Nominal Extensions to the Intertemporal Model

The models analysed in earlier notes are entirely in real terms: there is no role for money or nominal magnitudes. The essential role for money in an economy is as a ‘medium of exchange’. As its nominal price is fixed, it also defines the unit of account, and we can define the ‘price level’ (and its growth rate, inflation) as the price of goods that can be bought with money.

Will changes in the amount of money, or the rate by which it grows, influence the real economy, or just the price level and inflation rate? ‘Neutrality’ is defined as the first proposition (i.e. money has no effect on the real economy), ‘super neutrality’ is real independence from the money growth rate. One reason why these propositions might not hold is that money is, by definition, also a ‘store of value’, so it is part of wealth.

This note looks at two models. In the first both types of neutrality hold, but the model is useful primarily because it helps show us why neutrality may not hold. The second illustrates what has become known as the ‘Fiscal Theory of the Price Level’.

<u>The Sidrauski Model</u>

Take the infinite life model from our earlier note. We make one change: we introduce money issued by government, which is part of consumers wealth, and which generates utility because it provides ‘transactions services’. This is a rather indirect way of motivating the medium of exchange role for money, but it is analytically convenient. (For an excellent discussion, see chapter 4 of Blanchard and Fischer, starting with 4.5.)

Households now maximise

$$\int_{t=0}^{\infty} u(c_t, m_t) e^{-\theta t} dt \qquad (1.1)$$

where c is consumption and m are real money balances (nominal money M deflated by the price level P). Both first partial derivatives of utility are positive and both second partial derivatives are negative. There are two assets: capital (k) and money. The household’s budget constraint in per head terms is

$$c + dk/dt + nk + dm/dt + \pi m + nm = w + rk + x \qquad (1.2)$$

where w is the wage, π is the inflation rate, r is the real interest rate, and x are transfers from government (negative, lump sum, taxes). Compared to our earlier model, we now have three new terms on the left hand side. First, consumers must save a certain amount to equip the newborn with average money balances (nm), just as they do for capital. Second, inflation will erode the real value of money, so mπ must be saved to keep its real value in tact. Finally any additional saving can increase real money balances. Looking at this equation also makes it clear that the cost of holding money rather than capital is r + π, which is the nominal rate of interest.

The final right hand side term is how the government distributes the income it receives from printing money (called seigniorage). If we define the rate of growth of money as σ, then we have x = σm.

Optimising with the usual No-Ponzi games condition gives two key first order conditions:

$$\dot{u}'_t / u'_t = \theta + n - r \qquad (1.3)$$

$$u_m = u_c(\pi + r) \qquad (1.4)$$

The first is the Keynes/Ramsey rule, which is identical to the equation in a model without money. The second gives the marginal rate of substitution between consumption and money, which depends on the cost of holding money, the nominal rate of interest.

We can immediately see that the model is both neutral and super neutral. The steady state of the model has $f'(k) = \theta + n$ as before, so capital and output per head depend only on technology, preferences and population growth. Output is independent of the rate of growth of money σ. The second first order condition is easier to interpret if we assume

$$u(c,m) = \ln(c) + \chi \ln(m) \qquad (1.5)$$

so that we obtain

$$m = \chi c/(\pi + r) \qquad (1.6)$$

This is a demand for money function: demand increases with consumption (more exchange) and χ, and decreases as the cost of holding money rises. In steady state, using the Ramsey rule together with $\pi + n = \sigma$ (money growth equals the rate of growth of nominal output), we can see that an increase in money growth will reduce real money balances. However, as this equation determines m, neutrality must hold: any change in M will lead to a proportionate change in P.

If an increase in money growth raises inflation, which raises the cost of holding money, why does this have no knock-on effects on the rest of the economy? In particular, why does the reduction in wealth implied by lower m not matter? The reason is that wealth only influences consumption in the infinite life model to the extent that it yields a return. Money yields no return, so having less of it does not alter consumption. (In fact, in real terms, it has a negative return, because its real value falls with inflation, but this is paid back by the government as x.)

We can also get a well known welfare result originally due to Friedman. Although raising money growth is super neutral, it does decrease welfare, because lower m reduces utility. To maximise utility, therefore, we want an infinite m, which we can achieve in steady state by making money growth equal to $-\theta$.

What if the monetary authority fixes the nominal interest rate rather than the money stock? The money stock growth rate now becomes an endogenous variable, and the nominal interest rate is exogenous. To a considerable extent this change seems fine: as the real side is tied down in the usual way, the rate of inflation is now given by the difference between the nominal and real interest rates. However, what ties down the level of prices? As money is now endogenous, neither M nor P are defined. This is an example of price level indeterminacy.

