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**Paul Glasserman and H. Peyton Young**

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Manor Road Building, Manor Road, Oxford OX1 3UQ

# How Likely is Contagion in Financial Networks?

Paul Glasserman\* and H. Peyton Young†

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## Abstract

Interconnections among financial institutions create potential channels for contagion and amplification of shocks to the financial system. We estimate the extent to which interconnections increase expected losses, with minimal information about network topology, under a wide range of shock distributions. Expected losses from network effects are small without substantial heterogeneity in bank sizes and a high degree of reliance on interbank funding. They are also small unless shocks are magnified by some mechanism beyond simple spillover effects; these include bankruptcy costs, fire sales, and mark-to-market revaluations of assets. We illustrate the results with data on the European banking system.

Keywords: *systemic risk, contagion, financial network*

JEL: D85, G21

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\*Columbia Business School, Columbia University, pg20@columbia.edu

†Department of Economics, University of Oxford, peyton.young@economics.ox.ac.uk

# 1 Introduction

The interconnectedness of the modern financial system is widely viewed as having been a key contributing factor to the recent financial crisis. Due to the complex web of links between institutions, stresses to one part of the system can spread to others, leading to a system-wide threat to financial stability. Specific instances include the knock-on effects of the Lehman bankruptcy, potential losses to counterparties that would have resulted from a failure of the insurance company AIG, and more recently the exposure of European banks to the risk of sovereign default by some European countries. These and other examples highlight concerns that interconnectedness could pose a significant threat to the stability of the financial system.<sup>1</sup> Moreover there is a growing body of research that shows how this can happen in a theoretical sense.<sup>2</sup>

Although it is intuitively clear that interconnectedness has *some* effect on the transmission of shocks, it is less clear whether and how it significantly increases the likelihood and magnitude of losses compared to a financial system that is not interconnected. In this paper we propose a general framework for analyzing this question. There are in fact many different types of networks connecting parts of the financial system, including networks defined through ownership hierarchies, payment systems, derivatives contracts, brokerage relationships, and correlations in stock prices, among other examples. We focus on the network defined by liabilities between financial institutions. These payment obligations create the most direct channel for the spread of losses.

It turns out that one can derive general bounds on the effects of this source of interconnectedness with almost no information about the network topology: our bounds hold independently of the degree distribution, amount of connectivity, node centrality, average path length, and so forth. The topology-free property of our results is one of the main contributions of this work. Moreover, we impose probability distributions on the shocks to the nodes and show that the same bounds hold for a wide range of distributions, including beta, exponential, normal, and many others. This robustness is important because detailed information about interbank liabilities is often unavailable and the exact form of the shock distributions is subject to considerable uncertainty.

To model a network of payment obligations, we build on the elegant framework of Eisenberg and Noe (2001). The model specifies a set of nodes that represent financial institutions together with the obligations between them. Given an initial shock to the balance sheets of one or more nodes, one can compute a set of payments that clear the network by solving a fixed-point problem. This framework is

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<sup>1</sup>See, for example, Bank of England 2011, International Monetary Fund 2012, and Office of Financial Research 2012.

<sup>2</sup>See in particular Allen and Gale (2000), Upper and Worms (2002), Degryse and Nguyen (2004), Goodhart, Sunirand, and Tsomocos (2004), Elsinger, Lehar, and Summer (2006), Allen and Babus (2009), Gai and Kapadia (2010), Gai, Haldane, and Kapadia (2011), Haldane and May (2011), Upper (2011), Allen and Carletti (2011), Georg (2011), Rogers and Veraart (2012), Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013), and Elliott, Golub, and Jackson (2013).

very useful for analyzing how losses propagate through the financial system. A concrete example would be delinquencies in mortgage payments: if some fraction of a bank's mortgages are delinquent and it has insufficient reserves to cover the shortfall, then it will be unable to pay its creditors in full, who may be unable to pay their creditors in full, and so forth. The original shortfall in payments can cascade through the system, causing more and more banks to default through a domino effect. The Eisenberg-Noe framework shows how to compute a set of payments that clear the network, and it identifies which nodes default as a result of an initial shock to the system. The number and magnitude of such defaults depend on the topological structure of the network, and there is now a substantial literature characterizing those structures that tend to propagate default or alternatively that tend to dampen it (Gai and Kapadia 2010, Gai, Haldane, and Kapadia 2011, Haldane and May, 2011, Acemoglu, Ozdaglar, and Tahbaz-Salehi 2013, and Elliott, Golub, and Jackson 2013).

Much of this literature proceeds by examining the effects of fixed shocks applied to particular nodes rather than fully specifying the distribution that generates the shocks. In this paper we analyze the probability of contagion and the expected losses generated by contagion when the joint distribution of shocks is given. We then apply the framework to answer the following questions about the impact of network effects. First, how likely is it that a given set of banks will default due to contagion from another node, as compared to the likelihood that they default from direct shocks to their own assets? Second, how much does the network increase the probability and magnitude of losses compared to a situation where there are no connections?

To compare systems with and without interconnections, we proceed as follows. First, we define our nodes to be financial institutions that borrow and lend on a significant scale, which together with their obligations to one another constitute the *financial network*. In addition, such institutions borrow and lend to the nonfinancial sector, which is composed of investors, households, and nonfinancial firms. We compare this system to one without connections that is constructed as follows. We remove all of the obligations between the financial nodes while keeping their links with the nonfinancial sector unchanged. We also keep node equity values as before by creating, for each node, a fictitious outside asset (or liability) whose value equals the net value of the connections at that node that were removed. We then apply the same shock distributions to both systems, with the shocks to real assets originating in the external sector and the fictitious assets (if any) assumed to be impervious to shocks. We can ascertain how much the network connections contribute to increased defaults and losses by comparing the outcomes in the two systems.

One might suppose that the comparison hinges crucially on what shock distribution we use, but this turns out not to be the case: we show how to compute general bounds on the increased losses attributable to network contagion that hold under a wide variety of distributions. The bounds also hold whether the

shocks are independent or positively associated and thus capture the possibility that institutions have portfolios that are exposed to common shocks (see for example Caccioli et al. 2012).

Two key findings emerge from this analysis. First we compute the probability that default at a given node causes defaults at other nodes (via network spillovers), and compare this with the probability that all of these nodes default by direct shocks to their outside assets with no network transmission. We derive a general formula that shows when the latter probability is larger than the former, in which case we say that contagion is *weak*. A particular implication is that contagion is always weak unless there is substantial heterogeneity in node sizes as measured by their claims outside the financial sector. More generally, contagion will tend to be weak unless the originating node is large, highly leveraged, and – crucially – has a relatively high proportion of its obligations to other financial institutions as opposed to the nonfinancial sector. Second, the analysis shows that the total additional losses generated by network spillover effects are surprisingly small under a wide range of shock distributions for plausible values of model parameters. Both of these results are consistent with the empirical and simulation literature on network stress testing, which finds that contagion is quite difficult to generate through the interbank spillover of losses (Degryse and Nguyen 2004, Elsinger, Lehar, and Summer 2006, Furfine 2003, Georg 2011, Nier et al. 2007). Put differently, our results show that contagion through spillover effects becomes most significant under the conditions described in Yellen (2013), when financial institutions inflate their balance sheets by increasing leverage and expanding interbank claims backed by a fixed set of real assets.

These results do not imply that *all forms* of network contagion are unimportant; rather they show that simple spillover or “domino” effects have only a limited impact at realistic levels of payment obligations between banks. This leads us to examine other potential sources of contagion. The prior literature has focused on the role of fire sales, that is, the dumping of assets on the market in order to cover losses.<sup>3</sup> Here we shall focus on two alternative mechanisms that are more immediate extensions of the simple spillover mechanism in the Eisenberg-Noe model and thus allow a more immediate comparison: bankruptcy costs and losses of confidence.

Bankruptcy costs magnify the costs associated with default both directly, through costs like legal fees, and indirectly through delays in payments to creditors and disruptions to the provision of financial intermediation services necessary to the real economy. We model these effects in reduced form through a multiplier on losses when a node defaults. This approach allows us to estimate how much the probability of contagion, and the expected losses induced by contagion, increase as a function of bankruptcy costs. A somewhat surprising finding is that bankruptcy costs must be quite large in order to have an appreciable impact on expected losses as they propagate through the network.

A second mechanism that we believe to be of greater importance is crises of confidence in the credit

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<sup>3</sup>See Shleifer and Vishny (2011) for a survey and Cifuentes, Ferrucci, and Shin (2005) for an extension of the Eisenberg-Noe framework with fire sales.

quality of particular firms. If a firm's *perceived* ability to pay declines for whatever reason, then so does the market value of its liabilities. In a mark-to-market regime this reduction in value can spread to other firms that hold these liabilities among their assets. In other words, the mere possibility (rather than the actuality) of a default can lead to a general and widespread decline in valuations, which may in turn trigger actual defaults through mark-to-market losses.<sup>4</sup> This is an important phenomenon in practice: indeed it has been estimated that mark-to-market losses from credit quality deterioration exceeded losses from outright defaults in 2007-2009.<sup>5</sup>

We capture this idea by re-interpreting the Eisenberg-Noe framework as a *valuation model* rather than as a clearing model. Declines in confidence about the ability to pay at some nodes can spread to other nodes through a downward revaluation of their assets. This mechanism shows how a localized crisis of confidence can lead to widespread losses of value. Our analysis suggests that this channel of contagion is likely to be considerably more important than simple domino or spillover effects.

The rest of the paper is organized as follows. In Section 2 we present the basic Eisenberg-Noe framework and illustrate its operation through a series of simple examples. In Section 3 we introduce shock distributions explicitly. We then compare the probability that a given set of nodes default from simultaneous direct shocks to their outside assets, with the probability that they default indirectly by contagion from some other node. In Section 4 we examine the expected loss in value that is attributable to network contagion using the comparative framework described above. We show that one can obtain useful bounds on the losses attributable to the network with almost no knowledge of the specific network topology and under very general assumptions about the shock distributions. In Section 5 we introduce bankruptcy costs and show how to extend the preceding analysis to this case. Section 6 examines the effects of a deterioration in confidence at one or more institutions, such as occurred in the 2008-09 financial crisis. We show how such a loss of confidence can spread through the entire system due to mark-to-market declines in asset values. In the Appendix we illustrate the application of these ideas to the European banking system using data from European Banking Authority (2011).

## 2 Measuring systemic risk

### 2.1 The Eisenberg-Noe framework

The network model proposed by Eisenberg and Noe (2001) has three basic ingredients: a set of  $n$  nodes  $N = \{1, 2, \dots, n\}$ , an  $n \times n$  liabilities matrix  $\bar{P} = (\bar{p}_{ij})$  where  $\bar{p}_{ij} \geq 0$  represents the *payment due* from

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<sup>4</sup>This mechanism differs from a bank run, which could also be triggered by a loss of confidence. Mark-to-market losses spread when a lender continues to extend credit, whereas a run requires withdrawal of credit. In the seminal framework of Diamond and Dybvig (1983), a run is triggered by a demand for liquidity rather than a concern about credit quality.

<sup>5</sup>According to the Basel Committee on Banking Supervision, for example, roughly two thirds of losses attributed to counterparty credit risk were due to mark-to-market losses and only about one third of losses were due to actual defaults. See <http://www.bis.org/press/p110601.htm>.

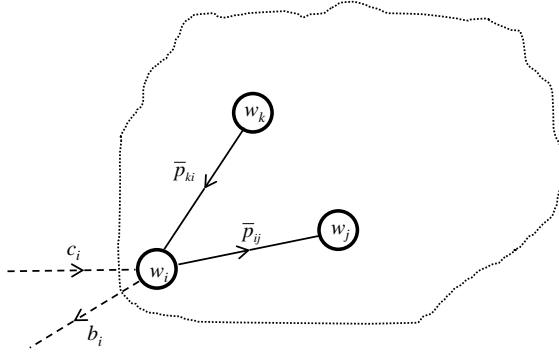


Figure 1: Node  $i$  has an obligation  $\bar{p}_{ij}$  to node  $j$ , a claim  $\bar{p}_{ki}$  on node  $k$ , outside assets  $c_i$ , and outside liabilities  $b_i$ , for a net worth of  $w_i$ .

node  $i$  to node  $j$ ,  $\bar{p}_{ii} = 0$ , and a vector  $c = (c_1, c_2, \dots, c_n) \in R_+^n$  where  $c_i \geq 0$  represents the value of *outside assets* held by node  $i$  in addition to its claims on other nodes in the network. Typically  $c_i$  consists of cash, securities, mortgages and other claims on entities outside the network. In addition each node  $i$  may have liabilities to entities outside the network; we let  $b_i \geq 0$  denote the sum of all such liabilities of  $i$ , which we assume have equal priority with  $i$ 's liabilities to other nodes in the network.

The *asset* side of node  $i$ 's balance sheet is given by  $c_i + \sum_{j \neq i} \bar{p}_{ji}$ , and the *liability* side is given by  $\bar{p}_i = b_i + \sum_{j \neq i} \bar{p}_{ij}$ . Its *net worth* is the difference

$$w_i = c_i + \sum_{j \neq i} \bar{p}_{ji} - \bar{p}_i. \quad (1)$$

The notation associated with a generic node  $i$  is illustrated in Figure 1. Inside the network (indicated by the dotted line), node  $i$  has an obligation  $\bar{p}_{ij}$  to node  $j$  and a claim  $\bar{p}_{ki}$  on node  $k$ . The figure also shows node  $i$ 's outside assets  $c_i$  and outside liabilities  $b_i$ . The difference between total assets and total liabilities is the node's net worth  $w_i$ .

