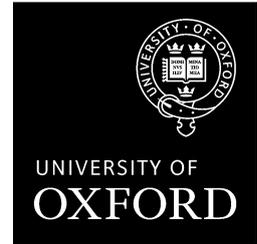


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Seven Principles for Managing Resource Wealth

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Abstract

This paper studies how capital-scarce countries should manage volatile resource income. Existing literature recommends that capital-scarce countries invest domestically, but that volatile resource income should be saved in a foreign sovereign wealth fund. I reconcile these by combining a stochastic model of precautionary savings with a deterministic model of a capital-scarce resource exporter. I show that capital-scarce countries should still establish a Volatility Fund, but it should be relatively smaller than in capital-abundant countries. The fund should be built before anticipated windfalls, partially invested domestically, and used as a source of income rather than a buffer against temporary shocks. To do so I develop a parsimonious framework that nests a variety of existing results as special cases, which are presented in seven principles. The first three apply to capital-abundant countries: i) Smooth consumption using a Future Generations Fund; ii) Build a Volatility Fund quickly, then leave it alone; and iii) Invest to stabilise the real exchange rate. The remaining four apply to capital-scarce countries: iv) Finance consumption and investment with oil; v) Use a temporary Parking Fund to improve absorption, vi) Invest part of the Volatility Fund domestically; and vii) Support private investment.

Keywords: Natural resources, oil, volatility, precautionary saving, capital scarcity, anticipation.

JEL Classification: D81, D91, E21, F34, H63, O13, Q32, Q33

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1 Introduction

Should governments consume their natural resource revenues, invest them in local capital or save in foreign assets? Existing work suggests that resource wealth should be saved in a sovereign wealth fund and consumed steadily. More should be saved if commodity prices are volatile and less if capital is scarce. This paper is the first to combine the stochastic literature on precautionary saving with recent deterministic literature on capital-scarcity. It shows that capital-scarce countries should save relatively less in a Volatility Fund, and invest more domestically, than capital-abundant ones. The fund should be built in advance if the windfall is anticipated, and be a source of income rather than a temporary buffer to smooth out oil shocks. The real exchange rate should appreciate steadily if capital is scarce, but be stabilised by investment if capital is abundant. To show this the paper develops an intuitive framework that nests a variety of existing results, which we present in seven principles.

Managing natural resources presents a major challenge for policymakers. Using global panel data from 1963 to 2003, Collier and Goderis (2009) estimate that for a country with commodity exports accounting for 35% of GDP, a 10% increase in commodity price leads to a 4 to 5% lower long run level of GDP per head.¹ The cause of this “resource curse” has been attributed to de-industrialization (or Dutch disease, see Corden and Neary, 1982), oil price volatility (Poelhekke and van der Ploeg, 2009), political instability and corruption (Sala-i-Martin and Subramanian, 2003 and Acemoglu et al., 2004), and environmental degradation (such as the case of Nauru in Hughes, 2004).

Addressing this challenge is at the forefront of the international policy debate. The International Monetary Fund (IMF) and World Bank regularly provide guidance to resource exporters (see Barnett and Ossowski, 2003, Baunsgaard et al., 2012, IMF, 2012 amongst others). Non-government organizations also advise policymakers on resource management, including the International Growth Centre and the Natural Resource Charter: recently adopted by the African Union Heads of State steering committee in 2012, and now being implemented in Nigeria.

The decision to consume, save or invest resource wealth lies at the heart of this debate. For many years the policy consensus was based on Friedman’s (1957) permanent income hypothesis: that a temporary income stream should be saved and only the permanent income from interest should be consumed. It assumes that capital is readily available, so domestic investment should not be affected by oil discoveries, as all profitable projects should already be financed by debt (the Fisher, 1930, separation theorem).² This underpins why 60% of resource-exporters save wealth abroad in a sovereign wealth fund (as advocated by Davis et al., 2001 and Barnett and Ossowski, 2003).³

¹There is a large body of literature on the resource curse, including Sachs and Warner (1999, 2001), Gylfason et al. (1999), Sala-i-Martin and Subramanian (2003) and Collier and Goderis (2009), and reviewed by van der Ploeg (2011).

²Hartwick (1977) first argued that revenues from exhaustible resources should be invested in domestic capital, to offset the declining stock of reserves. In essence, assets below the ground should be replaced by assets above the ground. However, this ignores access to foreign capital.

³Rather than using the permanent income hypothesis to justify a sovereign wealth fund, Hsieh (2003) uses payments from Alaska’s Permanent Fund to test the hypothesis, and confirms it by showing that households do smooth anticipated income.

Recent research is challenging the conventional wisdom. Oil prices are volatile, so there should be some precautionary savings in a Volatility Fund (see the review by Carroll and Kimball, 2008, and van den Bremer and van der Ploeg, 2013). Oil⁴ wealth also provides an important source of finance, particularly for capital-scarce developing countries. The rate of return on domestic investment in these countries is likely to be much higher than on offshore government bonds. Thus, some oil wealth should be directed towards investment (van der Ploeg and Venables, 2011, van der Ploeg, 2012). However, the effects of volatile oil prices if capital is scarce have not been studied.

This paper extends past work by considering how volatile oil prices will affect capital-scarce economies.⁵ If capital is scarce, then the government should save relatively less in a Volatility Fund. This is because such countries should already be saving a lot to overcome capital-scarcity. The benefit from additional saving will partly be outweighed by the costs of consuming less. Some of the Volatility Fund should also be invested domestically, as saving abroad (or repaying debt) will reduce borrowing costs and make more domestic investments viable. Investing some of the Volatility Fund domestically will equate the rate of return on saving abroad with that on investing at home. These results are presented in Principle 6.

This paper also adds new emphasis to existing results that may have been overlooked. Principles 1,2 and 4 show that policymakers should start preparing for volatility as soon as oil is discovered, even if it is received in the future. Principle 2 shows that the Volatility Fund should be treated as an additional source of permanent income, rather than a buffer that can be consumed when oil prices are low. This stems from the high persistence of oil price shocks. Principle 3 emphasises that the real exchange rate should be stabilised during an oil boom by investment, rather than just accumulating foreign reserves. Principle 7 emphasises that governments in developing countries should also support private capital if it is credit-constrained, to improve the productivity of public capital.

To do this the paper develops an intuitive framework that nests a variety of existing results, which we present in seven principles. Oil price volatility is captured using stochastic control, and its effect on the economy is summarised in two equations describing the expected path of consumption (the Euler equation) and total wealth (the budget constraint). This permits explicit results, and can be neatly explained using phase diagrams. The framework nests as special cases: the permanent income rule (Principle 1, Friedman, 1957); precautionary savings if capital is abundant and prices are volatile (Principle 2, van den Bremer and van der Ploeg, 2013); and saving for development if capital is scarce and prices are certain (Principle 4, van der Ploeg and Venables, 2011). Simple extensions capture the real exchange rate (Principle 3, van der Ploeg and Venables, 2013), and absorption constraints (Principle 5, van der Ploeg, 2012).

1.1 Characteristics of resource windfalls

Resource windfalls are a particular challenge for policymakers for a number of reasons. Resource revenues can be treated like a windfall of foreign currency. They are volatile,

⁴For convenience we will use “oil” as shorthand for resources more generally.

⁵“Capital-scarcity” is captured by assuming that there is a debt premium on interest rates. This mechanism is also used to “close” small open economies, inducing stationarity in general equilibrium models facing stochastic shocks (see Schmitt-Grohe and Uribe, 2003).

though the volatility diminishes as oil is extracted. The revenues are also usually anticipated, because of delays between discovery and production. Finally, the majority of resource-dependent economies are developing.

Resource revenues can be treated as a windfall of foreign currency. Commodity transactions, particularly oil, tend to be conducted in US dollars. Resource extraction is not particularly labour intensive, Norway's petroleum sector accounts for 21% of GDP but only 7% of employment, directly or indirectly (Statistics Norway, 2013). Extraction is more capital intensive, however in developing countries this tends to be imported by extraction firms, see Tullow Oil in Ghana. Resource wealth is therefore often modelled as an exogenous windfall (see review by van der Ploeg and Venables, 2012).

Resource revenues are also notoriously volatile, though this volatility is only temporary. Regnier (2007) finds that the prices of crude oil, refined petroleum, and natural gas are more volatile than 95% of products sold by US producers. Commodity price volatility has been used to explain the poor growth of resource dependent economies (see Ramey and Ramey, 1995, Gylfason et al., 1997, Blattman et al., 2007, Aghion et al., 2009, Poelhekke and Poelhekke, 2010 and van der Ploeg, 2010). However, this volatility is temporary, as the exposure to oil prices falls as oil is extracted. This is not particularly important for countries with extensive reserves: Venezuela, Canada and Iraq each have over 100 years left at current rates of production. However, if reserves will be exhausted in the near future then the change in exposure will be important: as in the next twenty years for Brazil, Oman and Brunei (BP, 2012). Policymakers face the challenge of what to do with a Volatility Fund if there is no more volatility.

Oil wealth is also usually anticipated. Once oil is discovered it can take years to build the rigs, pipelines and processing facilities necessary for production. For example, of the 400 offshore oil and gas fields discovered in the UK between 1957-2011, the mean time from discovery to production was 4.5 years (Wills, 2013). Policymakers face the challenge of what to do during the period of anticipation.

Finally, most resource-dependent economies are developing: 55% have a Human Development Index of low or medium (Baunsgaard et al., 2012). These countries face many challenges, in particular limited access to capital. Recent work has shown that some resource wealth should be invested domestically if capital is scarce, as the permanent income hypothesis does not apply (van der Ploeg and Venables, 2011). Policymakers in developing countries still face many challenges, including what to do before the windfall begins, whether to build a Volatility Fund, how big it should be, and what it should invest in.

1.2 The seven principles

This paper presents seven principles for managing natural resource wealth. The principles are presented in a single framework, and differ for developed and developing countries.

The first set of principles apply to developed, capital-abundant economies, such as Australia, Norway, the Netherlands and the UAE. The first priority of these countries

should be to save oil wealth for future generations (principle one) and against volatility (principle two). Investment should then be used to stabilise the real exchange rate (principle three). These principles are consistent with current policy in some, but not all, developed resource exporters.

The first principle is to “*Smooth consumption using a Future Generations Fund*”. A Future Generations Fund should be used to borrow before-, save during-, and finance consumption after the period of oil production. The size of the windfall will matter: if it is large or long, then more should be borrowed beforehand and less saved during. The fund is based on the permanent income hypothesis, and should be invested abroad if capital is abundant, as discussed above. This principle underpinned policy advice over the past decade,⁶ and provides a useful starting point for the analysis. The challenge of what assets these funds should invest in has been addressed by van den Bremer, van der Ploeg and Wills (2013).

The second principle is to “*Build a Volatility Fund quickly, then leave it alone*”, as oil price shocks are persistent. A Volatility Fund should be used as an additional source of income if prices are volatile. This based on precautionary savings.⁷ It has also been widely studied in a natural resource setting.⁸ This principle extends previous work in three ways. First, it focuses on analytical results. Second, it considers anticipated income and finds that the Volatility Fund should be prioritised at the start of the windfall. This includes saving (or borrowing less) before the windfall begins. Third, it shows that the fund should be treated as a source of permanent income, rather than a temporarily buffer. This is because oil price shocks are persistent, so if prices are low then they are likely to stay that way. The Volatility Fund should therefore not be consumed after oil has been extracted, but combined with the Future Generations Fund.

The third principle is to “*Invest to stabilise the real exchange rate, if capital is abundant*”. When some goods are not traded, the extra demand from an oil discovery will drive up their price and attract workers from traded sectors.⁹ Investing in the non-traded sector will meet higher demand with higher production, rather than higher prices. This principle extends previous work¹⁰ in two ways. First, by emphasising that a stable real exchange rate is a useful goal for guiding investment policy. Second, by showing that investment can limit Dutch disease if the initial capital stock is low, as fewer workers will need to be attracted from the traded sector. This only applies when capital is abundant. When capital is scarce the real exchange rate should slowly appreciate as capital is accumulated, as found in section 4.4.

⁶See Barnett and Ossowski, 2003, and IMF, 2012, which note that 39% of resource dependent economies were evaluated against the permanent income hypothesis on IMF visits.

⁷Leland (1968) gave one of the first formal treatments of precautionary savings which was followed by a number of papers including Sandmo (1970) and Zeldes (1986), and reviewed in Carroll and Kimball (2008). Kimball (1990a and 1990b) introduced “prudence” as a measure of precautionary savings. For a given utility function, $U(C(t))$, prudence is described by the coefficient of absolute prudence, $CAP(t) = -U'''(C(t))/U''(C(t))$, akin to that for risk aversion.

⁸In a natural resources setting recent work includes van der Ploeg (2010) in two periods, van den Bremer and Van der Ploeg (2013) in continuous time and with numerical simulations, and Bems and de Carvalho Filho (2011), Cherif and Hasanov (2011) and (2013), and Berg et al. (2012) in calibrated DSGE models.

⁹This is referred to as “Dutch disease”, see Corden and Neary (1982). This overturns the argument for saving all resource wealth abroad based on the Fisher separation theorem.

¹⁰Such as van der Ploeg and Venables, 2011, who focus on absorption constraints.

These principles support the policies of most, but not all, developed resource exporters. Nineteen of the twenty resource-dependent economies with a Human Development Index of “very high” or “high” have sovereign wealth funds (excluding Ecuador). These include measures to manage volatility, such as Norway’s fixed-deposit/fixed-withdrawal rule¹¹. The majority (75%) of these countries have pegged exchange rates, supported by stable non-traded goods prices such as Saudi Arabia since 1990 (Darvas, 2012). Sound policies have allowed countries such as Norway and the UAE to retain manufacturing sectors despite large oil exports (see Fasano, 2002 and Larsen, 2005). Lessons could still be taken from this analysis by countries such as Australia and Canada. These do not have national sovereign wealth funds, relying instead on state-based vehicles in Western Australia and Alberta. They have also seen high inflation in resource-rich states (ABS, 2013), which could be managed by further investment in the non-traded sector.

The next set of principles apply to developing countries with limited access to capital, like Ghana, Iraq and Nigeria. The first priority of developing countries should be to invest domestically (principle four). If investment cannot be absorbed, then a temporary Parking Fund should be used to hold revenues until they can be productively used (principle five). In these countries the Volatility Fund should not all be saved abroad, as domestic returns are high and the fund is an alternative source of income (principle six). Finally, if private capital is also constrained, then some resource revenues should be used to boost it (principle seven).

The fourth principle is to “*Finance consumption and investment with oil if capital is scarce*”. Developing countries face far higher costs of borrowing than developed ones. Ghana’s 2013 issue of ten-year, dollar denominated bonds yielded 8%, whilst ten-year US treasuries yielded 2.5%. At this cost of borrowing, many domestic investments will have a higher realised (as well as social) rate of return than foreign assets. Van der Ploeg and Venables (2012) show that oil revenues provide a cheaper source of finance for these countries, and so should be used to accelerate development. Consumption should also rise relatively more than capital-abundant economies, in the knowledge that future generations will be wealthier.

The fifth principle is to “*Use a temporary Parking Fund to improve absorption*”. An oil discovery should boost investment in capital-scarce countries. However, they may have difficulties absorbing a large influx of investment. This is because capital is needed to produce capital (as established by van der Ploeg, 2011 and van der Ploeg and Venables, 2011). A lot of investment may also lead to corruption and poor project choice, as in the island of Nauru (Hughes, 2004). Temporarily saving some funds abroad lets capital accumulate, so that future investment is more productive. As the Parking Fund is temporary, it should be managed separately from the more permanent Future Generations and Volatility Funds.

The sixth principle is to “*Invest part of the Volatility Fund domestically if capital is scarce*”. This shows how oil price volatility affects capital-scarce countries for the first time. Capital-scarce countries should build a smaller Volatility Fund than their capital-abundant neighbours. This is because the Future Generations Fund will be large, so there is less benefit from additional saving against volatility. The Volatility Fund should

¹¹van den Bremer, van der Ploeg and Wills (2013) show that such a rule, with a withdrawal rate slightly below the market rate of return, is consistent with precautionary savings.

be treated as an alternative source of income, so some of it should be invested in high-yielding domestic capital. This keeps the rate of return on foreign assets and domestic capital in balance.¹² Ghana recently established a Volatility Fund with its Resource Revenue Management Act (2011). It has priority over the Future Generations Fund, but receives 70% of sovereign wealth payments and does not invest domestically (see Figure 4.1).

The seventh and final principle is to “*Support private investment*”. Some developing resource exporters have a significant deficit in private capital. The highly centralised economies of Iraq and Libya both have capital stocks over 90% owned by the government (World Bank, 2012 and van der Ploeg, Venables and Wills, 2012). As there are diminishing returns to public capital, and private capital improves its productivity, some oil wealth should be used to alleviate private capital constraints. This can involve financing via the domestic banking sector to “top up” private capital, similar to the UK’s Funding for Lending scheme (Churm et al., 2012).

The last four principles provide support for current policy advice, including that provided by the Natural Resource Charter (NRC, 2013). The NRC is a collection of twelve precepts designed to guide policymakers in resource-rich developing countries. Those covering revenue management cover from Precept 7: “promote economic development through high levels of investment” to Precept 10: “facilitate private sector investment”. This paper adds further detail to how policy should be conducted during the anticipation phase of a windfall, and on how volatility should be managed in developing countries.

The rest of the paper establishes these principles by proceeding as follows. Section 2 introduces the small, partial equilibrium model at the heart of our analysis, and reduces it to two core equations: the Euler equation and the budget constraint. Section 3 then uses the model to investigate policy in a capital-abundant developed economy, yielding principles one, two and three. Section 4 extends this to the case of a capital-scarce developing economy, adding principles four to eight. Section 5 concludes with suggested extensions.

2 Model

This section introduces the framework underpinning our analysis. The model will be stochastic and in continuous time, to neatly capture the effects of volatility. To simplify the analysis it is partial equilibrium, so all consumption and output can be bought and sold at the world price.¹³ The model is extended in sections 3.3 and 4.4 to general equilibrium, to present important intuition that otherwise would be missed. This section will use Ito calculus to reduce the problem to two simple equations: one governing the expected evolution of consumption (the Euler equation); and the other governing the evolution of total assets (the budget constraint). The following sections then use these two equations to present the principles.

¹²van den Bremer and van der Ploeg (2013) illustrate precautionary savings in a two-period model with capital scarcity. We extend this to many periods, and include investment.

¹³It is also a common simplification, see van der Ploeg 2011, van der Ploeg and Venables, 2012 and 2013, van den Bremer and van der Ploeg, 2013.

Let us begin with a social planner that receives exogenous oil income, $P(t)O^i$, and non-oil income from production, $Y(K^i)$, and must choose how much to consume, $C^i(t)$, how much to invest, $I^i(t)$, in domestic capital, $K^i(t)$, and how much to save in foreign assets, $F^i(t)$. The superscript $i = [A, B, C]$ denotes the three phases the economy passes through after an oil discovery at time $t = 0$: Anticipation, where $O^A = 0$ for $0 \leq t < T_1$; Boom, where $O^B = O$ for $T_1 \leq t < T_2$; and Constant income, where $O^C = 0$ for $T_2 \leq t^C$. The planner's general problem can then be summarised as follows,

$$J(F^i, K^i, P, t) = \max_{C(t)} \left[\int_t^\infty U(C^i(\tau)) e^{-\rho(\tau-t)} d\tau \right] \quad (2.1)$$

s.t.

$$dF^i(t) = (r(F^i(t))F^i(t) + P(t)O^i + Y(K^i(t)) - C^i(t) - I^i(t))dt \quad (2.2)$$

$$dK^i(t) = (I^i(t) - \delta K^i(t))dt \quad (2.3)$$

$$dP(t) = \alpha P(t)dt + \sigma P(t)dZ(t) \quad (2.4)$$

where $J(\cdot)$ is the value function, $U(\cdot)$ is the utility function, ρ is the rate of time preference, $r(F(t))$ is the interest rate faced by the planner and which may depend on the level of assets/debt, δ is the rate of depreciation on domestic capital and Z is a Wiener process. In addition we will make four further assumptions regarding utility, interest rates, output and oil prices.

We will assume that utility exhibits constant absolute risk aversion (CARA), $U(C) = \frac{1}{-a}e^{-aC}$. This utility function is useful because it makes the effect of volatility on consumption very clear (and is especially conducive to the explicit solutions we are after, as noted by Kimball and Mankiw, 1989, Merton, 1990, and Kimball, 1990b). As the absolute degree of risk aversion - and by extension prudence - is constant, the effect of a given level of volatility on consumption will also be constant. It will not depend on the level of consumption itself. This is useful because the level of volatility faced by the planner will change over time, as oil is extracted and less remains exposed to volatile oil prices. With CARA preferences we are able to isolate the effects of these changes in volatility.¹⁴

Interest rates will have a linear premium on debt, given in equation 2.5. This is a tractable way to capture the capital scarcity facing developing countries.¹⁵ If the country is capital-scarce and indebted, $F(t) < 0$, then its cost of borrowing will rise according to $\omega > 0$. If the country is capital abundant or a net lender, $F(t) \geq 0$, then we assume that the country can borrow or lend freely at the world interest rate, $\omega = 0$. We assume that the world rate of interest is constant. In practice the prices of financial assets that

¹⁴In contrast, the common "constant relative risk aversion" (CRRA) utility function exhibits diminishing absolute risk aversion. So, the effect of oil price volatility in our model could fall for two reasons: because oil is being extracted, and because consumption is rising. This makes the results less clear, and is less amenable to explicit solutions. Another way to think of it is that that consumption is affected by the size of a volatile income stream under CARA, but by the share in total wealth of a volatile income stream under CRRA.

