

## MSC MACRO HANDOUT

### Optimality in a Simple Overlapping Generations Model

Consider two period lives. In the first, consumers receive a labour income ( $w$ ) and save, and they run down their savings ( $s$ ) in the second period. They leave no bequests. Utility is logarithmic. From the two period model we have

Keynes/Ramsey rule: 
$$U'(c_1) = U'(c_2) (1+r)/(1+\theta)$$

With  $U = \ln c$  this becomes

$$c_1 = c_2 (1 + \theta) / (1 + r) \quad (1)$$

The two budget constraints are

$$c_1 + s = w \quad s (1 + r) = c_2 \quad (2)$$

Combining (1) and (2) implies  $s = w/(2+\theta)$ . (3)

In the economy at any period in time there are  $N_1$  of the young generation, and  $N_2$  of the old. In aggregate

$$C_1 + C_2 + \Delta K = Y \quad (4)$$

In addition  $N_1 = N_2 (1+n)$ , so the above can be written in per head form as

$$c_1 + c_2 / (1+n) + \Delta k + nk = f(k) \quad (5)$$

where  $f(k) = Y/N_1$  (Recall only the first generation works.)

Finally we have the two first order conditions from profit maximisation

$$r = f'(k) \quad w = f(k) - rk \quad (6)$$

In steady state ( $\Delta k = 0$ ) substituting (6) and (2) in (5) and dividing by  $(n-r)$  gives

$$s = (1+n) k \quad (7)$$

To assess optimality, we need to compare the market allocation of consumption over time (controlled by  $r$ ) with the allocation chosen by a benevolent central planner who fixed  $k$  and  $c$  directly. In the family/infinite life case the central planner did not have to compare utility between generations, but here they do. If the central planner cared equally about each individual in each generation, then the Pareto Optimal allocation of capital would be given by the golden rule  $r = n$ . (Note that in this model  $\theta$  influences the allocation of consumption between periods for the same generation but not allocation between generations.) What would a market solution imply?

Suppose technology was given by a Cobb-Douglas function with depreciation

$$f(k) = Ak^\alpha - \delta k$$

From (6) we have

$$\begin{aligned} r &= \alpha Ak^{\alpha-1} - \delta = \alpha(f(k) / k + \delta) - \delta \\ &= \alpha(w / k + r) - \delta(1 - \alpha) = \alpha w / k(1 - \alpha) - \delta \end{aligned}$$

From the above we have  $s = w/(2+\theta) = (1+n)k$ , which implies  $w/k = (1+n)(2+\theta)$ . We then have

$$r = (1+n)(2+\theta)\alpha / (1-\alpha) - \delta$$

Only by chance will  $r = n$ . (Note that  $r$  is the interest rate over a generation – i.e. something like 30 years, not one year. The same for  $n$ .)

In general, therefore, the market solution will not be optimal. The level of capital generated under perfect competition will not be the level that maximises consumption treating each generation equally.

The term **dynamic inefficiency** is used to describe the case where the stock of capital is too high i.e. where  $r < n$ . In this case the government could intervene to increase current consumption, which would make everyone better off. The opposite case, where  $k$  is too low, would require a reduction in current consumption, which would harm the current generation at the expense of future generations. In that case, a Pareto improvement is not possible, so we do not describe this situation as inefficient.

In the case where  $k$  is too low, bequests would move the economy towards the efficient solution, because they increase saving. If  $k$  is too high, negative bequests would be required! As a result, allowing only positive bequests does not rule out dynamic inefficiency (Blanchard and Fischer, pp104-110).

How could the government tackle dynamic inefficiency? One possibility is to reduce savings by cutting taxes through borrowing. The same can be achieved through an unfunded social security scheme. In such a scheme, the current working generation transfers money to the current retired generation. The implicit rate of interest in the steady state is  $n$ , because population growth allows the retired to receive more than they put in when working. (When the scheme starts, the current retired generation get a windfall gain!) This moves the economy towards the golden rule. Social security contributions and tax cuts are like negative bequests.