Observe that  $i$ 's net worth is unrestricted in sign; if it is nonnegative then it corresponds to the book value of  $i$ 's equity. We call this "book value" because it is based on the nominal or face value of the liabilities  $\bar{p}_{ji}$ , rather than on "market" values that reflect the nodes' ability to pay. These market values depend on other nodes' ability to pay conditional on the realized value of their outside assets. To be specific, let each node's outside assets be subjected to a random shock that reduces the value of its outside assets, and hence its net worth. These are shocks to "fundamentals" that propagate through the network of financial obligations. Let  $X_i \in [0, c_i]$  be a random shock that reduces the value of  $i$ 's outside assets from  $c_i$  to  $c_i - X_i$ . After the shock,  $i$ 's net worth has become  $w_i - X_i$ . Let  $F(x_1, x_2, \dots, x_n)$  be the joint cumulative distribution function of these shocks; we shall consider specific classes of shock distributions in the next section. (We use  $X_i$  to denote a random variable and  $x_i$  to denote a particular

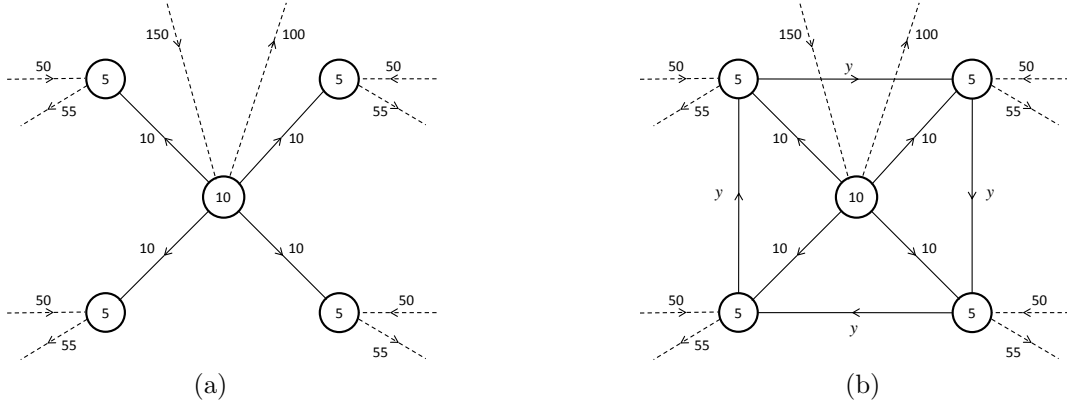


Figure 2: Two network examples.

realization.)

To illustrate the effect of a shock, we consider the numerical example in Figure 2(a), which follows the notational conventions of Figure 1. In particular, the central node has a net worth of 10 because it has 150 in outside assets, 100 in outside liabilities, and 40 in liabilities to other nodes inside the network. A shock of magnitude 10 to the outside assets erases the central node's net worth, but leaves it with just enough assets (140) to fully cover its liabilities. A shock of magnitude 80 leaves the central node with assets of 70, half the value of its liabilities. Under a *pro rata* allocation, each liability is cut in half, so each peripheral node receives a payment of 5, which is just enough to balance each peripheral node's assets and liabilities. Thus, in this case, the central node defaults but the peripheral nodes do not. A shock to the central node's outside assets greater than 80 would reduce the value of every node's assets below the value of its liabilities.

Figure 2(b) provides a more complex version of this example in which a cycle of obligations of size  $y$  runs through the peripheral nodes. To handle such cases, we need the notion of a clearing vector introduced by Eisenberg and Noe (2001).

The *relative liabilities matrix*  $A = (a_{ij})$  is the  $n \times n$  matrix with entries

$$a_{ij} = \begin{cases} \bar{p}_{ij}/\bar{p}_i, & \text{if } \bar{p}_i > 0 \\ 0, & \text{if } \bar{p}_i = 0 \end{cases} \quad (2)$$

Thus  $a_{ij}$  is the proportion of  $i$ 's obligations owed to node  $j$ . Since  $i$  may also owe entities in the external sector,  $\sum_{j \neq i} a_{ij} \leq 1$  for each  $i$ , that is,  $A$  is row substochastic.<sup>6</sup>

Given a shock realization  $x = (x_1, x_2, \dots, x_n) \geq 0$ , a *clearing vector*  $p(x) \in R_+^n$  is a solution to the system

$$p_i(x) = \bar{p}_i \wedge \left( \sum_j p_j(x) a_{ji} + c_i - x_i \right)_+. \quad (3)$$

<sup>6</sup>The row sums are all equal to 1 in Eisenberg and Noe (2001) because  $b_i = 0$  in their formulation.



As we shall subsequently show, the clearing vector is unique if the following condition holds: from every node  $i$  there exists a chain of positive obligations to some node  $k$  that has positive obligations to the external sector. (This amounts to saying that  $A$  has spectral radius less than 1.) We shall assume that this condition holds throughout the remainder of the paper.

## 2.2 Mark-to-market values

The usual way of interpreting  $p(x)$  is that it corresponds to the payments that balance the realized assets and liabilities at each node given that: i) debts take precedence over equity; and ii) all debts at a given node are written down *pro rata* when the net assets at that node (given the payments from others), is insufficient to meet its obligations. In the latter case the node is *in default*, and the *default set* is

$$D(p(x)) = \{i : p_i(x) < \bar{p}_i(x)\}. \quad (4)$$

However, a second (and, in our setting, preferable) way of interpreting  $p(x)$  is to see (3) as a *mark-to-market* valuation of all assets following a shock to the system. The nominal value  $\bar{p}_i$  of node  $i$ 's liabilities is marked down to  $p_i(x)$  as a consequence of the shock  $x$ , including its impact on other nodes. As in our discussion above of Figure 2(a), after marking-to-market, node  $i$ 's net worth is reduced from (1) to

$$c_i - x_i + \sum_{j \neq i} p_{ji}(x) - p_i(x). \quad (5)$$

The reduction in net worth reflects both the *direct* effect of the shock component  $x_i$  and the *indirect* effects of the full shock vector  $x$ . Note, however, that this is a statement about *values*; it does not require that the payments actually be made at the end of the period. *Under this interpretation  $p(x)$  provides a consistent re-valuation of the assets and liabilities of all the nodes when a shock  $x$  occurs.*

As shown by Eisenberg and Noe, a solution to (3) can be constructed iteratively as follows. Given a realized shock vector  $x$  define the mapping  $\Phi : R_+^n \rightarrow R_+^n$  as follows:

$$\forall i, \quad \Phi_i(p) = \bar{p}_i \wedge \left( \sum_j p_j a_{ji} + c_i - x_i \right)_+. \quad (6)$$

Starting with  $p^0 = \bar{p}$  let

$$p^1 = \Phi(p^0), \quad p^2 = \Phi(p^1), \dots \quad (7)$$

This iteration yields a monotone decreasing sequence  $p^0 \geq p^1 \geq p^2 \dots$ . Since it is bounded below it has a limit  $p'$ , and since  $\Phi$  is continuous  $p'$  satisfies (3). Hence it is a clearing vector.

We claim that  $p'$  is in fact the only solution to (3). Suppose by way of contradiction that there is another clearing vector, say  $p'' \neq p'$ . As shown by Eisenberg and Noe, the equity values of all nodes must be the same under the two vectors, that is,

$$p'A + (c - x) - p' = p''A + (c - x) - p''.$$

Rearranging terms it follows that

$$(p'' - p')A = p'' - p', \text{ where } p'' - p' \neq 0.$$

This means that the matrix  $A$  has eigenvalue 1, which is impossible because under our assumptions  $A$  has spectral radius less than 1.

### 3 Estimating the probability of contagion

Systemic risk can be usefully decomposed into two components: i) the probability that a given set of nodes  $D$  will default and ii) the loss in value conditional on  $D$  being the default set. This decomposition allows us to distinguish between two distinct phenomena: contagion and amplification. Contagion occurs when defaults by some nodes trigger defaults by other nodes through a domino effect. Amplification occurs when contagion stops but the losses among defaulting nodes keep escalating because of their indebtedness to one another. Roughly speaking the first effect corresponds to a “widening” of the crisis whereas the second corresponds to a “deepening” of the crisis. In this section we shall examine the probability of contagion; the next section deals with the amplification of losses due to network effects.

To estimate the probability of contagion we shall obviously need to make assumptions about the distribution of shocks. We claim, however, that we can estimate the *relative* probability of contagion versus simultaneous default with virtually no information about the network structure and relatively weak conditions on the shock distribution.

To formulate our results we shall need the following notation. Let  $\beta_i = \bar{p}_i / (b_i + \bar{p}_i)$  be the proportion of  $i$ 's liabilities to other entities in the financial system.<sup>7</sup> We can assume that  $\beta_i > 0$ , since otherwise node  $i$  would effectively be outside the financial system. Recall that  $w_i$  is  $i$ 's initial net worth (before a shock hits), and  $c_i$  is the initial value of its outside assets. The shock  $X_i$  to node  $i$  takes values in  $[0, c_i]$ . We shall assume that  $w_i > 0$ , since otherwise  $i$  would already be insolvent. We shall also assume that  $w_i \leq c_i$ , since otherwise  $i$  could never default directly through losses in its own outside assets. Define the ratio  $\lambda_i = c_i / w_i \geq 1$  to be the *leverage* of  $i$ 's outside assets. (This is not the same as  $i$ 's overall leverage, which in our terminology is the ratio of  $i$ 's total assets to  $i$ 's net worth.)

#### 3.1 A general bound on the probability of contagion

**Proposition 1.** *Suppose that only node  $i$  receives a shock, so that  $X_j = 0$  for all  $j \neq i$ . Suppose that no nodes are in default before the shock. Fix a set of nodes  $D$  not containing  $i$ . The probability that the*

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<sup>7</sup>Elliott, Golub, and Jackson (2013) have a similar measure which they call the level of integration. More broadly, Shin (2012) discusses the reliance of banks on wholesale funding as a contributor to financial crises, and  $\beta_i$  measures the degree of this reliance in our setting.

shock causes all nodes in  $D$  to default is at most

$$P(X_i \geq w_i + (1/\beta_i) \sum_{j \in D} w_j). \quad (8)$$

Moreover, contagion from  $i$  to  $D$  is impossible if

$$\sum_{j \in D} w_j/w_i > \beta_i(\lambda_i - 1). \quad (9)$$

The condition in (9) states that contagion from  $i$  to  $D$  is impossible if the total net worth of the nodes in  $D$  is sufficiently large relative to the net worth of  $i$  weighted by the exposure of the financial system to node  $i$ , as measured by  $\beta_i$ , and the vulnerability of  $i$  as measured by its leverage. A similar interpretation applies to (8).

Before proceeding to the proof, we illustrate the impossibility condition in (9) through the network in Figure 2(a). The central node is node  $i$ , meaning that the shock affects its outside assets, and the remaining nodes comprise  $D$ . The relevant parameter values are  $\beta_i = 2/7$ ,  $\lambda_i = 15$ , and the net worths are as indicated in the figure. The left side of (9) evaluates to 2 and the right side to 4, so the condition is violated, and, indeed, we saw earlier that contagion is possible with a shock greater than 80. However, a modification of the network that raises the sum of the net worths of the peripheral nodes above 40 makes contagion impossible. This holds, for example, if the outside liabilities of every peripheral node are reduced by more than 5, or if the outside liabilities of a single peripheral node are reduced by more than 20. This example also illustrates that (9) is tight in the sense that if the reverse strict inequality holds, then contagion is possible in this example with a sufficiently large shock.

*Proof of Proposition 1.* Let  $D(x) \equiv \bar{D}$  be the default set resulting from the shock vector  $X$ , whose coordinates are all zero except for  $X_i$ . By assumption  $i$  causes other nodes to default, hence  $i$  itself must default, that is,  $i \in \bar{D}$ . To prove (8) it suffices to show that

$$\beta_i(X_i - w_i) \geq \sum_{j \in D - \{i\}} w_j \geq \sum_{j \in D} w_j. \quad (10)$$

The second inequality in (10) follows from the assumption that no nodes are in default before the shock and the fact that we must have  $D \subseteq \bar{D} - \{i\}$  for all nodes in  $D$  to default.

For the first inequality in (10), define the *shortfall* at node  $j$  to be the difference  $s_j = \bar{p}_j - p_j$ . From (3) we see that the vector of shortfalls  $s$  satisfies

$$s = (sA - w + X)_+ \wedge \bar{p}.$$

By (4) we have  $s_j > 0$  for  $j \in \bar{D}$  and  $s_j = 0$  otherwise. We use a subscript  $\bar{D}$  as in  $s_{\bar{D}}$  or  $A_{\bar{D}}$  to restrict a vector or matrix to the entries corresponding to nodes in the set  $\bar{D}$ . Then the vector of shortfalls at

the nodes of  $\bar{D}$  satisfies

$$s_{\bar{D}} \leq s_{\bar{D}} A_{\bar{D}} - w_{\bar{D}} + X_{\bar{D}}, \quad (11)$$

hence

$$X_{\bar{D}} - w_{\bar{D}} \geq s_{\bar{D}}(I_{\bar{D}} - A_{\bar{D}}). \quad (12)$$

The vector  $s_{\bar{D}}$  is strictly positive in every coordinate. From the definition of  $\beta_j$  we also know that the  $j^{th}$  row sum of  $I_{\bar{D}} - A_{\bar{D}}$  is at least  $1 - \beta_j$ . Hence,

$$s_{\bar{D}}(I_{\bar{D}} - A_{\bar{D}}) \cdot \mathbf{1}_{\bar{D}} \geq \sum_{j \in \bar{D}} s_j(1 - \beta_j) \geq s_i(1 - \beta_i). \quad (13)$$

From (11) it follows that the shortfall at node  $i$  is at least as large as the initial amount by which  $i$  defaults, that is,

$$s_i \geq X_i - w_i > 0. \quad (14)$$

From (12)–(14) we conclude that

$$\sum_{j \in \bar{D}} (X_j - w_j) = X_i - w_i - \sum_{j \in \bar{D} - \{i\}} w_j \geq s_i(1 - \beta_i) \geq (X_i - w_i)(1 - \beta_i). \quad (15)$$

This establishes (10) and the first statement of the proposition. The second statement follows from the first by recalling that the shock to the outside assets cannot exceed their value, that is,  $X_i \leq c_i$ . Therefore by (8) the probability of contagion is zero if  $c_i < w_i + (1/\beta_i) \sum_{j \in D} w_j$ . Dividing through by  $w_i$  we see that this is equivalent to the condition  $\sum_{j \in D} w_j/w_i > \beta_i(\lambda_i - 1)$ .  $\square$

The preceding proposition relates the probability of contagion from a given node  $i$  to the net worth of the defaulting nodes in  $D$  relative to  $i$ 's net worth. The bounds are completely general and do not depend on the distribution of shocks or on the topology of the network. The critical parameters are  $\beta_i$ , the degree to which the triggering node is indebted to the financial sector, and  $\lambda_i$ , the degree of leverage of  $i$ 's outside assets.