¹⁵A similar debt premium is used for resource-rich developing countries by van der Ploeg and Venables, 2011 and 2012, and van den Bremer and van der Ploeg, 2013. A debt premium on interest rates is also commonly used as a tool to remove the unit-root in models of a small-open economy (Schmitt-Grohe and Uribe, 2003).

sovereign wealth funds invest in are also volatile. However, they tend to be less volatile than oil prices, and so we make the simplifying assumption that the world rate of interest is constant.

$$r(F(t)) = \begin{cases} r - \omega F(t) & \text{for } F(t) < 0 \\ r & \text{otherwise} \end{cases} \quad (2.5)$$

Output will be produced using domestic capital and a fixed stock of labour, and sold at a constant world price. We assume that domestic firms produce a single internationally traded good using technology, capital and labour, $Y(K(t)) = AK(t)^\alpha \bar{L}^{1-\alpha}$, where technology is constant and the supply of labour is fixed, $\bar{L} = 1$. This good will be sold at the constant world price, $P^* = 1$. It is also the same as the single good consumed by the social planner, $C^i(t)$. Assuming a constant world price is a useful simplification, and it lets us begin by ignoring the general equilibrium effects of changes in the relative price of domestic goods. This also makes the analysis consistent with work by van der Ploeg and Venables (2012) and van den Bremer and van der Ploeg (2013). In sections 3.3 and 4.4 we will take brief detours into general equilibrium, introducing a traded and a non-traded good to see how the decision to consume, invest and save changes.

Oil prices will follow a geometric Brownian motion with zero drift, $\alpha = 0$. In practice oil price shocks are very persistent with little drift.¹⁶ By setting the drift of oil prices to zero we make the analysis simpler and ensure the present value of the income stream is finite. As price shocks are persistent, a shock today will affect the price in all future periods. Positive or negative shocks are equally likely at any point in time. This means that the volatility of oil prices increases exponentially into the future.

Now we turn to solving the social planner's problem. Dropping superscripts for simplicity, the Hamilton-Jacobi-Bellman equation for the problem in 2.1 to 2.4 is,

$$\max_C \left[U(C(t))e^{-\rho t} + \frac{1}{dt} E_t[dJ(F, K, P, t)] \right] = 0 \quad (2.6)$$

where Ito's lemma can be used to express the right hand term as,

$$\begin{aligned} \frac{1}{dt} E_t[dJ(F, K, P, t)] &= J_F(r(F)F + PO + Y(K) - C - I) \\ &\quad + J_K(I - \delta K) + J_t + \frac{1}{2} J_{PP} \sigma^2 P^2 \end{aligned} \quad (2.7)$$

The first order conditions with respect to consumption, investment, foreign assets, and domestic capital can be summarised in the following two equations, derived in Appendix

¹⁶see Bems and de Carvalho Filho (2011) and van den Bremer and van der Ploeg (2013) who estimate $\alpha = 0.009$.

A, which describe the evolution of the marginal utility of foreign assets and equating the marginal product of capital with the interest paid on foreign borrowings,

$$\frac{E_t[dJ_F]/dt}{J_F} = -(r - 2\omega F) \quad (2.8)$$

$$r - 2\omega F = \alpha AK^{\alpha-1} - \delta \quad (2.9)$$

Domestic capital and foreign assets will be tied together by an optimal asset allocation condition, equation 2.9. This condition intuitively states that the social planner should allocate total assets so that the marginal benefit of paying down foreign debt (reducing both the stock of debt and the interest that is paid on it), equals the marginal benefit of investing more in domestic capital (boosting output). So, if consumption differs from income at any time, both domestic capital and foreign assets should increase or decrease together. Overall it can be seen that domestic capital increases more rapidly than foreign assets, so that its share in total assets, $\bar{S} = F + K$, will rise.

The value function must satisfy the optimality condition in 2.10. This is derived from the first order conditions in Appendix A,

$$-\frac{1}{a}J_F + \{J_t + J_F(r(F)F + PO + Y(K) - C - I) + J_K(I - \delta K) + \frac{1}{2}J_{PP}\sigma^2 P^2\} = 0 \quad (2.10)$$

Finding an explicit solution for the value function is not straight-forward, for three reasons. The first is the risk premium on debt. The second is the non-linear production function. The third is that the exposure to oil price volatility varies as oil is extracted. It is relatively straightforward to find the value function when volatile income is constant in size (under CARA preferences), or is constant as a share of total wealth (under CRRA preferences as discussed in Merton, 1990 and Chang, 2004). However, it is much more challenging when volatile income changes over time. In the next section we arrive at an explicit solution for the value function when oil prices are certain. When oil prices are uncertain we will take a different tack.

Rather than finding an explicit solution for the value function, we can still gain a lot of intuition by examining the dynamics of consumption and total assets, $\bar{S} = F + K$. These are given in equations 2.11 and 2.12 and derived in Appendix A. They are expressed in terms of linear deviations from a steady state, $S = \bar{S} - S^*$, where capital scarcity is overcome and oil income is exhausted, $F^* = 0$ and $O^* = 0$,

$$\frac{1}{dt}E[dC^i(t)] = (r - \rho - \frac{2\omega}{a}fS^i(t)) + \frac{1}{2}aP(t)^2C_P^i(t)^2\sigma^2 \quad (2.11)$$

$$\frac{1}{dt}dS^i(t) = rS^i(t) + P(t)O^i - C^i(t) + C^* \quad (2.12)$$

The expected dynamics of consumption are given by the Euler equation in 2.11. The first term describes the typical trade-off between consuming today, or saving at rate r

for future consumption, which is discounted at rate ρ . In addition, the desire to save will depend on total assets through the risk premium on debt, $F^i(t) = fS(t)$. This is based on a linear approximation around the steady state, $f = (1 + \frac{\omega}{\alpha(1-\alpha)}(\frac{\alpha}{r+\delta})^2)^{-1}$. The second term describes how oil price volatility affects the expected change in consumption. Higher volatility, σ , will delay consumption from today until tomorrow, in line with precautionary savings. The severity of this effect depends on the marginal propensity to consume from a change in the oil price, $C_P^i \equiv \partial C^i(t)/\partial P(t)$. This will depend on the size of the remaining windfall, as seen in the next section.

The expected dynamics of total assets are given by the budget constraint in 2.12. This uses the relationship tying foreign assets to capital in equation 2.9, and is linearised around the steady state level of assets, S^* . The term $rS^i(t)$ describes the rate of return to total assets near this steady state. This can actually be split into two components, as seen in Appendix A. The first is the rate of return earned on foreign assets, rf . The second is the marginal product of capital less depreciation, $(Y'(K^*) - \delta)(1 - f) = r(1 - f)$. Note that we do not assume that the share of foreign assets and domestic capital is constant. Rather, we assume that the share of capital in total assets increases linearly, rather than the non-linear relationship in equation 2.9. Also note that linearising does not remove the effect of the risk premium, ω . It will still appear in the Euler equation so the incentive to save, $\dot{C}^i(t) > 0$, will still increase with the level of debt, $F^i(t) = fS(t)$.

The rest of this paper will involve using these two equations to demonstrate the eight principles, both explicitly and with the use of diagrams. There will also be some short detours into general equilibrium in sections 3.3 and 4.4.

3 Oil discoveries in a developed country

This section considers how developed countries should manage their resource wealth. Of the forty-five countries identified by the IMF as resource-dependent, five have a Human Development Index (HDI) of “very high”: Bahrain, Brunei, Norway, Qatar and the UAE (IMF, 2012). A further fifteen have an HDI of “high”, while other developed countries have extensive natural resources but are not classed as dependent, including Australia, Canada, Israel, the Netherlands, the UK and the US. Despite their many differences these countries share a similar trait: ready access to international capital.

This section shows that the first consideration for developed, capital-abundant economies should be to share the windfall across generations. This will involve saving in offshore Future Generations and Volatility Funds, with the latter being prioritised in the early days of the windfall (principles one and two). There should also be investment in non-traded sectors like retail, health care and education, to limit inflation and stabilise the real exchange rate in the medium to long term (principle three).

These results are generally consistent with what we observe in practice. Of the developed countries classed as resource-dependent, all but Ecuador have established offshore sovereign wealth funds. These vary by size and use, from Norway’s fixed-deposit/fixed-withdrawal rule to Russia’s focus on consuming a moving average of past revenues. In addition, 75% of these economies have pegged exchange rates, which are supported by

stable non-traded goods prices. The lessons from this section tend to apply more to non-dependent economies like Australia and Canada. These have not established national sovereign wealth funds and rely instead on state-based vehicles. Inflation in places such as Perth, Western Australia and Edmonton, Alberta may be limited by further localised investment in the non-traded sector.

The section proceeds as follows. Section 3.1 establishes the first principle in the base case when capital is abundant and oil prices are certain. Section 3.2 establishes the second principle by letting the oil price fluctuate. Section 3.3 presents the third principle by introducing a non-traded sector and general equilibrium. This extension to general equilibrium is revisited in section 4.4.

3.1 The case for a Future Generations Fund

This section presents the first principle: “*Smooth consumption using a Future Generations Fund*”. This is based on the permanent income hypothesis and underpins the use of sovereign wealth funds in almost all resource-dependent economies. Investment will not be affected by an oil discovery, as all profitable projects should already be undertaken. We assume capital is abundant, oil prices are constant and goods are freely traded as a baseline. The analysis is based on two dynamic equations that give an explicit solution, which will be used in the following sections.¹⁷

The economy is governed by the dynamics of consumption and assets, equations 3.1 and 3.2. These follow from equations 2.11 and 2.12, if capital is abundant, $\omega = 0$, and prices are certain, $\sigma = 0$. Capital will be constant if interest rates are constant and goods can be freely bought and sold at the world price, $K = K^* = (\frac{r+\delta}{\alpha A})^{1/(\alpha-1)}$ from equation 2.9. Any change in total assets will be caused by foreign assets, so $f = 1$ and $S^i(t) = F^i(t)$. Therefore we concentrate on foreign assets in equation 3.2, which holds without any need for approximation. Note the notation $\dot{C}^i(t)$ rather than $\frac{1}{dt}E[dC^i]$, to make clear that we are treating the expected change in each variable as deterministic.

$$\dot{C}^i(t) = \frac{1}{a}(r - \rho) \quad (3.1)$$

$$\dot{F}^i(t) = rF^i(t) + PO^i - C^i(t) + C^* \quad (3.2)$$

The path of consumption and assets is found by solving this linear system. For simplicity we set the time rate of preference to equal the risk-free rate, $r = \rho$. Taking the time derivative of 3.2 and substituting in 3.1 gives the single differential equation, $\ddot{F}^i(t) - r\dot{F}^i(t) = 0$. This has the general solution,

$$C^i(t) = ra_2^i + PO^i + C^* \quad (3.3)$$

$$F^i(t) = a_1^i e^{rt^i} + a_2^i \quad (3.4)$$

¹⁷An alternative solution based on explicitly solving the value function in equation 2.6 is presented in Appendix B.

The parameters a_1^i and a_2^i will depend on the phase the oil boom is in: $i = [A, B, C]$, Anticipation, Boom or Constant income. They are found by imposing conditions at the start and end of each phase. The first condition is that each phase begins with the assets at the end of the last phase, $F(0) = a_1^A + a_2^A$, $F^A(T_1) = a_1^B + a_2^B$ and $F^B(T_2) = a_1^C + a_2^C$. The second condition is that consumption is chosen to move smoothly between phases. This requires a constant level of assets at the end of the Boom, to satisfy the transversality condition. Moving recursively through each phase these conditions give,

$$\begin{aligned} a_1^C &= 0 & ; & a_2^C = F(T_2) \\ a_1^B &= POe^{-r(T_2-T_1)}/r & ; & a_2^B = F(T_1) - a_1^B \\ a_1^A &= -PO(e^{-rT_1} - e^{-rT_2})/r & ; & a_2^A = F(0) - a_1^A \end{aligned} \quad (3.5)$$

The effect of oil prices on consumption will depend on the remaining value of oil, given in equation 3.6.¹⁸ This is found by combining equations 3.3 and 3.5. The marginal propensity to consume from a change in oil prices, $C_p^i(t)$, will be important in the following sections. It dictates how oil price volatility affects consumption. It is difficult to derive in the stochastic model, so will be approximated with the deterministic case (see also van den Bremer and van der Ploeg, 2013). It captures the changing effect of oil price volatility, increasing during the Anticipation phase and decreasing during the Boom, as illustrated in Figure 3.1 using the calibration in Appendix C.

$$\begin{aligned} C_P^C(t) &= 0 & \text{for } T_2 \leq t \\ C_P^B(t) &= O(1 - e^{-r(T_2-t)}) & \text{for } T_1 \leq t < T_2 \\ C_P^A(t) &= O(e^{-r(T_1-t)} - e^{-r(T_2-t)}) & \text{for } 0 \leq t < T_1 \end{aligned} \quad (3.6)$$

We can now present the first principle of managing resource windfalls,

Principle 1. “Smooth consumption using a Future Generations Fund”

i) A social planner facing a certain but temporary windfall, which can freely borrow at the world rate of interest, should smooth consumption based on the rates of return and time preference, $C^i(t) = \frac{1}{a}(r - \rho)$ for all t .

ii) Consumption will be a constant proportion of total wealth: the sum of foreign assets, oil wealth and steady state consumption, $C^i(t) = rW(t)$ where $W(t) = F(t) + V(t) + \frac{1}{r}C^$ and $r = \rho$.*

iii) This will involve borrowing before the windfall, saving during the windfall in a Future Generations Fund, and consuming the interest earned after the windfall, $\dot{F}^A(t) < 0$, $\dot{F}^B(t) > 0$, $\dot{F}^C(t) = 0$.

iv) Less will be saved from a long windfall, and more borrowed beforehand, $\frac{\partial}{\partial T_2} \dot{F}^i(t) < 0$ for $t < T_2$.

v) Investment will be independent of any oil discovery if capital is abundant and goods are freely traded, consistent with the Fisher separation theorem, $K = K^$.*

Proof. See Appendix 1. □

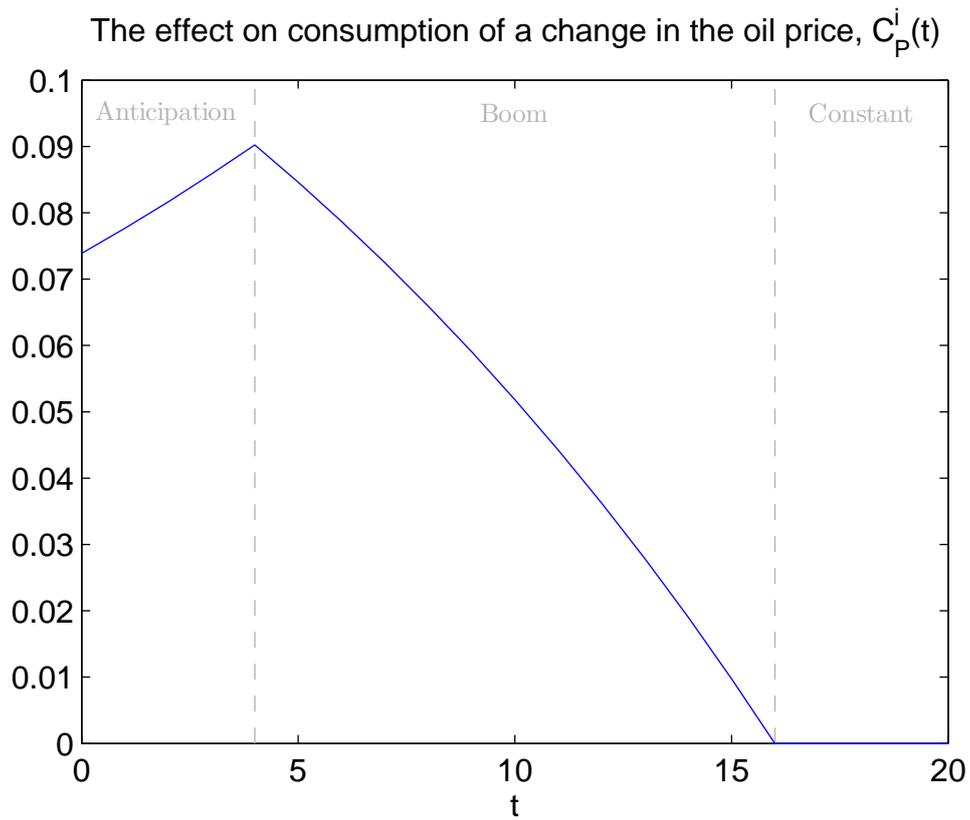


Figure 3.1: Consumption is most affected by oil price changes at the start of the Boom, when the present value of future revenues is highest. Consumption is less affected as oil is extracted and less remains exposed to price changes.

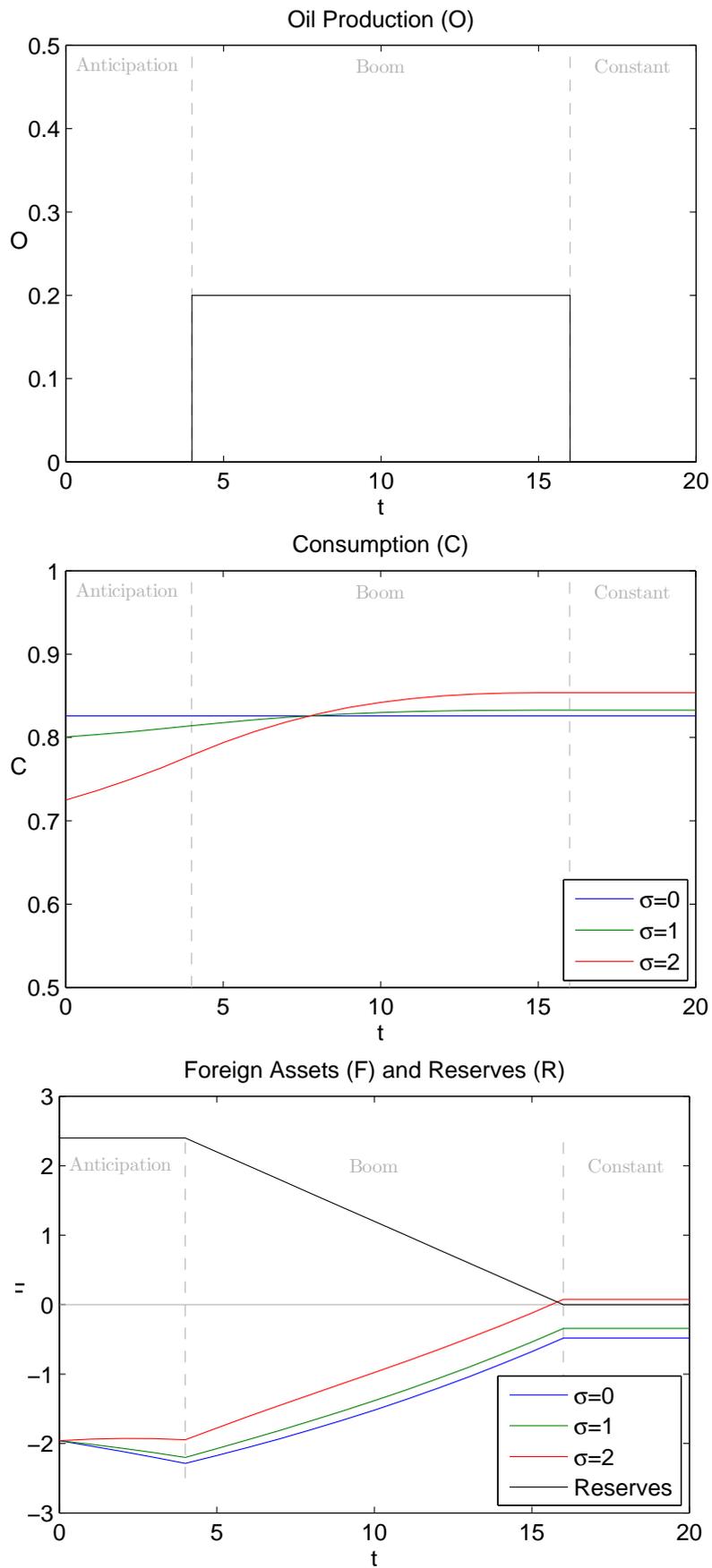


Figure 3.2: An oil boom should lead to borrowing before it begins, saving during, and consuming interest after it ends. Volatility will lead to additional precautionary savings (from equations, 3.3, 3.4, 3.9, and 3.10).

