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**THIRD-DEGREE PRICE DISCRIMINATION AND  
CONSUMER SURPLUS**

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# Third-Degree Price Discrimination and Consumer Surplus

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Abstract

This paper presents simple conditions for monopoly third-degree price discrimination to have negative or positive effects on aggregate consumer surplus. Consumer surplus is often reduced by discrimination, for example when total welfare (consumer surplus and profits) falls. Surplus increases with discrimination, however, in two cases: first, when the marginal revenues without discrimination are close together and inverse demand in the market where the price will fall with discrimination is more convex; second, when inverse demand functions are highly convex and the discriminatory prices are close together.

*Keywords:* third-degree price discrimination, monopoly, consumer surplus.

*JEL Classification:* D42, L12, L13.

## 1 Introduction

How are consumers affected by third-degree price discrimination by a monopolist? Total welfare, defined as consumer surplus plus profits, may rise or fall with discrimination. Aguirre, Cowan and Vickers (2010) present sufficient conditions for both possibilities. There are several reasons to consider

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the effect on consumer surplus on its own. Anti-trust agencies sometimes use consumer surplus, rather than total welfare, as the standard. The monopolist might be owned by foreigners, so its profits would normally be excluded from the measure of domestic welfare. Consumer organizations have a natural interest in the effect of discrimination. If discrimination can be shown to raise aggregate consumer surplus then there is a strong presumption in its favour, because this ensures that total welfare will rise.

The effect of discrimination on consumer surplus is analyzed here using a standard model. For simplicity, and without loss of generality, there are two markets. The firm supplies a positive quantity in each market when discrimination is not feasible, so discrimination does not open a new market. When discrimination is allowed (or becomes feasible) the price falls in one market and rises in the other, so consumers in the former gain while those in the latter lose. Demand in each market is independent of the price in the other market and marginal cost is constant.

One result is immediate. When discrimination reduces total welfare, consumer surplus must also fall (since profits increase when discrimination is allowed). A sufficient condition for total welfare to fall with discrimination is that total output does not increase. An example is when the demand functions are linear (Pigou, 1920). Some nonlinearity of demand and an increase in total output are necessary for surplus to rise. The curvature of demand - a measure of convexity analogous to relative risk aversion - plays a key role in the analysis of the effects of discrimination. Aguirre, Cowan and Vickers (2010) provide general curvature-based conditions for discrimination to reduce total welfare. It is natural to expect that discrimination will normally reduce consumer surplus.

Two analytical approaches are used. In the first the firm is modelled as choosing its quantities subject to a constraint on how far they may vary from the levels supplied without discrimination. Initially the firm sets the quantities associated with the non-discriminatory price. As the constraint is

relaxed the marginal effect on aggregate surplus can be determined. Surprisingly the main result from this approach is a positive one. For a broad set of demand functions, including those with constant curvature, if the effect of allowing a small amount of discrimination is positive then each additional increase in discrimination will also raise surplus. In general extra output raises consumer surplus by the difference between marginal utility and marginal expenditure, which equals price less the monopolist's marginal revenue. Full discrimination raises consumer surplus if, at the non-discriminatory price, the price-marginal revenue differences are close together and inverse-demand curvature (or convexity) is higher in the market where discrimination will reduce the price.

In the second approach the firm chooses how much consumer surplus to give to consumers in each market subject to a constraint on the variation of the surpluses from their levels without discrimination (and subject to the requirement that it uses linear pricing). This yields both a positive and a negative result: if (i) the demand curvatures are above 1.5, so demands are very convex, and (ii) the discriminatory prices are sufficiently close, then consumer surplus rises, while the opposite happens if both (i) and (ii) do not hold. Shi, Mai and Liu (1988) prove that if the curvature of demand is common to the markets and exceeds zero discrimination raises total output. Aguirre, Cowan and Vickers (2010) show that discrimination raises total welfare if common curvature is above unity (provided that discriminatory prices are close). The result here completes the story: the critical values of common demand curvature for positive effects on output, welfare and consumer surplus are zero, unity, and 1.5 respectively.

