Commodities and International Business Cycles

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Abstract

Output growth in the U.S., a commodity-importing country, increases its demand for imports of commodities and therefore commodity prices, which benefits commodity-exporting countries but has adverse effects on commodity-importing countries. Hence, in the data, U.S. output is more positively correlated with outputs of commodity-exporting countries than commodity-importing countries. In other words, U.S. business cycle comovements with other countries differ according to the countries’ commodity trade structures. Nevertheless, this has not been considered in the analyses of international business cycles. This paper addresses this by adding the commodity sector to the foreign country. In the model, the home country (the U.S.) produces tradable and nontradable goods and imports commodities, while the foreign country (commodity-exporting countries) produces commodities as well as tradable and nontradable goods and exports commodities. The model produces higher output correlation and lower consumption correlation than a standard two-country model. Notably, unlike standard models, the model yields output correlation that exceeds the consumption correlation, matching the ranking in the data and mitigating the ‘consumption correlation puzzle’. Complementarity between commodities and noncommodity goods in consumption plays a key role in generating this result.

1. Introduction

Commodities have peculiar features compared to other tradable goods. They are used as inputs in production in almost every sector as well as consumed by households. Other tradable goods, however, are typically only consumed or only used as inputs in production.
in a specific sector. For example, energy is not only consumed but also used as inputs in all sectors. On the contrary, jeans are consumed, but not used in production (they can be used in production, but very rarely). Semiconductors are only used in production in the electronic sector, and not consumed by households. Therefore, it is desirable to model the commodity sector as an independent sector from the tradable sector, in order to take into account these peculiar features of commodities.

Since commodities are consumed and used as inputs in production in nearly all sectors, changes in commodity prices affect both households and firms in the economy by causing fluctuations in both consumer prices and the marginal costs of firms. This means that commodity prices have more widespread effects on the economy than prices of other tradable goods. Since the 1970s when commodity prices were very volatile due to oil shocks, macroeconomists have regarded changes in commodity prices as a critical source of economic fluctuations and one that can affect many countries simultaneously (Blanchard and Galí (2007)). However, in spite of the important effects of commodity prices on the economy, many macro models have so far omitted commodities in their models.

Furthermore, each country has its own idiosyncratic commodity trade structure. For instance, countries like the U.S. are net commodity-importing countries, and countries like Canada are net commodity-exporting countries. Owing to different commodity trade structures, each country will be affected by the same shock in a different way. As a result, several papers studying North-South interactions such as Moutos and Vines (1989) have already emphasized that to properly capture macroeconomic interactions between countries in open economy macro models, the asymmetries of trade and production resulting from commodities should be considered.

More specifically, output growth triggered by a certain shock in a large economy like the U.S., a commodity-importing country, will increase its demand for imports of commodities for production and consumption. Accordingly, commodity prices will go up. The increased demand for imports of commodities and the higher prices will have positive influences on the economies of commodity-exporting countries. Conversely, the increased commodity prices will adversely affect the economies of commodity-importing countries. Hence, U.S. GDP would be expected to have stronger positive correlations with GDPs of commodity-exporting countries than those of commodity-importing countries. There are some studies in line with this hypothesis. For instance, using a two-country dynamic stochastic general equilibrium

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1 During 2006-2011 the average ratio of net commodity exports to GDP in the U.S. and Canada were -7.5% and 26.0%, respectively, according to OECD Trade in Value Added database. Following IMF’s commodity price index, this paper defines the commodity sector as agriculture, hunting, forestry, fishing, mining, quarrying, wood and basic metal industries.

2 In these models, the “Northern” economies (i.e. the developed countries) produce industrial goods, whilst the “Southern” economies (i.e. the developing countries) produce commodities.
(DSGE) model, Bodenstein, Erceg and Guerrieri (2011) conclude that an increase in oil price due to an oil demand or supply shock brings about a wealth transfer toward the oil-exporter and a fall in output in the oil-importer. Chen, Filardo, He and Zhu (2012) and Georgiadis (2016) also empirically show that an expansionary monetary policy shock in the U.S. raises the U.S. output and has a more positive effect on the commodity-exporting countries or regions than on the commodity-importing countries or regions.\(^3\)

In accordance with this hypothesis, the U.S. has higher positive GDP correlations with commodity-exporting countries than with commodity-importing countries in the data (see Section 2 for details). This suggests that when analysing U.S. international business cycles, different commodity trade structures of partner countries should be taken into account.

However, previous papers on international business cycles have not only omitted commodities and idiosyncratic commodity trade structures of countries in their models, but have also been unsuccessful in explaining certain observed patterns of international business cycles. One of the first papers in the literature is Backus, Kehoe and Kydland (1992), henceforth BKK. BKK extend a closed economy real business cycle (RBC) model to a two-country model. Their model, however, is not successful in replicating the basic properties of international business cycles. It predicts that the output correlation between countries is negative (whereas in the data it is positive), and that the consumption correlation between countries is very high and larger than the output correlation (whereas in the data the output correlation is greater than the consumption correlation). In addition, the correlation between net exports and output in the home country (the U.S.) is about zero (whereas in the data it is negative). In particular, higher consumption correlation than output correlation in standard international business cycle models, which is opposite to the ranking in the data, is referred to in the literature as the “consumption correlation puzzle.”\(^4\) Since the work of BKK, a number of papers have tried to tackle these puzzles by adding non-traded goods (Stockman and Tesar (1995), henceforth ST), informational frictions (Kehoe and Perri (2002)), the distribution sector (Corsetti, Dedola and Leduc (2008), henceforth CDL), etc. Some papers have tried to find factors causing the high output correlation between countries using a two-country DSGE model with sticky prices and financial frictions (see Faia (2007), Ueda (2012), etc.). All these literatures, however, have regarded commodities simply as part of tradable goods and have not considered idiosyncratic commodity trade structures of countries. Surprisingly, international business cycles between the U.S. and commodity-exporting countries have not yet been analysed in the international business cycle literature, although they are significantly different from those between the U.S. and commodity-importing countries. This

\(^3\) The former uses a global vector error-correction model and the latter uses a global vector autoregressive model.

\(^4\) Alternatively, the “consumption correlation puzzle” or the “BKK puzzle.”
paper addresses these shortcomings in the literature.

To analyse international business cycles between the U.S. and commodity-exporting countries, this paper introduces commodities and different commodity trade structures into the workhorse model with nontradable goods: the two-country, two-sector RBC model of ST. Specifically, in the model analysed here, the home country (the U.S.) produces tradable and nontradable goods and imports commodities, while the foreign country (commodity-exporting countries) produces commodities as well as tradable and nontradable goods and exports commodities. The primary differences of the model from standard models are that only the foreign country produces commodities, that commodities are used as an input in the production of tradable goods, and that commodities are consumed as a complement to noncommodity consumption goods.\footnote{The inventory or storage of commodities is not considered in the model. If commodities can be stored, the stored commodities can be used to meet the increased demand for commodities triggered by a positive shock to the U.S., which will lead to a fall in inventory of commodities. Nevertheless, the shock will increase commodity prices due to the increased demand. However, the positive effects on commodity-exporting countries will be slightly weaker than would be the case without the inventory, because in the case without the inventory, the U.S. will import commodities from commodity-exporting countries, which will also positively affect commodity-exporting countries. Some empirical studies such as Kilian and Murphy (2014) show that oil inventory changes have no significant effect on oil prices suggesting that omitting inventories may not matter greatly. Furthermore, if the U.S. has to refill the decreased inventories of commodities soon after using them to meet the increased demand for commodities, inventories will not make a significant difference, since the U.S. will import commodities from commodity-exporting countries to refill the inventories.}

The main goal of this paper is to examine whether the model can replicate main properties of business cycle comovements between the U.S. and commodity-exporting countries well with focus on output and consumption correlations; (i) to show that the model yields higher output correlation between the two countries than the ST model\footnote{Almost all features of the ST model are the same as the model of Stockman and Tesar (1995). The utility function of the households in the ST model is different from that in Stockman and Tesar (1995). The economy in Stockman and Tesar (1995) grows at a constant rate, while there is no growth in the ST model.} and (ii) that the model produces higher output correlation than consumption correlation (mitigates the consumption correlation puzzle). Introducing commodities with consideration for idiosyncratic commodity trade structures of countries into the ST model can indeed help strengthen the connection between outputs in the two countries and lower the consumption correlation, and hence mitigate the consumption correlation puzzle. The model assumes that there is complementarity between commodities and noncommodity goods in consumption; this plays an important role in my analysis.\footnote{There are many empirical studies showing that they are complement (see Cooper (2003), Bodenstein, Erceg and Guerrieri (2011), etc.).}

I now summarize my findings. As for the output correlation, positive productivity shocks in the home country bring about rises in the home country’s aggregate output and its demand for imports of foreign tradable goods and commodities. This, in turn, leads to increases in the real prices of foreign tradable goods and commodities. The rise in home demand for imports...
of foreign tradable goods is offset by their increased real prices. However, this substitution effect for commodities is muted because of the complementarity between commodities and noncommodity goods in consumption. Hence, there is a larger positive spillover to the foreign country, which raises the output correlation.

Commodity consumption, which is not in the ST model, together with the complementarity plays a key role in lowering the consumption correlation and therefore mitigating the consumption correlation puzzle. When a positive productivity shock hits the home tradable sector, home commodity consumption and real commodity prices increase, and the real price of home tradable goods falls. Since foreign noncommodity consumption rises thanks to a fall in the real price of home tradable goods, despite the rise in real commodity prices, foreign commodity consumption goes up due to the complementarity. Thus, with respect to the shock, home and foreign commodity consumptions are positively correlated. On the contrary, in response to a positive productivity shock in the home nontradable sector, home aggregate output and commodity consumption rise. The real prices of commodity and tradable goods increase, since the increased home aggregate output puts upward pressure on them. Foreign noncommodity consumption drops due to the increased real price of tradable goods. Thus, foreign commodity consumption falls as well due to the complementarity and the increased real commodity prices. This means that home and foreign commodity consumptions are negatively correlated with respect to the shock. As a result, the correlation between home and foreign commodity consumptions is much lower than that between home and foreign noncommodity consumptions. Hence, the existence of commodity consumption in the model and the complementarity decrease the consumption correlation compared to the ST model. This enables the output correlation to be higher than the consumption correlation in the model, i.e. the model can mitigate the consumption correlation puzzle.