The first principle makes the case for establishing a sovereign wealth fund, to replace the assets below the ground with assets above it, see the blue lines in Figure 3.2. The Hartwick (1977) rule is based on similar reasoning, replacing oil reserves with productive capital. However, this principle (part v) shows that capital should be constant, as all profitable investments can already be financed with abundant capital. Thus, oil income should be saved abroad in a sovereign wealth fund, and borrowing should smooth consumption before production begins.

Consumption will be smooth, but may not be constant. Part i) shows that if the rate of interest is higher than the discount rate then consumption will grow, $r > \rho$. The amount depends on the degree of intertemporal substitution, a , which is also the coefficient of absolute risk aversion.¹⁹ In contrast if the policymaker is impatient (the discount rate is high), then more will be consumed in the early years of the windfall. This may happen if they value future generations less than current generations, are concerned with the possibility of being removed from office, or expect output to grow.

Longer windfalls will involve less saving. This is because the income produced by a long windfall will be closer to its permanent income. In the limit, if a constant windfall becomes permanent then it makes sense to consume all income as it is received.

The effects of an oil discovery can also be illustrated using the phase diagram in Figure 3.3. The blue lines are the same as those in Figure 3.2, but directly compare the movement of consumption and foreign assets. The solid line illustrates where consumption exactly equals non-oil income, so assets are constant before and after the windfall, $\dot{F}^{A,C} = 0$ and $O^{A,C} = 0$ in equation 3.2. The economy starts on this line, at X_0 . On discovering oil consumption jumps up to X_0^A as households become wealthier. Consumption is higher than non-oil income, so assets are consumed and the economy moves to point X_1 at the beginning of the boom, $t = T_1$. During the boom households also receive oil income, so the solid line moves up to the dotted line, $\dot{F}^B = 0$ and $O^B > 0$ in equation 3.2. Consumption is below this level, so assets are accumulated until the end of the boom, $t = T_2$. Consumption will have been chosen so that the economy lands on the initial steady state line at this time, point X_2 .

3.2 The case for a Volatility Fund

This section presents the second principle, “*Build a Volatility Fund quickly and then leave it alone*”. The Volatility Fund is motivated by precautionary savings, and has been established in previous literature on natural resources.²⁰ It should be prioritized at the start of the windfall, when oil price risk is greatest. The fund will be larger when the windfall is long or prices are more volatile. The Volatility Fund should be treated as a source of income, rather than as a “shock absorber”, as oil price shocks are persistent. An example is Norway’s fixed withdrawal rule, which is consistent with precautionary savings for certain withdrawal rates (see van den Bremer, van der Ploeg and Wills, 2013).

¹⁸This is verified by solving the value function directly in Appendix B.

¹⁹This is a result of the constant absolute risk aversion utility function. An alternative is to use Epstein-Zin preferences, which separate intertemporal substitution from risk aversion.

²⁰see Kimball (1990), van der Ploeg (2010), van den Bremer and Van der Ploeg (2013), Bems and de Carvalho Filho (2011), Cherif and Hasanov (2011) and (2013), and Berg et al. (2012).

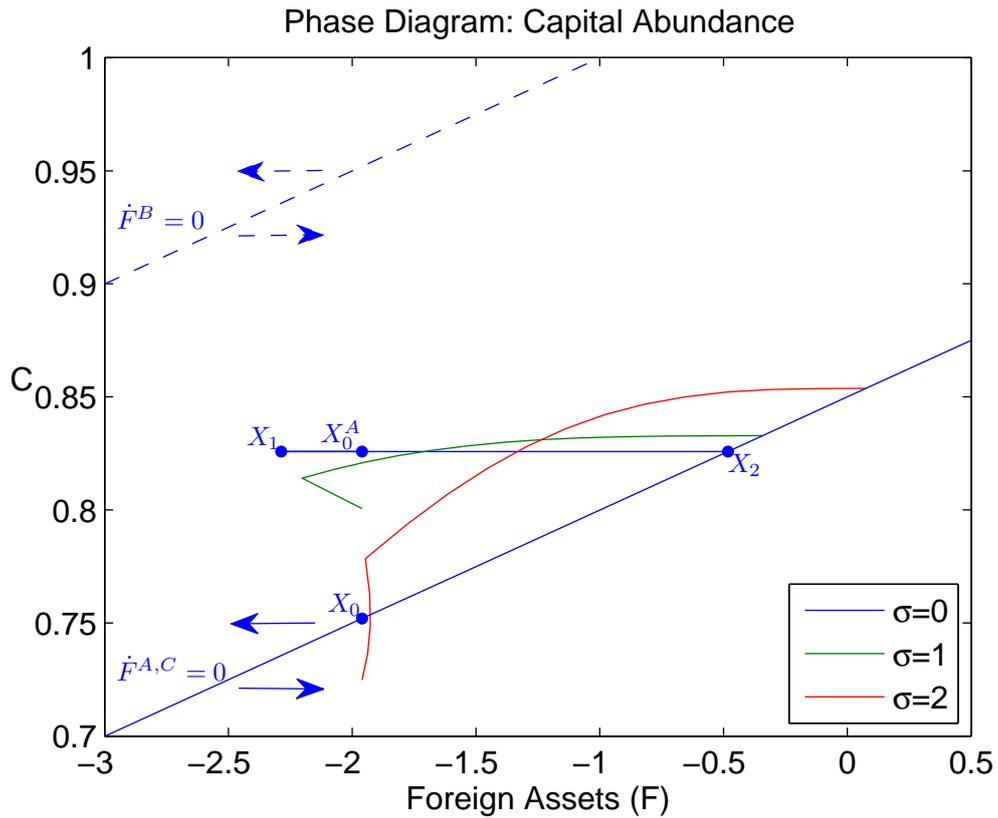


Figure 3.3: An anticipated windfall in a capital-abundant economy. Before and after the windfall the movement of consumption and assets is dictated by the solid blue line and arrows, $\dot{F}^{A,C} = 0$. During the windfall it is dictated by the dashed blue line and arrows, $\dot{F}^B = 0$. If prices are certain the economy will follow the path $[X_0, X_0^A, X_1, X_2]$. If prices are volatile (green and red), then consumption will also rise over time.

Consumption and assets will satisfy the two equations 2.11 and 2.12, when capital is abundant and oil prices are volatile. This follows from equations 2.11 and 2.12 with $\omega = 0$, $f = 1$ as capital remains constant, $i = [A, B, C]$ and $C_P^i(t)$ as defined in equation 3.6,

$$\dot{C}^i(t) = \frac{1}{a}(r - \rho) + \frac{1}{2}aP^2C_P^i(t)^2\sigma^2 \quad (3.7)$$

$$\dot{F}^i(t) = rF^i(t) + PO^i - C^i(t) + C^* \quad (3.8)$$

These two equations describe the expected dynamics of consumption and assets, and so we treat them as deterministic. The effects of volatility are still captured in the second term of the Euler equation, 3.7. Volatility will increase the change in consumption, all else being equal, giving rise to more savings in the near term, to fund more consumption in the future. The adjustment for volatility will decrease with time, as $C_P^i(t) \rightarrow 0$ in equation 3.6.

The paths of consumption and assets are found by solving this system of equations. The two equations can be summarised by a single differential equation $\ddot{F}^i(t) - r\dot{F}^i(t) = \frac{1}{2}aP^2\sigma^2C_P^i(t)^2$, as before.²¹ The general solution to this is,

$$F^i(t) = a_1^i e^{rt^i} + a_2^i + F_V^i(t) \quad (3.9)$$

$$C^i(t) = ra_2^i + PO + Y + C_V^i(t) \quad (3.10)$$

where $C_V^i(t) = rF_V^i(t) - \dot{F}_V^i(t)$. To aid our interpretation we choose a “particular solution”, $F_V^i(t)$, so that a_1^i, a_2^i are the same as before, in equation 3.5. This lets us split the assets accumulated by the social planner into two funds. The Future Generations Fund, $F_{FG}^i(t) = a_1^i e^{rt^i} + a_2^i$, holds the funds that are used to smooth consumption over time, in accordance with principle one. It need not be positive, such as borrowing during the Anticipation phase of a windfall. The Volatility Fund, $F_V^i(t)$, then captures all the funds that are accumulated in response to oil price volatility. The volatility fund will begin at zero before the oil discovery is announced, $F_V^i(0) = 0$, and will always be positive in expectation, $F_V^i(t) \geq 0$ for all t . This brings us to the second principle of managing resource wealth,

Principle 2. “Build a Volatility Fund quickly and then leave it alone”

Build a Volatility Fund quickly:

i) A social planner facing a volatile temporary windfall, who can freely borrow at the world rate of interest, should engage in precautionary savings and build up a “Volatility Fund”, in addition to the “Future Generations” fund in principle 1, $F_V^i(t) > 0$ for all $t > 0$.

²¹This is a non-homogeneous, second-order linear differential equation. If $F^i(t)$ and $F_V^i(t)$ are both solutions, then $\ddot{f}^i(t) - r\dot{f}^i(t) = 0$ for $f^i(t) \equiv F^i(t) - F_V^i(t)$ and the problem collapses to that discussed in section 3.1. The “particular solution” $F_V^i(t)$ can be found by the methods of undetermined coefficients or variation of parameters (see Robinson, 2004).

ii) The Volatility Fund should receive relatively more savings during the early years of the windfall, including during the Anticipation phase, $\dot{F}_V^A(t), \dot{F}_V^B(t) > 0$ and $\ddot{F}_V^A(t), \ddot{F}_V^B(t) < 0$ for $t < T_2$.

Then leave it alone:

iii) The assets in the Volatility Fund should not be consumed in the presence of a persistent negative shock to the oil price. Only interest should be consumed, $\frac{\partial}{\partial P_0} F_V^i(0) = 0$.

iv) As oil is extracted, the need for a Volatility Fund will diminish, $\dot{C}^B(t) \rightarrow 0$ as $t \rightarrow T_2$.

v) Any funds remaining in the Volatility Fund should be saved, and only the permanent income consumed, $\dot{F}_V^i(t) \geq 0$ for all $t \leq T_2$.

Proof. See Appendix 2. □

Principle two begins by emphasizing that the Volatility Fund should be prioritised at the start of the windfall, when oil price exposure is greatest. Consumption from an oil windfall will depend on two effects: the wealth effect and the precautionary effect. The wealth effect increases consumption, as an oil discovery increases lifetime wealth. The precautionary effect reduces consumption, as oil also makes income more volatile.

The Volatility Fund should be prepared before oil production even begins. During the Anticipation phase the Future Generations Fund will go into debt, borrowing against future income because of the wealth effect. This will be offset by the Volatility Fund, built because of the precautionary effect as illustrated in Figure 3.4.²² The volatility effect is strongest near the start of the windfall when its present value is highest, as seen in Figure 3.1. If the windfall is particularly long or volatile then CARA preferences permit the counter-intuitive result that consumption can fall below its pre-oil level, as seen in red in Figure 3.3. This follows from two particular characteristics of CARA preferences. First, under these preferences the effect of a given level of volatility on consumption is constant, regardless of the level of consumption. Second, CARA preferences also do not exclude the (nonsensical) possibility of consumption falling below zero, $C < 0$.²³ So, if the oil price is very volatile then the volatility effect may outweigh the wealth effect and marginal consumption from an oil discovery can be negative. This would not occur under other utility functions (such as CRRA). However, the assumption of CARA preferences remains crucial in making the model tractable, as discussed in Section 2.

Both the Volatility and the Future Generations Funds should accumulate during oil production. The Volatility Fund will receive relatively more at the start of the windfall, when the exposure to oil price volatility is highest. If the windfall is long or volatile then the Volatility Fund may receive more than the Future Generations Fund in absolute value. The need to save against future shocks will then fall as oil is extracted. After the windfall both the Volatility and Future Generations Funds can be combined, and the interest they earn should finance consumption in perpetuity.

²²In practice this may involve simply borrowing less in the Future Generations Fund, rather than borrowing in one fund, whilst saving in another.

²³This can only be prevented by choosing an appropriate values for the parameters and the starting level of consumption.

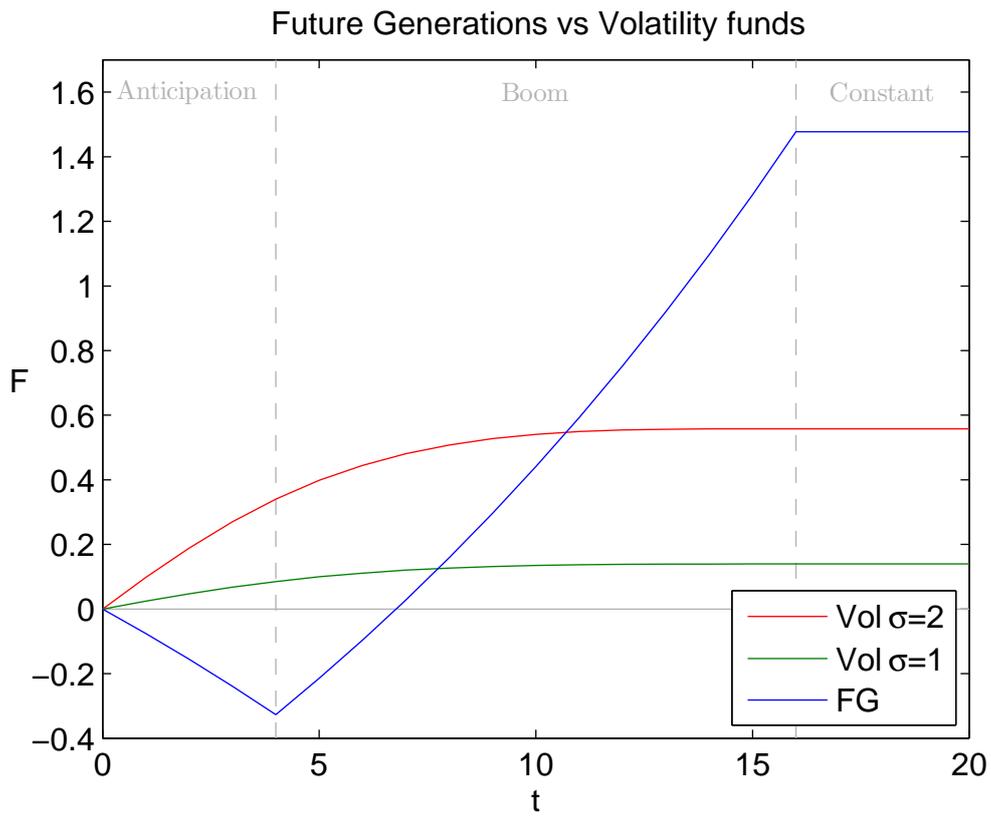


Figure 3.4: The Future Generations and Volatility Funds from the beginning of each stage of the windfall. The Volatility Fund will receive relatively more savings during the early years of the windfall, and may receive more savings in absolute value if the windfall is particularly long or volatile.

Principle two also emphasises that the Volatility Fund should be treated as a source of income, rather than as a “shock absorber”. It may be tempting for policymakers to dip into the Volatility Fund when oil prices are low. However, oil price shocks are very persistent, so if prices are low today then they are also likely to be low in the future.²⁴ Instead, when oil prices fall the social planner should re-evaluate the planned path for future consumption. In this case the income from a Volatility Fund will finance more consumption than would be possible otherwise. This is illustrated in the example in Figure 3.5, which compares consumption and assets with and without a Volatility Fund, after a negative shock to the oil price.

The need for a Volatility Fund will diminish over time, but its capital should not be consumed. The exposure to oil price volatility falls as oil is extracted. Policymakers may see this as an opportunity to consume the fund, once the threat of volatility has passed. However, principle two shows that the income from the Volatility Fund compensates future generations for the risk they have had to bear.²⁵ The Volatility and Future Generations Funds therefore behave the same way after the windfall ends, suggesting that they can be combined.

The role of the Volatility Fund is also illustrated in the green and red lines of the phase diagram in Figure 3.3, and the time paths in Figure 3.2. Volatility causes consumption to jump less when oil is discovered ($\sigma = 1$) and can even fall ($\sigma = 2$), because of the precautionary effect. Consumption will then rise steadily over time, both before and during the windfall, according to equation 3.7. The Future Generations Fund still goes into debt before the windfall, and accumulates during it. This is offset by the Volatility Fund, which grows throughout. At the end of the windfall the economy ends on the steady state line, $\dot{F}^{A,C} = 0$, above X_2 . The extra assets from the Volatility Fund will then finance higher consumption in perpetuity.

3.3 The case for investing to stabilise the real exchange rate

This section presents the third principle, “*Invest to stabilise the real exchange rate, if capital is abundant*”. Policy advice has been influenced by the separation result, that an oil discovery should not affect investment in capital abundant economies (for example Barnett and Ossowski, 2003, and IMF, 2012). However, this will not be true if some goods are non-traded. Demand for non-traded goods will drive up their relative price (a real appreciation),²⁶ and attract workers from the traded sector - causing it to shrink (Dutch disease). Policymakers should stabilise the real exchange rate by investing in the non-traded sector. While this may prevent a real appreciation, it will not necessarily prevent Dutch disease. Some resource exporters have maintained very stable real exchange rates for decades, such as Saudi Arabia since 1990 (Darvas, 2012).

To more accurately capture investment in capital-abundant economies we take a short detour into general equilibrium. Previous sections focused on the allocation of resources

²⁴see Bems and de Carvalho Filho (2011) and van den Bremer and van der Ploeg (2013) who estimate $\alpha = 0.009$.

²⁵The social planner prefers certain to uncertain income according to Jensen’s inequality.

²⁶Such as the rapid appreciation of Australia’s real exchange rate from 2002-2008 (Darvas, 2012).

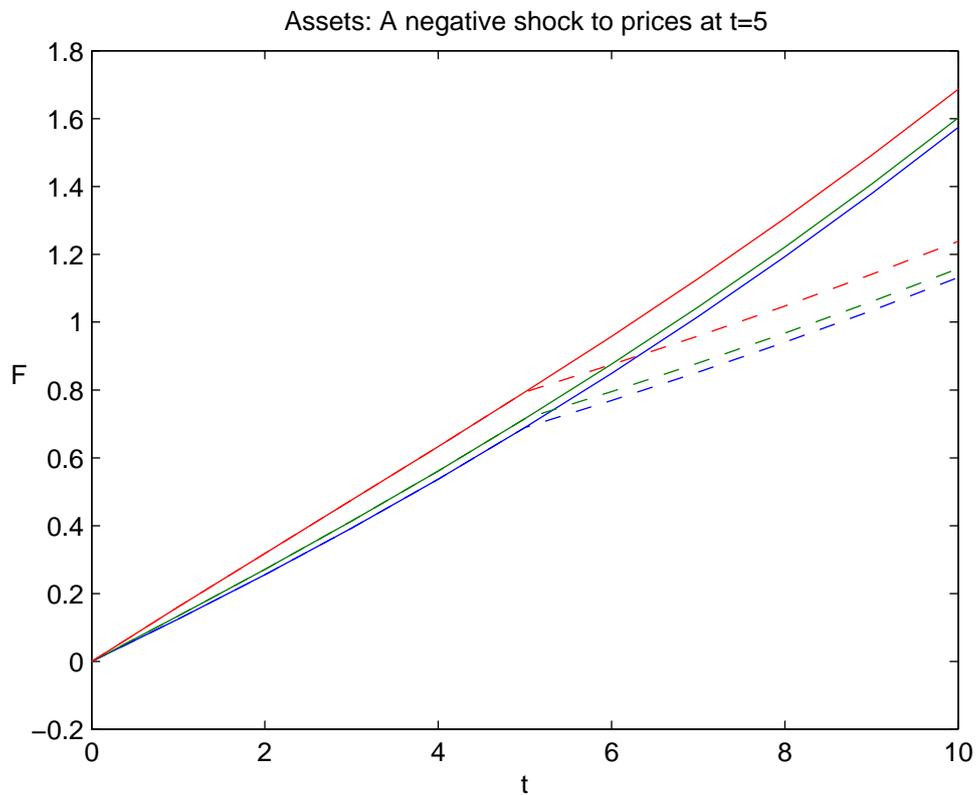
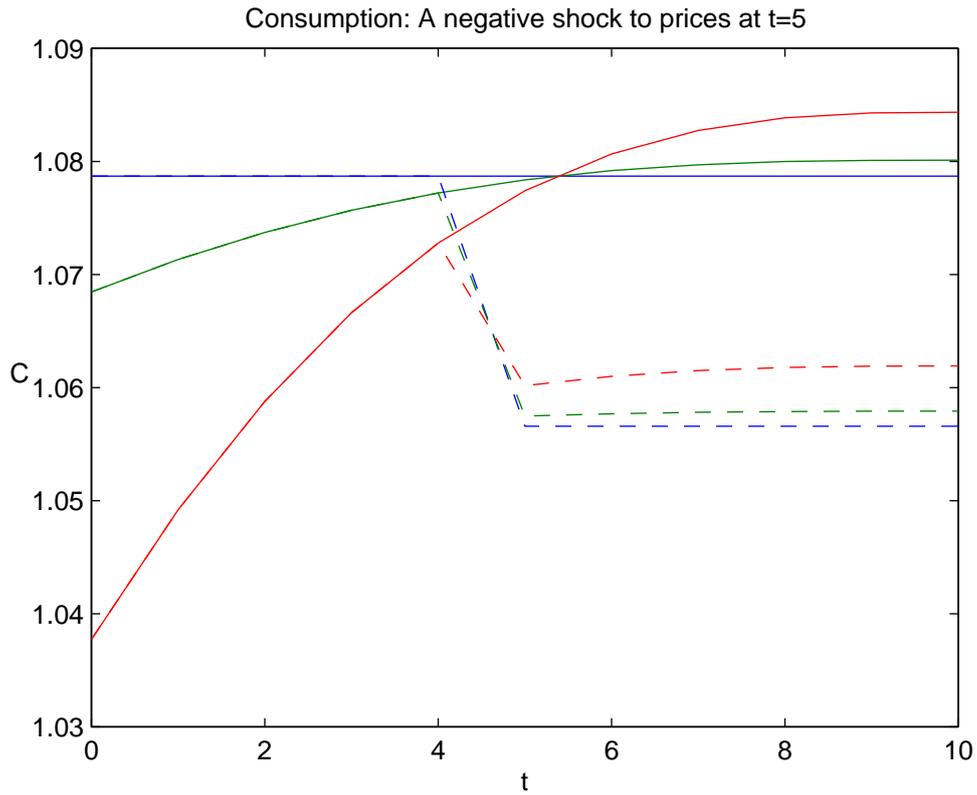


