

*ISSN 1471-0498*



**DEPARTMENT OF ECONOMICS**

**DISCUSSION PAPER SERIES**

**MARGINAL COST PRICING VERSUS INSURANCE**

**Simon Cowan**

Number 102

May 2002

Manor Road Building, Oxford OX1 3UQ

# Marginal cost pricing versus insurance

Simon Cowan\*

This version: May 2002

## Abstract

The regulator of a natural monopoly that sets a two-part tariff and whose marginal cost is stochastic will generally want the price to vary less than marginal cost when the lump-sum charge in the tariff is fixed. A trade-off exists between efficient pricing and an optimal allocation of risk. Pricing at marginal cost is only optimal when the consumer's marginal utility is independent of the price. When marginal utility increases with the price the mark-up falls monotonically as marginal cost rises. The lump-sum element of the tariff should exceed the fixed cost when demand is inelastic and equals the fixed cost only with unit elasticity. The model may also be applied to optimal commodity taxation.

JEL classification: L51, D42, H21

Keywords: price risk, regulation, Ramsey pricing

\* Department of Economics, Manor Road Building, Manor Road, Oxford OX1 3UQ, UK. Tel: +44 1865 281174, fax: +44 1865 271094, e-mail: [simon.cowan@economics.ox.ac.uk](mailto:simon.cowan@economics.ox.ac.uk).

## 1. Introduction

Should prices equal marginal costs? This paper examines this old question using a model of public utility regulation where marginal cost is stochastic and the price can change after marginal cost is realized. Marginal cost pricing is feasible, because the price is flexible and lump-sum transfers are available to cover fixed costs, but the main point of the paper is that it is not generally optimal. Consumers prefer prices to be smoothed when the marginal utility of income is affected by the price. Price smoothing implies that the price-cost mark-up is negatively related to marginal cost.

For marginal cost pricing to be undesirable, when it is feasible, there must be some market failure. The failure here is the absence of market mechanisms for offsetting price risk, such as insurance contracts and futures markets. Such markets would provide consumers with the ability to shift income across states of nature in response to price changes. Unfortunately households do not have access to these markets because transactions costs are high and because players need a constant supply of credit to enable them to meet margin calls (Gilbert, 1985). Of course the regulator might be able to mimic the absent insurance and futures markets by making the lump-sum charge in the two-part tariff state-contingent. Again this does not happen in reality. Regulatory agencies typically are required to ensure that tariffs are non-discriminatory, and interpret this duty to mean that prices should reflect costs. The lump-sum charge in the two part tariff is set to cover customer-related costs such as metering and billing as well as a share of the common costs of network provision.

Many electricity utilities have fuel adjustment clauses that allow them to pass through input price increases to final customers. Such clauses are common in the United States, Mexico, Argentina and the United Kingdom. In the natural gas industry gas purchase agreements work in the same way. One infamous case where there was no pass-through of costs, however, was in the restructured electricity market in California. The crisis in the system in 2000 and 2001 was largely caused by the ban on the regulated distribution companies passing through large increases in wholesale prices to retail consumers.

Our result that marginal cost pricing is not optimal when insurance and futures markets are absent is related to the optimal income tax result of Eaton and Rosen (1980). In their model the government chooses the income tax rate and the level of lump-sum taxation. Households are uncertain about their real wages and choose their hours of work before the uncertainty is resolved. It is optimal to set a positive income

tax rate, rather than to rely exclusively on lump-sum taxation, as this provides households with some insurance against wage risk. Newbery and Stiglitz (1981, Chapter 15) ask whether a commodity market subject to supply shocks will provide Pareto-efficient outcomes in the absence of insurance markets. A necessary condition for the spot market to be efficient on its own is that the consumers' marginal utility of income is not affected by price changes. In our model this condition is necessary and sufficient for pricing at marginal cost to be optimal.