The paper is organised as follows. Section 2 shows the main properties of business cycle comovements between the U.S. and commodity-exporting countries. Section 3 describes the model where the home country produces tradable and nontradable goods, whilst the foreign country produces commodities, and tradable and nontradable goods. Calibration is also presented in the section. Section 4 presents the model analysis. I first show the responses of the model to productivity shocks and relate the responses to the Dutch disease. I then use the model to analyse the business cycle statistics that are produced by productivity shocks. Finally, I present sensitivity analysis of the model to the newly introduced features in comparison with the ST model. Section 5 concludes.
2. Properties of U.S. business cycle comovement

In this section, I first present the annual U.S. GDP correlations with GDPs of 60 major countries to identify whether U.S. GDP correlations with GDPs of commodity-exporting countries are significantly higher than those with GDPs of commodity-importing countries. Then, international business cycle statistics between the U.S and commodity-exporting OECD countries, and between the U.S. and commodity-importing OECD countries from annual data of 35 OECD member countries for the period of 1990-2015 are shown, and the main properties of the former and the differences between the former and latter with focus on the output and consumption correlations are briefly explained.\textsuperscript{8}

2.1. U.S. GDP comovement with commodity-exporting countries

Since the U.S. is a large economy, changes in its demand for imports of commodities affect commodity prices. Specifically, an increase in its output will increase its demand for imports of commodities, and thus commodity prices increase. Rises in U.S. demand for imports of commodities and commodity prices will have a positive effect on outputs in commodity-exporting countries, whilst an increase in commodity prices will adversely affect outputs in commodity-importing countries. Hence, it would be expected that the U.S. will have stronger GDP comovement with commodity-exporting countries than with commodity-importing countries.

Figure 1: GDP correlations with U.S. GDP and ratios of net commodity exports to GDP

*Notes:* GDP correlations with U.S. GDP are calculated by using detrended annual GDPs during 1990-2015 with Hodrick-Prescott filter. The values of net commodity exports/GDP are average of the period 1995-2011. Brunei Darussalam and Saudi Arabia are omitted in this figure. 

*Sources:* IMF and OECD

\textsuperscript{8} See Appendix A for details of the data.
Figure 1 shows the relationship between U.S. GDP correlations with 60 major countries’ GDPs and the ratios of net commodity exports to GDP of the 60 countries. However, contrary to initial expectation it seems that there is no clear relationship between them.

Since according to Zimmermann (1997), business cycle differences across countries can be explained by various factors such as countries’ sizes and distance between countries, to analyse whether U.S. GDP correlations with commodity-exporting countries are higher than those with commodity-importing countries, one needs to control for factors other than the ratio of net commodity exports to GDP that have influences on GDP correlations. Therefore, to control for other factors affecting GDP comovement, I define two groups of countries: advanced countries like the U.S., and countries in Europe or North America that are culturally and geographically close to the U.S. Figure 2 clearly shows that there is a positive relationship between GDP correlations with U.S. GDP and the ratios of net commodity exports to GDP of these groups of countries. Within these groups, U.S. GDP comovements with commodity-exporting countries are stronger than those with commodity-importing countries.

Figure 2: GDP correlations with U.S. GDP and ratios of net commodity exports to GDP

Notes: GDP correlations with U.S. GDP are calculated by using detrended annual GDPs during 1990-2015 with Hodrick-Prescott filter. The values of net commodity exports/GDP are average of the period 1995-2011. Advanced countries are based on IMF’s classification. Russia and Turkey are not regarded as Europe here, since their territories are in both Europe and Asia and they are not geographically and culturally close to the U.S.

Sources: IMF and OECD

2.2. Main properties of U.S. business cycle comovement with commodity-exporting countries

Table 1 shows international business cycle statistics for the U.S.: the correlations between U.S. GDP, consumption and investment, and the same variables for other major countries, and commodity-exporting and commodity-importing OECD countries.
The correlations between U.S. GDP and GDPs in other countries are positive and high. Canada has very large correlations, 0.854. The correlation for Japan (0.408) and Germany (0.375) are relatively small. More importantly, the correlation between U.S. GDP and GDP of commodity-exporting OECD countries (0.742) is substantially higher than that between U.S. GDP and GDP of commodity-importing OECD countries (0.501).

U.S. consumption is also positively correlated with consumptions in other countries except Japan. However, the consumption correlations are lower than the GDP correlation. Specifically, the U.S. consumption correlations with consumptions of commodity-exporting (0.582) and commodity-importing (0.262) OECD countries are smaller than the U.S. GDP correlations with GDPs of commodity-exporting (0.742) and commodity-importing (0.501) OECD countries.

Table 1: Correlations with same U.S. variables

<table>
<thead>
<tr>
<th>Variables</th>
<th>OECD</th>
<th>Commodity-exporting countries</th>
<th>Commodity-importing countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>0.742</td>
<td>0.662</td>
<td>0.854</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.582</td>
<td>0.653</td>
<td>0.624</td>
</tr>
<tr>
<td>Investment</td>
<td>0.473</td>
<td>-0.098</td>
<td>0.634</td>
</tr>
</tbody>
</table>

*Notes: Statistics are computed from the detrended data with Hodrick-Prescott filter. Sample period is 1990-2015. ‘Exporting’ and ‘Importing’ mean trade-weighted averages of commodity-exporting and commodity-importing OECD member countries, respectively.*

*Source: OECD*

In sum, U.S. output is more strongly correlated with outputs of commodity-exporting countries than those of commodity-importing countries. The output correlation between the U.S. and commodity-exporting countries is higher than the consumption correlation between them. These are the main properties of international business cycles between the U.S. and commodity-exporting countries that the model in this paper tries to replicate.

3. The model

In this section, I describe the two-country model with commodities where different commodity trade structures between countries are considered. This research builds on the open economy macro models of several papers (BKK; ST; Backus and Crucini (2000); CDL). The model is mainly based on ST. I only add commodities to the two-country, two-sector RBC model of ST.

To be specific, there are two countries: the home country (the U.S.) and the foreign country (commodity-exporting countries). The home country produces tradable and nontradable
goods and imports commodities, while the foreign country produces commodities as well as tradable and nontradable goods and exports commodities. Capital is sector-specific. Labour is mobile between sectors but immobile between countries. Asset markets are complete.

The main differences of this model from ST are that commodities as a complement to noncommodity goods are in the consumption basket of households, that commodities are used in the production of tradable goods as an input, and that the foreign country produces commodities whilst the home country does not.

I next explain the details of the model.\textsuperscript{9} I focus on the equations for the home country. Unless stated otherwise, there are symmetric equations for the foreign country.

3.1. Household

The representative household in each country maximises the following utility.

$$U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t - \frac{L_t^{1+\chi}}{1+\chi} \right)$$

where $\mathbb{E}_0$ is the conditional expectations operation, $C_t$ is the composite of tradable, nontradable and commodity consumption goods, $L_t$ is labour supply by households, $\beta$ is the discount factor, and $\chi$ is the inverse of Frisch elasticity of labour supply.

The budget constraint\textsuperscript{10} is

$$C_t + p_{H,t} I_{T,t} + p_{N,t} I_{N,t} + \mathbb{E}_t \{ Q_{t+1}^*, D_{t+1}^* \} = W_{T,t} L_{T,t} + W_{N,t} L_{N,t} + r_{T,t} K_{T,t} + r_{N,t} K_{N,t} + D_t + \Pi_t$$

with $L_t = L_{T,t} + L_{N,t}$. $Q_{t+1}^*, D_{t+1}^*, W_t$ and $\Pi_t$ denote the stochastic discount factor for one-period ahead real pay-offs relevant to the home household, the real wage and the profits remitted by firms, respectively. The subscripts ‘$T$’, ‘$N$’ and ‘$X$’ denote the tradable, nontradable and commodity sectors, respectively, and the subscript ‘$H$’ means the tradable goods produced in the home country. $p$, $I$, $K$ and $r$ are the real prices of goods, the investment, the capital and the real rental rate of capital, respectively.

The law of motion for capital in each sector is given by:

$$K_{j,t+1} = (1-\delta) K_{j,t} + I_{j,t}, \quad j \in \{ T, N, X \}$$

\textsuperscript{9} For the equilibrium conditions of the model, see Appendix B.

