

### Real Business Cycle Models

The basic RBC model introduces three new elements to the Classical Growth model described in earlier handouts:

1. The model is stochastic, so we explicitly allow for uncertainty.
2. The key source of shocks to the model are technological shocks, which are autocorrelated (typically first order autoregressive).
3. Labour supply is not fixed.

This last departure means that households choose both consumption and labour supply. The household will maximise

$$U = \sum_{t=0}^{\infty} e^{-\theta t} u(c_t, 1-l_t) M_t$$

where  $U$  is total discounted utility,  $c$  is consumption,  $l$  is hours worked (so  $1-l$  is leisure),  $u()$  is the instantaneous utility function, and  $M$  are the number of people in each household. A specific form of  $u()$  that is often used is log-linear i.e.

$$u = \ln(c) + b \ln(1-l) \text{ for all } t, b > 0$$

The log-linear nature of  $u()$  implies that, if wages were the only source of income (i.e. no financial wealth), labour supply is independent of *permanent* changes in the real wage – the income and substitution effects offset each other. Higher income will raise consumption, and we have

$$c_t / (1-l_t) = w_t / b$$

(Romer, pages 155-158. Note Romer uses  $\rho$  rather than  $\theta$  for the rate of time preference).

However in an intertemporal framework changes in the *pattern* of wages will influence labour supply: it is sensible to supply more labour when wages are high than when they are low (Lucas and Rapping, 1969). In a two period model we have

$$\frac{1-l_1}{1-l_2} = \frac{1}{e^{-\theta(1+r)}} \frac{w_2}{w_1}$$

where labour supply in the two periods responds to the relative wage. The log utility function gives a unit elasticity of substitution. Note also that changes the rate of interest will also influence relative labour supply in the two periods.

Introducing uncertainty into the intertemporal optimisation problem gives us a variant of the Keynes-Ramsey rule:

$$\frac{1}{c_t} = e^{-\theta} E\left[\frac{1}{c_{t+1}}(1 + r_{t+1})\right]$$

This looks very like the KR rule with log utility we had before, but where future values are expected. However this now means that the covariance between consumption and interest rates will influence current consumption

Solving the model (Romer pages 159-162) shows that output will follow a second order autoregressive process, but in practice the first order technological autocorrelation dominates. This is sufficient, within a stochastic environment, to generate cyclical type behaviour. However the model also implies that labour supply is constant: despite the possibility of intertemporal substitution of labour, because technology shocks lead to offsetting movements in relative wages and interest rates which leave labour supply unchanged. The model predicts constant employment and pro-cyclical real wages, whereas in most cycles employment is pro-cyclical and real wages only weakly so. In addition, the saving rate in the model is constant, implying equal volatility in consumption and investment, whereas in reality the latter is much more volatile.

The RBC research program involves complicating the model to allow it to generate more realistic predictions about the stochastic properties of output, consumption, investment and employment. Romer (Section 4.6) discusses two examples: adding depreciation of capital (which makes saving and employment more responsive to shocks), and stochastic government spending (which breaks the link between output and the real wage).

In most of the literature that has developed from these foundations, the metric of success is to 'match' the relative pattern of variances and covariances observed in actual economies. Even if this programme is successful (and on this the jury is still out, particularly in relation to employment fluctuations), it is still subject to basic criticisms about the realism of its assumptions (Romer page 186-189).