Two simple models of imperfect competition

Here our focus moves away from the allocation of consumption and capital over time to the determination of the real wage in labour and goods markets. Formally, however, the only departure from the model in the earlier handouts is the assumption of imperfect competition in goods and labour markets. Our analysis is static.

Let the economy consist of a number of identical firms setting prices and employment conditional on a given nominal wage. We look at two alternative models of imperfect competition in the labour market.

1. Price setting under monopolistic competition

Suppose the demand curve faced by firms has the form

$$Y_i = Y(\frac{p_i}{p})Q \qquad Y' < 0 \tag{1.1}$$

where p is the aggregate (and hence competitors) price level and Q is aggregate demand. Firms treat p and Q as exogenous, which defines them as monopolistic competitors. The production function exhibits constant returns to scale:

$$Y_i = f(\frac{\mu N_i}{K_i})K_i \tag{1.2}$$

where μ represents labour embodied technical progress, N is labour, Y output and K capital. Profits for the individual firm are given by

$$\Pi_i = p_i Y_i - wN_i - rpK_i \tag{1.3}$$

where w is the nominal wage rate and r is the real cost of capital. We can maximise profits by forming the Lagrangian

$$L = \Pi_i + \lambda_1[Y_i - Y(\frac{p_i}{p})Q] + \lambda_2[Y_i - f(\frac{\mu N_i}{K_i})K_i] \tag{1.4}$$

and deriving the first order conditions

$$\begin{aligned} &p_i + \lambda_1 + \lambda_2 = 0 \\ &Y_i - \lambda_1 Y' Q / p = 0 \\ &-w - \lambda_2 f' \mu = 0 \end{aligned} \tag{1.5}$$

Substituting out for the multipliers gives

$$p_i + Y_i p / (Y'Q) = w / (f' \mu) \tag{1.6}$$

If we define the elasticity of product demand as

$$e = -\frac{\partial Y_i}{\partial p_i}\frac{p_i}{Y_i} = -Y'\frac{Q}{p}\frac{p_i}{Y_i} \tag{1.7}$$

(so e is always positive), then (6) becomes

$$p_i(1-\frac{1}{e})=\frac{w}{f'\mu} \qquad (1.8)$$

As we approach perfect competition, and e gets infinitely large, then the left hand side reduces to p_i. Equation (1.8) can then be rearranged to give the familiar real wage = marginal product of labour condition. In general, the elasticity e will depend on Q, but if (1.1) has the particular form

$$Y_i=(\frac{p_i}{p})^{-\eta}Q \qquad \eta>1 \qquad (1.9)$$

(so e=η) then the price mark-up (p_i/w) is independent of aggregate demand. In the special case of a Cobb Douglas production function where

$$f(.)=(\mu N_i/K_i)^a \qquad (1.10)$$

then we can write (4) as

$$p_i(1-1/e_i)=\frac{w}{a}\frac{N_i}{Y_i} \qquad (1.11)$$

Here a 1% increase in output per head will lead to a 1% fall in the mark up.

2. Monopolistic competition in the labour market

Suppose workers can set their own wage rate, but are subject to a demand curve of the form

$$N_i=N(\frac{w_i}{w}) \qquad N'<0 \qquad (2.1)$$

where w is the wage of competing workers. Workers maximise

$$U(c_i,1-N_i)=\ln(c_i)+b\ln(1-N_i) \qquad (2.2)$$

as in the handout on real business cycles, and lets assume a very simple budget constraint

$$N_iw_i=pc_i \qquad (2.3).$$

Maximising

$$L=\ln(c_i)+b\ln(1-N_i)+\lambda_1[N_i-N(\frac{w_i}{w})]+\lambda_2[N_iw_i-pc_i] \qquad (2.4)$$