The Appendix gives estimates of these parameters for large European banks, based on data from stress tests conducted by the European Banking Authority (2011). Among the 50 largest of these banks the average of the  $\lambda_i$  is 24.9, the average of our estimated  $\beta_i$  is 14.9%, and the average of the products  $\beta_i(\lambda_i - 1)$  is 3.2. Proposition 1 implies that contagion from a “typical” bank  $i$  cannot topple a set of banks  $D$  if the net worth of the latter is more than 3.2 times the net worth of the former, unless there are additional channels of contagion.<sup>8</sup>

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<sup>8</sup>On average, commercial banks in the United States are leveraged only about half as much as European banks, and their values of  $\beta_i$  are somewhat smaller (Federal Reserve Release H.8, Assets and Liabilities of Commercial Banks in the United States, 2012). This suggests that contagion is even less likely in the US financial sector than in Europe.

### 3.2 Contagion with proportional shocks

We can say a good deal more if we impose some structure on the distribution of shocks. The notion that contagion from  $i$  to  $D$  is *weak*, described informally in Section 1, can now be made precise by the condition

$$P(X_i \geq w_i + (1/\beta_i) \sum_{j \in D} w_j) \leq P(X_i > w_i) \prod_{j \in D} P(X_j > w_j). \quad (16)$$

The expression on the left bounds the probability that these nodes default solely through contagion from  $i$ , while the expression on the right is the probability that the nodes in  $D$  default through independent direct shocks. Contagion is weak if the latter probability is at least as large as the former. The assumption of independent direct shocks is somewhat unrealistic: in practice one would expect the shocks to different nodes to be positively associated. In this case, however, the probability of default from direct shocks is even larger, hence weak contagion covers this situation as well.

Let us assume that the losses at a given node  $i$  scale with the size of the portfolio  $c_i$ . Let us also assume that the distribution of these *relative* losses is the same for all nodes, and independent among nodes. Then there exists a distribution function  $H : [0, 1] \rightarrow [0, 1]$  such that

$$F(x_1, \dots, x_n) = \prod_{1 \leq i \leq n} H(x_i/c_i). \quad (17)$$

Beta distributions provide a flexible family with which to model the distribution of shocks as a fraction of outside assets. We work with beta densities of form

$$h_{p,q}(y) = \frac{y^{p-1}(1-y)^{q-1}}{B(p,q)}, \quad 0 \leq y \leq 1, \quad p, q \geq 1, \quad (18)$$

where  $B(p, q)$  is a normalizing constant. The subset with  $p = 1$  and  $q > 1$  has a decreasing density and seems the most realistic, but (18) is general enough to allow a mode anywhere in the unit interval. The case  $q = 1, p > 1$  has an increasing density and could be considered “heavy-tailed” in the sense that it assigns greater probability to greater losses, with losses capped at 100 percent of outside assets.<sup>9</sup>

**Theorem 1.** *Assume the shocks are i.i.d. beta distributed as in (18) and that the net worth of every node is initially nonnegative. Let  $D$  be a nonempty subset of nodes and let  $i \notin D$ . Contagion from  $i$  to  $D$  is impossible if*

$$\sum_{j \in D} w_j > w_i \beta_i (\lambda_i - 1) \quad (19)$$

and it is weak if

$$\sum_{j \in D} w_j \geq w_i \beta_i \sum_{j \in D} (\lambda_i - 1) / \lambda_j. \quad (20)$$

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<sup>9</sup>Bank capital requirements under Basel II and III standards rely on a family of loss distributions derived from a Gaussian copula model. As noted by Tasche (2008) and others, these distributions can be closely approximated by beta distributions.

As noted after Proposition 1, the condition in (19) states that contagion from  $i$  to  $D$  is impossible if the total net worth of the nodes in  $D$  is sufficiently large relative to the net worth of  $i$  weighted by the exposure of the financial system to node  $i$  and the vulnerability of  $i$  as measured by its leverage. The condition in (20) compares the total net worth of  $D$  relative to that of  $i$  with the leverage of  $i$  relative to that of the nodes in  $D$ . With other parameters held constant, increasing the relative net worth of  $D$  makes contagion weaker in the sense that it strengthens the inequality; increasing the leverage of  $i$  relative to that of nodes in  $D$  has the opposite effect. Importantly, the two effects are mediated by  $\beta_i$ , which measures the exposure of the financial system to node  $i$  — a lower  $\beta_i$  makes  $D$  less vulnerable to  $i$  and makes  $D$  less sensitive to the degree of leverage at  $i$ . Thus, (20) captures the effects of equity levels, leverage ratios, and the degree of reliance on interbank lending on the risk of contagion.

By recalling that  $\lambda_j = c_j/w_j$  we can rewrite (20) in the equivalent form

$$\sum_{j \in D} c_j \lambda_j^{-1} / \sum_{j \in D} \lambda_j^{-1} \geq c_i \beta_i (1 - \lambda_i^{-1}). \quad (21)$$

Written this way, the condition states that contagion from  $i$  to  $D$  is weak if the *average* size of the nodes in  $D$  weighted by their inverse leverage ratios (their capital ratios) is sufficiently large relative to  $i$ ; on the right side of the inequality,  $c_i \beta_i$  measures the financial system's exposure to node  $i$ 's outside assets, and the factor  $(1 - \lambda_i^{-1})$  is greater when node  $i$  is more highly leveraged. Inequality (21) is thus harder to satisfy, and  $D$  more vulnerable to contagion from  $i$ , if the large nodes in  $D$  are more highly leveraged, if node  $i$  draws more of its funding from the financial system, or if node  $i$  is more highly leveraged.

Through (21), a key implication of Theorem 1 is that without some heterogeneity, contagion will be weak *irrespective of the structure* of the interbank network:

**Corollary 1.** *Assume that all nodes hold the same amount of outside assets  $c_k \equiv c$ . Under the assumptions of Theorem 1, contagion is weak from any node to any other set of nodes.*

*Proof.* This follows from the fact that  $\beta_i(1 - \lambda_i^{-1}) < 1$ ; hence, if  $c_k \equiv c$ , inequality (21) holds for all  $i$  and  $D$ .  $\square$

In the Appendix we apply our framework to the 50 largest banks in the stress test data from the European Banking Authority. It turns out that contagion is weak in a wide variety of scenarios. In particular, we analyze the scenario in which one of the five largest European banks (as measured by assets) topples two other banks in the top 50. We find that the probability of such an event is less than the probability of direct default unless the two toppled banks are near the bottom of the list of 50.

In the example of Figure 2(a), with node  $i$  the central node, the left side of (20) evaluates to 20, and the right side evaluates to 16 because  $\beta_i = 2/7$ ,  $\lambda_i = 15$ , and each of the peripheral nodes has  $\lambda_j = 10$ . Thus, contagion is weak.

*Proof of Theorem 1.* Proposition 1 implies that contagion is weak from  $i$  to  $D$  if

$$P(X_i \geq w_i + (1/\beta_i) \sum_{j \in D} w_j) \leq P(X_i > w_i) \prod_{j \in D} P(X_j > w_j). \quad (22)$$

On the one hand this certainly holds if  $w_i + (1/\beta_i) \sum_{j \in D} w_j > c_i$ , for then contagion is impossible. In this case we obtain, as in (9),

$$\sum_{j \in D} w_j/w_i > \beta_i(\lambda_i - 1). \quad (23)$$

Suppose on the other hand that  $(w_i + (1/\beta_i) \sum_{j \in D} w_j) \leq c_i$ . By assumption the relative shocks  $X_k/c_k$  are independent and beta distributed as in (18). In the uniform case  $p = q = 1$ , (22) is equivalent to

$$[1 - (w_i/c_i + (1/\beta_i c_i) \sum_{j \in D} w_j)] \leq (1 - w_i/c_i) \prod_{j \in D} (1 - w_j/c_j), \quad (24)$$

We claim that (24) implies (22) for the full family of beta distributions in (19). To see why, first observe that the cumulative distribution  $H_{p,q}$  of  $h_{p,q}$  satisfies

$$1 - H_{p,q}(y) = H_{q,p}(1 - y).$$

Hence (22) holds if

$$H_{q,p}(1 - w_i/c_i - (1/\beta_i c_i) \sum_{j \in D} w_j) \leq H_{q,p}(1 - w_i/c_i) \prod_{j \in D} H_{q,p}(1 - w_j/c_j). \quad (25)$$

But (25) follows from (24) because beta distributions with  $p, q \geq 1$  have the submultiplicative property

$$H_{q,p}(xy) \leq H_{q,p}(x)H_{q,p}(y), \quad x, y \in [0, 1].$$

(See Proposition 4.1.2 of Wirch 1999; the application there has  $q \leq 1$ , but the proof remains valid for  $q \geq 1$ . The inequality can also be derived from Corollary 1 of Ramos Romero and Sordo Diaz 2001.) It therefore suffices to establish (25), which is equivalent to

$$(1/\beta_i c_i) \sum_{j \in D} w_j \geq (1 - w_i/c_i)(1 - \prod_{j \in D} (1 - w_j/c_j)). \quad (26)$$

Given any real numbers  $\theta_j \in [0, 1]$  we have the inequality

$$\prod_j (1 - \theta_j) \geq 1 - \sum_j \theta_j. \quad (27)$$

Hence a sufficient condition for (26) to hold is that

$$(1/\beta_i c_i) \sum_{j \in D} w_j \geq (1 - w_i/c_i)(\sum_{j \in D} w_j/c_j). \quad (28)$$

After rearranging terms and using the fact that  $\lambda_k = c_k/w_k$  for all  $k$ , we obtain (20). This concludes the proof of Theorem 1.  $\square$

From the argument following (25), it is evident that the same result holds if the shocks to each node  $j$  are distributed with parameters  $p_j, q_j$  in (18) with  $p_i \leq \min_{j \in D} p_j$  and  $q_i \geq \max_{j \in D} q_j$ .

As a further illustration of Theorem 1, suppose the nodes in  $D$  are numbered  $2, \dots, m$  and suppose node  $i = 1$  receives a shock. Further suppose the outside assets are ordered  $c_1 \geq c_2 \geq \dots \geq c_m$ ; because shocks are proportional to outside assets, the assumption that the node with the largest  $c_k$  receives the shock maximizes the chances of contagion to the other nodes.

**Corollary 2.** *If  $c_1 \geq c_2 \geq \dots \geq c_m$ , then contagion from node 1 to nodes  $2, \dots, m$  is weak if  $c_2 \geq \beta_1(c_1 - w_1)$  and  $c_j \geq (c_{j-1} - w_{j-1})$ ,  $j = 2, \dots, m$ . Contagion is impossible if, in addition,  $c_2 - c_m + w_m > \beta_1(c_1 - w_1)$ .*

This is a direct consequence of (24) and (26), hence we omit the details and comment on the interpretation. The lower bounds on the  $c_j$  ensure that the potential spillovers from other nodes cannot push the full set of nodes into default regardless of the network topology. Viewing the conditions in the corollary as lower bounds on the  $w_j$  suggests minimum capital requirements to ensure the resilience of the system, based on the relative sizes of banks.

Of course these results do not say that the network structure has *no* effect on the probability of contagion; indeed there is a considerable literature showing that it does (see among others Haldane and May 2011, Gai and Kapadia 2011, Georg, 2011). Rather it shows that in quite a few situations the probability of contagion will be lower than the probability of direct default, absent some channel of contagion beyond spillovers through payment obligations. We have already mentioned bankruptcy costs, fire sales, and mark-to-market losses as amplifying mechanisms. The model of Acemoglu, Ozdaglar, and Tahbaz-Salehi (2013) generates greater contagion by making debts to financial institutions subordinate to other payment obligations. With priority given to outside payments, shocks produce greater losses within the network.

### 3.3 Contagion with truncated shocks

In this section we shall show that the preceding results are not an artifact of the beta distribution: similar bounds hold for a variety of shock distributions. Under the beta distribution the probability is zero that a node loses all of its outside assets. One could easily imagine, however, that the probability of this event is positive. This situation can be modeled as follows. Let  $X_i^o \geq 0$  be a *primary shock* (potentially unbounded in size) and let  $X_i = c_i \wedge X_i^o$  be the resulting loss to  $i$ 's outside assets. For example  $X_i^o$  might represent a loss of income from an employment shock that completely wipes out  $i$ 's



outside assets. Assume that the primary shocks have a joint distribution function of form

$$F^o(x_1^o, \dots, x_n^o) = \prod_{1 \leq i \leq n} H^o(x_i^o/c_i), \quad (29)$$

where  $H^o$  is a distribution function on the nonnegative real line. In other words we assume that the shocks are i.i.d. and that a given shock  $x_i^o$  affects every dollar of outside assets  $c_i$  equally. (We shall consider a case of dependent shocks after the next result.)