Figure 3.5: Building up a Volatility Fund buffers consumption against negative price shocks. The expected path of consumption and assets before (solid) and after (dotted) an unanticipated shock to the oil price, from $P = 1$ to $P = 0.5$ at $t = 5$.

over time. This section focuses on the allocation at any given point in time. The social planner will now split consumption between goods to minimise expenditure,

$$\min_{C_N, C_T} P(t)C(t) = P_N(t)C_N(t) + P_T(t)C_T(t) \quad (3.11)$$

subject to,

$$C(t) = C_N^{a_N}(t)C_T^{a_T}(t) \cdot a_N^{-a_N} a_T^{-a_T} \quad (3.12)$$

$$P(t) = P_N^{a_N}(t)P_T^{a_T}(t) \quad (3.13)$$

Spending on each good will be a constant share of total consumption, $P_j(t)C_j(t) = a_j P(t)C(t)$ for $j = N, T$. The traded good will be freely bought and sold at the world price, $P_T = 1$. The non-traded good's price (which is also the real exchange rate, $P_N = e$) will adjust to equate domestic supply and demand, $C_N(t) = Y_N(t)$. Each good will be produced competitively using technology, public and private capital, and labour. Public and private capital will be freely available at the world rate of interest, r , and the wage will adjust to clear the labour market, $\bar{L} = L_N(t) + L_T(t)$. They are chosen to maximise profits,

$$\max_{K_{Gj}, K_{Pj}, L_j} \pi_j(t) = P_j(t)Y_j(t) - rK_{G,j} - rK_{P,j} - wL_j \quad (3.14)$$

where

$$Y_j(t) = AK_{G,j}^{\gamma_G} K_{P,j}^{\gamma_P} L_j^{1-\gamma_G-\gamma_P} \quad (3.15)$$

This gives rise to the third principle for managing resource windfalls,

Principle 3. “Invest to stabilise the real exchange rate, if capital is abundant”

i) When there is both a traded and a non-traded sector, an oil discovery should increase investment in the non-traded sector, and decrease investment in the traded sector, with the aim of stabilising the real exchange rate, $e = e^$.*

ii) This is not just a reallocation of capital. The total capital stock can increase or decrease, depending on the production technology of each good.

iii) During the Anticipation phase this will involve borrowing to finance both consumption and investment in the non-traded sector.

iv) Investing in the non-traded sector may limit the decline of the traded sector (Dutch disease) when the initial capital stock is low, but immigration is more effective.

Proof. See Appendix 3 □

Dutch disease describes the shrinking of the non-oil traded sector after an oil discovery, and is often linked to a real appreciation. When oil is discovered there will be an increase

in demand, as noted in principle one. This will be spread across both traded and non-traded goods. Traded goods can be imported, but non-traded goods must be produced domestically. Corden and Neary (1982) attribute the shrinking of the traded sector to both a resource movement and a spending effect. The resource movement effect describes capital and labour moving from the traded to the non-traded sector. The spending effect describes the increase in the price of non-traded goods to meet demand.

Principle three emphasises that investment should prevent the real exchange rate appreciating when oil is discovered. An oil discovery will create extra demand for non-traded goods, which can be met by buying more goods, or the same amount of goods at higher prices. The former is preferred. As capital is freely available on international markets, any rise in demand should be met by an inflow of capital and movement in labour from the traded sector. The amount will depend on the demand for non-traded goods and their capital intensity. The capital inflow should keep the price of non-traded goods, and in turn the real exchange rate, constant. This is consistent with van der Ploeg and Venables (2011), who also show that the real exchange rate will appreciate if capital is slow to adjust.

Investing in the non-traded sector may prevent a real appreciation, but it will not necessarily prevent Dutch disease. Since Corden and Neary (1982) Dutch disease has been closely associated with a real exchange rate appreciation. As principle three states that investment should stabilise the real exchange rate, it is tempting to think that it will also prevent Dutch disease. This is only possible if the initial capital stock is low. In that case, labour in both sectors will be very unproductive. A large amount of labour would need to move from the traded to the non-traded sector. So, by increasing the stock of capital in the traded sector, the labour that moves there becomes more productive and less will be required. However, promoting immigration would be more successful as it relaxes the constraint on labour at its source. This has been widely used in many Gulf states, including the UAE and Saudi Arabia, which have not seen a major appreciation of their real effective exchange rate as a result (Darvas, 2012 and Espinoza et al., 2013).

In practice, 75% of the world's developed resource exporters have fixed nominal exchange rates (IMF, 2012). Of the five with a Human Development Index of "Very High", Qatar, Bahrain and the UAE have an explicit US dollar peg, Brunei is pegged to the Singaporean dollar and only Norway follows an inflation target. Of the further fifteen with an HDI of "High", eleven have currency pegs. In maintaining these pegs most resource exporters have accumulated large reserves of foreign assets. In our framework this means accumulating foreign assets, F , which limits the rise in consumption, C , and in turn the appreciation of the real exchange rate. Principle three highlights that investment in the non-traded sector, K_N , should complement foreign saving to stabilise the real exchange rate. While a constant nominal exchange rate does not require a constant real exchange rate, it is easier to defend if investment is being used to stabilise prices in the non-traded sector.

These results are easily reconciled with the previous sections. Some of the oil windfall should be set aside for investment. The rest should be consumed according to the permanent income hypothesis. The investment will be chosen to stabilise the real exchange rate at this level of consumption. Formally, it involves choosing additional capital, K' , so that the price of non-traded goods stays constant, $P_N = P_N^*$, at some level of aggregate consumption, C ,

$$a_N C = P_N^{*1-a_N} Y_N(K_N^* + K'_N)$$

This aggregate level of consumption will be based on the permanent income hypothesis below, where total output is defined as, $Y(K^* + K') \equiv P_N Y_N(K_N^* + K'_N) + P_T Y_T(K_T^* + K'_T)$ and capital is allocated efficiently between each sector.

$$C = rF(0) + r(PO(1 - e^{-rT})\frac{1}{r} - K') + Y(K^* + K') - \delta(K^* + K')$$

4 Oil discoveries in a developing country

We now consider how policymakers in developing countries should manage resource wealth. Of the world's forty-five resource-dependent economies, the majority have an HDI of “low” or “medium”. They range from the Democratic Republic of Congo, with a GDP per capita (PPP) of less than US\$500, to Botswana with GDP per capita of nearly US\$17,000 (World Bank, 2012). To borrow from Tolstoy's *Anna Karenina*, while developed countries are alike, every developing country is developing in its own way. This analysis focuses on the shared challenge of capital scarcity. While developed countries can readily borrow at the world rate of interest, developing countries can not. Ghana is a case in point. Ghana is in the midst of an oil boom, having discovered it in 2007 and started production in 2010. In July 2013 Ghana sold US\$750 million of ten-year, dollar-denominated bonds at a yield of 8%, six years after their previous international issue. US government bonds were yielding 2.5% at the time. To capture capital scarcity we assume that developing countries face a debt premium on borrowing.²⁷

The first priority for developing countries should be to invest domestically (principle four). If investment cannot be absorbed, then a temporary Parking Fund should be used to hold revenues until they can be productively used (principle five). In these countries the Volatility Fund should not all be saved abroad, as domestic returns are high and the fund is an alternative source of income (principle six). Finally, if private capital is also constrained, then some resource revenues should be used to boost it (principle seven).

There remains scope for these principles to improve policy. Genuine savings rates²⁸ remain low in many developing resource-exporters, suggesting that more resource wealth needs to be saved. Most egregiously, eight resource-dependent countries have negative genuine savings rates.²⁹ Many developing countries also have a severe deficit in private capital. It is thought that less than 10% of the capital stock in Iraq and Libya is privately owned (van der Ploeg, Venables and Wills, 2012 and World Bank, 2012). There has been

²⁷This follows van der Ploeg and Venables (2011) who find empirical evidence that the log of annual average bond spreads is increasing in the ratio of public and publicly guaranteed debt to gross national income, *PPG/GNI*. They also do not find evidence for natural resource discoveries alone reducing the cost of borrowing.

²⁸Gross savings plus education spending less capital depreciation and resource depletion, as calculated by the World Bank (2008).

²⁹Angola, Bolivia, Chad, Equatorial Guinea, Guinea, Saudi Arabia, Sudan and Zambia.

Ghana's Petroleum Revenue Management Act (2011)

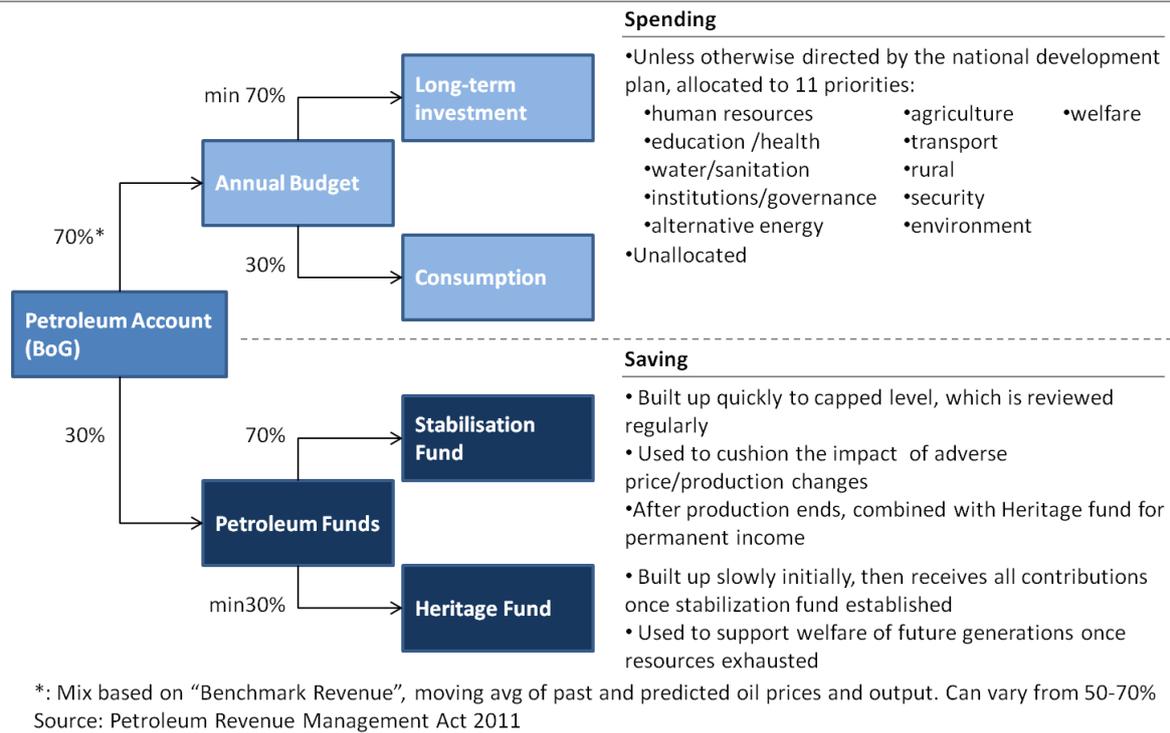


Figure 4.1: Ghana's Petroleum Revenue Management Act (2011) established one of the world's newest sovereign wealth funds.

recent progress in establishing sovereign wealth funds. Eight of the twenty-five developing resource-dependent economies now have funds, with five being established in the past three years.³⁰ Ghana is a recent example of this: its Petroleum Revenue Management Act (2011) directs 70% of the windfall to the annual budget and 30% to offshore "Stabilization" and "Heritage" funds, see Figure 4.1.

The principles are consistent with the Natural Resource Charter (NRC, 2013). The NRC is a set of twelve precepts for managing resource wealth, adopted by the African Union Heads of State steering committee in 2012. Principle four supports the NRC's Precept 7: "promote economic development through high levels of investment". Principles five and six add detail to Precept 8: "build up investment gradually and smooth revenue volatility". Principle seven is consistent with Precepts 9: "increase efficiency of public spending", and 10: "facilitate private sector investment". We extend the NRC by analysing the anticipation phase of a windfall, and how volatility affects capital-scarce economies.

The section proceeds as follows. Section 4.1 establishes principle four by extending the analysis in section 3.1 to the case when capital is scarce and oil prices are certain. Section 4.2 establishes principle five by briefly introducing absorption constraints. Section 4.3 presents principle six by extending previous literature to the case when capital is scarce and oil prices are volatile. Finally section 4.4 establishes principle seven by returning to the simple general equilibrium setting in section 3.3.

³⁰Ghana in 2011, Mongolia in 2011, Nigeria in 2011, Papua New Guinea in 2011 and Angola in 2012.

4.1 The case for using oil to accelerate development

This section presents principle four, “*Finance consumption and investment with oil if capital is scarce*”. Saving oil wealth for future generations will naturally be met with skepticism if current generations are suffering. If capital is scarce then the realised (and social) return on domestic investments will be higher than on foreign bonds. Investing oil wealth will accelerate development, justifying higher consumption in the short-term (see van der Ploeg and Venables, 2011). This principle is consistent with the impressive development experiences of Botswana, Bahrain, Qatar and the UAE, which combined foreign savings and domestic investment.

We consider two types of windfall: small and large. Small windfalls will not overcome capital scarcity before they are exhausted (following the characterisation in van der Ploeg and Venables, 2011). Large booms will, so the economy will begin to behave like those discussed in section 3 during the period of extraction.

4.1.1 Small oil discoveries

If the oil discovery is small then the social planner will have to contend with capital scarcity both during and after the windfall. Consumption and assets will follow equations 4.1 and 4.2, from equations 2.11 and 2.12 with $\sigma = 0$ and $\omega > 0$, for the duration of the windfall. When capital is scarce, repaying debt will reduce the cost of borrowing. Domestic capital will therefore be linked to foreign assets by the asset allocation condition, 2.9. This means that the share of capital in total assets will grow over time, so $f \neq 1$ and $F(t) = \frac{f}{1-f}(K(t) - K^*)$ as illustrated in Figure 4.2.

$$\dot{C}^i(t) = r - \rho - \frac{2\omega}{a}fS^i(t) \quad (4.1)$$

$$\dot{S}^i(t) = rS^i(t) + PO^i - C^i(t) + C^* \quad (4.2)$$

The paths of consumption and assets are found by solving equations 4.1 and 4.2. Extending the approach in section 3.1, these equations are summarised by a single differential equation, $\ddot{S}^i(t) - r\dot{S}^i(t) - \frac{2\omega}{a}fS^i(t) = 0$. The general solution is as follows, where t^i is measured from the beginning of each phase and, $\lambda_j = \frac{1}{2}r \pm \frac{1}{2}\sqrt{r^2 + 8\omega f/a}$, are the roots of the characteristic equation with $\lambda_1 < 0$ and $\lambda_2 > 0$,

$$S^i(t^i) = c_1^i e^{\lambda_1 t^i} + c_2^i e^{\lambda_2 t^i} \quad (4.3)$$

$$C^i(t^i) = c_1^i (r - \lambda_1) e^{\lambda_1 t^i} + c_2^i (r - \lambda_2) e^{\lambda_2 t^i} + PO^i + C^* \quad (4.4)$$

The coefficients, c_1^i and c_2^i , are found by imposing conditions at the beginning and end of each phase, $i = [A, B, C]$: Anticipation, Boom and Constant income. Each phase will begin with the assets left from the last one, $S(0) = c_1^A + c_2^A$, $S^A(T_1) = c_1^B + c_2^B$ and $S^B(T_2) = c_1^C + c_2^C$. During the Constant income phase, consumption and assets must stay

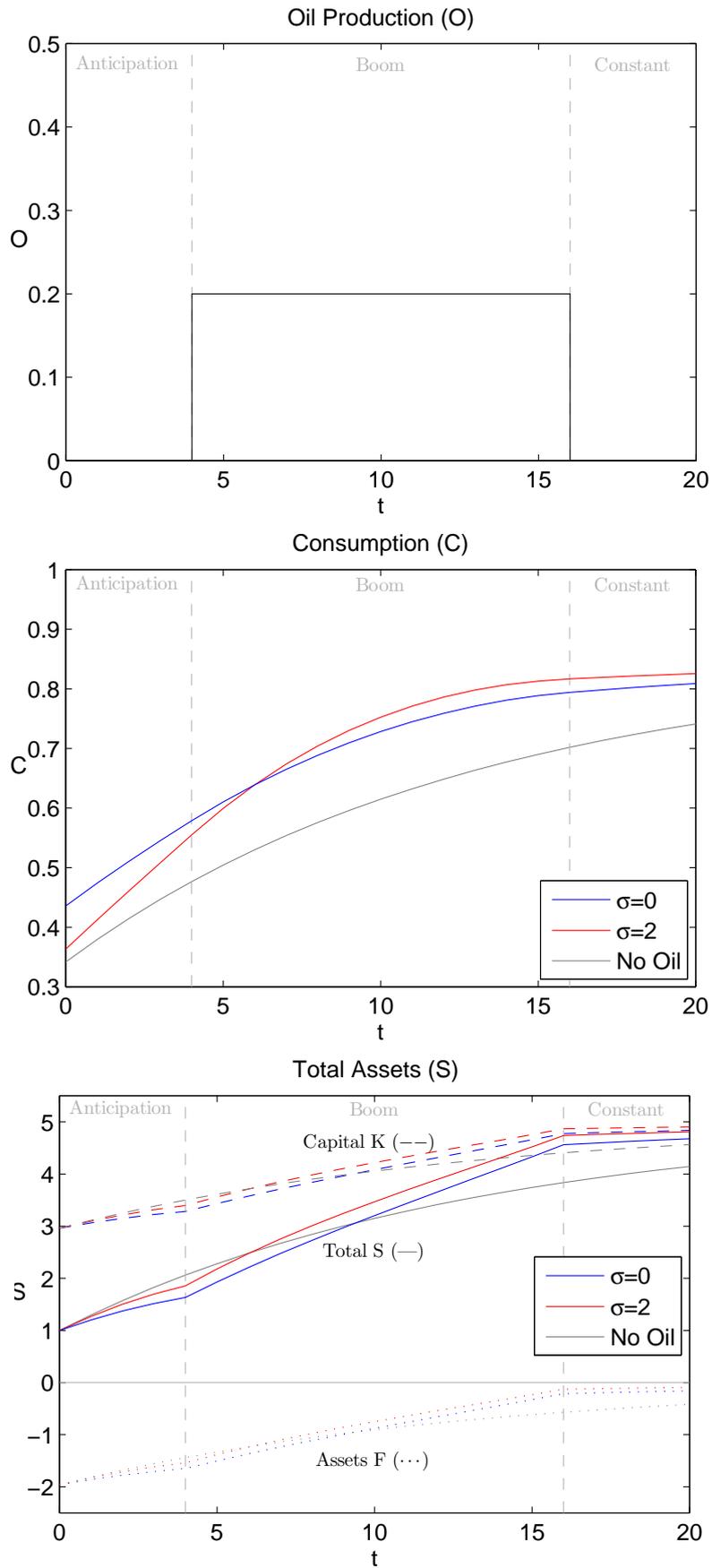


Figure 4.2: An oil boom will accelerate development if capital is scarce. Before the boom both capital and assets will fall, to smooth consumption. During the boom both accumulate quickly, financing an increase in consumption. Volatility leads to more savings (from equations 4.3, 4.4, 4.13, and 4.14). 28

on the stable saddle path, so $c_2^C = 0$. During the Boom phase consumption must be chosen so that the windfall ends with consumption and assets on the stable saddle path, $C^B(T_2) = (r - \lambda_1)S^B(T_2) + Y$. Recursively, consumption during the Anticipation phase will be chosen to smoothly enter the boom phase, so $C^A(T_1) = (r - \lambda_1)S^A(T_1) + PO(1 - e^{-\lambda_2(T_2 - T_1)}) + C^*$. Together these give the parameter values,

$$\begin{aligned} c_1^C &= S(T_2) & ; & & c_2^C &= 0 \\ c_1^B &= S(T_1) - c_2^B & ; & & c_2^B &= PO(\lambda_2 - \lambda_1)^{-1}e^{-\lambda_2(T_2 - T_1)} \\ c_1^A &= S(0) - c_2^A & ; & & c_2^A &= PO(\lambda_2 - \lambda_1)^{-1}(e^{-\lambda_2 T_2} - e^{-\lambda_2 T_1}) \end{aligned} \quad (4.5)$$

The permanent income hypothesis will not hold in a capital-scarce country. Instead, consumption will change over time and depend on the level of total assets, seen directly from 4.1. Before oil is discovered consumption will begin far below its permanent level, C^* . This is because the planner has an incentive to repay initial debts, F_0 , and reduce the rate of interest they face, $r(F(t))$. This brings us to our fourth and fifth principles,

Principle 4. “Finance consumption and investment with oil if capital is scarce”

i) Consumption in capital-scarce countries should begin lower than in capital abundant economies, $C_L^C(0) < C_H^C(0)$, but jump further when oil is discovered, $C_L^A(0) - C_L^C(0) > C_H^A(0) - C_H^C(0)$. It should still remain lower in absolute level, $C_L^A(0) < C_H^A(0)$.

ii) Borrowing in capital-scarce countries before a boom should initially be lower than capital abundant countries, $\bar{F}_L^A(0) > \bar{F}_H^A(0)$, and if they begin to borrow it will happen near the date of extraction, $\ddot{F}_L^A(t) < 0$ for $t < T_1$.

iii) Domestic capital in capital-scarce economies should grow over time, $\dot{K}_L^C(t) > 0$, be accelerated during an oil boom, $\dot{K}_L^B(t) > \dot{K}_L^C(t)$, and comprise an increasing share of the total capital stock over time, $\frac{d}{dt}(K(t)/\bar{S}(t)) > 0$.

