Anticipated Changes in Household Debt and Consumption

Isaac Gross

University of Oxford

November 6th, 2017

Link to Latest Version

Abstract

This paper evaluates how anticipated changes in household debt associated with the leveraged purchase of housing affects consumption. I build a heterogeneous agent model, calibrated to match the distribution of income and wealth within the US economy, in which households are subject to uninsurable income shocks and can save in both liquid and illiquid assets. I show that the marginal propensity to consume of households who are saving for a house deposit is negative as they decrease their consumption in anticipation of being credit constrained after they purchase a house. I verify the model’s predictions using micro-data from the PSID to show that (i) consumption falls in anticipation of and after increases in household debt and that (ii) households who are planning on purchasing housing stock have negative marginal propensities to consume. Finally, I use this model to examine the general equilibrium effects of tax credits for first home buyers and show that they can lead to a decrease in aggregate consumption.

JEL classification: D14, E21, E62.

Keywords: household consumption, housing demand, leverage.

---

1I cannot find enough words to thank my adviser Andrea Ferrero. I would also like to thank Guido Ascani, Francesco Zanetti, Martin Ellion, Steve Bond and the comments and discussion of the Oxford Macro group and participants at European Meeting of the Econometric Society. I would like to acknowledge the use of the University of Oxford Advanced Research Computing (ARC) facility in carrying out this work (http://dx.doi.org/10.5281/zenodo.22558). Please address correspondence to Isaac Gross, University of Oxford, Department of Economics, Manor Road, Oxford, OX1 3UQ, UK, email: isaac.gross-at- economics.ox.ac.uk; All remaining errors are my own.
1 Introduction

The recent financial crisis has highlighted the role that household debt has on consumption. A burgeoning literature has documented how high levels of household leverage were associated with deeper falls in consumption, and how debt overhang held back the subsequent recovery. While the deterioration of balance sheets during the crisis was largely unforeseen, most of the variation in household leverage revolves around the decision to purchase a house, which is generally anticipated years in advance. This paper will examine how these anticipated changes in household leverage affect non-durable consumption and the implications for housing policy.

In this paper I develop an Aiyagari-Huggett incomplete market model in which households have the ability to save in either a liquid asset, or an illiquid housing asset. I model the housing market in general equilibrium where supply from a construction sector is matched with demand from households and a real estate sector. Household’s adjust their stock of housing subject to a transaction cost which renders it optimal to make lumpy purchases in between periods of inaction. The price in housing adjusts to clear the market. To fund the purchase of housing households are able to take on mortgages subject to an endogenous debt limit.

I use this model to show how a household’s decision to increase their stock of housing results in the household becoming credit constrained leading to a fall in consumption. Anticipating this decline in consumption as they approach the adjustment point, households decrease their consumption even as their liquid wealth rises. This anticipatory saving effect is thus able to generate households with small, even negative, marginal propensities to consume. This implies that a one dollar increase in income, increases household savings by more than one-for-one and results in a decrease income.

I then use micro-data from the Panel Study of Income Dynamics (PSID) to verify two predictions of the model. First I show that households who expect to increase their stock of housing in the next two years have lower, even negative, marginal propensities to consume. Second I show that expected increases in a households leverage rate lead to lower levels of consumption using consumption regression.

In an application of the model I estimate the impact of temporary tax credits for first home buyers that were enacted during the financial crisis on the housing market and aggregate con-
sumption. I show that while the tax credits do boost prices and turnover in the housing market, the effect is concentrated at the time the tax credit expires. The tax credit “pulls forward” sales that would otherwise occurred in future periods. Finally, I show that by encouraging households to purchase more housing it increases the share of highly levered households within the economy which in turn leads to lower aggregate consumption.

These results are driven by frictions in the housing market and the financial sector. In a model of housing and non-durable consumption without borrowing constraints or transactions costs, households will optimize their consumption of housing and non-durable goods by keeping the marginal rate of substitution between the two constant. This implies an empirical positive covariance between consumption and housing as a households income and wealth increase over the life-cycle. However, transaction costs induce households to make lumpy, debt-financed purchases of housing. When a household chooses to borrow to finance the purchase of housing stock the interest that they face rises and the probability they will be credit constrained in the future increases. The combination of having to pay a higher interest rate on their mortgage, relative to what they earned previously on their liquid savings, and the desire to reduce the probability that they hit their credit constraint leads households to lower their level of consumption upon purchasing housing stock.

A household anticipating the fall in consumption that occurs upon purchasing a house will reduce it in the time prior to buying a house. For the majority of households, who are unlikely to purchase additional housing in the near term, this effect is small. However as a household approaches its optimal adjustment point the marginal propensity to buy (the increase in likelihood that they will buy over the next quarter given a small increase in liquid assets) increases. For households that are close to the adjustment point, and thus expect to purchase housing in the near term, this anticipatory saving effect is larger than the wealth and liquidity effects, which means that their marginal propensity to consume is negative. A small increase in income results in lower consumption and a greater than one-for-one increase in saving.

This mechanism is verified through two empirical exercises. First, I estimate households’ marginal propensities to consume from transitory income shocks using a semi-parametric method popularized by Kaplan and Violante (2010). I show that average marginal propensities to consume are lower for households that are more likely to buy additional stock as measured by
households self-reported expectations about moving\(^3\), matching the predictions of the model. Furthermore households who report that they are highly likely to purchase additional housing stock have a marginal propensity to consume that is significantly smaller than zero.

Secondly, I estimate a series of models of consumption growth motivated by Dynan (2012) to show that anticipated changes in household debt are associated with declines in household consumption in magnitudes that are consistent with the results from the model.

Finally, I use this model to estimate the impact of tax credits subsidizing first home buyers deployed during the financial crisis. Solving the model in general equilibrium I show that tax credits reduce aggregate consumption as they encourage higher leverage among households which represses consumption due to the financial frictions.

**Related Literature**

The model I present in this paper is related to several strands of the literature.

First there is a long theoretical literature that models how heterogeneous agents behave in the presence of incomplete markets. Introduced in Bewley (1983) and with early explorations by Huggett (1993) and Aiyagari (1994) these models explored how idiosyncratic shocks affect the distribution of wealth and income within the economy and how households achieve partial insurance via risk free assets. Further developments of this framework included extensions to multiple assets with different risk profiles, degrees of liquidity, and heterogeneity among household preferences.

More recent work by Achdou et al. (2017) has shown that by solving the model in continuous time, coupled with increases in computational resources, aggregate shocks can be solved keeping the entire wealth distribution as a state variable. Guerrieri and Iacoviello (2017) highlight the importance of this approach showing that aggregate shocks can have quantitatively different impacts on agents in different regions of the distribution. My model builds on this work expanding the two asset model of Kaplan et al. (2016) to allow for an endogenous collateral constraint governed by the nominal value of housing.

My approach to modeling the mortgage market is adapted from the wealth of literature on housing markets. Given the computational complexity, life-cycle models typically assume that house prices are purely exogenous (Fernandez-Villaverde and Krueger (2011); Iacoviello

\(^3\)These expectations are surveyed by the PSID and are positively correlated with future changes in both home ownership and household debt.
and Pavan (2013); Yang (2009)). A handful of papers develop models with aggregate shocks that impact the equilibrium price of housing, but they either require environments with very limited heterogeneity (Justiniano et al. (2015); Iacoviello and Pavan (2013)) or approximations which reduce its dimensionality as pioneered by Krusell and Smith (1998). However the Krusell and Smith aggregation result is dependent on the marginal propensity to consume to be largely invariant with respect to households wealth, or at least with deviations confined to households who own a relatively small proportion of the total assets. For more complicated models where this assumption does not hold the number of moments required for an accurate approximation can be orders of magnitude higher Ahn et al. (2017).

This paper also contributes to the empirical literature studying the link between a household’s balance sheet and non-durable consumption. A number of papers have used increasingly rich models with housing and debt to address aggregate questions and make cross sectional predictions, e.g., Carroll and Dunn (1997), Campbell and Cocco (2007), and Attanasio et al. (2012). In particular there has been a lot of focus on how exogenous price movements affect consumption, the profile of debt and consumption over the life-cycle and how mortgage debt affects a household’s marginal propensity to consume.

By contrast this paper focuses on how transaction costs affect non-durable consumption before and after a household adjusts its stock of housing. The closest approach in the literature is Martin (2003) who shows that households increase their spending on food before moving to a smaller house and decrease it before moving to a larger house. I replicate this analysis with data on broader consumption and extend it to measuring a household’s marginal propensity to consume just prior to moving.

This paper also contributes to the fast expanding literature that estimates heterogeneity in marginal propensities to consume across households. There is a wealth of literature that analyses how debt effects households’ marginal propensities to consume, using self-reported values (Aucdert (2017)), exogenous variation in fiscal transfers (Kaplan and Violante (2014)), lottery winnings (Fagereng et al. (2016)) and a semi-structural approach (Blundell et al. (2008)). In particular these methods consistently find that households with high levels of household debt and low access to liquid assets have high marginal propensities to consume.

Finally, in an application of the model I estimate the impact of tax credits for first home buyers. Given the sparse empirical data available there is limited research on their effects.
Dynan et al. (2013) find that while there is some evidence that the tax credit helped support house prices, there was no discernible positive impact of on broader economic activity. My work suggests these tax credits have a contradictory impact on aggregate consumption, potentially resolving this puzzle.

Outline

In the next section I present a general equilibrium model of the housing market under incomplete markets. In Section 3 I discuss the calibration of the model. Section 4 verifies the main findings of the model using micro-data from the PSID. In an application of the model Section 5 analyses the impact of the tax credits for first home buyers. I consider alternative specifications and assumptions in Section 6, before concluding.

