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THE DYNAMICS OF DELEGATED DECISION MAKING

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Abstract.

This paper investigates the behaviour of bodies or organizations, operating in a stochastic environment, where there is a delegated decision maker. A crucial decision is when to delegate to another decision maker. The problem may be intrapersonal, as occurs when there are endogenously changing tastes, or interpersonal where delegation is intuitionally necessary or where decision making is 'as if' there is delegation. This is possible if decision making is through voting - an existence theorem is given. Decisions lead to shifts in control involving option losses; forward-looking recognition of this leads to the endogenous creation of hysteresis. The fact that the behaviour of other agents leads to hysteresis makes it optimal for any single agent to introduce more hysteresis. Organisations or bodies with many possible decision makers operate, in subsets of the state space, in one of two regimes, one where hysteresis is small and the other where hysteresis is large.

1. Introduction

This paper investigates dynamic delegated decision-making. The dynamics come from an underlying stochastic environment. The interest of the paper is in entities defined by a set of heterogeneous preferences with choices corresponding to preference maximization by one particular preference – the delegated decision-maker. A main focus is the decision by a delegated decision-maker to delegate to an alternative preference: it is assumed that different decision-makers may have a comparative advantage in control in different states of nature. The stochastic environment introduces the notion that such transfer of control may arise whoever is the initial decision-maker and the optimal choice of one decision-maker depends, through foresight, upon the choices that potentially will be made by other decision-makers. Thus, behaviour is determined as the outcome of a game between different potential decision-makers.

We can characterize a number of cases where the above scenario is relevant:

Intrapersonal decision-making. Consider a situation of multiple selves where an individual is defined by a set of preferences, perhaps indexed by some function of past consumption decisions. Preferences and tastes are endogenous. The individual will use current preferences to determine choices and may recognize the fact that future decisions will be taken to optimize different preferences – the delegated decision-maker will change over time. This recognition gives rise to the so-called 'sophisticated planner's' problem as studied by Strotz (1956) and Phelps and Pollak (1968). One example is given by addiction where preferences are not fixed, as in Becker and Murphy (1988), but determined by the level of dependence to some drug. With a given level of dependence, the self with the appropriate preferences can maintain future 'control' by keeping the same level of dependence through time. However, a decision to become more dependent by increased consumption may produce short term gains but future decisions are then delegated to preferences relevant to greater dependency levels.

In many examples of changing tastes, the change in tastes is exogenous. For instance, in models of hyperbolic discounting, e.g. Laibson (1997), preferences are indexed by time. In such models, the solution to the sophisticated planner's problem is recursive, solved backwards from some last date. In models of endogenous tastes, the sophisticated planner's problem is more complicated. However, a standard model of addiction, say, would posit a static underlying environment and each level of dependence begets a unique incremental change in dependence. In this case, dependency paths will be monotonic and the recursive nature to the planner's problem is preserved. However, if the environment evolves, with preference for a drug changing over time then behaviour at one level of dependence affects behaviour at all levels of dependence and there is non-recursive strategic interaction between different selves.

Interpersonal Decision-Making with Explicit Delegation. In an organization or team, different individuals may have different preferences and delegation is one method of possible decision-making. Delegation could be desirable because it is recognized that some individuals have superior information on which to make choices, and such individuals may be different in different environments, which makes it desirable for the decision-maker to change over time. Delegation could also be an enforced structure, as occurs in team ball games like soccer: the player in possession of the ball is the delegated decision-maker and, as the state of play evolves, it can be desirable to pass the ball to another player. If other players hold on to the ball excessively then it becomes optimal for any player not to relinquish control and also hold on to the ball. Behaviour viewed as inefficient by all players could then result. However, if players have common interests, it turns out (Proposition 8 below) that inefficient 'hogging' of the ball cannot be sustained as an equilbrium.

Interpersonal Decision-Making with Implicit Delegation. Instead of delegating decision-making to one individual, a group may aggregate preferences through a mechanism like voting. Ownership of an organization may be decentralized and voting becomes one method to exercise joint control (Benham and Keefer (1991), Hansmann

(1996)). If ownership is endogenous then aggregate preferences of the group of owners will be endogenous. Under certain conditions, the aggregate voting preference will coincide with the preferences of one of the voters, e.g. under majority voting, a median voter result may apply. In this case, group behaviour will be 'as if' there is a delegated decision-maker and the decision-maker will be endogenous, given that the group of voters is endogenous. Thus, a model of delegated decision-making can apply to voting situations. A model of this for a static environment is presented in Roberts (1998). See also Jack and Lagunoff (2006) and Acemoglu, Egorov and Sonin (2012).

This paper analyzes situations of delegated or 'as if' delegated decision-making. The stochastic environment for the analysis is important because i) delegation to others involves a loss of control and this has an option value in a stochastic world; ii) the optimal decision by one to delegate to another depends upon the decisions that the other potentially will take and these will, in turn, depend upon the decisions of the one – the decision to delegate to others involves strategic interaction; iii) the behaviour of an entity like an organization will embody the richness of response to change rather than being a static choice. With regard to this dynamic behaviour, it will be shown that dynamic delegated decision-making can lead to sudden changes in behaviour, either because a change of delegated decision-maker may exogenously lead to a difference in behaviour or, more interestingly, because such changes in behaviour are endogenously created as a consequence of strategic interaction. A consequence of this sudden change in behaviour is that dynamic behaviour exhibits hysteresis –dynamic behaviour is incompatible with behaviour induced by single preference maximization.

The analysis starts in Sections 2 and 3 with an investigation of decision-making under majority voting. A major role of this analysis is to determine conditions under which majority voting is equivalent to delegated decision-making; one requirement for this will be that voting equilibria exist. From Section 4 onwards, the paper concentrates on a model, the reduced form from the earlier sections, of delegated or 'as if' delegated, decision-making. Section 4 determines some general properties of equilibria and these are applied in Section 5 to a model where the number of possible decision-makers is restricted to two. Hysteresis in behaviour is induced because preferences differ. More importantly, the existence of hysteresis gives an option value to retaining control and this generates further (strategic) hysteresis. The extent of this strategic effect is examined in Section 6 by considering a continuous/linear approximation to the model. Section 7 extends the analysis to environments with many decision-makers and this allows, in the limit, the discreteness between preferences to be reduced to zero. Under certain conditions, some equilibria – termed disconnected – will not exhibit hysteresis and sophisticated behaviour is equivalent to myopic and naïve behaviour. Under other conditions, myopic and naïve behaviour would induce sudden change and, under sophisticated behaviour, a strategic reaction to this induces hysteresis. Even when myopic and naïve behaviour would not induce sudden change, sophisticated behaviour may involve sudden changes of behaviour, inducing hysteresis. Concluding remarks are contained in Section 8.

2. The Framework

We will lay down a model of an organization where there is majority voting and decisions determine the composition of the electorate in the future. In the next section, it will be shown that behaviour is equivalent to that resulting from decisions being delegated to one particular individual. For the rest of the paper, the model may be interpreted as one of pure delegated decision-making.

