Commodities and International Business Cycles

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November 12, 2017

Abstract

I introduce commodities and different commodity trade structures of countries into an otherwise standard two-country model to analyse international business cycles between the U.S. and commodity-exporting countries. In the model, only the foreign country (commodity-exporting country) produces commodities and exports them to the home country (the U.S., commodity-importing country). The model produces closer international business cycle statistics to the data than a standard model. In particular, the output correlation between the two countries increases and the consumption correlation falls compared to a standard model. Notably, unlike standard models, the model yields output correlation that exceeds the consumption correlation, which mitigates the “consumption correlation puzzle” previously noted in the literature. Commodity consumption and complementarity between commodities and noncommodity goods in consumption play key roles in generating this result.

1. Introduction

The existing international business cycle literature does not specifically consider the role of commodities trade in international business cycles, despite the fact that commodity prices are a critical source of economic fluctuations. To address this shortcoming in the literature, I augment a standard international business cycle model with commodities and idiosyncratic commodity trade structures of countries. The model generates international business cycle statistics closer to data than a standard model by increasing the output correlation between the two countries and decreasing the consumption correlation between them. Furthermore,

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I am very grateful to my supervisors, Andrea Ferrero and David Vines, for their continuous support and guidance. I also thank Xiyu Jiao, Carlo Pizzinelli, Kenneth Rogoff, Segye Shin, Rick van der Ploeg and seminar participants at the University of Oxford and the Korea Energy Economics Institute for helpful comments. All errors are my own.
the model yields higher output correlation than consumption correlation, which is consistent with the ranking in the data. On the contrary, standard models typically produce the opposite prediction, which is referred in the literature as the “consumption correlation puzzle.” Therefore, adding commodities with consideration for different commodity trade structures of countries helps mitigate the consumption correlation puzzle.

Commodities have peculiar features compared to other tradable goods. They are used as inputs in production in almost every sector as well as consumed by households. Other tradable goods, however, are typically only consumed or only used as inputs in production in a specific sector. Due to these features of commodities, changes in commodity prices affect both households and firms in the economy, which means that commodity prices have more widespread effects on the economy than prices of other tradable goods. Thus, macroeconomists have regarded changes in commodity prices as a critical source of economic fluctuations and one that can affect many countries simultaneously (Blanchard and Galí (2007)). Furthermore, each country has its own idiosyncratic commodity trade structure. For instance, countries like the U.S. are net commodity-importing countries, and countries like Canada are net commodity-exporting countries. Therefore, each country will be affected by the same shock in a different way. As a result, several papers studying North-South interactions such as Moutos and Vines (1989) have already emphasized that to properly capture macroeconomic interactions between countries in open economy macro models, the asymmetries of trade and production resulting from commodities should be considered.

As an illustration, output growth triggered by a certain shock in the U.S., a commodity-importing country, will increase its demand for imports of commodities for production and consumption. Accordingly, commodity prices will go up. The increased demand for imports of commodities and the higher prices will have positive influences on the economies of commodity-exporting countries. Conversely, the increased commodity prices will adversely affect the economies of commodity-importing countries. Hence, U.S. GDP would be expected to have stronger positive correlations with GDPs of commodity-exporting countries than those of commodity-importing countries. There are some studies in line with this hypothesis. For instance, using a two-country DSGE model, Bodenstein, Erceg and Guerrieri (2011) conclude that an increase in oil price due to an oil demand or supply shock brings

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1 Alternatively, the “quantity anomaly” or the “BKK puzzle.”
2 For example, energy is not only consumed but also used as inputs in all sectors. On the contrary, jeans are consumed, but not used in production (they can be used in production, but very rarely). Semiconductors are only used in production in the electronic sector, and not consumed by households.
3 During 2006-2011 the average ratio of net commodity exports to GDP in the U.S. and Canada were -7.5% and 26.0%, respectively, according to OECD Trade in Value Added database. Following IMF’s commodity price index, this paper defines the commodity sector as agriculture, hunting, forestry, fishing, mining, quarrying, wood and basic metal industries.
4 In these models, the “Northern” economies (i.e. the developed countries) produce industrial goods, whilst the “Southern” economies (i.e. the developing countries) produce commodities.
about a wealth transfer toward the oil-exporter and a fall in output in the oil-importer. Chen, Filardo, He and Zhu (2012) and Georgiadis (2016) also empirically show that an expansionary monetary policy shock in the U.S. raises the U.S. output and has more positive effects on the commodity-exporting countries or regions than on the commodity-importing countries or regions.\(^5\)

In accordance with this hypothesis, the U.S. has higher positive GDP correlations with commodity-exporting countries than with commodity-importing countries in the data (see Section 2 for details). It may therefore be reasonable to analyse U.S. international business cycles with commodity-exporting countries and those with commodity-importing countries separately. Surprisingly, however, international business cycles between the U.S. and commodity-exporting countries have not specifically been analysed in the international business cycle literature, although they are significantly different from those between the U.S. and commodity-importing countries.

To analyse international business cycles between the U.S. and commodity-exporting countries, this paper introduces commodities and different commodity trade structures into the workhorse model with nontradable goods: the two-country, two-sector RBC model of Stockman and Tesar (1995), henceforth ST. The primary differences of the model from the ST model\(^6\) are that only the foreign country produces commodities, that commodities are used as an input in the production of tradable goods, and that commodities are consumed as a complement to noncommodity consumption goods.\(^7\)

The main goal of this paper is to examine whether the model can replicate main properties of business cycle comovements between the U.S. and commodity-exporting countries well with focus on output and consumption correlations; (i) to show that the model yields higher output correlation between the two countries than the ST model and (ii) that the model produces higher output correlation than consumption correlation (mitigates the consumption correlation puzzle). Introducing commodities with consideration for idiosyncratic commodity

\(^5\) The former uses a global vector error-correction model and the latter uses a global vector autoregressive model.

\(^6\) Almost all features of the ST model are the same as the model of Stockman and Tesar (1995). The utility function of the households in the ST model is different from that in Stockman and Tesar (1995). The economy in Stockman and Tesar (1995) grows at a constant rate, while there is no growth in the ST model.

\(^7\) The inventory or storage of commodities is not considered in the model. If commodities can be stored, the stored commodities can be used to meet the increased demand for commodities triggered by a positive shock to the U.S., which will lead to a fall in inventory of commodities. Nevertheless, the shock will increase commodity prices due to the increased demand. However, the positive effects on commodity-exporting countries will be slightly weaker than would be the case without the inventory, because in the case without the inventory, the U.S. will import commodities from commodity-exporting countries, which will also positively affect commodity-exporting countries. Some empirical studies such as Kilian and Murphy (2014) show that oil inventory changes have no significant effect on oil prices suggesting that omitting inventories may not matter greatly. Furthermore, if the U.S. has to refill the decreased inventories of commodities soon after using them to meet the increased demand for commodities, inventories will not make a significant difference, since the U.S. will import commodities from commodity-exporting countries to refill the inventories.
trade structures of countries into the ST model can indeed help strengthen the connection between outputs in the two countries and lower the consumption correlation, and hence mitigate the consumption correlation puzzle. In the model there is complementarity between commodities and noncommodity goods in consumption; this complementarity together with commodity consumption play important roles in my analysis.8

I now summarize my findings. As for the output correlation, positive productivity shocks in the home country bring about rises in the home country’s aggregate output and its demand for imports of foreign tradable goods and commodities. Thus, the real prices of foreign tradable goods and commodities increase. The rise in home demand for imports of foreign tradable goods is offset by their increased real prices. However, this substitution effect for commodities is muted because of the complementarity between commodities and noncommodity goods in consumption. Therefore, there is a larger positive spillover to the foreign country, which raises the output correlation.

Commodity consumption, which is not in the ST model, together with the complementarity plays a key role in lowering the consumption correlation and therefore mitigating the consumption correlation puzzle. When a positive productivity shock hits the home tradable sector, home commodity consumption and real commodity prices increase, and the real price of home tradable goods falls. Foreign noncommodity consumption rises thanks to a fall in the real price of home tradable goods. Despite the rise in real commodity prices, foreign commodity consumption goes up due to the complementarity. Thus, with respect to the shock, home and foreign commodity consumptions are positively correlated. On the contrary, in response to a positive productivity shock in the home nontradable sector, home aggregate output and commodity consumption rise. The real prices of commodity and tradable goods increase, since the increased home aggregate output puts upward pressure on them. Foreign noncommodity consumption drops due to the increased real price of tradable goods. Thus, foreign commodity consumption falls as well due to the complementarity and the increased real commodity prices. Accordingly, home and foreign commodity consumptions are negatively correlated with respect to the shock. As a result, the correlation between home and foreign commodity consumptions is lower than that between home and foreign noncommodity consumptions. Hence, the existence of commodity consumption in the model and the complementarity decrease the consumption correlation compared to the ST model, which enables the output correlation to be higher than the consumption correlation in the model, i.e. the model can mitigate the consumption correlation puzzle.