In general a random variable with distribution function  $G$  and density  $g$  is said to have an *increasing failure rate (IFR) distribution* if  $g(x)/(1 - G(x))$  is an increasing function of  $x$ . Examples of IFR distributions include all normal, exponential, and uniform distributions and, more generally, all log-concave distributions. Observe that truncating the shock can put mass at  $c_i$  and thus assign positive probability to a total loss of outside assets.

**Theorem 2.** *Assume the primary shocks are i.i.d. and IFR-distributed, and that the net worth of every node is initially nonnegative. Let  $D$  be a nonempty subset of nodes and let  $i \notin D$ . Contagion from  $i$  to  $D$  is impossible if*

$$\sum_{j \in D} w_j > w_i \beta_i (\lambda_i - 1)$$

and it is weak if

$$\sum_{j \in D} w_j > w_i \beta_i \sum_{j \in D} \lambda_i / \lambda_j. \quad (30)$$

**Corollary 3.** *Assume that all nodes hold the same amount of outside assets  $c_i \equiv c$ . Under the assumptions of Theorem 2, contagion is weak from any node to any other set of nodes.*

This is immediate upon rewriting (30) as

$$\sum_{j \in D} c_j \lambda_j^{-1} / \sum_{j \in D} \lambda_j^{-1} \geq \beta_i c_i.$$

*Proof of Theorem 2.* Through relabeling, we can assume that the source node for contagion is  $i = 1$  and that the infected nodes are  $D = \{2, 3, \dots, m\}$ . By Proposition 1 we know that contagion is weak from 1 to  $D$  if

$$P(X_1 > w_1 + (1/\beta_1) \sum_{2 \leq j \leq m} w_j) \leq P(X_1 > w_1) P(X_2 > w_2) \cdots P(X_m > w_m). \quad (31)$$

Since  $X_1 = c_1 \wedge X_1^o$ , the left-hand side is zero when  $w_1 + (1/\beta_1) \sum_{2 \leq j \leq m} w_j > c_1$ . Thus contagion is impossible if

$$\sum_{2 \leq j \leq m} w_j / w_1 > \beta_1 (\lambda_1 - 1). \quad (32)$$

Let us therefore assume that  $w_1 + (1/\beta_1) \sum_{2 \leq j \leq m} w_j \leq c_1$ . Define the random variables  $Y_i = X_i^o/c_i$ . Weak contagion from 1 to  $D$  holds if

$$\begin{aligned} P(Y_1 > w_1/c_1 + (1/\beta_1 c_1) \sum_{2 \leq j \leq m} w_j) &\leq P(Y_1 > w_1/c_1)P(Y_2 > w_2/c_2) \cdots P(Y_m > w_m/c_m) \\ &= P(Y_1 > w_1/c_1)P(Y_1 > w_2/c_2) \cdots P(Y_1 > w_m/c_m). \end{aligned} \quad (33)$$

where the latter follows from the assumption that the  $Y_i$  are i.i.d. By assumption  $Y_1$  is IFR, hence  $P(Y_1 > s + t | Y_1 > s) \leq P(Y_1 > t)$  for all  $s, t \geq 0$ . (See for example Barlow and Proschan 1975, p.159.) It follows that

$$\begin{aligned} P(Y_1 > \sum_{1 \leq k \leq m} w_k/c_k) &= P(Y_1 > w_1/c_1)P(Y_1 > w_1/c_1 + w_2/c_2 | Y_1 > w_1/c_1) \\ &\quad \cdots P(Y_1 > w_1/c_1 + w_2/c_2 + \cdots + w_m/c_m | Y_1 > w_1/c_1 + w_2/c_2 + \cdots + w_{m-1}/c_{m-1}) \\ &\leq P(Y_1 > w_1/c_1)P(Y_1 > w_2/c_2) \cdots P(Y_1 > w_m/c_m) \end{aligned}$$

Together with (33) this shows that contagion from 1 to  $D$  is weak provided that

$$P(Y_1 \geq w_1/c_1 + (1/\beta_1 c_1) \sum_{2 \leq j \leq m} w_j) \leq P(Y_1 > \sum_{1 \leq k \leq m} w_k/c_k). \quad (34)$$

This clearly holds if

$$w_1/c_1 + (1/\beta_1 c_1) \sum_{2 \leq j \leq m} w_j \geq \sum_{1 \leq k \leq m} w_k/c_k, \quad (35)$$

which is equivalent to

$$(1/\beta_1 c_1) \sum_{2 \leq j \leq m} w_j \geq \sum_{2 \leq j \leq m} w_j/c_j = \sum_{2 \leq j \leq m} \lambda_j^{-1}. \quad (36)$$

Since  $c_1 = \lambda_1 w_1$ , we can re-write (36) as

$$\sum_{2 \leq j \leq m} w_j/w_1 \geq \beta_1 \lambda_1 \sum_{2 \leq j \leq m} \lambda_j^{-1}. \quad (37)$$

We have therefore shown that if contagion from 1 to  $D = \{2, 3, \dots, m\}$  is possible at all, then (37) is a sufficient condition for weak contagion.  $\square$

From (37), we see that a simple sufficient condition for weak contagion is  $c_j \geq \beta_1 c_1$ ,  $j = 2, \dots, m$ , and the condition  $\sum_{j=2}^m w_j > \beta_1 (c_1 - w_1)$  makes contagion impossible.

The assumption of independent shocks to different nodes is conservative. If we assume that direct shocks to different nodes are positively dependent, as one would expect in practice, the bounds in Theorems 1 and 2 will be lower and the relative likelihood of default through contagion even smaller.

A particularly simple case arises when the primary shocks  $X_i$  are independent with a negative exponential distribution of form

$$f(x_i) = \mu e^{-\mu x_i}, \quad x_i \geq 0. \quad (38)$$

If we further assume that  $c_1 = \dots = c_m \equiv c$  and  $\beta_1 = 1$ , then the two probabilities compared in (33) are equal, both evaluating to  $\exp(-\mu \sum_{j \in D} w_j/c)$ . In this sense, the exponential distribution is a borderline case in which the probability of a set of defaults from a single shock is roughly equal to the probability from multiple independent shocks. We say “roughly” because the left side of (33) is an upper bound on the probability of default through contagion, and in practice the  $\beta_i$  are substantially smaller than 1. In the example of Figure 2(a), we have seen that contagion from the central node requires a shock greater than 80, which has probability  $\exp(-80\mu)$  under an exponential distribution. For direct defaults, it suffices to have shocks greater than 5 at the peripheral nodes and a shock greater than 10 at the central node, which has probability  $\exp(-30\mu)$  given i.i.d. exponential shocks.

If the primary shocks have a Pareto-like tail, meaning that

$$P(X_i > x) \sim ax^{-\mu} \quad (39)$$

for some positive constants  $a$  and  $\mu$  (or, more generally, a regularly varying tail), then the probability that a single shock will exceed  $\sum_{j \in D} w_j/c$  will be greater than the probability that the nodes in  $D$  default through multiple independent shocks, at least at large levels of the  $w_j$ . However, introducing some dependence can offset this effect, as we now illustrate. To focus on the issue at hand, we take  $c = 1$  and  $\beta_i = 1$ .

To consider a specific and relatively simple case, let  $Y_1, \dots, Y_m$  be independent random variables, each distributed as  $t_\nu$ , the Student  $t$  distribution with  $\nu > 2$  degrees of freedom. Let  $\tilde{Y}_1, \dots, \tilde{Y}_m$  have a standard multivariate Student  $t$  distribution with  $t_\nu$  marginals.<sup>10</sup> The  $\tilde{Y}_j$  are uncorrelated but not independent. To make the shocks positive, set  $X_j = Y_j^2$  and  $\tilde{X}_j = \tilde{Y}_j^2$ . Each  $X_j$  and  $\tilde{X}_j$  has a Pareto-like tail that decays with a power of  $\nu/2$ .

**Proposition 2.** *With independent shocks  $X_j$ ,*

$$P(X_i > \sum_{j=1}^m w_j) \geq \prod_{j=1}^m P(X_j > w_j)$$

*for all sufficiently large  $w_j$ ,  $j = 1, \dots, m$ . With dependent shocks*

$$P(\tilde{X}_i > \sum_{j=1}^m w_j) \leq P(\tilde{X}_j > w_j, j = 1, \dots, m)$$

*for all  $w_j \geq 0$ ,  $j = 1, \dots, m$ .*

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<sup>10</sup>More explicitly,  $(\tilde{Y}_1, \dots, \tilde{Y}_m)$  has the distribution of  $(Z_1, \dots, Z_m)/\sqrt{\chi_v^2/\nu}$ , where the  $Z_i$  are independent standard normal random variables and  $\chi_v^2$  has a chi-square distribution with  $v$  degrees of freedom and is independent of the  $Z_i$ .

*Proof of Proposition 2.* The first statement follows from applying (39) to both sides of the inequality. The second statement is an application of Bound II for the  $F$  distribution on p.1196 of Marshall and Olkin (1974).  $\square$

Thus, even with heavy-tailed shocks, we may find that default of a set of nodes through contagion from a single shock is less likely than default through direct shocks to individual nodes if the shocks are dependent.

## 4 Amplification of losses due to network effects

The preceding analysis dealt with the impact of default by a single node (the source) on another set of nodes (the target). Here we shall examine the impact of shocks on the entire system, including multiple and simultaneous defaults. To carry out such an analysis, we need to have a measure of the total systemic impact of a shock. There appears to be no commonly accepted measure of systemic impact in the prior literature. Eisenberg and Noe (2001) suggest that it is the number of waves of default that a given shock induces in the network. Other authors have suggested that the systemic impact should be measured by the aggregate loss of bank capital; see for example Cont, Moussa, and Santos (2010). Still others have proposed the total loss in value of only those nodes external to the financial sector, i.e. firms and households.

Here we shall take the systemic impact of a shock to be *the total loss in value summed over all nodes, including nodes corresponding to financial entities as well as those representing firms and households*. This measure is easily stated in terms of the model variables. Given a shock realization  $x$ , the total reduction in asset values is

$$\sum_i x_i + S(x) \text{ where } S(x) = \sum_i (\bar{p}_i - p_i(x)). \quad (40)$$

The term  $|x| = \sum_i x_i$  is the direct loss in value from reductions in payments by the external sector. The term  $S(x)$  is the indirect loss in value from reductions in payments by the nodes to other nodes and to the external sector. An overall measure of the riskiness of the system is the *expected loss in value*

$$L = \int (|x| + S(x)) dF(x). \quad (41)$$

The question we wish to examine is what proportion of these losses can be attributed to connections between institutions as opposed to characteristics of individual banks. To analyze this issue let  $x$  be a shock and let  $D = D(x)$  be the set of nodes that defaults given  $x$ . Under our assumptions this set is unique because the clearing vector is unique. To avoid notational clutter we shall suppress  $x$  in the ensuing discussion.

As in the proof of Propostion 1, define the *shortfall* in payments at node  $i$  to be  $s_i = \bar{p}_i - p_i$ , where  $p$  is the clearing vector. By definition of  $D$ ,

$$\begin{aligned} s_i &> 0 \text{ for all } i \in D \\ s_i &= 0 \text{ for all } i \notin D \end{aligned} \tag{42}$$

Also as in the proof of Proposition 1, let  $A_D$  be the  $|D| \times |D|$  matrix obtained by restricting the relative liabilities matrix  $A$  to  $D$ , and let  $I_D$  be the  $|D| \times |D|$  identity matrix. Similarly let  $s_D$  be the vector of shortfalls  $s_i$  corresponding to the nodes in  $D$ , let  $w_D$  be the corresponding net worth vector defined in (1), and let  $x_D$  be the corresponding vector of shocks. The clearing condition (3) implies the following *shortfall equation*, provided no node is entirely wiped out — that is, provided  $s_i < \bar{p}_i$ , for all  $i$ :

$$s_D A_D - (w_D - x_D) = s_D. \tag{43}$$

Allowing the possibility that some  $s_i = \bar{p}_i$ , the left side is an upper bound on the right side. Recall that  $A_D$  is substochastic, that is, every row sum is at most unity. Moreover, by assumption, there exists a chain of obligations from any given node  $k$  to a node having strictly positive obligations to the external sector. It follows that  $\lim_{k \rightarrow \infty} A_D^k = 0_D$ , hence  $I_D - A_D$  is invertible and

$$[I_D - A_D]^{-1} = I_D + A_D + A_D^2 + \dots \tag{44}$$

From (43) and (44) we conclude that

$$s_D = (x_D - w_D)[I_D + A_D + A_D^2 + \dots]. \tag{45}$$

Given a shock  $x$  with resulting default set  $D = D(x)$ , define the vector  $u(x) \in R_+^n$  such that

$$u_D(x) = [I_D + A_D + A_D^2 + \dots] \cdot 1_D, \quad u_i(x) = 0 \text{ for all } i \notin D. \tag{46}$$

Combining (40), (45), and (46) shows that total losses given a shock  $x$  can be written in the form

$$L(x) = \sum_i (x_i \wedge w_i) + \sum_i (x_i - w_i) u_i(x). \tag{47}$$

The first term represents the direct losses to equity at each node and the second term represents the total shortfall in payments summed over all of the nodes. The right side becomes an upper bound on  $L(x)$  if  $s_i = \bar{p}_i$  for some  $i \in D(x)$ .