2 The Model

In this section I introduce a two asset, incomplete markets model. My main innovation is to augment Kaplan, Moll, and Violante (2016)’s (KMV) rich representation of household consumption and saving behavior with the ability of households to borrow against the nominal value of their stock of illiquid housing. Households face uninsurable idiosyncratic shocks to labor income and can self insure through saving in either housing and liquid assets. Outside of the household and housing sector, the rest of the model is kept as parsimonious as possible. To economize on computational time I solve the model in continuous time using the upwind approximation outlined in Achdou et al. (2017).

2.1 Households

The economy is populated by a continuum of households who are differentiated by their holdings of liquid assets $b$, their ownership of illiquid housing $a$, and their idiosyncratic labor productivity $z$. I assume that time is continuous and that there is no aggregate uncertainty, thus at each point in time $t$ the economy is governed by the joint distribution of the households $\mu_t (b, a, z)$. Households die off with a fixed probability $\lambda$ and are replaced by new households with zero net wealth ($a = b = 0$) and the mean level of income.$^4$

$^4$The stochastic death of households is not required to solve the model, but is critical to matching the high share of renters with no liquid assets observed within the economy.
Households must optimize the expected present value of their by choosing the optimal path of their of labor supply, \( l_t \), non-durable consumption \( c_t \) and their ownership of housing \( h_t \). Each household takes prices, wages, interest rates and governmental transfers, \( \Phi = \Phi \{ w_t, r^b_t, a^p_t, \tau_t, T_t \} \), which are taken as given. Preferences are time-separable and conditional on surviving the future is discounted at a rate \( \rho \geq 0 \):

\[
U_t = E_t \int_t^\infty e^{-(\rho+\lambda)t} u(c_t, h_t, l_t) \, dt,
\]

where the expectation is taken over realizations of idiosyncratic productivity shocks.

The flow utility is increasing and strictly concave in \( c \) and \( h \) and decreasing and strictly convex in \( l \). I assume the functional form

\[
\begin{align*}
    u_t &= \left\{ \frac{c_t - z_t G(l_t)^{1-\zeta} h_t^{\zeta}}{1-\gamma} \right\}^{1-\gamma} - 1 \\
    G(l_t) &= \phi z_t \frac{l_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}},
\end{align*}
\]

where the curvature parameter \( \gamma \) determines households’ risk aversion and inter-temporal elasticity of substitution, \( \zeta \) is the weight placed on housing relative to non-durable consumption, and \( \eta \) is the Frisch elasticity of labor supply. I assume a functional form for flow utility that follows Greenwood et al. (1988), which ensures there is no wealth effect on labor supply, \( l_t \). Thus all households, regardless of their assets, will supply the same amount of labor which is solely a function of the wage per effective unit of labor, \( w_t \), and does not vary with idiosyncratic labor productivity shocks, \( z_t \).

Households maximize the present value of expected utility subject to four constraints. The first is the budget constraint, which governs the evolution of liquid assets, \( b_t \), in the form of a risk-free bond

\[
\dot{b}_t = z_t w_t l_t + r_t b_t - c_t - c_t^{rent} p_t^{rent} - k(a_t, a'_t) - p_t^a \Delta a_t - T_t
\]

Housing assets are illiquid as households must pay a fee \( k(a_t, a'_t) \) in order to buy or sell an amount of housing. I set this friction as a fixed percentage of the current, \( a_t \), and new level housing, \( a'_t \). I denote any new purchases or sales of housing \( \Delta a_t = a'_t - a_t \). Note that while the model is written in continuous time, any purchase of housing stock and the accompanying

\[\text{This assumption creates a direct mapping between productivity and earnings, which facilitates the calibration of the exogenous productivity process. I relax this assumption in Section 5.}\]
change in liquid assets occurs instantaneously. Each household’s stock of housing depreciates over time at a rate $\delta$, and thus the stock of housing evolves according to

$$\dot{a}_t = (1 - \delta) a_t + \Delta a_t$$ (4)

The interest rate paid on the liquid asset varies depending on whether the household is a saver, who deposits their funds with the financial firms, or a net borrower. The spread between saving and borrowing occurs due to the cost of financial intermediation and is detailed below.

$$r_t = \begin{cases} r_t^+ & \text{if } b_t > 0 \\ r_t^+ + r_t^{spread} & \text{if } b_t \leq 0 \end{cases}$$ (5)

Each household’s consumption of housing services is defined by the stock of housing that they own. Households that do not own housing must purchase housing via the rental market. However, rental units only grant a fraction, $\theta^r$, of the benefit to households relative to owned stock.

$$h_t = \begin{cases} \theta^r c_t^r & \text{if } c_t^r > 0 \\ a_t & \text{if } c_t^r \leq 0 \end{cases}$$ (6)

Finally, each household can borrow against a proportion of the total value of the stock of housing they own.

$$b_t \geq -Ma_t p_t^a$$ (7)

$$a_t > 0$$ (8)

Households maximize 1 subject to (3)-8, taking as given the path for the real wage $\{w_t\}_{t \geq 0}$, the interest rate on liquid assets $\{r_t^b\}_{t \geq 0}$, the price of housing relative to non-durable goods $\{p_t^a\}_{t \geq 0}$, and taxes and fiscal transfers $\{\tau_t, T_t\}_{t \geq 0}$. As described below, the paths for the price variables are determined by the market clearing conditions for labor, liquid assets and housing. In Appendix A I describe how the households’ optimization problem can be solved recursively with a Hamilton-Jacobi-Bellman equation. The resulting steady state solution yields decision
rules governing household behavior as a function of liquid assets, illiquid housing, productivity and relative prices for non-durable consumption $c(a, b, z; \Phi)$, labor supply $l(a, b, z; \Phi)$, and the housing adjustment rule $\alpha(a, b, z; \Phi)$. Outside of the steady state these rules will be time-varying and respond to the time path of aggregate prices $\Phi_t = \{w_t, r_t, p_t, \tau_t, T_t\}_{t \geq 0}$. These optimal decision rules imply drift paths for households’ holdings of liquid assets and housing, which when combined with the exogenous stochastic process for $z$, can be used to calculate the stationary joint distribution of assets and income $\mu_t (a, b, z, \Phi)$. In Appendix A I also outline the Kolmogorov forward equation that defines the evolution of this distribution over time.

2.2 Firms

Final Goods Firms

A continuum of final goods firms produce goods for consumption, $y_{j,t}$, by using effective units of labor $n_{j,t}$ in a linear production function. These firms are perfectly competitive, take the real effective wage, $w_t$, as given and have zero profit. Their prices are perfectly flexible and are defined as the numaire. They maximize profits

$$\Pi_t^{goods} = y_t - w_t n_t,$$

subject to the production function

$$y_t = \zeta_t n_t,$$

where $\zeta_t$ is an aggregate labor productivity shock.

Real Estate Firms

Real estate firms borrow from financial firms to fund the purchase of housing, which is rented out to households that do not own their own home. Real estate firms operate in a perfectly competitive market, and thus the rental price is pinned down by the cost of borrowing, a fixed cost of providing rental services, the price of purchasing house stock and any expected capital appreciation driven by the drift in the price of housing. I assume that real estate firms are able to borrow against the full value of their housing stock. The zero profit condition pins down the price of rental stock as the function of the cost of housing, capital gain from housing and the
interest rate

\[ p_t^{\text{rent}} = \phi^{\text{rent}} + (r^b_t + \delta) p_t^a - \dot{p}_t^a, \]  

(11)

where \( \phi^{\text{rent}} \) is the fixed cost of providing rental services.

Financial Firms

Financial firms intermediate saving and borrowing within the economy subject to a fixed cost, \( r^i_t \), in the form of a spread between the interest rate on deposits and loans. Households make liquid deposits in financial firms, which are then loaned out to households who require a mortgage or to real estate firms. Finally, I assume that these financial firms operate in a competitive environment and thus earn zero profits

\[ \Pi_t^{\text{financial}} = \left( r^b_t + r^i_t \right) \left( \int_{b<0} 1 \, d\mu_t + \int_{a=0} \phi^{\text{rent}} d\mu_t \right) - r^b_t \int_{b>0} d\mu_t. \]

(12)

Construction Firms

Finally, a continuum of construction firms produce new units of housing for sale to both households and real estate firms. These firms are perfectly competitive and produce houses using labor, \( n_t^a \), and housing permits, \( \bar{L} \), purchased from the government. This ensures that the price elasticity of supply of housing is finite, and is akin to assuming investment adjustment costs in the housing sector. They maximize profits

\[ \Pi_t^{\text{construction}} = p_t^a I A_t - w_t n_t^a - p_t^L \bar{L} \]

subject to the production function

\[ I A_t = \zeta_t^a (n_t^a)^\omega \bar{L}^{1-\omega}. \]

(14)

Thus the price elasticity of supply is given by \( \frac{\omega}{1-\omega} \).

I follow Favilukis et al. (2017) by assuming that the government sells the land permits at the market clearing price. This ensures that all rents from the fixed supply of housing permits accrue to the government and the construction sector makes no profit in equilibrium.

Maximizing 13 subject to 14 generates the housing investment function
IA_t = (ω_t^a)^{\frac{1}{L}} \bar{L} \tag{15}

2.3 Government

The fiscal authority funds an exogenous level of government spending and fiscal transfers to households via a combination of lump-sum payments and taxes on labor income.

\[ T_t = \tau_t w_t l_t + \tilde{T}_t \tag{16} \]

Any fiscal deficit must be financed by issuing debt to households in the form of liquid assets. I assume that the government are able to borrow directly from savers and are not subject to the friction generated by the financial intermediaries. Thus the government’s budget position is given (in deficit terms) by

\[ \dot{B}_t^{gov} = r_t^b B_t^{gov} - \int T_t d\mu_t - p_t^L \bar{L} + G_t \tag{17} \]

2.4 Market Clearing Conditions

An equilibrium in this economy is defined as paths for individual household and firm choice variables \( \{a_t, b_t, c_t, c_t^{rent}, \alpha_t, l_t, n_t, n_t^a\}_{t \geq 0} \) and aggregate prices \( \{w_t, r_t^b, p_t^b, \tau_t, T_t\}_{t \geq 0} \) such that households and firms optimize their objective functions subject to their respective constraints, taking prices as given, at all time \( t \) and that distribution of households, \( \mu_t (a_t, b_t, l_t) \), is such that (i) the goods market, (ii) the market for liquid assets, (iii) the market for housing and (iv) the labor market all clear.