Among the various literatures on international business cycles, this paper is related to research that attempt to explain the discrepancies between the data and standard models’

8 There are many empirical studies showing that they are complement (see Cooper (2003), Bodenstein, Erceg and Guerrieri (2011), etc.).
predictions by adding new elements under productivity shocks. The first paper on interna-
tional business cycles is Backus, Kehoe and Kydland (1992), henceforth BKK. Since their
seminal work, a number of papers have introduced models that can explain the discrepan-
cies by adding non-traded goods (ST), informational frictions (Kehoe and Perri (2002)), the
distribution sector (Corsetti, Dedola and Leduc (2008), henceforth CDL), etc. Some papers
have tried to find factors causing the high output correlation between countries; similar finan-
cial market structures of countries (Faia (2007)), banking globalisation (Ueda (2012)), and
so on. Among these papers, this paper is closest to ST and CDL in which there are tradable
and nontradable sectors and sectoral productivity shocks are used in generating international
business cycle statistics.

The paper is organised as follows. Section 2 shows the main properties of business cycle
comovements between the U.S. and commodity-exporting countries. Section 3 describes the
model. Calibration is also presented in this section. Section 4 presents the model analysis. I
first show the business cycle statistics that the model produces. I then provide the responses
of the model to productivity shocks and relate them to the Dutch disease. Finally, I present
sensitivity analysis of the model to the newly introduced features in comparison with the ST
model. Section 5 concludes.

2. Properties of U.S. business cycle comovement

In this section, I first present the annual U.S. GDP correlations with GDPs of 60 major
countries to identify whether U.S. GDP correlations with GDPs of commodity-exporting
countries are significantly higher than those with GDPs of commodity-importing countries.
Then, I show international business cycle statistics between the U.S and commodity-exporting
OECD countries, and between the U.S. and commodity-importing OECD countries from
annual data of 35 OECD member countries for the period of 1990-2015, and briefly explain
the main properties of the former and the differences between the former and the latter with
focus on the output and consumption correlations.\footnote{See Appendix A for details of the data.}

2.1. U.S. GDP comovement with commodity-exporting countries

Since the U.S. is a large economy, changes in its demand for imports of commodities af-
fect commodity prices. Specifically, an increase in its output will increase its demand for
imports of commodities, and thus commodity prices increase. Rises in U.S. demand for
imports of commodities and commodity prices will have a positive effect on outputs in
commodity-exporting countries, whilst an increase in commodity prices will adversely af-
fect outputs in commodity-importing countries. Hence, we would expect that the U.S. will have stronger GDP comovement with commodity-exporting countries than with commodity-importing countries.

Figure 1: GDP correlations with U.S. GDP and ratios of net commodity exports to GDP

Notes: GDP correlations with U.S. GDP are calculated by using detrended annual GDPs during 1990-2015 with Hodrick-Prescott filter. The values of net commodity exports/GDP are average of the period 1995-2011. Brunei Darussalam and Saudi Arabia are omitted in this figure.

Sources: IMF and OECD

Figure 1 shows the relationship between U.S. GDP correlations with 60 major countries’ GDPs and the ratios of net commodity exports to GDP of the 60 countries. However, contrary to initial expectation, there seems to be no clear relationship between the two measures.

Since according to Zimmermann (1997), business cycle differences across countries can be explained by various factors such as countries’ sizes and distance between countries, to analyse whether U.S. GDP correlations with commodity-exporting countries are higher than those with commodity-importing countries I control for factors other than the ratio of net commodity exports to GDP that have influences on GDP correlations. To do this, I define two groups of countries: advanced countries like the U.S., and countries in Europe or North America that are culturally and geographically close to the U.S. Figure 2 clearly shows that there is a positive relationship between GDP correlations with U.S. GDP and the ratios of net commodity exports to GDP of these groups of countries. Within these groups, U.S. GDP comovements with commodity-exporting countries are stronger than those with commodity-importing countries.
Figure 2: GDP correlations with U.S. GDP and ratios of net commodity exports to GDP

Notes: GDP correlations with U.S. GDP are calculated by using detrended annual GDPs during 1990-2015 with Hodrick-Prescott filter. The values of net commodity exports/GDP are average of the period 1995-2011. Advanced countries are based on IMF’s classification. Russia and Turkey are not regarded as Europe here, since their territories are in both Europe and Asia and they are not geographically and culturally close to the U.S.
Sources: IMF and OECD

2.2. Main properties of U.S. business cycle comovement with commodity-exporting countries

Table 1 shows international business cycle statistics for the U.S.: the correlations between U.S. GDP, consumption and investment, and the same variables for other major countries, and commodity-exporting and commodity-importing OECD countries.

The correlations between U.S. GDP and GDPs in other countries are positive and high. Canada has very large correlations, 0.854. The correlation for Japan (0.408) and Germany (0.375) are relatively small. More importantly, the correlation between U.S. GDP and GDP of commodity-exporting OECD countries (0.738) is substantially higher than that between U.S. GDP and GDP of commodity-importing OECD countries (0.502).

U.S. consumption is also positively correlated with consumptions in other countries except Japan. However, the consumption correlations are lower than the GDP correlation. Specifically, the U.S. consumption correlations with consumptions of commodity-exporting (0.584) and commodity-importing (0.263) OECD countries are smaller than the U.S. GDP correlations with GDPs of commodity-exporting (0.738) and commodity-importing (0.502) OECD countries.
### Table 1: Correlations with same U.S. variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>OECD Exporting</th>
<th>OECD Importing</th>
<th>Commodity-exporting countries Exporting</th>
<th>Commodity-importing countries Exporting</th>
<th>Commodity-exporting countries Importing</th>
<th>Commodity-importing countries Importing</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.738</td>
<td>0.502</td>
<td>0.662</td>
<td>0.854</td>
<td>0.408</td>
<td>0.375</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.584</td>
<td>0.263</td>
<td>0.653</td>
<td>0.624</td>
<td>-0.163</td>
<td>0.100</td>
</tr>
<tr>
<td>Investment</td>
<td>0.471</td>
<td>0.513</td>
<td>-0.098</td>
<td>0.634</td>
<td>0.687</td>
<td>0.340</td>
</tr>
</tbody>
</table>

*Notes:* Statistics are computed from the detrended data with Hodrick-Prescott filter. Sample period is 1990-2015. ‘Exporting’ and ‘Importing’ mean trade-weighted averages of commodity-exporting and commodity-importing OECD member countries, respectively.

*Source:* OECD

In sum, U.S. output is more strongly correlated with outputs of commodity-exporting countries than those of commodity-importing countries. The output correlation between the U.S. and commodity-exporting countries is higher than the consumption correlation between them. These are the main properties of international business cycles between the U.S. and commodity-exporting countries that the model in this paper tries to replicate.

### 3. The model

In this section, I describe the two-country model with commodities in which different commodity trade structures between countries are considered. This research builds on the open economy macro models of several papers (BKK; ST; Backus and Crucini (2000); CDL). The model is mainly based on ST. I only add commodities to the two-country, two-sector RBC model of ST.

To be specific, there are two countries: the home country (the U.S.) and the foreign country (commodity-exporting countries). The home country produces tradable and nontradable goods and imports commodities, while the foreign country produces commodities as well as tradable and nontradable goods and exports commodities. Capital is sector-specific. Labour is mobile between sectors but immobile between countries. Asset markets are complete.

The main differences of this model from ST are that commodities as a complement to noncommodity goods are in the consumption basket of households, that commodities are used in the production of tradable goods as an input, and that the foreign country produces commodities whilst the home country does not.

I next explain the details of the model. Unless stated otherwise, there are symmetric equations for the foreign country.

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10 For the equilibrium conditions of the model, see Appendix B.
3.1. Household

The representative household in each country maximizes the following utility.

\[ U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t - \frac{L_t^{1+\chi}}{1 + \chi} \right) \]  

where \( \mathbb{E}_0 \) is the conditional expectations operation, \( C_t \) is the composite of tradable, nontradable and commodity consumption goods, \( L_t \) is labour supply by households, \( \beta \) is the discount factor, and \( \chi \) is the inverse of Frisch elasticity of labour supply.

The budget constraint\(^{11}\) is

\[ C_t + p_H t_L_T,t + p_{N,t} t_{N,t} + \mathbb{E}_t \{ \varphi_{t, t+1} D_{t+1} \} = W_{T,t} t_L_T,t + W_{N,t} t_{N,t} + r_{T,t} t_K_T,t + r_{N,t} t_K_N,t + D_t + \Pi_t \]

with \( L_t = L_{T,t} + L_{N,t} \). \( \varphi_{t, t+1} \), \( D_{t+1} \), \( W_t \) and \( \Pi_t \) denote the stochastic discount factor for one-period ahead real pay-offs relevant to the home household, the real pay-off in period \( t + 1 \) of the portfolio held at the end of period \( t \), the real wage and the profits remitted by firms, respectively. The subscripts ‘\( T \)’, ‘\( N \)’ and ‘\( X \)’ denote the tradable, nontradable and commodity sectors, respectively, and the subscript ‘\( H \)’ means the tradable goods produced in the home country. \( p, I, K \) and \( r \) are the real prices of goods, the investment, the capital and the real rental rate of capital, respectively.