Time is discrete. At any date t, the attributes of an organization are captured by an integer valued variable x_t which is the size of the electorate in the organization at date t - there is a one-to-one relationship between possible organization attributes and electorate size. The electorate is drawn from a finite set X of infinitely lived individuals, $X = \{1,2,...,\overline{x}\}$. It is assumed that there is a fixed system of expansion or contraction in the organization so that when the organization is of size x_t, the electorate is given by $\{1,2,...,x_t\}$. The per unit utility or payoff to voter ξ is given by $u(x,s,\xi)$ where s, the state of the world, evolves over time according to a Markov process. This is taken to be a simple process where the set of states of the world is assumed finite, $S=(s_{\min},...,s_{\max})$, and transitions can only occur between adjacent states:

$$\Pr\{s_{t+1} \in (s\text{-}1,\ s,\ s+1) \mid \ s_t = s)\} = 1$$

The transition probabilities may depend upon s but are assumed to be independent of x. At date T, individual ξ has an objective function given by

$$V_T = E\left[\sum_{T}^{\infty} \delta^{(t-T)}\ u(x_t,\ s_t,\ \xi)]\right] \tag{1}$$

We restrict preferences in the following way. First, a change in s has a similar effect on everybody with regard to their preference for size.

Monotonicity:

It is assumed that, for all $x > x'$, $s > s'$:

$$u(x,\ s,\xi) - u(x',\ s,\ \xi) \quad \geq \quad u(x,\ s',\ \xi) - u(x',\ s',\ \xi). \tag{2}$$

This implies a preference for a larger organization when s is larger. With regard to the preferences of different individuals, we assume that a *single-crossing* condition is satisfied.

Single Crossing:

For any $\xi \neq \xi'$,

either for all s, for all $x > x'$:

$$u(x, s, \xi) - u(x', s, \xi) > u(x, s, \xi') - u(x', s, \xi') \tag{3}$$

or for all s, for all $x > x'$:

$$u(x, s, \xi) - u(x', s, \xi) < u(x, s, \xi') - u(x', s, \xi'). \tag{4}$$

This condition implies that agents may be ordered by their preference for greater size of the organization, ie., there is a labelling of agents $r(\xi)$ such that $r(\xi) > r(\xi')$ implies that, when $x > x'$,

$$u(x, s, \xi) - u(x', s, \xi) > u(x, s, \xi') - u(x', s, \xi'). \tag{5}$$

The single-crossing condition is sufficient to ensure the existence of a size which is a Condorcet winner in a one-period version of the present problem (Roberts (1977), Rothstein (1990), Gans and Smart (1996)).

The size of the organization at t+1 is determined by the voters at date t. The organization is assumed to change only incrementally. With size x_t at date t, the electorate first vote for a minimal reduction in size (to the maximum possible size in X below x_t). There is then further vote for a minimal increase in size and, again, this is adopted only if there is a majority vote in favour. If there is a vote to increase from a size just below x_t then there is no incentive to vote to reduce from x_t. We therefore adopt a 'trembling hand' assumption that a vote for a reduction is based on the assumption that it is not immediately reversed.

This restriction to sequential pairwise voting ensures that there are no existence problems created by the possible intransitivity of majority rule. However, apart from ensuring that there are no large 'jumps' in the size of the organization, the restriction is relatively weak. Given the stochastic structure of s, it is natural to restrict attention to markovian behaviour so that decisions to decrease or increase size are markovian. Transitions are governed by two functions D(x,s) and U(x,s) such that

$$D(x^k, s) = x^k \text{ or } x^{k-1}$$

$$U(x^k, s) = x^k \text{ or } x^{k+1}.$$

The evolution of organization size is governed by a function e(.) given by

$$x_{t+1} = e(x_t, s_t)$$

where

$$e(x, s) = U(D(x, s),s) \ . \tag{6}$$

With markovian transitions, expected discounted utility (1) will be a function of present size and the state of the world:

$$V_t = V(x_t, s_t, \xi) \ . \tag{7}$$

We are interested in transition rules governed by majority rule so we permit transition rules only if they can be generated by majority vote:

$$\text{(i)}\ \ U(x, s) \neq x \text{ iff } \#\{\xi: \xi \le x \ \&\ V(U(x, s),s, \xi) > V(x, s, \xi)\}$$

$$> \#(\xi{:}\xi \le x \ \&\ V(U(x, s), s, \xi) < V(x, s, \xi)\} \tag{8}$$

$$\text{(ii)}\ \ D(x,s) \neq x \text{ iff } \#\{\xi{:}\xi \le x \ \&\ V(D(x, s), s, \xi) > V(U(x, s),s, \xi)\}$$

$$> \#\{\xi{:}\xi \le x \ \&\ V(D(x, s), s, \xi) < V(U(x, s),s, \xi)\} \ . \tag{9}$$

Our analysis now concentrates on the properties of the functions D and U so that the overall behaviour of the organization, and its response to underlying changes in s, can be analysed.

3. Basic Results

We start by investigating the voting behaviour of different individuals. Our first result is concerned with what has been forced upon the evolution function by the protocol of voting:

Lemma 1. The evolution function e is (weakly) monotonic in organization size, i.e., for all $x > x'$, for all s:

$$e(x, s) \geq e(x', s).$$

Proof: As $x \geq x'$ implies that $D(x, s) \geq D(x', s)$ and $U(x, s) \geq U(x', s)$, we have $e(x, s) = U(D(x, s), s) \geq U(D(x',s), s) = e(x', s)$. ■

This result permits an analysis of how different voters perceive present choices, given that size will then evolve.

Lemma 2. Assume that $r(\xi) > r(\xi')$ and $x > x'$. Then

$$V(x, s, \xi') \geq V(x', s, \xi') \Rightarrow V(x, s, \xi) \geq V(x', s, \xi) \tag{10}$$

and

$$V(x', s, \xi) \geq V(x, s, \xi) \Rightarrow V(x', s, \xi') \geq V(x, s, \xi') . \tag{11}$$

Proof: Both implications are proved similarly and we look at (10). Using Lemma 1, we know that if initial size is x rather than x′, future size will always be weakly greater. If, for some realisation of the state of the world $(s_0,s_1,...)$, the path of organization sizes is $(x,x_1,...)$ starting from x and $(x',x'_1,...)$ starting from x' then (5) gives

$$\Sigma\delta^t\ u(x_t, s_t, \xi') - \Sigma\delta^t\ u(x', s_t, \xi')$$

$$< \Sigma\delta^t \; u(x_t, s_t, \xi) - \Sigma\delta^t \; u(x'_t, s_t, \xi) \; .$$

Taking expectations over states of the world gives (10). ■

Lemma 2 is very useful in understanding the outcome of organization decision-making. If some individual ξ' wishes the organization to increase in size then this will be shared by all voters ξ such that $r(\xi) > r(\xi')$. Thus, if voters are ordered by r, there will be a connected set of 'low' r voters who will wish for a size decrease, at most one voter who will wish for no change, and a connected set of 'high' r voters who will wish for an increase.