Section 2 presents a graphical example to illustrate the role of the difference in demand curvatures. Section 3 sets up the model of pricing with and without discrimination. Section 4 contains the analysis of the quantity-restriction technique, yielding the positive result that stresses the role of the difference in demand curvatures. Section 5 uses the technique based on

limiting surplus-variation to find results that emphasize the level of demand curvature. Conclusions are in Section 6.

## 2 Example

Standard monopoly theory and existing results on price discrimination are used to illustrate graphically how the difference in demand curvature determines the effect of discrimination on consumer surplus. Both markets have linear demand functions initially, and both are served when there is no discrimination. Total output is the same with and without discrimination, by Pigou's 1920 result, so both total welfare and aggregate consumer surplus fall with discrimination. Now take a concave transformation of demand in the market with the lower price elasticity (so this is the market where the discriminatory price will be high). Figure 1 illustrates this market with two alternative demands and their associated marginal revenues. The new demand curve,  $p_2(q)$ , is tangential to the old one,  $p_1(q)$ , at the non-discriminatory price and quantity ( $\bar{p}$  and  $\bar{q}$ ). The marginal revenue curves,  $MR_1$  and  $MR_2$ , intersect at the non-discriminatory quantity because of the tangency of the demands - the price, quantity and demand slope are the same for both demands at this quantity. Marginal cost,  $c$ , is zero. The non-discriminatory price and the discriminatory outcome in the other, low-price, market are unaffected by this transformation.

Joan Robinson (1933) showed graphically that if demand in the high-price market is strictly concave and that in the low-price market is linear then total output is higher with discrimination than without.<sup>2</sup> This implies that the discriminatory quantity for the new demand,  $q_2$ , is higher than that for the old demand,  $q_1$ . Both are determined by the usual marginal revenue equals marginal cost condition. The transformed (inverse) demand function

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<sup>2</sup>Joan Robinson's argument was a local one. A formal proof of this result that applies globally was provided by Schmalensee (1981). See also Aguirre, Cowan and Vickers (2010, Proposition 4).

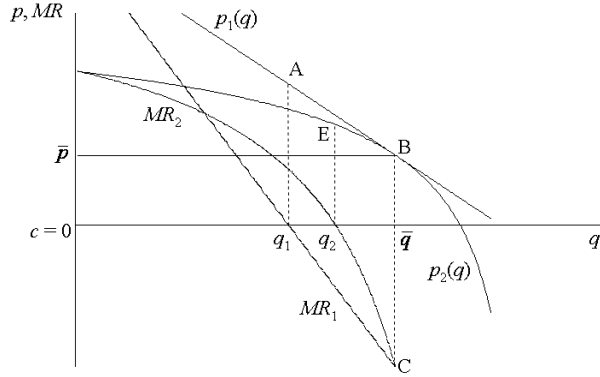


Figure 1: Increased concavity in the high-price market

has constant curvature, with  $\frac{-qp_2''(q)}{p_2'(q)}$  being independent of  $q$ . This ensures that in between  $q_2$  and  $\bar{q}$  marginal revenue for the new demand is above that of the old demand.<sup>3</sup>

An infinitesimal rise in output increases consumer surplus by marginal utility minus marginal expenditure, i.e. the price less marginal revenue. The effect of a discrete change is the integral of the price-marginal revenue differences over the relevant quantities. Thus the loss of surplus in this market from the quantity reduction that discrimination induces is  $EBCq_2$  with the new demand. This is smaller than the loss of surplus with the linear demand of  $ABCq_1$ , both because  $q_2$  is higher than  $q_1$  and because the price-marginal revenue difference is lower with the transformed demand in this region.

The concave transformation thus reduces the loss of surplus in the high-

<sup>3</sup>With constant curvature the new marginal revenue curve is strictly concave, because it inherits the same curvature as its demand function.

price market without affecting the increase in surplus in the low-price market, and so makes it more likely that total surplus will rise. As the degree of concavity increases  $q_2$  moves closer to  $\bar{q}$  and eventually demand becomes rectangular (demand equals  $\bar{q}$  for  $p \leq \bar{p}$  and zero otherwise). At this point there is no loss in surplus in the high-price market, because the discriminatory price in the high-price market is just the non-discriminatory price,  $\bar{p}$ . In between the cases of linear demand (zero concavity) and rectangular demand (infinite concavity) the effect of discrimination on total consumer surplus changes from negative to positive.