The law of motion for capital in each sector is given by:

\[ K_{j,t+1} = (1 - \delta) K_{j,t} + I_{j,t}, \ j \in \{ T, N, X \} \]  

The composite of consumption goods, \( C_t \), is defined as the following constant elasticity of substitution (CES) aggregator function of noncommodity consumption goods, \( C_{NC,t} \), and commodity consumption goods, \( C_{X,t} \).

\[ C_t = \left\{ \begin{array}{l} (1 - \gamma_0) \frac{1}{\eta_0} C_{NC,t}^{\eta_0 - 1} + \gamma_0 \frac{1}{\eta_0} C_{X,t}^{\eta_0 - 1} \end{array} \right\}^{\frac{\eta_0}{\eta_0 - 1}} \]  

The noncommodity consumption goods, in turn, are similarly defined by tradable, \( C_{T,t} \), and nontradable consumption goods, \( C_{N,t} \):

\[ C_{NC,t} = \left\{ \begin{array}{l} (1 - \gamma_1) \frac{1}{\eta_1} C_{T,t}^{\eta_1 - 1} + \gamma_1 \frac{1}{\eta_1} C_{N,t}^{\eta_1 - 1} \end{array} \right\}^{\frac{\eta_1}{\eta_1 - 1}} \]  

\(^{11}\) Since the commodity sector exists in the foreign country, the household’s budget constraint in the foreign country is \( C_t^* + p_{H,t}^* t_L_T,t^* + p_{N,t}^* t_{N,t}^* + p_{X,t}^* t_{X,t}^* + \mathbb{E}_t \{ \varphi_{t, t+1} D_{t+1}^* \} = W_{T,t}^* t_L_T,t^* + W_{N,t}^* t_{N,t}^* + W_{X,t}^* t_{X,t}^* + r_{T,t}^* t_K_T,t^* + r_{N,t}^* t_K_N,t^* + D_t^* + \Pi_t^* \) with \( L_t^* = L_{T,t}^* + L_{N,t}^* + L_{X,t}^* \). The superscript ‘\( ^* \)’ denotes the variables in the foreign country.
The tradable consumption goods consists of tradable consumption goods produced in the home country, \(C_{H,t}\), and tradable consumption goods produced in the foreign country, \(C_{F,t}\).

\[
C_{T,t} = \left\{\left(1 - \gamma_2\right)^{\frac{1}{\eta_2}}C_{H,t}^{-\frac{\eta_2-1}{\eta_2}} + \gamma_2^{\frac{1}{\eta_2}}C_{F,t}^{-\frac{\eta_2-1}{\eta_2}}\right\}^{\frac{\eta_2}{\eta_2-1}} \quad (6)
\]

The corresponding price indices, \(P_t\) (CPI), \(P_{NC,t}\) (price of noncommodity consumption goods) and \(P_{T,t}\) (price of tradable consumption goods), are following.\(^{12}\)

\[
P_t = \left\{(1 - \gamma_0)P_{NC,t}^{-\eta_0} + \gamma_0 P_{X,t}^{-\eta_0}\right\}^{\frac{1}{1-\eta_0}} \quad (7)
\]

\[
P_{NC,t} = \left\{(1 - \gamma_1)P_{T,t}^{-\eta_1} + \gamma_1 P_{N,t}^{-\eta_1}\right\}^{\frac{1}{1-\eta_1}} \quad (8)
\]

\[
P_{T,t} = \left\{(1 - \gamma_2)P_{H,t}^{-\eta_2} + \gamma_2 P_{F,t}^{-\eta_2}\right\}^{\frac{1}{1-\eta_2}} \quad (9)
\]

where \(P_{X,t}\) is the price of commodity consumption goods, \(P_{N,t}\) is the price of nontradable consumption goods, \(P_{H,t}\) is the price of tradable consumption goods produced in the home country, and \(P_{F,t}\) is the price of tradable consumption goods produced in the foreign country.

### 3.2. Firms

Nontradable good and commodity producers use labour and capital as inputs for the productions, while tradable good producers use commodities as well as labour and capital in production. Both countries produce tradable and nontradable goods with the same production function, but only the foreign country produces commodities.

#### Nontradable good Producers

Nontradable good producers use labour and capital in production, and both countries produce nontradable goods with the identical production function.

\[
Y_{N,t} = A_{N,t}K_{N,t}^{\alpha_N}L_{N,t}^{1-\alpha_N} \quad (10)
\]

where \(Y_{N,t}\) is the output of nontradable goods, \(A_{N,t}\) is the productivity in the nontradable production, \(K_{N,t}\) is the capital used in the nontradable production, \(L_{N,t}\) is the labour, and \(1 - \alpha_N\) is the labour share of income.

#### Tradable good Producers

Both countries produce tradable goods by using capital, labour and commodities as in-
puts. The production function of tradable goods is a nested CES as in Kim and Loungani (1992) and Backus and Crucini (2000).

\[ Y_{H,t} = A_{T,t} \left\{ (1 - a)K_{T,t}^{\frac{\nu - 1}{\nu}} + ax_t^{\frac{\nu - 1}{\nu}} \right\}^{\frac{\nu}{\nu - 1}}L_{T,t}^{1 - \alpha} \]  

\[ (11) \]

where \( Y_{H,t}, A_{T,t}, K_{T,t}, x_t \) and \( L_{T,t} \) denote the output, productivity, capital, commodities and labour for the production of tradable goods, respectively. \( 1 - \alpha \) is the labour share of income. \( a \) and \( \nu \) determine the importance of commodities and the elasticity of substitution between capital and commodities.

**Commodity Producers**

Commodity producers use capital and labour as inputs in production, and only the foreign country produces commodities. The production function is Cobb-Douglas as in Arora and Gomis-Porqueras (2011) and Huynh (2016).

\[ Y^*_{X,t} = A^*_X K^*_X L^*_X^{1 - \alpha} \]  

\[ (12) \]

where \( Y^*_{X,t}, A^*_X, K^*_X \) and \( L^*_X \) are the output, productivity, capital and labour for the commodity production, respectively. \( 1 - \alpha_X \) is the labour share of income.

**3.3. Resource constraints**

Since output in the nontradable sector in the home country has to be the same as home country’s demand for nontradable goods and its investment and output in the tradable sector in the home country must equal the world demand for the tradable goods produced in the home country and its investment, the resource constraints of the home country can be written as:\(^{14}\)

\[ Y_{N,t} = C_{N,t} + I_{N,t} \]  

\[ Y_{H,t} = C^*_{H,t} + C_{H,t} + I_{T,t} \]  

\[ (13) \]

\[ (14) \]

\(^{13}\) Some papers such as Bodenstein, Erceg and Guerrieri (2011) assume that commodities are endowments rather than goods produced. If commodities are endowments, commodity prices should be exogenous. However, the important channel for changes in commodity prices and output in the commodity sector to respond to productivity shocks in the model will be omitted if commodity prices are exogenous (see Section 4 for details). Therefore, I assume that the commodities are produced rather than being endowments.

\(^{14}\) Since output in the commodity sector in the foreign country should be equal to the world demand for commodities and its investment, the resource constraint of the commodity sector in the foreign country is

\[ Y^*_{X,t} = C_{X,t} + C^*_{X,t} + x_t + x^*_t + I^*_{X,t}. \]
3.4. Calibration

Table 2 lists the parameter values for the model which are based on U.S. annual data. I assume symmetry between the two countries following standard international business cycle literature such as BKK, ST and CDL, with exception of the commodity sector.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.96</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\chi$</td>
<td>3</td>
<td>Inverse of Frisch elasticity of labour supply</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.1</td>
<td>Depreciation rate of capital</td>
</tr>
<tr>
<td>$1 - \alpha_N$</td>
<td>0.56</td>
<td>Labour share in the nontradable sector</td>
</tr>
<tr>
<td>$1 - \alpha$</td>
<td>0.61</td>
<td>Labour share in the tradable sector</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.999</td>
<td>Elasticity of substitution between commodities and capital in the tradable production</td>
</tr>
<tr>
<td>$a$</td>
<td>0.1823</td>
<td>Parameter related to the weight of commodities in the tradable production</td>
</tr>
<tr>
<td>$1 - \alpha_X$</td>
<td>0.6568</td>
<td>Labour share in the commodity sector</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>0.112</td>
<td>Weight of commodities in the composite of consumption goods</td>
</tr>
<tr>
<td>$\eta_0$</td>
<td>0.3</td>
<td>Elasticity of substitution between commodity and noncommodity consumption goods</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>0.5631</td>
<td>Weight of nontradable goods in the noncommodity consumption goods</td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.44</td>
<td>Elasticity of substitution between tradable and nontradable goods</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.3557</td>
<td>Weight of foreign tradable goods in the tradable consumption goods in the home country</td>
</tr>
<tr>
<td>$\gamma_2^*$</td>
<td>0.7749</td>
<td>Weight of home tradable goods in the tradable consumption goods in the foreign country</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>1.1</td>
<td>Elasticity of substitution between home and foreign tradable goods</td>
</tr>
</tbody>
</table>

I borrow certain parameter values from the standard literature. The discount factor, $\beta$, is 0.96 following ST and CDL. The inverse of Frisch elasticity of labour supply, $\chi$, is set to 3 as in Galí and Monacelli (2005). The depreciation rates of capitals for the tradable and nontradable productions, $\delta$, are 0.1 as in ST and CDL. I assume that the depreciation rate of capital for the commodity production is also 0.1 as in Arora and Gomis-Porqueras (2011) and Huynh (2016), using the same depreciation rate of capital for energy production as in the production in other sectors. Following ST, I set the labour shares of income in the nontradable ($1 - \alpha_N$) and tradable ($1 - \alpha$) sectors at 0.56 and 0.61 respectively.