These discussions suggest the pivotal role of the median voter, where median is defined relative to the ordering of voters by r. If x is odd then there is a unique voter that is midway in the r ranking. Starting at x, (9) implies that a reduction in size is possible only if the median voter strictly prefers the reduction (and the new median voter at D(x,s) does not wish for an increase) and (8) implies that an increase in size occurs only if the median voter strictly prefers the increase. When x is even, there are two possibilities for the median voter, ξ and ξ' with $r(\xi) > r(\xi')$. A decrease can occur only if it is strictly desired by ξ' and not strictly objected to by ξ and an increase can occur only if it is strictly desired by ξ and not strictly objected to by ξ'. We let m(x) denote the median voter at x, uniquely defined when x is odd, one of two possibilities when x is even.

Proposition 1. An increase or decrease in organization size occurs if and only if it is strictly desired by the median voter (when size is odd) or strictly by one median voter and weakly by the other (when size is even).

Subject to minor caveats, Proposition 1 says that decisions about the size of the organization are as if they are made by the median voter of the organization. As size changes, so does the median voter and the behaviour of the organization is like that of a single agent who has endogenous preferences. Our analysis of the behaviour of organizations is applicable to such single-agent situations.

Equilibrium in this model can be viewed as the equilibrium of a normal form game. The set of XxS is the set of "players" with preferences given by the expected discounted utility of the median voter in that state. The strategy set consists of {down, not down, up, not up}. Such a finite normal form game has a Nash equilibrium in mixed strategies which equates to the existence of a mixed strategy markov voting equilibrium. The argument follows that in Roberts (1998, Proposition 6):

Proposition 2. In the model, a majority voting equilibrium exists with markovian voting strategies. In the equilibrium, behaviour is 'as if' determined by a median voter.

The voting behaviour of rational voters when size is x is strongly dependent upon behaviour at different sizes. If there is upward movement from size x_{i+1} only in states of the world below some s* then voters at x_i must vote for an increase when s is below s* if they wish to have the possibility of size moving above x_{i+1}. To say much about behaviour at one size it is necessary to look at behaviour at all sizes.

4. Properties of Equilibria

In the voting model, if the present choice of the electorate is x^i then control of the organization is 'delegated' to an individual with utility function u(·, ·,m)). We can, from now on, concentrate on the analysis of *pure delegated decision-making*: x is the choice variable at each date and this determines the delegated decision maker m(x) who determines when increases or decreases in x occur. Any individual preference

$u(\cdot, \cdot, m)$ has the property that payoff depends upon choices made and the exogenously determined state of the world. As m is determined given x, the dependence of individual preference on x can arise because there is utility payoff directly from x and/or payoff from the person who is the delegated decision-maker. For instance, the function u may show a direct bias for each agent to be the delegated decision maker. This payoff is independent of the indirect return to being the delegated decision-maker and therefore the one able to exercise control.[1]

With a pure delegation problem, the model can be interpreted as being intrapersonal, in which case dependence of u on m relfects endogenous tastes, or interpersonal, in which case dependence of u on m reflects heterogeneity within the group of potential decision-makers.

Consider the decision facing m(x). If there is a decision which involves a change of delegated decision-maker then m(x) gives up the 'option value' of (limited) control; for instance, if $m(x^i)$ gives control to $m(x^{i+1})$ then this may prove to be a bad decision if the state of the world deteriorates. However $m(x^i)$ cannot claim back control from $m(x^{i+1})$. Because control is of value it is likely that there will be connected subsets of S where an agent chooses not to give up control. Define $\underline{C}^i$ and $\overline{C}^i$ as follows:

$$\underline{C}^i = \{s:\ s \in\ S\ \&\ U(x^i, s) = x^i\} \tag{12}$$

$$\overline{C}^i = \{s,\ s \in S\ \&\ D(x^i, s) = x^i\} \tag{13}$$

When there is non-upward or non-downward behaviour we can say something about voting behaviour at adjacent sizes:

Proposition 3.

[1] The function u could also depend upon choices y which do not determine the delegation process: v(x, y, s, m). If y(x, s, m) is the optimal choice by m then u can be defined as v(x, y (x, s, m), s, m).

(i) Let s$\in$ int $\underline{C}^{i+1}$. If m(x^i) is some median voter then

$$u(x^i,s,m(x^i)) > u(x^{i+1},s,m(x^i)) \;\Rightarrow\; U(x^i,s) = x^i$$

(ii) Let s$\in$ int $\overline{C}^{i-1}$. If m(x^i) is some median voter then

$$u(x^i,s,m(x^i)) > u(x^{i-1},s,m(x^i)) \;\Rightarrow\; D(x^i,s) = x^i$$

<u>Proof</u>: Both parts are similarly proved and we demonstrate (i). If s$\in$ int $\underline{C}^{i+1}$, we have

$$V(x^{i+1},s,m(x^i)) = u(x^{i+1},s,m(x^i)) + \delta[Pr(s\text{-}1/s)\ V(x^{i+1},s\text{-}1,m(x^i)) + Pr(s/s)\ V(x^{i+1},s,m(x^i)) + Pr(s\text{+}1/s)\ V(x^{i+1},s\text{+}1,m(x^i))] \tag{14}$$

whereas

$$V(x^i,s,m(x^i)) \;\geq\; u(x^i,s,m(x^i)) + \delta[Pr(s\text{-}1/s)\ V(x^{i+1},s\text{-}1,m(x^i)) + Pr(s/s)\ V(x^{i+1},s,m(x^i)) + Pr((s\text{+}1/s)\ V(x^{i+1},s\text{+}1,m(x^i))]\ . \tag{15}$$

The inequality in (15) follows because the right-hand side of (15) is expected utility resulting from a move to x^{i+1} after one period irrespective of its desirability and the left-hand side is based upon a move being made after one-period which follows a voting rule which, by Proposition 1, is dictated by the preferences of m(x^i). Subtracting (14) from (15) gives

$$V(x^i,s,m(x^i)) - V(x^{i+1},s,m(x^i)) \;\geq\; u(x^i,s,m(x^i)) - u(x^{i+1},s,m(x^i)) \tag{16}$$

If the right-hand side of (16) is positive then the left-hand side is positive and Proposition 1 gives U(x^i,s) = x^i. ■

Proposition 3 implies that if further changes in size are not immediately induced by an initial change in size then changes are made only if it is in the direct one-period interests of the median voter. An alternative way of viewing this is that the delegated decision-maker has an option value with regard to increases or decreases in size. If there is no reason to change size because of further changes that will be induced then the option value is only worth being forsaken if there is a direct utility gain from change.