The innovation in our model is that we allow the regulator to choose how much to change the price when marginal cost shifts. In general it is optimal to allow partial pass-through of marginal costs. Complete price stabilization (as in California) is inefficient, except when demand is perfectly inelastic, and the opposite extreme of full pass-through (which holds when fuel adjustment clauses are used) is only optimal when marginal utility is independent of the price. Thus in general price should not equal marginal cost and effectively there is cross-subsidization across states of nature. We also find that under reasonable conditions the lump-sum charge in the two-part tariff should exceed the fixed costs and thus expected operating revenue should be below expected operating costs. The consumer is prepared to pay a larger lump-sum charge than is necessary to finance the fixed costs in order to have lower average prices. This result would not arise in a model without risk.

The literature that sees regulation as a principal-agent problem suggests that the price should not respond fully to changes in observed costs when there is asymmetric information about the composition of costs – see Laffont and Tirole (1993) for the definitive statement of this view. In the model discussed by Armstrong *et al.* (1994) there is a trade-off between providing insurance for the firm against risk (which requires full pass-through of observed cost changes), and giving an incentive for productive efficiency (which requires the price to be fixed). Partial pass-through is optimal in general. In our paper information is symmetric and there are thus no incentive issues. Nevertheless a similar conclusion is reached – in general it is optimal for prices not to be fully responsive to costs.

In Section 2 the attitudes of the consumer and the firm to price risk are discussed. Section 3 considers how insurance and futures markets, when available, allow the consumer to offset price risk. Section 4 contains the main analysis of price smoothing for the case where insurance and futures markets do not exist. Section 5 considers alternative applications of the model. Section 6 concludes.

## 2. Attitudes of consumers and the firm to price risk

The consumer purchases  $q$  units of the good supplied by the regulated firm and  $x$  units of the alternative good, a Hicksian composite commodity whose price,  $p_0$ , is normalized to unity. Consumption decisions are made after the unit price of the regulated good,  $p$ , is known. The consumer faces a two-part tariff for the regulated good with  $A$  being the lump-sum charge. Preferences are represented by the von-Neumann-Morgenstern indirect utility function  $V(p, m - A)$  where  $m$  is exogenous income and  $m - A$  is net income. Roy's Identity implies  $V_p = -q(p, m - A)V_m$  with subscripts denoting partial derivatives and  $q(p, m - A)$  being the uncompensated demand function. We assume non-satiation ( $V_m > 0$ ) and that the consumer is risk averse ( $V_{mm} < 0$ ) but not infinitely so. The substitution effect between  $q$  and  $x$  is assumed to be strictly negative, rather than just non-positive, to rule out the case of perfect complements. From the Slutsky equation the substitution effect is  $q_p + q.q_m$ . It follows that  $-\varepsilon + s\eta < 0$  where  $\varepsilon \equiv -pq_p/q$  is the price elasticity of uncompensated demand,  $s \equiv pq/(m - A)$  is share of net income spent on the good and  $\eta \equiv (m - A)q_m/q$  is the income elasticity of demand. We also assume that uncompensated demand is decreasing in the price ( $q_p < 0$ ), which is guaranteed if demand is not too inferior.

The attitude of the consumer to price risk depends on the sign of  $V_{pp}$ . If this is positive  $V$  is convex in  $p$  and the consumer prefers variable prices to a stable price equal to the mean, while a negative sign implies the opposite. Turnovsky *et al.* (1980) define the consumer's preference for price risk, analogously to relative risk aversion for income risk, as  $-pV_{pp}/V_p$ . This has the same sign as  $V_{pp}$ . Differentiating  $V_p = -qV_m$  with respect to  $p$  gives:

$$\frac{-pV_{pp}}{V_p} = \varepsilon - p \frac{V_{mp}}{V_m} \quad (1)$$

The right-hand side of (1) is the difference between the price elasticity of demand and the elasticity of the marginal utility of income with respect to price. The consumer prefers not to bear price risk, and instead wants to have the price stabilized at the expected value, if and only the price elasticity of marginal utility exceeds the price elasticity of demand.

