduces the average benefit from a given level of investment in specific training. Thus fewer employee types receive any specific training at all than in a first-best world.

Investment in specific training may also remove the discontinuity in the joint payoffs at the start of a job between marginal types with a relational contract and marginal
types without a relational contract that occurs in the reinterpretation by Bulow and Summers (1986) of the Shapiro and Stiglitz (1984) model. The reason lies in the cost of the investment. After this cost has been incurred, there is indeed a jump in the joint payoff from the job with unverifiable effort over that with verifiable effort. That is essential for sustaining the relational contract required to induce effort in the former. But at that stage, the cost of investment in specific training is sunk and so does not affect the joint payoff gain to continuing the relationship, which is what is crucial for the relationship to be sustained. Thus if, at the stage the amount of investment is determined, it is optimal to train all employee types for whom the job with unverifiable effort gives a higher joint payoff conditional on optimal investment than the job with verifiable effort, there need be no jump in joint payoff at the margin between the jobs. The possibility of investment in specific training then removes discontinuities in the joint payoffs from the two different types of jobs.

Investment in specific training may even remove the need for a jump in productivity between the marginal employee receiving specific training and the marginal employee not receiving it, as shown by an example in which the marginal product of effort is the same in the two different types of jobs. The reason lies in the endogenous determination of effort. Effort in the job with unverifiable effort is reduced below the first-best level for types for which the incentive compatibility constraint binds. With effort in that job not inherently more productive than in the job with verifiable effort, optimal effort is lower in the former. The increase in productivity that results from specific training is precisely offset by the reduction in effort below the first-best level.

In summary, the return to investment in specific training is higher for employees in jobs that rely on a relational contract to sustain performance than it would be if pay
could be made explicitly performance-related. Moreover, under plausible conditions, the possibility of such investment removes the discontinuity in the joint payoff to employer and employee between jobs making use of relational contracts and those not doing so. It may even remove the discontinuity in their productivity.

Appendix A  Proofs

Proof of Proposition 1.

To simplify notation in the proof, drop the argument $I$ because the proposition specifies a given $I$ and let $u(\theta)$ denote the payoff to the employee of type $\theta$ in each period of a stationary relational contract. Then, if both parties stick to the contract,

$$u(\theta) = w(\theta) + b(e(\theta), \theta) - c(e(\theta)). \tag{20}$$

So, having observed $\theta$, the payoff to the employee from sticking to the contract for ever, conditional on the employer doing so, is

$$u(\theta) + \frac{\delta}{1-\delta} u(\theta). \tag{21}$$

For the employer, let $v(\theta)$ denote the payoff in each period of a stationary relational contract with an employee of type $\theta$. Then, if both parties stick to the contract,

$$v(\theta) = y(e(\theta), \theta) - w(\theta) - b(e(\theta), \theta). \tag{22}$$

So, having observed $\theta$, the payoff to the employer from sticking to the contract for ever, conditional on the employee doing so, is

$$v(\theta) + \frac{\delta}{1-\delta} v(\theta). \tag{23}$$

Necessity. By setting $e_t = 0$ and quitting the following period, the employee can, even with no bonus, guarantee a payoff of

$$w(\theta) + \frac{\delta}{1-\delta} u(\theta). \tag{24}$$

A necessary condition for an employee of type $\theta$ to stick to the contract is that the payoff in (21) is no less than that in (24). With the use of (20), that condition can be written

$$\frac{\delta}{1-\delta} [u(\theta) - u(\theta)] \geq c(e(\theta)) - b(e(\theta), \theta). \tag{25}$$

Once the employee has incurred effort $e(\theta)$, the employer can, by setting $b_t = 0$ and
quitting the following period, guarantee a payoff of

\[ y(e(\theta), \theta) - w(\theta) + \frac{\delta}{1 - \delta} v(\theta). \]  

(26)

A necessary condition for the employer to stick to the contract for \( \theta \) is that the payoff in (23) is no less than that in (26). With the use of (22), that condition can be written

\[ \frac{\delta}{1 - \delta} [v(\theta) - w(\theta)] \geq b(e(\theta), \theta). \]  

(27)

As both (25) and (27) are necessary, so is their sum, which gives the combined necessary condition

\[ \frac{\delta}{1 - \delta} [u(\theta) + v(\theta) - (u(\theta) + v(\theta))] \geq c(e(\theta)). \]

Substitution for \( u(\theta) \) from (20), for \( v(\theta) \) from (22) and for \( u(\theta) + v(\theta) \) by \( s(\theta) \) and re-arrangement gives (7), which is, therefore, also necessary. Moreover, (7) can be satisfied only if (6) is satisfied, so (6) is necessary too.