The elasticity of substitution between capital and commodities in the tradable production, $\nu$, is 0.999 following Kim and Loungani (1992). They consider two cases, $\nu = 0.999$ and $\nu = 0.588$. They refer to the former case as the “Cobb-Douglas” case and the latter as the “CES” case. Considering that the Cobb-Douglas case replicates the business cycle data better than the CES case in Kim and Loungani (1992), this model uses $\nu = 0.999$. Nevertheless, since $\nu$ is a new parameter from the introduction of commodities to the ST model, various values of $\nu$ ranging from 0.5 to 1 based on previous studies are examined to check whether changes in $\nu$ lead to different simulation results of the model. I assume that the steady state capital/commodity ratio in the tradable production, $K_T/x$, is 31.55 which is different from
Kim and Loungani (1992) and others.\textsuperscript{15} Kim and Loungani (1992) use a value of 50 as the ratio. However, in this model, firms use all types of commodities rather than only energy. Because the weight of energy in the IMF’s commodity price index is 0.631, $50 \times 0.631 = 31.55$ as the steady state capital/commodity ratio is appropriate. The parameter related to the weight of commodities in the tradable production, $a$, is set to 0.1823. This value is determined by the values of $\nu$, $K_T/x$, $\beta$ and $\delta$.\textsuperscript{16} Based on the assumption that real wage in each sector is identical in the model due to labour mobility between sectors, the labour share of income in the commodity sector, $1-\alpha_X$, is set to 0.6568. These parameter values are consistent with the steady state ratio of investment to output in each sector, $I_N/Y_N = 0.31$ and $I_T/Y_H = 0.23$, and the steady state ratio of aggregate investment to aggregate output, $I/Y = 0.27$ which is close to the value in the standard RBC literature (0.25). Accordingly, the steady state ratio of output in the nontradable sector to aggregate output, $Y_N/Y$, is 0.51 which closely matches the average ratio of output in the service sector\textsuperscript{17} to GDP for 1997-2015\textsuperscript{18} from the NIPA data (0.54).\textsuperscript{19}

The parameters associated with consumption are as follows. The weight of commodity consumption goods in the composite of consumption goods, $\gamma_0$, is set to 0.112 which is consistent with the NIPA data, the average ratio of commodity consumption\textsuperscript{20} to total consumption for 1990-2015. Since $\gamma_0$ is newly introduced due to the commodity consumption, various values of $\gamma_0$, from the minimum (0.1) to maximum (0.13) for 1990-2015, are examined in the sensitivity analysis. The elasticity of substitution between commodity and noncommodity consumption goods, $\eta_0$, is 0.3 following Natal (2012). Because $\eta_0$ does not appear in the ST model, a variety of values of $\eta_0$ in previous studies, from 0.1 to 1, are used to see whether the main results of this paper are sensitive to change in $\eta_0$. The weight of nontradable consumption goods in the noncommodity consumption goods, $\gamma_1$, is 0.5631 which is different from ST. Since commodities are part of tradable goods in ST, $\gamma_1=0.5631$ is consistent with the fact that the steady state ratio of consumption of all tradable goods (commodities and tradable goods in the model) to nontradable consumption is 1, which is equal to ST’s

\textsuperscript{15} Variables without time subscript ‘$t$’ denote their steady state values.
\textsuperscript{16} From the first order conditions of households and firms in steady state, $a = \frac{1}{1+(1/\beta-1+\delta)(K_T/x)^{\frac{1}{\nu}}}$.
\textsuperscript{17} I take the service sector to be wholesale and retail trades, transportation and warehousing, finance, insurance, real estate, rental, leasing, professional and business services, educational services, health care, social assistance, arts, entertainment, recreation, accommodation, food services, and other services (except government).
\textsuperscript{18} Because the NIPA data on real gross output by industry is only available from 1997, I use the data for 1997-2015.
\textsuperscript{19} Since both countries have identical tradable and nontradable sectors, $I_N^*/Y_N^*$ and $I_T^*/Y_T^*$ are the same as those in the home country. For the foreign commodity sector, the values of $\alpha_X$, $\delta$ and $\beta$ are consistent with $I_N^*/Y_N^* = 0.24$. Foreign commodity output is determined by the market clearing condition, which implies $Y_N^*/Y_* = 0.52$, $Y_T^*/Y_* = 0.21$ and $Y_N^*/Y_* = 0.27$.
\textsuperscript{20} Gasoline and other energy goods, plus food and beverages purchased for off-premises consumption...
assumption. The elasticity of substitution between tradable and nontradable consumption goods, \( \eta_1 \), is 0.44 which is borrowed from ST. The weight of tradable consumption goods produced in the foreign country in the tradable consumption goods in the home country, \( \gamma_2 \), is 0.3557. Considering that commodities are also tradable goods, under \( \gamma_2 = 0.3557 \) the steady state ratio of imported commodity and tradable consumption goods to total tradable consumption goods in the home country is 0.5, i.e. \( (C_X + C_F)/(C_T + C_X) = 0.5 \). This ratio is consistent with ST in which the steady state ratio of imported tradable consumption goods to total tradable consumption goods is 0.5. The weight of tradable consumption goods produced in the home country in the tradable consumption goods in the foreign country, \( \gamma_2^* \), is 0.7749, which is a result of the assumptions that in the steady state households’ labour supply in the home country is the same as that in the foreign country (\( L = L^* \)) and that the steady state values of aggregate consumptions in both countries are equal (\( C = C^* \)). Under these assumptions, from the first order condition of households’ problems in steady state, the steady state real wages in both countries are equal. The elasticity of substitution between home and foreign tradable consumption goods, \( \eta_2 \), is set to 1.1 following Bodenstein, Erceg and Guerrieri (2011). These parameter values are set to equate the steady state ratio of aggregate consumption to aggregate output in the home country, \( C/Y \), to 0.7 which is the same as the value in the standard RBC literature (0.7) and close to the average ratio of private consumption expenditure to GDP for 1990-2015 from the NIPA data (0.66). This implies that \( C^*/Y^* = 0.71 \).

Finally, productivities in the tradable and nontradable sectors in the home and the foreign countries follow AR(1) process as in BKK, ST and CDL.

\[
A_t = \rho A_{t-1}^T + \varepsilon_t
\]  

(15)

where \( A_t = [A_{T,t}, A^*_{T,t}, A_{N,t}, A^*_{N,t}]^T \) and \( \varepsilon_t = [\varepsilon_{T,t}, \varepsilon^*_{T,t}, \varepsilon_{N,t}, \varepsilon^*_{N,t}]^T \) is the vector of productivity shocks. As in BKK and CDL, to estimate the process I use annual data of “Index of output per hour in manufacturing” and “Index of output per hour in private services” for the period 1970-2015 from BLS as the productivities in the home tradable and nontradable sectors, and as the productivities in the foreign tradable and nontradable sectors average of commodity-exporting OECD countries’ indices of manufacturing output and output in services divided by sectoral total employment for the period 1970-2015 from the OECD STAN sectoral database are used.\(^{21}\) The coefficient matrix, \( \rho \), and the variance-covariance matrix of productivity

\(^{21}\) The data for the Netherlands and Norway is used for the estimation, since the data for other commodity-exporting OECD countries is not available for the period.
shocks, \( \text{Var}(\varepsilon_t) \) (in per cent), are

\[
\rho = \begin{pmatrix}
0.85 & -0.05 & 0.10 & 0.16 \\
-0.05 & 0.85 & 0.16 & 0.10 \\
0.01 & -0.04 & 0.94 & 0.01 \\
-0.04 & 0.01 & 0.01 & 0.94
\end{pmatrix}, \quad \text{Var}(\varepsilon_t) = \begin{pmatrix}
0.0463 & 0.0067 & 0.0027 & 0.0014 \\
0.0067 & 0.0463 & 0.0014 & 0.0027 \\
0.0027 & 0.0014 & 0.0036 & 0.0004 \\
0.0014 & 0.0027 & 0.0004 & 0.0036
\end{pmatrix}
\]

The values for the elements in the above matrices are similar to those in CDL.

4. Model analysis

In this section, I further assume that the productivity in the commodity production is fixed, while the productivities in the tradable and nontradable sectors are stochastic as in ST. This is because the main aim of this paper is to show that instead of adding additional shocks, adding commodities and idiosyncratic commodity trade structures of countries to the ST model can help produce better international business cycle statistics compared to the ST model.

4.1. International business cycles

Table 3 shows the business cycle statistics from the model and the ST model driven by productivity shocks, in terms of the relative standard deviations to output, correlations with output of the home country, and correlations between the home and the foreign countries. Nonetheless, since the main aim of this paper is to show that the output correlation in the model is higher than the consumption correlation in the model and the output correlation in the ST model, I focus on the output and consumption correlations between the two countries.

The ST model is unsuccessful in replicating the main properties of business cycle comovement between the U.S. and commodity-exporting countries explained in Section 2. Specifically, the output correlation between the two countries is too low compared to the data. The consumption correlation between the two is too high and greater than the output correlation. These are very different from the data, but typical in open economy macro models.

The model, however, produces much closer international business cycle statistics to the data than the ST model.

Specifically, the output correlation in the model is higher than that in the ST model and closer to the data. This result is consistent with the empirical finding in Section 2 that GDPs of commodity-exporting countries are more positively correlated with the U.S. GDP than those of commodity-importing countries. The consumption correlation decreases compared to the ST model. As a result, in the model the output correlation is greater than
the consumption correlation, i.e. the consumption correlation puzzle is mitigated. In short, the model produces better business cycle comovement between the U.S. and commodity-exporting countries in terms of the output and consumption correlations compared to the ST model.