The final goods market clears if the total production by final goods firms is equal to consumption, government services, and the total resources lost to the frictions related to the housing sector, the financial firms and the real estate sector.

\[ Y_t = \int_{a_t \neq a'_t} k (a_t, a'_t) d\mu_t + \int_{b < 0} r_t^{spread} d\mu_t + \int_{a = 0} \phi_t^{rent} d\mu_t + C_t + G_t \tag{18} \]

The liquid asset market clears when net household savings, \( B_t^{net} \), are equal to the funds borrowed by the real estate sector and the fiscal authority.
\[ \int b \, d\mu_t = B^\text{net}_t = p_t^A C^\text{rent}_t + B^\text{gov}_t \]  (19)

The market for housing must clear when total inflows from additional construction is equal to the aggregate depreciation plus the change in total demand by both home owners and renters.

\[ I A_t = \delta A_t + \int \dot{a}_t + \dot{c}_t^\text{rent} \, d\mu_t \]  (20)

Finally, the labor market clears when the total labor demanded by the final goods and construction sectors equals the effective supply from households.

\[ N_t + N_t^c = \int zl (a, b, z) \, d\mu_t \]  (21)

### 3 Calibration

#### Household Preferences

I calibrate the household utility function to match key moments in the data. I set the disutility of labor \( \phi \) such that hours worked is equal to \( \frac{1}{3} \) of the time endowment in equilibrium. I set the weight on housing in the utility function to match the average expenditure share on housing at 0.16. I set the curvature parameter, \( \gamma \), which governs the risk aversion and inter-temporal elasticity of substitution to 1. Given these parameter values, the average value for the inter-temporal elasticity of substitution is approximately 0.7.\(^6\)

Finally, I set the rate of household deaths \( \lambda \) to \( \frac{1}{180} \) such that the expected lifespan for a newly created household is 45 years.

#### Income Process

The household earnings process is modeled using a “jump-diffusion” process following Kaplan et al. (2016) which can produce realistic income dynamics even when approximated over a discrete grid. Since all workers face the same wage per effective unit of labor and choose to supply the same amount of labor, a set of moments of households labor income can be used to estimate the

\(^6\)The inter-temporal elasticity of substitution is defined as \( \frac{c_{uc}}{c_{uc}^G} \). Given our assumption on the utility function, this corresponds to \( \frac{c-G(l)}{(\gamma l^\gamma (1-\gamma))c \Delta t} \) which varies across the distribution of households.
labor productivity process $z_{it}$. Given the continuous time setting, the income process is governed by the size, persistence and frequency of the shocks, in contrast with discrete-time environments in which the frequency is defined by the assumed period length. I split the aggregate log-earnings process, $z_{it}$, into two components

$$z_{it} = z_{1,it} + z_{2,it}, \quad (22)$$

where each component, $z_{j,it}$, evolves according to

$$dz_{j,it} = -\beta_j z_{j,it}dt + dJ_{it}, \quad (23)$$

where $dJ_{it}$ is the jump process which arrives at a rate $\alpha_j$, such that over a small time period $dt$ the probability that a jump occurs is $\alpha_j dt$ and the probability that a jump does not occur is $(1 - \alpha_j) dt$. Conditional on a jump occurring, the new level for the component, $z_{j,it}$, is drawn from a normal distribution with zero mean and variance $\sigma_j^2$. Thus

$$dJ_{j,it} = -z_{j,it} + \epsilon_{j,it} \quad \text{with} \quad \epsilon_{j,it} \sim N(0, \sigma_j^2)$$

To calibrate the parameters of the earnings process I use a simulated method of moments to match data from Guvenen et al. (2015).

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Process - Continuous Time</th>
<th>Process - Discreteised Grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance - log earning</td>
<td>0.70</td>
<td>0.70</td>
<td>0.72</td>
</tr>
<tr>
<td>Variance - 1yr change</td>
<td>0.23</td>
<td>0.23</td>
<td>0.21</td>
</tr>
<tr>
<td>Variance - 5yr change</td>
<td>0.46</td>
<td>0.46</td>
<td>0.48</td>
</tr>
<tr>
<td>Kurtosis - 1yr change</td>
<td>17.8</td>
<td>16.5</td>
<td>17.3</td>
</tr>
<tr>
<td>Kurtosis - 5yr change</td>
<td>11.6</td>
<td>12.1</td>
<td>11.6</td>
</tr>
<tr>
<td>Share of 1yr change &lt; 10%</td>
<td>0.54</td>
<td>0.56</td>
<td>0.59</td>
</tr>
<tr>
<td>Share of 1yr change &lt; 20%</td>
<td>0.71</td>
<td>0.67</td>
<td>0.61</td>
</tr>
<tr>
<td>Share of 1yr change &lt; 50%</td>
<td>0.86</td>
<td>0.85</td>
<td>0.89</td>
</tr>
</tbody>
</table>

The estimates of the parameters suggest the presence of two kinds of shocks. The first is an infrequent, but persistent career shock which occurs on average every 38 years and has a half-life of 18 years. The second is a more temporary shock which arrives roughly every 3 years, but
largely dissipates within a quarter.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Career Component $j = 1$</th>
<th>Transitory Component $j = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival Rate</td>
<td>0.70</td>
<td>0.72</td>
</tr>
<tr>
<td>Mean Reversion</td>
<td>0.67</td>
<td>0.61</td>
</tr>
<tr>
<td>St Deviation of Jump</td>
<td>0.85</td>
<td>0.89</td>
</tr>
</tbody>
</table>

### 3.1 Housing Market

I calibrate the parameters related to the housing market to match key long-run relationships in the US housing market. The fixed cost faced by households when adjusting their housing stock is set to 7 per cent of the value of the house price. This produces a housing market in which 9.2 per cent of the house stock is turned over annually. This compares with roughly 10 per cent in the data as estimated by Ngai et al. (2016).

The elasticity of housing supply is determined by the exponent in the production function of the construction sector, $\omega$, which I set to generate an elasticity of 1.5 per cent, which is the median value of elasticities estimated by Saiz (2010) across US cities. I calibrate the size of the construction sector to be 5 per cent on total output within the economy, matching its long-run share of gross value added.

The remaining parameters $(p^a, \theta^r, \rho, s^h, \phi_{rent})$ are set to match five key moments in the data.

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share of owners with a mortgage</td>
<td>0.55</td>
<td>0.57</td>
</tr>
<tr>
<td>Share of renters</td>
<td>0.38</td>
<td>0.38</td>
</tr>
<tr>
<td>Mean net worth - income ratio</td>
<td>5.2</td>
<td>5.5</td>
</tr>
<tr>
<td>Average ratio earnings owners - renters</td>
<td>2.1</td>
<td>2.2</td>
</tr>
<tr>
<td>Median loan-to-value ratio</td>
<td>0.66</td>
<td>0.63</td>
</tr>
</tbody>
</table>

We match these moments by simulating the model over a grid of parameters, and choosing the set which minimises the difference between the five parameters.

Given that the majority of lending within the model occurs between savers and mortgagees, I calibrate the financial friction to match the long-run average spread between 30-year fixed rate mortgages and the 3-month Treasury Bill. Since 1985 this spread has averaged 3.5 per
cent. Finally, the real interest rate and the real wage are internally calibrated to clear the labor markets.

3.2 Equilibrium

The households' optimal consumption and saving decisions and the evolution of the joint distribution of their income, housing and liquid assets can be summarized by a Hamilton-Jacobi-Bellman equation and a Kolmogorov Forward equation. The method of approximating these equations is outlined in Appendix A.

How well does this model match the distribution of wealth within the economy? In Table 4 I show some key moments related to the ownership of housing and total household net worth in the model.

<table>
<thead>
<tr>
<th>Table 4: Selected untargeted moments</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing owned by top 1%</td>
<td>19.43%</td>
<td>8.5%</td>
</tr>
<tr>
<td>Housing owned by top 10%</td>
<td>64.7%</td>
<td>55.3%</td>
</tr>
<tr>
<td>Housing owned by top 20%</td>
<td>79.4%</td>
<td>73.0%</td>
</tr>
<tr>
<td>Gini coefficient - housing</td>
<td>0.73</td>
<td>0.65</td>
</tr>
<tr>
<td>Net worth owned by top 1%</td>
<td>35%</td>
<td>19.5%</td>
</tr>
<tr>
<td>Net worth owned by top 10%</td>
<td>88%</td>
<td>76.3%</td>
</tr>
<tr>
<td>Net worth owned by top 20%</td>
<td>96%</td>
<td>91.2%</td>
</tr>
<tr>
<td>Gini coefficient - total net worth</td>
<td>0.85</td>
<td>0.79</td>
</tr>
<tr>
<td>Aggregate debt to house value</td>
<td>0.42</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Source: 2004 Survey of Consumer Finance

On the whole, the model matches the data fairly well with the notable exception of the top end of the distribution. While the model generates significant degrees of inequality across both liquid assets and housing, it fails to match the large shares of wealth owned by the top 1 percent of households.

3.3 Marginal Propensities to Consume and Household leverage

How well does this heterogeneous approach to households match the empirical findings on households' marginal propensities to consume (MPC) Of the range of empirical studies that examine this topic, the most compelling use exogenous variation in fiscal payments (Kaplan and Violante (2014); Broda and Parker (2014)), and lottery winnings (Fagereng et al. (2016)). These studies
suggest that households spend 15-25 per cent of one-off payments in the quarter that they are received.