Proposition 3 is a result saying that there is change only if there is direct utility gain. The following result presents a partial converse to this result.

<u>Proposition 4</u>.

(i) Let

$$[\underline{s},\overline{s}] \subseteq \underline{C}^{i+1},\ u(x^{i+1},\underline{s},m(x^i)) \geq u(x^i,\underline{s},m(x^i)),\ U(x^i,\underline{s}) = U(x^i,\overline{s}) = x^{i+1},$$

$$\text{and } D(x^i,s) = x^i,\ s \in [\underline{s},\overline{s}]\ .\ \ \text{Then } U(x^i,s){=}x^{i+1} \text{ for all } s \in [\underline{s},\overline{s}].$$

(ii) Let

$$[\underline{s},\overline{s}] \subseteq \overline{C}^{i-1},\ u(x^{i-1},\overline{s},m(x^i)) \geq u(x^i,\overline{s},m(x^i)),\ D(x^i,\underline{s}) = D(x^i,\overline{s}) = x^{i-1}$$

$$\text{and } u(x^i,s) = x^i,\ s \in [\underline{s},\overline{s}].\ \text{Then } D(x^i,s){=}x^{i-1} \text{ for all } s \in [\underline{s},\overline{s}].$$

<u>Proof</u>: Again we concentrate on part (i). If s$\in$ int$[\underline{s},\overline{s}]$ then

$$V(x^i,s,m(x^i)) = u(x^i,s,m(x^i)) + \delta E[\max\,(V(x^{i+1},\ \cdot\,,m(x^i)),\ V(x^i,\cdot\,,m(x^i)))] \quad (17)$$

as $D(x^i,s) = x_i$. Equation (14) applies so that

$$V(x^{i+1},s,m(x^i)) - V(x^i,s,m(x^i)) = u(x^{i+1},s,m(x^i)) - u(x^i,s,m(x^i))$$

$$+ \delta E[\min (V(x^{i+1}, \cdot, m(x^i)) - V(x^i, \cdot, m(x^i)), 0] \ . \tag{18}$$

If the left-hand side of (18) is ever negative then it must reach a negative minimum at some s*. Then

$$V(x^{i+1},s^*,m(x^i)) - V(x^i,s^*,m(x^i)) \geq u(x^{i+1},s^*,m(x^i)) - u(x^i,s^*,m(x^i))$$

$$+ \delta[V(x^{i+1},s^*,m(x^i)) - V(x^i,s^*,m(x^i))] \tag{19}$$

which gives, on rearrangement,

$$0 \geq (1-\delta)[V(x^{i-1},s^*,m(x^i)) - V(x^i,s^*,m(x^i))] \geq u(x^{i+1},s^*,m(x^i)) - u(x^i,s^*,m(x^i)) \ . \tag{20}$$

However, the right-hand side of (20) is strictly positive, using $s^* > \underline{s}$ and (2), which is a contradiction. Therefore, the left-hand side of (18) is strictly positive throughout the interior of the interval and $U(x_i,s) = x^{i+1}$ for all $s\epsilon[\underline{s},\overline{s}]$.■

This result is encumbered by the conditions that are assumed to be satisfied. However, if, for instance, $s \in \text{int } \underline{C}^{i+1}$ then Proposition 3 implies that $u(x^{i+1},\underline{s}, m(x^i)) \geq u(x^i,\underline{s},m(x^i))$ directly; furthermore, Proposition 3 gives straightforward conditions for when $D(x^i,s) = x^i$. The basic force of Proposition 4 is that if the movement from x^i to x^{i+1} is certain to occur in the future and there is a direct utility gain to the decision-maker from change then it is optimal to make the change without delay.

A minor amendment to the proof of Proposition 4 allows us to say something about the highest and lowest states of the world.

<u>Proposition 5.</u>

(i) Let $[\underline{s},s_{\max}] \subseteq \underline{C}^{i+1}$, $u(x^{i+1},\underline{s},m(x^i)) \geq u(x^i,\underline{s},m(x^i))$, $U(x^i,\underline{s}) = x^{i+1}$ and $D(x^i,s) = x^i$, $s \in [\underline{s},s_{\max}]$. Then $U(x^i,s) = x^{i+1}$ for all $s \in [\underline{s},s_{\max}]$.

(ii) Let $[s_{\min},\overline{s}] \subseteq \overline{C}^{i-1}$, $u(x^{i-1},\overline{s},m(x^i)) \geq u(x^i,\overline{s},m(x^i))$, $D(x^i,\overline{s}) = x^{i-1}$ and $U(x^i,s) = x^i$, $s \in [s_{\min},\overline{s}]$. Then $D(x^i,s) = x^{i-1}$ for all $s \in [s_{\min},\overline{s}]$.

Proof.

Taking part (i), the proof to Proposition 4 applies if $V(x^{i+1},s,m(x^i)) - V(x^i,s,m(x^i))$ reaches an internal minimum. If it achieves a negative minimum at $s^*=s_{max}$ then (19) still applies giving rise to the same contradiction.■

The implications of Propositions 3-5 are that, if further induced changes can be ignored in the region under consideration, there is upward movement in delegation only if there is a direct one-period utility gain and that if there upward movement in some state of the world s then there will also be such votes when the state is above s.

5. Simple Organizations

This section examines simple organizations or bodies where x can take on only one of two values, low or high. Later sections investigate complex organizations where there are many possible values. It will turn out that the behaviour of complex organizations is either, in essence, the same as simple organizations or it is radically different with very different properties. This will allow the behaviour of complex organizations to be classified into one of two very different regimes.

With only two possible sizes, Propositions 3-5 have power because if x^L and x^H, $x^L < x^H$, are the two sizes then $\underline{C}^H = \overline{C}^L = S$, from (12) and (13). In Proposition 3, $s \in \text{int } \underline{C}^{i+1}$ is used only to ensure that the state next period is also in $\underline{C}^{i+1}$. Therefore, applying the proof of Proposition 2 and appealing to Proposition 4 gives

Proposition 6

There exist s^L, s^H such that

(i) $U(x^L,s) = x^H$ iff $s \geq s^L$ where $u(x^H,s^L,m(x^L)) > u(x^L,s^L,m(x^L))$.

(ii) $D(x^H,s) = x^L$ iff $s \leq s^H$ where $u(x^L,s^H,m(x^H)) > u(x^H,s^H,m(x^H))$.

Thus, there are two critical states of the world, one, s^L, above which there is upward movement and one, s^H, below which there is downward movement. $m(x^L)$ determines s^L and $m(x^H)$ determines s^H. Equilibrium becomes the Nash equilibrium choices of s^L and s^H with objective functions given by expected discounted utility.

The relative sizes of s^L and s^H are important in determining the dynamic structure of the organization:

1. When s is below both s^L and s^H, the organization moves to or remains at a low size, above both s^L and s^H it moves to or remains at a high size.