gives

$$\frac{1}{c_i}-\lambda_2p=0$$

$$-b\frac{1}{1-N_i}+\lambda_1+\lambda_2w_i=0 \qquad (2.5)$$

$$-\lambda_1N'/w-\lambda_2N_i=0$$

Eliminating the multipliers, and rearranging, gives

$$c_i/(1-N_i)=\frac{w_i}{bp}(1-\frac{1}{d}) \qquad (2.6)$$

where

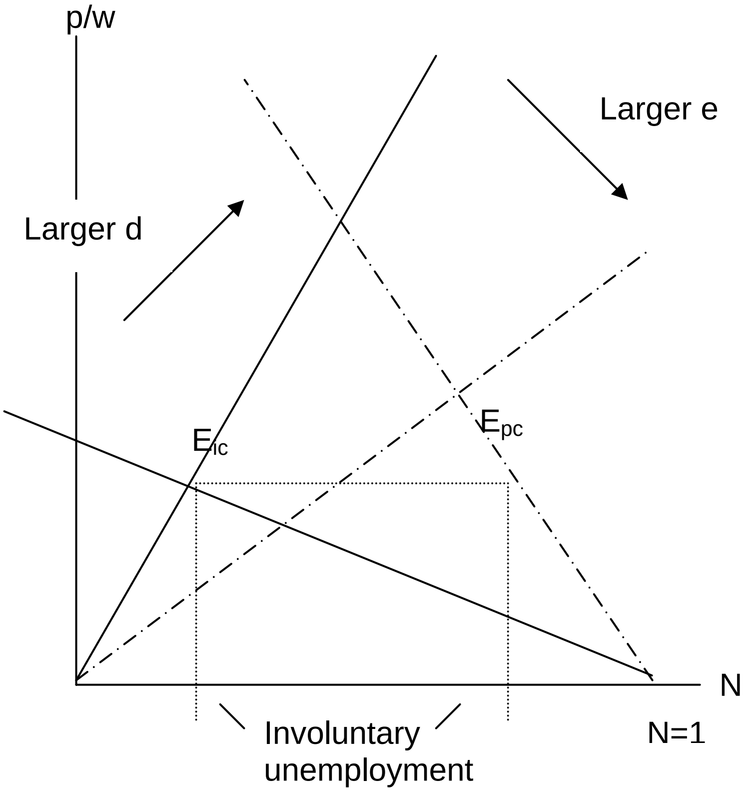
$$d = -\frac{N_i}{w}\frac{w_i}{N_i} > 1 \quad (2.7)$$

is the elasticity of labour demand. This is the same expression we had in the RBC handout except for the term involving d, and as d gets very large we tend to the perfectly competitive case.

Suppose now that all firms and workers are identical. We have price and wage equations, from (1.11) and (2.6)

$$\frac{p}{w}(1-1/e) = \frac{1}{a}\frac{N}{Y}$$
$$\frac{p}{w} = \frac{1-N}{bc}(1-\frac{1}{d}) \quad (2.8)$$

We can represent these in mark-up and employment space. If demand elasticities are constant, these are both straight lines. The dash-dot lines are the perfectly competitive limit. The price setting lines pivot about the origin, increasing in slope as demand gets less elastic. (Imperfect competition always raises the mark-up.) The wage setting lines pivot about N=1, getting shallower as labour demand gets less elastic. Note that employment is always lower under imperfect competition, but the real wage can go either way. One measure of involuntary unemployment is the difference between labour supply at the real wage at E_{ic} (where a horizontal line from E_{ic} hits the perfectly competitive wage curve) and employment at E_{ic}.