The MPC can be thought of as the impact of a one-off increase in liquid wealth. I thus define the MPC of a payment of $x$ additional dollars over a length $\tau$ periods as

$$MPC_x^\tau (a,b,z) = \frac{C_{\tau} (a,b+x,z) - C_{\tau} (a,b,z)}{x},$$

where $C_{\tau} (a,b,z)$ is the sum of expected consumption of an individual household over the next $\tau$ periods.

$$C_{\tau} (a,b,z) = E \left[ \int_0^\tau c(a_t, b_t, z_t) \ dt | a_0 = a, b_0 = b, z_0 = z \right]$$

This formula can be solved using the Feynman-Kac formula as outlined in Appendix B.\(^7\)

The unconditional quarterly MPC in the model is 19 per cent, which is consistent with the range of empirical results. However this unconditional mean masks a high level of heterogeneity across the distribution of wealth among households. Figure 1 shows the distribution of MPC as a function of households’ liquid and illiquid assets.

Figure 1: Household Heterogeneity

We present the distribution of marginal propensities to consume from two different views to display the multiple peaks and troughs in the surface.

There are three notable features of the distribution in MPCs in Figure 1. The first is the spike in MPCs as households approach the borrowing constraint which is governed by the nominal

\(^7\)An alternative strategy would be to estimate the impact of a small increase in the transitory component of the households’ productivity process. This alternative strategy produces qualitatively similar results, but with larger simulation error due to the coarser grid used to approximate the productivity process.
value of their housing stock. The second is the high MPCs for households that own some amount of housing stock but have zero liquid wealth. The third notable feature is the fall in MPCs, often to negative levels, of households who are close to the point at which they would optimally choose to adjust their stock of housing.

The first two features are well documented in previous research. Broda and Parker (2014) find that households with limited access to liquid funds had a significant increase in consumption in response to the 2008 fiscal stimulus payments. In addition, Kaplan, Violante, and Weidner (2014) use a semi-structural approach to measure marginal propensities to consume and find that households with limited access to liquid assets have significantly higher marginal propensities to consume, even if they have high levels of illiquid assets. These results are consistent with the existence of wealthy hand-to-mouth households. Fagereng et al. (2016) examine MPCs out of lottery prizes using Norwegian administrative data. They find that MPCs vary with the amount of households’ liquid assets and that households with close to zero liquid assets have high MPCs even if they are wealthy in terms of their illiquid asset position.

The existence of households with a negative MPC is a more novel finding. The key to understanding this result is examining how a household’s decision to adjust its stock of housing affects its incentives to consume non-durable goods.

Since changes in household’s ownership of housing is subject to transaction costs, it will be optimal for households to make lumpy purchases, waiting longer before deciding to interact with the market and adjusting by a larger amount when they do so. These large, lumpy purchases require financing in the form of mortgages, which will place post-purchase households closer to the endogenous credit constraint and subject to the higher interest rate faced by borrowers relative to their previous position as savers.

These two effects are partially offset by the complementarity between housing and the consumption of non-durable goods in the households’ utility function. However this effect is quantitatively small.

The combination of a higher probability of being credit constrained and the increase in interest rates faced by the household, will increase the incentive for households to repair their balance sheet by reducing spending and paying back debt. Thus, on average, households decrease their consumption by 6 per cent conditional on buying additional housing stock (Figure 2).

Prior to purchasing additional housing stock, households anticipating the impending increase
Figure 2: Change in non-durable consumption conditional on buying additional housing stock in debt and corresponding fall in non-durable consumption increase their savings rate to better smooth consumption in future periods. This anticipatory saving effect is stronger the higher the probability that households will find it optimal to buy a house in the near future.

Thus when households receive a small increase in liquid wealth there are three effects on household consumption. The wealth and liquidity effects both lead to an increase in the optimal rate of consumption. However by increasing the probability that the household will choose to buy a house in future periods, the anticipatory saving effect will cause households to decrease their consumption. For the vast majority of households, the first two effects outweigh the third as only a relatively small share of households adjust their housing stock annually.

The same logic holds for households that are close to selling a portion of their housing stock. When a household decides to reduce its stock of housing, it uses the proceeds from the sale to pay back its mortgage debt, thus reducing the probability that it will become credit constrained in the future. This increase in liquid assets will lead to the household increasing its non-durable consumption after reducing its stock of housing. Anticipating this increase in consumption, households will optimally choose to increase their consumption as their portfolio of assets approaches the adjustment point. Since a small increase in liquid assets moves such a household further away from the adjustment point, this will result in the household lowering
their level of consumption as the probability that they will sell a portion of their housing stock decreases.

While this anticipatory saving effect reduces households’ MPC, whether they adjust their stock of housing higher or lower, the size of the effect is asymmetric. This is because the transaction costs associated with adjusting the stock of housing reduces a household’s wealth regardless of whether they are buying or selling. This decrease in wealth ensures that the decrease in consumption associated with the purchase of additional stock is larger than the increase in consumption associated with selling the same amount. Thus the anticipatory saving effect is stronger for households who expect to buy housing compared with those who expect to sell.

This effect is illustrated in Figure 3 which shows how a household’s consumption varies with respect to liquid assets for a given level of housing and income. If the households level of liquid assets become to high (low) it buys (sells) housing stock which results in a fall (rise) in non-durable consumption post adjustment. The household anticipates the possibility of this adjustment occurring and begins to smoothing consumption towards the post adjustment level. This results in consumption falling (rising) as liquid wealth rises (falls) close to the optimal adjustment point which in turn implies that the marginal propensity to consume is negative.

Figure 3: Consumption as a function of liquid assets

In practice, this phenomenon is caused by the combination of the cost of adjusting housing stock and the interest rate spread between borrowers and savers. When renters decide to enter the housing market they change from being savers to borrowers. This increases the interest rate
they face when their liquid asset position shifts and causes them to decrease their level of non-
durable consumption. If the stock of housing can be frictionlessly adjusted then households are
able to continuously adjust their ownership of housing and any debt as needed. A household’s
ability to slowly change the mix of their asset portfolio eliminates the large jumps in debt that
are necessary in the presence of adjustment costs and allows consumption of both housing and
non-durable goods to increase as household wealth and income increases. Alternatively, if the
financial friction is removed then the interest rate households face remains unchanged even after
they take out a mortgage reducing the change in consumption.

These results share a similar pattern to other models of illiquid assets and incomplete markets.

4 Debt and Consumption: Quantitative Results

In this section I compare the predictions from the general equilibrium model against empirical
results derived from household surveys. To compute the key cross-section moments I require
household-level panel data on income, consumption and household debt. There are a variety
of techniques in the literature used to calculate parameters such as households’ MPC, the most
commonly used are the US Panel Study of Income Dynamics (PSID) and the US Consumer Ex-
penditure Survey. However since the Consumer Expenditure Survey does not follow households
who move address, it excludes those who buy or sell their home - the main population of interest.
I thus use the PSID to verify the predictions.

4.1 Measuring the MPC

The first exercise I conduct is to compare the predicted marginal propensity to consume of the
model with the data. The model generates three broad predictions concerning the distribution
of household MPC: (i) that households close to their borrowing constraint have a high MPC;
(ii) that households who have close to zero liquid wealth have a higher MPC; and (iii) that
households who are close to buying adjusting their stock of housing have a low, or even negative,
MPC. As there is already considerable empirical evidence supporting the first two predictions in
the literature as outlined in my literature review, I will focus on the third more novel prediction.

To estimate the MPCs I follow the identification strategy of Blundell, Pistaferri, and Preston
2008, (BPP) and popularized by Kaplan and Violante (2010). BPP show that by assuming
household income is governed by a process with a permanent and an i.i.d. component, then
given appropriate theoretical restrictions, an estimator for households’ MPC can be calculated
with panel data on consumption and income.\(^8\)

The estimator for the transmission of transitory income shocks to consumption is given by

\[
\hat{MPC} = \frac{\text{cov}(\Delta c_{i,t}, \Delta y_{i,t+1})}{\text{var}(\Delta y_{i,t}, \Delta y_{i,t+1})}
\]

which can be estimated using an instrumental variable regression, where \(\Delta c_{i,t}\) is regressed on
\(\Delta y_{i,t}\) instrumented by \(\Delta y_{i,t+1}\) with panel data of at least three periods.

Kaplan and Violante (2010) show that this estimator is highly robust at identifying the true
MPC in a wide variety of models, including models in which there is a forecastable component
of future income. Berger et al. (2015) further show that this method remains robust in the
presence of a housing market with transaction costs and the option to rent. I replicate the
exercises conducted by Kaplan and Violante (2010) and show that the true MPC is reliably
measured in my model of jump-diffusion income processes.

4.1.1 PSID Data

While the PSID started collecting data in 1968, it only started surveying non-food consumption
in 1999. While it is possible to impute aggregate consumption by inverting the demand for
food using data from the Consumer Expenditure Survey Blundell et al. (2004), this measure can
have high levels of measurement error as shown by Attanasio and Weber (1995). Accordingly,
I calculate a broader measure of consumption which is measured bi-annually from 1999 to 2015.
In this sub-sample the PSID surveys over 70 per cent of all consumption items available in the
Consumer Expenditure Survey and includes expenditure on food, utilities, gasoline, car repairs,
public transportation, childcare, education and medical services. Following Kaplan and Violante
(2010), I first purge the data of non-model features by regressing log consumption and log income
on year & cohort dummies and a range of demographic variables.

\(^8\)Two assumptions are required for the estimator to be consistent. The first is that households have no
information about future shocks, the second is that house
4.1.2 Measuring Expected Changes in Debt

To verify the model’s predictions on the heterogeneity of households’ MPC, I need to identify households who plan on adjusting their housing stock in the near future. Using actual changes in household balance sheets may bias our results due to unexpected shocks which affect both household consumption and their decision to take on debt.\(^9\)

To control for this potential bias I use households’ surveyed expectation that they will move in the next couple of years. Specifically the PSID asks households

A51. Do you think you might move in the next couple of years?