Table 3: Business cycle statistics

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Model</th>
<th>ST</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S.D. relative to GDP in home</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.986</td>
<td>0.457</td>
<td>0.432</td>
</tr>
<tr>
<td>Investment</td>
<td>3.389</td>
<td>3.059</td>
<td>2.622</td>
</tr>
<tr>
<td>Real net exports over GDP</td>
<td>0.361</td>
<td>0.135</td>
<td>0.042</td>
</tr>
<tr>
<td><strong>Correlation with GDP in home</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.938</td>
<td>0.849</td>
<td>0.803</td>
</tr>
<tr>
<td>Investment</td>
<td>0.957</td>
<td>0.985</td>
<td>0.977</td>
</tr>
<tr>
<td>Real net exports over GDP</td>
<td>-0.766</td>
<td>-0.969</td>
<td>0.067</td>
</tr>
<tr>
<td><strong>Correlation between home and foreign</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.738</td>
<td>0.789</td>
<td>0.658</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.584</td>
<td>0.740</td>
<td>0.774</td>
</tr>
<tr>
<td>Investment</td>
<td>0.471</td>
<td>0.638</td>
<td>0.531</td>
</tr>
</tbody>
</table>

Notes: The correlation between home and foreign from the data is trade-weighted average correlation between the same variables of the U.S. and commodity-exporting OECD countries. Sample period is 1990-2015. Source: OECD

As for other business cycle statistics, overall the model generates better business cycle statistics than the ST model, although it produces worse correlations between home output and investment, and between the two countries’ investments; the standard deviations of consumption, investment and real net exports over GDP relative to GDP in the home country and the correlations of consumption and net exports over output with GDP in the home country are all improved compared to the ST model.

4.2. Features of the model that generates the output and consumption correlation outcomes

In this section, I explain how the model generates better international business cycle statistics than the ST model with focus on the output and consumption correlations. Better international business cycle statistics in the model compared to the ST model are primarily because of the complementarity between commodity and noncommodity consumption goods and commodity consumption, which is not present in the ST model.

With regard to the output correlation, positive productivity shocks in the home country raise its aggregate output and consumption, which leads to an increase in its demand for imports of foreign tradable goods and commodities. The increased demand causes rises in the
real prices of foreign tradable goods and commodities. The rise in home demand for imports of foreign tradable goods is offset by their increased real prices. For commodities, however, although real commodity prices increase, the home country imports more commodities thanks to the complementarity between commodity and noncommodity consumption goods ($\eta_0 < 1$). Therefore, output in the foreign commodity sector rises, and thus there is a larger positive spillover to the foreign country compared to the ST model in which the foreign country does not have the commodity sector. As a result, the output correlation in the model becomes greater than that in the ST model.

With regard to the consumption correlation, when a positive productivity shock hits the home tradable sector, home commodity consumption and real commodity prices rise, and the real price of home tradable goods falls. The decrease in the real price of home tradable goods leads to a fall in the real price of tradable goods in the foreign country. Foreign noncommodity consumption increases due to the fall in the real price of tradable goods. Despite the rise in real commodity prices, foreign commodity consumption goes up owing to the complementarity between commodity and noncommodity consumption goods. On the contrary, in response to a positive productivity shock in the home nontradable sector, home commodity consumption and real commodity prices rise. Since the shock causes a fall in the real price of nontradable goods in the home country, the real prices of home and foreign tradable goods rise. The real price of tradable goods in the foreign country goes up, which causes a drop in foreign noncommodity consumption. Thus, foreign commodity consumption falls due to the complementarity and increased real commodity prices, i.e. home and foreign commodity consumptions are positively correlated with respect to productivity shocks in the tradable sector, whilst they are negatively correlated with respect to productivity shocks in the nontradable sector. This negative correlation with respect to productivity shocks in the nontradable sector enables the correlation between home and foreign commodity consumptions to be lower than that between home and foreign noncommodity consumptions in the model. Therefore, the model generates lower consumption correlation compared to the ST model in which there is no commodity consumption.

4.2. Impulse responses

In order to understand how the model yields better international business cycle statistics in a more straightforward manner, I conduct impulse response analysis to productivity shocks in this section. Specifically, I study how variables in the model and the ST model respond to

\footnote{Since productivity shocks to the nontradable sector are far weaker than those to the tradable sector as usually shown in previous studies, the consumption correlation between home and foreign commodity consumptions is positive. The standard deviation of the latter is about 3.5 times larger than the former which is implied in variance-covariance matrix of productivity shocks, $\text{Var}(\varepsilon_t)$.}
either a positive 1% tradable or nontradable productivity shocks in the home country, with focus on outputs and consumptions. In addition, for deeper intuition, I relate the impulse responses of the foreign country to a shock raising foreign commodity output to the “Dutch disease.” For simplicity, I abstract from the productivity spillover implied in the coefficient matrix of productivities and variance-covariance matrix of productivity shocks only in impulse responses, i.e. the off-diagonal elements of $\rho$ and $\text{Var}(\varepsilon_t)$ are all zero.$^{23}$

4.2.1. A positive 1% productivity shock in the home tradable sector

Figure 3 shows the responses of the model and the ST model to a positive 1% productivity shock in the home tradable sector.

First, consider the home country. In both models, a positive productivity shock in the home tradable sector increases output, $Y_H$, and consumption, $C_H$, in the sector and decreases the real prices of tradable goods in the home country, $p_T$, mainly due to a large fall in tradable goods produced in the home country, $p_H$. However, the increase in $Y_H$ in the ST model is greater than that in the model. This is because, differently from the ST model, in the model an increase in $Y_H$ puts upward pressure on demands for commodities as well as nontradable goods, which raises real commodity prices, $p_X$ and $p_X^*$, and the real price of nontradable goods, $p_N$. Since commodities are used in the production of tradable goods as an input, a rise in $p_X$ means an increase in the marginal costs of the firms in the tradable sector. Thus, the shock has a slightly bigger positive effect on $Y_H$ in the ST model which does not have commodities than in the model. Nevertheless, consumption of tradable goods, $C_T$, in the model rises by more than that in the ST model due to a bigger fall in $p_T$. Because in the model due to commodities $P_t = \left\{ (1 - \gamma_0)P_{NC,t}^{1-\eta_0} + \gamma_0 P_{X,t}^{1-\eta_0} \right\}^{1/\eta_0}$, which can be expressed as $1 = (1 - \gamma_0)P_{NC,t}^{1-\eta_0} + \gamma_0 P_{X,t}^{1-\eta_0}$ by dividing by $P_t$, and $p_X$ goes up, the real price of noncommodity goods, $p_{NC}$, falls. In the model $P_{NC,t}^{1-\eta_1} = (1 - \gamma_1)P_{T,t}^{1-\eta_1} + \gamma_1 P_{N,t}^{1-\eta_1}$, whilst in the ST model which does not have commodities $1 = (1 - \gamma_1)P_{T,t}^{1-\eta_1} + \gamma_1 P_{N,t}^{1-\eta_1}$. Therefore, $p_N$ in the model increases by less than that in the ST model, but $p_T$ in the model falls by more than that in the ST model. Despite the increased $p_N$, consumption of nontradable goods, $C_N$, in the model rises slightly thanks to the complementarity between $C_T$ and $C_N$ ($\eta_1 < 1$). On the contrary, $C_N$ in the ST model goes down slightly, since in spite of the complementarity a rise in $C_T$ is smaller and an increase in $p_N$ is bigger compared to the model. Hence, a fall in output in the nontradable sector, $Y_N$, in the model is smaller than that in the ST model. Because of rises in $C_T$ and $C_N$, noncommodity consumption, $C_{NC}$, increases, which leads to a rise in commodity consumption, $C_X$, in the model despite the increased $p_X$ due to the complementarity between

$^{23}$ Impulse response analysis with the productivity spillover results in all shocks hitting at the same time, which makes it hard to understand how a certain variable in the model responds to a certain shock.
\( C_{NC} \) and \( C_X (\eta_0 < 1) \). A rise in \( Y_H \) leads to an increase in demand for commodities in the production in the home tradable sector, \( x \). Finally, from \( p_{T,t}^{1-\eta_2} = (1 - \gamma_2)p_{H,t}^{1-\eta_2} + \gamma_2 p_{F,t}^{1-\eta_2} \), since \( p_T \) in the model goes down by more than that in the ST model, \( p_H \) in the model fall by more compared to the ST model. Hence, the rise in \( C_H \) in the model is greater than the ST model. Due to a larger fall in \( p_H \) than \( p_F \), consumption of foreign tradable goods, \( C_F \), drops. Overall, aggregate output and consumption in the home country, \( Y \) and \( C \), increase, and the increase in \( Y \) of the model is smaller than that of the ST model while the increase in \( C \) of the model is bigger than that of the ST model.

