

Giovannini 1988: Long run Comparative Statics

Consumers of generation s maximise

$$\int_{v=t}^{\infty} \ln [c_d(s, v)^\alpha c_f(s, v)^{1-\alpha}] e^{(p+\theta)(t-v)} dv \quad (1)$$

subject to

$$\dot{a}(s, v) = (r(v) + p)a(s, v) + w(s, v) - t(s, v) - c_d(s, v) - c_f(s, v)/e(v)$$

where

$$a(s, v) = k(s, v) + b(s, v) + f(s, v) / e(v)$$

c_d	consumption of domestic goods
c_f	consumption of overseas goods
a	financial assets
r	real interest rate
w	labour income
t	taxes
e	real exchange rate (+ appreciation)
k	capital, valued in domestic currency
b	real government debt
f	overseas assets

This is the model of perpetual youth, except there are two goods (home, overseas) with Cobb Douglas preferences between the two, and overseas financial assets

Solution:

$$\begin{aligned} c_d(s, t) &= \alpha(p + \theta)(h(s, t) + a(s, t)) \\ c_f(s, t)/e(t) &= (1 - \alpha)(p + \theta)(h(s, t) + a(s, t)) \end{aligned} \quad (2)$$

$$h(s, t) = \int_{v=t}^{\infty} (w(s, v) - t(s, v)) e^{-\int_{u=t}^v (r(u)+p) du} dv \quad (3)$$

where h is human wealth. Consumption of domestically produced goods is a fixed proportion of total wealth. The same is true for overseas produced goods, except that an appreciation makes these goods cheaper, so more are consumed.

This gives aggregate consumption functions of the form:

$$C = (p+\theta) (A + H), \quad C_d = \alpha C, \quad C_f/e = (1-\alpha)C \quad p+\theta > r \quad (4)$$

The firm's decision is unchanged from earlier handouts, as long as all capital goods are domestically produced (so the real cost of capital is just the real interest rate). In steady state, UIP implies that domestic real interest rates are fixed at world levels, which in turn implies that the capital stock will not change as long as world rates and domestic technology are unchanged. We have

$$y = c_d + g + x(e) \quad (5)$$

where y is total output, g is government spending, and x is exports, which are a function of the real exchange rate (see imperfect competition assumption above). In steady state, human wealth is given by

$$h^* = (y^* - t^*) / (r^* + p^*) \quad (6)$$

where we use an asterisk to denote steady state values. The consumers and governments budget constraints are in steady state

$$c^* = r^* a^* + y^* - t^* \quad (7)$$

$$g^* - t^* + r^* b^* = 0 \quad (8)$$

We can also derive the balance of payments identity in steady state

$$x^*(e) - c_f^*/e^* + r^* f^*/e^* = 0 \quad (9)$$

It is sometimes said that the real exchange rate moves to ensure (5) holds, or (9) holds, but in reality it is just one endogenous variable in a simultaneous system.

Combining (7) and (8) with the consumption function gives

$$(\theta + p - r^*)(a^* + h^*) = p h^* \quad \text{or} \quad (\theta + p - r^*) = (r^* - \theta) h^* \quad (10)$$

Thus the Blanchard Yaari model implies a very simple long run relationship between financial wealth and human capital. As $r^* > \theta$, then we require $p + \theta > r^*$ for positive wealth.

Combining this with the other equations gives

$$y^* (1 - \Omega) = g^* (1 - \Omega) - r^* b^* \Omega + x(e^*) \quad (11)$$

where

$$\Omega = \alpha(p + \theta)p / ((p + \theta - r^*)(r^* + p)) > 0$$

If capital is unchanged in steady state, then for fixed labour supply so will output via the production function, so $\Delta y^* = 0$. The effect of an increase in government debt is unambiguously to raise exports and hence depreciate the real exchange rate. Higher debt requires additional taxes which reduces steady state consumption, which reduces the demand for domestic output, so external demand must rise to meet unchanged supply.

The long run impact of a change in government spending is ambiguous. Higher spending raises the demand for domestic output, but higher taxes reduces demand because consumption falls. In fact consumption will fall by more than the rise in taxes,

because lower human wealth reduces financial wealth (see 10). If all of consumption was on domestic goods, then the net effect would be a reduction in the demand for domestic output, and the exchange rate would depreciate. (Set $\alpha=1$, and then it is possible to show that, as $r>\theta$, $\Omega>1$.) However, as some consumption expenditure goes overseas, the reduction in the demand for domestic output from this source may not offset the additional demand from government spending (which we have assumed all goes on domestic output.)

Finally note that the long run impact of a positive, additive productivity shock or labour supply increase is the opposite of extra government spending.