The sign of the effect of a price increase on marginal utility,  $V_{mp}$ , is important. To obtain a version of the price elasticity of marginal utility in (1) using familiar concepts differentiate  $V_p = -qV_m$  with respect to  $m$  to give:

$$-p \frac{V_{mp}}{V_m} = s(\eta - r) \quad (2)$$

where  $r \equiv -(m - A)V_{mm}/V_m$  is the standard coefficient of relative risk aversion (for income risk). Equation (2) implies that the sign of  $V_{mp}$  is the opposite of the sign of  $\eta - r$ . The final version of (1) is:

$$\frac{-pV_{pp}}{V_p} = \varepsilon + s(\eta - r) \quad (3)$$

The consumer likes price risk more as the price and income elasticities of demand increase, while a rise in relative risk aversion reduces the preference for price risk. If relative risk aversion exceeds the income elasticity of demand (which occurs when  $V_{mp} > 0$ ) then a rise in the share of spending on the product reduces the preference for price risk. Thus if the product is a necessity poorer consumers may be expected to have less preference for price risk than richer consumers because the budget share of the former is higher. When the utility function is separable, i.e.  $V = f(p) + h(m - A)$ , the cross-partial derivative  $V_{mp}$  equals 0 and price risk is preferred. Rogerson (1980) shows that in this case expected consumer surplus is a valid welfare measure as long as the variability in price arises from the supply-side – see also Stennek (1999).

In general the marginal utility of income is affected by changes in the price of at least one good. To show this write utility as a function of both prices and income, i.e.  $V(p_0, p, m)$ . Recall that the indirect utility function is homogeneous of degree zero in *all* prices and income since there is no money illusion. This implies that marginal utility is homogeneous of degree  $-1$ , and applying Euler's theorem gives:

$$p_0 V_{mp_0} + p V_{mp} + m V_{mm} = -V_m.$$

Suppose that  $V_m$  is independent of both prices. This implies that  $-mV_{mm}/V_m = 1$ , so relative risk aversion equals unity. The indirect utility function takes the form  $V =$

$f(p_0, p) + \ln(m)$  and Roy's Identity then implies that the income elasticities of demand for both products must be unity and thus the indifference map is homothetic. Since we know from empirical studies of demand that preferences are not homothetic we can rule out the case that both cross-partial derivatives are zero. Thus we expect  $p_0 V_{mp_0} + p V_{mp} \neq 0$ . Note that the argument is one-way. When both cross-partial derivatives are zero tastes must be homothetic, but when tastes are homothetic the cross-partials need not be zero.

This argument does not prove that  $V_{mp} \neq 0$ , i.e. that a price increase in the regulated sector has an effect on marginal utility. More direct evidence is needed. We expect the income elasticity of demand for a utility service to be at most unity. Giuliano and Turnovsky (2000) summarize the empirical evidence about the value of relative risk aversion, and they conclude that  $r$  is likely to be in the range 2 – 5. Constantinides *et al.* (2002) use values for  $r$  of 4 and 6 in their simulations. Gilbert (1985) uses a value of  $r = 2.5$  in his estimates of the benefits of commodity price stabilization. This provides some evidence that the right-hand side of (2) is negative and thus that  $V_{mp} > 0$ .

The firm is risk neutral and its welfare is represented by expected profits. Marginal cost is a continuous random variable, denoted by  $c$ , with a known distribution function  $F(c)$  and density  $f(c)$  on positive support  $[\underline{c}, \bar{c}]$ .<sup>1</sup> The firm also incurs a fixed cost of  $K$ . We first see how the firm, without facing any risk on the cost side, would respond to volatility in the price. A risk-neutral monopoly is indifferent to variation in *profits* around the mean, but is averse to *price* risk when its profit function is concave in the price.<sup>2</sup> Equivalently the firm prefers the price to be stabilized around its arithmetic mean if  $\Pi_{pp}$  is negative. We define a measure of the firm's preference for price risk as:

$$\frac{p \Pi_{pp}}{\Pi_p} = -\varepsilon - \frac{c\varepsilon + (p-c)p\varepsilon_p}{p \left[ 1 - \left( \frac{p-c}{p} \right) \varepsilon \right]}. \quad (4)$$

The term in square brackets in the denominator is proportional to  $\Pi_p$  and we show later that it is always positive. Intuitively the regulator always sets the mark-up,  $(p -$

<sup>1</sup> The main results hold for a discrete distribution as well.

