MSC Macro Handout - The Obstfeld-Rogoff Model

This open economy model introduces a number of new features. We have two countries interacting, so neither is 'small'. Money is introduced into the utility function. PPP holds for consumer prices, but not producer prices (i.e. price of national output). We have a variety of goods, and consumers hold CES preferences between them. We formally log-linearise around a steady state. The model incorporates nominal rigidity in a highly stylised, but analytically simple way. None of these additions is conceptually difficult: the main problem is keeping touch with the notation. To help, in some places I depart from the original, but these departures are explicitly noted.

The best reference for this model is Chapter 10 of Obstfeld & Rogoff's textbook, Foundations of International Macroeconomics (Ist Ed), and the page number references are to this. The model was first published in 'Exchange rate dynamics redux', Journal of Political Economy (1995) 103, 624-60. It has since becomes a standard starting point for many open economy journal articles.

Model

Set Up

Assumes perfect foresight. There is a continuum of individual monopolistic producers/consumers, a fraction n of which are in the 'home' country, with the remained in the overseas country. Each produces a single differentiated good, sold under monopolistic competitive conditions. There is no capital, but labour supply is a choice variable.

Consumers

All individuals have identical preferences. The representative consumer maximises

$$U = \sum_{s=t}^{\infty} \beta^{s-t} [\log C_s + \chi \log(M_s / P_s) - \kappa y_s{}^2 / 2] \qquad (1)$$

β is the rate of time preference (using the paper's original notation). C is a consumption bundle given by

$$C = [\int_0^1 c(z)^{(\theta-1)/\theta} dz]^{\theta/(\theta-1)} \qquad (2)$$

where c(z) is consumption of good z, and $\theta>1$. (Note capital letters denote aggregation across goods, not individuals.) In this CES type formulation, θ turns out to be the elasticity of demand. The aggregate price index based on (2) is

$$P = [\int_0^1 p(z)^{1-\theta} dz]^{1/(1-\theta)} \qquad (3)$$

This index solves the problem of choosing c(z) to minimise expenditure subject to C=1. (O&R page 662.) The other two terms in utility are real money balances, and a term reflecting disutility from labour supply, based on returns to scale of ½. (O&R page 662.) Note that a fall in κ implies an increase in productivity.

The only asset the consumer can hold is a real bond (denominated in terms of the consumer price bundle), which pays a real rate of interest r. These bonds are denominated in terms of consumer prices. This is an important assumption, for reasons noted under trade below. Their budget constraint is then

$$P_t B_{t+1} + M_t = P_t(1+r_t)B_t + M_{t-1} + p_t y_t - P_t C_t - P_t \tau_t \qquad (4)$$

where r_t is the real interest rate on bonds held between t-1 and t, p is the price of the individual's good y, and τ are lump sum taxes. (Note the difference between the price of an individual good p and the aggregate price P.)

Production

The individual's demand for good z is given by (O&R p664)

$$c(z) = [\frac{p(z)}{P}]^{-\theta} C \qquad (5)$$

Demand for any particular good depends on aggregate demand and relative prices. As all consumers are identical, we can write a (world) demand for good z as

$$y(z) = [\frac{p(z)}{P}]^{-\theta} C^W \qquad (6)$$

where $C^w = nC^h + (1-n)C^f$, using the superscripts w for world, f for overseas and h for home. (O&R use * for overseas and nothing for home.)

Trade

We assume no barriers to trade, so the law of one price holds for any particular good i.e.

$p^h(z) = ep^f(z)$

where e is the home currency price of foreign currency (+ is a depreciation). Because preferences are equal for all consumers, we can also write the same expression for aggregate prices i.e. $P^h = eP^f$ (O&R p663). This is PPP for the

consumption bundle, not output. However it has a crucial consequence – real interest rates across the two countries, defined in terms of consumer prices, must be the same. Recall that under real UIP, the real interest rate differential equals the expected change in the real exchange rate. PPP implies a constant real exchange rate, so real interest rate differentials must be zero.
As there is no capital or government, we have

$$nB^h + (1-n)B^f = 0 \qquad (7)$$

i.e. the sum of domestic and overseas bonds are zero. If B^h is positive, this means that home residents hold assets issued by overseas residents.