2. When s^H exceeds s^L there is an attempt to move in both directions which is resolved, by the convention of our protocol, by remaining, or moving to, the high state. Thus, the size of the organization is determined entirely by the current state of the world.

3. When s^L exceeds s^H, the organization stays at whichever size in which it finds itself whenever s is in the interval $[s^H,s^L]$ and $[s^H,s^L]$ describes the extent of hysteresis. To see this, assume that the organization starts at the low size. When state s^L is reached, it moves to the high size; however, a downturn in s does not lead to a reverse in size until s^L is reached - a classic situation of hysteresis.

The "jump" states s^L and s^H may also be compared with the behaviour of the organization that would be optimal from the viewpoint of each agent. To simplify the presentation, let us assume that there are states where the median voters are indifferent, in terms of one period utility, between the two sizes. Define $\widehat{s}^L$ and $\widehat{s}^H$ as follows:

$$u(x^H,\widehat{s}^L,m(x^L)) = u(x^L,\widehat{s}^L,m(x^L))$$

$$u(x^H,\widehat{s}^H,m(x^H)) = u(x^L,\widehat{s}^H,m(x^H)) \ .$$

It is then trivial to see that $m(x^L)$, for instance, would like the organization to be large when s exceeds $\widehat{s}^L$. Similarly, $m(x^H)$ would like the organization to be large when s exceeds $\widehat{s}^H$. Both agents would prefer that the organization does not exhibit hysteresis. We first note conditions under which s^L is less than s^H and there is no hysteresis.

Proposition 7. If $s^L < s^H$ in equilibrium then $s^L = \widehat{s}^L$ and $s^H = \widehat{s}^H$. Note that $\widehat{s}^L < \widehat{s}^H$ iff $r(m(x^L)) > r(m(x^H))$, i.e., the delegated decision-maker for a small organization has a greater preference for larger organizations than the decision-maker with a large organization.

Proof. Consider the choice of s^L. As $s^L < s^H$, size falls to x^L in the states surrounding s^L. Thus, for $s = s^L$-1, s^L:

$$V(x^H,s,m(x^L)) = u(x^H,s,m(x^L)) + \delta \underset{s'}{E}[\max(V(x^H,s',m(x^L)),\ V(x^L,s',m(x^L))] \quad (21)$$

$$V(x^L,s,m(x^L)) = u(x^L,s,m(x^L)) + \delta \underset{s'}{E}[\max(V(x^H,s',m(x^L)),\ V(x^L,s',m(x^L))] \quad (22)$$

If $s^L < \widehat{s}^L$ then (22) exceeds (21) with $s=s^L$ which implies that s^L is not optimal. If $s^L > \widehat{s}^L$ then $s^L\text{-}1 \geq \widehat{s}^L$ and (21) exceeds (22) at $s=s^L$-1 which again implies that s^L is not optimal. Thus $s^L=\widehat{s}^L$. A similar proof applies for $s^H=\widehat{s}^H$ where one must allow for the fact that, around s^H, a reduction to x^L at date t will be reversed at t+1 which cannot be re-reversed until t+2. The computation is straightforward. ■

Proposition 7 describes the perverse 'grass is always greener' case where, other things being equal, there is a preference for outcomes that imply that the other agent should be the decision-maker. The other two cases to examine are $\widehat{s}^L = \widehat{s}^H$ and, what is likely to be the most interesting, $\widehat{s}^L > \widehat{s}^H$ which implies, by Proposition 7 , that $s^L > s^H$.

If $\widehat{s}^L = \widehat{s}^H$ then the preferences of the decision-maker at x^L are the same as that at x^H. Proposition 7 implies that $s^L \geq s^H$. Consider the payoff to an agent of choosing x^L or x^H. This will be given as in (21) and (22) and $V(x^H, s, m) \geq V(x^L, s, m)$ if and only if $u(x^H, s, m) \geq u(x^L, s, m)$. Thus $s^L = \widehat{s}^L = \widehat{s}^H = s^H$ and behaviour of the organization will be efficient (defined by the common preferences of the decision makers). We thus have

Proposition 8. If the preferences of the decision-maker at x^L coincide with those of the decision-maker at x^H then the organization behaves efficiently, i.e. the two decision-makers cannot miscoordinate their behaviour.

Now, taking the case where $\widehat{s}^L > \widehat{s}^H$, let us consider the 'reaction functions' $s^L(s^H)$ and $s^H(s^L)$ which describe the optimal behaviour of an agent given the behaviour of the other agent. Concentrating on $s^L(s^H)$, we first note that $s^L \geq \widehat{s}^L$, from Proposition 3, and $s^L(\widehat{s}^L) = \widehat{s}^L$, as this would lead to the behaviour most desired by $m(x^L)$. Given $s^H \leq \widehat{s}^H$ define $W(s,s^H)$ as follows:

$$W(s,s^H) = \begin{cases} V(x^H,s,m(x^L)) - V(x^L,s,m(x^L)), & s > s^H \\ 0, & s \leq s^H \end{cases} \tag{23}$$

The interpretation of W is that it is the incremental value to $m(x^L)$ of being in a large rather than a small organization. At s^H, this incremental value as zero. The optimal value is defined as the value of s where W first becomes positive. Assume that $s < \min(s^L(s^H), s^L(s^H\text{-}1))$. From the definition of V, we have, for $s > s^H$:

$$Z(s) = W(s,s^H) - W(s,s^H\text{-}1) = \delta \mathop{E}_{s'} [W(s',s^H) - W(s',s^H\text{-}1)] \tag{24}$$

with $Z(s^H) = W(s^H,s^H) - W(s^H,s^H\text{-}1) > 0$ as $s^H < \widehat{s}^L$ and $Z(s^H\text{-}1) = 0$. The function Z(s) cannot reach a maximum (in which case $Z^{\max} \leq \delta Z^{\max} < Z^{\max}$); thus $W(\cdot,s^H)$

becomes positive before W($\cdot$,s^H-1). This implies that s^L(s^H) $\leq$ s^L(s^H-1). Performing a similar argument for the function s^H gives us:

Proposition 9.

The reaction functions s$^L(\cdot)$ and s$^H(\cdot)$ are decreasing functions.

This result implies that hysteresis when the organization is large encourages hysteresis when the organization is small and vice versa. Thus, there is an element of strategic hysteresis that is induced by the strategic interaction of agents, this hysteresis being greater than what is compatible with efficiency.

This is illustrated by portraying the reaction curves (see Figure 1). The reaction curves are downward sloping and intersect at (s$^L_*$, s$^H_*$). Efficiency demands that s^L $\leq$ $\widehat{s}^L$ and s^H $\geq$ $\widehat{s}^H$ so that the region [$\widehat{s}^H$,$\widehat{s}^L$] is the maximum hysteresis region compatible with efficiency. As s$^L_*$ $\geq$ $\widehat{s}^L$ and s$^H_*$ $\leq$ $\widehat{s}^H$, regions of strategic hysteresis are generated in equilibrium.