We call the coefficient  $u_i = u_i(x)$  the *depth* of node  $i$  in  $D = D(x)$ . The rationale for this terminology is as follows. Consider a Markov chain on  $D$  with transition matrix  $A_D$ . For each  $i \in D$ ,  $u_i$  is the expected number of periods before exiting  $D$ , starting from node  $i$ .<sup>11</sup> Expression (46) shows that the node depths measure the *amplification* of losses due to interconnections among nodes in the default set.

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<sup>11</sup>Liu and Staum (2012) show that the node depths can be used to characterize the gradient of the clearing vector  $p(x)$  with respect to the asset values.

We remark that the concept of node depth is dual to the notion of eigenvector centrality in the networks literature (see for example Newman 2010). To see the connection let us restart the Markov chain uniformly in  $D$  whenever it exits  $D$ . This modified chain has an ergodic distribution proportional to  $1_D \cdot [I_D + A_D + A_D^2 + \dots]$ , and its ergodic distribution measures the centrality of the nodes in  $D$ . It follows that node depth with respect to  $A_D$  corresponds to centrality with respect to the *transpose* of  $A_D$ .

Although they are related algebraically, the two concepts are quite different. To see why let us return to the example of Figure 2(b). Suppose that node 1 (the central node) suffers a shock  $x_1 > 80$ . This causes all nodes to default, that is, the default set is  $D' = \{1, 2, 3, 4, 5\}$ . Consider any node  $j > 1$ . In the Markov chain described above the expected waiting time to exit the set  $D'$ , starting from node  $j$ , is given by the recursion  $u_j = 1 + \beta_j u_j$ , which implies

$$u_j = 1/(1 - \beta_j) = 1 + y/55. \quad (48)$$

From node 1 the expected waiting time satisfies the recursion

$$u_1 = 1(100/140) + (1 + u_j)(40/140). \quad (49)$$

Hence

$$u_1 = 9/7 + 2y/385. \quad (50)$$

Comparing (48) and (50) we find that node 1 is deeper than the other nodes ( $u_1 > u_j$ ) for  $0 \leq y < 22$  and shallower than the other nodes for  $y > 22$ . In contrast, node 1 has lower eigenvector centrality than the other nodes for all  $y \geq 0$  because it cannot be reached directly from any other node.

The magnitude of the node depths in a default set can be bounded as follows. In the social networks literature a set  $D$  is said to be  $\alpha$ -*cohesive* if every node in  $D$  has at least  $\alpha$  of its obligations to other nodes in  $D$ , that is,  $\sum_{j \in D} a_{ij} \geq \alpha$  for every  $i \in D$  (Morris, 2000). The *cohesiveness* of  $D$  is the maximum such  $\alpha$ , which we shall denote by  $\alpha_D$ . From (46) it follows that

$$\forall i \in D, \quad u_i \geq 1/(1 - \alpha_D). \quad (51)$$

Thus the more cohesive the default set, the greater the depth of the nodes in the default set and the greater the amplification of the associated shock.

Similarly we can bound the node depths from above. Recall that  $\beta_i$  is the proportion of  $i$ 's obligations to other nodes in the financial system. Letting  $\beta_D = \max\{\beta_i : i \in D\}$  we obtain the upper bound, assuming  $\beta_D < 1$ ,

$$\forall i \in D, \quad u_i \leq 1/(1 - \beta_D). \quad (52)$$

The bounds in (51) and (52) depend on the default set  $D$ , which depends on the shock  $x$ . A *uniform* upper bound is given by

$$\forall i, \quad u_i \leq 1/(1 - \beta^+) \quad \text{where } \beta^+ = \max \beta_i, \quad (53)$$

assuming  $\beta^+ < 1$ .

We are now in a position to compare the expected systemic losses in a given network of interconnections, and the expected systemic losses without such interconnections. As before, fix a set of  $n$  nodes  $N = \{1, 2, \dots, n\}$ , a vector of outside assets  $c = (c_1, c_2, \dots, c_n) \in \mathbb{R}_+^n$  and a vector of outside liabilities  $b = (b_1, b_2, \dots, b_n) \in \mathbb{R}_+^n$ . Assume the net worth  $w_i$  of node  $i$  is nonnegative before a shock is realized. Interconnections are determined by the  $n \times n$  liabilities matrix  $\bar{P} = (\bar{p}_{ij})$ .

Let us compare this situation with the following: eliminate all connections between nodes, that is, let  $\bar{P}^o$  be the  $n \times n$  matrix of zeroes. Each node  $i$  still has outside assets  $c_i$  and outside liabilities  $b_i$ . To keep their net worths unchanged, we introduce “fictitious” outside assets and liabilities to balance the books. In particular if  $c_i - b_i < w_i$  we give  $i$  a new class of outside assets in the amount  $c'_i = w_i - (c_i - b_i)$ . If  $c_i - b_i > w_i$  we give  $i$  a new class of outside liabilities in the amount  $b'_i = w_i - (c_i - b_i)$ . We shall assume that these new assets are completely safe (they are not subject to shocks), and that the new liabilities have the same priority as all other liabilities.

Let  $F(x_1, \dots, x_n)$  be a joint shock distribution that is *homogeneous in assets*, that is,  $F(x_1, \dots, x_n) = G(x_1/c_1, \dots, x_n/c_n)$  where  $G$  is a symmetric c.d.f. (Unlike in the preceding results on contagion we do not assume that the shocks are independent.) We say that  $F$  is IFR if its marginal distributions are IFR; this is equivalent to saying that the marginals of  $G$  are IFR. Given  $F$ , let  $\bar{L}$  be the expected total losses in the original network and let  $\bar{L}^o$  be the expected total losses when the connections are removed as described above.

**Theorem 3.** *Let  $N(b, c, w, \bar{P})$  be a financial system and let  $N^o$  be the analogous system with all the connections removed. Assume that the shock distribution is homogeneous in assets and IFR. Let  $\beta^+ = \max_i \beta_i < 1$  and let  $\delta_i = P(X_i \geq w_i)$ . The ratio of expected losses in the original network to the expected losses in  $N^o$  is at most*

$$\frac{\bar{L}}{\bar{L}^o} \leq 1 + \frac{\sum \delta_i c_i}{(1 - \beta^+) \sum c_i}. \quad (54)$$

*Proof.* By assumption the marginals of  $F$  are IFR distributed. A general property of IFR distributions is that “new is better than used in expectation,” that is,

$$\forall i \forall w_i \geq 0, \quad E[X_i - w_i | X_i \geq w_i] \leq E[X_i] \quad (55)$$

(Barlow and Proschan 1975, p.159). It follows that

$$\forall i \forall w_i \geq 0, \quad E[(X_i - w_i)^+] \leq P(X_i \geq w_i)E[X_i] = \delta_i E[X_i]. \quad (56)$$

By (47) we know that the total expected losses  $\bar{L}$  can be bounded as

$$\bar{L} \leq \sum_i E[X_i \wedge w_i] + E[\sum_i (X_i - w_i)u_i(X)]. \quad (57)$$

From (53) we know that  $u_i \leq 1/(1 - \beta^+)$  for all  $i$ ; furthermore we clearly have  $X_i - w_i \leq (X_i - w_i)^+$  for all  $i$ . Therefore

$$\bar{L} \leq \sum_i E[X_i \wedge w_i] + (1 - \beta^+)^{-1} \sum_i E[(X_i - w_i)^+]. \quad (58)$$

From this and (56) it follows that

$$\begin{aligned} \bar{L} &\leq \sum_i E[X_i \wedge w_i] + (1 - \beta^+)^{-1} \sum \delta_i E[X_i] \\ &\leq \sum_i E[X_i] + (1 - \beta^+)^{-1} \sum \delta_i E[X_i]. \end{aligned} \quad (59)$$

When the network connections are excised, the expected loss is simply the expected sum of the shocks, that is,  $\bar{L}^o = \sum_i E[X_i]$ . By the assumption of homogeneity in assets we know that  $E[X_i] \propto c_i$  for all  $i$ . We conclude from this and (59) that

$$\bar{L}/\bar{L}^o \leq 1 + \frac{\sum \delta_i c_i}{(1 - \beta^+) \sum c_i}.$$

This completes the proof of Theorem 3.  $\square$

Theorem 3 shows that the increase in losses due to network interconnections will be very small unless  $\beta^+$  is close to 1 or the default rates of some banks (weighted by their outside asset base) is large. Moreover this statement holds even when the shocks are dependent, say due to common exposures, and it holds independently of the network structure.

Some might argue that write-downs of purely financial obligations should not be counted as part of the systemic loss. This is tantamount to truncating node depth at 1 and thus leads to an even smaller upper bound in Theorem 3.

To illustrate the result concretely, consider the EBA data. In this case  $\beta^+ = \max\{\beta_i\} = 0.43$  where the maximum occurs for Dexia bank. Regulatory restrictions on assets imply that any given bank is unlikely to fail due to direct default; indeed current regulatory standards seek to make the direct default probability of any individual bank smaller than 0.1% per year. Let us make the more conservative assumption that the probability of default is at most 1%, that is,  $\max\{\delta_i\} < 0.01$ . If this standard is met, then no matter what the interconnections might be among the banks in the data set, we conclude from Theorem 3 that the additional expected loss attributable to the network is at most  $.01/(1 - .43)$ , which is about 1.7%. In other words the actual network of connections cannot increase the expected losses by very much under the assumptions of the theorem.



## 5 Bankruptcy costs

In this section and the next we enrich the basic framework by incorporating additional mechanisms through which losses propagate from one node to another. We begin by adding bankruptcy costs. The equilibrium condition (3) implicitly assumes that if node  $i$ 's assets fall short of its liabilities by 1 unit, then the total claims on node  $i$  are simply marked down by 1 unit below the face value of  $\bar{p}_i$ . In practice, the insolvency of node  $i$  is likely to produce deadweight losses that have a knock-on effect on the shortfall at node  $i$  and at other nodes.

Cifuentes, Ferrucci, and Shin (2005), Battiston et al. (2012), Cont, Moussa, and Santos (2010), and Rogers and Veraart (2012) all use some form of liquidation cost or recovery rate at default in their analyses. Cifuentes, Ferrucci, and Shin (2005) distinguish between liquid and illiquid assets and introduce an external demand function to determine a recovery rate on illiquid assets. Elliott, Golub, and Jackson (2013) attach a fixed cost to bankruptcy. Elsinger, Lehar, and Summer (2006) present simulation results illustrating the effect of bankruptcy costs but without an explicit model. The mechanism we use appears to be the simplest and the closest to the original Eisenberg–Noe setting, which facilitates the analysis of its impact on contagion.

### 5.1 Shortfalls with bankruptcy costs

In the absence of bankruptcy costs, when a node fails its remaining assets are simply divided among its creditors. To capture costs of bankruptcy that go beyond the immediate reduction in payments, we introduce a multiplier  $\gamma \geq 0$  and suppose that upon a node's failure its assets are further reduced by

$$\gamma \left[ \bar{p}_i - (c_i + \sum_{j \neq i} p_j a_{ji} - x_i) \right], \quad (60)$$

up to a maximum reduction at which the assets are entirely wiped out. The term in square brackets is the difference between node  $i$ 's obligations  $\bar{p}_i$  and its remaining assets. This difference measures the severity of the failure, and the factor  $\gamma$  multiplies the severity to generate the knock-on effect of bankruptcy above and beyond the immediate cost to node  $i$ 's creditors. We can think of the expression in (60) as an amount of value destroyed or paid out to a fictitious bankruptcy node upon the failure of node  $i$ .

The resulting condition for a payment vector replaces (3) with

$$p_i = \bar{p}_i \wedge \left[ (1 + \gamma)(c_i + \sum_{j \neq i} p_j a_{ji} - x_i) - \gamma \bar{p}_i \right]_+. \quad (61)$$

Written in terms of shortfalls  $s_i = \bar{p}_i - p_i$ , this becomes

$$s_i = (1 + \gamma) \left[ \sum_{j \neq i} s_j a_{ji} - w_i + x_i \right]_+ \wedge \bar{p}_i. \quad (62)$$

Here we see explicitly how the bankruptcy cost factor  $\gamma$  magnifies the shortfalls.

Let  $\Phi_\gamma^s$  denote the mapping from the vector  $s$  on the right side of (62) to the vector  $s$  on the left side, and let  $\Phi_\gamma^p$  similarly denote the mapping of  $p$  defined by (61). We have the following:

**Proposition 3.** (a) For any  $\gamma \geq 0$ , the mapping  $\Phi_\gamma^s$  is monotone increasing, bounded, and continuous on  $R_+^n$ , and the mapping  $\Phi_\gamma^p$  is monotone decreasing, bounded, and continuous. It follows that  $\Phi_\gamma^s$  has a least fixed point  $s$  and  $\Phi_\gamma^p$  has a greatest fixed point  $p = \bar{p} - s$ . (b) For any  $s$ ,  $\Phi_\gamma^s(s)$  is increasing in  $\gamma$ , and for any  $p$ ,  $\Phi_\gamma^p(p)$  is decreasing in  $\gamma$ . Consequently, the set of default nodes under the minimal  $s$  and maximal  $p$  is increasing in  $\gamma$ . The fixed point  $s$  (and  $p$ ) is unique if  $(1 + \gamma)A$  has spectral radius less than 1.

*Proof of Proposition 3.* Part (a) follows from the argument in Theorem 1 of Eisenberg and Noe (2001).