**Sufficiency.** For (7) satisfied, there certainly exists a \( b(e(\theta), \theta) \in [0, c(e(\theta))] \) for each \( \theta \) such that (25) and (27) are both satisfied. For such \( b(e(\theta), \theta) \), the individual rationality conditions

\[ E_{\theta} u(\theta) - u(\theta) \geq 0 \]

(28)

\[ E_{\theta} v(\theta) - v(\theta) \geq 0 \]

(29)

are also satisfied. To establish that sticking to the contract is a best response for both parties, it remains only to show that the payoffs following deviation specified in (24) and (26), \( \delta u(\theta) / (1 - \delta) \) and \( \delta v(\theta) / (1 - \delta) \), are equilibrium payoffs. That is immediate if the strategies for the parties specify that play following deviation by either is for both to end the relationship because, if the strategy for either is to end the relationship it is a best response of the other also to end the relationship given \( u(\theta), v(\theta) \geq 0. \)

**Proof of Proposition 2.** If first-best effort given \((I, \theta), e^*(I, \theta)\), satisfies (7), it is clearly optimal to set \( \hat{e}(I, \theta) = e^*(I, \theta) \). Assumption 1 ensures that, if \( e^*(I, \theta) \) does not satisfy (7), no higher effort does, so \( \hat{e}(I, \theta) < e^*(I, \theta) \). In that case it is optimal to set \( \hat{e}(I, \theta) \) at the highest effort that satisfies (7). \[ \square \]

**Proof of Corollary.** For \( y(e, I, \theta) \) additively separable in \( e \) and \( \theta \), \( e^*(I, \theta) \) is independent of \( \theta \). Then, because \( y(e, I, \theta) \) is strictly decreasing in \( \theta \) for \( e > 0 \), the left-hand side of (7) is strictly decreasing in \( \theta \) for \( e = e^*(I, \theta) \). Thus, if (7) is satisfied for \( \theta = \tilde{\theta}(I) \) and \( e = e^*(I, \tilde{\theta}(I)) \), it is certainly satisfied for \( \theta \) and \( e = e^*(I, \theta) \) for all \( \theta \leq \tilde{\theta}(I) \). So \( \hat{e}(I, \theta) = e^*(I, \theta) \), for \( \theta \in [\tilde{\theta}, \tilde{\theta}(I)] \). \[ \square \]

**Proof of Proposition 3.** Suppose, contrary to the claim in the proposition, \( \hat{I}(\theta) \leq
$I^*(\theta)$ for $\hat{e}(\hat{I}(\theta), \theta) < e^*(\hat{I}(\theta), \theta)$. For the pair $(\hat{e}(\hat{I}(\theta), \theta), \hat{I}(\theta))$ to be optimal for $\theta$, it must be that the pair $(e^*(I^*(\theta), \theta), I^*(\theta))$ does not satisfy (7), so (7) is a binding constraint for $\theta$. With $y_2(e, I, \theta) > 0$, $\hat{I}(\theta) \leq I^*(\theta)$ implies that the left-hand side of (7) is no larger for $\hat{I}(\theta)$ than for $I^*(\theta)$ for given $e$ and so, therefore, is the maximum effort that satisfies (7). Thus $\hat{e}(\hat{I}(\theta), \theta) \leq \hat{e}(I^*(\theta), \theta)$. Moreover, by Proposition 2, $\hat{e}(I^*(\theta), \theta) \leq e^*(I^*(\theta), \theta)$, so $\hat{e}(\hat{I}(\theta), \theta) \leq e^*(\hat{I}(\theta), \theta)$.