\textsuperscript{10} Since the commodity sector exists in the foreign country, the household’s budget constraint in the foreign country is $C_t + p_{F,t} I_{T,t} + p_{N,t} I_{N,t} + p_X I_{X,t} + \mathbb{E}_t \{ Q_{t+1}^*, D_{t+1}^* \} = W_{T,t} L_{T,t} + W_{N,t} L_{N,t} + W_X L_X + r_{T,t} K_{T,t} + r_{N,t} K_{N,t} + r_X K_X + D_t + \Pi_t$ with $L_t^* = L_{T,t}^* + L_{N,t}^* + L_X^*$. The superscript ‘*’ denotes the variables in the foreign country.
The composite of consumption goods, $C_t$, is defined as the following constant elasticity of substitution (CES) aggregator function of noncommodity consumption goods, $C_{NC,t}$, and commodity consumption goods, $C_{X,t}$.

$$C_t = \left\{ (1 - \gamma_0)^{\frac{1}{\eta_0}} C_{NC,t}^{\frac{\eta_0 - 1}{\eta_0}} + \gamma_0^{\frac{1}{\eta_0}} C_{X,t}^{\frac{\eta_0 - 1}{\eta_0}} \right\}^{\frac{\eta_0}{\eta_0 - 1}} \tag{4}$$

The noncommodity consumption goods, in turn, are similarly defined by tradable, $C_{T,t}$, and nontradable consumption goods, $C_{N,t}$:

$$C_{NC,t} = \left\{ (1 - \gamma_1)^{\frac{1}{\eta_1}} C_{T,t}^{\frac{\eta_1 - 1}{\eta_1}} + \gamma_1^{\frac{1}{\eta_1}} C_{N,t}^{\frac{\eta_1 - 1}{\eta_1}} \right\}^{\frac{\eta_1}{\eta_1 - 1}} \tag{5}$$

The tradable consumption goods consists of tradable consumption goods produced in the home country, $C_{H,t}$, and tradable consumption goods produced in the foreign country, $C_{F,t}$.

$$C_{T,t} = \left\{ (1 - \gamma_2)^{\frac{1}{\eta_2}} C_{H,t}^{\frac{\eta_2 - 1}{\eta_2}} + \gamma_2^{\frac{1}{\eta_2}} C_{F,t}^{\frac{\eta_2 - 1}{\eta_2}} \right\}^{\frac{\eta_2}{\eta_2 - 1}} \tag{6}$$

The corresponding price indices, $P_t$ (CPI), $P_{NC,t}$ (price of noncommodity consumption goods) and $P_{T,t}$ (price of tradable consumption goods), are following.\(^\text{11}\)

$$P_t = \left\{ (1 - \gamma_0) P_{NC,t}^{1 - \eta_0} + \gamma_0 P_{X,t}^{1 - \eta_0} \right\}^{\frac{1}{1 - \eta_0}} \tag{7}$$

$$P_{NC,t} = \left\{ (1 - \gamma_1) P_{T,t}^{1 - \eta_1} + \gamma_1 P_{N,t}^{1 - \eta_1} \right\}^{\frac{1}{1 - \eta_1}} \tag{8}$$

$$P_{T,t} = \left\{ (1 - \gamma_2) P_{H,t}^{1 - \eta_2} + \gamma_2 P_{F,t}^{1 - \eta_2} \right\}^{\frac{1}{1 - \eta_2}} \tag{9}$$

where $P_{X,t}$ is the price of commodity consumption goods, $P_{N,t}$ is the price of nontradable consumption goods, $P_{H,t}$ is the price of tradable consumption goods produced in the home country, and $P_{F,t}$ is the price of tradable consumption goods produced in the foreign country.

3.2. Firms

Nontradable good and commodity producers use labour and capital as inputs for the productions, while tradable good producers use commodities as well as labour and capital in production. Both countries produce tradable and nontradable goods with the same production function, but only the foreign country produces commodities.

\(^\text{11}\) The uppercase letter $P$ denotes nominal prices.
Nontradable good Producers

Nontradable good producers use labour and capital in production, and both countries produce nontradable goods with the identical production function.

\[ Y_{N,t} = A_{N,t} K_{N,t}^{\alpha_N} L_{N,t}^{1-\alpha_N} \] (10)

where \( Y_{N,t} \) is the output of nontradable goods, \( A_{N,t} \) is the productivity in the nontradable production, \( K_{N,t} \) is the capital used in the nontradable production, \( L_{N,t} \) is the labour, and \( 1 - \alpha_N \) is the labour share of income.

 Tradable good Producers

Both countries produce tradable goods by using capital, labour and commodities as inputs. The production function of tradable goods is a nested CES as in Kim and Loungani (1992) and Backus and Crucini (2000).

\[ Y_{H,t} = A_{T,t} \left\{ (1 - a) K_{T,t}^{\nu-1} + a x_t^{\nu-1} \right\}^{\frac{1}{\nu-1}} L_{T,t}^{1-\alpha} \] (11)

where \( Y_{H,t} \), \( A_{T,t} \), \( K_{T,t} \), \( x_t \) and \( L_{T,t} \) denote the output, productivity, capital, commodities and labour for the production of tradable goods, respectively. \( 1 - \alpha \) is the labour share of income. \( a \) and \( \nu \) determine the importance of commodities and the elasticity of substitution between capital and commodities.

Commodity Producers

Commodity producers use capital and labour as inputs in production, and only the foreign country produces commodities. The production function is Cobb-Douglas as in Arora and Gomis-Porqueras (2011) and Huynh (2016).

\[ Y_{X,t}^* = A_{X,t}^* K_{X,t}^{\alpha_X} L_{X,t}^{1-\alpha_X} \] (12)

where \( Y_{X,t}^* \), \( A_{X,t}^* \), \( K_{X,t} \) and \( L_{X,t} \) are the output, productivity, capital and labour for the commodity production, respectively. \( 1 - \alpha_X \) is the labour share of income.

\[ \text{12 Some papers such as Bodenstein, Erceg and Guerrieri (2011) assume that commodities are endowments rather than goods produced. If commodities are endowments, commodity prices should be exogenous. However, the important channel for changes in commodity prices and output in the commodity sector to respond to productivity shocks in the model will be omitted if commodity prices are exogenous (see Section 4 for details). Therefore, I assume that the commodities are produced rather than being endowments.} \]
3.3. Resource constraints

Since output in the nontradable sector in the home country has to be the same as home country’s demand for nontradable goods and its investment and output in the tradable sector in the home country must equal the world demand for the tradable goods produced in the home country and its investment, the resource constraints of the home country can be written as:

\[ Y_{N,t} = C_{N,t} + I_{N,t} \]  

\[ Y_{H,t} = C_{H,t} + C^*_{H,t} + I_{T,t} \]  

(13)

(14)

3.4. Calibration

Table 2 lists the parameter values for the model which are based on U.S. annual data. I assume symmetry between the two countries following standard international business cycle literature such as ST and CDL, with exception of the commodity sector.

Table 2: Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>0.96</td>
<td>Discount factor</td>
</tr>
<tr>
<td>( \chi )</td>
<td>3</td>
<td>Inverse of Frisch elasticity of labour supply</td>
</tr>
<tr>
<td>( \delta )</td>
<td>0.1</td>
<td>Depreciation rate of capital</td>
</tr>
<tr>
<td>( 1 - \alpha_N )</td>
<td>0.56</td>
<td>Labour share in the nontradable sector</td>
</tr>
<tr>
<td>( 1 - \alpha )</td>
<td>0.61</td>
<td>Labour share in the tradable sector</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.999</td>
<td>Elasticity of substitution between commodities and capital in the tradable production</td>
</tr>
<tr>
<td>( a )</td>
<td>0.1823</td>
<td>Parameter related to the weight of commodities in the tradable production</td>
</tr>
<tr>
<td>( 1 - \alpha_X )</td>
<td>0.6568</td>
<td>Labour share in the commodity sector</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>0.112</td>
<td>Weight of commodities in the composite of consumption goods</td>
</tr>
<tr>
<td>( \eta_0 )</td>
<td>0.3</td>
<td>Elasticity of substitution between commodity and noncommodity consumption goods</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>0.5631</td>
<td>Weight of nontradable goods in the noncommodity consumption goods</td>
</tr>
<tr>
<td>( \eta_1 )</td>
<td>0.44</td>
<td>Elasticity of substitution between tradable and nontradable goods</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>0.3557</td>
<td>Weight of foreign tradable goods in the tradable consumption goods in the home country</td>
</tr>
<tr>
<td>( \gamma^*_2 )</td>
<td>0.7749</td>
<td>Weight of home tradable goods in the tradable consumption goods in the foreign country</td>
</tr>
<tr>
<td>( \eta_2 )</td>
<td>1.1</td>
<td>Elasticity of substitution between home and foreign tradable goods</td>
</tr>
</tbody>
</table>

Certain parameter values are borrowed from the standard literature. The discount factor, \( \beta \), is assumed to be 0.96 following ST and CDL. The inverse of Frisch elasticity of labour supply, \( \chi \), is set to 3 as in Galí and Monacelli (2005). The depreciation rates of capitals for the tradable and nontradable productions, \( \delta \), are 0.1 as in ST. The depreciation rate of capital for

\[ Y^*_X,t = C^*_X,t + C^*_X,t + X_t + X^*_t + I^*_X,t. \]

---

13 Since output in the commodity sector in the foreign country should be equal to the world demand for commodities and its investment, the resource constraint of the commodity sector in the foreign country is

\[ Y^*_X,t = C^*_X,t + C^*_X,t + x_t + x^*_t + I^*_X,t. \]
the commodity production is also assumed to be 0.1 as in Arora and Gomis-Porqueras (2011) and Huynh (2016) using the same depreciation rate of capital for the energy production as the production of other sectors. Following ST, the labour shares of income in the nontradable \((1 - \alpha_N)\) and tradable \((1 - \alpha)\) sectors are chosen to be 0.56 and 0.61, respectively.