<sup>2</sup> This differs from the standard result that competitive risk-neutral firms like price variability because they can adjust the quantity produced (Oi, 1961). The difference is that here the firm is a monopoly and faces a downward-sloping demand curve.

$c)/p$ , below the unregulated profit-maximizing level  $1/\varepsilon$ . When price equals marginal cost (4) is negative as long as the price elasticity of demand is strictly negative. Another sufficient for concavity and thus for (4) to be negative is that  $\varepsilon$  is constant. A special case is when  $\varepsilon = 1$  and  $p\Pi_{pp}/\Pi_p = -2$ .

The following assumption implies that the consumer and the firm together prefer the unit price to be stable when there is no exogenous source of risk:

$$\text{Assumption 1. } \frac{-pV_{pp}}{V_p} + \frac{p\Pi_{pp}}{\Pi_p} < 0.$$

This is a weak assumption and usually it follows from more basic assumptions. For example in the special case where marginal cost pricing is optimal the sum of the price risk preference coefficients is  $-\varepsilon$  which is negative since we are ruling out Giffen goods. Another set of sufficient conditions for Assumption 1 to hold is that the elasticity is constant and  $V_{mp} \geq 0$ , so  $s(\eta - r) \leq 0$ . The assumption ensures that the pass-through coefficient has the appropriate sign and plays a role in guaranteeing that the second-order conditions are satisfied.

### 3. Insurance against price risk and future markets

Before presenting the main argument we examine what happens in the counterfactual case where the regulator is able to adjust the lump-sum charge to offset price risk while maintaining marginal cost pricing. Suppose first that the regulator can mimic a first-best insurance market. The regulator's problem is to choose  $A$  for each value of  $c$  to maximize  $E[V(c, m - A)]$  subject to the constraint that  $E[A] = K$ . The first order condition implies that the marginal utility of income,  $V_m$ , is constant across states of the world. Take the central case where  $V_{mp} > 0$ . When marginal cost and thus the price is high the regulator cuts the lump-sum charge to give the consumer compensation to offset the impact of the price rise on marginal utility, while a low realization of  $c$  would call for a rise in  $A$ . Barzel and Suen (1992) apply this type of insurance to a standard model of demand to show that Giffen goods will not be observed. Such insurance does not exist in practice. Regulators do not adjust lump-sum charges in this way and instead set lump-sum charges in the two-part tariffs to cover the relevant costs. It is implausible that a competitive market would offer this type of insurance

when the random variable against which insurance is offered is not a market price but is controlled by the regulator.

The second mechanism is a futures market, which offers an alternative, albeit imperfect, way for the consumer to hedge against price risk. Gilbert (1985) compares the roles of futures markets and price stabilization for primary commodities.<sup>3</sup> Contracts for differences in the UK electricity market, which allow companies supplying electricity to final customers to hedge the price risk inherent in the competitive wholesale market, are effectively futures contracts. Such contracts allow the holder to shift income between states. If the consumer has bought  $h$  futures contracts at a futures price of  $z$  then her net income when the spot price is  $c$  is  $(c - z)h + m - K$ . The futures contract offers more restricted hedging possibilities than first-best insurance since income is restricted to change linearly with the spot price. The holder of a futures contract receives income from the seller of the contract which is proportional to the difference between the spot price and the futures price,  $c - z$ . The consumer chooses the number of contracts,  $h$ , to maximize expected utility  $E[V(c, (c - z)h + m - K)]$ . The first order condition is  $E[V_m(c - z)] = 0$ . If the futures market is unbiased then  $z = E[c]$  and the first order condition implies that the covariance of marginal utility with the spot price is zero. As in the case of insurance the consumer would not want to hedge when  $V_{mp} = 0$ . Again such a futures market is unlikely to exist when a regulator controls the spot price – the reason they do exist in the UK market is that the generation market is competitive and unregulated.<sup>4</sup> Even when futures markets do exist they are likely to be unavailable to households because transactions costs are large and because of the credit required to meet margin calls.