For part (b), write  $v_i = c_i + (pA)_i - x_i$  and observe that

$$\Phi_\gamma^p(p)_i = \begin{cases} v_i - \gamma(\bar{p}_i - v_i), & v_i < \bar{p}_i; \\ \bar{p}_i & \text{otherwise,} \end{cases}$$

from which the monotonicity in  $\gamma$  follows. The maximal fixed-point is the limit of iterations of  $\Phi_\gamma^p$  starting from  $\bar{p}$  by the argument in Section 3 of Eisenberg and Noe (2001). If  $\gamma_1 \leq \gamma_2$ , then the iterates of  $\Phi_{\gamma_1}^p$  are greater than those of  $\Phi_{\gamma_2}^p$ , so their maximal fixed-points are ordered the same way. But then the set of nodes  $i$  for which  $p_i < \bar{p}_i$  at the maximal fixed-point for  $\gamma_1$  must be contained within that for  $\gamma_2$ . The same argument works for  $\Phi_\gamma^s$ . Uniqueness follows as in the case without bankruptcy costs.  $\square$

This result confirms that bankruptcy costs expand the set of defaults (i.e., increase contagion) resulting from a given shock realization  $x$  while otherwise leaving the basic structure of the model unchanged. In what follows, we examine the amplifying effect of bankruptcy costs conditional on a default set  $D$ , and we then compare costs with and without network effects. The factor of  $1 + \gamma$  already points to the amplifying effect of bankruptcy costs, but we can take the analysis further.<sup>12</sup>

Suppose, for simplicity, that the maximum shortfall of  $\bar{p}_i$  in (62) is not binding on any of the nodes in the default set. In other words, the shocks are large enough to generate defaults, but not so large as to entirely wipe out asset value at any node. In this case, we have

$$s_D = (1 + \gamma)[s_D A_D - w_D + x_D].$$

If we further assume that  $I_D - (1 + \gamma)A_D$  is invertible, then

$$s_D = (1 + \gamma)(x_D - w_D)[I_D - (1 + \gamma)A_D]^{-1}$$

and the systemic shortfall is

$$S(x) = s_D \cdot u_D(\gamma) = (1 + \gamma)(x_D - w_D) \cdot u_D(\gamma), \tag{63}$$

<sup>12</sup>A corresponding comparison is possible using partial recoveries at default, as in Rogers and Veraart (2012). This leads to qualitatively similar results, provided claims on other banks are kept to a realistic fraction of a bank's total assets.

where the modified node depth vector  $u_D(\gamma)$  is given by  $[I_D - (1 + \gamma)A_D]^{-1} \cdot 1_D$ . If  $(1 + \gamma)A_D$  has spectral radius less than 1, then

$$u_D(\gamma) = [I_D + (1 + \gamma)A_D + (1 + \gamma)^2 A_D^2 + \dots] \cdot 1_D.$$

The representation in (63) reveals two effects from introducing bankruptcy costs. The first is an immediate or local impact of multiplying  $x_i - w_i$  by  $1 + \gamma$ ; every element of  $s_D$  is positive, so the outer factor of  $1 + \gamma$  increases the total shortfall. The second and more important effect is through the increased node depth. In particular, letting  $\alpha_D$  denote the cohesiveness of  $D$  as before, we can now lower-bound the depth of each node by  $1/[1 - (1 + \gamma)\alpha_D]$ .<sup>13</sup> This makes explicit how bankruptcy costs deepen the losses at defaulted nodes and increase total losses to the system. By the argument in Theorem 3, we get the following comparison of losses with and without interconnections:

**Corollary 4.** *In the setting of Theorem 3, if we introduce bankruptcy costs satisfying  $(1 + \gamma)\beta^+ < 1$ , then*

$$\frac{\bar{L}}{\bar{L}^o} \leq 1 + \frac{\sum \delta_i c_i}{[1 - (1 + \gamma)\beta^+] \sum c_i}.$$

In the EBA data in the appendix, we have  $\beta^+ = 0.43$ . If we set  $\gamma$  at the rather large value of 0.5 and continue to assume  $\delta_i \leq 0.01$ , the corollary gives an upper bound of 1.042. In other words, even with large bankruptcy costs, the additional expected loss attributable to the network is at most 4.2%.

## 5.2 Example

We saw previously that in the example of Figure 2(a) we need a shock greater than 80 to the outside assets of the central node in order to have contagion to all other nodes. Under a beta distribution with parameters  $p = 1$  and  $q \geq 1$ , this has probability  $(1 - 80/150)^q = (7/15)^q$ . If we assume i.i.d. proportional shocks to the outside assets of all nodes, then all nodes default directly with probability

$$[(1 - w_1/c_1) \cdots (1 - w_5/c_5)]^q = [(14/15) \cdot 0.9^4]^q \approx 0.61^q > (7/15)^q.$$

Thus contagion is weak.

Now introduce a bankruptcy cost factor of  $\gamma = 0.5$  and consider a shock of 57 or larger to the central node. The shock creates a shortfall of at least 47 at the central node that gets magnified by 50% to a shortfall of at least 70.5 after bankruptcy costs. The central node's total liabilities are 140, so the result is that each peripheral node receives less than half of what it is due from the central node, and this is sufficient to push every peripheral node into default. Thus, with bankruptcy costs, the

<sup>13</sup>If the spectral radius of  $(1 + \gamma)A$  is less than 1, then  $(1 + \gamma)\alpha_D < 1$ .

probability of contagion is at least  $(1 - (57/150))^q = 0.62^q$ , which is now greater than the probability of direct defaults through independent shocks. A similar comparison holds with truncated exponentially distributed shocks.

This example illustrates how bankruptcy costs increase the probability of contagion. However, it is noteworthy that  $\gamma$  needs to be quite large to overcome the effect of weak contagion.

## 6 Confidence, credit quality and mark-to-market losses

In the previous section, we demonstrated how bankruptcy costs can amplify losses. In fact, a borrower's deteriorating credit quality can create mark-to-market losses for a lender well before the point of default. Indeed, by some estimates, these types of losses substantially exceeded losses from outright default during 2007-2009. We introduce a mechanism for adding this feature to a network model and show that it too magnifies contagion. See Harris, Herz, and Nissim (2012) for a broad discussion of accounting practices as sources of systemic risk.

The mechanism we introduce is illustrated in Figure 3, which shows how the value of node  $i$ 's total liabilities changes with the level  $z$  of node  $i$ 's assets. The figure shows the special case of a piecewise linear relationship. More generally, let  $r(z)$  be the *reduced value* of liabilities at a node as a function of asset level  $z$ , where  $r(z)$  is increasing, continuous, and

$$\begin{aligned} 0 \leq r(z) \leq \bar{p}_i, & \quad \text{for } z < (1+k)\bar{p}_i; \\ r(z) = \bar{p}_i, & \quad \text{for } z \geq (1+k)\bar{p}_i. \end{aligned}$$

Let  $R(z_1, \dots, z_n) = (r(z_1), \dots, r(z_n))$ . Given a shock  $x = (x_1, \dots, x_n)$ , the clearing vector  $p(z)$  solves

$$p(x) = R(c + p(x)A + w - x).$$

Our conditions on  $r$  ensure the existence of such a fixed point by an argument similar to the one in Section 2.2.

The effect of credit quality deterioration begins at a much higher asset level of  $z = (1+k)\bar{p}_i$  than does default. Think of  $k$  as measuring a capital cushion: node  $i$ 's credit quality is impaired once its net worth (the difference between its assets and liabilities) falls below the cushion. At this point, the value of  $i$ 's liabilities begins to decrease, reflecting the mark-to-market impact of  $i$ 's deteriorating credit quality.

In the example of Figure 2(a), suppose the central node is at its minimum capital cushion before experiencing a shock; in other words  $(1+k)\bar{p}_i = 150$ , which implies that  $k = 1/14$ . A shock of 5 to the central node's outside assets reduces the total value of its liabilities by  $5\eta$ , so each peripheral node incurs a mark-to-market loss of  $5\eta/14$ . In contrast, in the original model with  $\eta = 0$ , the peripheral nodes do not experience a loss unless the shock to the central node exceeds 10.

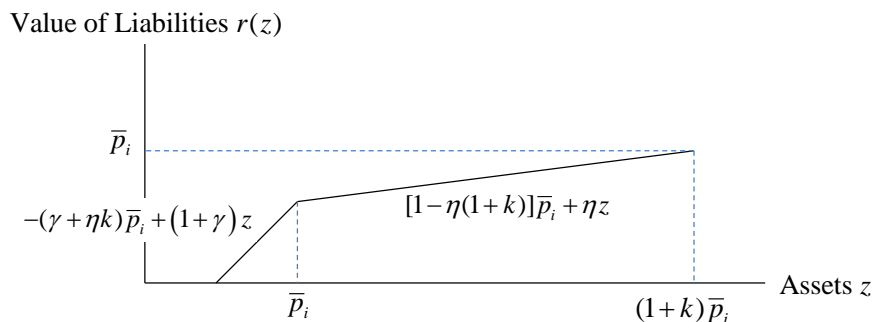


Figure 3: Liability value as a function of asset level in the presence of bankruptcy costs and credit quality.

As this example illustrates, mark-to-market losses from credit deterioration and bankruptcy costs operate in qualitatively different ways, even though both increase overall losses. Bankruptcy costs amplify the propagation of shortfall. In contrast, accounting for credit quality effectively increases the linkages between nodes. It does so by propagating losses at higher levels of asset values, rather than by amplifying the losses as they are propagated. Shocks can also be propagated through prices due to common exposures or fire sales that go beyond the network of direct obligations, as in Allen and Gale (2000), Cifuentes, Ferrucci, and Shin (2005), and Caccioli et al. (2012).

## 7 Concluding Remarks

In this paper we have argued that it is relatively difficult to generate contagion solely through spillover losses in a network of payment obligations. For contagion to occur, a shock to one node must lead to losses at another set of nodes sufficiently large to wipe out their initial equity (or net worth). This means that either the initial shock must be large (and therefore improbable) relative to the net worth of the infecting node, or that the net worth of the infected nodes must be small. More generally, the probability of contagion depends on the comparative net worths of the various nodes, their level of leverage, and the extent to which the infecting node has obligations to the rest of the financial sector. As Theorems 1 and 2 show, one can compare the probability of contagion and direct default under a wide range of shock distributions without knowing the topological details of the network.

The network structure matters more for the amplification effect, in which losses among defaulting nodes multiply because of their obligations to one another. The degree of amplification is captured by the concept of node depth, which is the expected number of periods it takes to exit from the default set from a given starting point. This clearly depends on the specific structure of the interbank obligations, and is dual to the concept of eigenvector centrality in the networks literature. As Theorem 3 shows one can place an upper bound on the amplification effect based on banks' maximum connectedness with the

rest of the financial system (the parameter  $\beta^+$ ) without knowing any details of the connections within the system.

The network structure takes on added importance for both contagion and amplification once we introduce bankruptcy costs and mark-to-market reductions in credit quality. Bankruptcy costs steepen the losses at defaulted nodes, thereby increasing the likelihood that defaults will spread to other nodes. These losses are further amplified by feedback effects, thus increasing the system-wide loss in value. By contrast, reductions in credit quality have the effect of marking down asset values in advance of default. This process is akin to a slippery slope: once some node suffers a deterioration in its balance sheet, its mark-to-market value decreases, which reduces the value of the nodes to which it has obligations, causing their balance sheets to deteriorate. The result can be a system-wide reduction in value that was triggered solely by a loss of confidence rather than an actual default.

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## Appendix: Application to European Banks

To provide some insight into how our theoretical results apply in practice, we draw on data from banks that participated in the European Banking Authority’s (EBA) 2011 stress test. Detailed information on interbank exposures needed to calibrate a full network model is not publicly available. But our results do not depend on detailed network structure, so the information disclosed with the results of the stress test give us most of what we need.

Ninety banks in 21 countries participated in the stress test. For each, the EBA reports total assets and equity values as of the end of 2010. In addition, the EBA reports each bank’s total exposure at default (EAD) to other financial institutions. The EAD measures a bank’s total claims on all other banks, so we take this as the size of each bank’s in-network assets. Subtracting this value from total assets gives us  $c_i$ , the size of the bank’s outside assets. For  $w_i$  we use the equity values reported by the EBA, which then allows us to calculate  $\lambda_i = c_i/w_i$ .

The only remaining parameter we need is  $\beta_i$ , the fraction of a bank’s liabilities owed to other banks. This information is not included in the EBA summary, nor is it consistently reported by banks in their financial statements. As a rough indication, we assume that each bank’s in-network liabilities equal its in-network assets (though we will see that our results are fairly robust to this assumption).<sup>14</sup> This gives us  $\beta_i = \text{EAD}/(\text{assets-equity})$ .

Some of the smallest banks have problematic data, so as a simple rule we omit the ten smallest. We also omit any countries with only a single participating bank. This leaves us with 76 banks, of which the 50 largest are included in Table 1. For all 76 banks, Table 2 includes assets, EAD, and equity as reported by the EBA (in millions of euros), and our derived values for  $c_i, w_i, \beta_i, \lambda_i$ . The banks are listed by asset size in Table 1 and grouped by country in Table 2.

In Table 1, we examine the potential for contagion from the failure of each of the five largest banks, BNP Paribas, Deutsche Bank, HSBC, Barclays, and Credit Agricole. Taking each of these in turn as the triggering bank, we then take the default set  $D$  to be consecutive pairs of banks. The first default set under BNP Paribas consists of Deutsche Bank and HSBC, the next default set consists of HSBC and Barclays, and so on.

Under “WR” (weak ratio), we report the ratio of the left side of inequality (20) to the right side. Contagion is weak whenever this ratio exceeds 1, as it does in most cases in the table. Contagion fails to be weak only when the banks in the default set are much smaller than the triggering bank. Moreover, the ratio reported for each bank shows how much greater  $\beta_i$  would have to be to reverse the direction of inequality (20). For example, the first ratio listed under BNP Paribas, corresponding to the default

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<sup>14</sup>Based on Federal Reserve Release H.8, the average value of  $\beta_i$  for commercial banks in the U.S. is about 3%, so our estimates for European banks would appear to be conservative.

of Deutsche Bank and HSBC, is 18.64, based on a  $\beta_i$  value of 4.6% (see Table 2). This tells us that the  $\beta_i$  value would have to be at least  $18.64 \times 4.6\% = 85.7\%$  for the weak contagion condition to be violated. In this sense, the overall pattern in Table 1 is robust to our estimated values of  $\beta_i$ . Expanding the default sets generally makes contagion weaker because of the relative magnitudes of  $w_i$  and  $\lambda_i^{-1}$ ; see (20) and Table 2.