The elasticity of substitution between capital and commodities in the tradable production, \(\nu\), is assumed to be 0.999 following Kim and Loungani (1992). They consider two cases, \(\nu = 0.999\) and \(\nu = 0.588\). They refer to the former case as the “Cobb-Douglas” case and the latter as the “CES” case. Considering that the Cobb-Douglas case replicates the business cycle data better than the CES case in Kim and Loungani (1992), this model uses \(\nu = 0.999\). Nevertheless, since \(\nu\) is a new parameter from the introduction of commodities to the ST model, various values of \(\nu\) ranging from 0.5 to 1 based on previous studies are examined to check whether changes in \(\nu\) lead to different simulation results of the model. I assume that the steady state capital/commodity ratio in the tradable production, \(K_T/x\), is 31.55 which is different from Kim and Loungani (1992) and others.\(^\text{14}\) Kim and Loungani (1992) use a value of 50 as the ratio. However, in this model, firms use all types of commodities rather than only energy. Because the weight of energy in the IMF’s commodity price index is 0.631, \(50 \times 0.631 = 31.55\) as the steady state capital/commodity ratio is appropriate. The parameter related to the weight of commodities in the tradable production, \(a\), is set to 0.1823. This value is determined by the values of \(\nu\), \(K_T/x\), \(\beta\) and \(\delta\).\(^\text{15}\) Based on the assumption that real wage in each sector is identical in the model due to labour mobility between sectors, the labour share of income in the commodity sector, \(1 - \alpha_X\), is set to 0.6568. These parameter values are consistent with the steady state ratio of investment to output in each sector, \(I_N/Y_N = 0.31\) and \(I_T/Y_H = 0.23\), and the steady state ratio of aggregate investment to aggregate output, \(I/Y = 0.27\) which is very close to the value in the standard RBC literature (0.25). Accordingly, the steady state ratio of output in the nontradable sector to aggregate output, \(Y_N/Y\), is 0.51 which closely matches the average ratio of output in the service sector\(^\text{16}\) to GDP for 1997-2015\(^\text{17}\) from the NIPA data (0.54).\(^\text{18}\)

The parameters associated with consumption are as follows. The weight of commod-

\(^{14}\) Variables without time subscript ‘t’ denote their steady state values.

\(^{15}\) From the first order conditions of households and firms in steady state, \(a = \frac{1}{1+(1/\beta-1+\delta)(K_T/x)^\nu}.\)

\(^{16}\) I take the service sector to be wholesale and retail trades, transportation and warehousing, finance, insurance, real estate, rental, leasing, professional and business services, educational services, health care, social assistance, arts, entertainment, recreation, accommodation, food services, and other services (except government).

\(^{17}\) Because the NIPA data on real gross output by industry is only available from 1997, I use the data for 1997-2015.

\(^{18}\) Since both countries have identical tradable and nontradable sectors, \(I_N^*/Y_N^*\) and \(I_T^*/Y_T^*\) are the same as those in the home country. For the foreign commodity sector, the values of \(\alpha_X\), \(\delta\) and \(\beta\) are consistent with \(I_X^*/Y_X^* = 0.24\). Foreign commodity output is determined by the market clearing condition, which implies \(Y_N^*/Y^* = 0.52\), \(Y_N^*/Y^* = 0.21\) and \(Y_N^*/Y^* = 0.27\).
ity consumption goods in the composite of consumption goods, $\gamma_0$, is set to 0.112 which is consistent with the NIPA data, the average ratio of commodity consumption\(^{19}\) to total consumption for 1990-2015. Since $\gamma_0$ is newly introduced due to the commodity consumption, various values of $\gamma_0$, from the minimum (0.1) to maximum (0.13) for 1990-2015, are examined in the sensitivity analysis. The elasticity of substitution between commodity and noncommodity consumption goods, $\eta_0$, is 0.3 following Natal (2012). Because $\eta_0$ does not appear in the ST model, a variety of values of $\eta_0$ in previous studies, from 0.1 to 1, are used to see whether the main results of this paper are sensitive to change in $\eta_0$. The weight of nontradable consumption goods in the noncommodity consumption goods, $\gamma_1$, is 0.5631 which is different from ST. Since commodities are part of tradable goods in ST, $\gamma_1$=0.5631 is consistent with the fact that the ratio of consumption of all tradable goods (commodities and tradable goods in the model) to nontradable consumption is 1, which is equal to ST’s assumption. The elasticity of substitution between tradable and nontradable consumption goods, $\eta_1$, is 0.44 which is borrowed from ST. The weight of tradable consumption goods produced in the foreign country in the tradable consumption goods in the home country, $\gamma_2$, is 0.3557. Considering that commodities are also tradable goods, under $\gamma_2$=0.3557 the steady state ratio of imported commodity and tradable consumption goods to total tradable consumption goods in the home country is 0.5, i.e. $(C_X + C_F)/(C_T + C_X) = 0.5$. This is consistent with ST where the steady state ratio of imported tradable consumption goods to total tradable consumption goods is 0.5. The weight of tradable consumption goods produced in the home country in the tradable consumption goods in the foreign country, $\gamma_2^*$, is 0.7749, which is a result of the assumptions that in the steady state households’ labour supply in the home country is the same as that in the foreign country ($L = L^*$) and that the steady state values of aggregate consumptions in both countries are equal ($C = C^*$). Under these assumptions, from the first order condition of households’ problems in steady state, the steady state real wages in both countries are equal. The elasticity of substitution between home and foreign tradable consumption goods, $\eta_2$, is set to 1.1 following Bodenstein, Erceg and Guerrieri (2011). These parameter values are set to equate the steady state ratio of aggregate consumption to aggregate output in the home country, $C/Y$, to 0.7 which is the same as the average of the value in the standard RBC literature (0.7) and close to the average ratio of private consumption expenditure to GDP for 1990-2015 from the NIPA data (0.66). This implies that $C^*/Y^* = 0.71$.

Finally, productivities in the tradable and nontradable sectors in the home and the foreign countries follow AR(1) process as in BKK, ST and CDL.

\[
A_t = \rho A^T_{t-1} + \varepsilon_t
\]  

\(^{19}\)Gasoline and other energy goods, plus food and beverages purchased for off-premises consumption
where $A_t = [A_{T,t}, A_{T,t}^*, A_{N,t}, A_{N,t}^*]^T$ and $\varepsilon_t = [\varepsilon_T, \varepsilon_T^*, \varepsilon_{N,t}, \varepsilon_{N,t}^*]^T$ is the vector of productivity shocks. As in BKK and CDL, to estimate the process I use annual data of “Index of output per hour in manufacturing” and “Index of output per hour in private services” for the period 1970-2015 from BLS as the productivities in the home tradable and nontradable sectors, and as the productivities in the foreign tradable and nontradable sectors average of commodity-exporting OECD countries’ indices of manufacturing output and output in services divided by sectoral total employment for the period 1970-2015 from the OECD STAN sectoral database are used.\footnote{Since the data for Australia, Canada, Chile and Mexico is not available, only the data for Denmark, the Netherlands and Norway is used for the estimation.} The coefficient matrix, $\rho$, and the variance-covariance matrix of productivity shocks, $\text{Var}(\varepsilon_t)$ (in per cent), are

$$
\rho = \begin{pmatrix}
0.87 & -0.04 & 0.10 & 0.14 \\
-0.04 & 0.87 & 0.14 & 0.10 \\
0.01 & -0.03 & 0.95 & 0.05 \\
-0.03 & 0.01 & 0.05 & 0.95
\end{pmatrix}, \quad 
\text{Var}(\varepsilon_t) = \begin{pmatrix}
0.0453 & 0.0049 & 0.0027 & 0.0023 \\
0.0049 & 0.0453 & 0.0023 & 0.0027 \\
0.0027 & 0.0023 & 0.0036 & 0.0006 \\
0.0023 & 0.0027 & 0.0006 & 0.0036
\end{pmatrix}
$$

The values for the elements in the above matrices are similar to those in CDL.

4. Model analysis

In this section, I further assume that the productivity in the commodity production is fixed, while the productivities in the tradable and nontradable sectors are stochastic as in ST. This is because the main aim of this paper is to show that instead of adding additional shocks, adding commodities and idiosyncratic commodity trade structures of countries to the ST model can help produce better international business cycle statistics compared to the ST model.

4.1. Impulse responses

In order to analyse how home and foreign variables in the model fluctuate differently from those in the ST model in response to productivity shocks and thus to understand how the international business cycles statistics that the model produces are improved compared to those that the ST model generates, in this section I study how variables in the model and the ST model respond to a positive 1% tradable and nontradable productivity shocks in the home country. In addition, in order to obtain deeper intuition, I relate the impulse responses of the foreign country to a shock raising foreign commodity output to the “Dutch disease.” For simplicity, I abstract from the productivity spillover implied in the coefficient
matrix of productivities and variance-covariance matrix of productivity shocks only in impulse responses, i.e. the off-diagonal elements of $\rho$ and $\text{Var}(\varepsilon_t)$ are all zero. I will use this impulse responses to explain my findings about international business cycle statistics.

4.1.1. A positive 1% productivity shock in the home tradable sector

Figure 3 shows the responses of the model and the ST model to a positive 1% productivity shock in the home tradable sector.

First, consider the home country. In both models, a positive productivity shock in the home tradable sector increases output, $Y_H$, and consumption, $C_H$, in the sector and decreases the real prices of tradable goods in the home country, $p_T$, mainly due to a large fall in tradable goods produced in the home country, $p_H$. However, the increase in $Y_H$ in the ST model is greater than that in the model. This is because, differently from the ST model, in the model an increase in $Y_H$ puts upward pressure on demands for commodities as well as nontradable goods, which raises real commodity prices, $p_X$ and $p^*_X$, and the real price of nontradable goods, $p_N$. Since commodities are used in the production of tradable goods as an input, a rise in $p_X$ means an increase in the marginal costs of the firms in the tradable sector. Thus, the shock has a slightly bigger positive effect on $Y_H$ in the ST model which does not have commodities than in the model. Nevertheless, consumption of tradable goods, $C_T$, in the model rises by more than that in the ST model due to a bigger fall in $p_T$. Because in the model

$$p_1 = (1 - \gamma_0) p_{NC,t}^{1-\eta_0} + \gamma_0 p_X^{1-\eta_0}$$

which can be expressed as

$$1 = (1 - \gamma_0) p_{NC,t}^{1-\eta_0} + \gamma_0 p_X^{1-\eta_0}$$

dividing by $p_1$, and $p_X$ goes up, the real price of noncommodity goods, $p_{NC}$, falls. In the model

$$p_{NC,t}^{1-\eta_0} = (1 - \gamma_1) p_{TC,t}^{1-\eta_1} + \gamma_1 p_{NC,t}^{1-\eta_1}$$