### 3 Pricing with and without discrimination

Direct utility functions are quasi-linear. Consumer surplus in a market, as a function of the price,  $p$ , is  $v(p)$ . Without loss of generality there are two markets, labelled initially 1 and 2. (Market subscripts are used only when necessary.) Roy's Identity for quasi-linear utility gives  $v'(p) = -q(p)$  where  $q(p)$  is demand. At the margin a price increase causes a loss of surplus equal to the additional cost of buying the original quantity. Demand is twice-differentiable and strictly decreasing in the price. Inverse demand is  $p(q)$ . The price elasticity of demand is  $\eta(q) = -\frac{p}{qp'(q)}$  and demand curvature, the elasticity of the slope of inverse demand, is  $\sigma(q) = -\frac{qp''(q)}{p'(q)}$ . A positive value of  $\sigma$  means that demand is strictly convex.<sup>4</sup> Marginal revenue,  $MR(q) = p(q) + qp'(q)$ , is assumed to be decreasing in output, which holds if and only if  $2 - \sigma > 0$ . Demand must not be too convex.

Marginal cost,  $c \geq 0$ , is constant and demand is positive and finite when price equals marginal cost. Profit is  $\Pi(q) = (p(q) - c)q$  and  $\Pi''(q) = p'(q)(2 - \sigma) < 0$ . The marginal revenue equals marginal cost condition,  $MR(q^*) = c$ , implicitly defines the unique discriminatory quantity,  $q^*$ , and the associated

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<sup>4</sup>In this paper "demand curvature" means  $\sigma$ , i.e. the curvature of inverse demand, rather than the curvature of direct demand (which may be defined as  $\sigma\eta$ ).

price  $p^* = p(q^*)$ , where the star denotes the discriminatory outcome. The Lerner index is  $L \equiv \frac{p-c}{p}$ , and with discrimination  $L(p^*)\eta(p^*) = 1$ . Profit, as a function of price,  $\pi(p) = \Pi(q(p))$ , is single-peaked with an interior maximum at the discriminatory price.<sup>5</sup> When the same price is set in the two markets the aggregate profit function,  $\pi_1(p) + \pi_2(p)$ , might not be single-peaked if the individual demands are convex. Theorem 1 of Nahata, Ostaszewski and Sahoo (1990), however, states that if each  $\pi_i(p)$  has an interior single-peak the non-discriminatory price,  $\bar{p}$ , is bounded above and below by the discriminatory prices in the two markets. If not and the price was, say, above the higher discriminatory price then a price reduction would raise profits in each market and the original price would not have maximized aggregate profits.

At the non-discriminatory price,  $\bar{p}$ , the first-order condition

$$\pi'_1(\bar{p}) + \pi'_2(\bar{p}) = \frac{\Pi'_1(\bar{q}_1)}{p'_1(\bar{q}_1)} + \frac{\Pi'_2(\bar{q}_2)}{p'_2(\bar{q}_2)} = 0 \quad (1)$$

holds, where the second expression uses the non-discriminatory quantities  $\{\bar{q}_1, \bar{q}_2\}$ . It is assumed that at the non-discriminatory price both markets are served with positive quantities so new markets are not opened by price discrimination. Since  $\pi'_i(\bar{p}) = \bar{q}_i[1 - L(\bar{p})\eta_i(\bar{p})]$  the market where discrimination will cause the price to fall is the one with the higher price elasticity at the non-discriminatory price.<sup>6</sup> Joan Robinson (1933) introduced the terminology that the market with the low discriminatory price is weak, and the high-price market is strong. From now on the subscripts  $w$  and  $s$  are used where necessary to denote the weak and strong markets respectively, with prices  $p_s^* > \bar{p} > p_w^*$ .

Consumer surplus as a function of quantity is  $v(p(q))$  and  $\frac{d}{dq}v(p(q)) =$

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<sup>5</sup>This follows because  $\pi'(p^*) = \Pi'(q^*)q'(p^*) = 0$  and so  $\pi''(p^*) = \Pi''(q^*)(q'(p^*))^2 < 0$ .

