Intertemporal Consumer Choice: 2 period problem

Maximise

$$U = u(c_1) + u(c_2)(1+\theta)^{-1} \qquad \textbf{(1)}$$

subject to

$$c_1 + c_2(1+r)^{-1} = D \qquad \textbf{(2)}$$

where D is discounted two period income. (If the consumer receives y_1 in period 1 and y_2 in period 2, then $D = y_1 + y_2(1+r)^{-1}$, assuming no initial or end period wealth.) θ is the rate of time preference (impatience).

Lagrangian[1]

$$L(c_1, c_2, \lambda) = u(c_1) + u(c_2)(1+\theta)^{-1} + \lambda[D - c_1 - c_2(1+r)^{-1}] \qquad (3)$$

To obtain first order conditions, partially differentiate (3) with respect to choice variables [c_1 and c_2] and the Lagrange multiplier [λ], and set to zero.

First Order Conditions:

$$\partial L / \partial c_1 = u'(c_1) - \lambda = 0$$

$$\partial L / \partial c_2 = u'(c_2) / (1+\theta) - \lambda / (1+r) = 0$$

$$c_1 + c_2(1+r)^{-1} = D$$

Combining the first two to eliminate λ gives

$$(1+\theta)u'(c_1) / u'(c_2) = (1+r)$$

The left hand side is the marginal rate of substitution, and the right hand side the marginal rate of transformation from production. This is the Keynes/Ramsey rule for a two period model, and tells us about the optimal dynamic path for consumption, while the budget constraint gives us the overall 'lifetime' level of consumption.

With log utility, we have

$$u(c) = \log(c) \quad u'(c) = 1/c \quad c_1 / c_2 = (1+\theta)/(1+r)$$

For the equivalent infinite horizon problem in continuous time, solved using Hamiltonians, see Blanchard and Fischer pages 39-40, and next handout.

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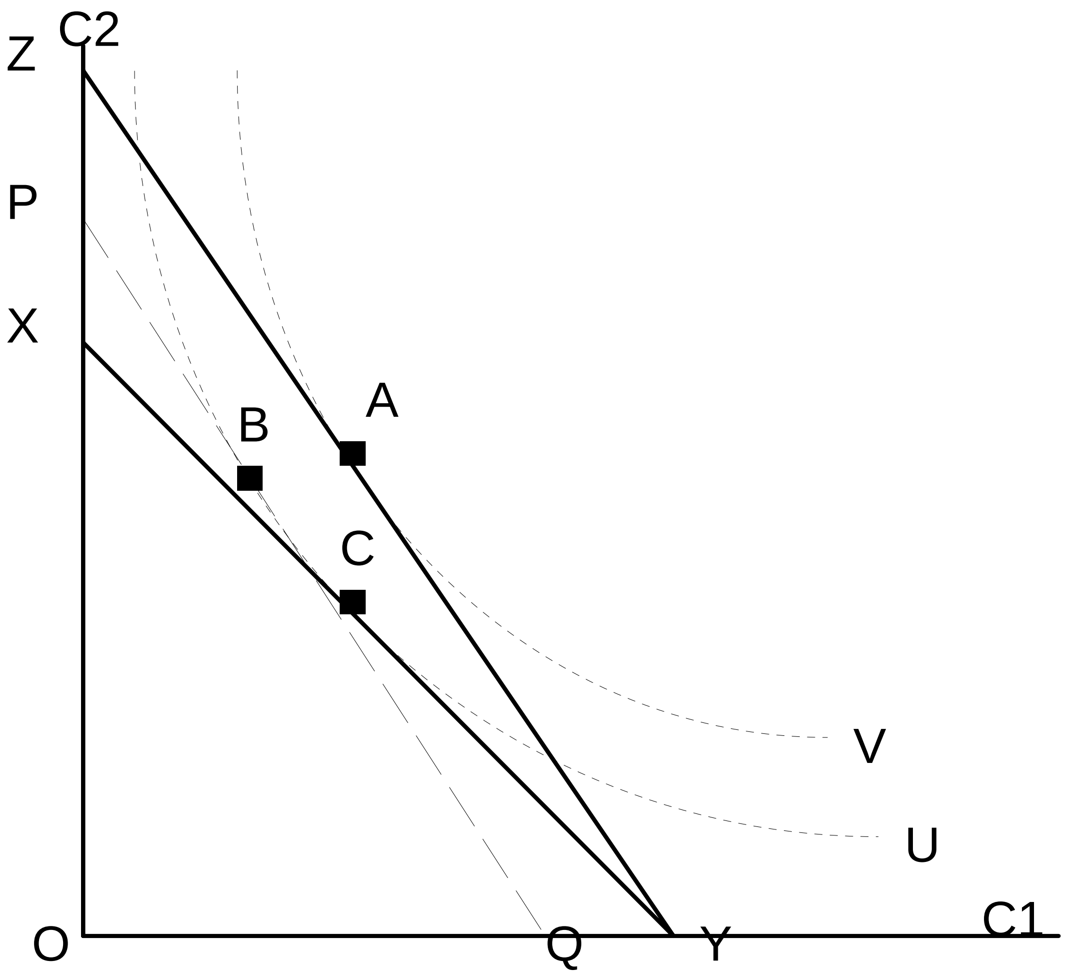
[1] Expression to be maximised (in this case utility) plus any constraints multiplied by 'Lagrange multipliers' (in this case the budget constraint with λ)

The Two Period Consumption Problem: raising interest rates holding discounted lifetime resources constant



- D = Discounted labour income   Interest rate moves from r to s, s > r
- Length OY = D   Length OX = (1+r) D   Length OZ = (1+s) D
- Slope YZ = Slope PQ = -(1+s)   Slope YX = - (1+r)
- U is tangent to XY at C, and PQ at B   V is tangent to YZ at A
- C1 = period 1 consumption,   C2 = period 2 consumption

Start at C. Rise in interest rates: substitution effect C to B, substitution and income effect C to A. C1 at B is less than C1 at C, but C1 at A could be more or less than at C. Note higher interest rates may also reduce discounted lifetime resources (ZY shifts towards origin).