Under “LR” (for likelihood ratio), we report the relative probability of failure through independent direct shocks and through contagion, calculated as the ratio of the right side of (22) to the left side. This is the ratio of probabilities under the assumption of a uniform distribution  $p = q = 1$ , which is conservative. An asterisk indicates that the ratio is infinite because default through contagion is impossible.<sup>15</sup> The value of 6.68 reported under BNP Paribas for the default set consisting of Banco Santander and Societe Generale indicates that the probability of default through independent shocks is 6.68 times more likely than default through contagion. Raising the LR values in the table to a power of  $q > 1$  gives the corresponding ratio of probabilities under a shock distribution having parameters  $p = 1$  and  $q$ .<sup>16</sup>

Table 2 reports a similar analysis by country. Within each country, we first consider the possibility that the failure of a bank causes the next two largest banks to default – the next two largest banks constitute the default set  $D$ . (In the case of Belgium, there is only one bank to include in  $D$ .) For each bank, the column labeled “Ratio for Next Two Banks” reports the ratio of the left side of inequality (20) to the right side. As in Table 1, contagion is weak whenever this ratio exceeds 1. The table shows that the ratio is greater than 1 in every case. As in Table 1, the magnitude of the ratio also tells us how much larger  $\beta_i$  would have to be to reverse the inequality in (20).

The last column of the table reports the corresponding test for weak contagion but now holding  $D$  fixed as the two smallest banks in each country group. We now see several cases in which the ratio is less than 1 – for example, when we take Deutsche Bank to be the triggering bank and the two smallest banks in the German group (Landesbank Berlin and DekaBank Deutsch Girozentrale) as the default set we get a ratio of 0.5 in the last column.

The results in Tables 1 and 2 reflect the observations we made following Theorem 1 about the relative magnitudes needed for the various model parameters in order that contagion not be weak, and they show that the parameter ranges implying weak contagion are indeed meaningful in practice.

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<sup>15</sup>In deriving (32), we use the bound in (38), which is conservative. Thus, LR must be greater than 1 whenever WR is, and WR can be finite even when LR is infinite and default through contagion is impossible.

<sup>16</sup>We noted previously that a beta distribution can be used to approximate the Gaussian copula loss distribution used in Basel capital standards. We find that the best fit occurs at values of  $q$  around 20 or larger. Raising the likelihood ratios in the table to the  $q$ th power thus has a major impact.

|                                      | From BNP Paribas |       | From Deutsche Bank |      | From HSBC |      | From Barclays |      | From Credit Agricole |       |
|--------------------------------------|------------------|-------|--------------------|------|-----------|------|---------------|------|----------------------|-------|
|                                      | WR               | LR    | WR                 | LR   | WR        | LR   | WR            | LR   | WR                   | LR    |
| BNP PARIBAS                          |                  |       |                    |      |           |      |               |      |                      |       |
| DEUTSCHE BANK AG                     |                  |       |                    |      |           |      |               |      |                      |       |
| HSBC HOLDINGS plc                    | 18.64            | *     |                    |      |           |      |               |      |                      |       |
| BARCLAYS plc                         | 18.63            | *     | 9.21               | 3.89 |           |      |               |      |                      |       |
| CREDIT AGRICOLE                      | 17.83            | *     | 8.81               | 2.01 | 8.27      | 1.88 | 27.23         |      |                      |       |
| BANCO SANTANDER S.A.                 | 14.91            | *     | 7.37               | 1.98 | 6.92      | 1.85 | 24.62         | *    |                      |       |
| SOCIETE GENERALE                     | 12.52            | 6.68  | 6.19               | 1.62 | 5.81      | 1.55 | 20.67         | *    | 13.66                | 15.32 |
| LLOYDS BANKING GROUP plc             | 11.23            | 7.70  | 5.55               | 1.63 | 5.21      | 1.56 | 18.55         | *    | 12.26                | 23.52 |
| BPCE                                 | 11.29            | 12.77 | 5.58               | 1.70 | 5.24      | 1.62 | 18.65         | *    | 12.32                | *     |
| ING BANK NV                          | 10.32            | 3.44  | 5.10               | 1.46 | 4.79      | 1.41 | 17.05         | *    | 11.27                | 4.57  |
| UNICREDIT S.p.A                      | 9.55             | 4.06  | 4.72               | 1.49 | 4.43      | 1.44 | 15.77         | *    | 10.42                | 5.90  |
| COMMERZBANK AG                       | 8.47             | 3.33  | 4.18               | 1.43 | 3.93      | 1.38 | 13.98         | *    | 9.24                 | 4.39  |
| RABOBANK NEDERLAND                   | 6.96             | 2.48  | 3.44               | 1.33 | 3.23      | 1.29 | 11.49         | *    | 7.59                 | 2.94  |
| ROYAL BANK OF SCOTLAND GROUP plc     | 6.06             | *     | 2.99               | 1.67 | 2.81      | 1.58 | 10.01         | *    | 6.61                 | *     |
| INTESA SANPAOLO S.p.A                | 5.70             | 72.67 | 2.82               | 1.63 | 2.64      | 1.54 | 9.41          | *    | 6.22                 | *     |
| DEXIA                                | 4.59             | 1.79  | 2.27               | 1.19 | 2.13      | 1.16 | 7.58          | 5.19 | 5.01                 | 1.97  |
| Nordea Bank AB (publ)                | 4.52             | 1.57  | 2.23               | 1.15 | 2.10      | 1.13 | 7.46          | 2.96 | 4.93                 | 1.68  |
| BANCO BILBAO VIZCAYA ARGENTARIA      | 5.24             | 1.85  | 2.59               | 1.21 | 2.43      | 1.19 | 8.65          | 5.82 | 5.72                 | 2.05  |
| DANSKE BANK                          | 4.47             | 1.66  | 2.21               | 1.16 | 2.08      | 1.14 | 7.39          | 3.72 | 4.88                 | 1.80  |
| ABN AMRO BANK NV                     | 3.91             | 1.33  | 1.93               | 1.09 | 1.81      | 1.08 | 6.46          | 1.85 | 4.27                 | 1.38  |
| Landesbank Baden-Württemberg         | 3.36             | 1.23  | 1.66               | 1.06 | 1.56      | 1.05 | 5.55          | 1.57 | 3.67                 | 1.27  |
| Hypo Real Estate Holding AG, Münche  | 3.00             | 1.15  | 1.48               | 1.03 | 1.39      | 1.03 | 4.96          | 1.34 | 3.28                 | 1.17  |
| BFA-BANKIA                           | 3.38             | 1.21  | 1.67               | 1.05 | 1.57      | 1.04 | 5.59          | 1.49 | 3.69                 | 1.24  |
| DZ BANK AG Dt. Zentral-Genossenschaf | 2.83             | 1.21  | 1.40               | 1.04 | 1.31      | 1.03 | 4.67          | 1.54 | 3.08                 | 1.25  |
| Bayerische Landesbank                | 2.39             | 1.16  | 1.18               | 1.02 | 1.11      | 1.01 | 3.94          | 1.42 | 2.61                 | 1.20  |
| KBC BANK                             | 2.73             | 1.24  | 1.35               | 1.04 | 1.27      | 1.03 | 4.50          | 1.63 | 2.98                 | 1.28  |
| CAJA DE AHORROS Y PENSIONES DE BA    | 3.03             | 1.24  | 1.50               | 1.05 | 1.41      | 1.04 | 5.01          | 1.63 | 3.31                 | 1.29  |
| BANCA MONTE DEI PASCHI DI SIENA S    | 2.96             | 1.17  | 1.46               | 1.04 | 1.38      | 1.03 | 4.89          | 1.40 | 3.23                 | 1.20  |
| Svenska Handelsbanken AB (publ)      | 2.61             | 1.13  | 1.29               | 1.02 | 1.21      | 1.02 | 4.31          | 1.30 | 2.85                 | 1.15  |
| Norddeutsche Landesbank -GZ          | 2.13             | 1.09  | 1.05               | 1.00 | 0.99      | 1.00 | 3.52          | 1.22 | 2.33                 | 1.11  |
| Skandinaviska Enskilda Banken AB (pu | 1.96             | 1.09  | 0.97               | 1.00 | 0.91      | 0.99 | 3.24          | 1.25 | 2.14                 | 1.11  |
| DnB NOR Bank ASA                     | 2.26             | 1.16  | 1.12               | 1.02 | 1.05      | 1.01 | 3.73          | 1.44 | 2.46                 | 1.20  |
| Erste Bank Group (EBG)               | 2.22             | 1.17  | 1.10               | 1.01 | 1.03      | 1.01 | 3.66          | 1.47 | 2.42                 | 1.21  |
| WestLB AG, Düsseldorf                | 1.91             | 1.10  | 0.94               | 1.00 | 0.88      | 0.99 | 3.15          | 1.27 | 2.08                 | 1.12  |
| Swedbank AB (publ)                   | 1.82             | 1.07  | 0.90               | 0.99 | 0.84      | 0.99 | 3.00          | 1.19 | 1.98                 | 1.09  |
| Nykredit                             | 1.98             | 1.10  | 0.98               | 1.00 | 0.92      | 0.99 | 3.27          | 1.26 | 2.16                 | 1.12  |
| BANK OF IRELAND                      | 1.76             | 1.08  | 0.87               | 0.99 | 0.82      | 0.98 | 2.91          | 1.24 | 1.92                 | 1.10  |
| HSH Nordbank AG, Hamburg             | 1.63             | 1.06  | 0.80               | 0.98 | 0.76      | 0.98 | 2.69          | 1.18 | 1.78                 | 1.08  |
| BANCO POPOLARE - S.C.                | 1.58             | 1.05  | 0.78               | 0.98 | 0.73      | 0.98 | 2.61          | 1.15 | 1.73                 | 1.06  |
| Landesbank Berlin AG                 | 1.21             | 1.03  | 0.60               | 0.96 | 0.56      | 0.95 | 1.99          | 1.13 | 1.32                 | 1.04  |
| ALLIED IRISH BANKS PLC               | 1.12             | 1.01  | 0.55               | 0.96 | 0.52      | 0.96 | 1.85          | 1.10 | 1.22                 | 1.03  |
| Raiffeisen Bank International (RBI)  | 1.23             | 1.03  | 0.61               | 0.96 | 0.57      | 0.95 | 2.04          | 1.14 | 1.35                 | 1.05  |
| UNIONE DI BANCHE ITALIANE SCPA (U    | 1.22             | 1.04  | 0.60               | 0.95 | 0.57      | 0.94 | 2.02          | 1.19 | 1.33                 | 1.06  |
| DekaBank Deutsche Girozentrale, Fran | 1.19             | 1.02  | 0.59               | 0.96 | 0.55      | 0.96 | 1.96          | 1.12 | 1.30                 | 1.04  |
| BANCO POPULAR ESPAÑOL, S.A.          | 1.21             | 1.03  | 0.60               | 0.96 | 0.56      | 0.96 | 2.00          | 1.12 | 1.32                 | 1.04  |
| CAIXA GERAL DE DEPÓSITOS, SA         | 1.27             | 1.04  | 0.63               | 0.96 | 0.59      | 0.95 | 2.10          | 1.18 | 1.39                 | 1.06  |
| NATIONAL BANK OF GREECE              | 1.25             | 1.05  | 0.62               | 0.95 | 0.58      | 0.94 | 2.07          | 1.21 | 1.37                 | 1.07  |
| BANCO COMERCIAL PORTUGUÊS, SA (f     | 1.21             | 1.03  | 0.60               | 0.95 | 0.56      | 0.95 | 2.00          | 1.15 | 1.32                 | 1.05  |
| BANCO DE SABADELL, S.A.              | 1.08             | 1.01  | 0.53               | 0.96 | 0.50      | 0.96 | 1.78          | 1.07 | 1.17                 | 1.02  |
| EFG EUROBANK ERGASIAS S.A.           | 1.01             | 1.00  | 0.50               | 0.95 | 0.47      | 0.95 | 1.66          | 1.07 | 1.10                 | 1.01  |
| ESPÍRITO SANTO FINANCIAL GROUP, S    | 0.92             | 0.99  | 0.46               | 0.94 | 0.43      | 0.94 | 1.52          | 1.07 | 1.01                 | 1.00  |

Table 1: Results based on 2011 EBA stress test data. We consider contagion from each of the five largest banks across the top to pairs of banks in consecutive rows. A weak ratio (WR) value greater than 1 indicates that contagion is weak. Each LR value is a likelihood ratio for default through independent shocks to default through contagion, assuming a uniform fractional shock.