If a household replies in the affirmative, this question is followed with

A52. Would you say you definitely will move, probably will move, or are you more uncertain?

This variable, when lagged, is orthogonal to contemporaneous shocks and highly correlated with future decisions to buy new houses as shown by Table 5.

<table>
<thead>
<tr>
<th>Share of households that:</th>
<th>N</th>
<th>subsequently move</th>
<th>move into a larger house</th>
<th>move into a similar sized house</th>
<th>move into a smaller house</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do not expect to move</td>
<td>40,183</td>
<td>0.17</td>
<td>0.80</td>
<td>0.04</td>
<td>0.16</td>
</tr>
<tr>
<td>Expectation - uncertain</td>
<td>4,075</td>
<td>0.34</td>
<td>0.84</td>
<td>0.03</td>
<td>0.13</td>
</tr>
<tr>
<td>Expectation - probably</td>
<td>7,154</td>
<td>0.50</td>
<td>0.86</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>Expectation - definitely</td>
<td>10,004</td>
<td>0.72</td>
<td>0.87</td>
<td>0.03</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 5: PSID Surveyed Expectations

4.1.3 Results

In order to compute the MPC I must first group households into buckets. I try three different strategies to identify households who anticipate buying or selling housing stock. First, I group households by their self-reported expectations in the previous survey. The second approach is to estimate a logit model of the probability a household will move, regressed on household lagged

\(^9\)For example an unexpected persistent, positive income shock may lead households to increase both consumption and their willingness to hold debt.
expectations, demographic variables and lags of family income and employment. The equation provides us with a continuous measure of the likelihood that a household will move. Households are then grouped into quartiles based on this estimated likelihood of moving. Although the majority of households who adjust their stock of housing increase their stock, both of these strategies combine households regardless of whether they are net buyers or net sellers. To separate out the minority of households who move into smaller houses, my third approach groups households by whether they actually did move in the subsequent period, and whether the house they moved into had a smaller, unchanged or larger number of rooms.

Figure 4 shows the estimated MPCs of households according to their expectations about the likelihood of moving in the next couple of years.

![MPC and household expectations](image)

Figure 4: MPC and household expectations

These results show that households with a higher expectation of moving house consistently have a lower MPC. Indeed for households that report that they “Definitely expect to move” in subsequent periods or are in the top quartile of the estimated equation, the MPC is statistically below zero as predicted by the model. Furthermore when households are grouped by their actual change in housing tenure, I show that households which this effect is stronger for those households who move into larger houses.

The intuition behind this result is illuminated by breaking down the covariances that the BPP estimator is comprised of. The correlation between $\Delta y_{t+1}$ and $\Delta c_t$ is positive for the population as a whole, but is negative for the subset of households who previously reported that they had a high probability of moving. This change in sign occurs because households who expect to move, but subsequently receive a negative income shock, are more likely to delay or cancel their
planned purchase of housing. By choosing not to adjust their stock of housing they remain free from mortgage debt and with a high level of liquid assets which results in a higher level of consumption.

![MPC and Actual Change in Housing Tenure](image)

**Figure 5: MPC and changes in household tenure**

While the standard errors around the estimates produced with this method are large, the covariance between anticipated increases in debt and households’ MPC, and the negative MPC for those who are highly likely to buy additional housing stock, are consistent with the results of the model.

The identification of households with a negative MPC is a novel result. The closest related work is Martin (2003) who shows a similar effect in a partial equilibrium model of the housing market with transaction costs. There are several possible explanations for this. First, the share of households who adjust their stock of housing in any given period is quite small. As empirical studies usually compare households across quantiles, unless the analysis deliberately isolates such households it is likely that they will be subsumed by the majority of households that do not adjust their stock of housing and have a conventionally non-negative MPC. Secondly, because the optimal adjustment point varies with income, even analyses using administrative data such as Fagereng et al. (2016), which is able to partition households by their balance sheets in finer gradients, may fail to find this effect. Finally, while the finding that some households have a negative MPC is new to the literature, these results are consistent with the broader conclusions on the negative correlation between households’ MPC and their ownership of liquid assets.
4.2 Consumption Growth Regressions

The second exercise I conduct is to estimate how anticipated changes in debt affect the level of consumption. To estimate this effect I adapt the augmented Euler equation used by Dynan (2012) to model how changes in debt affect growth in consumption:

$$\Delta \log c_{i,t} = \beta_0 + \beta_r r_{t-1} + \beta_y \Delta \log y_{i,t} + \beta_{Debt} \Delta D_{i,t} + \beta_W \Delta \log NW_{i,t} + \beta X_{i,t} + \varepsilon_{i,t}$$

Where $c_{i,t}$ is the log of real consumption, $r_t$ is the real interest rate, $y_{i,t}$ is the household’s real income, $D_{i,t}$ is the level of debt-to-asset of each household and $X_{i,t}$ is a set of demographic factors. Note that since the PSID is a biannual survey I define $\Delta$ such that $\Delta x_t = x_t - x_{t-2}$.

In order to identify the impact of expected debt on household consumption I instrument the debt-to-asset ratio with households’ lagged expectation of the probability that they will move along with lagged variables for income, consumption and demographic variables.

Since I only have data on actual changes in household debt, I need to be careful that I identify only the effect of expected changes, and not the impact of changes of debt motivated by contemporaneous shocks. It is relatively straightforward to extend Hansen and Singleton (1982) to show that if the set of instruments used to identify the endogenous variable contain only information from time $t - 2$ then an instrumental variable regression will produce a consistent estimator of the expected change in debt. This result is driven by the assumption of rational expectations, in which i.i.d deviations from prior expectations are orthogonal to information available at time $t - 2$. A sketch of this of extension is outline in Appendix C.\textsuperscript{10} Thus by using the household’s self reported expectation of moving as a instrument for changes in debt, I will recover a consistent estimator of the effect of an expected change in debt on household consumption. It should be note that while this estimator is consistent what ever the true parameters of the model are, it is only unbiased when the marginal effect of an expected increase in debt is equal to that of unexpected change (i.e. $\beta_1 = \beta_2$ in the context of above proposition). However simulating the model under a wide range of parameters suggest that this bias will be quantitatively small given our sample size.

\textsuperscript{10}It should be noted that while the IV regression will provide a consistent estimator for the effect of expected changes in household debt, it will not be unbiased. However simulations suggest that this bias will be quantitatively small in our sample size over a broad range of possible parameter values.
<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔDebt-Asset&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>-0.043 *</td>
<td>-0.043 *</td>
<td>-0.046 *</td>
<td>-0.051 *</td>
<td>0.011</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δ&lt;sub&gt;i,t&lt;/sub&gt;IMortgage</td>
<td></td>
<td>-0.019***</td>
<td>-0.018***</td>
<td>-0.015***</td>
<td>-0.021***</td>
<td>0.0141</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Δlog Debt&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.015**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔFamily Size&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>0.015</td>
<td>0.015</td>
<td>0.012</td>
<td>0.020</td>
<td>0.079***</td>
<td>-0.027</td>
<td>-0.023</td>
<td>-0.0219</td>
<td>-0.0372</td>
<td>0.105***</td>
</tr>
<tr>
<td>r&lt;sub&gt;t-2&lt;/sub&gt;</td>
<td>-0.009***</td>
<td>-0.009***</td>
<td>-0.009***</td>
<td>0.000</td>
<td>-0.004*</td>
<td>-0.00369*</td>
<td>-0.00361*</td>
<td>-0.00397*</td>
<td>0.00432</td>
<td>0.00182</td>
</tr>
<tr>
<td>Δlogy&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>-0.005</td>
<td>-0.005</td>
<td>-0.005</td>
<td>0.013***</td>
<td>-0.0230***</td>
<td>-0.0228***</td>
<td>-0.0261***</td>
<td>0.0199***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔlogW&lt;sub&gt;i,t&lt;/sub&gt;</td>
<td>-0.002</td>
<td>-0.005*</td>
<td>0.000</td>
<td></td>
<td>-0.00306*</td>
<td>-0.00224</td>
<td>0.00183***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>25545</td>
<td>25545</td>
<td>25157</td>
<td>11601</td>
<td>25157</td>
<td>28532</td>
<td>28532</td>
<td>28068</td>
<td>13552</td>
<td>27690</td>
</tr>
</tbody>
</table>

Table 6: Regression Table

Source: Authors Regressions.
Dependent Variable is the change in log of real consumption.
All regressions instrument contemporaneous variables are using lags of endogenous variables and households expectation of moving, unless specifically stated otherwise. Sample includes households with consecutive, complete set of interviews from 1999 to 2015. Also included but not reported are a constant, controls for the age and education of the household head. The following approximation for logs is used to log \( \left( x_t + (x_t^2 + 1)^{\frac{1}{2}} \right) \) following the practice in the empirical literature to down-weight large values and include households with negative wealth. All nominal variables are detrended by the core PCE index.
* The variable Δ<sub>i,t</sub>IMortgage is equal to 1 if the household \( i \) obtains a mortgage at time \( t \) and 0 otherwise.
These results show that anticipated increases in household debt have a significant impact on household consumption, even when it is anticipated years in advance across a range of measures and specifications.

To compare these estimates to the predictions of the model I calculate the average change in debt and leverage that occurs when household’s adjust their stock of housing in the model and multiple these estimates by the predictions within the model. For example, my estimates suggest that an increase in the mortgage-value ratio by 0.1 would lead to decrease in the level of consumption around 0.45 per cent. Within the sample the average purchase of a house is associated with an increase in the debt to asset ratio of 0.62. Thus for the average home buyer these estimate imply that consumption falls by 2.8 per cent due to the increase in mortgage debt. This is consistent to the quantitative outcomes of the model that predict a fall in consumption of approximately 4 per cent upon upgrading housing stock.