#### 4. Price smoothing

A simple method of coping with price risk when insurance and futures markets are unavailable is to stabilize the price. The regulator's problem is to choose a constant  $p$  and  $A$  to maximize  $V(p, m - A)$  subject to the constraint that  $A - K + (p - E[c])q(p, m - A) = 0$ , which means that expected profits are zero. The solution is  $p = E[c]$  and  $A =$

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<sup>3</sup> Besley (1989) has a similar analysis of the consumer's demand for futures contracts.

<sup>4</sup> Borenstein (2002, pages 194-195) states that the forward market in California "never achieved sufficient volume to be considered a reliable market. The utilities purchased nearly all of their power in the Power Exchange day-ahead market."

$K$ .<sup>5</sup> Does the consumer prefer such a constant price to having it vary one-for-one with marginal cost? The answer is that the consumer is better off with this contract if and only if indirect utility is concave in  $p$ . This holds when (3) is negative and requires relative risk aversion to be sufficiently high. Otherwise the consumer prefers to the price to fluctuate with marginal cost.

Of course when consumers differ in their risk attitudes it would be desirable to allow consumers to choose between two tariffs, one with a fixed price and one with marginal cost pricing. One example of such a choice in a competitive context is the mortgage market. Households who are especially risk averse have mortgage contracts characterized by a fixed interest rate and redemption penalties (to discourage them from switching to a variable rate when the short-term rate falls below the contracted interest rate). Less risk-averse households have mortgages with variable rates which track the spot rate. Campbell and Cocco (2001) present a theoretical analysis of the choice between such mortgages and Dhillon *et al.* (1987) assess the empirical evidence on household characteristics that affect the choice.

A fixed price contract is not the best way to deal with price risk. It is preferable to relate the unit price to realized marginal cost – in other words to have some pass-through of costs. Only in the special case where demand is perfectly inelastic would it be optimal to stabilize the price fully, because price changes are identical to income changes and the consumer is averse to income risk.

We now explore in general how much pass-through is optimal. The regulator's problem is to choose  $p$  for each value of  $c$  and a constant value of  $A$  to maximize expected utility subject to the constraint that expected profits are zero. This yields a function  $p(c)$  that relates the price to observed marginal cost and an optimal value of  $A$  (which need not equal  $K$ ). The Lagrangian for the regulator's problem is:

$$\int_{\underline{c}}^{\bar{c}} [V(p, m - A)] f(c) dc + \lambda \left[ A - K + \int_{\underline{c}}^{\bar{c}} (p - c) q(p, m - A) f(c) dc \right].$$

The first order condition for the choice of  $p$  for a particular value of  $c$ , when divided by  $f(c)$ , is:

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<sup>5</sup> This is formally the same as the standard two-part pricing problem with marginal cost given by  $E(c)$ . The strictly negative substitution effect ensures that  $p = E[c]$ , the constraint then implies  $A = K$  and the second order conditions hold with Assumption 1 and with  $V_{mm} < 0$ .

$$V_p + \lambda\{q + (p - c)q_p\} = 0. \quad (5)$$

This confirms that  $\Pi_p \equiv q + (p - c)q_p > 0$  since  $V_p < 0$ . The first order condition for the choice of  $A$  is:

$$-E[V_m] + \lambda\{1 - E[(p - c)q_m]\} = 0. \quad (6)$$

An increase in  $A$  cuts expected utility and raises expected profits directly by 1, but it also impacts on profits indirectly through the income effect when  $p \neq c$ . The third equation characterizing the solution is the constraint,  $A - K + E[(p - c)q] = 0$ . As is typical in such models in public economics (see Myles, 1995, pp. 113-14) we assume that the second order conditions hold. Assumption 1 effectively ensures that there is no problem about second order conditions.