<sup>6</sup>A parallel issue to the one addressed here is how large is the increase in profits induced by allowing discrimination. The upper bound developed by Malueg and Snyder (2006) implies that profit (excluding any fixed cost) will double at most with discrimination in the current model. Beard and Stern (2008) show that an upper bound for consumer surplus with monopoly pricing exists when there is a choke price at which demand is zero.

$-qp'(q) = p - MR(q)$ , the price-marginal revenue difference.<sup>7</sup> From (1)  $MR_w(\bar{q}_w) > c > MR_s(\bar{q}_s)$ , which is why the firm raises quantity in the weak market and cuts quantity in the strong market when it is able to do so. For small changes in quantities,  $dq_w$  and  $dq_s$ , the change in aggregate consumer surplus from the no-discrimination position is  $[\bar{p} - MR_w(\bar{q}_w)]dq_w + [\bar{p} - MR_s(\bar{q}_s)]dq_s$ . Since  $\bar{p} - MR_s(\bar{q}_s) > \bar{p} - MR_w(\bar{q}_w)$  the increase in output in the weak market must exceed the reduction in output in the strong market for surplus to rise. Using  $\bar{p} - MR_i(\bar{q}_i) = \frac{\bar{p}}{\eta_i(\bar{p})}$ , the condition for surplus to rise locally is that the ratio of the output changes exceeds the ratio of the price elasticities:  $\frac{dq_w}{-dq_s} > \frac{\eta_w(\bar{p})}{\eta_s(\bar{p})}$ . To say something about the global effect of discrimination on surplus requires knowledge of the demand curvatures as well as the elasticities.

## 4 The difference in demand curvatures

The first method used is the one introduced in Section IV of Aguirre, Cowan and Vickers (2010) to consider the total welfare effect. This is an adaptation of the price-restriction approach of Schmalensee (1981) and Holmes (1989).<sup>8</sup> The firm chooses its quantities to maximize aggregate profits,  $\sum_{i \in \{w,s\}} \Pi_i(q_i)$ , subject to the linear constraint  $\sum \Pi'_i(\bar{q}_i)(q_i - \bar{q}_i) \leq t$ , where  $t \geq 0$  is the amount of quantity-variation or discrimination allowed. The first-order condition for market  $i$  is  $\Pi'_i(q_i) = \lambda \Pi'_i(\bar{q}_i)$  where  $\lambda \geq 0$  is the multiplier. When  $t = 0$  the non-discriminatory quantities are set. As the constraint is relaxed the change in quantity is

$$q'_i(t) = \lambda'(t) \frac{\Pi'_i(\bar{q}_i)}{\Pi''_i(\bar{q}_i)} = \lambda'(t) \frac{\Pi'_i(\bar{q}_i)}{p'_i(\bar{q}_i)(2 - \sigma_i)} \quad (2)$$

<sup>7</sup>Bulow and Klemperer (2009) use this relationship to analyze the effect of price controls on consumer surplus when there is proportional rationing.

<sup>8</sup>The price-restriction technique can be used to derive some, but not all, of the results in this section (Example 1 and Proposition 2).

which has the same sign as  $\Pi'_i(\bar{q}_i)$  since  $\lambda'(t)$  is negative.<sup>9</sup> Equation (2) confirms that the quantity rises if marginal revenue exceeds marginal cost at the non-discriminatory quantity. Define  $t^*$  as full discrimination, so  $\lambda(t^*) = 0$  since the constraint does not bind (just).

Aggregate consumer surplus is  $V(t) \equiv \sum v_i(p_i(q_i(t)))$ . The effect on consumer surplus of more discrimination is  $V'(t) = \sum (p_i - MR_i)q'_i(t) = -\sum q_i p'_i(q_i)q'_i(t)$ . Using (2) this is

$$V'(t) = -\lambda'(t) \sum \frac{q_i}{2 - \sigma_i} \Pi'_i(\bar{q}_i), \quad (3)$$

which has the same sign as  $y(t) \equiv \sum \frac{q_i}{2 - \sigma_i} \Pi'_i(\bar{q}_i)$ . Assume now:

**The quantity-curvature condition.**  $\frac{q}{2 - \sigma(q)}$  is increasing in  $q$ .

This holds when  $\sigma$  is constant (special cases are constant-elasticity demand, log-linear inverse demand and linear demand), and when  $\sigma(q)$  is increasing. The condition implies two monotonicity results that are central to the analysis: (i) if  $V'(0) \geq 0$  then  $V'(t) > 0$  for  $0 < t \leq t^*$ ; and (ii) if  $V'(t^*) \leq 0$  then  $V'(t) < 0$  for  $0 \leq t < t^*$  (where  $V'(t^*)$  is the left-derivative). There are two steps involved in proving these two results. First, the quantity-curvature condition and the fact that  $q'_i(t)\Pi'_i(\bar{q}_i) > 0$  together imply  $y'(t) = \sum \frac{d}{dq_i} \left( \frac{q_i}{2 - \sigma_i} \right) q'_i(t)\Pi'_i(\bar{q}_i) > 0$ . Second, if  $V(t)$  has a stationary point at  $\tau$ , so  $V'(\tau) = -\lambda'(\tau)y(\tau) = 0$  and thus  $y(\tau) = 0$ , then  $V''(\tau) = -\lambda''(\tau)y(\tau) - \lambda'(\tau)y'(\tau) = -\lambda'(\tau)y'(\tau) > 0$ . This eliminates the possibility that surplus first rises and then falls with the amount of discrimination. Thus if surplus is constant or rising at  $t = 0$  then it continues to rise, and if surplus is constant or falling at  $t^*$  then it decreases everywhere as  $t$  goes from 0 to  $t^*$ .

At the non-discriminatory price the marginal effect on the quantity in

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<sup>9</sup>Differentiating the constraint and using (2) gives  $\lambda'(t) = \left[ \sum \frac{[\Pi'_i(\bar{q}_i)]^2}{\Pi'_i(q_i)} \right]^{-1} < 0$ .

market  $i$  is  $q'_i(0) = \lambda'(0) \frac{\Pi'_i(\bar{q}_i)}{p'_i(\bar{q}_i)} \frac{1}{(2-\sigma_i)}$ . Using (1), the effect on aggregate surplus at the margin,  $V'(0) = \sum(\bar{p} - MR_i(\bar{q}_i))q'_i(0)$ , has the same sign as  $\frac{\bar{p} - MR_w(\bar{q}_w)}{2 - \sigma_w(\bar{q}_w)} - \frac{\bar{p} - MR_s(\bar{q}_s)}{2 - \sigma_s(\bar{q}_s)}$ . This gives the main result.

**Proposition 1.** *Consumer surplus increases with discrimination if (i)  $\frac{\bar{p} - MR_w(\bar{q}_w)}{2 - \sigma_w(\bar{q}_w)} \geq \frac{\bar{p} - MR_s(\bar{q}_s)}{2 - \sigma_s(\bar{q}_s)}$  (so the marginal revenues are close together and demand curvature is higher in the weak market at the non-discriminatory price) and (ii) the quantity-curvature condition holds.*

Condition (i) states that at the margin the quantity changes, which are proportional to  $\frac{1}{2-\sigma_i}$ , offset the effect of the smaller price-marginal revenue difference in the weak market. The quantity-curvature condition then ensures that surplus continues to rise. Condition (i) is equivalent to  $\eta_s(\bar{p})(2 - \sigma_s(\bar{q}_s)) \geq \eta_w(\bar{p})(2 - \sigma_w(\bar{q}_w))$ , which is in a form that might be testable empirically since it depends only the elasticities and curvatures at the non-discriminatory price. The consumer surplus effect is more likely to be positive the closer are the price elasticities (equivalently, the closer are the marginal revenues) and the greater the difference between the curvatures. If  $\sigma_s \geq \sigma_w$ , for example when both demands have constant elasticities or both are linear, condition (i) is not satisfied.<sup>10</sup>

To illustrate Proposition 1 suppose that market 1 has a linear demand,  $p_1 = a - q$  for  $a > c$ , and market 2 has a log-linear inverse demand,  $p_2 = A - b \log(q)$  with  $b > 0$ . The curvatures are  $\sigma_1 = 0$  and  $\sigma_2 = 1$  and the elasticities are  $\eta_1 = \frac{p}{a-p}$  and  $\eta_2 = \frac{p}{b}$ . Proposition 1 applies if, at the non-discriminatory price,  $\eta_2(\bar{p}) > \eta_1(\bar{p}) \geq \frac{\eta_2(\bar{p})}{2}$  with the first inequality being the condition for the log-linear market to be the weak one, and the second being the condition for  $V'(0)$  to be non-negative.