I show that using a more comprehensive measure of consumption and a range of specifications upholds the results from Martin (2003). Namely, that households that choose to purchase additional housing stock decrease their non-durable consumption. This result accords with the predictions of the model and highlights the importance of transaction costs and financial frictions when modeling household’s consumption choices.

5 Fiscal Transfers and the Housing Market

Subsidies for home ownership are a common feature of housing markets (Andrews et al., 2011). These policies range from subsidized access to credit, tax preferences for household debt and outright transfers to home buyers. While many of these are subsidies are structural in nature, a common component of the fiscal response to the global financial crisis was to expand these programs to help stabilize housing markets and stimulate demand within the economy (Table 7).

The results from the previous sections raise the question as to whether a transfer payment or tax credit to these almost home-owners may decrease aggregate consumption in general equilibrium and thus be contractionary in nature. This is an important policy question as tax credits to households saving for their first deposit were enacted during the financial crisis. While these programs were primarily designed to help boost the housing and credit markets, it was also assumed that they would help boost the broader economy and increase output (Dynan et al., 2013).
Table 7: Selected Transfers targeted at First Home Buyers

<table>
<thead>
<tr>
<th>Country</th>
<th>Size (% of median home price)</th>
<th>Dates</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>$7,500-8,000 (3.5%)</td>
<td>April 2008 - June 2010</td>
<td>Initially an interest free loan, it was converted into an outright grant for first home buyers.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2012 - Present</td>
<td>“Right to Buy” discounts were introduced in 1980, but greatly expanded in 2012.</td>
</tr>
<tr>
<td></td>
<td>Discount of up to $140,000 for public housing tenants to buy their home (35-70% depending on length of tenancy)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>$11,200 (4 %)</td>
<td>October 2008 - September 2009</td>
<td>This is in addition to $6,000 permanently available.</td>
</tr>
<tr>
<td>Canada</td>
<td>$700 (0.3%)</td>
<td>January 2009 - Present</td>
<td></td>
</tr>
</tbody>
</table>

I consider two policy scenarios. The first a unconditional transfer to renters who are on the margin of purchasing a house. The second is a conditional tax credit which is available to all renters who purchase a purchase additional housing stock with a one year window.

5.1 Unconditional Transfers

To estimate the impact of these programs I model the impact of a $8,000 dollar debt-funded transfer to renters who are sub-marginal first home buyers. I model this transfer as a one-off increase in liquid wealth to households who are currently renting and are on the margin of becoming a home owner. The transfer is funded by the government by an increase in debt, while tax rates remain unchanged. I then solve for the impact of this transfer in equilibrium allowing prices to adjust to clear markets.

I solve for the equilibrium path for the model as it returns to steady state after the initial transfer. Figure 6 shows the impact from this policy on the housing market and the broader economy. Unsurprisingly the transfer increases both prices and the stock of housing owned by households, as the transfer to renters enables them to purchase more housing and sooner then they otherwise would have. This leads to an increase in household leverage within the model which results in households decrease their consumption via the mechanism that I explored in when prices and wages are held constant.
5.2 Conditional Transfers

An alternative policy design is to offer tax credits when renters purchase a house. I model this scenario by announcing a policy that will give renters a $8,000 payment if they purchase a house in the next four quarters. This policy differs from a straight fiscal transfer in two ways. First, by applying the tax credit over a longer period it allows more renters to take advantage of it magnifying its impact on the goods and housing market. The second is that it provides the opportunity for renters to adjust their behavior in anticipation of taking advantage of the credit before it expires.

If the tax policy is open to all renters who buy a house within a set period, in this case one year, the general equilibrium impact of the policy has two distinct phases. From the time the tax credit is announced renters decrease their consumption of both non-durable goods and rental stock in order to increase their stock of savings high enough to enable them to buy a house while the credit is available. This temporary incentive to purchase a house thus results in a fall in consumption while households try to increase their stock of savings. When the tax credit is due to expire renters’ incentive to buy a house, rather then delay and continue saving, spikes and demand for housing increases dramatically. This leads to an increase in purchases, house prices and interest rates on debt.
Models the impact of a $8,000 payment to renters conditional on buying housing stock within one year.

After the tax credit has expired the economy slowly returns to equilibrium. The overall effect is to effect a shift in housing from the real estate sector to owners, due to the incentive provided by the temporary tax credit which pulls forward housing purchases from future periods. The incentive to pull forward the purchase of housing has the opposite effect on non-durable consumption which is pushed back as households prioritize their investment in housing. Thus despite the state intention of policy makers to stimulate the broader economy, I find that this tax credit has a contractionary impact on aggregate consumption and output.

Previously empirical studies of these programs Dynan et al. (2013) find that while the tax credit affected the price of housing, they failed to find a significant positive impact on the broader economy. These results suggest that one possible resolution of this puzzle is the tax credit merely encourages households to substitute consumption across time and goods, pulling forward the purchase of housing and pushing back the consumption of non-durable goods. This finding is consistent with Mian and Sufi (2010) who the evaluated other temporary tax credits such as the “cash for clunkers” program, which offered tax credits for owners trading in fuel inefficient cars, and show that it had a limited effect on overall demand.
6 Sensitivity Analysis

6.1 Partial Borrowing Constraints

In the baseline version of the model I assumed that mortgage debt taken on by households is taken on a rolling basis. This implies that fluctuations in the price of housing will change the endogenous borrowing constraint faced by households. However, as discussed by Iacoviello and Pavan (2013) the assumption that the collateral constraint binds at all times is at odds with the reality that loan-to-value constraints only bind for agents that refinance. In practice, falls in the price of housing do not tighten the credit constraints of the majority of households who choose not to refinance. To relax this assumption and to better match the lagged correlation between measures of aggregate debt and house prices, I relax this assumption to see how it affects the baseline results.

Concretely, I assume that the endogenous credit constraint applies directly to households that choose to adjust their stock of housing. For households who do not choose to adjust their stock of housing, I assume that stock of debt can be maintained at its current level even if the value of their housing stock falls due to a decline in house prices. This introduces an asymmetry into the relationship between house prices and the debt constraint. If house prices fall then households are not forced to immediately reduce their mortgage, but rather can maintain the size of the debt making repayments that cover the interest only. If house prices rise then households have the option of refinancing their mortgage and increasing their stock of debt. I assume that a household can costlessly refinance its current stock of housing. Thus, the borrowing constraint becomes

\[
\begin{align*}
    b_t &\geq -Ma_t p_t^a & \text{if } I_{\alpha_t} = 1 \\
    b_t &\geq \min \{-Ma_t p_t^a, b_t\} & \text{if } I_{\alpha_t} = 0
\end{align*}
\]

(24)

This changes both the steady-state of the model, and its response to shocks around the equilibrium. While the price of housing is constant in the steady state, this alternative borrowing constraint loosens restrictions on households for whom the depreciation of their stock of housing
would otherwise cause the debt constraint to bind. In equilibrium this relaxation allows households to take on greater levels of debt due to the reduced probability of being credit constrained. Accordingly, the equilibrium interest rate is marginally lower.

The impact of this assumption on the estimated impact of the tax credit for is marginal as house prices generally increase in response to the policy.

6.2 Heterogeneous Labor Supply

Previous research has indicated that households vary labor supply in response to changes in their balance sheet Attanasio et al. (2012). In the baseline model labor supply is invariant to households asset portfolio. Here I relax this assumption by considering a utility function in which the disutility from supplying labor is separable from consumption

\[ u_t = \left( \frac{c_t^{1-\gamma} h_t^{\gamma}}{1-\gamma} \right)^{1-\gamma} - \phi z_t^{l+\frac{1}{\eta}}. \]  

(25)

Under this alternative function for flow utility, a household’s optimal supply of labor will set by

\[ z_t w_t \frac{\partial u_t}{\partial c_t} = - \frac{\partial u_t}{\partial l_t}, \]

which in turn implies that it each household’s optimal labor supply \( l_t = l_t (a, b, z) \) will vary according to household’s asset portfolio, though not with the productivity shock. To reflect the change in household preferences I re-calibrate the parameters within the utility function to match their original targets.

When I assume that labor supply varies with households assets I create a second margin that households can adjust to respond to the change in debt that occurs when they adjust their stock of housing. Specifically, households increase their supply of labor in anticipation and after buying additional stock. The reasoning behind this choice is the same as that of non-durable consumption, namely that households increase their labor supply to reduce the probability their will become bound by their credit constraint and reduce the time spent paying the higher interest rate on their mortgage. The same logic applies in reverse to households who anticipate decreasing their stock of housing.

Despite this second margin with which households can use to off-set fluctuations in the level of
their non-durable consumption, the effect on marginal propensities to consume is quantitatively small given reasonable calibrations for the Frisch elasticity of supply.

7 Conclusion

This paper establishes that the presence of financial frictions and transaction costs in the housing market generate quantitatively important distortions in households consumption of non-durable goods. In particular I showed that these frictions create an anticipatory saving effect, by which households increase their savings in the expectation that they will face higher interest rates and be more likely to be credit constrained after making a debt-financed purchase of housing.

My results suggest that this effect is quantitatively meaningful. For households close their optimal point of adjustment an increase in liquid assets will lead to a decrease in consumption as the anticipatory saving effect outweighs the wealth and liquidity effects. I provide empirical evidence for these results using micro-data from the PSID to show households MPC is negatively correlated with the probability that a household will move, and that those who report that they definitely expect to move have a negative MPC.

This finding has important implications for the use of fiscal incentives in the housing market as a counter-cyclical policy tool. My general equilibrium model shows that tax credits for first home buyers decrease aggregate consumption due to an increase in mortgage debt leads. I however find that these transfers to boost prices and sales in the housing market, though much of this is the result of demand being pulled forward. While our model indicates that such policies lower output and employment, my model of the financial system is relatively simplistic. In the context of a bursting house price bubble and a financial crisis, a policy that helps stabilize house prices may be welfare-improving despite the decrease in consumption due to higher household leverage.