Equation (5) and Roy's Identity imply that the mark-up is:

$$\frac{p - c}{p} = \left(1 - \frac{V_m}{\lambda}\right) \frac{1}{\varepsilon}. \quad (7)$$

This is a Ramsey pricing formula since each value of  $c$  can define a separate market. The mark-up is inversely related to the price elasticity but also depends on the way marginal utility changes with  $p$ . In general  $V_m$  depends on  $p$  and the mark-up should vary with the value of  $c$ . Note that the multiplier,  $\lambda$ , is constant as  $c$  varies. From (6) the multiplier is  $\lambda = E[V_m] / \{1 - E[(p - c)q_m]\}$ .

The first step is to show that the pass-through coefficient is positive. Differentiating (5) gives the slope of  $p(c)$ :

$$p'(c) = \frac{pq_p / \Pi_p}{\frac{-pV_{pp}}{V_p} + \frac{p\Pi_{pp}}{\Pi_p}}. \quad (8)$$

By Assumption 1 the denominator of (8) is negative and since the numerator is also negative it follows that  $p'(c) > 0$ . Only with zero price elasticity, which we have

assumed does not hold, would it be optimal to keep the price fixed in the face of changes in marginal cost. If we multiply the pass-through coefficient in (8) by  $c$  and divide by  $p$  we obtain  $cp'(c)/p$ , which is the elasticity of price with respect to marginal cost. This expression will be useful later.

We can now state the condition under which marginal cost pricing is optimal.

*Proposition 1.* Setting price equal to marginal cost in each state is optimal if and only if indirect utility is additively separable in the price and in net income, so the marginal utility of income is independent of the price.

*Proof.* With a separable utility function  $V_m$  is constant as  $p$  changes so  $E(V_m) = V_m$ . The first order conditions (5) and (6) hold with  $p = c$  in all states and thus with  $V_m = \lambda$ . To show that  $V_{mp} = 0$  is necessary suppose that  $p = c$  in all states. It follows from (6) that  $\lambda = E(V_m)$ . With (7) this implies that  $V_m = E(V_m)$  when  $p = c$ . But  $V_m$  in general depends on  $p$ . It is only constant when either  $V_{mp} = 0$  or when  $p'(c) = 0$  (which does not hold).  $\square$

The key to Proposition 1 is that there is no consumer demand for insurance or hedging against price risk when the marginal utility of income is constant. The necessary and sufficient condition in Proposition 1 is the same as the condition for expected consumer surplus to be a valid measure of welfare.

We now see how the mark-up,  $(p - c)/p$ , should vary with marginal cost when utility is not separable. It is assumed from now on that  $V_{mp} > 0$  and that  $\varepsilon$  is constant. Inspection of equation (7) shows that when  $c$  rises, and therefore from (8)  $p$  increases, the term in brackets is reduced when  $V_{mp} > 0$ . This implies:

*Proposition 2.* When the marginal utility of income rises with the unit price and the demand elasticity is constant the mark-up falls monotonically as marginal cost rises.

The mark-up is negatively related to marginal cost when  $V_{mp} > 0$  because the consumer wants some insurance. The regulator has a trade-off between maximizing *ex post* efficiency, which calls for price to equal marginal cost, and optimally allocating risk *ex ante* in the absence of other insurance mechanisms, which requires prices to be smoothed when  $V_{mp} > 0$ . Note that Proposition 2 does not require consumers to dislike

price risk, that is  $-pV_{pp}/V_p$  in (3) need not be negative. What matters is that marginal utility is positively affected by the price.

An implication of the fact that the mark-up declines as  $c$  rises is that the elasticity of price with respect to marginal cost,  $cp'(c)/p$ , is less than one. The expression for this elasticity comes from (8) and uses (3) and (4) with the assumption that the price elasticity is constant:

$$\frac{cp'(c)}{p} = \frac{\varepsilon}{\varepsilon + s(r - \eta)(1 - \varepsilon\mu)/(1 - \mu)} \quad (9)$$