For $\hat{I}(\theta)$ strictly positive, (12) holds and so $\hat{e}(\hat{I}(\theta), \theta) > 0$. Thus, by the specification of $e^*(I, \theta)$ in (2) and strict concavity, the upper line of (11) is strictly positive for $\hat{e}(\hat{I}(\theta), \theta) < e^*(\hat{I}(\theta), \theta)$, so

$$\frac{1}{1-\delta}y_2(e, I, \theta) - C'(\hat{I}(\theta)) < 0.$$  

Hence, with $\hat{e}(\hat{I}(\theta), \theta) \leq e^*(I^*(\theta), \theta)$ and $y_{12}(e, I, \theta) \leq 0$,

$$\frac{1}{1-\delta}y_2(e^*(I^*(\theta), \theta), \hat{I}(\theta), \theta) - C'(\hat{I}(\theta)) < 0. \quad (30)$$

Under the strict concavity in Assumption 2, the left-hand side of this is strictly decreasing in $\hat{I}(\theta)$. So, for $I^*(\theta)$ satisfying (4), (30) implies $\hat{I}(\theta) > I^*(\theta)$, which contradicts the supposition that $\hat{I}(\theta) \leq I^*(\theta)$ and is thus sufficient to establish the proposition.  

**Proof of Proposition 4.** Consider maximization of the objective function in (1) for given $\theta$. Let $V^*(\theta)$ denote the maximum value function when there is no constraint (for which $(e^*(I^*(\theta), \theta), I^*(\theta))$ is optimal) and $\hat{V}(\theta)$ the maximum value function when constraint (7) is added (for which $(\hat{e}(\hat{I}(\theta), \theta), \hat{I}(\theta))$ is optimal). Also, let $\underline{V}(\theta)$ denote the higher of the payoff gain conditional on $I = 0$ and 0 (because $g(\theta) / (1-\delta)$ is available from the NT job). Then $\hat{\theta} < \hat{\theta}$ implies $\hat{V}(\hat{\theta}) = \underline{V}(\theta)$.

Addition of a constraint to a maximization problem leaves optimal choices unchanged if these satisfy the constraint but otherwise alters some optimal choice and strictly reduces the maximum value. For $\theta = \theta$, the first possibility corresponds to $\hat{I}(\theta) = I^*(\theta)$ and $\hat{e}(\hat{I}(\theta), \theta) = e^*(I^*(\theta), \theta)$, in which case $\hat{V}(\theta) = V^*(\theta)$ and so $\theta^* = \theta$. The second possibility corresponds to $\hat{I}(\theta) = I^*(\theta)$ and/or $\hat{e}(\hat{I}(\theta), \theta) = e^*(I^*(\theta), \theta)$, in which case $\hat{V}(\theta) < V^*(\theta)$. With $\hat{V}(\theta) = \underline{V}(\theta)$, then $V^*(\theta) > \underline{V}(\theta)$, so it is worthwhile having $I^*(\theta) > 0$ for some $\theta > \hat{\theta}$ and thus $\theta^* > \hat{\theta}$. So, in all cases, $\hat{\theta} \leq \theta^*$ and, if $\hat{I}(\theta) = I^*(\theta)$, $\hat{\theta} < \theta^*$.  

**Proof of Proposition 5.** By Assumption 1 and the envelope theorem, $\hat{S}(\hat{I}(\theta), \theta)$ is differentiable and hence continuous for $\hat{I}(\theta) > 0$. By hypothesis, for some $\theta \in [\theta, \hat{\theta}]$, $\hat{S}(\hat{I}(\theta), \theta) \geq 0$. With all $\theta$ in the $T$ job having $\hat{I}(\theta) > 0$, $\hat{\theta}$ defined as in (13), no $\theta > \hat{\theta}$ is employed in the $T$ job. But, by hypothesis, a relational contract in the $T$ job would have been feasible for $\hat{\theta} + d\theta$ with $\hat{I}(\hat{\theta} + d\theta) > 0$ as $d\theta \rightarrow 0+$ and was not chosen, so
it must be that $\hat{S}(\hat{I}(\hat{\theta} + d\theta), \hat{\theta} + d\theta) \leq 0$. But with $\hat{S}(\hat{I}(\theta), \theta)$ continuous, that implies $\hat{S}(\hat{I}(\hat{\theta}), \hat{\theta}) = 0$. 

References