Maximisation

First Order Conditions

$$C_t = \beta(1+r_{t+1})C_{t+1} \qquad (8)$$

The standard Keynes/Ramsey rule with log utility (intertemporal substiution elasticity is unity).

$$M_t / P_t = \chi C_t (1+i_{t+1})/i_{t+1} \qquad (9)$$

where i is the nominal interest rate defined as $1+i_t=P_{t+1}(1+r_{t+1})/P_t$. Note money demand depends on consumption, not output.

$$y_t^{(1+\theta)/\theta} = \frac{\vartheta - 1}{\theta\kappa}(C_t^W)^{1/\theta}(\frac{1}{C_t}) \qquad (10)$$

This last equation is in effect a labour supply equation, given the demand curve. There is also a transversality condition (O&R p666).

Steady State

Denote steady state variables by an underlining (O&R use overlining). From (7) we have

$$\overline{r} = (1-\beta)/\beta = \delta \qquad (11)$$

where δ is the rate of time preference. The budget constraint implies (using the above)

$$\overline{C} = \delta\overline{B} + \overline{py/P} \qquad (12)$$

where we recall that B are bond holdings (which can be negative). In general the equilibrium need not be symmetric if B^h is not zero. If overseas residents are borrowing from domestic residents, domestic consumption will be higher, and they will produce less (to enjoy more liesure), raising p^h/P. In the symmetric steady state where $B^h=0$, we have

$$\bar{y} = (\frac{\theta - 1}{\theta\kappa})^{1/2} \quad (13)$$

This is less than the welfare maximising level of output because of monopoly power (O&R p668).

Log-linearise around steady state

We now log-linearise around the symetrical steady state. (See O&R page 503-4 to see how this is done, in the context of a different model.) To avoid introducing more notation, from now on the variable x_t will denote (using the previous notation) $dx_t / \bar{x}_t$, which is d(log x_t) near equilibrium. (O&R use a bold font.)

Aggregating (3) for home and overseas implies

$$P_t^h = np_t^h + (1-n)(e_t + p_t^f) \quad (14h)$$

$$P_t^f = n(p_t^h - e_t) + (1-n)p_t^f \quad (14f)$$

The aggregate consumption price in the home country is the sum of all home produced goods (assumed identical) and overseas goods. (In O&R notation, P becomes **p**, and p^h is denoted as p(h).) We have, as before, aggregate PPP:

$$e_t = P_t^h - P_t^f \quad (15)$$

The demand curves become

$$y_t^h = \theta(P_t^h - p_t^h) + C_t^w \quad (16h)$$

and a similar equation replacing h with f. A world goods market clearing condition is

$$C_t^w = nC_t^h + (1-n)C_t^f = ny_t^h + (1-n)y_t^f = Y_t^w \quad (17)$$

The labour/liesure trade-off becomes

$$(\theta + 1)y_t^h = -\theta C_t^h + C_t^w \quad (18h)$$

with a similar equation for overseas. Thus if home consumption rises relative to the world average, output falls. The consumer Euler equation becomes

$$C_{t+1}^{h} = C_{t}^{h} + \frac{\delta}{1+\delta} r_{t+1} \qquad (19h)$$

with a similar equation for overseas. (Note r is equal across countries – see above.)

Money demand is

$$M_{t}^{h} - P_{t}^{h} = C_{t}^{h} - \frac{r_{t+1}}{1+\delta} - \frac{P_{t+1}^{h} - P_{t}^{h}}{\delta} \qquad (20h)$$

with, again, a similar equation for overseas. A log-linearised version of the steady state budget constraint (12) is

$$C^{h} = \delta B^{h} + p^{h} + y^{h} - P^{h} \qquad (21h)$$

where we exclude time subscripts because this only holds in steady state. (Deriving this, you first differentiate (12), divide through by steady state C, and then note that steady state B is zero, so steady state consumption equals income.