Proof. See Appendix 4 □

This principle shows that capital-scarcity changes the way policymakers should respond to an oil discovery, illustrated in Figure 4.2.

Principle four highlights how capital-scarcity changes the trade-off between consuming today and consuming tomorrow. Capital-scarcity makes debt particularly costly. The first part of the principle states that before oil is discovered capital-scarce countries (L) should prioritise repaying debt and investing in capital: they should save more. Consumption should therefore be lower than a similar country with ready access to capital (H). It is tempting to extend this intuition to oil discoveries: that the poor should save more from marginal oil revenues. However, this is not the case.

Principle four also states that a capital-scarce country should consume relatively more from an oil boom than a capital abundant one. This happens because some oil revenues will be used to repay debt and accumulate capital. Future generations will benefit from this, which justifies higher consumption in the short term in the interests of sharing the

benefits. It should be noted that the initial jump in consumption is dictated by the debt burden, ω , rather than the level of debt itself, $\partial(C_j^A(0) - C_j^C(0))/\partial F(0) = 0$ as shown in Appendix 4.

The second part of principle four states that policymakers should borrow less before a windfall when capital is scarce. This happens for two reasons. The first is that borrowing is more costly, so less should be done. The second is that the policymaker also can use the capital stock. It may be better to let some capital depreciate, rather than access expensive foreign borrowing. This follows from the desire to keep the marginal product of capital in line with the cost of borrowing, in equation 2.9. It is interesting that there is any borrowing before a windfall at all. If the debt burden, ω , is sufficiently large - or the windfall small - then the capital-scarce country will not borrow. Consumption should begin so low that the entire jump can be financed directly from permanent income, Y . The need to borrow should also be highest just before production begins, as consumption should be steadily increasing in capital-scarce countries, $\dot{C}_L^i(t) > 0$.

The third part of this principle states that policymakers should accelerate investment during an oil boom. Part of an oil boom should be saved in foreign assets to relax the debt premium on interest rates, $r(F) = r - \omega F$. This means that capital should also grow. At every point in time, the policymaker should direct their savings to where it will earn the highest return: foreign assets or domestic capital. The marginal product of capital will therefore equal the cost of borrowing, 2.9. Domestic capital will also increase its share in total assets, as debt is repaid and capital accumulates.

The phase diagram in Figure 4.3 illustrates the effects of an oil discovery when capital is scarce. When the country is a net borrower, $S(t) < 0$, the economy will be governed by equations 4.1 and 4.2. These are summarised by the two blue steady state lines, $\dot{C}_D = 0$ and $\dot{F}^{A,C} = 0$ when prices are deterministic (D). These intersect at the point where all foreign debt is repaid, so $F^i = 0$ and $\bar{S}^i = K^i = S^* = (\frac{\alpha}{r+\delta})$ from Appendix A. In the absence of any shocks the social planner will choose to be on the stable (deterministic) saddle path, DD^1 , which ensures the economy will move towards the steady state. This path is a line with slope $(r - \lambda_1)$, and converges to the line $\dot{S}^{A,C} = 0$ as $\omega \rightarrow 0$. When the country is a net saver, $S(t) > 0$, the economy behaves as in Figure 3.3 and anywhere on the line $\dot{S}^{A,C} = 0$ above zero is a steady state.

Before oil is discovered the economy will begin on the stable saddle path, X_0 . When oil is discovered, consumption will immediately jump and then continue to grow, X_0^A . If the windfall is small, then consumption will remain below $\dot{S}^{A,C} = 0$ and the economy will continue to accumulate assets by repaying foreign debt and investing in domestic capital, F and K . Alternatively, if the windfall is large or the debt burden small, then consumption may jump above the line $\dot{S}^{A,C} = 0$ and the planner will deplete assets: both foreign and domestic.

When the Boom begins the planner will save, X_1 . The line $\dot{S}^{A,C} = 0$ will shift up, to $\dot{S}^B = 0$. The dynamics of the economy will now be dictated by a new steady state. Consumption will be below this, so total assets will grow: repaying debt and investing in capital. If the windfall is small, then the planner will have chosen consumption to arrive on the line DD^1 at the end of the Boom, X_2 . If the windfall is large, then the planner will repay all the debt and begin to accumulate assets in a sovereign wealth fund, ending the boom somewhere on $\dot{S}^{A,C} = 0$ above zero. This is discussed in the next section.

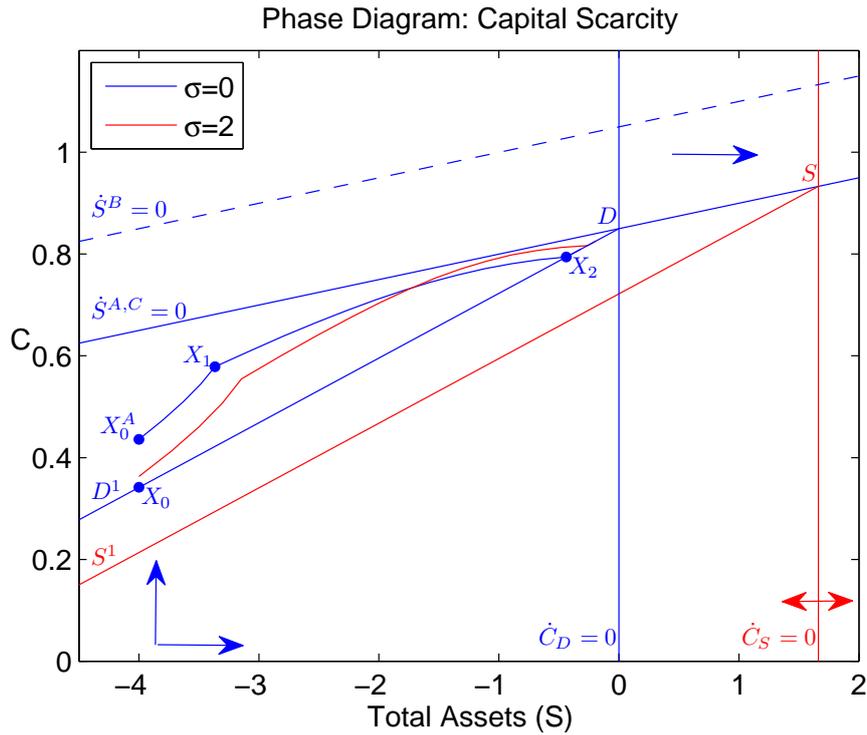


Figure 4.3: An anticipated windfall in a capital-scarce economy. Before and after the windfall the movement of consumption and assets is dictated by the solid blue lines and arrows. During the windfall the line $\dot{S}^{A,C} = 0$ shifts up to $\dot{S}^B = 0$. If oil prices are certain then the dynamics are dictated by the blue arrows and the economy follows the path $[X_0, X_0^A, X_1, X_2]$. If prices are volatile then the steady state line $\dot{C}_S = 0$ will move to the right before-, and return to $\dot{C}_D = 0$ during the windfall. This changes the path of consumption and assets.

After the boom, consumption will eventually exceed that in a capital abundant economy for the same initial level of debt. In a capital abundant economy there is no incentive to repay debt. In a capital-scarce economy, the incentive to reduce the debt premium will mean that eventually the economy will converge to the steady state, D .

4.1.2 Large oil discoveries

If the discovery is large then all debt should be repaid before accumulating a Future Generations Fund. Large windfalls will repay all debt before they are exhausted.³¹ As the economy will no longer be capital-constrained it will behave as in section 3.1. Consumption will stop growing and assets will accumulate. The economy will end the boom with a positive Future Generations Fund, on the steady state line $\dot{F}^{A,C} = 0$ above zero in Figure 4.3. The planner's behaviour will be a combination of the analysis in section 4.1.1 and section 3.1.

4.2 The case for a Parking Fund

This section presents principle five, “*Use a temporary Parking Fund to improve absorption*”. Oil wealth should increase investment if capital is scarce. However, this investment may be difficult to absorb. A major challenge is the need to use non-traded capital to build capital. Roads are built more efficiently with a motor grader (physical capital), and it requires teachers (human capital) to teach new teachers (see van der Ploeg, 2012). A lot of investment at once may also lead to corruption and poor project choice. An example is Nauru, an island nation of 10,000 people and 8 square miles in the central Pacific. At independence from Australia in 1968 Nauru had phosphate reserves of AU\$ 500,000 per person (1968 currency, Hughes, 2004). In 2004 communal net assets were estimated to be AU\$ 3,000 per person, after a succession of failed projects including an unsuccessful West-End musical, a cruise ship that did not leave port and a yellow Lamborghini for the police chief (despite a single coastal road with 25mph speed limit, McDaniel and Gowdy, 1999).

The Parking Fund will alleviate absorption constraints. In practice absorption constraints are measured by the IMF using the Public Investment Management Index (PIMI) (Dabla-Norris et al., 2011). We follow van der Ploeg (2012) and van der Ploeg and Venables (2013) and assume that absorption depends on the amount of investment and the stock of capital. For every I bricks of investment laid there will be $I + \frac{1}{2}\phi I^2/K$ bricks purchased. The social planner's budget constraint in equation 2.2 becomes,

$$dF^i = (r(F^i)F^i + PO^i + Y(K^i) - C^i - (I^i + \frac{1}{2}\phi I^i{}^2/K^i))dt \quad (4.6)$$

Optimal investment will depend on the ratio of the marginal utilities of capital and foreign assets, $q = J_K/J_F$. When absorption is not constrained this ratio equals one, $q = 1$

³¹Formally, this will happen when $F(0)(\lambda_2 - \lambda_1) + PO(e^{-\lambda_1 T_2} - e^{-\lambda_1 T_1} + e^{-\lambda_2 T_1} - e^{-\lambda_2 T_2}) > 0$, which is satisfied for larger, longer windfalls and lower initial levels of debt.

from equation A.2 in Appendix A. When absorption is constrained the ratio satisfies the following conditions, derived in Appendix 5 and giving rise to principle five,

$$q = 1 + \phi \frac{I}{K} \quad (4.7)$$

$$\dot{q} = q(r - \omega F + \delta) - Y_K - 0.5 \frac{1}{\phi} (q - 1)^2 \quad (4.8)$$

Principle 5. “If capital is scarce and absorption is constrained, use a temporary Parking Fund abroad”

i) In capital scarce countries with absorption constraints, part of an oil discovery should be accumulated in a temporary offshore Parking Fund while absorption constraints are relaxed, $F_P^i(t) > 0$ if $\phi > 0$.

ii) In capital abundant countries an oil discovery will not affect the size of any Parking Fund.

Proof. See Appendix 5. □

Principle five introduces a third type of sovereign wealth fund, the Parking Fund. Oil discoveries should stimulate investment if capital is scarce. However, if absorption is constrained then investment may not be productive, seen in the ratio $q = 1 + \phi \frac{I}{K}$. If there are absorption constraints and investment is positive, $\phi, I > 0$, then this ratio will be greater than one, $J_K > J_F$. Absorption constraints will increase the optimal amount of foreign assets relative to capital. This should be achieved with an offshore Parking Fund, to hold oil wealth while capital is built and absorption constraints are relaxed, as found in van der Ploeg (2012) and van der Ploeg and Venables (2013). The need for the Parking Fund will diminish as capital is accumulated, so it is inherently temporary. It will therefore require a different legal structure to the Future Generations and Volatility Funds, which are intended to be permanent.

Oil discoveries will not affect the Parking Fund when capital is abundant. The analysis in section 3.1 showed that an oil discovery should not affect investment, if capital is abundant and all goods are traded. All positive net present value projects should be developed regardless of the source of funding. The investment path may be altered by absorption constraints, but the discovery of oil will not change it further. Of course, if some goods are non-traded then an oil discovery should boost investment in that sector, as discussed in principle three. This would require an additional Parking Fund, as discussed in Van der Ploeg (2011).

4.3 The case for using the Volatility Fund for investment

This section presents the sixth principle, “Invest part of the Volatility Fund domestically if capital is scarce”. This extends previous work by considering how capital-scarce economies should respond to oil price volatility. The Volatility Fund will be an alternative source of income if the commodity price falls, from principle two. If the Volatility Fund is only

held abroad then the cost of borrowing will fall, but there will be unfunded profitable investments at home. It is better to invest part of fund domestically, and keep the rates of return on foreign saving and domestic investment in line. The illiquidity of domestic investment should not be a concern as the Volatility Fund is a source of income. It is not a buffer to temporarily support consumption when oil prices are low, because oil price shocks are persistent.

4.3.1 Small oil discoveries

We start by concentrating on a small windfall, so capital will still be scarce when it is exhausted. If the windfall is volatile, then consumption and assets will evolve according to the two equations 2.11 and 2.12, reproduced below. Again let $r = \rho$ for simplicity.

$$\dot{C}^i(t) = r - \rho - \frac{2\omega}{a}fS^i(t) + \frac{1}{2}aP^2C_P^i(t)^2\sigma^2 \quad (4.9)$$

$$\dot{S}^i(t) = rS^i(t) + PO^i - C^i(t) + C^* \quad (4.10)$$

It will be useful to define the instantaneous “steady state” of the system. At any point in time this state will govern the behaviour of consumption and total assets, analogous to the steady state in section 4.1. However, it will move over time, as the remaining exposure to volatility changes, through $C_P^i(t)$. It is given by the two equations,

$$S_S^i(t) = \frac{1}{4\omega f}a^2P^2C_P^i(t)^2\sigma^2 \quad (4.11)$$

$$C_S^i(t) = rS_S^i(t) + PO^i + C^* \quad (4.12)$$

The dynamics of consumption and assets are found by solving equations 4.9 and 4.10. The dynamics around the steady state are summarised by a single differential equation, $\ddot{S}^i(t) - r\dot{S}^i(t) - \frac{2\omega f}{a}S^i(t) = \frac{1}{2}aP^2C_P^i(t)^2\sigma^2$. As this is a non-homogeneous second-order linear differential equation, it must be solved relative to a reference point. The solution is of the form:

$$S^i(t) = c_1^i e^{\lambda_1 t} + c_2^i e^{\lambda_2 t} + S_V^i(t) \quad (4.13)$$

$$C^i(t) = c_1^i (r - \lambda_1) e^{\lambda_1 t} + c_2^i (r - \lambda_2) e^{\lambda_2 t} + C_V^i(t) \quad (4.14)$$

where $C_V^i(t) = rS_V^i(t) - \dot{S}_V^i(t)$. We choose the coefficients, c_1^i, c_2^i , to equal those in equation 4.5, following the approach in section 3.2. This allows total assets to be separated into those accumulated for Future Generations, and those to manage Volatility, $S^i(t) = S_{FG}^i(t) + S_V^i(t)$. Some assets would be accumulated without an oil windfall, to overcome capital scarcity. These are included in the Future Generations assets. The Volatility assets then capture the adjustment to manage oil price volatility. Both sets of assets should be allocated between a foreign fund and domestic capital, $S_{FG}^i(t) = F_{FG}^i(t) + K_{FG}^i(t)$ and $S_V^i(t) = F_V^i(t) + K_V^i(t)$. This ensures that the asset allocation condition in equation 2.9 is satisfied, and gives rise to the next principle,

Principle 6. “When capital is scarce, direct some precautionary savings to investment”

i) Capital-scarce countries should accumulate less in an offshore Volatility Fund than capital abundant economies, $F_{V,L}^i < F_{V,H}^i$, despite the fund accelerating the expected rate of development.

ii) Capital-scarce countries should also direct some precautionary savings from an oil boom to domestic investment, $K_V^i(t) > 0$.

Proof. See Appendix 6. □

The sixth principle states that capital-scarcity reduces the precautionary motive, as illustrated in 4.2. This is because policymakers in these countries should already be saving a great deal, to overcome capital scarcity. We include this in the Future Generations assets, which grow much more quickly when capital is scarce as stated in principle four. There will be less additional benefit from saving more in a Volatility Fund.³² Capital-scarce countries should therefore have a large Future Generations Fund, and a relatively small Volatility Fund.

Capital-scarce countries should have less precautionary savings, despite the additional benefits of reducing capital scarcity. Saving will accelerate the pace of development if capital is scarce, reducing the debt premium on interest rates. Policymakers may see this as an additional incentive to save against volatility. However, together the precautionary and development incentives to save are less than the sum of their parts. This is because utility is concave in consumption, and the benefits to saving are realised in the future, whilst the costs are borne in the present.

Principle six also states that part of the Volatility Fund should be invested domestically, if capital is scarce. Holding all of the Volatility assets in an offshore sovereign wealth fund, $F_V > 0$ and $K_V = 0$, will reduce the cost of capital, $r(F) = r - \omega(F_{FG} + F_V)$. More domestic investment will be profitable at this lower cost of capital. Thus, some of the Volatility Funds should be invested domestically to keep the marginal products of capital and foreign assets equal, from equation 2.9. Domestic capital is less liquid than foreign assets. This should not be a major concern as the Volatility Fund should be treated as a source of permanent income, rather than a “shock absorber”. This is because oil price shocks are persistent, as argued in principle two.

The phase diagram in Figure 3.3 also illustrates the effects of oil price volatility. The stochastic case (red, subscript S) illustrates the dynamics in equations 4.14 and 4.13. Volatility introduces a crucial difference to the deterministic case in section 4.1. The vertical “steady state” line will move, $\dot{C}_S = 0$ in equation 4.11. This happens because the exposure to oil price volatility changes over time as oil is extracted, $C_P^i(t)$. Therefore, the level of assets the social planner is “targeting” also changes, illustrated in the stable saddle path SS^1 . If the windfall is immediate and permanent, then this line will immediately jump to its new level and stay there.

³²Note that this analysis assumes prudence is constant, regardless of consumption. This makes the effects of volatility more clear. If prudence is higher at low levels of consumption (such as if relative risk aversion is constant), then higher prudence may outweigh the costs of lower consumption.

When oil is discovered, the policymaker becomes exposed to oil price volatility. The line $\dot{C}_S = 0$ will jump to the right as the policymaker targets a higher level of total assets, because of precautionary savings. The line continues to move right until oil production begins, when the exposure to oil price volatility is highest. The stable saddle path, SS^1 , moves to the right as well. This results in consumption being less than the deterministic case, because of precautionary savings.

When oil production begins, the policymaker's exposure to volatility will fall. The line $\dot{C}_S = 0$ will move to the left throughout the Boom phase, as the amount of subsoil oil exposed to volatile prices falls. At the same time the line $\dot{S}^{A,C} = 0$ shifts up to $\dot{S}^B = 0$. At the end of the Boom the line will return to meet the deterministic steady state line, $\dot{C}_D = 0$ at $t = T_2$, as there is no further exposure to volatility. Higher savings throughout the boom means that the economy meets the line DD^1 at a higher level of both consumption and assets.