where  $\mu \equiv (p - c)/p$  is the mark-up. When  $V_{mp} > 0$ , so  $r > \eta$ , the denominator exceeds the numerator in (9). In general the elasticity depends on the mark-up and is not constant. Some tendencies are clear however. As  $\varepsilon$  gets closer to zero the cost elasticity approaches zero. This is not surprising since risk sharing is the only regulatory objective when  $\varepsilon = 0$  (there are no deadweight triangles to minimize) and the price should be fixed to insure the risk-averse consumer fully. Similarly as either relative risk aversion or the expenditure share increases the cost elasticity tends to fall. This claim can be made precise with a Cobb-Douglas utility function with constant relative risk aversion. Utility is  $V = (m - A)^{1-r} p^{-s(1-r)}/(1 - r)$  for  $r \neq 1$  and  $V = \ln(m - A) - s \ln(p)$  for  $r = 1$ . Cobb-Douglas preferences imply that the price and income elasticities are unity. The cost elasticity can be found by substituting  $\varepsilon = 1$  and  $\eta = 1$  into (9) giving  $cp'(c)/p = 1/[1 + s(r - 1)]$ , which does not depend on the mark-up. As  $r$  increases the elasticity falls, and similarly as  $s$  rises (when  $r > 1$ ) the elasticity falls.

We now explore whether it is optimal to set the lump-sum charge equal to the fixed cost. To do this it helps to define operating profits as  $(p - c)q$ . The participation constraint implies that  $A = K - E[(p - c)q]$ . Expected operating profits are:

$$E[(p - c)q] = E[\mu pq] = Cov(\mu, pq) + E[\mu]E[pq]. \quad (10)$$

The first equality follows from the definition of the mark-up and the second is implied by the definition of the covariance. Now we assume that the income elasticity, as well

as the price elasticity, is constant.<sup>6</sup> To find an expression for the expected mark-up,  $E[\mu]$ , we substitute for  $\lambda$  from equation (6) into (7) and use the fact that  $E[(p - c)q_m] = E[\mu s \eta] = \eta E[\mu p q] / (m - A)$  by definition of the budget share  $s$  and using the constancy of  $\eta$ . This gives

$$E[\mu] = \frac{\eta E[\mu p q]}{\varepsilon(m - A)}. \quad (11)$$

The expected mark-up thus has the same sign as expected operating profits for  $\eta > 0$  and is zero when there is no income effect in demand. Substituting (11) into (10) and rearranging gives the final expression for expected operating profits:

$$E[(p - c)q] = \frac{\text{Cov}(\mu, pq)}{1 - E[s]\eta / \varepsilon}. \quad (12)$$

In Section 2 we noted that the negative substitution effect entails  $-\varepsilon + s\eta < 0$  so the denominator in (12) is positive. Thus expected operating profit has the same sign as the covariance of the mark-up with revenue,  $pq$ . It is straightforward to find the sign of this covariance. From Proposition 2 we know the mark-up falls as  $c$  increases. The effect on revenue when  $c$  (and thus  $p$ ) rises is positive when the elasticity is below unity. Thus the mark-up and revenue are negatively correlated when  $\varepsilon < 1$  and positively correlated with elastic demand. When the demand function has unit elasticity revenue is fixed and the covariance is zero. We have the following result.

*Proposition 3.* When  $V_{mp} > 0$  and the price and income elasticities are constant expected operating profit has the same sign as  $\varepsilon - 1$ , and the difference between the lump-sum charge and the fixed cost has the opposite sign.

Only in the special case of unit-elastic demand (which holds, for example, with Cobb-Douglas preferences) should expected operating costs be covered exactly by expected revenue from unit prices. There is no requirement in general to have each element of costs (fixed and variable) being covered by their associated revenues. In the realistic

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<sup>6</sup> A slightly more general assumption that also works is that the income elasticity of demand is independent of  $p$ , though it may vary with net income.

case where the elasticity is below unity the consumer benefits from offering a higher lump-sum charge in return for lower prices, even though this means that expected operating profits are negative.