Comparative statics: flexible prices

In this version of the model, B is not an endogenous variable. (This is a consequence of assuming infinite lives.) Indeed it is the only factor leading to differences between the two countries. We now analyse the impact of a change in the distribution of wealth. If this change is permanent, then the model will immediately jump to a new equilibrium.

We now add steady state versions of equations (14) to (20) and solve the model. It is easiest to do this by first solving for the differences between countries, and then solving for the aggregate. Denote $x^{d} = x^{h} - x^{f}$.

Demand curves:

$$y^{d} = \theta(e - p^{d}) \qquad (22)$$

Labour/leisure:

$$y^{d} = -\frac{\theta}{1+\theta} C^{d} \qquad (23)$$

Budget constraint:

$$C^{d} = (\frac{1}{1-n})\delta B^{h} + y^{d} - (e - p^{d}) \qquad (24)$$

(You need to be slightly careful in deriving this last equation.) Note that a depreciation has two effects on the budget constraint: income rises because output rises (from 22), but there is a negative terms of trade effect. However, as $\theta>1$, the former must dominate.

We now have three equations in three unknowns, with B as the only exogenous variable. Combining all three gives

$$C^d = (\frac{1}{1-n})(\frac{1+\theta}{2\theta})\delta B^h \qquad (25)$$

and

$$p^h - e - p^f = (\frac{1}{1-n})\delta B^h / 2\theta \qquad (26)$$

A wealth transfer (increase in B^h) raises consumption at home relative to overseas. This reduces relative home output (more leisure goes with more consumption), raising home prices and improving home terms of trade (the left hand side of 26).

It is straightforward to show (O&R p673) that world output and consumption are unchanged by a change in B. This solves for all real variables. We can then use the money demand equations to determine prices, and the nominal exchange rate (O&R p673). In particular we have for each country

$$P = M - C \qquad (28)$$

as M is fixed, so steady state inflation is zero, and

$$e = M^d - C^d \qquad (29)$$

It is straightforward to show that we get a standard money neutrality result, and (29) combined with the fact that C^d is independent of M, implies that changes in money lead to proportionate changes in the exchange rate.

Comparative statics: sticky prices

Now suppose output prices are fixed for one period, and consider a permanent unanticipated change in the home money supply. As prices are fully flexible by period 2, then period 2 will be the steady state.

By fixing prices in period 1, we must drop two equations: the output/labour supply equations (18). Output now follows demand. In addition, B can now change, because in period 1 we are not in a steady state and so an individual in a country can run a current account deficit:

$$B_{t+1}^h - B_t^h = r_t B_t^h + p_t^h y_t^h / P_t^h - C_t^h \qquad (30)$$

Log-linearising in period 1 gives

$$B_2^h = y_1^h - C_1^h - (1-n)e_1 \qquad (31)$$

because B_1 is preset, as are prices, and by using (14). Thus any changes in period 1 will lead to current account changes that will alter the distribution of world wealth, which in turn will have real steady state effects. Money will no longer be neutral!

Using (19) implies that

$$C_1^d = C_2^d$$

Relative consumption changes in period 1 by the same as it changes in period 2 = steady state. Why? Because all consumers are identical, they will respond in the same way to changes in real interest rates, and real rates are equal for both countries.[1]

Using (20) plus PPP gives

$$M_1^d - e_1 = C_1^d - (e_2 - e_1)/\delta \qquad (32)$$

Using (29) for e_2, plus the lack of dynamics in consumption, allows us to use the above to show that

$$e^2 = M_2^d - C_2^d \qquad \text{and} \qquad (33)$$

$e_1 = e_2$

The exchange rate also jumps in period 1 to its steady state value. We do not get overshooting. The reason for this is simple – PPP implies that real interest rates cannot differ between countries, so there is no scope for changes in relative real interest rates influencing the exchange rate, as it does in the Dornbusch model.