6. A Continuous/Linear Approximation

To investigate the regions of strategic hysteresis further, it is necessary to make say more about the reaction functions.

We make two approximations which will allow us to explicitly solve for reaction curves:

1. The direct utility gain from changing organization size is approximately linear in s. Concretely, we assume

$$\text{u}(\text{x}^H,\text{s},\text{m}(\text{x}^L)) - \text{u}(\text{x}^L,\text{s},\text{m}(\text{x}^L)) = \alpha^L(\text{s-}\widehat{\text{s}}^L) \qquad (25)$$

with a similar expression for m(x^H).

2. Time periods are small so that time can be approximated by a continuous variable. Concretely, the stochastic process underlying s is assumed to be a Wiener process with per unit variance σ^2. Per unit discounting is captured by a discount rate r.

The expected discounted utility functions may be expressed as differential equations. Taking the continuous time versions of (14) and applying these to (23) allow us to obtain an expression for the incremental value to m(x^L) from having size x^H over x^L:

$$0 = \alpha^L\ (s\text{-}\widehat{s}^L)\ \text{-}\ rW + \frac{\sigma^2}{2}\ W_{ss} \tag{26}$$

There are three boundary conditions which permit us to determine the critical s^L:

$$\begin{aligned} W(s^H,s^H) &= 0 \\ W(s^L,s^H) &= 0 \\ W_s(s^L,s^H) &= 0 \end{aligned} \tag{27}$$

The first two conditions do not require comment. The third is a "smooth-pasting" condition (Dixit and Pindyck, 1995) which is standard in optimal stopping problems and captures the fact that s^L is optimally chosen to maximize V(x^L,s,m(x^L)) - a discussion of this condition is found in Dixit and Pindyck (1995).

Equation (26) can be solved to give

$$W = \frac{\alpha^L(s\text{-}\widehat{s}^L)}{r} + \gamma_1 e^{-\lambda s} + \gamma_2 e^{\lambda s} \tag{28}$$

where γ_1 and γ_2 are constants which may be determined from (27) and λ is given by

$$\lambda = (\frac{2r}{\sigma^2})^{\frac{1}{2}} \tag{29}$$

so that $1/\lambda$ is a measure of underlying uncertainty. Performing the manipulations gives

$$2(s^H\text{-}\widehat{s}^L) = (s^L\text{-}\widehat{s}^L\text{-}\frac{1}{\lambda})\ e^{-\lambda(s^H\text{-}s^L)} + (s^L\text{-}\widehat{s}^L+\frac{1}{\lambda})\ e^{\lambda(s^H\text{-}s^L)}. \tag{30}$$

We can also write a reaction function in terms of hyperbolic functions:

$$(s^H\text{-}\widehat{s}^L) = (s^L\text{-}\widehat{s}^L)\cosh(s^H\text{-}s^L) + \frac{1}{\lambda}\sinh(s^H\text{-}s^L) \tag{31}$$

To get an idea of the form of the implied reaction function, we look at extremes. First, when s^H is very low, implying that there is a very large amount of hysteresis when the organization is of size x^H, s^H-s^L will be large and negative and (30) gives

$$s^H \rightarrow -\infty \Rightarrow s^L \rightarrow \widehat{s}^L + (\frac{\sigma^2}{2r})^{\frac{1}{2}} \tag{32}$$

Thus, the maximum amount of strategic hysteresis created by $m(x^L)$ is given by $(\frac{\sigma^2}{2r})^{\frac{1}{2}}$ which is directly proportional to underlying uncertainty. At the other extreme, when s^H is close to s^L (which is relevant only if $\widehat{s}^L$-$\widehat{s}^H$ is small) then a third-order Taylor expansion of (30) yields after manipulation:

$$s^L = \frac{3\widehat{s}^L}{2} - \frac{s^H}{2} \tag{33}$$

This has the surprising implication that the slope of the reaction function (minus one-half) is independent of all parameters in the model. A similar expression for the other reaction function is

$$s^H = \frac{3\widehat{s}^H}{2} - \frac{s^L}{2} \tag{34}$$

and the two reaction functions solve to give equilibrium hysteresis:

$$s^L\text{-}s^H = 3(\widehat{s}^L - \widehat{s}^H) \tag{35}$$

This implies that strategic hysteresis triples the region of maximal initial hysteresis. These results can be collected in the following result:

Proposition 10. With the continuity/linearity assumptions, the equilibrium region of hysteresis $[s^L\text{-}s^H]$ is given by:

(i) When $\widehat{s}^L$ - $\widehat{s}^H$ is small (and positive):

$$s^L\text{-}s^H = 3(\widehat{s}^L\text{-}\widehat{s}^H)$$

(ii) When $\widehat{s}^L$ - $\widehat{s}^H$ is large (and positive):

$$\mathrm{s}^L\text{-}\mathrm{s}^H = (\widehat{\mathrm{s}}^L\text{-}\widehat{\mathrm{s}}^H) + 2(\frac{\sigma^2}{2\mathrm{r}})^{\frac{1}{2}}$$

The definiteness of the result for small ($\widehat{s}^L$-$\widehat{s}^H$) is remarkable. If the two possible organization sizes are close together then this will lead to small ($\widehat{s}^L$ - $\widehat{s}^H$) and the linearity of direct utility gain will be a good approximation. The fact that the hysteresis region can be expanded threefold shows the potential importance of strategic elements.

7. Complex Organizations

Complex organizations have more than two and possibly many size levels. If the size most preferred by the median voter is diminishing with organization size then Proposition 7 can be generalized to show that there will be no hysteresis. We concentrate on the 'regular' case where the delegated decision maker's most preferred size is increasing with the size of the organization (recall that the decision maker is endogenous to size). Our initial interest is in cases where the movement between any two adjacent size levels is, in essence, like the movement between the two levels of a simple organization.

We define:

Disconnected Equilibrium: An equilibrium is disconnected if

1. there is an interval {s: $\mathrm{s}^d(\mathrm{x}) < \mathrm{s} < \mathrm{s}^u(\mathrm{x})$} over which there is a vote for no change with a vote for reduction (increase) occurring at $\mathrm{s}^d(\mathrm{x})$ ($\mathrm{s}^u(\mathrm{x})$) and
2. the functions $\mathrm{s}^d(\mathrm{x})$ and $\mathrm{s}^u(\mathrm{x})$ are strictly increasing in x.