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<sup>10</sup>The constant-elasticity inverse demand function is  $p = Aq^{-1/\eta}$  with  $\eta > 1$ . This has  $\sigma = 1 + 1/\eta$ , which is lower in the weak market.

Aguirre, Cowan and Vickers (2010) give two alternative sets of conditions for total welfare to be higher with discrimination. In the first the discriminatory prices are close together with higher curvature in the weak market. In the second demand curvatures are constant and above unity. Jointly these conditions ensure that consumer surplus rises because they imply that both conditions in Proposition 1 hold. This is shown in the following example.

*Example 1.* Suppose that the discriminatory prices are close together with  $\frac{p_w^* - c}{2 - \sigma_w^*} \geq \frac{p_s^* - c}{2 - \sigma_s^*}$  (so  $\sigma_w > \sigma_s$ ), and both demand curvatures are constant with  $\sigma_i \geq 1$ . Thus  $\sigma_w > \sigma_s \geq 1$  and  $\frac{d}{dp}[p - MR(q(p))] = \sigma - 1$  is positive in the weak market and non-negative in the strong market. It follows that  $\bar{p} - MR_w(\bar{q}_w) > p_w^* - MR_w(q_w^*) = p_w^* - c$  and  $p_s^* - c = p_s^* - MR_s(q_s^*) \geq \bar{p} - MR_s(\bar{q}_s)$ . Since the curvatures are constant both conditions in Proposition 1 hold.

A negative result for surplus is obtained using the fact that if  $V'(t^*) \leq 0$  then  $V'(t) < 0$  for all  $t < t^*$ .

**Proposition 2.** *Consumer surplus falls with discrimination if (i) demand functions are concave and the quantity-curvature condition holds, and (ii)  $\frac{p_w^* - c}{2 - \sigma_w^*} \leq \frac{p_s^* - c}{2 - \sigma_s^*}$ .*

*Proof.* The quantity-curvature condition implies that if  $y(t^*) \leq 0$  then  $y(t) < 0$ . Using the first-order condition for discriminatory pricing gives  $y(t^*) = \pi'_s(\bar{p}) \left[ \left( \frac{p_w^* - c}{2 - \sigma_w^*} \right) \frac{p'_w(\bar{q}_w)}{p'_w(q_w)} - \left( \frac{p_s^* - c}{2 - \sigma_s^*} \right) \frac{p'_s(\bar{q}_s)}{p'_s(q_s)} \right]$ . Concavity of demand implies  $\frac{p'_w(\bar{q}_w)}{p'_w(q_w)} \leq 1$  and  $\frac{p'_s(\bar{q}_s)}{p'_s(q_s)} \geq 1$ , so with condition (ii)  $y(t^*) \leq 0$ .

An example of Proposition 2 is when  $0 \geq \sigma_s \geq \sigma_w$ , which itself implies condition (ii). A special case is when both demands are linear. Cheung and Wang (1994) show that with this ranking of the curvatures total output

does not rise. Proposition 2 is more general. If (ii) holds and demands are concave surplus falls whatever happens to output. As the example in Section 2 indicates, however, surplus can rise with concave demands if condition (ii) does not hold.

Proposition 1 states a positive result for consumer surplus that depends on the difference between the demand curvatures. Together Example 1 and Proposition 2 indicate that the level of curvature is also important for the surplus effect. To explore this further a new approach is used.

