Using Rational Expectations to Solve Stochastic Forward Looking Equations

Properties of Rational Expectations

1. Rational expectations errors conditioned on available information have zero means

2. Rational expectations errors are uncorrelated with (orthogonal to) the available information set, or any subset of the available information set

A corollary of 2 is that RE errors are uncorrelated with their own past values (errors NOT serially correlated)

Proofs

Define the rational expectations error e_t as

$$e_t = x_t - E[x_t | I_{t-1}]$$

We want to find $E[e_t|I_{t-1}]$. Taking expectations of both sides implies

$$E[e_t | I_{t-1}] = E[x_t | I_{t-1}] - E\{E[x_t | I_{t-1}] | I_{t-1}\} \qquad (1)$$

To evaluate the last term we need the 'law of iterated expectations', which states that

$$E\{E[x_t | I_{t-1}] | S_{t-1}\} = E[x_t | S_{t-1}]$$
$$S \subseteq I$$

As S can equal I, then the law proves the right hand side of (1) is zero.

To show the absence of serial correlation, consider

$$E[e_t e_{t-1} | I_{t-1}] = e_{t-1} E[e_t | I_{t-1}]$$

as e_{t-i} is part of I_{t-1}. But we have already shown that $E[e_t|I_{t-1}] = 0$

Solving Stochastic Forward Looking Equations

1. <u>Single lead, no inertia</u>

There are many solution methods for equations of the type

$$y_t = a E[y_{t+1} | I_t] + c x_t \tag{1}$$

(see Blanchard and Fischer, Chapter 5, appendix), where a and c are parameters. By taking the expectation of (1) for y_{t+1}, and using the Law of Iterated Projections, we can substitute $E[y_{t+2} | I_t]$ for $E[y_{t+1} | I_t]$. Repeating the process T times gives

$$y_t = c\sum_{i=0}^{T} a^i E[x_{t+i}|I_t] + a^{T+1}E[y_{t+T+1}|I_t] \tag{2}$$

If x does not grow 'too fast' in relation to a, then the first term will converge. In this case a solution to (1) is

$$y_t = c\sum_{i=0}^{\infty} a^i E[x_{t+i}|I_t] \tag{3}$$

You can check that if (3) converges then the second term in (2) disappears as T tends to infinity.

It is crucial to specify the contents of I_t when examining the response of y to x (see also B&F 5.1).

Unfortunately another solution to (1) is

$$y_t = c\sum_{i=0}^{\infty} a^i E[x_{t+i}|I_t] + b_t$$

where $E[b_t] = b_{t-1}/a$. (Check this by substitution in (1).) In this case the second term in (2) no longer disappears, and y 'explodes'. We can often rule this possibility out by imposing a 'transversality condition'. However solutions where this is not possible could provide a model of speculative bubbles.

We can also implement the reverse procedure, and go from (3) to (1). For example, A Blanchard/Yaari type consumption model with no initial financial wealth and logarithmic utility has the form

$$C_t = kE[Y_t + \delta Y_{t+1} + \delta^2 Y_{t+2} + \ldots. | I_{t-1}]$$

where $E[Y_t]$ is expected labour income, and d is a discount factor.
We assume the discounted income stream converges. Agents consume a constant proportion of their human wealth.

If expectations are rational, then we can lead this equation one period.

$$C_{t+1} = kE[Y_{t+1} + \delta Y_{t+2} + \delta^2 Y_{t+3} + \ldots.. | I_t]$$

If we apply the law of iterated projections, we obtain

$$E[C_{t+1} | I_{t-1}] = kE[Y_{t+1} + \delta Y_{t+2} + \delta^2 Y_{t+3} + \ldots.. | I_{t-1}]$$

Multiplying by d and subtracting from the first equation gives

$$C_t = \delta E[C_{t+1} | I_{t-1}] + kE[Y_t | I_{t-1}]$$

2. Quadratic Adjustment Costs

Suppose an individual firm sets prices by minimising

$$E\{\sum_{i=0}^{\infty}[(p_{t+i} - p*_{t+i})^2 + c(p_{t+i} - p_{t+i-1})^2]$$

where c > 0, and p* represents the 'target' price. We ignore discounting for simplicity. Differentiating with respect to p_0 gives

$$(p_0 - p_0^*) + c\,(p_0 - p_{-1}) - c\,(E[p_{+1}] - p_0) = 0$$

or $$c\,E[p_{+1}] - (2c+1)\,p_0 + c\,p_{-1} = -p^*_0 \qquad (1)$$

We can derive similar expressions for $E[p_{+2}]$ etc. We can again use repeated substitution to derive a solution of the form

$$p_t = \lambda p_{t-1} + (1-\lambda)^2 \sum_{i=0}^{\infty} \lambda^i E[p*_{t+1} | I_{t-1}] \qquad (2)$$

where λ is a function of c, and $0<\lambda<1$. Prices depend on expectations about the future, but also their own past value. The more important inertia (λ) is, the more forward looking agents are.