More broadly, these results suggest there is strong empirical support for heterogeneity in households marginal propensities to consume and that these have significant implications for macroeconomic policy. Beyond the implications for fiscal transfers, my work on the relationship between debt and consumption raises questions - such as how monetary policy transmission may be affected - which should I leave as avenue for future work.
References


A Details on Model Solution

A.1 Model Approximation

I present the households’ HJB equation and Kolmogorov forward equation for the evolution of the distribution of households $\mu$ over their holdings of liquid assets, $b$, housing stock, $a$, and income, $z$. We first solve for HJB equation assuming that households choose not to adjust their stock of housing. The stationary version of the households HJB equation is given by

\[(\rho + \lambda) V^s (a,b,z) = \max_{c,l,c',\alpha} u (c,h,l) + V^s_b [zw (1 - \tau) l + r (b) b - c - \epsilon^rent p^rent + T] \tag{26} \]

\[+ V^s_a (\delta^a a) + V^s_z (-\beta z) + \lambda \int_{-\infty}^{\infty} (V^s (a,b,x) - V^s (a,b,z)) \phi (x) dx, \]

where the value function is determined by the current flow of utility and the expected change in state variables.

The optimal drift in liquid assets is equal to the level of household savings which is governed by the budget constraint. Since we assume that households do not buy or sell housing stock the optimal drift in housing stock is driven solely by depreciation. Finally, the expected change in household income is determined by the mean reversion and the expectation of idiosyncratic “jumps” which cause log-income to jump to a new level drawn from normal distribution of density $\phi$ with variance $\sigma^2$.

We assume that if household choose to adjust their stock of housing then they will choose the optimal portfolio of assets subject to their budget constraint

\[V^a (a,b,z) = \max V^s (a',b',z) \tag{27} \]

\[\text{s.t.} \]

\[p^a a + b = p^a a' + b' - k (a,a'). \]

The household re-allocates their wealth across the two assets subject to paying the cost of adjustment, $k (a,a')$.

The households’ final value function is given by
\[ V(a, b, z) = \max \{ V^a(a, b, z), V^s(a, b, z) \}, \]

such that households choose to 'stay' or 'adjust' depending on which value function is larger. The adjustment function can then be defined as

\[
f^{\text{adjust}} = \begin{cases} 
1 & \text{if } V(a, b, z) = V^a(a, b, z) \\
0 & \text{if } V(a, b, z) = V^s(a, b, z), 
\end{cases} \tag{28}
\]

and, conditional paying the adjustment cost, the optimal portfolio, \( a^*(a, b, z) \) and \( b^*(a, b, z) \), is solved by maximising 27.

The households’ optimal choice variables \((c_t, c_t^{\text{rent}}, l_t)\) are determined by this value function and the inverse of the utility function

\[
c_{i,j,k} = \left( \frac{\partial u}{\partial c} \right)^{-1} \left[ \frac{\partial V(a, b, z)}{\partial b} \right] \tag{29}
\]
\[
c_{i,j,k}^{\text{rent}} = \left( \frac{\partial u}{\partial h} \right)^{-1} \left[ \frac{p_t}{\partial r} \left( \frac{\partial V(a, b, z)}{\partial b} \right) \right] \tag{30}
\]
\[
l_{i,j,k} = \left( \frac{\partial u}{\partial l} \right)^{-1} \left[ w_t \left( \frac{\partial V(a, b, z)}{\partial b} \right) \right] \tag{31}
\]

The evolution of the joint distribution of households is described by a Kolmogorov forward equation. I denote the density of households over the joint-distribution \( g(a, b, z, t) \) corresponding to the density function \( \mu_t(a, b, z) \). Furthermore denote the optimal drift function for liquid assets as \( s(a, b, z) \). Then the stationary density must satisfy the Kolmogorov forward equations

\[
0 = -\partial_b \left( s(a, b, z) g(a, b, z) \right) - \partial_a \left( \delta^a g(a, b, z) \right) + \int f^{\text{adjust}} \delta(a - a^*) \delta(b - b^*) \, d\mu_t \\
- \partial_z \left( -\beta z g(a, b, z) \right) - \alpha g(a, b, z) + \alpha \phi(z) \int_{-\infty}^{\infty} g(a, b, x) \, dx \\
- \lambda g(a, b, z) + \lambda \delta(a - a_0) \delta(b - b_0) \delta(z - z_0), \tag{32}
\]

where \( \delta \) is the Dirac delta function and \((a_0, b_0, z_0)\) are the birth level of assets and income.
This equation equates inflows and outflows of households across each point in the joint-distribution. The first line measures the outflow as household increase (decrease) their stock of liquid assets through saving (dis-saving). The second line relates to changes in households stock of housing via depreciation and the decision to buy or sell. The third line describes how exogenous changes in household productivity impact the distribution. Finally, the stochastic death and the birth of households with zero wealth is described in the last line.

### A.2 Steady State

I solve (26) and (32) expanding on the finite difference method outlined by Achdou et al. (2017). To solve the HJB function for households who choose not to adjust their stock of housing, I approximate the function \( V^s(a, b, z) \) on \( I \times J \times K \) discrete equi-spaced points. I denote the grid points \( b_i, i = 1, \ldots I, a_j, j = 1, \ldots, J, z_k, k = 1, \ldots K \), and define the discretised value function for households that do not adjust their stock of housing

\[
v_{i,j,k} = V^s(a_j, b_i, z_k).
\]

I approximate the derivatives for \( b \) with either a forward or backward difference approximation

\[
\frac{\partial v_{i,j,k}}{\partial b} \approx v^F_b = \frac{v_{i+1,j,k} - v_{i,j,k}}{\Delta h^+_i}, \\
\frac{\partial v_{i,j,k}}{\partial b} \approx v^B_b = \frac{v_{i,j,k} - v_{i-1,j,k}}{\Delta h^-_i}, \tag{33}
\]

and use an upwind method to choose when to use the each approximation. I use the forward (backward) approximation whenever the drift of \( b \) is positive (negative). Since the drift terms for housing and income are always negative (due to depreciation and mean reversion) I use the backward difference approximation universally for these variables. As outline by Achdou et al. (2017) this approximation allows us to solve for the discretised value function using a process of iteration.
\[
\frac{v^{n+1}_{i,j,k} - v^n_{i,j,k}}{\Delta \text{step}} + (\rho + \lambda) v^{n+1}_{i,j,k} = u\left(c^n_{i,j,k}, h^n_{i,j,k}, l^n_{i,j,k}\right) \\
+ v^n_b \left[ z w_t (1 - \tau) h^n_{i,j,k} + r (b_t - c^n_{i,j,k} - rent_n P_t + T_t) \right] \\
+ v^n_a (\delta a_j) + v^n_z (-\beta z_k) + \lambda \int_{-\infty}^{\infty} (v^n_{i,j,x} - v^n_{i,j,k}) \phi(x) \, dx,
\]

which can in turn be written as

\[
\left[ \left( \frac{1}{\Delta \text{step}} + \rho + \lambda \right) I - A \right] v^{n+1}_{i,j,k} = u\left(c^n_{i,j,k}, h^n_{i,j,k}, l^n_{i,j,k}\right) + \frac{1}{\Delta \text{step}} v^n_{i,j,k},
\]

where \( \left(c^n_{i,j,k}, h^n_{i,j,k}, l^n_{i,j,k}\right) \) are the choice variables optimised by the households under the value function \( v^n_{i,j,k} \) and \( A^n \) is a \((I \times J \times K)^2\) square matrix which summarises the exogenous productivity process, depreciation of housing stock and optimal household saving.

The HJB equation is then solved for with the following algorithm.

1. Start with a guess for the value function \( v^0_{i,j,k} \)

2. Compute the derivatives of \( v^n_{i,j,k} \) using (33) and the upwind approximation.

3. Compute the optimal household choices \( \left(c^n_{i,j,k}, h^n_{i,j,k}, l^n_{i,j,k}\right) \) using the first order conditions \( X \) and the approximated derivatives of \( v^n_{i,j,k} \)

4. Find \( v^{n+1}_{i,j,k} \) from (34)

5. Compute the value function for households conditional of moving with a

\[
\left(v^*_{i,j,k}\right)^{n+\frac{1}{2}} = \max_{a', b'} v^{n+\frac{1}{2}}_{i,j,k} \text{ s.t. } p^a a + b = p^a a' + b' - k \left(a, a'\right).
\]

6. Given \( v^{n+\frac{1}{2}}_{i,j,k} \) and \( \left(v^*_{i,j,k}\right)^{n+\frac{1}{2}} \) calculate

\[
v^n_{i,j,k} = \max \left\{ v^{n+\frac{1}{2}}_{i,j,k}, \left(v^*_{i,j,k}\right)^{n+\frac{1}{2}} \right\}
\]

7. Repeat steps 2 through 6 until the difference between \( v^{n+1}_{i,j,k} \) and \( v^n_{i,j,k} \) is below a given threshold.
Once the discretised value function has been calculated we can solve the Kolmogorov Forward equation. In the absence of adjustments in the stock of housing the joint distribution can be conveniently be calculated by the linear system

\[ 0 = A'g, \]

where \( A \) is the transpose of the transition matrix in (34) as shown by Achdou et al. (2017), with a minor adjustment for the stochastic birth and death process.\(^{11}\)

To account for adjustments in the stock of housing we need to modified this transition matrix to

\[
C_{m,n} = \begin{cases} 
0, & \text{if } I^{\text{adjust}}(n) = 1 \\
A_{m,n} + \sum_{i=1}^{I^{\text{adjust}}(n)=1} A_{m,n} & \text{if } I^{\text{adjust}}(n) = 0, \ m'(m) = n \\
A_{m,n} & \text{if } I^{\text{adjust}}(n) = 0, \ m'(m) \neq n,
\end{cases}
\]

(35)

where \( m' \) is the optimal adjustment target conditional on a household deciding to change its stock of housing. \( I^{\text{adjust}} \) is a vector which indicates if it is optimal for household to adjust.

