MSC MACRO HANDOUT

A Neoclassical growth model with a fixed savings rate

Suppose the economy consists of N individuals, all of whom wish to work a fixed number of hours. (There is no labour supply choice.) With constant returns to scale, we can write a production function in capital (K) and labour (N)

$$Y = F(K,N)$$

in per head terms as

$$y = f(k) \qquad (1)$$

where y = Y/N and k = K/N.

If profits = PN (y - w - rk) where P is the nominal price level, w is the *real* wage and r is the <u>real</u> interest rate, then profit maximisation implies

$$f'(k) = r \qquad (2)$$

$$f(k) - kf'(k) = w \qquad (3)$$

For the individual firm, r is exogenous, so (2) ‘tells’ the firm the level of capital per head to employ. (Note r has nothing to do with monetary policy – we assume that the economy is superneutral and that the time frame of the model is such that any changes to nominal interest rates made by government are offset by changes in the inflation rate.)

Suppose the economy is made up of a number of identical families, and these families act as independent units when deciding how much to consume and save. Although the number of families in the economy remains fixed, the number of individuals in each family grows at a rate n each year. If there are M individuals in a family, then a family’s budget constraint is

$$Mc + d(aM)/dt = wM + raM$$

where c is average consumption per head, a are assets per head, and d(aM)/dt represents aggregate saving. If all family members are treated equally, then recalling that (dM/dt)/M = n this can be written as

$$c + da/dt + na = w + ra \qquad (4)$$

The term ‘na’ represents the saving required to give new family members the average level of assets. For each family, w and r are exogenous, and the only decision in each period is how much to consume (c) or save (i.e. da/dt). The level of family assets (a) is the result of cumulated savings decisions. In a steady state *if* r > n then a higher level of assets allows a higher level of consumption, but the family may have had to sacrifice consumption for saving to achieve a high level of assets.

Suppose the only asset available is capital in firms, so a = k. Assume no depreciation of capital, so *total* (gross) investment is equal to the change in the aggregate capital stock. The output = expenditure identity then implies

$$c + dk/dt + nk = y \qquad (5)$$

Suppose the savings rate (= (y-c)/y) is constant and equal to s. This implies

$$sy = dk/dt + nk \qquad (6)$$

In a steady state where capital per head is constant (dk/dt = 0) then

$$sf(k) = nk \qquad (7)$$

- individuals save just enough to provide resources (capital) for the newly born. (nk can be described as 'break even investment' – investment required to keep k constant. Increasing k itself is often called 'capital deepening'. Adding capital depreciation – see Romer – increases the amount of break even investment.)

For a given rate of population growth and savings rate, and a well behaved production function, there is a unique level of capital per head (and hence output per head) which satisfies equation (7). From equations (2) and (3) we then know the rate of interest and the real wage. We can think of the rate of interest as adjusting to ensure the savings/asset decisions of consumers are compatible with the capital/investment decisions of firms. The wage adjusts to ensure full employment and this determines the overall 'scale' of the economy. The aggregate economy grows at the rate of population growth (n).

Equation (6) rewritten as

$$dk/dt = sf(k) - nk$$

describes the dynamics of the model. If the saving rate suddenly and permanently increases, then $dk/dt>0$ and capital accumulates. If $n/s > f'(k) = r$ then the new equilibrium involves more capital. As capital increases, then $f''(k)<0$ ensures that this extra capital has a diminishing effect on output, so dk/dt gradually declines and we reach a new equilibrium (see diagram from class, or Romer pages 16-17).

Finally we can ask the following question: if we could choose a savings rate, what would be the choice that maximised the steady state level of consumption? From (7), we can see that choosing s is akin to choosing k. If we differentiate the steady state version of (5) with respect to capital, and set $dc/dk=0$ for a maximum, we obtain

$$f'(k) \ (= r) \ = n$$

This is called the 'golden rule' level of capital.

Optional Exercise

What difference would depreciation or labour augmenting technical progress make to this analysis (Romer Chapter 1)?