The main restriction imposed is that the bounds are strictly, as opposed to weakly, increasing. Consider two adjacent size levels, x^L and x^H, $\mathrm{x}^L < \mathrm{x}^H$. Then the value to m($x^L$) of being at size x^H rather than x^L is given by (23) which may be restated as

$$W(s) = \begin{cases} V(x^H,s,m(x^L)) - V(x^L,s,m(x^L)), & s^d(x^H) < s < s^u(x^H) \\ 0, & s \leq s^d(s^H) \end{cases} \tag{1}$$

The equilibrium value of $s^u(x^L)$ lies, by the assumption of disconnectedness, within the set of states defined by (36) and it is determined by the condition that W is strictly positive if a jump is delayed for a further period. W is governed by a difference equation

$$W(s) = u(x^H,s,m(x^L)) - u(x^L,s,m(x^L)) + \delta \underset{s'}{E} W(s') \tag{37}$$

which is determined entirely by the direct utility gain to m(x^L) from moving to x^H from x^L. Thus, the equilibrium choice of $s^u(x^L)$ is dependent upon $s^d(x^H)$ but is independent of behaviour at other size levels: behaviour between two size levels is exactly the same as in simple organizations.

<u>Proposition 11</u>. If the equilibrium in a complex organization is disconnected then the critical states $s^u(x^L)$ and $s^d(x^H)$ for movement between adjacent states are the same as the equilibrium states for movement in a simple organization where only the two sizes x^L and x^H are feasible.

Although this result is unsatisfactory because disconnectedness is a feature of equilibrium, it does allow us to divide possible equilibria into two classes (disconnected and not disconnected) and, for disconnected equilibria, to apply the results of the last section.

To examine the nature of equilibrium further, we will now assume that x^L and x^H are descriptions of the smallest and largest organization size and that fine gradations between x^L and x^H are feasible together with fine gradations between possible decision makers. For instance, indexing by n, the smallest size may be nx^L with each size of the form $(n\text{-}m)x^L + mx^H$ feasible. To ensure that the principal effect of a larger n is the change in the number of possible sizes, we assume that the delegated decision-maker when size is nx, $m^n(nx)$, is given by m(x) for some function m and that one-period utility for the agent takes the form

$$u^n(nx,s,m^n(nx)) = u(x,s,m(x)) \tag{38}$$

We can therefore scale size so that, for any n, the feasible range is $[x^L,x^H]$ and preferences of the median voter at any size are independent of n. We assume that the functions u and m in (38) are appropriately differentiable so that the new possibilities opened up by increasing n are not too different to the possibilities when n is smaller. The per unit gain to m(x) from moving to $x + \Delta$ where $\Delta = (x^H\text{-}x^L)/n$ is given by

$$g = u(x + \Delta, s,m(x)) - u(x,s,m(x)) \tag{39}$$

so, for large n, this gain is zero at $\hat{s}^L$ given by

$$u_1\ (x + \Delta/2, \hat{s}^L,m(x)) = 0\ . \tag{40}$$

The gain is zero for $m(x+\Delta)$ when s equals $\hat{s}^H$ given by

$$u_1\ (x + \Delta/2, \hat{s}^H,m(x + \Delta)) = 0 \tag{41}$$

The difference between $\hat{s}^H$ and $\hat{s}^L$, the region of non-strategic hysteresis between x and $x + \Delta$ is approximated by

$$\hat{s}^L - \hat{s}^H = \left[\frac{u_{13}m_x}{u_{12}}\right] \cdot \Delta \tag{42}$$

which is positive, given that we are looking at the 'regular' case where the decision-maker at higher sizes has a greater preference of higher sizes, i.e. $u_{13}m_x > 0$.

With a disconnected equilibrium, we may apply the results of the last section to determine the extent of the strategic hysteresis between two size levels. Given (42), the intersection of the two reaction curves determining movement between x and x + Δ moves closer to the 45^o line in Figure 1 which implies that the region of strategic hysteresis diminishes with n. More concretely, as the critical values s_*^H and s_*^L will be close to $\widehat{s}^L$ and $\widehat{s}^H$, the per unit gain (39) will be approximately linear in s and Proposition 10 may be applied directly.

Proposition 12. If equilibrium is disconnected and s is a continuous stochastic variable then, for large n, the region of strategic hysteresis is three times the size of the region of non-strategic hysteresis and each are proportional to 1/n. With n large, the extent of hysteresis tends to zero.

Proposition 12 suggests that the hysteresis that was described in our analysis of simple organizations was related to the discreteness of choice faced by the decision-maker. However, the condition of disconnectedness is a condition which says that equilibrium does not build in discreteness by, for instance, a domino effect of size changes induced by a single change where decision-makers at several sizes above some x rush to increase size in some state of the world where the decision-maker at x is indifferent about such a change. For instance, if $\widehat{s}^L$ in (40) is a diminishing function of x then s_*^L will also be diminishing which is incompatible with disconnectedness. Differentiating (40) gives

$$(u_{11} + u_{13}m_x) + u_{12}\,\frac{d\widehat{s}}{dx} = 0 \tag{43}$$

so that a requirement for disconnectedness is

$$u_{11} + u_{13}m_x < 0 \tag{44}$$

which says that the decision-maker's preferences do not change too rapidly as organization size changes. If (44) is violated then the organization 'jumps' between two sizes and significant hysteresis is reintroduced at the end 'sizes'.

Even without this domino effect, there is the possibility of a non-disconnected equilibrium because there may be incentives for an agent to deviate from an ‘equilibrium’ that is disconnected. Assume that the same structure is in place as that which led to Proposition 12 and assume that n is large. If the equilibrium is disconnected then, in state s, the size of the organization will be given by x(s) where

$$u_1(x(s),\ s,\ m(x(s))) = 0 \tag{45}$$

Differentiating (45) gives

$$\frac{dx}{ds} = \frac{-\ u_{12}}{[u_{11} + u_{13}m_x]} \tag{46}$$

and, if there are no natural “jumps” induced by the domino effects, the denominator is negative. Now consider a particular agent $m^*=m(x(s^*))$. The per unit gain to this individual of abandoning the equilibrium just described and instead keeping x constant until s deviates from s^* by some distance is given by

$$g(s) = u(x(s^*),s,m^*) - u(x(s),s,m^*) \tag{47}$$

Note that $g(s^*)=0$ and $g_s(s^*)=0$ (recalling (45)). The second derivative, evaluated at s^*, is given by

$$g_{ss}(s^*) = -\ [u_{11}\frac{dx}{ds} + 2u_{12}]\frac{dx}{ds} \tag{48}$$

Using (46) this may be written as

$$g_{ss}(s^*) = (\frac{dx}{ds})^2\ (u_{11} + 2u_{13}m_x) \tag{49}$$

If (49) is positive then there is positive per unit gain from not allowing x to follow the path x(s) which means that the equilibrium cannot be disconnected. We thus have:

Proposition 13. If the preferences of the delegated decision-maker change sufficiently rapidly as the organization changes size, ie., $2u_{13}m_x > -u_{11}$, then equilibrium is not disconnected and the organization size is held constant over intervals of states of the world.

We thus see that if decision-maker's preferences change rapidly ($u_{13}m_x > -\frac{1}{2}u_{11}$) then it is optimal to keep the organization at the constant size over a range of states of the world, and so induce strategic hysteresis. With continuous parameter changes in the model, the equilibrium can move discontinuously, from one of no hysteresis to significant hysteresis. If the preference change is rapid enough ($u_{13}m_x > -u_{11}$) then natural 'jumps' are introduced into the process, irrespective of strategic elements, and the discreteness of simple organizations returns.