## 5 The level of demand curvature

Suppose now that the firm chooses how much surplus to leave to consumers (while still being restricted to linear pricing). Armstrong and Vickers (2001) model firms as suppliers of surplus to consumers in their analysis of competitive price discrimination. Let  $p(v)$  be the price associated with surplus  $v$ , with  $p'(v) = \frac{1}{v'(p)} = -\frac{1}{q(p)}$ . The profit function is now  $\Pi(v) = \pi(p(v))$  and  $\Pi'(v) = -\frac{\pi'(p)}{q} = L\eta - 1$ . Throughout this section it is assumed that  $\sigma$  is constant in each market.<sup>11</sup> It follows that  $\frac{d}{dp}L\eta > 0$  and that profit is strictly concave in surplus (see the Appendix, part 1). In empirical applications the Lerner index times the elasticity,  $L\eta$ , is often taken to be the measure of market power, so it is natural to have this measure rising as the price increases.

The firm maximizes  $\sum \Pi_i(v_i)$  subject to  $\sum \Pi'_i(\bar{v}_i)(v_i - \bar{v}_i) \leq r$ , where  $r \geq 0$  is the amount of discrimination and  $\bar{v}_i$  is the non-discriminatory surplus in market  $i$ . Define  $r^*$  as the amount of discrimination at which the constraint no longer binds. The first-order condition for market  $i$  is  $\Pi'_i(v_i) = \mu \Pi'_i(\bar{v}_i)$ ,

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<sup>11</sup>When demands have constant curvature the slope of inverse demand divided by the slope of marginal revenue,  $1/(2 - \sigma)$ , is constant (Bulow and Pfleiderer, 1983). This ratio also equals both the monopoly cost-passthrough coefficient and the ratio of monopoly consumer surplus to profit (see Weyl and Fabinger, 2009, for a general analysis of pass-through).

where  $\mu$  is the multiplier. The marginal effect of discrimination on surplus in market  $i$  is  $v'_i(r) = \mu'(r) \frac{\Pi'_i(\bar{v}_i)}{\Pi''_i(v_i)}$ , which has the same sign as  $\Pi'_i(\bar{v}_i)$  since  $\mu'(r) < 0$  (given  $\Pi''_i(v_i) < 0$ ). Surplus rises in the market where the marginal profitability of surplus is positive. Adding across the markets gives the marginal effect of discrimination on aggregate surplus

$$V'(r) = \mu'(r) \sum \frac{\Pi'_i(\bar{v}_i)}{\Pi''_i(v_i)} \quad (4)$$

which has the sign of  $z(r) \equiv -\sum \frac{\Pi'_i(\bar{v}_i)}{\Pi''_i(v_i)}$ . To determine the sign use  $z(r) = z(r^*) - \int_r^{r^*} z'(u) du$ . Sufficient conditions for  $z(r) > 0$ , so discrimination are that  $z(r^*) \geq 0$  and  $z'(r) < 0$ , and sufficient for  $z(r) < 0$  are  $z(r^*) \leq 0$  and  $z'(r) > 0$ . The first entails that discrimination raises surplus, while the second gives a negative result for surplus.

**Proposition 3.** (a) *If (i) demand curvatures are constant and above 1.5, and (ii)  $\frac{(p_w^* - c) q_w^*}{(2 - \sigma_w^*) \bar{q}_w} \geq \frac{(p_s^* - c) q_s^*}{(2 - \sigma_s^*) \bar{q}_s}$ , then consumer surplus rises with discrimination.* (b) *If demand curvatures are constant and below 1.5, and  $\frac{(p_w^* - c) q_w^*}{(2 - \sigma_w^*) \bar{q}_w} \leq \frac{(p_s^* - c) q_s^*}{(2 - \sigma_s^*) \bar{q}_s}$ , then consumer surplus falls with discrimination.*

Proof. See the Appendix, part 2.

Condition (ii) in part (a) ensures that  $V'(r^*) \geq 0$ . It says that the discriminatory prices are fairly close to each other. If  $\frac{p_w^* - c}{2 - \sigma_w^*} \geq \frac{p_s^* - c}{2 - \sigma_s^*}$  then (ii) holds, but (ii) may hold otherwise and in particular when  $\sigma_w = \sigma_s$ . Part (a) of the proposition means that there is a trio of positive results when curvature is common. Output rises if  $\sigma > 0$  (Shih, Mai and Liu, 1988). Welfare rises if  $\sigma > 1$  and the discriminatory prices are not far apart (Aguirre, Cowan and Vickers, 2010). Part (a) shows that consumer surplus rises if  $\sigma > 1.5$  and the discriminatory prices are fairly close. Part (b) of the proposition, however, indicates that often consumer surplus falls. When demands are concave

