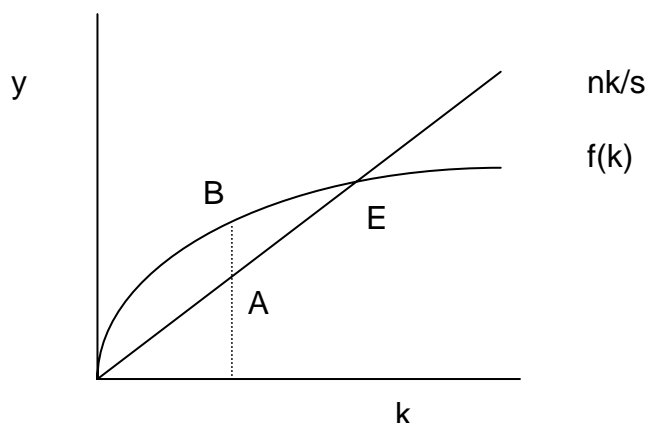


Stability in the growth model with fixed savings

This is seen very easily diagrammatically.



If we start from point B (we are always on the production function), then the amount of output required to keep capital at this level is given by A. As a result, a proportion of the extra output AB will be saved, leading to an increase in capital per head. This is true anywhere to the left of E, and the opposite is true to the right of E. So the model is stable. But note we need the production function to have a diminishing marginal product i.e. for $f''(k) < 0$.

To show the same point mathematically, consider the dynamic equation for investment

$$dk / dt = sf(k) - nk$$

We can take a first order Taylor expansion of any function $g(k)$ around the equilibrium k^* :

$$g(k) = g(k^*) + g'(k^*)(k - k^*) + O[k^2]$$

Now denote $dk / dt = g(k)$. By definition $g(k^*) = 0$. We get

$$dk / dt = (sf'(k^*) - n)(k - k^*) + O[k^2]$$

When $k > k^*$, we want k to be falling. This will be true if $f'(k^*) < n/s$, which says that the slope of the production function is less than the savings/investment line around the equilibrium.