As prices are fixed in the first period, we have from (22)

$$y_1^d = \theta e_1 \qquad (34)$$

Using (31) and its overseas equivalent implies

$$B_2^h = (1-n)(y_1^d - C_1^d - e_1) \qquad (35)$$

But using (25) allows us to relate the long run (=short run) consumption differential to this wealth transfer. Combining all this gives

$$e = \frac{\delta(1+\theta)+2\theta}{\theta\delta(1+\theta)+2\theta} M_1^d \qquad (36)$$

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[1] From UIP, we know that real (in terms of consumer price inflation) interest rate differentials are equal to the expected change in the real exchange rate (defined in terms of consumer prices). But under PPP, this real exchange rate is constant, so real interest rate differentials are zero. Note that although consumers benefit from an increase in money in period 1, they lose to an inflation tax in period 2.

An increase in home money leads to a depreciation, but less than proportionate. The reason for this is as follows, As prices are initially fixed, and output is below its perfectly competitive level, then higher money will raise output. As home produces more goods, it runs a current account surplus. This raises its long run wealth, raising consumption also in the long run. From the flex price version, we know this appreciates the exchange rate, because higher consumption goes with more leisure, lower output and higher prices. As the initial jump in the exchange rate is also its long run level, then this real appreciation partially offsets the initial depreciation.

In the long run, world real variables do not change as we noted above. In the short run we can derive (O&R p 682)

$$r_1 = -\frac{1+\delta}{\delta} M^w \qquad (37)$$

The real interest rate falls by an amount depending on the size of the home country. Given the Euler equations for consumption, this means world output and consumption rise in period 1.

Is the non-neutrality in this model interesting? In one sense it is a special case produced by assuming infinite lives, because only then is wealth indeterminate and subject to hysteresis effects. However even without infinite lives, short run Keynesian effects could lead to highly persistent consumption effects. It is also the case that the size of these long-run non-neutralities on output are second order (by an amount equal to the rate of interest) compared to the first round effects.

Note that there are two ‘transmission mechanisms’ between the two periods in this model. One is through asset accumulation, whereby shocks in the first period influence the steady state in the second period. The second works in the reverse direction (via rational expectations), where the nominal exchange rate in the second period determines the exchange rate in the first.

We can use the model to examine productivity changes by decreasing the parameter κ. (O&R p696-) If the productivity increase is temporary (period 1 only), the effect is trivial, because output is demand determined. All the productivity gain will go to increases in leisure: the same output will be produced with less labour. (The κ parameter only enters the one equation that is dropped because output is demand determined.) A permanent increase in global productivity will raise global output, but by not as much, because consumers will take some of this gain as higher leisure. The more interesting case is a permanent increase in home productivity.

In the long run, this will raise home output, and home's terms of trade (their real exchange rate) must depreciate so that the extra goods can be sold. Home consumption rises relative to overseas consumption in both periods, which given (33) implies a fall in e, which is an appreciation. The intuition behind this appreciation, which occurs in both the short and the long run, can be seen from considering period 1. A short run increase in home consumption (anticipating higher future output) leads to an increase in money demand. As output prices are fixed, the only way this extra demand can be accommodated with fixed nominal supply is if consumer prices fall, implying an appreciation.

It is also relatively straightforward to add government to this model (p700-). Note that, unlike our earlier open economy models, government spending is assumed to be split between countries in the same way as private consumption. As a result, and increase in only home government spending will result in higher demand for both countries products. However only home's taxes rise, so relative home consumption will fall.

There are a number of interesting results to note. First, higher spending has a long run positive output effect. This is because higher taxes lead consumers to substitute work for leisure. In the short run, as we have already noted, a rise in home government spending will produce a nominal (and real: prices are fixed) depreciation, which will raise home output (demand determined) relative to overseas. In the long run relative home output also rises, but this is a supply side effect, induced by home residents giving up leisure because of higher taxes.