Proposition 3 bears some resemblance to a result of Britto (1980). He examines a closed-economy general equilibrium model with two sectors and multiplicative technology risk for one sector. The question is whether the introduction of risk causes resources to be directed away from or towards the risky sector. The answer is that when producers in the risky sector are risk neutral resource allocation is the same as when there is no risk when the price elasticity of demand in the risky sector is unity, while resources are shifted into the risky sector when the price elasticity is constant and below unity.

## 5. Other applications

To fix ideas we have used the example of a regulated utility. We now discuss two other applications. First, the model may be applied to the case of commodity taxation. Here  $c$  is the marginal social cost of a commodity such as oil, with  $p - c$  being the per-unit indirect tax on oil consumption and  $A$  being the fixed lump-sum tax. The government's revenue requirement is  $K$ . Movements in the world price of oil shift  $c$ . Should the government adjust the commodity tax when the world price of oil changes? This question received considerable attention in Europe in 2000 when there were widespread protests against fuel taxes after world oil prices had tripled in eighteen months, and several governments made concessions on fuel tax rates. To simplify matters we shall assume that any environmental externalities associated with oil consumption are taken care of separately, perhaps by a fixed Pigouvian tax whose revenue is ring-fenced and used for other environmental projects.

If Proposition 1 applies then there should be no indirect tax on oil (other than the Pigouvian tax to offset negative externalities). With no demand for insurance it is optimal to set  $A = K$  and thus to use lump-sum taxation exclusively. Proposition 2 implies that when  $V_{mp} > 0$  the indirect tax should be adjusted down when the world price of oil is high, and should be raised when the world oil price is low. Proposition 3 implies that if demand for oil is inelastic then there should be on average a subsidy for oil consumption financed by setting the lump-sum tax above the level of required revenue. The model is easily extended to cover the case where the lump-sum tax is set separately and is below the revenue requirement  $K$  so indirect taxation must generate

positive expected revenue. When  $V_{mp} = 0$  we can see from equation (7) that it is optimal to have a constant positive mark-up – in other words there should be a Value Added Tax. But when  $V_{mp} > 0$  the tax rate should be cut when the world price of oil rises.

A second possible application has  $c$  varying across customers rather than across states of the world. The idea is that some consumers cost more to supply than other groups, for example transportation costs and the absence of economies of density might entail higher costs of supplying rural communities than urban ones with services such as energy supply or postal deliveries and collections. Marginal cost pricing entails that the revenue from each group of consumers should exactly cover the incremental cost. The analogy to a first-best insurance market in this case is a government that optimally redistributes income via lump-sum transfers and taxes. A utilitarian government would ensure that the marginal utility of income is equalized across consumers. In the absence of such redistribution marginal utilities will differ and there is room for adjusting the prices that the government does control in order to offset this effect. Intuitively rural consumers should face prices that do not fully reflect the costs that are incurred, while urban consumers should be the ones providing the cross-subsidy.

## **6. Conclusion**

In the introduction to his famous analysis of optimal commodity taxation Ramsey (1927, page 47) says “I propose to neglect altogether questions of distribution and considerations arising from the differences in the marginal utility of money to different people”. The model presented in this paper explicitly allows for differences in marginal utility (across states of the world) caused by the lack of first-best insurance markets and futures markets. The paper has both positive and normative implications. The positive aspect is that we explain why prices in regulated markets do not always respond fully to cost shocks. There are several normative aspects. First, we have shown that the Californian option of a completely fixed price is unlikely to be optimal. Second, the opposite case of full cost pass-through, or a fuel adjustment clause, as applied to many energy utilities around the world, is also generally sub-optimal when there are no mechanisms for consumers to offset the resulting price risk.

Two issues not covered in the paper present potentially fruitful lines of research. First, we have not allowed for metering costs. When these are significant the

argument for a completely stable price is strengthened. Second, we have focussed on supply-side or cost risk. In many energy markets there are also demand uncertainties, with load factors varying stochastically according to the time of day, the business cycle and the external temperature. There is a large literature on peak-load pricing (see Crew *et al.*, 1995, for a survey), but one issue that does not appear to have been addressed is whether the usual policy prescription of pricing at short-run marginal cost is appropriate when the marginal utility of income is affected by price changes.

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