|       |   |           |         | c_i            | w_i    | beta_i              | lambda_i              |                          |                          |
|-------|---|-----------|---------|----------------|--------|---------------------|-----------------------|--------------------------|--------------------------|
|       | Bank Number and Name                    | Assets    | EAD     | Outside Assets | Equity | EAD/(Assets-Equity) | Outside Assets/Equity | Ratio for Next Two Banks | Ratio for Last Two Banks |
| AT001 | Erste Bank Group (EBG)                  | 205,938   | 25,044  | 180,894        | 10,507 | 12.8%               | 17.2                  |                          |                          |
| AT002 | Raiffeisen Bank International (RBI)     | 131,173   | 30,361  | 100,812        | 7,641  | 24.6%               | 13.2                  |                          |                          |
| AT003 | Oesterreichische Volksbank AG           | 44,745    | 10,788  | 33,957         | 1,765  | 25.1%               | 19.2                  | 3.4                      |                          |
| BE004 | DEXIA                                   | 548,135   | 228,211 | 319,924        | 17,002 | 43.0%               | 18.8                  |                          |                          |
| BE005 | KBC BANK                                | 276,723   | 23,871  | 252,852        | 11,705 | 9.0%                | 21.6                  | 35.7                     |                          |
| CY006 | MARFIN POPULAR BANK PUBLIC CO LTD       | 42,580    | 7,907   | 34,673         | 2,015  | 19.5%               | 17.2                  |                          |                          |
| CY007 | BANK OF CYPRUS PUBLIC CO LTD            | 41,996    | 7,294   | 34,702         | 2,134  | 18.3%               | 16.3                  | 1.6                      |                          |
| DE017 | DEUTSCHE BANK AG                        | 1,905,630 | 194,399 | 1,711,231      | 30,361 | 10.4%               | 56.4                  |                          | 0.5                      |
| DE018 | COMMERZBANK AG                          | 771,201   | 138,190 | 633,011        | 26,728 | 18.6%               | 23.7                  |                          | 0.8                      |
| DE019 | Landesbank Baden-Württemberg            | 374,413   | 133,906 | 240,507        | 9,838  | 36.7%               | 24.4                  | 2.5                      | 1.0                      |
| DE020 | DZ BANK AG Dt. Zentral-Genossenschaf    | 323,578   | 135,860 | 187,718        | 7,299  | 43.0%               | 25.7                  | 1.9                      | 1.1                      |
| DE021 | Bayerische Landesbank                   | 316,354   | 97,336  | 219,018        | 11,501 | 31.9%               | 19.0                  | 2.4                      | 1.3                      |
| DE022 | Norddeutsche Landesbank -GZ             | 228,586   | 91,217  | 137,369        | 3,974  | 40.6%               | 34.6                  | 2.5                      | 1.6                      |
| DE023 | Hypo Real Estate Holding AG, Münche     | 328,119   | 29,084  | 299,035        | 5,539  | 9.0%                | 54.0                  | 3.0                      | 3.3                      |
| DE024 | WestLB AG, Düsseldorf                   | 191,523   | 58,128  | 133,395        | 4,218  | 31.0%               | 31.6                  | 3.6                      | 2.2                      |
| DE025 | HSH Nordbank AG, Hamburg                | 150,930   | 9,532   | 141,398        | 4,434  | 6.5%                | 31.9                  | 5.2                      | 9.7                      |
| DE027 | Landesbank Berlin AG                    | 133,861   | 49,253  | 84,608         | 5,162  | 38.3%               | 16.4                  | 2.6                      |                          |
| DE028 | DekaBank Deutsche Girozentrale, Fran    | 130,304   | 41,255  | 89,049         | 3,359  | 32.5%               | 26.5                  | 9.7                      |                          |
| DK008 | DANSKE BANK                             | 402,555   | 75,894  | 326,661        | 14,576 | 19.6%               | 22.4                  |                          | 0.4                      |
| DK011 | Nykredit                                | 175,888   | 8,597   | 167,291        | 6,633  | 5.1%                | 25.2                  |                          | 2.7                      |
| DK009 | Jyske Bank                              | 32,752    | 4,674   | 28,078         | 1,699  | 15.1%               | 16.5                  | 1.4                      |                          |
| DK010 | Sydbank                                 | 20,238    | 3,670   | 16,568         | 1,231  | 19.3%               | 13.5                  | 2.7                      |                          |
| ES059 | BANCO SANTANDER S.A.                    | 1,223,267 | 51,407  | 1,171,860      | 41,998 | 4.4%                | 27.9                  |                          | 0.4                      |
| ES060 | BANCO BILBAO VIZCAYA ARGENTARIA S       | 540,936   | 110,474 | 430,462        | 24,939 | 21.4%               | 17.3                  |                          | 0.2                      |
| ES061 | BFA-BANKIA                              | 327,930   | 39,517  | 288,414        | 13,864 | 12.6%               | 20.8                  | 7.4                      | 0.6                      |
| ES062 | CAJA DE AHORROS Y PENSIONES DE BA       | 275,856   | 5,510   | 270,346        | 11,109 | 2.1%                | 24.3                  | 3.2                      | 3.6                      |
| ES064 | BANCO POPULAR ESPAÑOL, S.A.             | 129,183   | 14,810  | 114,373        | 6,699  | 12.1%               | 17.1                  | 5.2                      | 1.5                      |
| ES065 | BANCO DE SABADELL, S.A.                 | 96,703    | 3,678   | 93,025         | 3,507  | 3.9%                | 26.5                  | 19.6                     | 5.5                      |
| ES066 | CAIXA D'ESTALVIS DE CATALUNYA, TAR      | 76,014    | 8,219   | 67,795         | 3,104  | 11.3%               | 21.8                  | 6.1                      | 2.7                      |
| ES067 | CAIXA DE AFORROS DE GALICIA, VIGO,      | 73,319    | 2,948   | 70,371         | 2,849  | 4.2%                | 24.7                  | 19.5                     | 6.9                      |
| ES083 | CAJA DE AHORROS DEL MEDITERRANE         | 72,034    | 4,981   | 67,053         | 1,843  | 7.1%                | 36.4                  | 9.5                      | 4.2                      |
| ES071 | GRUPO BANCA CIVICA                      | 71,055    | 7,419   | 63,636         | 3,688  | 11.0%               | 17.3                  | 22.9                     | 3.0                      |
| ES068 | GRUPO BMN                               | 69,760    | 7,660   | 62,101         | 3,304  | 11.5%               | 18.8                  | 13.6                     | 2.9                      |
| ES063 | EFFIBANK                                | 54,523    | 4,124   | 50,399         | 2,656  | 8.0%                | 19.0                  | 8.5                      | 5.2                      |
| ES069 | BANKINTER, S.A.                         | 53,476    | 2,141   | 51,335         | 1,920  | 4.2%                | 26.7                  | 7.5                      | 9.5                      |
| ES070 | CAJA ESPAÑA DE INVERSIONES, SALAM       | 45,656    | 7,235   | 38,422         | 2,076  | 16.6%               | 18.5                  | 11.5                     | 3.2                      |
| ES075 | GRUPO BBK                               | 44,628    | 1,924   | 42,704         | 2,982  | 4.6%                | 14.3                  | 19.9                     | 10.7                     |
| ES072 | CAJA DE AHORROS Y M.P. DE ZARAGOZ       | 42,716    | 1,779   | 40,938         | 2,299  | 4.4%                | 17.8                  | 6.9                      | 11.5                     |
| ES073 | MONTE DE PIEDAD Y CAJA DE AHORRO        | 34,263    | 2,599   | 31,664         | 2,501  | 8.2%                | 12.7                  | 19.4                     | 8.2                      |
| ES074 | BANCO PASTOR, S.A.                      | 31,135    | 1,665   | 29,470         | 1,395  | 5.6%                | 21.1                  | 18.1                     | 12.5                     |
| ES076 | CAIXA D'ESTALVIS UNIO DE CAIXES DE I    | 28,310    | 1,802   | 26,508         | 1,065  | 6.6%                | 24.9                  | 11.8                     | 11.6                     |
| ES077 | CAJA DE AHORROS Y M.P. DE GIPUZKO       | 20,786    | 285     | 20,501         | 1,935  | 1.5%                | 10.6                  | 14.2                     |                          |
| ES078 | GRUPO CAJAS                             | 20,144    | 1,926   | 18,218         | 1,164  | 10.1%               | 15.7                  | 11.6                     |                          |
| FR013 | BNP PARIBAS                             | 1,998,157 | 90,328  | 1,907,829      | 55,352 | 4.6%                | 34.5                  |                          | 11.1                     |
| FR014 | CREDIT AGRICOLE                         | 1,503,621 | 83,713  | 1,419,908      | 46,277 | 5.7%                | 30.7                  |                          | 12.2                     |
| FR016 | SOCIETE GENERALE                        | 1,051,323 | 100,013 | 951,310        | 27,824 | 9.8%                | 34.2                  | 13.9                     |                          |
| FR015 | BPCE                                    | 1,000,695 | 34,983  | 965,712        | 31,943 | 3.6%                | 30.2                  | 12.2                     |                          |
| GB089 | HSBC HOLDINGS plc                       | 1,783,199 | 212,092 | 1,571,107      | 86,900 | 12.5%               | 18.1                  |                          | 3.5                      |
| GB090 | BARCLAYS plc                            | 1,725,709 | 53,873  | 1,671,836      | 46,232 | 3.2%                | 36.2                  |                          | 12.3                     |
| GB091 | LLOYDS BANKING GROUP plc                | 1,006,082 | 29,233  | 976,849        | 47,984 | 3.1%                | 20.4                  | 6.6                      |                          |
| GB088 | ROYAL BANK OF SCOTLAND GROUP plc        | 607,351   | 105,506 | 501,845        | 58,982 | 19.2%               | 8.5                   | 12.3                     |                          |
| GR031 | NATIONAL BANK OF GREECE                 | 118,832   | 8,608   | 110,224        | 8,153  | 7.8%                | 13.5                  |                          | 6.0                      |
| GR030 | EGF EUROBANK ERGASIAS S.A.              | 85,885    | 3,838   | 82,047         | 4,296  | 4.7%                | 19.1                  |                          | 12.9                     |
| GR032 | ALPHA BANK                              | 66,798    | 3,492   | 63,306         | 5,275  | 5.7%                | 12.0                  | 8.9                      | 14.4                     |
| GR033 | PIRAEUS BANK GROUP                      | 57,680    | 1,581   | 56,099         | 3,039  | 2.9%                | 18.5                  | 16.5                     |                          |
| GR034 | AGRICULTURAL BANK OF GREECE S.A. (      | 31,221    | 1,657   | 29,564         | 792    | 5.4%                | 37.3                  | 14.4                     |                          |
| IE038 | BANK OF IRELAND                         | 156,712   | 17,254  | 139,458        | 7,037  | 11.5%               | 19.8                  |                          | 4.9                      |
| IE037 | ALLIED IRISH BANKS PLC                  | 131,311   | 11,277  | 120,034        | 3,669  | 8.8%                | 32.7                  |                          |                          |
| IE039 | IRISH LIFE AND PERMANENT                | 46,743    | 6,127   | 40,616         | 1,681  | 13.6%               | 24.2                  | 4.9                      |                          |
| IT041 | UNICREDIT S.p.A                         | 929,488   | 106,707 | 822,781        | 35,702 | 11.9%               | 23.0                  |                          | 1.3                      |
| IT040 | INTESA SANPAOLO S.p.A                   | 576,962   | 109,909 | 467,053        | 26,159 | 20.0%               | 17.9                  |                          | 1.4                      |
| IT042 | BANCA MONTE DEI PASCHI DI SIENA S.      | 244,279   | 12,074  | 232,205        | 6,301  | 5.1%                | 36.9                  | 4.2                      | 10.4                     |
| IT043 | BANCO POPOLARE - S.C.                   | 140,043   | 7,602   | 132,440        | 5,474  | 5.6%                | 24.2                  | 2.0                      |                          |
| IT044 | UNIONE DI BANCHE ITALIANE SCPA (UE      | 130,559   | 19,793  | 110,766        | 6,559  | 16.0%               | 16.9                  | 10.4                     |                          |
| NL047 | ING BANK NV                             | 933,073   | 111,756 | 821,317        | 30,895 | 12.0%               | 26.6                  |                          | 2.5                      |
| NL048 | RABOBANK NEDERLAND                      | 607,483   | 37,538  | 569,945        | 27,725 | 6.2%                | 20.6                  |                          | 7.2                      |
| NL049 | ABN AMRO BANK NV                        | 379,599   | 29,196  | 350,403        | 11,574 | 7.7%                | 30.3                  | 5.1                      |                          |
| NL050 | SNS BANK NV                             | 78,918    | 388     | 78,530         | 1,782  | 0.5%                | 44.1                  | 7.2                      |                          |
| PT053 | CAIXA GERAL DE DEPÓSITOS, SA            | 119,318   | 14,221  | 105,097        | 6,510  | 11.9%               | 16.1                  |                          | 5.0                      |
| PT054 | BANCO COMERCIAL PORTUGUÊS, SA (B        | 100,010   | 7,690   | 92,320         | 3,521  | 7.7%                | 26.2                  |                          | 8.5                      |
| PT055 | ESPÍRITO SANTO FINANCIAL GROUP, SA      | 85,644    | 8,690   | 76,955         | 4,520  | 10.1%               | 17.0                  | 7.1                      |                          |
| PT056 | Banco BPI, SA                           | 43,826    | 5,463   | 38,364         | 2,133  | 12.5%               | 18.0                  | 8.5                      |                          |
| SE084 | Nordea Bank AB (publ)                   | 542,853   | 61,448  | 481,405        | 19,103 | 11.3%               | 25.2                  |                          | 3.7                      |
| SE085 | Skandinaviska Enskilda Banken AB (publ) | 212,240   | 25,955  | 186,284        | 9,604  | 12.2%               | 19.4                  |                          | 9.0                      |
| SE086 | Svenska Handelsbanken AB (publ)         | 240,202   | 20,870  | 219,332        | 8,209  | 8.7%                | 26.7                  | 3.8                      |                          |
| SE087 | Swedbank AB (publ)                      | 191,365   | 17,358  | 174,007        | 7,352  | 9.1%                | 23.7                  | 9.0                      |                          |

Table 2: Results based on 2011 EBA stress test data. In the last two columns, a ratio greater than 1 indicates weak contagion from a bank to the next two largest banks in the same country and to the two smallest banks in the country group, respectively.