while in the ST model which does not have commodities

$$1 = (1 - \gamma_1) p_{TC,t}^{1-\eta_1} + \gamma_1 p_{NC,t}^{1-\eta_1}$$

Therefore, $p_N$ in the model increase by less than that in the ST model, but $p_T$ in the model falls by more than that in the ST model. Despite the increased $p_N$, consumption of nontradable goods, $C_N$, in the model rises slightly thanks to the complementarity between $C_T$ and $C_N$ ($\eta_1 < 1$). On the contrary, $C_N$ in the ST model goes down slightly, since in spite of the complementarity a rise in $C_T$ is smaller and an increase in $p_N$ is bigger compared to the model. Hence, a fall in output in the nontradable sector, $Y_N$, in the model is smaller than the ST model. Because of rises in $C_T$ and $C_N$, noncommodity consumption, $C_{NC}$, increases, which leads to a rise in commodity consumption, $C_X$, in the model despite the increased $p_X$ due to the complementarity between $C_{NC}$ and $C_X$ ($\eta_0 < 1$). A rise in $Y_H$ leads to an increase in demand for commodities in the production in the home tradable sector, $x$. Finally, from

$$p_T^{1-\eta_2} = (1 - \gamma_2) p_H^{1-\eta_2} + \gamma_2 p_{F,t}^{1-\eta_2}$$

since $p_T$ in the model goes down by more than that in the ST model, $p_H$ in the model fall by more compared to the ST model. Hence, the rise in $C_H$ in the model is greater than the ST model. Due to a larger fall in $p_H$ than $p_F$, consumption of foreign tradable goods, $C_F$,
drops. Overall, aggregate output and consumption in the home country, $Y$ and $C$, increase, and the increase in $Y$ of the model is smaller than that of the ST model while the increase in $C$ of the model is bigger than that of the ST model.

As for the foreign country, a fall in $p_T$ and a rise in $p_F$ lead to a decrease in $p_T^*$ and an increase in $p_F^*$, respectively. Since the fall in $p_T$ and the rise in $p_F$ in the model are bigger than those in the ST model, the decrease in $p_T^*$ and the increase in $p_F^*$ in the model are also larger than those in the ST model. Thus, $C_T^*$ in the model jumps by more than that in the ST model, and $C_F^*$ in the model drops by more than that in the ST model, which brings about a greater fall in $Y_F^*$ in the model compared to that in the ST model. Since $p_T^*$ in the model decrease by more than that in the ST model, a rise in $p_N^*$ in the model is larger than that in the ST model. In spite of an increase $p_N^*$, $C_N^*$ rises thanks to the complementarity between $C_T^*$ and $C_N^*$. Despite a greater increase in $C_T^*$ in the model, a bigger rise in $p_N^*$ in the model compared to the ST model causes $C_N^*$ in the model to increase by slightly less than that in the ST model. Accordingly, $Y_N^*$ in the model jumps by less compared to the ST model. $C_{NC}^*$ increases owing to rises in $C_T^*$ and $C_N^*$, and thus despite the increased $p_X^*$, $C_X^*$ rises thanks to the complementarity between $C_{NC}^*$ and $C_X^*$. $x^*$ falls because of a decrease in $Y_F^*$. Due to rises in $C_X$, $C_X^*$ and $x$, foreign commodity output, $Y_X^*$, goes up. Overall, $Y^*$ and $C^*$ increase. However, rises in $Y^*$ and $C^*$ in the model are bigger than those in the ST model mainly due to increases in $Y_X^*$ and $C_X^*$ that are not in the ST model.
(g) F nontradable output  (h) F nontradable consumption  (i) F tradable output

(j) F tradable consumption  (k) F commodity output  (l) Commodity consumption

(m) F aggregate output  (n) F aggregate consumption  (o) Real prices of nontradables

(p) Real prices of tradables  (q) Real commodity prices  (r) Real exchange rates

Figure 3: Responses to a positive productivity shock in the home tradable sector

Notes: ‘H’ and ‘F’ mean the home and the foreign countries, respectively.
Application to the “Dutch disease”

The responses of the foreign country to the shock can be related to a typical example of the Dutch disease model of Corden and Neary (1982). According to them, a boom in the commodity sector can have effects on the economy in two ways: (i) the “resource movement effect” where the boom in the commodity sector causes a rise in the marginal product of labour (the mobile factor) in the sector and thus draws labour, and (ii) the “spending effect” where the increased real income resulting from the boom leads to an increase in consumption of nontradables. The spending effect results in an increase in prices of nontradable goods. Since prices of tradable goods are set by the world (they consider a small open economy), real exchange rates appreciate. Consequently, the boom leads to a rise in output in the nontradable sector but a fall in output in the tradable sector.

All aspects in their model can be shown by the responses of the foreign country in the model to the positive shock in the home tradable sector, except that real exchange rates do not appreciate since the foreign country is not a small open economy in this model.

![Graphs](image)

(a) F marginal product of labour  (b) F labour in each sector  (c) F output in each sector

Figure 4: Responses of the model to a positive productivity shock in the home tradable sector

Notes: ‘F’ means the foreign country.

Figure 4 shows the responses of the foreign country in the model to a positive 1% productivity shock in the home tradable sector. The shock leads to an increase in home demand for commodities. Since the home country imports commodities from the foreign country, the shock results in a boom in the foreign commodity sector (see (k) in Figure 3). The boom leads to a rise in the marginal product of labour in the sector, and thus the sector draws labour from other sectors. Labour in the sector jumps. This explains the resource movement effect well. With regard to the spending effect, the boom triggered by the shock brings about a rise in consumption of nontradable goods in the foreign country (see (h) in Figure 3), which causes an increase in the real price of nontradable goods (see (o) in Figure 3). Accordingly, labour in the foreign nontradable sector rises. Since the foreign commodity and nontradable sectors draw labour, labour in the foreign tradable sector falls. Therefore, the boom causes a rise in output in the foreign nontradable sector and a fall in output in the foreign tradable
sector (see (g) and (i) in Figure 3).

4.1.2. A positive 1% productivity shock in the home nontradable sector

Figure 5 presents responses of the model and the ST model to a positive 1% productivity shock in the home nontradable sector. Since the responses of each sector in both models are very similar, I do not compare responses of each sector in the two models. Instead, I focus on comparing responses of aggregate variables in the two models.

With regard to the home country, in response to a positive productivity shock in the home nontradable sector, output, $Y_N$, and consumption, $C_N$, in the sector rise, and its real price, $p_N$, falls. Since an increase in $Y_N$ puts upward pressure on the real prices of tradable goods produced in both countries and commodities in both countries, $p_H$, $p_F$, $p^*_H$, $p^*_F$, $p_X$ and $p^*_X$ all rise. Thus, the real prices of tradable goods in the home country, $p_T$, increases. Despite the increased $p_T$, home tradable consumption, $C_T$, rises thanks to the complementarity between $C_N$ and $C_T$ ($\eta_1 < 1$). Due to the rise in $C_T$, $C_H$ and $C_F$ increase. Because the nontradable sector draws labour thanks to the shock, labour in the tradable sector falls, which leads to a slight fall in output in the sector, $Y_H$. A fall in $Y_H$ causes a decrease in commodity input for the production in the sector, $x$. Despite the increased $p_X$, commodity consumption in the home country, $C_X$, goes up owing the complementarity between $C_{NC}$ and $C_X$. Consequently, home aggregate output and consumption, $Y$ and $C$, jump in response to the shock, but the increase in $Y$ in the model is slightly lower than that in the ST model because of the slightly bigger fall in $Y_H$ in the model than that in the ST model. $C$ in both models increase by almost the same degree.

Now, consider the foreign country. Rises in $p^*_F$ and $p^*_H$, and therefore $p^*_T$, cause decreases in $C^*_H$ and $C^*_F$ and thus $C^*_T$. A rise in $p^*_T$ brings about a fall in $p^*_N$. Although $p^*_N$ falls, $C^*_N$ decreases due to the complementarity between $C^*_T$ and $C^*_N$. Due to falls in $C^*_T$ and $C^*_N$, $C^*_{NC}$ decreases. Owing to the complementarity between $C^*_X$ and $C^*_{NC}$ and a rise in $p^*_X$, $C^*_X$ also falls. Thanks to the increased $C_F$, demand for capital in the foreign tradable sector increases, which brings about a rise in its investment, $I^*_T$. Therefore, output in the foreign tradable sector, $Y^*_F$, goes up, which leads to an increase in $x^*$. Increases in $C_X$ and $x^*$ cause a jump in output in the foreign commodity sector, $Y^*_X$. A fall in $C^*_X$ brings about a decrease in $Y^*_X$. As a result, $Y^*$ increases, but $C^*$ falls. Owing to a rise in $Y^*_X$ that is not in the ST model, $Y^*$ in the model increases by more than that in the ST model does, and due to a fall in $C^*_X$ which is not in the ST model, $C^*$ in the model decreases by more than that in the ST model.
The model
The ST model

(a) H nontradable output
(b) H nontradable consumption
(c) H tradable output

(d) H tradable consumption
(e) H aggregate output
(f) H aggregate consumption

(g) F nontradable output
(h) F nontradable consumption
(i) F tradable output

(j) F tradable consumption
(k) F commodity output
(l) Commodity consumption
Figure 5: Responses to a positive productivity shock in the home nontradable sector

Notes: ‘H’ and ‘F’ mean the home and the foreign countries, respectively.