4.3.2 Large oil discoveries

If the discovery is large and volatile, then all debt should be repaid and the desired capital stock reached, before accumulating a Future Generations Fund. A large discovery will start in capital scarcity, and finish in capital abundance. At the start of the windfall the economy will behave as in section 4.3.1. At some point during the windfall total assets will accumulate far enough that debt is repaid and capital-scarcity is relaxed, $F^B(t^*) = 0$ at some point $T_1 < t^* < T_2$. For the rest of the windfall the economy will behave as if it were capital-abundant, from section 3.2. The vertical steady state line, $\dot{C}_S = 0$, will disappear as capital is no longer scarce, $\omega = 0$ in equation 4.11. Thus, the social planner never actually reaches this "target". Consumption will be chosen to grow smoothly throughout the windfall and finish on the steady state line at the end of the boom, $\dot{S}^{A,C} = 0$ at $t = T_2$ in equation 4.12.

4.4 The case for investing in private capital

This section presents the final principle, "*Support private investment*". Capital-scarcity does not just apply to public investment; the private sector may also require capital. Iraq and Libya are a case in point. After decades of centralised government, over 90% of capital is publicly owned (van der Ploeg, Venables and Wills, 2012). Private access to capital will vary by country, and by sector. While large agricultural businesses may have ready access to foreign direct investment, small service industries will usually not.³³ An oil discovery is a source of cheap finance for the government. If the government's cost of borrowing is less than the private sector's, then it should direct some investment to private capital. This in turn boosts the productivity of public capital. Public financing of private capital should be done indirectly and transparently. This may be done by financing the domestic banking sector as in the UK's Funding for Lending scheme.

³³The lack of access to finance in small service industries and smallholder agriculture has been partly addressed in some countries by the expansion of micro-finance institutions.

To establish this principle we consider three cases of private capital scarcity: no domestic market in capital; a domestic market with limited access to foreign capital; and a domestic market with ready access to foreign capital. This is done in the general equilibrium setting introduced in section 3.3, which includes both public and private capital. We now assume that the government is capital scarce, so $r_G(F) = r - \omega_G F$. If there is no domestic market in capital then the marginal product of capital will differ in the non-traded and traded sectors, $Y_{K,N} \neq Y_{K,T}$. If there is a domestic market in capital then firms in both sectors can borrow and lend from domestic banks at the same rate of interest, r_P .³⁴ If the banks have limited access to foreign capital we assume the private sector is more capital scarce than the government, $r_P(F) = r - \omega_P F$ where $\omega_P > \omega_G$. If there is ready access to foreign capital then $r_P = r$. This leads us to the final principle,

Principle 7. “Support private investment”

i) If a capital scarce country has no domestic private capital market, the government should promote its development to increase the productivity of public investment.

ii) If the country has a private capital market with limited access to foreign capital, then the government should “top-up” private capital by lending through the domestic banking sector.

iii) If the country has a private capital market and ready access to foreign investment, then the government should invest relatively more in public capital.

iv) Investment in capital scarce countries should allow the real exchange rate to steadily appreciate as borrowing constraints are relaxed and capital is accumulated, according to the Rybczynski theorem.

Proof. See Appendix 7 □

Principle seven begins by showing that the productivity of public capital depends on an efficient allocation of private capital. The marginal product of public capital in a sector depends on its amount of technology, private capital and labour. The most efficient allocation of these factors across sectors equates their marginal products. If there is no domestic market for private capital, then there is no mechanism to ensure this will happen. As a result the overall productivity of public capital will suffer.

The second part of principle seven shows that an oil windfall should be used to increase investment in private capital, if the government has better access to finance than the private sector. An oil windfall should promote additional investment if capital is scarce, as stated in principle four. If the government has a given amount of capital to allocate between different uses, it is best to equate their marginal products. An increase in public investment will also promote “co-investment” by the private sector, as the productivity of private capital will have improved. If the government can then “top-up” private investment at a cheaper cost of finance, this will lift the productivity of public capital and boost output. This is similar to the UK’s Funding for Lending scheme, where the Bank of

³⁴We abstract from any monitoring costs that would introduce a spread between borrowing and lending (see for example Curdia and Woodford, 2010).

England offered discounted financing to private banks, if they could demonstrate it was being used to increase lending (Churm et al., 2012).

The third part of principle seven shows that ready access to capital in the private sector should boost investment in the public sector. The productivity of public capital depends on the amount of private capital, and vice versa. If the government installs public capital, it will encourage private investment. This in turn increases the productivity of public capital, encouraging further investment.

The final part of principle seven shows that the real exchange rate in capital-scarce countries should rise as capital is accumulated. This is a counterpart to principle three, which stated that the real exchange rate should be stable in capital-abundant economies. An oil discovery will help alleviate capital scarcity, reducing the cost of borrowing over time. The amount of capital in the economy will rise accordingly. If we assume that the traded sector is more capital intensive than the non-traded sector, then an increase in capital will reduce the relative price of the traded good, according to the Rybczynski theorem. This is achieved by an increase in the relative price of the non-traded good, which is a real appreciation.

5 Conclusion

This paper considers whether policymakers should spend, save or invest an oil windfall. It extends existing literature by showing that capital-scarce countries should do relatively less precautionary saving in a Volatility Fund, and invest some of it domestically. This Volatility Fund should be built in advance if the windfall is anticipated, and treated as a source of income, rather than a temporary buffer for smoothing out oil shocks. Capital-scarce countries should also invest so that the real exchange rate appreciates slowly, and support private capital if it is also constrained. In contrast to current policy advice, we show that even capital-abundant economies should invest part of their windfall, to stabilise the real exchange rate. To do this the paper develops a framework that nests a variety of existing results, which we present in seven principles.

1. Smooth consumption using a Future Generations Fund
2. Build a Volatility Fund quickly, then leave it alone
3. Invest to stabilise the real exchange rate, if capital is abundant
4. Finance consumption and investment with oil if capital is scarce
5. Use a temporary Parking Fund to improve absorption
6. Invest part of the Volatility Fund domestically if capital is scarce
7. Support private investment

These principles also suggest a number of extensions on investment volatility and political economy. Principle six shows that capital scarcity reduces the precautionary motive and encourages investment in domestic capital. Further work should also allow for volatile investment returns and absorption constraints. A second extension could introduce political economy. Some capital-abundant democratic countries (such as Australia and Canada) do not have national sovereign wealth funds, while non-democratic (Saudi Arabia and the UAE), and capital-scarce democratic countries (such as Ghana and Botswana) do. Norway is a conspicuous exception. The uncertainty surrounding election may shorten the government's investment horizon, and reduce its incentive to save. Developing countries may still have an incentive to save because of the desire to overcome capital-scarcity. A formal treatment of this could be the subject of future work.

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Appendix A: Consumption and Total Assets

This appendix derives the equations governing the expected dynamics of consumption and total assets. We begin by taking the first-order conditions of the Hamilton-Jacobi-Bellman equation in 2.6 and 2.7. This is done with respect to consumption, investment, foreign

assets, and domestic capital, where A.3 and A.4 make use of the envelope condition,

$$U'(C)e^{-\rho t} - J_F = 0 \quad (\text{A.1})$$

$$-J_F + J_K = 0 \quad (\text{A.2})$$

$$0 + J_{tF} + J_{FF}(r(F)F + PO + Y(K) - C - I) + J_F(r + r_FF) = 0 \quad (\text{A.3})$$

$$\begin{aligned} & + J_{KF}(I - \delta K) + \frac{1}{2}J_{PPF}\sigma^2 P^2 \\ J_{tK} + J_{FK}(r(F)F + PO + Y(K) - C - I) + Y_K J_F & = 0 \quad (\text{A.4}) \\ & + J_{KK}(I - \delta K) - \delta J_K + \frac{1}{2}J_{PPK}\sigma^2 P^2 \end{aligned}$$

The results in equations 2.8 and 2.9 come from combining equations A.2, A.3 and A.4 with $dJ_s = J_{sF}dF + J_{sK}dK + J_{sP}dP + J_{st}dt + \frac{1}{2}J_{sPP}dP^2$ for $s = [F, K]$.

The optimality condition in equation 2.10 comes from substituting A.1 and 2.7 into 2.6, and using $U(C) = \frac{1}{-a}e^{-aC}$.

The social planner's problem can be summarised in terms of the overall evolution of total assets, $\bar{S} = F + K$, as foreign assets and capital are linked by equation 2.9. The evolution of total assets, equation A.5 below, comes from substituting 2.3 into 2.2. Total assets will be constant at the steady state, S^* , when capital scarcity is overcome and oil income is exhausted, $F^* = 0$ and $O^* = 0$. We define technology so that output is standardised at this point, $A = (\frac{r+\delta}{\alpha})^\alpha$ so $Y^* = 1$ and $K^* = (\frac{\alpha}{r+\delta})$.

$$d\bar{S}^i(t) = (r(F^i(t))F^i(t) + P(t)O^i + Y(K^i(t)) - C^i(t) - \delta K(t))dt \quad (\text{A.5})$$

To analyse the dynamics of consumption and total assets we will need to linearise this equation. We do this around S^* to give the following, where $S = \bar{S} - S^*$.

$$dS^i(t) = \left(rF^i(t) + P(t)O^i + Y'(K^*)(K^i(t) - K^*) - (C^i(t) - C^*) - \delta(K^i(t) - K^*) \right) dt \quad (\text{A.6})$$

Both foreign assets and domestic capital can be expressed as a share of total assets. At any point in time the optimal level of domestic capital can be linked to the level of foreign assets, $K(F) = (\frac{r+\delta-\omega F}{\alpha})^{1/(\alpha-1)}A^{-1/(\alpha-1)}$ from equation 2.9. Therefore, we can express total assets as a non-linear function of foreign assets, $S = S(F)$. However, we need an expression for foreign assets as a function of total assets, $F = F(S)$. The only way to proceed with a tractable solution is to linearise $S(F)$, which we do around the deterministic steady state S^* . Linearisation yields $S = f^{-1}(F - F^*)$ where $f^{-1} = (1 + \frac{\omega}{\alpha(1-\alpha)}(\frac{\alpha}{r+\delta})^2)$. Using this, $\bar{S} = F + K$ and A.6 gives the relationship,

$$\frac{1}{dt}dS^i(t) = \theta S^i(t) + P(t)O^i - C^i(t) + C^* \quad (\text{A.7})$$

where $\theta = rf + (Y'(K^*) - \delta)(1 - f)$ is the overall rate of return on total assets, and $C^* = Y^* - \delta K^*$ is the steady state level of consumption. Finally, making use of

equation 2.9 at the steady state gives $r = Y'(K^*) - \delta$, so $\theta = r$ and we have the budget constraint in equation 2.12. This simple expression is possible because we have linearised the relationship between foreign assets and domestic capital in equation 2.9 to give, $F = f/(1 - f)(K - K^*)$.

The Euler equation in 2.11 is found by taking the expected rate of change with respect to time of A.1, given in A.8 below, and combining it with expressions 2.8, $U(C) = \frac{1}{-a}e^{-aC}$, and $S = f^{-1}(F - F^*)$.

$$\frac{E[dJ_F]/dt}{J_F} = \frac{U''(C)}{U'(C)}E[dC]/dt - \rho + \frac{1}{2}\frac{U'''(C)}{U'(C)}C_P^2\sigma^2P^2dt \quad (\text{A.8})$$

Appendix B: The Permanent Income Hypothesis

This appendix derives an explicit expression for the value function, $J(F, K, P, t)$ in equation 2.1. This is done in the special case when preferences are CARA, $U(C) = \frac{1}{-a}\exp(-aC)$, oil prices are certain, $\sigma = 0$, and the interest rate is constant, $\omega = 0$. The value function must satisfy the optimality condition in equation 2.10, which reduces to,

$$-\frac{1}{a}J_F + \{J_t dt + J_F(rF + PO + Y - C)\} = 0 \quad (\text{A.9})$$

Let us define total wealth at time t , $W(t)$, to be the sum of foreign assets, $F(t)$, the present value of oil income $V^i(t)$, and the present value of permanent non-oil income less depreciation $C^*/r = (Y^* - \delta K^*)/r$. The present value of oil income is found by discounting at the risk-free rate, and will depend on whether the economy is in the Anticipation, Boom or Constant income stage of the oil windfall,

$$\begin{aligned} V^C(t) &= 0 && \text{for } T_2 \leq t \\ V^B(t) &= PO(1 - e^{-r(T_2-t)})/r && \text{for } T_1 \leq t < T_2 \\ V^A(t) &= PO(e^{-r(T_1-t)} - e^{-r(T_2-t)})/r && \text{for } 0 \leq t < T_1 \end{aligned} \quad (\text{A.10})$$

The explicit form of the value function will be as follows, which can be verified by substitution into the partial differential equation A.9,

$$\begin{aligned} J(F, P, t) &= \frac{-A}{a}e^{-ra(W(t))-\rho t} && (\text{A.11}) \\ \text{where } A &= \frac{1}{r}\exp((r - \rho)/r) \end{aligned}$$

Consumption will satisfy the permanent income hypothesis. From equation A.1, consumption will be a fixed proportion of total wealth, $C(t) = rW(t)$ when $r = \rho$. This means that as oil wealth $V(t)$ is extracted, it is converted into foreign assets $F(t)$, and only the permanent income is consumed. The partial derivatives $\partial C^i(t)/\partial P(t)$ are the same as those in equation 3.6.

Parameter	Value	Parameter	Value
a	1.00	f	0.49
ρ	0.05	$F_j(0)$	-1.96
$Y(K^*)$	1.00	P_0	1.00
α	0.40	O	0.2
δ	0.03	T_1	4
ω	0.01	T_2	16
$S_L(0)$	-4.00	σ	Various

Table A.1: Calibration

Appendix C: Calibration

The following values were used in the calibrated simulations:

Appendix 1: Proof of Principle 1

i) A social planner facing a certain but temporary windfall, which can freely borrow at the world rate of interest, should smooth consumption based on the rates of return and time preference, $\dot{C}^i(t) = \frac{1}{a}(r - \rho)$ for all t .

This follows directly from equation 3.1.

ii) Consumption will be a constant proportion of total wealth: the sum of foreign assets, oil wealth and steady state consumption, $C^i(t) = rW(t)$ where $W(t) = F(t) + V(t) + \frac{1}{r}C^*$ and $r = \rho$.

The present value of oil income is given in equation A.10. Combining this with equations 3.3 and 3.5 gives the result.

iii) This will involve borrowing before the windfall, saving during the windfall in a “Future Generations” fund, and consuming the interest earned after the windfall, $\dot{F}^A(t) < 0$, $\dot{F}^B(t) > 0$, $\dot{F}^C(t) = 0$.

From $F^i(t) = a_1^i e^{rt} + a_2^i$ and equation 3.5, assets will evolve according to,

$$\begin{aligned}
F^C(t) &= F(T_2) && \text{for } T_2 \leq t \\
F^B(t) &= F(T_1) + PO(e^{rt} - e^{rT_1})e^{-rT_2}/r && \text{for } T_1 \leq t < T_2 \\
F^A(t) &= F(0) + PO(e^{rt} - 1)(e^{-rT_2} - e^{-rT_1})/r && \text{for } 0 \leq t < T_1
\end{aligned} \tag{A.12}$$

Therefore, $\dot{F}^A(t) = POe^{rt}(e^{-rT_2} - e^{-rT_1}) < 0$, $\dot{F}^B(t) = POe^{-r(T_2-t)} > 0$ and $\dot{F}^C(t) = 0$.

iv) Less will be saved from a long windfall, and more borrowed beforehand, $\frac{\partial}{\partial T_2} \dot{F}^i(t) < 0$ for $t < T_2$.

$\dot{F}^A(t) < 0$ is the amount borrowed in period $0 \leq t < T_1$. From the results in ii), $d\dot{F}^A(t)/dT_2 = -rPOe^{-r(T_2-t)} < 0$, so a longer windfall will cause the social planner to borrow more before the windfall. $\dot{F}^B(t) > 0$ is the amount saved in period $T_1 \leq t < T_2$. Also from part ii), $d\dot{F}^B(t)/dT_2 = -rPOe^{-r(T_2-t)} < 0$, so a longer windfall reduces the amount saved each period during the windfall.

v) Investment will be independent of any oil discovery if capital is abundant and goods are freely traded, consistent with the Fisher separation theorem, $K = K^*$.

When interest rates are constant and goods can be freely bought and sold at the world price, then $\omega = 0$ in equation 2.9. Capital will therefore be constant, $K = K^* = (\frac{r+\delta}{\alpha A})^{1/(\alpha-1)}$. Technology is defined to normalise $Y(K^*) = 1$ so $K^* = \frac{\alpha}{r+\delta}$.

Appendix 2: Proof of Principle 2

The particular solution to equation 3.9, so that a_1^i, a_2^i are defined in equation 3.5, is,

$$F_V^C(t) = F_V^B(T_2) \quad (\text{A.13})$$

$$F_V^B(t) = \frac{1}{2}aP^2O^2\sigma^2\frac{1}{r^2}\left(t^B B_1 - B_2(1 + r(T_2 - T_1 - t^B))(e^{rt^B} - 1) - B_3(e^{2rt^B} - 1)\right) + F_V^A(T_1) \quad (\text{A.14})$$

$$F_V^A(t) = \frac{1}{2}aP^2O^2\sigma^2\frac{1}{r^2}\left(A_1(e^{rt} - 1) - A_2(e^{2rt} - 1)\right) \quad (\text{A.15})$$

where $t^B = t - T_1$, $B_1 = r(1 + 2e^{-r(T_2-T_1)})$, $B_2 = 2e^{-r(T_2-T_1)}$, $B_3 = \frac{1}{2}e^{-2r(T_2-T_1)}$, $A_1 = 2\left((e^{-rT_1} - e^{-rT_2}) - (T_2 - T_1)re^{-rT_2}\right)$, $A_2 = \frac{1}{2}(e^{-rT_1} - e^{-rT_2})^2$, and all $A_j, B_j \geq 0$. This can be verified by substitution into $\ddot{F}^i(t) - r\dot{F}^i(t) = \frac{1}{2}aP^2\sigma^2C_P^i(t)^2$.

The proof of this principle relies on the following two Lemmas,

Lemma 1. *The balance of the Volatility Fund never decreases in expectation, $\dot{F}_V^i(t) > 0$ for all $t > 0$.*

Proof. The time derivative of the fund during each phase is,

$$\dot{F}_V^C(t) = 0 \quad (\text{A.16})$$

$$\dot{F}_V^B(t) = \frac{1}{2}aP^2O^2\sigma^2\frac{1}{r^2}\left(B_1 - rB_2(1 + (T_2 - T_1 - t^B)re^{rt^B}) - B_32re^{2rt^B}\right) \quad (\text{A.17})$$

$$\dot{F}_V^A(t) = \frac{1}{2}aP^2O^2\sigma^2\frac{1}{r^2}\left(A_1re^{rt} - A_22re^{2rt}\right) \quad (\text{A.18})$$

During the Anticipation phase we begin by showing that $\dot{F}_V^A(T_1) > 0$,

$$\begin{aligned}
\dot{F}_V^A(T_1) &= \frac{1}{2}aP^2O^2\sigma^2\frac{1}{r^2}\left(A_1re^{rT_1} - A_22re^{2rT_1}\right) \\
&= \frac{1}{2}aP^2O^2\sigma^2\frac{1}{r}\left(1 - (2r(T_2 - T_1) + e^{-r(T_2-T_1)})e^{-r(T_2-T_1)}\right)
\end{aligned}$$

Now, when $T_2 = T_1$ then $\dot{F}_V^A(T_1) = 0$. Also, by l'Hopital's rule, $\lim_{(T_2-T_1)\rightarrow\infty}\dot{F}_V^A(T_1) = \frac{1}{2}aP^2O^2\sigma^2\frac{1}{r} > 0$. As $\ddot{F}_V^A(T_1) < 0$ for all T_1 then $\dot{F}_V^A(T_1) > 0$. It is also easy to see that $\dot{F}_V^A(t) > \dot{F}_V^A(T_1)$ for all $t < T_1$. Thus, $\dot{F}_V^A(t) > 0$.

During the Boom phase it is straightforward to show that $\dot{F}_V^B(T_2) > 0$ by substitution (that is when $t^B = T_2 - T_1$). Furthermore, $\dot{F}_V^B(t) > \dot{F}_V^B(T_2)$ for all $t < T_2$. Thus, $\dot{F}_V^B(t) > 0$. \square

Lemma 2. *The rate of increase of the Volatility Fund slows over time, $\ddot{F}_V^A(t), \ddot{F}_V^B(t) < 0$ for $t < T_2$.*

Proof. The second derivative of $F_V^i(t)$ with respect to time is,

$$\ddot{F}_V^C(t) = 0 \tag{A.19}$$

$$\ddot{F}_V^B(t) = \frac{1}{2}aP^2O^2\sigma^2\left(B_2(1 - r(T_2 - T_1 - t^B))e^{rt^B} - B_34e^{2rt^B}\right) \tag{A.20}$$

$$\ddot{F}_V^A(t) = \frac{1}{2}aP^2O^2\sigma^2\left(A_1e^{rt} - A_24e^{2rt}\right) \tag{A.21}$$

During the Anticipation phase we begin by showing that $\ddot{F}_V^A(0) < 0$. When $t = 0$,

$$\ddot{F}_V^A(0) = aP^2O^2\sigma^2\left((e^{-rT_1} - e^{-rT_2}) - (T_2 - T_1)re^{-rT_2} - (e^{-rT_1} - e^{-rT_2})^2\right)$$

If $T_2 = T_1$ then $\ddot{F}_V^A(0) = 0$. So, if $T_2 > T_1$ then $\ddot{F}_V^A(0) < 0$. Now, we also know from inspection of equation A.21 that $\ddot{F}_V^A(t) < \ddot{F}_V^A(0)$ for all $0 < t < T_1$. Therefore $\ddot{F}_V^A(t) < 0$ for all $0 < t < T_1$.

