Ricardian Equivalence in The Model of Perpetual Youth

Set up:

Each individual has a probability of dying at any moment. Each existing generation decreases in size through time as some of its members die. Births are such that the aggregate population is static (n=0). There are no bequests: the current generation is not concerned with, or is not able to help, future generations.

Assumptions:

1. Probability of death constant = p. This implies each generation's size decreases at the rate p

2. Each consumer has a contract with an insurance company such that the company receives their wealth on death in return for a premium paid to the consumer. If the insurance company makes zero profits the premium will be p.

Key differences from Infinite Life model

1. Rate at which utility is discounted is θ + p
2. The 'effective' rate of interest is r + p, because of annuity payments
3. For individuals, the Keynes Ramsey rule still holds: consumption grows at the rate of interest less the rate of time preference. The impact of greater discounting is exactly offset by the extra return on wealth. (Individuals make no allowance for the new born.)
4. In equilibrium, r>θ. This means that individuals save some of their labour income, allowing them to build up assets. As individuals get older, their assets grow, which enables their consumption to grow.
5. In the economy as a whole, although wealth for those individuals still alive increases each period, this is offset by the impact of some (wealthy) individuals dying, and being replaced by newborn with no wealth.

If utility is logrithmic, we obtain the following aggregate equations (see Blanchard & Fischer Ch.3 for derivation, with their α set to zero).

$$c_t = (p+\theta)(h_t + a_t)$$

$$\dot{a}_t = ra_t + w_t - \tau_t - c_t$$

$$h_t = \int_{t=0}^{\infty} (w_t - \tau_t) e^{-(r+p)t} dt$$

or

$$\dot{h}_t = (r+p)h_t - w_t + \tau_t$$

An important implication of this model is that consumers decisions imply a relationship between labour income and wealth. We can think of consumers having a target level of wealth (given income etc): as a result, a windfall change in wealth will be gradually run down in order to reach this target.[1]

[1] The relationship is

Suppose the government cuts taxes now (period 0) by an amount -ΔT financed through borrowing, and then pays off the extra borrowing plus the accrued interest in period t=s.

$$\Delta T_s = \Delta T e^{rs}$$

(For example, £100 invested at an interest rate of 5% (r = .05) will be worth £100 times exp(0.25) = £128.4 after five years.)

The effect on human wealth is

$$\Delta h_t = \Delta T - \Delta T_s e^{-(p+r)s}$$
$$= \Delta T (1 - e^{rs} e^{-(p+r)s})$$
$$= \Delta T (1 - e^{-ps})$$

which is >0 if p>0.

Thus any deficit financing (which is equivalent to shifting the path of taxes into the future) will lead to a change in human wealth. Some of the tax burden will be shifted on to future, unborn generations. However only currently living generations consume: they can pay the future tax bill by setting aside less than the tax cut, because their savings receive a return r+p, not r.

Although this means that Ricardian Equivalence does not hold exactly, the size of the propensity to consume out of the tax cut may still be small. Suppose s=10 (years), p = 0.02 and θ = 0.03. The change in human wealth is about 18% of the size of the tax cut. However from the equation for consumption we see that only 5% of any change in human wealth is consumed each period, so the marginal propensity to consume from the tax cut is 1%!

If f(k) is total output, then

$$f(k) = y + rk$$

Assume capital is the only asset. Equation (2) becomes

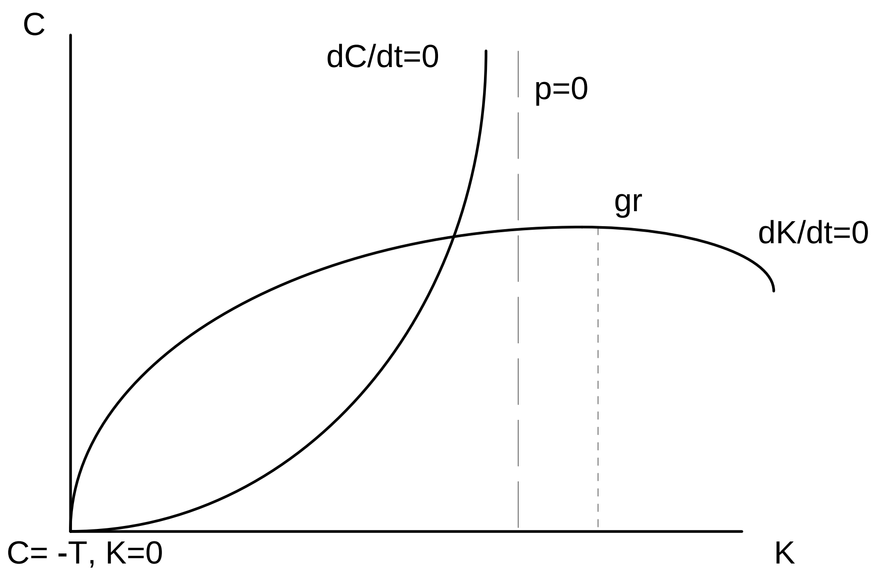
$$\dot{k}_t = f(k_t) - c_t - \tau_t$$

$$a / w = \frac{r - \theta}{(r + p)(p + \theta - r)}$$

Differentiating (1), substituting in (2) and (3), and then using (1) to eliminate H, gives

$$\dot{c}_t = (r - \theta)c_t - p(p + \theta)k_t$$

Drawing the two steady state curves in consumption and capital space gives



Note that, in general (p>0), steady state consumption now varies with capital. Only in the special case of p=0 do we get the vertical line from the infinite life model (where r =θ, as population growth is zero). Thus a positive probability of death raises the steady state interest rate, and reduces the steady state capital stock (and output).

We continue to get a saddle path equilibrium. However a permanent balanced budget increase in spending no longer leads to an immediate downward jump in consumption straight to a new equilibrium, with no change in the capital stock. The constant capital line still shifts downwards by an amount equal to the increase in spending, but the initial downward jump in consumption is less than in steady state, and capital gradually decreases to a new lower level.

Overall, this model represents a very tractable alternative to the infinite life model, with the latter as a special case. It allows for departures from Ricardian Equivalence, although these are small. It also has an important advantage when we come to model an open economy, as we shall see in a later handout.