4.2. International business cycles

Table 3 shows the business cycle statistics from the model and the ST model driven by productivity shocks, in terms of the relative standard deviations to output, correlations with output of the home country, and correlations between the home and the foreign countries. Nonetheless, since the main aim of this paper is to show that the output correlation in the model is higher than the consumption correlation in the model and the output correlation in the ST model, I focus on the output and consumption correlations between the two countries. The ST model is unsuccessful in replicating the main properties of business cycle comovement between the U.S. and commodity-exporting countries explained in Section 2. Specifically, the output correlation between the two countries is too low compared to the data. The consumption correlation between the two is too high and greater than the output correlation. These are very different from the data, but typical in open economy macro models.

The model, however, produces much closer international business cycle statistics to the data than the ST model.
Table 3: Business cycle statistics

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Data</th>
<th>Model</th>
<th>ST</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>S.D. relative to GDP in home</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.986</td>
<td>0.498</td>
<td>0.466</td>
</tr>
<tr>
<td>Investment</td>
<td>3.389</td>
<td>2.987</td>
<td>2.553</td>
</tr>
<tr>
<td>Real net exports over GDP</td>
<td>0.361</td>
<td>0.135</td>
<td>0.046</td>
</tr>
<tr>
<td><strong>Correlation with GDP in home</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption</td>
<td>0.938</td>
<td>0.857</td>
<td>0.809</td>
</tr>
<tr>
<td>Investment</td>
<td>0.957</td>
<td>0.981</td>
<td>0.973</td>
</tr>
<tr>
<td>Real net exports over GDP</td>
<td>-0.766</td>
<td>-0.975</td>
<td>0.102</td>
</tr>
<tr>
<td><strong>Correlation between home and foreign</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>0.742</td>
<td>0.790</td>
<td>0.659</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.582</td>
<td>0.780</td>
<td>0.813</td>
</tr>
<tr>
<td>Investment</td>
<td>0.473</td>
<td>0.599</td>
<td>0.484</td>
</tr>
</tbody>
</table>

Notes: The correlation between home and foreign from the data is trade-weighted average correlation between the same variables of the U.S. and commodity-exporting OECD countries. Sample period is 1990-2015.

Source: OECD

With regard to the output correlation, it becomes higher than that in the ST model. This is consistent with the empirical finding in Section 2 that GDPs of commodity-exporting countries are more positively correlated with the U.S. GDP than those of commodity-importing countries. The intuition for the high output correlation in the model can be seen in the impulse responses. Any positive productivity shock in the home country increases its aggregate output and raises home demand for imports of commodities and real commodity prices, which brings about a rise in output in the foreign commodity sector. In other words, since output in the foreign commodity sector and aggregate output in the home country are positively correlated with respect to productivity shocks, including the commodity sector only in the foreign country helps raise the output correlation. More formally, positive productivity shocks in the home country raise its aggregate output and consumption, which leads to an increase in its demand for imports of foreign tradable goods and commodities. This causes rises in the real prices of foreign tradable goods and commodities. The rise in home demand for imports of foreign tradable goods is offset by their increased real prices. However, this substitution effect for commodities is muted thanks to the complementarity between commodity and noncommodity consumption goods ($\eta_0 < 1$). Therefore, exports of commodities from the foreign country rise, and thus there is a larger positive spillover to the foreign country compared to the ST model where the foreign country does not have the commodity sector. As a result, the output correlation in the model becomes 0.790 which is greater than that in the ST model (0.659) and close to the data (0.742).

With regard to the consumption correlation, it decreases compared to the ST model. Accordingly, the output correlation is higher than the consumption correlation, i.e. the consumption correlation puzzle is mitigated. The intuition for the fall in the consumption
correlation can be seen in the impulse responses of commodity consumption which is not in the ST model. When a positive productivity shock hits the home tradable sector, home commodity consumption and real commodity prices rise, and foreign noncommodity consumption increases as well. Despite the rise in real commodity prices, foreign commodity consumption goes up due to the complementarity between commodity and noncommodity consumption goods. On the contrary, in response to a positive productivity shock in the home nontradable sector, home commodity consumption and real commodity prices rise, but foreign noncommodity consumption drops. Thus, foreign commodity consumption falls due to the complementarity and increased real commodity prices, i.e. home and foreign commodity consumptions are positively correlated with respect to productivity shocks in the tradable sector, whilst they are negatively correlated with respect to productivity shocks in the nontradable sector. Due to the negative correlation between them with respect to productivity shocks in the nontradable sector, the correlation between home and foreign commodity consumptions (0.642) is much lower than that between home and foreign noncommodity consumptions (0.972) in the model, which enables the model to generate lower consumption correlation compared to the ST model. Consequently, the cross-country correlation of aggregate consumption is 0.780 in the model which is smaller than that in the ST model (0.813) and the output correlation in the model (0.790). Nevertheless, the consumption correlation is still higher than the data (0.582).

Furthermore, the model also produces improved business cycle statistics compared to the ST model in many respects. To be specific, the standard deviation of consumption relative to GDP in the home country in the ST model is 0.466 which is much lower than the data (0.986). However, its value in the model is 0.498, closer to the data than that in the ST model. The relative standard deviation of investment to GDP in the home country in the data (3.389) is high but that in the ST model (2.553) is too low in comparison with the data. It becomes higher in the model (2.987) than in the ST model. The relative standard deviation of real net exports over GDP to GDP in the home country in the model (0.135) is higher and closer to the data (0.361) than that in the ST model (0.046). The correlation of consumption with GDP in the home country in the model (0.857) is greater and closer to the data (0.938) than that in the ST model (0.809). The correlation between home output and net exports over output is negative in the data (-0.766), but it is positive in the ST model (0.102). However, the correlation in the model (-0.975) is negative and closer to the data compared to the ST model. On the contrary, the model generates worse correlations between home output and investment, and between the two countries’ investments. Specifically, the

\[ \text{Var}(\varepsilon_t) \]

\[ \text{Var}(\varepsilon_t) \]

---

21 Since productivity shocks to the nontradable sector are far weaker than those to the tradable sector as usually shown in previous studies, the consumption correlation between home and foreign commodity consumptions is positive. The standard deviation of the latter is about 3.5 times larger than the former which is implied in variance-covariance matrix of productivity shocks, \( \text{Var}(\varepsilon_t) \).
correlation between home output and its investment is 0.981 in the model which is greater than that in the ST model (0.973) and the data (0.957). The investment correlation between the two countries in the model is 0.599 which is higher than that in the ST model (0.484) and the data (0.473).

Considering that the model produces better business cycle comovement between the U.S. and commodity-exporting countries from the perspectives of the output and consumption correlations compared to the ST model, incorporating commodities and idiosyncratic commodity trade structures of countries into an open economy macro model is very useful. The introduction of these features into a standard two-country model raises the output correlation and mitigates the consumption correlation puzzle.

4.3. Sensitivity analysis

In the model, the parameters for the elasticity of substitution between commodity and non-commodity consumption goods ($\eta_0$), the weight of commodity consumption goods in the composite consumption goods ($\gamma_0$), and the elasticity of substitution between commodities and capital in the production of tradable goods ($\nu$) are new parameters due to the introduction of commodities compared to the ST model. Thus, whether the main results of the model (higher output correlation than the consumption correlation and the output correlation in the ST model) are sensitive to changes in the values of these three parameters needs to be assessed. To do so, I set reasonable ranges of the parameter values. Specifically, $\eta_0 \in [0.1, 1]$ which is based on previous studies. $\gamma_0 \in [0.1, 0.13]$ which matches the minimum and maximum values in the NIPA data for 1990-2015. I set a range of $\nu \in [0.5, 1]$ again based on previous studies. Figure 6 shows the output and consumption correlations between the two countries according to changes in the values of these three parameters.

**Elasticity of substitution between commodity and noncommodity goods ($\eta_0$)**

(a) in Figure 6 shows the output and consumption correlations between the two countries according to changes in the elasticity of substitution between commodity and noncommodity consumption goods ($\eta_0$).

As already shown, a rise in the output correlation and a fall in the consumption correlation are due to the complementarity between commodity and noncommodity consumption goods. A rise in $\eta_0$ means weaker complementarity. Hence, it would be expected that a rise in $\eta_0$ will decrease the output correlation and increase the consumption correlation, holding other

\[22\] In fact, the parameter related to the weight of commodities in the production of tradable goods (a) is also new one. However, since the discount factor ($\beta$), the depreciation rate ($\delta$), the steady state ratio of capital to commodities in the tradable production ($K_T/x$) and $\nu$ determine its value, the assessment of the sensitivity to changes in its value is not conducted.
factors constant. However, since $\eta_0$ is directly related to commodity prices (see Equation (7)), changes in it affect the responses of real commodity prices to the productivity shocks, which also have impacts on the output and consumption correlations. Consequently, as $\eta_0$ rises, changes in the output correlation are not noticeable, but the consumption correlation increases. Nevertheless, as can be seen in (a) of Figure 6, the fluctuation is small, and thus as long as $\eta_0$ is in reasonable ranges, the main results of this paper hold.

**Weight of commodity goods in the composite consumption goods ($\gamma_0$)**

Now consider the changes in the weight of commodity consumption goods in the composite consumption goods ($\gamma_0$). (b) in Figure 6 presents the output and consumption correlations in the model according to the various values of $\gamma_0$.

Conversely to $\eta_0$ in which commodity consumption is decreasing, commodity consumption is increasing in $\gamma_0$. This can be seen in the households’ optimal commodity consumption allocation, $C_{X,t} = \gamma_0 p_{X,t}^{\eta_0} C_t$. Thus, an increase in $\gamma_0$ and a fall in $\eta_0$ have very similar effects on the output and consumption correlations, i.e. it would be expected that a rise in $\gamma_0$ will increase the output correlation and decrease the consumption correlation, holding other factors constant. Although like $\eta_0$, $\gamma_0$ is directly related to commodity prices (see Equation (7)), the impact of changes in $\gamma_0$ on real commodity prices is much weaker than $\eta_0$. As a result, consistently with the initial expectation, as $\gamma_0$ increases, the output correlation goes up and the consumption correlation falls. Overall, the output correlation is always higher than that in the ST model and the consumption correlation as long as $\gamma_0$ is in a reasonable interval.