As for the foreign country, a fall in \( p_T \) and a rise in \( p_F \) lead to a decrease in \( p_T^* \) and an increase in \( p_F^* \), respectively. Since the fall in \( p_T \) and the rise in \( p_F \) in the model are bigger than those in the ST model, the decrease in \( p_T^* \) and the increase in \( p_F^* \) in the model are also larger than those in the ST model. Thus, \( C_T^* \) in the model jumps by more than that in the ST model, and \( C_T^* \) in the model drops by more than that in the ST model, which brings about a greater fall in \( Y_F^* \) in the model compared to that in the ST model. Since \( p_T^* \) in the model decrease by more than that in the ST model, a rise in \( p_N^* \) in the model is larger than that in the ST model. In spite of an increase \( p_N^* \), \( C_N^* \) rises thanks to the complementarity between \( C_T^* \) and \( C_N^* \). Despite a greater increase in \( C_T^* \) in the model, a bigger rise in \( p_N^* \) in the model compared to the ST model causes \( C_N^* \) in the model to increase by slightly less than that in the ST model. Accordingly, \( Y_N^* \) in the model jumps by less compared to the ST model. \( C_{NC}^* \) increases owing to rises in \( C_T^* \) and \( C_N^* \), and thus despite the increased \( p_N^* \), \( C_X^* \) rises thanks to the complementarity between \( C_{NC}^* \) and \( C_X^* \). \( x^* \) falls because of a decrease in \( Y_F^* \). Due to rises in \( C_X \), \( C_X^* \) and \( x \), foreign commodity output, \( Y_X^* \), goes up. Overall, \( Y^* \) and \( C^* \) increase. However, rises in \( Y^* \) and \( C^* \) in the model are bigger than those in the ST model mainly due to increases in \( Y_X^* \) and \( C_X^* \) that are not in the ST model.

![Graphs](a) H nontradable output  (b) H nontradable consumption  (c) H tradable output
(d) H tradable consumption  
(e) H aggregate output  
(f) H aggregate consumption  

(g) F nontradable output  
(h) F nontradable consumption  
(i) F tradable output  

(j) F tradable consumption  
(k) F commodity output  
(l) Commodity consumption  

(m) F aggregate output  
(n) F aggregate consumption  
(o) Real prices of nontradables
Application to the “Dutch disease”

The responses of the foreign country to the shock can be related to a typical example of the Dutch disease model of Corden and Neary (1982). According to them, a boom in the commodity sector can have effects on the economy in two ways: (i) the “resource movement effect” in which the boom in the commodity sector causes a rise in the marginal product of labour (the mobile factor) in the sector and thus draws labour, and (ii) the “spending effect” in which the increased real income resulting from the boom leads to an increase in consumption of nontradables. The spending effect results in an increase in prices of nontradable goods. Since prices of tradable goods are set by the world (they consider a small open economy), real exchange rates appreciate. Consequently, the boom leads to a rise in output in the nontradable sector but a fall in output in the tradable sector.

We can show all aspects of the Dutch disease in the responses of the foreign country in the model to the positive shock in the home tradable sector, with the exception that real exchange rates do not appreciate since the foreign country is not a small open economy in this model.

Figure 3: Responses to a positive productivity shock in the home tradable sector

Note: ‘H’ and ‘F’ mean the home and the foreign countries, respectively.

Figure 4: Responses of the model to a positive productivity shock in the home tradable sector

Note: ‘F’ means the foreign country.
Figure 4 shows the responses of the foreign country in the model to a positive 1% productivity shock in the home tradable sector. The shock leads to an increase in home demand for commodities. Since the home country imports commodities from the foreign country, the shock results in a boom in the foreign commodity sector (see (k) in Figure 3). The boom leads to a rise in the marginal product of labour in the sector, and thus the sector draws labour from other sectors. Labour in the sector jumps. This explains the resource movement effect well. With regard to the spending effect, the boom triggered by the shock brings about a rise in consumption of nontradable goods in the foreign country (see (h) in Figure 3), which causes an increase in the real price of nontradable goods (see (o) in Figure 3). Accordingly, labour in the foreign nontradable sector rises. Since the foreign commodity and nontradable sectors draw labour, labour in the foreign tradable sector falls. Therefore, the boom causes a rise in output in the foreign nontradable sector and a fall in output in the foreign tradable sector (see (g) and (i) in Figure 3).

4.2.2. A positive 1% productivity shock in the home nontradable sector

Figure 5 presents responses of the model and the ST model to a positive 1% productivity shock in the home nontradable sector. Since the responses of each sector in both models are very similar, I do not compare responses of each sector in the two models. Instead, I focus on comparing responses of aggregate variables in the two models.

With regard to the home country, in response to a positive productivity shock in the home nontradable sector, output, $Y_N$, and consumption, $C_N$, in the sector rise, and its real price, $p_N$, falls. Since an increase in $Y_N$ puts upward pressure on the real prices of tradable goods produced in both countries and commodities in both countries, $p_H$, $p_F$, $p_H^*$, $p_F^*$, $p_X$ and $p_X^*$ all rise. Thus, the real prices of tradable goods in the home country, $p_T$, increases. Despite the increased $p_T$, home tradable consumption, $C_T$, rises thanks to the complementarity between $C_N$ and $C_T$ ($\eta_1 < 1$). Due to the rise in $C_T$, $C_H$ and $C_F$ increase. Because the nontradable sector draws labour thanks to the shock, labour in the tradable sector falls, which leads to a slight fall in output in the sector, $Y_H$. A fall in $Y_H$ causes a decrease in commodity input for the production in the sector, $x$. Despite the increased $p_X$, commodity consumption in the home country, $C_X$, goes up owing the complementarity between $C_{NC}$ and $C_X$. Consequently, home aggregate output and consumption, $Y$ and $C$, jump in response to the shock, but the increase in $Y$ in the model is slightly lower than that in the ST model because of the slightly bigger fall in $Y_H$ in the model than that in the ST model. $C$ in both models increase by almost the same degree.

Now, consider the foreign country. Rises in $p_F^*$ and $p_H^*$, and therefore $p_T^*$ cause decreases in $C_H^*$ and $C_F^*$ and thus $C_T^*$. A rise in $p_T^*$ brings about a fall in $p_N^*$. Although $p_N^*$ falls, $C_N^*$ decreases due to the complementarity between $C_T^*$ and $C_N^*$. Due to falls in $C_T^*$ and $C_N^*$, $C_{NC}^*$
decreases. Owing to the complementarity between $C_X^*$ and $C_{NC}^*$ and a rise in $p_X^*$, $C_X^*$ also falls. Thanks to the increased $C_F$, demand for capital in the foreign tradable sector increases, which brings about a rise in its investment, $I_T^*$. Therefore, output in the foreign tradable sector, $Y_F^*$, goes up, which leads to an increase in $x^*$. Increases in $C_X^*$ and $x^*$ cause a jump in output in the foreign commodity sector, $Y_X^*$. A fall in $C_N^*$ brings about a decrease in $Y_N^*$. As a result, $Y^*$ increases, but $C^*$ falls. Owing to a rise in $Y_X^*$ that is not in the ST model, $Y^*$ in the model increases by more than that in the ST model does, and due to a fall in $C_X^*$ which is not in the ST model, $C^*$ in the model decreases by more than that in the ST model.

(a) H nontradable output  (b) H nontradable consumption  (c) H tradable output  
(d) H tradable consumption  (e) H aggregate output  (f) H aggregate consumption  
(g) F nontradable output  (h) F nontradable consumption  (i) F tradable output
Figure 5: Responses to a positive productivity shock in the home nontradable sector

(j) F tradable consumption  (k) F commodity output  (l) Commodity consumption

(m) F aggregate output  (n) F aggregate consumption  (o) Real prices of nontradables

(p) Real prices of tradables  (q) Real commodity prices  (r) Real exchange rates

Note: ‘H’ and ‘F’ mean the home and the foreign countries, respectively.

4.3. Sensitivity analysis

In the model, the parameters for the elasticity of substitution between commodity and non-commodity consumption goods \( (\eta_0) \), the weight of commodity consumption goods in the composite consumption goods \( (\gamma_0) \), and the elasticity of substitution between commodities and capital in the production of tradable goods \( (\nu) \) are new parameters due to the introduction of commodities compared to the ST model.\(^{24}\) Thus, whether the main results of

\(^{24}\) In fact, the parameter related to the weight of commodities in the production of tradable goods \( (a) \) is also new one. However, since the discount factor \( (\beta) \), the depreciation rate \( (\delta) \), the steady state ratio of
the model (higher output correlation than the consumption correlation and the output correlation in the ST model) are sensitive to changes in the values of these three parameters needs to be assessed. To do so, I set reasonable ranges of the parameter values. Specifically, \( \eta_0 \in [0, 1] \) which is based on previous studies. \( \gamma_0 \in [0.1, 0.13] \) which matches the minimum and maximum values in the NIPA data for 1990-2015. I set a range of \( \nu \in [0.5, 1] \) again based on previous studies. Figure 6 shows the output and consumption correlations between the two countries according to changes in the values of these three parameters.

**Elasticity of substitution between commodity and noncommodity goods** 

(a) in Figure 6 shows the output and consumption correlations between the two countries according to changes in the elasticity of substitution between commodity and noncommodity consumption goods (\( \eta_0 \)).

As already shown, a rise in the output correlation and a fall in the consumption correlation are due to the complementarity between commodity and noncommodity consumption goods. A rise in \( \eta_0 \) means weaker complementarity. Hence, we would expect that a rise in \( \eta_0 \) will decrease the output correlation and increase the consumption correlation, holding other factors constant. However, since \( \eta_0 \) is directly related to commodity prices (see Equation (7)), changes in it affect the responses of real commodity prices to the productivity shocks, which also have impacts on the output and consumption correlations. Consequently, as \( \eta_0 \) rises, changes in the output correlation are not noticeable, but the consumption correlation increases. Nevertheless, as can be seen in (a) of Figure 6, the fluctuations are small, and thus as long as \( \eta_0 \) is in reasonable ranges, the main results of this paper hold.