part (b) is less general than Proposition 2, because  $\frac{(p_w^*-c) q_w^*}{(2-\sigma_w^*) \bar{q}_w} \leq \frac{(p_s^*-c) q_s^*}{(2-\sigma_s^*) \bar{q}_s}$  implies  $\frac{p_w^*-c}{2-\sigma_w^*} < \frac{p_s^*-c}{2-\sigma_s^*}$  and constant curvature is a stronger assumption than the quantity-curvature condition. So the value-added of part (b) is when demands are convex.

Proposition 3 may not be useful in practice if initially there is no discrimination, because it depends on knowledge of the discriminatory prices. It may, however, be useful if the starting point is discrimination and a ban is proposed.

## 6 Conclusion

This paper has presented conditions for consumer surplus to rise or fall with third-degree price discrimination. While surplus may rise, in many cases discrimination will reduce consumer surplus. This is to be expected since surplus rises only when total welfare increases. Consumer surplus rises either with sufficiently high curvature of demand or, perhaps more plausibly, when there is a large difference in the curvatures across markets.

# Appendix

## 1. Proof that $L\eta$ rises with the price when curvature is constant.

In general  $\frac{d}{dp}L\eta = \frac{\eta}{p}[1 + L\eta(1 - \sigma)]$ . This is positive for  $\sigma \leq 1$ . To show that it is also positive for constant  $\sigma > 1$  take the generic inverse demand  $p = a + b\frac{q^{1-\sigma}}{\sigma-1}$  with  $b > 0$  and  $\sigma > 1$ . Given the assumption that demand is positive and finite when price equals marginal cost  $c = a + b\frac{q(c)^{1-\sigma}}{\sigma-1} > a$ . For all prices  $\eta = \frac{pq^{\sigma-1}}{b}$  so  $1 + L\eta(1 - \sigma) = \frac{(c-a)(\sigma-1)q^{\sigma-1}}{b} > 0$ .

## 2. Proof of Proposition 3.

The effect of more discrimination on  $z(r) \equiv -\sum \frac{\Pi'_i(\bar{v}_i)}{\Pi''_i(v_i)}$  is

$$z'(r) = \sum \frac{\Pi'''_i(v_i)}{[\Pi''_i(v_i)]^2} \Pi'_i(\bar{v}_i) v'_i(r).$$

Given  $\Pi'_i(\bar{v}_i) v'_i(r) > 0$ , if  $\Pi'''_i(v_i) < 0$  in both markets then  $z'(r) < 0$  and if both  $\Pi'''_i(v_i)$  are positive  $z'(r)$  is positive. Differentiating  $\Pi''(v) = -\frac{\eta}{pq}[1 + L\eta(1 - \sigma)]$  with respect to  $v$ , and using  $\frac{p}{\eta} = p - MR$ , gives

$$\Pi'''(v) = \frac{[1 + L\eta(1 - \sigma)]}{(p - MR)^2 q^2} (3 - 2\sigma),$$

so  $\Pi'''(v)$  has the sign of  $3 - 2\sigma$ . Thus  $z'(r) < 0$  if  $\sigma_i > 1.5$  in both markets, and  $z'(r) > 0$  if  $\sigma_i < 1.5$ . For the sign of  $z(r^*)$  use  $\Pi'(\bar{v}) = -\frac{\pi'(\bar{p})}{q}$  and  $\Pi''(v^*) = -\frac{(2-\sigma)}{\pi^*}$ , so  $z(r^*) = \pi'_s(\bar{p}) \left[ \frac{(p_w^* - c) q_w^*}{(2-\sigma_w^*) q_w} - \frac{(p_s^* - c) q_s^*}{(2-\sigma_s^*) q_s} \right]$ . Thus  $z(r^*) \geq 0$  in (a) and  $z(r^*) \leq 0$  in (b). Since  $z'(r) < 0$  in (a) and  $z'(r) > 0$  in (b),  $z(r) > 0$  in (a) and  $z(r) < 0$  in (b).

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