In addition, since according to the EIA the U.S. could become a net energy-exporting country by 2026, meaning that the U.S. is likely to be a minor commodity-importing country rather than a commodity-exporting country, whether the main results of the model change in the case where the home country imports commodities to a much lesser extent needs to be assessed. Because the lower $\gamma_0$, the smaller the home country’s imports of commodities from the foreign country, this can be assessed by reducing $\gamma_0$ greatly. Specifically, when $\gamma_0=0.095$, the model generates almost equal output and consumption correlations (0.782 for the output correlation and 0.785 for the consumption correlation). $\gamma_0=0.03$ causes the model to produce 0.760 for the output correlation and 0.809 for the consumption correlation. In sum, if the home country shifts from a major commodity-importing country to a minor commodity-importing country, the output correlation in the model is higher than that in

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23 According to the EIA’s Annual Energy Outlook 2017, the EIA expects that the U.S. could become a net energy-exporting country by 2026. Since the U.S. is a net importing country of other commodities such as basic metal and wood, even if the U.S. becomes a net energy-exporting country, it will not become a net commodity-exporting country. Rather, the U.S. is likely to shift from a major commodity-importing country to a minor commodity-importing country.
the ST model (0.659) and the consumption correlation in the model is smaller than that in the ST model (0.813). In other words, even if the home country shifts from a major commodity-importing country to a minor commodity-importing country, the model yields closer international business cycle statistics to the data than the ST model.

**Elasticity of substitution between commodities and capital in production** ($\nu$)

As the elasticity of substitution between commodities and capital in the production of tradable goods ($\nu$) goes up, both output and consumption correlations fall slowly (see (c) in Figure 6).

As for the output correlation, when $\nu$ is high, it is easier for commodity input to be substituted by capital input, which means weaker complementarity between the two. Furthermore, $\nu$ is not directly related to commodity prices. Therefore, when $\nu$ is high, the increased home demand for imports of commodities for the tradable production in response to positive productivity shocks are offset by the increased real commodity prices by more than when $\nu$ is small. This leads foreign commodity output to rise by less, which brings about a smaller increase in foreign aggregate output, i.e. the larger $\nu$ is, the smaller the positive spillover to the foreign country from positive productivity shocks. Therefore, the output correlation decreases as $\nu$ becomes greater.

With regard to the consumption correlation, $\nu$ is not directly related to commodity consumption, and as already shown in the previous section, changes in commodity consumption in response to productivity shocks in the nontradable sector decrease the consumption correlation. Therefore, although $\nu$ increases, changes in commodity consumption is very marginal. Nevertheless, when $\nu$ is high, foreign noncommodity consumption goes up by less due to a smaller rise in its aggregate output compared to the case of low $\nu$, which leads to a fall in the consumption correlation. Consequently, high $\nu$ results in less positively correlated consumptions of the two countries.

![Figure 6: Sensitivity analysis](image-url)
Overall, the main results of this paper are maintained in a reasonable interval for $\nu$.

5. Conclusion

This paper builds a two-country model with commodities by extending the two-country, two-sector RBC model of Stockman and Tesar (1995) (ST model), taking into account idiosyncratic commodity trade structures of the countries to analyse international business cycles between the U.S. and commodity-exporting countries. The model in this paper performs better in replicating the cycles between the U.S. and commodity-exporting countries than the ST model where commodities and different commodity trade structures of the countries are not considered. Furthermore, it is empirically shown that the higher the ratio of net commodity exports to GDP a country has, the stronger the GDP comovement with U.S. GDP. The simulation results of the model are consistent with this empirical finding.

Specifically, this paper considers two models: the ST model where the two countries are symmetric and produce tradable and nontradable goods, and the model where the two countries have different commodity trade structures and only the foreign country produces commodities (since the U.S., the home country, is a net commodity-importing country).

The output correlation in the model is larger than that in the ST model. This is because the complementarity between commodity and noncommodity consumption goods mutes the substitution effects for commodities exported by the foreign country due to the increased commodity prices stemming from positive productivity shocks in the home country. Accordingly, commodity exports from the foreign country rise, which enables the shocks to have a larger positive spillover to the foreign country. Thus, outputs of the two countries are more positively correlated in the model compared to those in the ST model. This is very consistent with the empirical finding in Section 2 that GDPs of commodity-exporting countries are more positively correlated with U.S. GDP than those of commodity-importing countries.

Compared to the ST model, the consumption correlation in the model falls. This is mainly due to the existence of commodity consumption and the complementarity between commodity and noncommodity consumption goods. Owing to the complementarity, home and foreign commodity consumptions have a positive correlation with respect to productivity shocks in the tradable sector, while they have a negative correlation with respect to productivity shocks in the nontradable sector. The negative correlation with respect to productivity shocks in the nontradable sector enables the correlation between home and foreign commodity consumptions to be much lower than that between home and foreign noncommodity consumptions in the model. This brings about lower consumption correlation in the model than that in the ST model. As a result, in the model the output correlation between the two countries is higher than the consumption correlation, i.e. the consumption correlation puzzle is mitigated.
Finally, the sensitivity analysis of the model's results with regards to the parameters in the newly introduced features, as compared to the ST model, show that the main results of this paper hold as long as the parameter values are in reasonable ranges.

Adding commodities to open economy macro models with consideration for different commodity trade structures of countries therefore appears to be useful, since it helps to explain key features of international business cycle correlations between the U.S. and commodity-exporting countries. The framework that this paper proposes is therefore promising for further research on the international macroeconomic interactions between the U.S. and commodity-exporting countries, especially the international transmissions of monetary and fiscal policies.
References


Appendix

A. Data sources

This section describes the data used in this paper. The data is from two sources: OECD and IMF.

Figures 1 and 2 The 60 countries are those whose data of commodity exports and imports are available in the OECD Trade in Value Added.

Annual real GDPs for computing GDP correlations are taken from the IMF World Economic Outlook, since annual real GDP data of some countries such as Argentina and Bulgaria among the 60 countries are not available in the OECD National Accounts. Net commodity exports are the gross exports minus imports of agriculture, hunting, forestry, fishing, mining, quarrying, wood and basic metal industries in the OECD Trade in Value Added, and nominal GDP is drawn from the IMF World Economic Outlook. The ratio of net commodity exports to GDP is average of dividing net commodity exports by nominal GDP for the period 1995-2011.

Tables 1 and 3 Annual real GDP, real private consumption and real private fixed investment of OECD countries are taken from OECD National Accounts. Since the pre 1990 data for most Eastern European countries are not available, the data cover the period 1990-2015 with the exception of Estonia which covers 1993-2015, Hungary which covers 1991-2015, Latvia which covers 1994-2015, and Slovak Republic which covers 1992-2015.

For the trade-weighted average, the average trade weight of an OECD country i for the period 1995-2011 is constructed by summing exports from an OECD country i to the U.S. and imports from the U.S. to the OECD country i, and dividing by the total OECD countries’ trade with the U.S. The trade data is drawn from OECD Trade in Value Added.

Commodity-exporting OECD countries are Australia, Canada, Chile, Denmark, Mexico, the Netherlands and Norway whose ratios of net commodity exports to GDP are positive.
B. Equilibrium conditions of the model

In this section, I present the equilibrium system of the model described in the section 3.

- Home households' intertemporal problem:
  \[ 1 = \beta E_t \left[ \frac{C_t}{C_{t+1}} R_t \right] \quad (16) \]
  \[ W_{T,t} = L_t^X C_t \quad (17) \]
  \[ W_{N,t} = L_t^X C_t \quad (18) \]
  \[ C_t = \vartheta e_t C_t^* \quad (19) \]
  
  with \( R_t = \{(1 - \delta)p_{H,t+1} + r_{T,t+1}\}/p_{H,t} = \{(1 - \delta)p_{N,t+1} + r_{N,t+1}\}/p_{N,t} \) and \( L_t = L_{T,t} + L_{N,t} \)

- Foreign households' intertemporal problem:
  \[ 1 = \beta E_t \left[ \frac{C_t^*}{C_{t+1}^*} R_t^* \right] \quad (20) \]
  \[ W_{T,t}^* = L_t^{*X} C_t^* \quad (21) \]
  \[ W_{N,t}^* = L_t^{*X} C_t^* \quad (22) \]
  \[ W_{X,t}^* = L_t^{*X} C_t^* \quad (23) \]
  
  with \( R_t^* = \{(1 - \delta)p_{F,t+1}^* + r_{T,t+1}^*\}/p_{F,t}^* = \{(1 - \delta)p_{N,t+1}^* + r_{N,t+1}^*\}/p_{N,t}^* = \{(1 - \delta)p_{X,t+1}^* + r_{X,t+1}^*\}/p_{X,t}^* \) and \( L_t^* = L_{T,t}^* + L_{N,t}^* + L_{X,t}^* \)

- Home households' consumption allocation:
  \[ C_{NC,t} = (1 - \gamma_0)p_{NC,t}^{-\eta_0} C_t \quad (24) \]
  \[ C_{X,t} = \gamma_0 p_{X,t}^{-\eta_0} C_t \quad (25) \]
  \[ C_{T,t} = (1 - \gamma_1) \left( \frac{p_{T,t}}{p_{NC,t}} \right)^{-\eta_1} C_{NC,t} \quad (26) \]
  \[ C_{N,t} = \gamma_1 \left( \frac{p_{N,t}}{p_{NC,t}} \right)^{-\eta_1} C_{NC,t} \quad (27) \]
  \[ C_{H,t} = (1 - \gamma_2) \left( \frac{p_{H,t}}{p_{T,t}} \right)^{-\eta_2} C_{T,t} \quad (28) \]
  \[ C_{F,t} = \gamma_2 \left( \frac{p_{F,t}}{p_{T,t}} \right)^{-\eta_2} C_{T,t} \quad (29) \]