**Weight of commodity goods in the composite consumption goods** 

Now consider the changes in the weight of commodity consumption goods in the composite consumption goods (\( \gamma_0 \)). (b) in Figure 6 presents the output and consumption correlations in the model according to the various values of \( \gamma_0 \).

Conversely to \( \eta_0 \) in which commodity consumption is decreasing, commodity consumption is increasing in \( \gamma_0 \). This can be seen in the households’ optimal commodity consumption allocation, \( C_{X,t} = \gamma_0 p_{X,t} C_t \). Thus, an increase in \( \gamma_0 \) and a fall in \( \eta_0 \) have very similar effects on the output and consumption correlations, i.e. we would expect that a rise in \( \gamma_0 \) will increase the output correlation and decrease the consumption correlation, holding other factors constant. Although like \( \eta_0 \), \( \gamma_0 \) is directly related to commodity prices (see Equation (7)), the impact of changes in \( \gamma_0 \) on real commodity prices is much weaker than \( \eta_0 \). As a result, consistently with the expectation, as \( \gamma_0 \) increases, the output correlation goes up and

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capital to commodities in the tradable production \( (K_T/x) \) and \( \nu \) determine its value, the assessment of the sensitivity to changes in its value is not conducted.
the consumption correlation falls. Overall, the output correlation is always higher than that in the ST model and the consumption correlation in the model as long as $\gamma_0$ is in a reasonable interval.

In addition, since according to the EIA the U.S. could become a net energy-exporting country by 2026, meaning that the U.S. is likely to be a minor commodity-importing country rather than a commodity-exporting country\textsuperscript{25}, whether the main results of the model change in the case in which the home country imports commodities to a much lesser extent needs to be assessed. Because the lower $\gamma_0$, the smaller the home country’s imports of commodities from the foreign country, this can be assessed by reducing $\gamma_0$ greatly. Specifically, I consider two cases; 33% and 50% falls in $\gamma_0$ ($\gamma_0=0.075, 0.056$). When $\gamma_0$ falls by 33% ($\gamma_0=0.075$), the model generates higher output correlation (0.772) than consumption correlation (0.759). $\gamma_0=0.056$ causes almost equal output and consumption correlations (0.765 for the output correlation and 0.769 for the consumption correlation). In sum, if the home country shifts from a major commodity-importing country to a minor commodity-importing country, the output correlation in the model is higher than that in the ST model (0.658) and the consumption correlation in the model is smaller than that in the ST model (0.774). In other words, the model yields closer international business cycle statistics to the data than the ST model, regardless of whether the home country is a major or a minor commodity-importing country.

**Elasticity of substitution between commodities and capital in production ($\nu$)**

As the elasticity of substitution between commodities and capital in the production of tradable goods ($\nu$) goes up, both output and consumption correlations fall slowly (see (c) in Figure 6).

As for the output correlation, when $\nu$ is high, it is easier for commodity input to be substituted by capital input, which means weaker complementarity between the two. Furthermore, $\nu$ is not directly related to commodity prices. Therefore, when $\nu$ is high, the increased home demand for imports of commodities for the tradable production in response to positive productivity shocks is offset by the increased real commodity prices by more than when $\nu$ is small. This leads foreign commodity output to rise by less, which brings about a smaller increase in foreign aggregate output, i.e. the larger $\nu$ is, the smaller the positive spillover to the foreign country from positive productivity shocks. Therefore, the output correlation decreases as $\nu$ becomes greater.

With regard to the consumption correlation, $\nu$ is not directly related to commodity con-

\textsuperscript{25} According to the EIA’s Annual Energy Outlook 2017, the EIA expects that the U.S. could become a net energy-exporting country by 2026. Since the U.S. is a net importing country of other commodities such as basic metal and wood, even if the U.S. becomes a net energy-exporting country, it will not become a net commodity-exporting country. Rather, the U.S. is likely to shift from a major commodity-importing country to a minor commodity-importing country.
sumption, and changes in commodity consumption in response to productivity shocks in the nontradable sector decrease the consumption correlation. Therefore, although $\nu$ increases, changes in commodity consumption is very marginal. Nevertheless, when $\nu$ is high, foreign noncommodity consumption goes up by less due to a smaller rise in its aggregate output compared to the case of low $\nu$, which leads to a fall in the consumption correlation. Consequently, high $\nu$ results in less positively correlated consumptions of the two countries.

Overall, the main results of this paper are maintained in a reasonable interval for $\nu$.

Figure 6: Sensitivity analysis

5. Conclusion

In this paper, I set up a two-country model with commodities and idiosyncratic commodity trade structures of countries to analyse international business cycles between the U.S. and commodity-exporting countries. Consistently with the empirical finding, the output correlation in the model is higher than that in a standard international business cycle model (the ST model). In particular, the model generates output correlation that is greater than the consumption correlation unlike standard models, and thus mitigates the consumption correlation puzzle. The complementarity between commodity and noncommodity consumption goods enables productivity shocks to have stronger spillovers to the other country. Commodity consumptions in both countries, which are not present in the ST model, are less correlated compared to noncommodity consumptions. Thus, the complementarity and commodity consumption play important roles in yielding these results. The model also explains the typical Dutch disease by showing that a boom in the commodity sector leads to a rise in output in the nontradable sector and a fall in output in the tradable sector. Finally, the sensitivity analysis of the model’s results with regards to the parameters in the newly introduced features, as compared to the ST model, show that the main results of this paper hold as long as the parameter values are in reasonable ranges.
Adding commodities to open economy macro models with consideration for different commodity trade structures of countries therefore appears to be useful, since it helps to explain key features of international business cycle correlations between the U.S. and commodity-exporting countries. The framework that this paper proposes is therefore promising for further research on the international macroeconomic interactions between the U.S. and commodity-exporting countries, especially the international transmissions of monetary and fiscal policies.
References


Appendix

A. Data sources

This section describes the data used in this paper. The data is from two sources: OECD and IMF.

Figures 1 and 2  The 60 countries are those whose data of commodity exports and imports are available in the OECD Trade in Value Added.

Annual real GDPs for computing GDP correlations are taken from the IMF World Economic Outlook, since annual real GDP data of some countries such as Argentina and Bulgaria among the 60 countries are not available in the OECD National Accounts. Net commodity exports are the gross exports minus imports of agriculture, hunting, forestry, fishing, mining, quarrying, wood and basic metal industries in the OECD Trade in Value Added, and nominal GDP is drawn from the IMF World Economic Outlook. The ratio of net commodity exports to GDP is average of dividing net commodity exports by nominal GDP for the period 1995-2011.

Tables 1 and 3  Annual real GDP, real private consumption and real private fixed investment of OECD countries are taken from OECD National Accounts. Since the pre 1990 data for most Eastern European countries are not available, the data cover the period 1990-2015 with the exception of Estonia which covers 1993-2015, Hungary which covers 1991-2015, Latvia which covers 1994-2015, and Slovak Republic which covers 1992-2015.

For the trade-weighted average, the average trade weight of an OECD country $i$ for the period 1995-2011 is constructed by summing exports from an OECD country $i$ to the U.S. and imports from the U.S. to the OECD country $i$, and dividing by the total OECD countries’ trade with the U.S. The trade data is drawn from OECD Trade in Value Added.

Commodity-exporting OECD countries are Australia, Canada, Chile, Estonia, Finland, Iceland, Latvia, Luxembourg, Mexico, the Netherlands, New Zealand and Norway whose ratios of net commodity exports to GDP are positive.
B. Equilibrium conditions of the model

In this section, I present the equilibrium system of the model described in the section 3.

- Home households' intertemporal problem:
  \[ 1 = \beta E_t \left[ \frac{C_t}{C_{t+1}} R_t \right] \]  
  \[ W_{T,t} = L_t^X C_t \]  
  \[ W_{N,t} = L_t^X C_t \]  
  \[ C_t = \vartheta e_t C_t^* \]

with \( R_t = \{(1 - \delta)p_{H,t+1} + r_{T,t+1}\}/p_{H,t} = \{(1 - \delta)p_{N,t+1} + r_{N,t+1}\}/p_{N,t} \) and \( L_t = L_{T,t} + L_{N,t} \)

- Foreign households' intertemporal problem:
  \[ 1 = \beta E_t \left[ \frac{C_t^*}{C_{t+1}^*} R_t^* \right] \]  
  \[ W_{T,t}^* = L_t^X C_t^* \]  
  \[ W_{N,t}^* = L_t^X C_t^* \]  
  \[ W_{X,t}^* = L_t^X C_t^* \]

with \( R_t^* = \{(1 - \delta)p_{F,t+1} + r_{T,t+1}\}/p_{F,t} = \{(1 - \delta)p_{N,t+1} + r_{N,t+1}\}/p_{N,t} = \{(1 - \delta)p_{X,t+1} + r_{X,t+1}\}/p_{X,t} \) and \( L_t^* = L_{T,t}^* + L_{N,t}^* + L_{X,t}^* \)

- Home households' consumption allocation:
  \[ C_{NC,t} = (1 - \gamma_0)p_{NC,t}^{-\eta_0} C_t \]  
  \[ C_{X,t} = \gamma_0 p_{X,t}^{-\eta_0} C_t \]  
  \[ C_{T,t} = (1 - \gamma_1) \left( \frac{p_{T,t}}{p_{NC,t}} \right)^{-\eta_1} C_{NC,t} \]  
  \[ C_{N,t} = \gamma_1 \left( \frac{p_{N,t}}{p_{NC,t}} \right)^{-\eta_1} C_{NC,t} \]  
  \[ C_{H,t} = (1 - \gamma_2) \left( \frac{p_{H,t}}{p_{T,t}} \right)^{-\eta_2} C_{T,t} \]  
  \[ C_{F,t} = \gamma_2 \left( \frac{p_{F,t}}{p_{T,t}} \right)^{-\eta_2} C_{T,t} \]