Let us consider an interpretation of these results for the intrapersonal problem of addiction. A non-addict may not wish to take some drug, even though instantaneous 'utility' is increased, because it is recognized that he will then change his preferences and increase consumption still further, so becoming addicted. This is the domino effect which arises when (44) is violated. However, Proposition 12 says that, even when preferences fail to induce domino effects, the non-addict will still wish to be cautious; in essence, because the world may change in the future, the non-addict recognizes that mild consumption of the drug now is more likely to lead to sequences of events where addiction will follow. The condition in Proposition 12 is sufficient for the introduction of rigidity, not necessary.

Finally, we can say something about behaviour at sizes adjacent to ones where rigidity is put into the system by holding size constant. First, if x^i is a size where there is rigidity in not allowing size to rise then the response at x^{i+1} will, following our analysis of simple organizations, be to increase the region of states of the world where size is not permitted to fall. Thus strategic hysteresis will augment the initial effect. The effects at x^{i-1} are different. Because size will not be permitted to rise above x^i until s is sufficiently high, the median voter at x^{i-1} is in a position similar to the median voter at the low size in a simple organization and the hysteresis effect between x^{i-1} and x^i may be quite small.

8. Concluding Remarks

This paper has examined the dynamic behaviour of an organization or body governed by many decision-makers. Dynamic behaviour has been uncovered by positing an underlying stochastic structure for the economy. We have shown that, under a reasonable set of conditions, voting is equivalent to delegated decision-making.. Our results are applicable to that of a single decision-maker with endogenous preferences, brought about through addiction for instance, who is operating in a stochastic environment. Another example of a similar problem is that which arises when a decision-maker chooses actions including the action of when to pass control to another agent who may be better at taking good decisions in particular environments. Team-ball games like soccer have some of these characteristics, the decision-maker being the agent in present control of the ball.

The dynamic behaviour of organizations can be quite complicated. In organizations where any change in size has to be quite large, there is a natural hysteresis effect induced by the fact that preferences are endogenous. We have shown that this hysteresis effect is increased by the strategic interaction of decision making. When changes in size can be small, it is possible that induced hysteresis is unimportant because there is minimal natural hysteresis. However, hysteresis can enter through two routes. First, through domino-like effects, a small incremental change may directly induce much larger changes and this takes us back to a situation like one where any change must be large. Second, even without this domino-like effect, it may be optimal for a decision maker to retain control in the knowledge that in so doing, a domino-like effect will be induced. This retention of control then induces strategic hysteresis which makes the behaviour of an organization very different to one governed by a single-decision maker with fixed preferences.

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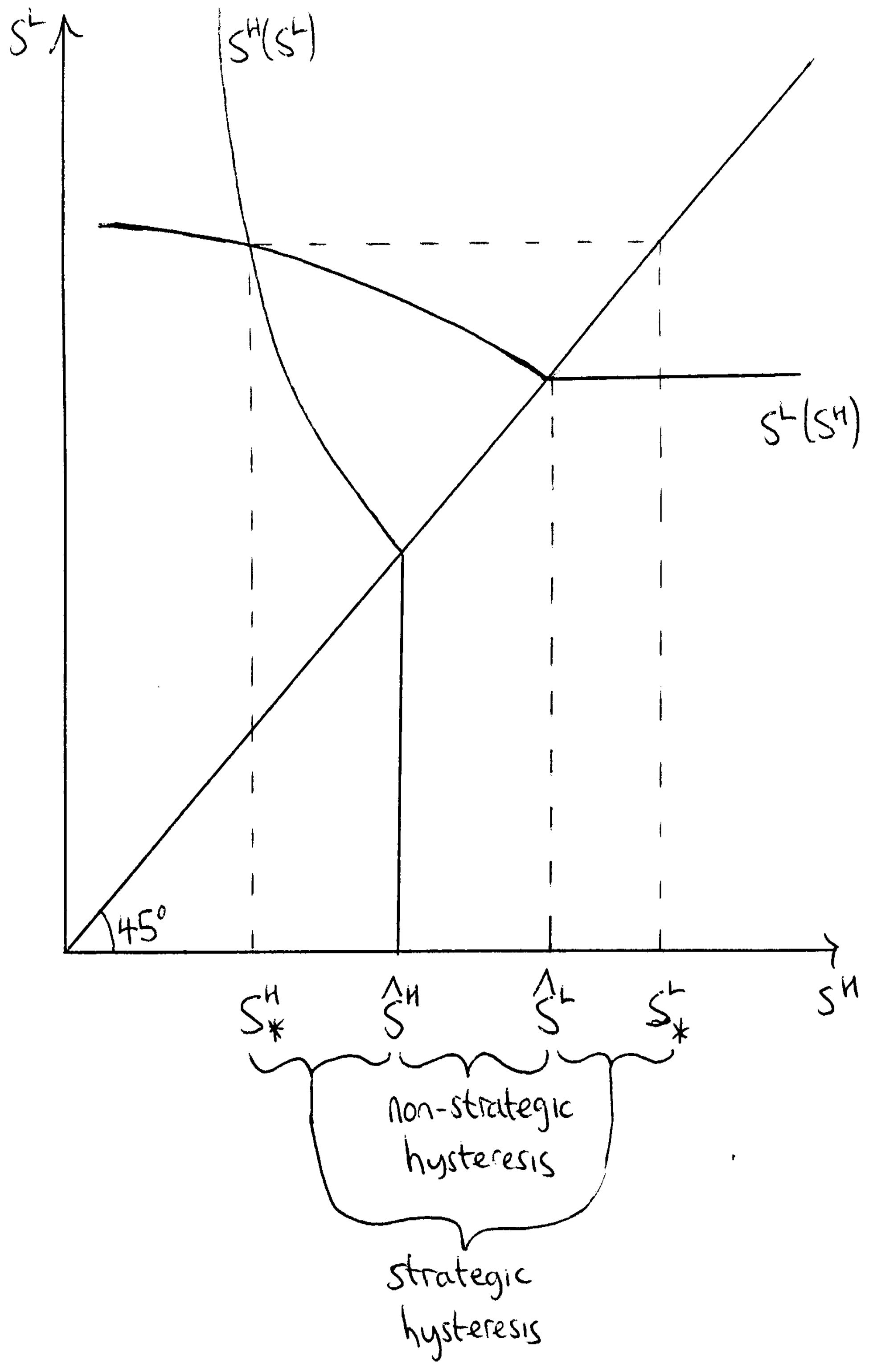

S^L
$S^H(S^L)$
$S^L(S^H)$
45°
S^H
S^H_*
$\hat{S}^H$
$\hat{S}^L$
S^L_*
non-strategic hysteresis
strategic hysteresis

Figure 1.