- Foreign households' consumption allocation:
\[ C_{NC,t}^* = (1 - \gamma_0)p_{NC,t}^{1-\eta_0}C_t^* \]  \hspace{1cm} (30)
\[ C_{X,t}^* = \gamma_0 p_{X,t}^{1-\eta_0}C_t^* \]  \hspace{1cm} (31)
\[ C_{T,t}^* = (1 - \gamma_1) \left( \frac{p_{T,t}^*}{p_{NC,t}^*} \right)^{-\eta_1} C_{NC,t}^* \]  \hspace{1cm} (32)
\[ C_{N,t}^* = \gamma_1 \left( \frac{p_{N,t}^*}{p_{NC,t}^*} \right)^{-\eta_1} C_{NC,t}^* \]  \hspace{1cm} (33)
\[ C_{F,t}^* = (1 - \gamma_2) \left( \frac{p_{F,t}^*}{p_{T,t}^*} \right)^{-\eta_2} C_{T,t}^* \]  \hspace{1cm} (34)
\[ C_{H,t}^* = \gamma_2 \left( \frac{p_{H,t}^*}{p_{T,t}^*} \right)^{-\eta_2} C_{T,t}^* \]  \hspace{1cm} (35)

- Home price indices:
\[ 1 = (1 - \gamma_0)p_{NC,t}^{1-\eta_0} + \gamma_0 p_{X,t}^{1-\eta_0} \]  \hspace{1cm} (36)
\[ p_{NC,t}^{1-\eta_1} = (1 - \gamma_1)p_{T,t}^{1-\eta_1} + \gamma_1 p_{N,t}^{1-\eta_1} \]  \hspace{1cm} (37)
\[ p_{T,t}^{1-\eta_2} = (1 - \gamma_2)p_{H,t}^{1-\eta_2} + \gamma_2 p_{F,t}^{1-\eta_2} \]  \hspace{1cm} (38)

- Foreign price indices:
\[ 1 = (1 - \gamma_0)p_{NC,t}^{1-\eta_0} + \gamma_0 p_{X,t}^{1-\eta_0} \]  \hspace{1cm} (39)
\[ p_{NC,t}^{1-\eta_1} = (1 - \gamma_1)p_{T,t}^{1-\eta_1} + \gamma_1 p_{N,t}^{1-\eta_1} \]  \hspace{1cm} (40)
\[ p_{T,t}^{1-\eta_2} = (1 - \gamma_2)p_{H,t}^{1-\eta_2} + \gamma_2 p_{F,t}^{1-\eta_2} \]  \hspace{1cm} (41)

- Laws of one price:
\[ p_{H,t} = e_t p_{H,t}^* \]  \hspace{1cm} (42)
\[ p_{F,t} = e_t p_{F,t}^* \]  \hspace{1cm} (43)
\[ p_{X,t} = e_t p_{X,t}^* \]  \hspace{1cm} (44)

- Home nontradable good producers’ optimality conditions:
\[ Y_{N,t} = A_{N,t}K_{N,t}^{\alpha_N}L_{N,t}^{1-\alpha_N} \]  \hspace{1cm} (45)
\[ W_{N,t} = (1 - \alpha_N)p_{N,t}Y_{N,t} \]  \hspace{1cm} (46)
\[ R_{t-1} = \frac{1}{p_{N,t-1}} \left\{ (1 - \delta)p_{N,t} + \alpha_N p_{N,t} Y_{N,t} \right\} \]  \hspace{1cm} (47)
• Foreign nontradable good producers’ optimality conditions:

\begin{align*}
Y_{N,t}^* &= A_{N,t}\star N_{N,t}^{1-\alpha_N} \quad (48) \\
W_{N,t}^* &= (1 - \alpha_N)p_{N,t}Y_{N,t}^* \quad (49) \\
R_{t-1}^* &= \frac{1}{p_{N,t-1}^*}\{(1 - \delta)p_{N,t}^* + \alpha_N p_{N,t}^* \frac{Y_{N,t}^*}{K_{N,t}^*}\} \quad (50)
\end{align*}

• Home tradable good producers’ optimality conditions:

\begin{align*}
Y_{H,t} &= A_{T,t}\star T_{T,t}^{\nu-1} \quad (51) \\
W_{T,t} &= (1 - \alpha)p_{H,t}Y_{H,t} \quad (52) \\
R_{t-1} &= \frac{1}{p_{H,t-1}}\{(1 - \delta)p_{H,t} + \alpha(1 - a)p_{H,t}K_{T,t}^{-\frac{1}{\nu}} \frac{Y_{H,t}}{(1 - a)K_{T,t}^{\nu-1} + ax_t^{\nu-1}}\} \quad (53)
\end{align*}

\begin{align*}
p_{X,t} &= a\alpha p_{H,t}x_t^{-\frac{1}{\nu}} \frac{Y_{H,t}}{(1 - a)K_{T,t}^{\nu-1} + ax_t^{\nu-1}} \quad (54)
\end{align*}

• Foreign tradable good producers’ optimality conditions:

\begin{align*}
Y_{F,t}^* &= A_{F,t}\star F_{T,t}^{\nu-1} \quad (55) \\
W_{F,t}^* &= (1 - \alpha)p_{F,t}Y_{F,t}^* \quad (56) \\
R_{t-1}^* &= \frac{1}{p_{F,t-1}^*}\{(1 - \delta)p_{F,t}^* + \alpha(1 - a)p_{F,t}^*K_{T,t}^{-\frac{1}{\nu}} \frac{Y_{F,t}^*}{(1 - a)K_{T,t}^{\nu-1} + ax_t^{\nu-1}}\} \quad (57)
\end{align*}

\begin{align*}
p_{X,t}^* &= a\alpha p_{F,t}^*x_t^{-\frac{1}{\nu}} \frac{Y_{F,t}^*}{(1 - a)K_{T,t}^{\nu-1} + ax_t^{\nu-1}} \quad (58)
\end{align*}

• Foreign commodity producers’ optimality conditions:

\begin{align*}
Y_{X,t}^* &= A_{X,t}\star X_{X,t}^{1-\alpha_X} \quad (59) \\
W_{X,t}^* &= (1 - \alpha_X)p_{X,t}Y_{X,t}^* \quad (60) \\
R_{t-1} &= \frac{1}{p_{X,t-1}^*}\{(1 - \delta)p_{X,t}^* + \alpha_X p_{X,t}^* \frac{Y_{X,t}^*}{K_{X,t}^*}\} \quad (61)
\end{align*}

• Laws of motion for capital:

\begin{align*}
K_{N,t+1} &= (1 - \delta)K_{N,t} + I_{N,t} \quad (62)
\end{align*}
\[K_{N,t+1}^* = (1 - \delta)K_{N,t}^* + I_{N,t} \]  
(63)

\[K_{T,t+1}^* = (1 - \delta)K_{T,t}^* + I_{T,t} \]  
(64)

\[K_{T,t+1}^{**} = (1 - \delta)K_{T,t}^{**} + I_{T,t}^* \]  
(65)

\[K_{X,t+1}^{**} = (1 - \delta)K_{X,t}^{**} + I_{X,t}^* \]  
(66)

- Resource constraints:

\[Y_{N,t} = C_{N,t} + I_{N,t} \]  
(67)

\[Y_{H,t} = C_{H,t} + C_{H,t}^* + I_{T,t} \]  
(68)

\[Y_{N,t}^* = C_{N,t}^* + I_{N,t}^* \]  
(69)

\[Y_{F,t}^* = C_{F,t}^* + C_{F,t}^* + I_{T,t}^* \]  
(70)

\[Y_{X,t}^* = C_{X,t}^* + C_{X,t}^* + x_t + x_t^* + I_{X,t}^* \]  
(71)

- Productivity shocks

\[\Delta_t = \rho \Delta_{t-1}^T + \varepsilon_t \]  
(72)
C. The ST model

The equilibrium conditions under the ST model are given by equations (16)-(22), (28), (29), (34), (35), (38), (41)-(43), (45)-(50), (52), (56), (62)-(65), (67)-(70) and (72), and the following 10 equations.

\[ C_{T,t} = (1 - \gamma_1)p_{T,t}^{-\eta_1}C_t \]  
\[ C_{N,t} = \gamma_1 p_{N,t}^{-\eta_1}C_t \]  
\[ C^*_{T,t} = (1 - \gamma_1)p_{T,t}^{*^{-\eta_1}}C^*_t \]  
\[ C^*_{N,t} = \gamma_1 p_{N,t}^{*^{-\eta_1}}C^*_t \]  
\[ 1 = (1 - \gamma_1)p_{T,t}^{1-\eta_1} + \gamma_1 p_{N,t}^{1-\eta_1} \]  
\[ 1 = (1 - \gamma_1)p_{T,t}^{1-\eta_1} + \gamma_1 p_{N,t}^{1-\eta_1} \]  
\[ Y_{H,t} = A_{T,t}K_{T,t}^\alpha L_{T,t}^{1-\alpha} \]  
\[ R_{t-1} = \frac{1}{p_{H,t-1}}\{ (1 - \delta)p_{H,t} + \alpha p_{H,t} \frac{Y_{H,t}}{K_{T,t}} \} \]  
\[ Y^*_{R,t} = A_{T,t}K_{T,t}^\alpha L_{T,t}^{1-\alpha} \]  
\[ R^*_{t-1} = \frac{1}{p_{F,t-1}}\{ (1 - \delta)p_{F,t}^* + \alpha p_{F,t}^* \frac{Y_{F,t}^*}{K_{T,t}^*} \} \]

Moreover, since there are no commodities in the ST model, \( \gamma_2 = \gamma_2^* = 0.5 \) and \( L_t^* = L_{T,t}^* + L_{N,t}^* \).