- Foreign households' consumption allocation:
\[ C_{NC,t}^* = (1 - \gamma_0)p_{NC,t}^{\star - \eta_0}C_t^* \]  
\[ C_{X,t}^* = \gamma_0 p_{X,t}^{\star - \eta_0}C_t^* \]  
\[ C_{T,t}^* = (1 - \gamma_1) \left( \frac{p_{T,t}^*}{p_{NC,t}^*} \right)^{-\eta_1} C_{NC,t}^* \]  
\[ C_{N,t}^* = \gamma_1 \left( \frac{p_{N,t}^*}{p_{NC,t}^*} \right)^{-\eta_1} C_{NC,t}^* \]  
\[ C_{F,t}^* = (1 - \gamma_2^*) \left( \frac{p_{F,t}^*}{p_{T,t}^*} \right)^{-\eta_2} C_{T,t}^* \]  
\[ C_{H,t}^* = \gamma_2^* \left( \frac{p_{H,t}^*}{p_{T,t}^*} \right)^{-\eta_2} C_{T,t}^* \]  

- **Home price indices:**

\[ 1 = (1 - \gamma_0)p_{NC,t}^{1 - \eta_0} + \gamma_0 p_{X,t}^{1 - \eta_0} \]  
\[ p_{NC,t}^{1 - \eta_1} = (1 - \gamma_1)p_{T,t}^{1 - \eta_1} + \gamma_1 p_{N,t}^{1 - \eta_1} \]  
\[ p_{T,t}^{1 - \eta_2} = (1 - \gamma_2)p_{H,t}^{1 - \eta_2} + \gamma_2 p_{F,t}^{1 - \eta_2} \]

- **Foreign price indices:**

\[ 1 = (1 - \gamma_0)p_{NC,t}^{\star 1 - \eta_0} + \gamma_0 p_{X,t}^{\star 1 - \eta_0} \]  
\[ p_{NC,t}^{\star 1 - \eta_1} = (1 - \gamma_1)p_{T,t}^{\star 1 - \eta_1} + \gamma_1 p_{N,t}^{\star 1 - \eta_1} \]  
\[ p_{T,t}^{\star 1 - \eta_2} = (1 - \gamma_2)p_{H,t}^{\star 1 - \eta_2} + \gamma_2 p_{F,t}^{\star 1 - \eta_2} \]

- **Laws of one price:**

\[ p_{H,t} = e_t p_{H,t}^* \]  
\[ p_{F,t} = e_t p_{F,t}^* \]  
\[ p_{X,t} = e_t p_{X,t}^* \]

- **Home nontradable good producers' optimality conditions:**

\[ Y_{N,t} = A_{N,t}K_{N,t}^{\alpha_N}L_{N,t}^{1 - \alpha_N} \]  
\[ W_{N,t} = (1 - \alpha_N)p_{N,t}Y_{N,t}^* \]  
\[ R_{t-1} = \frac{1}{p_{N,t-1}} \left\{ (1 - \delta)p_{N,t} + \alpha_N p_{N,t} \frac{Y_{N,t}}{K_{N,t}} \right\} \]
- Foreign nontradable good producers’ optimality conditions:

\[ Y_{N,t}^* = A_{N,t}K_{N,t}^{\alpha_N} L_{N,t}^{1-\alpha_N} \]  
\[ W_{N,t}^* = (1 - \alpha_N)p_{N,t}^{*} Y_{N,t}^* \]  
\[ R_{t-1}^* = \frac{1}{p_{N,t-1}^*} \{ (1 - \delta)p_{N,t}^* + \alpha_N p_{N,t}^* Y_{N,t}^* / K_{N,t}^* \} \]

- Home tradable good producers’ optimality conditions:

\[ Y_{H,t} = A_{T,t} \{(1-a)K_{T,t}^{\nu-1} + ax_t^{\nu-1}\} L_{T,t}^{1-\alpha} \]  
\[ W_{T,t} = (1 - \alpha)p_{H,t} Y_{H,t} / L_{T,t} \]  
\[ R_{t-1} = \frac{1}{p_{H,t-1}} \{ (1 - \delta)p_{H,t} + \alpha(1-a)p_{H,t} K_{T,t}^{-\frac{1}{\nu}} Y_{H,t} / (1-a)K_{T,t}^{\nu-1} + ax_t^{\nu-1} \} \]

\[ p_x = a\alpha p_{H,t} x_t^{\nu-1} Y_{H,t} / (1-a)K_{T,t}^{\nu-1} + ax_t^{\nu-1} \]

- Foreign tradable good producers’ optimality conditions:

\[ Y_{F,t}^* = A_{T,t} \{(1-a)K_{T,t}^{\nu-1} + ax_t^{\nu-1}\} L_{T,t}^{1-\alpha} \]  
\[ W_{T,t}^* = (1 - \alpha)p_{F,t}^{*} Y_{H,t} / L_{T,t}^* \]  
\[ R_{t-1}^* = \frac{1}{p_{F,t-1}^*} \{ (1 - \delta)p_{F,t}^* + \alpha(1-a)p_{F,t} K_{T,t}^{-\frac{1}{\nu}} Y_{F,t}^* / (1-a)K_{T,t}^{\nu-1} + ax_t^{\nu-1} \} \]

\[ p_x^* = a\alpha p_{F,t} x_t^{\nu-1} Y_{F,t}^* / (1-a)K_{T,t}^{\nu-1} + ax_t^{\nu-1} \]

- Foreign commodity producers’ optimality conditions:

\[ Y_{X,t}^* = A_{X,t}^{*} K_{X,t}^{*\alpha_X} L_{X,t}^{*1-\alpha_X} \]  
\[ W_{X,t}^* = (1 - \alpha_X)p_{X,t}^{*} Y_{X,t}^* \]  
\[ R_{t-1}^* = \frac{1}{p_{X,t-1}^{*}} \{ (1 - \delta)p_{X,t}^{*} + \alpha_X p_{X,t}^{*} Y_{X,t}^* / K_{X,t}^* \} \]

- Laws of motion for capital:

\[ K_{N,t+1} = (1 - \delta)K_{N,t} + I_{N,t} \]
\[ K_{N,t+1}^* = (1 - \delta)K_{N,t}^* + I_{N,t} \quad (63) \]
\[ K_{T,t+1}^* = (1 - \delta)K_{T,t}^* + I_{T,t} \quad (64) \]
\[ K_{T,t+1}^* = (1 - \delta)K_{T,t}^* + I_{T,t}^* \quad (65) \]
\[ K_{X,t+1}^* = (1 - \delta)K_{X,t}^* + I_{X,t}^* \quad (66) \]

- Resource constraints:

\[ Y_{N,t} = C_{N,t} + I_{N,t} \quad (67) \]
\[ Y_{H,t} = C_{H,t} + C_{H,t}^* + I_{T,t} \quad (68) \]
\[ Y_{N,t}^* = C_{N,t}^* + I_{N,t} \quad (69) \]
\[ Y_{F,t}^* = C_{F,t}^* + C_{F,t} + I_{T,t} \quad (70) \]
\[ Y_{X,t}^* = C_{X,t} + C_{X,t}^* + x_t + x_t^* + I_{X,t} \quad (71) \]

- Productivity shocks

\[ A_t = \rho A_{t-1}^T + \varepsilon_t \quad (72) \]
C. The ST model

The equilibrium conditions under the ST model are given by equations (16)-(22), (28), (29), (34), (35), (38), (41)-(43), (45)-(50), (52), (56), (62)-(65), (67)-(70) and (72), and the following 10 equations.

\[
C_{T,t} = (1 - \gamma_1) p_{T,t}^{-\eta_1} C_t \\
C_{N,t} = \gamma_1 p_{N,t}^{-\eta_1} C_t \\
C_{T,t}^* = (1 - \gamma_1) p_{T,t}^{*\eta_1} C_t^* \\
C_{N,t}^* = \gamma_1 p_{N,t}^{*\eta_1} C_t^* \\
1 = (1 - \gamma_1) p_{T,t}^{1-\eta_1} + \gamma_1 p_{N,t}^{1-\eta_1} \\
1 = (1 - \gamma_1) p_{T,t}^{1-\eta_1} + \gamma_1 p_{N,t}^{1-\eta_1} \\
Y_{H,t} = A_{T,t} K_{T,t}^\alpha L_{T,t}^{1-\alpha} \\
R_{t-1} = \frac{1}{p_{H,t-1}} \left\{ (1 - \delta) p_{H,t} + \alpha p_{H,t} \frac{Y_{H,t}}{K_{T,t}} \right\} \\
Y_{F,t}^* = A_{T,t}^* K_{T,t}^* L_{T,t}^{1-\alpha} \\
R_{t-1}^* = \frac{1}{p_{F,t-1}} \left\{ (1 - \delta) p_{F,t}^* + \alpha p_{F,t}^* \frac{Y_{F,t}^*}{K_{T,t}^*} \right\}
\]

Moreover, since there are no commodities in the ST model, \( \gamma_2 = \gamma_2^* = 0.5 \) and \( L_t^* = L_{T,t}^* + L_{N,t}^* \).