The Tobin Effect

We know that, once we abandon the infinite life model (or the assumption of Barro bequests), then consumption does depend of the level of wealth. Consumers will still spend any ‘excess’ wealth gradually over time, but they will not hold on to it forever. (In the infinite life model there is no concept of optimal wealth, so it does not make sense to say consumers have excess wealth.) In the Blanchard/Yaari model this is because they discount future income at a rate above the rate of interest.

In this case, an increase in money growth which reduces the demand for money will lead consumers to accumulate more capital in an attempt to restore their wealth. The model will no longer be super neutral. This is known as the Tobin effect.

There may be many other reasons why economies are not super neutral in practice. On potentially important, but rather dull, reason is that taxes are levied on nominal interest payments, not real interest flows. Therefore an equal rise in the nominal interest rate and inflation will leave consumers worse off, because they will be paying higher taxes.

A model with money and bonds but without capital

The model above did not include government debt. However we know that in the infinite life model changes in government debt just alters future taxes, and Ricardian Equivalence holds. However this result was derived for a real economy in which, implicitly, all government debt was indexed. In reality most government debt, like money, is fixed in nominal terms. We will now show that this offers an interesting way of avoiding the Ricardian Equivalence result.[1]

To simplify, we ignore capital, and we continue to assume a fixed labour supply. As a result, output per head is fixed by assumption. Consumers utility is as before. Their budget constraint becomes

$$c + db/dt + nb + dm/dt + \pi m + nm = w + rb + x \qquad (2.1)$$

where b is real debt. The variable x can now change because of changes in debt as well as changes in money.

The result of optimisation is exactly as in the previous model: as far as the consumer is concerned, we have simply replaced the source of assets from k to b. So the Keynes Ramsey rule still holds, and we still obtain the money demand function (1.6).

The government's intertemporal budget constraint is

$$b_0 + \int x_t e^{-(r-n)t} dt = \int m_t \sigma e^{-(r-n)t} dt \qquad (2.2)$$

In other words, the discounted sum of seigniorage revenue less the discounted sum of transfers must equal the initial debt stock. (In the earlier model, seigniorage = transfers in each period: as there was no debt, the government budget was balanced each period. Now the present value of transfers has to be below the present value of seigniorage to allow for interest payments on the initial debt stock.)

Until now, we have used (2.2) to say that the government must choose its policies (i.e. the path of x), so that this intertemporal budget constraint is satisfied, otherwise the model would be explosive in debt. This was because the real value of initial debt was given to us (i.e predetermined)[2]. Now, however, only the nominal stock of debt B_0 (where $B_0 = b_0/P_0$) is predetermined. So another possibility arises. The fiscal authorities could choose any path for x, and the initial price level would jump to a level such that (2.2) held. This is the Fiscal Theory of the Price Level.

This possibility is clearest when the monetary authorities fixed the nominal interest rate. We noted above that this left the initial price level indeterminate. (Once

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[1] In fact, nominal debt is not necessary to get Fiscal Theory type results. If we allow for nominal inertia, the real interest rate is no longer independent of policy. In (2.2) below, the intertemporal budget constraint can be satisfied by a suitable path for real interest rates: see Leith and Wren-Lewis, Economic Journal (2000).

[2] Also it was because debt was not net wealth, so the real interest rate was independent of it – see earlier footnote.

the initial price level is known, all future prices are given by the inflation rate.) This difficulty could be ‘solved’ by the Fiscal Theory, which determines precisely the initial price level. Note that Ricardian Equivalence no longer holds. The government could increase transfers at some date, and never raise transfers to compensate. This policy would no longer produce an explosive debt spiral, because the initial price level would rise to deflate initial nominal debt, and therefore the real value of all future debt interest payments.

The contrast with the previous model appears stark. In that model, monetary policy determined prices completely, and fiscal policy (x) had little effect on anything. In this set up, fiscal policy has a powerful influence on the price level. (Note, however, that monetary policy still determines the inflation rate by fixing nominal interest rates.)

The fiscal theory is controversial. Some economists have suggested that governments will always respect their intertemporal budget constraint, or if they did not, their debt would be discounted by the market. Controversy increases if the monetary authorities return to fixing nominal money, but the fiscal authorities continue to set arbitrary paths for x. Does the money stock determine the price level, or equation (2.2)?

This debate is not an academic curiosity. If the Fiscal Theory is a possibility, then it becomes one powerful reason why the monetary authorities might want to influence what fiscal authorities do. Some have interpreted the Stability and Growth pact of EMU in this light, although with significant reservations.