3. A bargaining model of wage setting

The model of the labour market above is not very realistic. Consider instead a simple bargaining model in which each union (j) only attempt to maximise wages. Assume a Nash set up where firms and unions effectively maximise a weighted average of the union's utility (U_j) and firms' profits (Π_j), where in both cases we are concerned with differences from a fall-back level if no bargain is struck. This can be represented as

$$\max \quad (U_j - Z)^g \Pi_j \tag{3.1}$$

where Z is the fall back level of utility for all union members, and we assume fall back profits are zero. The parameter g reflects the relative power of the union in the bargain. Maximisation with respect to the union's wage w_j implies

$$\Pi_j g \frac{dU_j}{dw_j} + (U_j - Z)\frac{d\Pi_j}{dw_j} \tag{3.2}$$

If firms subsequently maximise profits subject to this wage (the 'right to manage' assumption), then we always have $d\Pi_j/dw_j = - N_j$. (As profits are a function of wages and employment, we can write

$$\frac{d\Pi_j}{dw_j} = \frac{\partial \Pi_j}{\partial w_j} + \frac{\partial \Pi_j}{\partial N_j}\frac{dN_j}{dw_j}$$

But the penultimate term must be zero given profit maximisation.[1]

We take the union's utility to be real take home pay, which we can write as $w_j(1-t)/p$, where t is some tax rate. If the bargain is not struck, we assume the average union member will either get a job outside the firm (with the aggregate nominal wage w) or become unemployed, receiving a state benefit wb(1-t), where b is the benefit to wage ratio. The probability of unemployment occurring is v, which is a positive function of the aggregate unemployment rate u. Thus

$$U_j - Z = \frac{w_j}{p}(1-t) - (1-v(u))\frac{w}{p}(1-t) - v(u)\frac{bw(1-t)}{p} \tag{3.3}$$

Solving (3.2) gives

$$\Pi_j g(1-t)/p = (U_j - Z)N_j \tag{3.4}$$

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[1] This is a specific example of the envelope theorem: see, for example, Akerlof, G and Yellen, J (1985), Can Small Deviations from Rationality Make Significant Differences to Economic Equilibria , American Economic Review, 75,708-720

Now (and only now!) we aggregate over firms and unions, equating outside and inside wages:

$$\frac{\Pi}{N} = v(u)w(1-b)/g \tag{3.5}$$

Substituting out for profits implies

$$\frac{w}{p}(1+v(u)\frac{1-b}{g}) = \frac{Y}{N} - \frac{rK}{pN} \tag{3.6}$$

In the Cobb Douglas special case it is easy to show that profit maximisation implies

$$rKp = wN\frac{1-a}{a}$$

This allows us to simplify (3.6) as

$$\frac{w}{p}[1+v(u)(1-b)/g+\frac{1-a}{a}] = \frac{Y}{N} \tag{3.7}$$

(3.6) and (3.7) are rather different to an aggregate version of 2.6. Output replaces consumption, but more importantly individual preferences (b) are replaced by union power, and we have additional terms in aggregate unemployment and benefits. The unemployment term makes it difficult to know how to draw this curve in real wage/employment space, but an algebraic analysis is revealing. From section 1 we have

$$f'\mu(1-1/e) = \frac{w}{p} \tag{3.8}$$

Combining this with (3.6) implies

$$f'\mu(1-1/e)(1+v(u)(1-b)/g) = \frac{Y}{N} - \frac{rK}{pN} \tag{3.9}$$

Equation (3.9) can be considered as an equation for the equilibrium or 'natural' rate of unemployment: the level of unemployment at which the real wage implied by price setting is equal to the real wage set in the labour market. The natural rate depends on benefits, union power, the goods elasticity of demand and factor proportions, but not on the tax rate or aggregate demand (Q). A rise in the benefit ratio (b) or a rise in union power (g) will require, ceteris paribus, a rise in unemployment. An increase in the ‘degree of monopoly’ in the goods market caused by a reduction in e will also lead to a rise in equilibrium unemployment.

In (3.9), a change in the rate of interest can influence unemployment through changes in capital costs and factor proportions. However in the Cobb Douglas case, where we combine (3.7) and (3.8), we get

$$f'\mu(1-1/e)(1+v(u)(1-b)/g)+\frac{1-a}{a}=\frac{Y}{N}$$

which can be further simplified to

$$(1-1/e)(1+v(u)(1-b)/g+\frac{1-a}{a}=\frac{1}{a}$$

In this case the natural rate is independent of the rate of interest as well as technical progress.