During the Boom phase we begin by showing that $\ddot{F}_V^B(T_1) < 0$ for $r < \frac{2}{3}$ (that is, when $t^B = 0$),

$$\ddot{F}_V^B(T_1) = \frac{1}{2}aP^2O^2\sigma^2\left(r(1 + 2e^{-r(T_2-T_1)})(1 - r(T_2 - T_1)) - 2e^{-2r(T_2-T_1)}\right)$$

If $T_2 = T_1$ then $\ddot{F}_V^B(T_1) < 0$ if $r < \frac{2}{3}$. If $T_2 > T_1$ then $\ddot{F}_V^B(T_1)$ will become more negative. We can also see from inspection of equation A.20 that $\ddot{F}_V^B(t) < \ddot{F}_V^B(T_1)$ for all $T_1 < t < T_2$. Therefore $\ddot{F}_V^B(t) < 0$ for all $T_1 < t < T_2$. \square

Turning now to the parts of principle two:

Build a Volatility Fund quickly:

i) A social planner facing a volatile temporary windfall, who can freely borrow at the world rate of interest, should engage in precautionary savings and build up a “Volatility Fund”, in addition to the “Future Generations” fund in principle 1, $F_V^i(t) > 0$ for all $t > 0$.

The Volatility Fund begins at $F_V^A(0) = 0$, as prior to discovering oil there is no exposure to volatility. The expected balance of the fund is also always increasing, $\dot{F}_V^i(t) > 0$ for all $0 < t < T_2$ by Lemma 1. Therefore $F_V^i(t) > 0$ for all $t > 0$.

ii) The Volatility Fund should receive relatively more savings during the early years of the windfall, including during the Anticipation phase, $\dot{F}_V^A(t), \dot{F}_V^B(t) > 0$ and $\ddot{F}_V^A(t), \ddot{F}_V^B(t) < 0$ for $t < T_2$.

The balance of the Volatility Fund increases for all $0 < t < T_2$, $\dot{F}_V^A(t), \dot{F}_V^B(t) > 0$ by Lemma 1. However, the rate it increases will diminish over time, $\ddot{F}_V^A(t), \ddot{F}_V^B(t) < 0$ for $t < T_2$ by Lemma 2. Thus, it will receive relatively more savings in the early years of the windfall.

Then leave it alone:

iii) The assets in the Volatility Fund should not be consumed in the presence of a persistent negative shock to the oil price. Only interest should be consumed, $\frac{\partial}{\partial P_0} F_V^i(0) = 0$.

We know that $F_V^i(0) = 0$. Thus, a contemporaneous oil price shock will not affect the stock of assets in the Volatility Fund, only the subsequent path of consumption.

iv) As oil is extracted, the need for a Volatility Fund will diminish, $\dot{C}^B(t) \rightarrow 0$ as $t \rightarrow T_2$.

The marginal effect of a change in oil prices on consumption falls as oil is extracted, $C_P^B(t) = O(1 - e^{-r(T_2-t)}) \rightarrow 0$ as $t \rightarrow T_2$ from equation 3.6. The result then follows directly from equation 3.7, capturing a diminishing incentive for precautionary savings.

v) Any funds remaining in the Volatility Fund should be saved, and only the permanent income consumed, $\dot{F}_V^i(t) \geq 0$ for all $t \leq T_2$.

This follows from Lemma 1.

Appendix 3: Proof of Principle 3

i) When there is both a traded and a non-traded sector, an oil discovery should increase investment in the non-traded sector, and decrease investment in the traded sector, with the aim of stabilising the real exchange rate, $e = e^*$.

Firms in this model choose the mix of capital and labour to maximise profit, from equation 3.14. This gives rise to the following first order conditions, and the following Lemma,

$$\gamma_G P_j Y_j / K_{G,j} = r_G \quad (\text{A.22})$$

$$\gamma_P P_j Y_j / K_{P,j} = r_P \quad (\text{A.23})$$

$$(1 - \gamma_G - \gamma_P) P_j Y_j / L_j = w \quad (\text{A.24})$$

Lemma 3. *When the traded good is sold on international markets for a constant price, capital is freely allocated at constant interest rates, and there are constant returns to scale; both the wage and the price of non-traded goods will also be constant, $w = w^*$ and $P_N = P_N^*$. They are also independent of the aggregate level of consumption, C . This follows from the factor-price equalization theory.*

Proof. Production in each sector is given by $Y_j = AK_{G,j}^{\gamma_G} K_{P,j}^{\gamma_P} L_j^{1-\gamma_G-\gamma_P}$. When capital is freely traded on world markets, the only scarce factor of production is labour. If a unit of labour moves into a sector, it will be accompanied by both private and public capital, which have been freely borrowed at the world rate of interest. This ensures that the marginal product of each factor equals its price. The amount of capital in each sector can therefore be expressed in terms of labour, using equations A.22 to A.24,

$$K_{G,j} = wL_j/P_j \frac{\gamma_G/r}{(1 - \gamma_G - \gamma_P)} \quad (\text{A.25})$$

$$K_{P,j} = wL_j/P_j \frac{\gamma_P/r}{(1 - \gamma_G - \gamma_P)} \quad (\text{A.26})$$

Output in each sector can also be expressed in terms of labour, making use of the constant returns to scale assumption and substituting A.25 and A.26 into the production function,

$$Y_j = A_j L_j b (w/P_j)^{\gamma_1 + \gamma_2} \quad (\text{A.27})$$

where $b = \gamma_1^{\gamma_G} \gamma_2^{\gamma_P} (r(1 - \gamma_G - \gamma_P))^{-\gamma_G - \gamma_P}$. The marginal product of labour is essentially constant, as it will be accompanied by capital. The marginal product of labour also equals the wage, $(1 - \gamma_G - \gamma_P) P_j Y_j / L_j = w$. Combining this with equation A.27 gives,

$$w/P_j = c_j \quad (\text{A.28})$$

where $c_j = (A_j (\gamma_G/r)^{\gamma_G} (\gamma_P/r)^{\gamma_P})^{1/(1-\gamma_G-\gamma_P)} (1 - \gamma_G - \gamma_P)$.

In the traded sector, the price is set on world markets, $P_T = 1$, so the wage will be dictated by the price of the traded good, the world rate of interest and the production technology in the traded sector $w = w^* = c_T$.

In the non-traded sector, the price will be set to equate demand and supply for the non-traded good. As the non-traded sector must compete with the traded sector for labour, the price of non-traded goods will also be constant - even if the production technology in each sector differs, $P_N = c_T/c_N$. These results stem from Samuelson's (1948) factor-price equalization theory. \square

The social planner will increase aggregate consumption when oil is discovered, as stated in principle one. Equilibrium in the non-traded sector requires that $P_N^{1-a_N} Y_N = a_N C$. An increase in consumption must be met by either a rise in prices or a rise in output. The price of non-traded goods should be constant, from lemma 3. Higher aggregate consumption should therefore be met by higher output. This can only be achieved with a movement of labour from the traded sector, and investment in both public and private capital. The reduction in labour in the traded sector will also cause the sector's capital stock to fall, according to equations A.25 and A.26. We assume that this faces no restrictions. Thus, discovering oil will cause investment in the traded sector, with the aim of maintaining a constant real exchange rate, $e = e^* = c_N/c_T$.

ii) This is not just a reallocation of capital. The total capital stock can increase or decrease, depending on the production technology of each good.

The total stock of public and private capital in the economy is given by, $K_k = K_{k,N} + K_{k,T}$ for $k = G, P$. From equations A.25 and A.26 we have,

$$K_k = w \sum_{j=N,T} L_j / P_j \frac{\gamma_{k,j}/r}{(1 - \gamma_{G,j} - \gamma_{P,j})}$$

So, the allocation of labour, $\bar{L} = L_N + L_T$ will affect the total stock of both public and private capital, unless both sectors use exactly the same production technologies (so that $P_N = P_T = 1$).

iii) During the Anticipation phase this will involve borrowing to finance both consumption and investment in the non-traded sector.

On discovering oil the social planner will immediately increase consumption as stated in principle one. Consumption will remain at this new level in perpetuity, in the absence of volatility. Higher consumption will immediately increase demand for both traded and non-traded goods. To prevent prices rising this will require an immediate investment in the non-traded sector, I_N , as stated in part i). If this exceeds the amount of capital that can be released by the traded sector then it will reduce the stock of foreign assets, F , according to the budget constraint in equation 2.2.

iv) Investing in the non-traded sector may limit the decline of the traded sector (Dutch disease) when the initial capital stock is low, but immigration is more effective.

To show this we compare how an increase in consumption affects the labour market, when capital is abundant and when it is fixed.

When capital is abundant, an increase in consumption will have a constant effect on labour. In this case investment will ensure that the real exchange rate is constant, $P_N = P_N^*$ from Lemma 1. Therefore, an increase in consumption must be met by an increase in output of the non-traded good, $C = P_N^{1-a_N} Y_N / a_N$. Using this, equation A.27 and $\frac{dL_N}{dC} \cdot \frac{dC}{dL_N} = 1$ we get,

$$\frac{dL_N}{dC} = \frac{a_N}{A_N b} w^{*(-\gamma_G - \gamma_P)} P_N^{*(1-a_N - \gamma_G - \gamma_P)} \quad (\text{A.29})$$

So when capital is abundant, an exogenous increase in consumption will have a constant effect on labour (unless all labour is already employed in the non-traded sector, which we ignore).

When capital is fixed, an increase in consumption will have a varying effect on labour, depending on the initial level of consumption. The wage will still be driven by the marginal product of labour in the traded sector, $w = (1 - \gamma_G - \gamma_P) k_T (1 - L_N)^{-\gamma_G - \gamma_P}$, using the constant, $k_T = A_T K_{G,T}^{\gamma_G} K_{P,T}^{\gamma_P}$, for simplicity. In this case an increase in aggregate consumption will be met by both a higher price and quantity of the non-traded good, $C = P_N^{1-a} Y_N / a$. To isolate the effect on the labour market we combine this with the marginal product of labour in the non-traded and the traded sectors, equation A.24, to give,

$$\begin{aligned} w &= (1 - \gamma_G - \gamma_P) (aC / Y_N)^{1/(1-a)} (Y_N / L_N) \\ C &= \frac{1}{a} k_N^a k_T^{(1-a)} L_N^{(1-a\gamma_G - a\gamma_P)} (1 - L_N)^{-(1-a)(\gamma_G + \gamma_P)} \end{aligned}$$

The effect of a change in consumption on the labour market when capital is fixed, L'_N , will be,

$$\frac{dL'_N}{dC} = C^{-1} \frac{(1 - L'_N) L'_N}{(1 - a\gamma_G - a\gamma_P) - L'_N (1 - \gamma_G - \gamma_P)} \quad (\text{A.30})$$

Thus, the effect of a change in consumption on the labour market when capital is fixed will depend on the initial level of consumption. This in turn is a function of the initial capital stock, and the associated allocation of labour between the traded and non-traded sectors. If the initial capital stock is sufficiently low, then an increase in consumption will require a large movement of labour from the traded to the non-traded sector when capital is fixed. Investing in the non-traded sector may then reduce the marginal effect of consumption on labour, $\frac{dL'_N}{dC} > \frac{dL_N}{dC}$. Output in each sector will be driven by the reallocation of labour. We have $\frac{dY'_T}{dC} = \frac{dY'_T}{dL'_N} \cdot \frac{dL'_N}{dC} < \frac{dY_T}{dC} = \frac{dY_T}{dL_N} \cdot \frac{dL_N}{dC}$ if the initial capital

stock is sufficiently low, as $\frac{dY_T'}{dL_N}, \frac{dY_T}{dL_N} < 0$. So, under some parametrisations, investing in the non-traded sector can limit the decline of the traded sector.

Immigration is more effective at limiting the decline of the traded sector. The contraction of the traded sector is driven by the limited availability of labour. Consider instead the case of ready access to immigrant labour. If the social planner sets the wage at $w = w^*$, such as a reference wage paid to government employees, total labour will freely adjust to ensure that its marginal product equals the wage in each sector. The decline in the traded sector described in part i), driven by the reallocation of labour from the traded to the non-traded sector, would be avoided as immigrant workers fill the demand for labour in the non-traded sector.

Appendix 4: Proof of Principle 4

i) Consumption in capital-scarce countries should begin lower than in capital abundant economies, $C_L^C(0) < C_H^C(0)$, but jump further when oil is discovered, $C_L^A(0) - C_L^C(0) > C_H^A(0) - C_H^C(0)$. It should still remain lower in absolute level though, $C_L^A(0) < C_H^A(0)$.

Consumption in capital scarce economies should begin lower, $C_L^C(0) < C_H^C(0)$. Phase C describes the economy in the absence of oil. If capital is scarce (L), consumption and assets will lie on the stable saddle path during this phase, $C_L^C(t) = (r - \lambda_1)S_L^C(t) + C^*$, from equations 4.3, 4.4 and 4.5. If capital is abundant (H) they will lie on the steady state line, $C_H^C(t) = rS_H^C(t) + C^*$, from equation 3.2 with $\bar{F} = O = 0$. As $\lambda_1 < 0$ and $S_L^C(0) < S_H^C(0) < 0$ (as the capital stock will also be lower when capital is scarce), $C_L^C(0) < C_H^C(0)$ for any given level of C^* .

Consumption in capital scarce economies should jump further when oil is discovered, $C_L^A(0) - C_L^C(0) > C_H^A(0) - C_H^C(0)$. First we note that $C_j^C(0)$ is the level of consumption given the starting level of assets, $S_j(0)$, during the Constant income phase for $j = [L, H]$. This is the level of consumption that would hold at time $t = 0$ if there is no windfall. Now, we have,

$$\begin{aligned} C_L^A(0) - C_L^C(0) &= (r - \lambda_1)S_L(0) + PO(e^{-\lambda_2 T_1} - e^{-\lambda_2 T_2}) + C^* \\ &\quad - S_L(0)(r - \lambda_1) - C^* \\ &= PO(e^{-\lambda_2 T_1} - e^{-\lambda_2 T_2}) \end{aligned}$$

$$\begin{aligned} C_H^A(0) - C_H^C(0) &= rS_H(0) + PO(e^{-r T_1} - e^{-r T_2}) + C^* - rS_H(0) - C^* \\ &= PO(e^{-r T_1} - e^{-r T_2}) \end{aligned}$$

Now, $(e^{-r T_1} - e^{-r T_2}) - (e^{-\lambda_2 T_1} - e^{-\lambda_2 T_2}) < 0$, for $\omega > 0$ as $\lambda_2 = \frac{1}{2}r \pm \frac{1}{2}\sqrt{r^2 + 8\omega f/a}$. So on discovering oil, consumption will jump further in a capital-scarce than a capital abundant economy.

However, consumption in capital scarce economies will remain lower in absolute level, $C_L^A(0) < C_H^A(0)$. Let the difference between consumption in capital abundant and capital-scarce economies be, $c^i(t) = C_H^i(t) - C_L^i(t)$. The associated difference in assets is $s^i(t) = S_H^i(t) - S_L^i(t)$. The difference in consumption will satisfy,

$$\begin{aligned}\dot{c}^i(t) &= \dot{C}_H^i(t) - \dot{C}_L^i(t) \\ &= \frac{2\omega}{a} f S_L^i(t) \\ &= -\frac{2\omega}{a} f s^i(t) + \frac{2\omega}{a} f S_H^i(t) \\ &< 0\end{aligned}$$

So, consumption in the capital-scarce economy grows more quickly than consumption in the capital abundant economy. The difference in assets will satisfy $\dot{s}^i(t) = r s^i(t) - c^i(t)$. These dynamics are summarised by the differential equation $\dot{s}^i(t) - r s^i(t) - \frac{2\omega}{a} s^i(t) = -\frac{2\omega}{a} f S_H^i(t)$. The explicit solution to this is,

$$s^i(t) = k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t} + a_1 e^{rt} + a_2 \quad (\text{A.31})$$

$$\begin{aligned}c^i(t) &= k_1 e^{\lambda_1 t} + k_2 e^{\lambda_2 t} + S_H^A(t) \\ c^i(t) &= (r - \lambda_1) k_1 e^{\lambda_1 t} + (r - \lambda_2) k_2 e^{\lambda_2 t} + r a_2\end{aligned} \quad (\text{A.32})$$

To find the coefficients k_1, k_2 we impose two constraints. First, at $t = 0$, $0 = k_1 + k_2 + S_L(0)$ and $s(0) > 0$ as the initial domestic capital stock will be lower when capital is scarce. Second, the difference between the capital-scarce and capital abundant economies will disappear if foreign assets ever reach zero. So, if for any point in time $t = t^* > 0$, $S_H^i(t^*) = S_L^i(t^*) = s^i(t^*) = 0$, then the difference in consumption will also be zero, $c^i(t^*) = 0$. This gives $k_1 = -S_L(0) - k_2$ and $k_2 = r a_2 (\lambda_2 - \lambda_1)^{-1} e^{-\lambda_2 t^*}$. Substituting this into equation A.32 gives $c^A(0) = \lambda_1 S_L(0) + PO(e^{-rT_1} - e^{-rT_2})(1 - e^{-\lambda_2 t^*}) > 0$. Thus, the absolute level of consumption on discovering oil in a capital-scarce economy will be lower than in a capital abundant economy, $C_H^A(0) > C_L^A(0)$.

ii) Borrowing in capital-scarce countries before a boom should initially be lower than capital abundant countries, $\dot{F}_L^A(0) > \dot{F}_H^A(0)$, and if they begin to borrow it will happen near the date of extraction, $\dot{F}_L^A(t) < 0$ for $t < T_1$.

From part i) we have $C_H^A(0) > C_L^A(0)$. As $\dot{S}_j^A(t) = r S_j^A(t) + C^* - C_j^A(t)$ for $j = [L, H]$, so $\dot{S}_H^A(0) < \dot{S}_L^A(0)$. We also know that $F_H^i(t) = S_H^i(t)$ and $F_L^i(t) = f S_L^i(t)$ where $f < 1$. So, $\dot{F}_L^A(0) > \dot{F}_H^A(0)$ and capital scarce countries will initially borrow less.

As the Anticipation phase proceeds, the amount saved by a capital-scarce economy will decrease (or the rate of borrowing will increase). From equation 4.3 we have $S^i(t^i) = c_1^i e^{\lambda_1 t^i} + c_2^i e^{\lambda_2 t^i}$ and $\dot{S}^A(t) = c_1^A \lambda_1^2 e^{\lambda_1 t} + c_2^A \lambda_2^2 e^{\lambda_2 t}$. We know that $c_1^A = S(0) - c_2^A$ and $c_2^A = -PO(e^{-\lambda_2 T_1} - e^{-\lambda_2 T_2})(\lambda_2 - \lambda_1)^{-1} < 0$ from equation 4.5. If a windfall is small, its present value will be much less than is needed to overcome capital scarcity, $PO(e^{-rT_1} - e^{-rT_2})/r \ll |S(0)|$. This is because the country will also be saving from income Y and we assume that $S(T_2) < 0$. As $c_2^A \approx -PO(e^{-rT_1} - e^{-rT_2})/r$ and $S(0) < 0$ we have,

$c_1^A = S(0) - c_2^A < 0$ for a small windfall. So, $\ddot{S}^A(t) = c_1^A \lambda_1^2 e^{\lambda_1 t} + c_2^A \lambda_2^2 e^{\lambda_2 t} < 0$ and in turn $\dot{F}^A(t) < 0$.

iii) Domestic capital in capital-scarce economies should grow over time, $\dot{K}_L^C(t) > 0$, be accelerated during an oil boom, $\dot{K}_L^B(t) > \dot{K}_L^C(t)$, and comprise an increasing share of the total capital stock over time, $\frac{d}{dt}(K(t)/\bar{S}(t)) > 0$.

Domestic capital should grow over time, $\dot{K}_L^C(t) > 0$. In equation 4.3 we see that $\dot{S}^C(t) > 0$ for all t . We also know that $S^C(t) = (1-f)(K^C(t) - K^*)$. As $f < 1$ we have $\dot{K}_L^C(t) > 0$.

The growth of domestic capital will be accelerated during an oil boom, $\dot{K}_L^B(t) > \dot{K}_L^C(t)$. From equation 4.3 we have $\dot{S}^i(t) = c_1^i \lambda_1 e^{\lambda_1 t} + c_2^i \lambda_2 e^{\lambda_2 t}$. Let us consider how total assets evolve with or without an oil boom, starting at the same initial level of assets S_0 . From equation 4.5 we would have $c_1^C = S_0$, $c_2^C = 0$, $c_1^B = S_0 - c_2^B$ and $c_2^B > 0$. So, $\dot{S}^B(t) > \dot{S}^C(t)$ as $c_1^B < c_1^C$ and $\lambda_1 < 0$, and $c_2^B > c_2^C$ and $\lambda_2 > 0$. Therefore $\dot{K}^B(t) > \dot{K}^C(t)$.