The first row indicates that any point \( n \) that lies in the adjustment region will have a density of 0. Instead, the second row says that any households who would otherwise transition into the adjustment region instead moves to the optimal adjustment target. Finally, any point that is not in the adjustment region, nor the optimal target for a point in the region, remains the same as the original transition matrix.\(^{12}\) The density is thus given by

\[ 0 = C'g, \]

where \( g(a,b,z) \) is normalised such that

\[ 1 = \int_{a}^{a'} \int_{b}^{b'} \int_{z}^{z'} g(a,b,z) \ da \ db \ dz. \]

Once we have solved for the density of households, \( g(a,b,z) \), we can easily solve for the

\(^{11}\)Specifically, I add, \( \lambda \) to each element of the row associate with the birth position \((a_0, b_0, z_0)\) and subject \( \lambda \) from each diagonal element of \( A \).

\(^{12}\)To ensure the matrix \( C \) is not singular we also set \( C_{m,m} = -\epsilon \) for some small \( \epsilon > 0 \) for all \( I^{\text{adjust}}(m) = 1 \). This ensures that all points in the adjustment region have a defined density of zero.
aggregate demand for liquid, $B_t$, and illiquid, $A_t$, assets

$$B_t = \int_{0}^{a_j} \int_{b_i}^{b_l} \int_{z_1}^{z_K} b g(a, b, z) \, da \, db \, dz$$

$$A_t = \int_{0}^{a_j} \int_{b_i}^{b_l} \int_{z_1}^{z_K} a g(a, b, z) \, da \, db \, dz.$$

### A.3 Transition Dynamics

Outside of the steady state, when changes in prices, wages or interest rates vary over the time, the time-dependent HJB equation is required

$$(\rho + \lambda) V^n(a, b, z, t) = \max_{c, h, l, \alpha} \left[ z_t w_t (1 - \tau) l + r_t (b - c) - c^{rent} p_t^{rent} + T_t \right]$$

$$+ V^n_a(\delta a) + V^n_z(-\beta z) + \lambda \int_{-\infty}^{\infty} (V^n(a, b, x) - V^n(a, b, z)) \phi(x) \, dx$$

$$+ V^n_t(a, b, z, t),$$

where the last term accounts for changes in the value function over time. Solving this function over the same discrete grid as in the steady state results in the approximation

$$(\rho + \lambda) v_{i,j,k}^n = u \left( c_{i,j,k}^{n+1}, h_{i,j,k}^{n+1}, l_{i,j,k}^{n+1} \right) + v^n_b \left[ z_t w_t (1 - \tau) l_{i,j,k}^n + r_t (b_t - c_{i,j,k}^n - c^{rent,n} p_t^{rent} + T_t \right]$$

$$+ v^n_a(\delta a) + v^n_z(-\beta z) + \lambda \int_{-\infty}^{\infty} (v_{i,j,x}^n - v_{i,j,k}^n) \phi(x) \, dx$$

$$+ \frac{v_{i,j,k}^{n+1} - v_{i,j,k}^n}{\Delta t},$$

where $v_{i,j,k}^n$ is now short-hand for $v(a_j, b_i, z_k, t^n)$ and the terminal condition $v_{i,j,k}^N$ is equal to the steady state solution $v_{i,j,k}$. This can be written as

$$\left( \rho + \lambda - \left[ A^{n+1} + \frac{1}{\Delta t} \right] \right) v_{i,j,k}^n = u \left( c_{i,j,k}^{n+1}, h_{i,j,k}^{n+1}, l_{i,j,k}^{n+1} \right) + \frac{1}{\Delta t} \left( v_{i,j,k}^{n+1} \right),$$

which yields a solution for $v_{i,j,k}^n$ given $v_{i,j,k}^{n+1}$.

Similarly the joint distribution may vary over time, thus outside of steady state the time-
dependent Kolmogorov equation is

\[
\partial_t g(a, b, z, t) = -\partial_b \left(s(a, b, z) g(a, b, z)\right)
- \partial_a (\delta^a g(a, b, z)) + \int \text{I}^{\text{adjust}} \delta(a - a^*) \delta(b - b^*) d\mu_t
- \partial_z (-\beta z g(a, b, z)) - \alpha g(a, b, z) + \alpha \phi(z) \int_{-\infty}^{\infty} g(a, b, x) \, dx
- \lambda g(a, b, z) + \lambda \delta(a - a_0) \delta(b - b_0) \delta(z - z_0).
\]

Solving this equation over the same discrete grid requires a two step n

\[
g^{n+1} = \left(\mathbf{I} - \Delta t (C^n)\right)^{-1} g^n,
\]

where \(C^n\) is the transition matrix from (37) adjusted for households who choose to adjust their stock of housing.

Once we have solved for the discrete-time steady state value function, \(v(a, b, z)\), and density \(g(a, b, z)\) it is relatively straightforward to solve for transition dynamics in response to a “MIT” shock - an unanticipated shock followed by a deterministic transition back to the (potentially new) steady state. To solve for the transition path we use the following algorithm.

1. Guess a path for wages, \(w_t\), interest rates, \(r_t^b\), and house prices, \(p_t^h\).
2. Given the path of wages, interest rates and prices solve the HJB backwards in time, using \(v_{i,j,k}\) as the terminal condition.
3. Using this time path for the value function, calculate the optimal rules for household consumption and housing stock adjustment for each time period.
4. Given the optimal saving and adjustment rules solve the time-dependent Kolmogorov forward equation, using the steady state as the initial condition \(g^0\).
5. Given the evolution of the density of households calculate the aggregate household supply of labour and demand for liquid and illiquid assets over time.
6. Update the path for wages, interest rates and house prices such that
(a) $w'_t = w_t - \xi_w (N_t + N^c_t - \int zl(a, b, z, t) \, d\mu_t)$, such that the wage falls (rises) when there is excess labour supply (demand).

(b) $(r^b_t)' = r^b_t - \xi_r (\int b \, d\mu_t - p^A_t C^\text{rent}_t + B^\text{gov}_t)$, such that the interest rate falls (rises) when there is an excess supply (demand) of liquid assets.

(c) $(p^a_t)' = p^a_t - \xi_a (IA_t - \delta A_t + \int \dot{a}_t + \dot{c}^\text{rent}_t \, d\mu_t)$, such that the price of houses falls (rises) when there is an excess supply (demand) of housing.

7. Repeat steps 2 through 6 until the sum of the changes in three time path of prices $(w_t, r^b_t, p^a_t)$ converges.

B Marginal Propensities to Consume

In this continuous time model I follow Achetou et al. (2017) in their definition of the marginal propensity to consume.

**Definition 1.** The marginal propensity to consume over a period $\tau$ for an individual with the state vector $(a, b, z)$ is given by

$$MPC^x_{\tau} (a, b, z) = \frac{C_{\tau} (a, b + x, z) - C_{\tau} (a, b, z)}{x},$$

(40)

where $C_{\tau} (a, b, z)$ is the sum of expected consumption of an individual household over the next $\tau$ periods.

$$C_{\tau} (a, b, z) = E \left[ \int_0^\tau c (a_t, b_t, z_t) \, dt | a_0 = a, b_0 = b, z_0 = z \right]$$

(41)

This conditional expectation can be calculated using the Feynman-Kac formula. This formula links the conditional expectations of a stochastic process and the solution to partial differential equations.

C Rational Expectations and Instrumental Variables

**Proposition 2.** Given a lagged instrument, an IV regression will produce a consistent estimator of the marginal effect of the expected change in the endogenous variable, ignoring the marginal effect of contemporaneous changes.
Proof. Consider an endogenous variable which can be decomposed into a expected component based on information available at time $t - 2$ and a unexpected component, $\xi_t$

$$X_t = E_{t-2}X_t + \xi_t$$

Such that

$$E_{t-2}(\xi_t) = 0$$

Assume that in the true data generating process each component may have a different impact on the dependent variable, $Y_t$, such that

$$Y_t = \beta_1 E_{t-2}X_t + \beta_2 \xi_t + u_t$$

Now assume I have a instrument available, $Z_{t-2}$, that I wish to use to identify the marginal effect of the expected component, $\beta_1$, that is

Informative

$$E(Z_{t-2}E_{t-2}X_t) \neq 0$$

Valid

$$E(Z_{t-2}u_t) = 0$$

Only comprises information known at time $t - 2$ such that

$$E(Z_{t-2}\xi_t) = 0$$

Consider the regression of $Y_t$ on $X_t$ where $X_t$ is instrumented with $Z_{t-2}$

$$\hat{\beta}_{IV} = \frac{\frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) (Y_i)}{\frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) (X_i)}$$

Therefore
\[ p \lim_{N \to \infty} \hat{\beta}_{IV} = \frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) (Y_t) \]
\[ = \frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) (E_{t-2}X_t) + \beta_2 \frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) \xi_t + \frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) u_t \]
\[ = p \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) (E_{t-2}X_t) + p \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) (\xi_t) \]

By the law of large numbers

\[ p \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) E_{t-2}X = E ((Z_{t-2}) E_{t-2}X) \neq 0 \]
\[ p \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) (\xi_t) = E ((Z_{t-2}) (\xi_t)) = 0 \]
\[ p \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (Z_{t-2}) u_t = E ((Z_{t-2}) u_t) = 0 \]

Therefore

\[ p \lim_{N \to \infty} \hat{\beta}_{IV} = \beta_1 \]

Thus assuming that the instrument is valid and informative the IV estimator will be a consistent estimator of the marginal effect of the expected change in the independent variable.  \(\square\)