Domestic capital will also comprise an increasing share of the total capital stock over time, $\frac{d}{dt}(K(t)/\bar{S}(t)) > 0$. We know that $\bar{S}(t) = F(t) + K(t)$. We also can see that $\dot{F}(t) = \frac{\alpha(1-\alpha)}{2\omega} AK(t)^{\alpha-2} \dot{K}(t)$ from equation 2.9. At $K(t) = K^* = \frac{\alpha}{r+\delta}$ and using $A = (\frac{r+\delta}{\alpha})^\alpha$ from Appendix A, $\frac{\alpha(1-\alpha)}{2\omega} AK(t)^{\alpha-2} = \frac{(1-\alpha)(r+\delta)^2}{2\omega \alpha}$. For $\alpha \approx (1-\alpha)$ and r, δ and ω of similar order of magnitude, $\frac{\alpha(1-\alpha)}{2\omega} AK^{\alpha-2} < 1$. So, near K^* domestic capital grows more quickly than foreign assets, $\dot{F}(t) < \dot{K}(t)$. Domestic capital will therefore increase its share in total assets, $\frac{d}{dt}(K(t)/\bar{S}(t)) > 0$. This will not necessarily hold if $K(t)$ is significantly below K^* , which we abstract from in our linearised analysis.

Appendix 5: Proof of Principle 5

i) In capital scarce countries with absorption constraints, part of an oil discovery should be accumulated in a temporary offshore Parking Fund while absorption constraints are relaxed, $F_P^i(t) > 0$ if $\phi > 0$.

We begin by replacing the social planner's budget constraint in equation 2.2 with equation 4.6, and finding the first order conditions to the social planner's problem, replacing those in equations A.1 to A.4,

$$U'(C)e^{-\rho t} - J_F = 0 \quad (\text{A.33})$$

$$-J_F(1 + \phi I/K) + J_K = 0 \quad (\text{A.34})$$

$$-J_F(Y_K + 0.5\phi I^2 K^{-2}) + \delta J_K = \frac{1}{dt} E[dJ_K(F, P, K, t)] \quad (\text{A.35})$$

$$-(r - 2\omega F)J_F = \frac{1}{dt} E[dJ_F(F, P, K, t)] \quad (\text{A.36})$$

Now let us introduce the ratio of the marginal utility from an extra unit of capital to that of foreign assets, $q = J_K/J_F$. Using this new variable we can summarise equations A.34 to A.36 in equations 4.7 and 4.8, reproduced below,

$$\begin{aligned}
q &= 1 + \phi \frac{I}{K} \\
\dot{q} &= q(r - \omega F + \delta) - Y_K - 0.5 \frac{1}{\phi} (q - 1)^2
\end{aligned}$$

The first equation shows that the marginal utility from an extra unit of installed capital will be greater than for an extra unit of foreign assets, $q > 1$ as $\phi I/K > 0$. In the absence of absorption constraints the marginal utilities are equal, $q = 1$ when $\phi = 0$ from equation A.2. Marginal utility is diminishing in both capital and assets, $J_{KK}, J_{FF} < 0$. This is also consistent with equations A.35 and A.36, where the marginal utilities fall over time as capital and foreign assets are accumulated, $\frac{1}{dt}E[dJ_K], \frac{1}{dt}E[dJ_F] < 0$. So, for a given stock of assets, $\bar{S} = F + K$, when $\phi > 0$ then $J_K > J_F$ and there will be relatively more foreign assets and less domestic capital than when $\phi = 0$ and $J_F = J_K$. We call the additional foreign assets in the presence of absorption constraints a Parking Fund, which is the difference between foreign assets when $\phi > 0$ and $\phi = 0$, $F_P^i(t) = F_\phi^i(t) - F^i(t) > 0$. Illustrations of the size of this fund are provided in van der Ploeg (2012).

The parking fund will be temporary. As capital is accumulated absorption constraints will disappear, the marginal utilities of foreign assets and capital will converge and the Parking Fund will disappear, as $K \rightarrow \infty$, $q \rightarrow 1$ and $F_P^i(t) \rightarrow 0$.

ii) In capital abundant countries an oil discovery will not affect the size of any Parking Fund.

In capital abundant economies there is no debt premium on interest, $\omega = 0$. In this case the equations 4.7, 4.8 and 2.3 form a system of three dynamic equations in three variables, I, K and q . This system can be solved without any reference to consumption, foreign assets or oil. Thus, the separation theorem in principle 1 holds and an oil discovery will not alter the path of investment in a capital abundant economy.

Appendix 6: Proof of Principle 6

We begin with the following expressions for $S_V^i(t)$:

$$S_V^C(t) = S_V^B(T_2) \tag{A.37}$$

$$\begin{aligned}
S_V^B(t) &= \frac{1}{4\omega f} a^2 P^2 O^2 \sigma^2 \left(B_1(e^{2rt^b} - e^{\lambda_1 t^b}) - B_2(e^{rt^b} - e^{\lambda_1 t^b}) \right. \\
&\quad \left. - B_3(e^{\lambda_2 t^b} - e^{\lambda_1 t^b}) + (1 - e^{\lambda_1 t^b}) \right) \tag{A.38}
\end{aligned}$$

$$S_V^A(t) = \frac{1}{4\omega f} a^2 P^2 O^2 \sigma^2 \left(A_1(e^{2rt} - e^{\lambda_1 t}) - A_2(e^{\lambda_2 t} - e^{\lambda_1 t}) \right) \tag{A.39}$$

Where $t^b = t - T_1$, $B_1 = \frac{\omega f/a}{\omega f/a - r^2} e^{-2r(T_2 - T_1)}$, $B_2 = 2e^{-r(T_2 - T_1)}$, $B_3 = \frac{r^2}{\omega f/a - r^2} \frac{2r - \lambda_1}{\lambda_2 - \lambda_1} e^{-\lambda_2(T_2 - T_1)}$, $A_1 = \frac{\omega f/a}{\omega f/a - r^2} (e^{-rT_1} - e^{-rT_2})^2$ and $A_2 = \frac{e^{-\lambda_2 T_1}}{(\lambda_2 - \lambda_1)} \left\{ \frac{(2r - \lambda_1)}{\omega f/a - r^2} r^2 \left(1 - 2e^{-r(T_2 - T_1)} + e^{-\lambda_2(T_2 - T_1)} \right) + \right.$

$2r(1 - e^{-r(T_2 - T_1)}) \left. \vphantom{2r(1 - e^{-r(T_2 - T_1)})} \right\}$, where all $B_j, A_k > 0$ for $\omega f/a - r^2 > 0$. Also, we have, $C_V^i(t) = rS_V^i(t) - \dot{S}_V^i(t)$. This can be verified by substitution into equations 4.9 and 4.10.

i) Capital-scarce countries should accumulate less in an offshore Volatility Fund than capital abundant economies, $F_{V,L}^i < F_{V,H}^i$, despite the fund accelerating the expected rate of development.

The Volatility Fund is the difference between the total level of foreign assets in a stochastic and a deterministic setting (the Future Generations Fund), $F_{V,j}^i = F_j^i - F_{FG,j}^i$, for both capital-scarce (L) and capital-abundant (H) economies, $j = [L, H]$. This Volatility Fund will be a fraction of the total level of assets accumulated to manage volatility, $F_{V,j}^i = fS_{V,j}^i$, which satisfies the dynamic equation, $\dot{S}_{V,j}^i = rS_{V,j}^i + PO^i + C^* - C_{V,j}^i$ for both capital abundant and capital-scarce economies. The volatility effect on consumption, $C_{V,j}^i = C_j^i - C_{FG,j}^i$, will satisfy the following equations, for capital abundant and capital-scarce economies respectively,

$$\begin{aligned} \dot{C}_{V,H}^i &= \dot{C}_H^i - \dot{C}_{FG,H}^i \\ &= \frac{1}{2}aP^2C_P^i(t)^2\sigma^2 \end{aligned} \quad (\text{A.40})$$

$$\begin{aligned} \dot{C}_{V,L}^i &= \dot{C}_L^i - \dot{C}_{FG,L}^i \\ &= \left\{ -\frac{2\omega f}{a}S_L^i + \frac{1}{2}aP^2C_P^i(t)^2\sigma^2 \right\} - \left\{ -\frac{2\omega f}{a}S_{FG,L}^i \right\} \\ &= -\frac{2\omega f}{a}S_{V,L}^i + \frac{1}{2}aP^2C_P^i(t)^2\sigma^2 \end{aligned} \quad (\text{A.41})$$

The rate of change of the volatility adjustment to consumption is higher in capital-abundant economies, $\dot{C}_{V,L}^i(t) < \dot{C}_{V,H}^i(t)$ for all $t > 0$. This follows from equations A.37 to A.39, which show that $S_{V,j}^i(t) > 0$ for all $t > 0$, and equation A.41. This is consistent with precautionary savings building up a positive Volatility Fund.

Therefore, the adjustment to consumption from volatility will be greater in capital-abundant economies during the early years of the windfall, $C_{V,H}^i(t) < C_{V,L}^i(t) < 0$ for t less than some t^V . The volatility effect on total assets is zero before oil is discovered, for both capital-scarce and capital-abundant economies, $S_{V,j}(0) = 0$ for $j = [L, H]$. The ‘‘budget constraint’’ tying together the volatility adjustments to consumption and total assets is also the same in both capital-abundant and capital-scarce economies, $\dot{S}_{V,j}^i = rS_{V,j}^i + PO^i + C^* - C_{V,j}^i$ for $j = [L, H]$. If both the initial state and the budget constraint are the same in both countries, but the consumption adjustment grows more quickly in the capital-abundant economy, then its initial level must be lower. So, $C_{V,H}^i(t) < C_{V,L}^i(t) < 0$.

As a result, capital-abundant economies will accumulate a larger volatility fund than capital-scarce economies, $F_{V,H}^i > F_{V,L}^i$. The overall amount of assets accumulated to manage volatility will grow faster in a capital-abundant country, $\dot{S}_{V,H}^i > \dot{S}_{V,L}^i$. All of these assets in a capital-abundant economy are accumulated in a Volatility Fund, $F_{V,H} = S_{V,H}$, while only a fraction are in a capital-scarce economy, $F_{V,L} = fS_{V,L}$ where $f \in [0, 1]$. Thus, capital-abundant economies will accumulate a larger Volatility Fund than capital-scarce economies. Over time, the larger Volatility Fund in the capital-abundant country will

generate interest payments, which will finance a larger volatility effect on consumption, $C_{V,H}^i(t) > C_{V,L}^i(t)$ for $t > t^V$.

This holds despite the Volatility Fund accelerating the pace of development in expectation. This follows directly from equation 4.9, where volatility increases the rate of change of consumption $\frac{\partial}{\partial \sigma} \dot{C}^i(t) > 0$. This means that consumption will be lower in the short-run, to finance precautionary savings. These precautionary savings lead to the accumulation of a Volatility Fund, $F_V^i(t) > 0$ from equations A.37 to A.39, in addition to the future generations fund. This additional Volatility Fund will reduce capital scarcity, and generate income that can finance a higher level of consumption in the future.

ii) Capital-scarce countries should also direct some precautionary savings from an oil boom to domestic investment, $K_V^i(t) > 0$.

From equations A.39 to A.37 we have that $S_V^i(t) > 0$ for all $t > 0$. We also know that $K_V^i(t) - K_V^{*i} = (1 - f)S_V^i(t)$ where $f \in [0, 1]$. Finally, we know that the volatility adjustment to the capital stock in the deterministic steady state will be zero, as we assume that there is no volatile income at this point, $K_V^{*i} = 0$. So, a volatile oil windfall should increase the stock of capital, $K_V^i(t) > 0$.

Appendix 7: Proof of Principle 7

i) If a capital scarce country has no domestic private capital market, the government should promote its development to increase the productivity of public investment.

The optimal allocation of a fixed quantity of any factor of production between two sectors is to equate its marginal product in each sector. Consider the problem of allocating a fixed amount of public and private capital and labour across two sectors, $j = N, T$, to maximise revenues,

$$\begin{aligned} \max_{K_{G,j}, K_{P,j}, L_j} \quad & \sum_{j=N,T} P_j A_j K_{G,j}^{\gamma_G} K_{P,j}^{\gamma_P} L_j^{(1-\gamma_G-\gamma_P)} \\ \text{s.t.} \quad & \\ 1 \quad & = \quad K_{G,N} + K_{G,T} \\ 1 \quad & = \quad K_{P,N} + K_{P,T} \\ 1 \quad & = \quad L_N + L_T \end{aligned}$$

The optimal allocation of every factor will equate its marginal product in each sector. For private capital this gives us $\gamma_P P_N Y_N / K_{P,N} = \gamma_P P_T Y_T / K_{P,T}$. This maximises revenues, and in turn the productivity of public investment.

If a country does not have a domestic capital market, then there is nothing to guarantee that the marginal product of private capital in each sector will be equal. Firms in each sector will only be able to invest from retained earnings. The decision to invest or save would be based on equating the marginal product of capital with the marginal utility of

consumption by the owner of the firm, based on a condition similar to $U'(C)e^{-\rho t} = J_K$ from equations A.1 and A.2. The amount of investment would therefore be based on the existing capital stock in each sector.

However, if a domestic market for private capital can be established, then private capital will be allocated efficiently. Consider the profit maximisation problem of each firm in the traded and non-traded sector, assuming that the government is allocating public capital efficiently, $\gamma_G P_j Y_j / K_{G,j} = r_G$

$$\max_{K_{P,j}, L_j} \pi_j = P_j Y_j - r_G K_{G,j} - r_P K_{P,j} - w L_j$$

This gives the first order condition for each firm, $\gamma_P P_j Y_j / K_{P,j} = r_P$. If there is a well-functioning domestic capital market then both firms will face the same r_P , so capital will be allocated to equate its marginal product in each sector.

ii) If the country has a private capital market with limited access to foreign capital, then the government should “top-up” private capital by lending through the domestic banking sector.

We begin with production before oil is discovered, and assume that public and private capital are allocated efficiently. Their marginal product will equal their respective costs of borrowing, for $j = N, T$,

$$\begin{aligned} \gamma_G P_j Y_j / K_{G,j} &= r - \omega_G F \\ \gamma_P P_j Y_j / K_{P,j} &= r - \omega_P F \end{aligned}$$

Now consider production after oil is discovered. As noted in principle 4, at any point in time the government will have more capital to allocate between each sector, $K' = K'_N + K'_T$, and will seek to maximise output,

$$\begin{aligned} \max_{K'_{G,j}, K'_{P,j}} \quad & \sum_{j=N,T} P_j A_j (K_{G,j} + K'_{G,j})^{\gamma_G} (K_{P,j} + K'_{P,j} + K''_{P,j})^{\gamma_P} L_j^{(1-\gamma_G-\gamma_P)} \\ \text{s.t.} \quad & \\ K' &= K'_{G,N} + K'_{G,T} + K'_{P,N} + K'_{P,T} \end{aligned}$$

Investment by the public sector will increase the productivity of private capital, so there will be some co-investment by the private sector, $K''_{P,j}$. The private sector will allocate its capital, $K''_{P,j}$, so that its marginal product equals the cost of borrowing. The government will then “top-up” private capital to equate its marginal product with that of public capital. So,

$$\begin{aligned} \gamma_P P_j Y_j / (K_{P,j} + K''_{P,j}) &= r_P \\ \gamma_P P_j Y_j / (K_{P,j} + K''_{P,j} + K'_{P,j}) &= r_G \\ \gamma_G P_j Y_j / (K_{G,j} + (K'_j - K'_{P,j})) &= r_G \end{aligned}$$

Combining the first and last gives $K''_{P,j} = \frac{r_G/\gamma_G}{r_P/\gamma_P}(K_{G,j} + K'_{G,j}) - K_{P,j}$, so private co-investment is increasing in public investment. Combining all three gives,

$$K'_{P,j} = \frac{(r_P - r_G)\gamma_P}{(r_P - r_G)\gamma_P + r_P\gamma_G}(K_{G,j} + K'_j)$$

So, if $r_P = r_G$ then the additional investment by the government in the private sector is zero. As capital scarcity in the private sector increases relative to the public sector, $r_P > r_G$, the optimal government “top-up” to the private sector increases.

iii) If the country has a private capital market and ready access to foreign investment, then the government should invest relatively more in public capital.

Let us now assume that the resource exporter has a well functioning domestic capital market, with foreign direct investment that is financed at the world rate of interest, r .

In this setting there will be more investment in private capital. As the cost of private capital is lower, $r < r - \omega_G F$ for $F < 0$, its marginal product in each sector will also be lower. As there are diminishing returns to private capital, this requires the level of private capital to be higher. Thus, there will be more private capital if it can be financed cheaply at the world interest rate.

More private capital increases the marginal product of public capital. The marginal product of public capital is, $MPK_{G,j} = \gamma_G P_j A_j K_{G,j}^{\gamma_G-1} K_{P,j}^{\gamma_P} L_j^{1-\gamma_G-\gamma_P}$. Thus, a higher stock of private capital, $K'_{P,j} > K_{P,j}$, will increase the marginal product of public capital, $MPK'_{G,j} > MPK_{G,j}$.

If the government can borrow at a particular rate of interest, then a higher marginal product of public capital will require more investment in public capital. The government can borrow at the rate of interest, $r_G = r - \omega_G F$. For a given rate of interest and two different levels of private capital, $\gamma_G P_j A_j K_{G,j}^{\gamma_G-1} K_{P,j}^{\gamma_P} L_j^{1-\gamma_G-\gamma_P} = \gamma_G P_j A_j K_{G,j}^{\gamma_G-1} K_{P,j}^{\gamma_P} L_j^{1-\gamma_G-\gamma_P} = r_G$, then if $K'_{P,j} > K_{P,j}$, $K'_{G,j} > K_{G,j}$ as $\gamma_G \in [0, 1]$.

iv) Investment in capital scarce countries should allow the real exchange rate to steadily appreciate as borrowing constraints are relaxed and capital is accumulated, according to the Rybczynski theorem.

Let us return to the setting in part ii) of this principle, where public and private capital at any point in time are financed at the respective rates of interest, $r_G(F(t)) < r_P(F(t))$. Following the reasoning in Lemma 3, public and private capital will accompany labour as it moves between sectors,

$$K_{G,j} = wL_j/P_j \frac{\gamma_G/r_G(F(t))}{(1 - \gamma_G - \gamma_P)} \quad (\text{A.42})$$

$$K_{P,j} = wL_j/P_j \frac{\gamma_P/r_P(F(t))}{(1 - \gamma_G - \gamma_P)} \quad (\text{A.43})$$

Output in each sector will be,

$$Y_j = A_j L_j b(F(t)) (w/P_j)^{\gamma_1 + \gamma_2} \quad (\text{A.44})$$

where $b(F(t)) = (\gamma_G/r_G(F(t)))^{\gamma_G} (\gamma_P/r_P(F(t)))^{\gamma_P} (1 - \gamma_G - \gamma_P)^{-\gamma_G - \gamma_P}$. In addition we know that the marginal product of labour equals the wage, $(1 - \gamma_G - \gamma_P)P_j Y_j/L_j = w$. Combining this with equation A.44 gives,

$$w/P_j = c_j(F(t)) \quad (\text{A.45})$$

where $c_j(F(t)) = (A_j (\gamma_G/r_G(F(t)))^{\gamma_G} (\gamma_P/r_P(F(t)))^{\gamma_P})^{1/(1-\gamma_G-\gamma_P)} (1 - \gamma_G - \gamma_P)$, and,

$$c'_j(F) = c_j(F) \left(\frac{-\gamma_G r'_G(F)}{(1 - \gamma_G - \gamma_P) r_G(F)} + \frac{-\gamma_P r'_P(F)}{(1 - \gamma_G - \gamma_P) r_P(F)} \right) \quad (\text{A.46})$$

In the traded sector $P_T = 1$, so at any point in time the wage will be a function of the level of foreign assets, via the cost of borrowing, $w^*(F(t)) = c_T(F(t))$. Over time, as the cost of borrowing falls there will be more capital employed in the traded sector. The marginal product of labour will therefore rise, and the wage will rise with it.

In the non-traded sector, $P_N^*(F(t)) = c_T(F(t))/c_N(F(t))$. If the production technology in each sector is the same, then $P_N^* = 1$ for any value of $F(t)$, and the real exchange rate should stay constant. However, if the production technology differs then $P_N^*(F(t))$ will change as foreign debt is repaid, the cost of borrowing falls and capital accumulates. If the traded sector is more capital intensive, $\gamma_{j,T} > \gamma_{j,N}$ for $j = N, T$, then the price of the non-traded good will rise as debt is repaid. This is a real appreciation, and follows from the Rybczynski (1955) theorem.